Mathematics for Finance

by

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Professor Harvey teaches investment analysis and portfolio management. He has held visiting appointments at the Stockholm School of Economics, the Helsinki School of Economics and the Graduate School of Business at the University of Chicago.

Harvey is an internationally recognized expert in global risk management. He sits on the Board of Directors of Torstar Corporation, an international media concern with publishing operations in 12 countries. In addition, he is a consultant to the World Bank. Harvey also advises portfolio managers on global asset allocation strategies.

Harvey is the recipient of numerous awards. He is the 1993-94 Batterymarch Fellow. This annual award is given to the person that is most likely to establish a new area of research in finance. In 1990, he received the R. L. Rosenthal Award for Innovation in Investment Management. The Association for Investment Management and Research has recently honored him with a Graham and Dodd Scroll in recognition of excellence in financial writing. and the American Finance Association awarded him a Smith-Breeden prize for his publication "The World Price of Covariance Risk."

Professor Harvey's work on the implication of changing risk for both domestic and international asset allocation has recently been featured at leading universities such as Stanford, UCLA, Berkeley, MIT, Chicago, Princeton, and Northwestern. He has been invited to present his ideas in Canada, Finland, France, Greece, Italy, the Netherlands, Sweden, Switzerland and Turkey.

His current work focuses on investment opportunities in emerging stock markets. His paper, "Predictable Risk and Returns in Emerging Markets" recently won the 1993 American Association of Individual Investors Best Paper in Investments Award.
0. The Plan

1. Stock return indices.

2. Continuous compounding.

3. Mean, volatility and covariance.

4. Regression analysis.
1. Stock Return Indices

The percentage return on stock $j$ is:

- $\%\text{Return}_j = 100 \times \frac{\text{New Price} - \text{Old Price}}{\text{Old Price}} = 100 \times \frac{S_{j,t+1} - S_{j,t}}{S_{j,t}}$

where $S_j$ is the price of stock $j$.

If a dividend is paid, the formula becomes:

- $\%\text{Return}_j = 100 \times \frac{S_{j,t+1} - S_{j,t} + \text{Dividend}_j}{S_{j,t}}$

The formula for the percentage change in the bond return is identical – the dividend may be replaced with a coupon.

- $\%\text{Return}_j = 100 \times \frac{B_{j,t+1} - B_{j,t} + \text{Coupon}_j}{B_{j,t}}$

where $B_j$ is the price of bond $j$. 
Consider a fund that consists of three oil stocks: Exxon, Chevron and Amoco. Each stock has a different price and a different number of shares held. Suppose we purchase the stocks at the following prices.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Price per share</th>
<th>Number of shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon (XON)</td>
<td>$65</td>
<td>1000</td>
</tr>
<tr>
<td>Chevron (CHV)</td>
<td>$90</td>
<td>500</td>
</tr>
<tr>
<td>Amoco (AN)</td>
<td>$53</td>
<td>400</td>
</tr>
</tbody>
</table>

How are returns computed for this portfolio?

The percentage return on the portfolio would be

\[
\text{%Portfolio Return}_j = 100 \times \frac{\text{New Total Value} - \text{Beginning Total Value}}{\text{Beginning Total Value}}
\]

\[
= 100 \times \frac{V_{j,t+1} - V_{j,t}}{V_{j,t}}
\]

where the Beginning Total Value is:

\[(1000 \times 65) + (500 \times 90) + (400 \times 53) = \]
1. Stock Return Indices

Suppose the prices change:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Price per share</th>
<th>Number of shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon (XON)</td>
<td>$63</td>
<td>1000</td>
</tr>
<tr>
<td>Chevron (CHV)</td>
<td>$93</td>
<td>500</td>
</tr>
<tr>
<td>Amoco (AN)</td>
<td>$60</td>
<td>400</td>
</tr>
</tbody>
</table>

What is the percentage return on the portfolio:
There are three main methods to calculate an index return: value weighting, price weighting and equal weighting.

- The value-weighting method is used in the S&P 500, NYSE, AMEX and NASDAQ indices.

- The price-weighting method is used in the Dow Jones and MMI indices.

- The equally-weighting method is used in the Value Line index.

I. Value-Weighted Index:

The index price level is calculated by multiplying the price of each security, $S_i$, by the total number of shares outstanding, $N_i$, and then dividing by some arbitrary number called the “divisor” $D$.

\[
VW\text{ Index}_t = \frac{\sum_{i=1}^{\text{stocks}} N_i S_i}{D}
\]

This is very similar to valuing our portfolio of oil stocks except we normalize by “D.”
Let's create a value-weighted index from our portfolio of three stocks. Set $D = 1312$. Calculate the beginning value-weighted index level.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Price per share</th>
<th>Number of shares</th>
<th>Value of position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon (XON)</td>
<td>$65</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>Chevron (CHV)</td>
<td>$90</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>Amoco (AN)</td>
<td>$53</td>
<td>400</td>
<td></td>
</tr>
</tbody>
</table>

- Total Value=$______
- VW Index Value=$______

And the new index value:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Price per share</th>
<th>Number of shares</th>
<th>Value of position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon (XON)</td>
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<td></td>
</tr>
<tr>
<td>Amoco (AN)</td>
<td>$60</td>
<td>400</td>
<td></td>
</tr>
</tbody>
</table>

- Total Value=$______
- VW Index Value=$______
The **percentage** return on the index:

\[
\text{%VW Index Return} = 100 \times \frac{\text{New Index} - \text{Beginning Index}}{\text{Beginning Index}}
\]

\[= 100 \times \frac{I_{vw,t+1} - I_{vw,t}}{I_{vw,t}}\]

For our index:

\[
\text{%VW Index Return} = ____
\]

**Important**: The value-weighted index returns mimics the returns to a buy and hold strategy.

Note the relation between the percentage VW Index return and the percentage return to our original portfolio!
II. Price-Weighted Index:

The index price level is calculated by adding the prices of each security $S$ in the portfolio. The final number is divided by a "divisor" $D$.

$$\text{PW Index}_t = \frac{\sum_{i=1}^{\text{stocks}} S_i}{D}$$

Notice, that the key difference is the omission of $N_i$ – the number of shares outstanding.

- Tootsie Roll (TR) trading at $66$ gets a bigger weight than General Motors (GM) trading at $46$!

- Price-weighted indices give a greater than proportional weight to smaller cap equities.

Let's calculate the price-weighted index level for our portfolio of oil stocks. Assume $D = 2.08$
The **beginning** index value:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Price per share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon (XON)</td>
<td>$65</td>
</tr>
<tr>
<td>Chevron (CHV)</td>
<td>$90</td>
</tr>
<tr>
<td>Amoco (AN)</td>
<td>$53</td>
</tr>
</tbody>
</table>

- Total Share Value=$_____
- PW Index Value=$_____

And the **new** index value:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Price per share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon (XON)</td>
<td>$63</td>
</tr>
<tr>
<td>Chevron (CHV)</td>
<td>$93</td>
</tr>
<tr>
<td>Amoco (AN)</td>
<td>$60</td>
</tr>
</tbody>
</table>

- Total Share Value=$_____
- PW Index Value=$_____


1. Stock Return Indices

The percentage return on the PW index:

\[ \text{%PW Index Return} = 100 \times \frac{\text{New Index} - \text{Beginning Index}}{\text{Beginning Index}} \]

\[ = 100 \times \frac{I_{pw,t+1} - I_{pw,t}}{I_{pw,t}} \]

For our index:

\[ \text{%PW Index Return} = \ldots \]

**Important**: Return assumes you hold equal numbers of shares in each security.

Note the difference between the PW return and the return on the VW index and on our original portfolio!

⇒ PW index gives greater than proportional weight to the small cap stocks.
III. Equally-Weighted Index:

To get an equally-weighted index return, just average the returns of the individual securities!

<table>
<thead>
<tr>
<th>Stock</th>
<th>Beginning price</th>
<th>New price</th>
<th>Percentage return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon (XON)</td>
<td>$65</td>
<td>$63</td>
<td></td>
</tr>
<tr>
<td>Chevron (CHV)</td>
<td>$90</td>
<td>$93</td>
<td></td>
</tr>
<tr>
<td>Amoco (AN)</td>
<td>$53</td>
<td>$60</td>
<td></td>
</tr>
</tbody>
</table>

%EW Return = ______

- EW Index return assumes equal investment in each security.

- EW Index return does not represent the returns to a buy and hold strategy unless the portfolio is rebalanced each period.
Let's review the formulae for present value and future value:

Present Value:

The present value of a payment of $100 \ T$ periods from now is:

$$PV = \frac{100}{(1 + r)^T}$$

where $r$ is the effective periodic interest rate over the $T$ periods.

Future Value:

The future value in period $T$ of a payment of $100$ today is:

$$FV = 100(1 + r)^T$$

where $r$ is the effective periodic interest rate over the $T$ periods.
The number of times that you compound can have a big effect.

Suppose a bank offers 6% deposit rate and you choose the number of times your $100 million is compounded.

- **Annual**: Deposit \times (1 + r) = $100\text{mill} \times 1.06
- **Semi-annual**: Deposit \times (1 + r/2)^2 = $100\text{mill} \times 1.03^2
- **Monthly**: Deposit \times (1 + r/12)^{12} = $100\text{mill} \times 1.005^{12}

Let's trace the returns on our deposit

<table>
<thead>
<tr>
<th>Period</th>
<th>Formula</th>
<th>Calc.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual</td>
<td>( (1 + r) ) ( (1 + \frac{r}{4})^4 ) ( (1 + \frac{r}{12})^{12} ) ( (1 + \frac{r}{365})^{365} ) ( (1 + \frac{r}{8760})^{8760} ) ( e^{0.06} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
We can use the HP Business Calculator for this.

<table>
<thead>
<tr>
<th>N</th>
<th>I%YR</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>OTHER</th>
</tr>
</thead>
</table>

- Choose **OTHER**.

- Enter 12; Press arrow below **P/YR**

- Press **EXIT** button

- Enter 12; Press arrow below **N**

- Enter 6; Press arrow below **I%YR**

- Enter -100; Press arrow below **PV**

⇒ Press arrow below **FV** to obtain solution
Did you ever wonder where $e$ comes from – the “natural exponent”?

- $e = 2.7182818 \ldots$

- **PRESS MATH button**

<table>
<thead>
<tr>
<th>LOG</th>
<th>$10^X$</th>
<th>LN</th>
<th>EXP</th>
<th>N!</th>
<th>PI</th>
</tr>
</thead>
</table>

- Enter .06; Press arrow below **EXP**

By the way, by definition,

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$
2. Continuous Compounding

Problem:

The native Americans sold Manhattan in 1626 for $24. Suppose they could invest at 6% (compounded once per year) and that they are tax exempt. Was the sale a wise one? What is the value if interest was continuously compounded?

\[ N = 367; \quad r = .06; \quad PV = 24; \quad \text{Find } FV? \]

\[ FV = $24 \times (1.06)^{367} = \underline{\underline{\underline{}}} \]

The continuously compounded value,

\[ FV = $24 \times e^{.06 \times 367} = \underline{\underline{\underline{}}} \]
2. Continuous Compounding

Natural Log (LN) and the Natural Exponent (e):

\( e \) is the base of the Natural Log, i.e. \( \text{LOG}_e = LN \)

Hence, \( LN(e) = 1 \) or \( LN(e^x) = x \)

Continuous rates of returns

- Last year the Dow was at 3000 and today it is 3300. What is the continuously compounded rate of return?

**Step by step**

\[
T = 1; \quad PV = 3000; \quad FV = 3300; \quad r =? \]

\[
FV = PV \times e^{rT}
\]

\[
3300 = 3000 \times e^r \quad \text{since } T = 1
\]

\[
3300/3000 = e^r
\]

We must solve for \( r \).
2. Continuous Compounding

\[
\frac{3300}{3000} = e^r
\]

\[
LN\left(\frac{3300}{3000}\right) = LN(e^r)
\]

\[
LN(1.1) = LN(e^r)
\]

Hence,

\[
r = LN(1.1) =
\]

In general, the continuously compounded rate of return is:

\[
\text{Continuous Rate of Return} = LN \left( \frac{\text{New Price}}{\text{Old Price}} \right)
\]

⇒ We will use continuous rates of return in the Black-Scholes Formula.
Mean:

The mean is a measure of the expected value of a random variable $x$.

Volatility:

The volatility is a measure of the expected dispersion of the random variable $x$ around its mean.

Covariance:

The covariance is a measure of how random variables $x$ and $y$ interact. If they both move in same direction, on average, they have positive covariance. If they move in opposite directions, they have negative covariance.

Correlation:

The correlation has the same interpretation as the covariance. The correlation is the covariance scaled by the volatility of $x$ and $y$ so that it falls in the range of -1 to +1.
Let's consider a year of monthly returns on the S&P500 and three stocks. [Data are constructed.]

<table>
<thead>
<tr>
<th>Month</th>
<th>S&amp;P500</th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Stock 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.05</td>
<td>.06</td>
<td>.10</td>
<td>-.05</td>
</tr>
<tr>
<td>2</td>
<td>.05</td>
<td>.04</td>
<td>.15</td>
<td>-.06</td>
</tr>
<tr>
<td>3</td>
<td>.10</td>
<td>.11</td>
<td>.15</td>
<td>-.09</td>
</tr>
<tr>
<td>4</td>
<td>.15</td>
<td>.14</td>
<td>.30</td>
<td>-.14</td>
</tr>
<tr>
<td>5</td>
<td>.05</td>
<td>.06</td>
<td>.10</td>
<td>-.06</td>
</tr>
<tr>
<td>6</td>
<td>-.10</td>
<td>-.11</td>
<td>-.20</td>
<td>.11</td>
</tr>
<tr>
<td>7</td>
<td>.05</td>
<td>.06</td>
<td>.05</td>
<td>-.04</td>
</tr>
<tr>
<td>8</td>
<td>.10</td>
<td>.09</td>
<td>.25</td>
<td>-.09</td>
</tr>
<tr>
<td>9</td>
<td>.20</td>
<td>.21</td>
<td>.30</td>
<td>-.19</td>
</tr>
<tr>
<td>10</td>
<td>-.10</td>
<td>-.11</td>
<td>-.20</td>
<td>.11</td>
</tr>
<tr>
<td>11</td>
<td>.05</td>
<td>.06</td>
<td>.15</td>
<td>-.04</td>
</tr>
<tr>
<td>12</td>
<td>.05</td>
<td>.04</td>
<td>.05</td>
<td>-.06</td>
</tr>
</tbody>
</table>

One measure of the mean is the average:

\[
\bar{x} = \frac{\sum_{i=1}^{12} x_i}{12}
\]

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P500</th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Stock 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean=</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This mean is based on the historical performance.
We can also learn a lot by graphing the distribution of the returns in a histogram. Let's put each return into a class.

<table>
<thead>
<tr>
<th>Range</th>
<th>S&amp;P500</th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Stock 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.225→-.175</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-.175→-.125</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-.125→-.075</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-.075→-.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-.025→.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.025→.075</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.075→.125</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.125→.175</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.175→.225</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.225→.275</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.275→.325</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations:

- Mean is near center of histograms
- It is obvious that mean of stock 3 is lower than 1.
- Alternate definition of mean:

\[
\text{Mean} = \sum_{i=1}^{Num} \text{Prob}_i \times x_i
\]

That is, the mean is the sum of the value of \( x \) at each possible event \( i \) multiplied by the probability of the events occurring. The probabilities sum to one.
Another observation from the histogram is that stock 2’s returns are much more disperse than stock 1’s. Stock 2 has higher volatility.

Definition

\[
\text{Sample Variance} = \sigma^2 = \sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N - 1}
\]

That is, the variance, \( \sigma^2 \), (\( \sigma \)=Greek “sigma”) is the sum of squared deviations divided by the number of observations minus one. [We lose “one degree of freedom” because we have already estimated one parameter – the mean].

Definition

\[
\text{Standard Deviation} = \sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1}}
\]

That is, the standard deviation, \( \sigma \), – or the volatility – is the square root of the variance.
Let's calculate the volatility of the S&P 500 based on the historical data.

<table>
<thead>
<tr>
<th>Month=$i$</th>
<th>S&amp;P</th>
<th>$(x_i - \bar{x})$</th>
<th>$(x_i - \bar{x})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.05</td>
<td>-.0042</td>
<td>.0000176</td>
</tr>
<tr>
<td>2</td>
<td>.05</td>
<td>-.0042</td>
<td>.0000176</td>
</tr>
<tr>
<td>3</td>
<td>.10</td>
<td>.0458</td>
<td>.0020976</td>
</tr>
<tr>
<td>4</td>
<td>.15</td>
<td>.0958</td>
<td>.0091776</td>
</tr>
<tr>
<td>5</td>
<td>.05</td>
<td>-.0042</td>
<td>.0000176</td>
</tr>
<tr>
<td>6</td>
<td>-.10</td>
<td>-.1542</td>
<td>.0237776</td>
</tr>
<tr>
<td>7</td>
<td>.05</td>
<td>-.0042</td>
<td>.0000176</td>
</tr>
<tr>
<td>8</td>
<td>.10</td>
<td>.0458</td>
<td>.0020976</td>
</tr>
<tr>
<td>9</td>
<td>.20</td>
<td>.1458</td>
<td>.0212576</td>
</tr>
<tr>
<td>10</td>
<td>-.10</td>
<td>-.1542</td>
<td>.0237776</td>
</tr>
<tr>
<td>11</td>
<td>.05</td>
<td>-.0042</td>
<td>.0000176</td>
</tr>
<tr>
<td>12</td>
<td>.05</td>
<td>-.0042</td>
<td>.0000176</td>
</tr>
</tbody>
</table>

Mean | .0542 | 0 | .0074810

Variance = .0074810

Standard Deviation or Volatility = .0864927

Before we elaborate on the interpretation of the volatility, let's review the normal distribution.
The Black-Scholes formula makes use of the normal distribution. This is the most widely used distribution in statistics – the so-called "bell distribution."

- A probability distribution tells us the same thing as the histogram – what the probability is of obtaining a result in a particular range.

- If we had expanded the number of months that we used in calculating the histogram, it might start to look like a bell.

The **Standard Normal** distribution is a normal distribution with a mean of zero and a standard deviation of one.
Questions

What is the probability of obtaining a value less than 0?

\[ N(x < 0) = 50\% \]

What is the probability of obtaining a value between -1 and +1? [Plus or minus one standard deviation from the mean]

\[ N(-1 < x < 1) = 68.3\% \]

What is the probability of obtaining a value between -2 and +2? [Plus or minus two standard deviation from the mean]

\[ N(-2 < x < 2) = 95.4\% \]

What is the probability of obtaining a value less than 2? [Two standard deviations below the mean]

\[ N(x < 2) = 97.7\% \]
Now we are in a position to give more interpretation to the volatility.

If the continuous (log) S&P 500 returns are normally distributed with:

- Mean = .0542
- Standard Deviation = .0865,

then we can say that 95% of the time, the S&P return should be in the range of .0542 ± (2 × .0865).

- That is, the volatility and the mean and the distribution give us a way to make statements about the future expected returns.

- For example, there is less than a 5% chance that the S&P will gain or lose 25% in any given month – because 25% is beyond two standard deviations from the mean.

*Both the normal distribution and the volatility play a critical role in option pricing models.*

⇒ We use the distribution to make statements about the expected future outcome of the stock return – which is critical for valuing options.
Definition

Sample Covariance  =  \sigma_{xy} = \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{N - 1}

That is, the deviation from the mean of \( x \) is multiplied by the deviation from the mean of \( y \).

- If \( x \) is above its mean (deviation positive) when \( y \) is above its mean (deviation positive), the product is positive.

- If \( x \) is below its mean (deviation negative) when \( y \) is below its mean (deviation negative), the product is positive.

\Rightarrow \text{Hence, } x \text{ and } y \text{ move in similar directions and the covariance is positive.}

- If \( x \) is above its mean (deviation positive) when \( y \) is below its mean (deviation negative), the product is negative.

- If \( x \) is below its mean (deviation negative) when \( y \) is above its mean (deviation positive), the product is negative.

\Rightarrow \text{Hence, } x \text{ and } y \text{ move in opposite directions and the covariance is negative.}
Definition

Sample Correlation = \( \rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \)

That is, the correlation is the covariance divided by the product of the respective volatilities (or standard deviations).
Let’s consider two scatter plots: Stock 1 and the S&P; Stock 3 and the S&P

- **Stock 1 and the S&P have positive correlation.** When the S&P is above its mean, so is stock 1 (on average).

- **Stock 3 and the S&P have negative correlation.** When the S&P is above its mean, stock 3 is below its mean (on average).
Notice that the slope (rise over run) of a line we eyeball through figure 1 is about 1. [If the S&P is up 10%, stock 1 is up about 10%, i.e. 1:1.]

Notice that the slope (rise over run) of a line we eyeball through figure 2 is about -1. [If the S&P is up 10%, stock 3 is down about 10%, i.e. 1:-1.]

You have just done regression (in your head).

A Linear Regression tries to fit the best line in the scatter of $y$ and $x$.

- "Best" is measured by minimizing the square of the horizontal distance between the line and the point.

- This is why regression is called “least squares.”

The regression equation is:

$$y_i = \alpha + \beta x_i + \epsilon_i$$

where $\alpha$ is the intercept, $\beta$ is the slope and $\epsilon$ is the horizontal distance (or the residual).
4. Regression Analysis

Obviously, the regression slope, $\beta$, is related to correlation.

**Definition**

\[
\text{Regression slope } = \beta = \frac{\sigma_{xy}}{\sigma_x^2}
\]

That is, the slope is just the covariance divided by the variance of $x$.

What about stock 2 and the S&P?

The slope here is about 2.

- When the S&P goes up by 10%, stock 2 goes up (on average) by 20%.
Hedging interpretation of $\beta$

Your portfolio is worth $100,000 and the distribution of returns looks very much like stock 2 (i.e. $\beta=2$).

Given increased uncertainty about the economy, you decide to hedge by either selling the S&P 500 futures or purchasing put options.

Suppose you sell $100,000 worth of the S&P 500 futures or buy puts on $100,000 of the S&P.

Outcome:

- S&P drops by 10%.
- Good news, you collect $10,000 profit on your hedge.
- Bad news, your portfolio lost 20% or $20,000
- You are a loser to the tune of $10,000.

**Your mistake was not to take the $\beta$ into account.** A beta of 2 means that if the market drops by 1% your stock will drop by 2% on average. Hence, you should have sold $200,000 of the S&P.

$\Rightarrow$ The beta tells us how much to hedge.