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Autoregressive Conditional Skewness

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We present a new methodology for estimating time-varying conditional skewness. Our model allows for changing means and variances and uses a maximum likelihood framework with instruments and assumes a noncentral t-distribution. We apply this method to daily, weekly and monthly stock returns, and find that conditional skewness is important. In particular, we show that the evidence of asymmetric variance is consistent with conditional skewness. Inclusion of conditional skewness also impacts the persistence in conditional variance.

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I. Introduction

Skewness, asymmetry in distribution, is found in many important economic variables such as stock index returns and exchange rate changes, see for example, Harvey and Siddique (1998) for analysis of U.S. monthly stock returns. Negative skewness in returns can be viewed as the phenomenon where, after the returns have been standardized by subtracting the mean, negative returns of a given magnitude have higher probabilities than positive returns of the same magnitude or vice-versa. This can be measured through the third moment about the mean.

The second moment of returns, variance, has been the subject of a large literature in finance. Variance of returns has been widely used as a proxy for risk in financial returns. Therefore, the properties of variance by itself as well as the relation between expected return and variance have been important topics in asset pricing. Campbell (1987), Harvey (1989), Nelson (1991), Campbell and Hentschel (1992), Hentschel (1995), Glosten, Jagannathan, and Runkle (1993), and Wu (1998) have focused on the intertemporal relation between return and risk where risk is measured in the form of variance or covariance. An important concern has been the sign and magnitude of this tradeoff.

The generalized autoregressive conditional heteroskedasticity (GARCH) class of models, including the exponential GARCH (EGARCH) specification, have been the most widely used models in modeling time-series variation in conditional variance. Persistence and asymmetry in variance are two stylized facts that have emerged from the models of conditional volatility. Persistence refers to the tendency where high conditional variance is followed by high conditional variance. Asymmetry in variance, i.e., the observation that conditional variance depends on the sign of the innovation to the conditional mean has been documented in asymmetric variance models used in Nelson (1991), Glosten, Jagannathan, and Runkle (1993) and Engle and Ng (1993). These studies find that conditional variance and innovations have an inverse relation: conditional variance increases if the innovation in the mean is negative and decreases if the innovation is positive.

The fourth moment of financial returns, kurtosis, has drawn substantial attention as well. This has been primarily because kurtosis can be related to the variance of variance and, thus, can be used as a diagnostic for the correct specification of the return and variance dynamics.

In contrast, skewness, the third moment, has drawn far less scrutiny in empirical asset pricing, though skewness in financial markets appears to vary through time and also appears to possess systematic relation to expected returns and variance. The time-series variation in skewness can be viewed as analogous to heteroskedasticity.

This paper studies the conditional skewness of asset returns, and extends the traditional GARCH(1,1) model by explicitly modeling the conditional second and third moments jointly. Specifically, we present a framework for modeling and estimating time-varying volatility and skewness using a maximum likelihood approach assuming that the errors from the mean have a non-central conditional t distribution. We then use this method to model daily and monthly index returns for the U.S., Germany, and Japan, and weekly returns for Chile, Mexico, Taiwan, and Thailand; concurrently estimating conditional mean, variance and skewness. We also present a bivariate model of estimating coskewness and covariance in a GARCH-like framework. We find significant presence of conditional skewness and a significant impact of skewness on the estimated dynamics of conditional volatility. Our results suggest that conditional volatility is much less persistent after including conditional skewness in the modeling framework and asymmetric variance appears to disappear when skewness is included.

The dynamics of moments over time also appear to be intimately tied with frequency, seasonality and aggregation in returns. Daily and monthly returns on the same asset appear to have quite different properties. Aggregation of individual stocks into larger portfolios also appears to have substantial impact on the properties of conditional variance. Finally, seasonal effects affect the findings on behavior of moments as well. This includes the well-known January effect in the conditional mean along with less familiar day of the week effects in daily returns.

A third important question, the relation between the conditional mean and conditional variance, has been answered with conflicting findings. Campbell and Hentschel (1992), French, Schwert, and Stambaugh (1987), and Chan, Karolyi, and Stulz (1992) have found either an insignificant or positive relation whereas Glosten, Jagannathan, and Runkle (1993), Campbell (1987), Pagan and Hong (1991), and Nelson (1991) find a negative relation. Wu (1998), using a more general specification of the Campbell and Hentschel (1992) models, finds a substantially more negative relation between expected returns and volatility.

Estimation of time-varying moments is important for testing asset pricing models that impose restrictions across moments. Estimation of time-varying skewness may also be important in implementing models in option pricing. The presence of skewness can also affect the time-series properties of the conditional mean and variance. Skewness in the returns of financial assets can arise from many sources. Brennan (1993) points out that managers have an option-like features in their compensation. The impact of financial distress on firms and the choice of projects can also induce skewness in the returns. More fundamentally, skewness can be induced through asymmetric risk preferences in investors. However, we do not study the causes of skewness in this paper.

In the following sections, we lay out the model for estimating time-varying conditional skewness in returns, document the substantial variations and seasonalities in skewness and empirically examine the impact that inclusion of conditional skewness has on the properties of conditional variance and the relation between return and conditional variance. We also carry out diagnostic tests of our model.

II. The Model

We use the residuals from the mean to estimate the conditional variance and skewness of asset returns. The residual from the conditional mean is

$$\epsilon_{t+1} = r_{M,t+1} - \boldsymbol{\alpha}'\mathbf{Z}_t \tag{1}$$

where $\boldsymbol{\alpha}'\mathbf{Z}_t$ is the conditional mean, r_M is the variable to be modeled, which in our case is the excess return on the market index, and $\mathbf{Z}_t$ are the instruments in $\boldsymbol{\Omega}_t$, the full information set.

Since our primary focus is on modeling the conditional variance and skewness, we use a GARCH-M specification for the conditional mean effectively using conditional variance of $r_{M,t}$ as an instrument. This is consistent with the specification used in much of the literature on persistence and asymmetry in variance.[1] Our specifications for conditional variance and skewness are GARCH(1,1) in the terminology of the ARCH/GARCH literature introduced by Engle (1982) and Bollerslev (1986). The initial GARCH specification assumed that returns come from a conditionally normal distribution. However, stock market returns have thicker tails than conditional normal distributions would imply. Bollerslev (1987) assumes that the returns come from a central-t distribution. The central-t distribution permits thicker tails but is still symmetric like the normal distribution.

However, none of these models accommodate time-varying conditional skewness in returns. We assume that the excess returns $r_{M,t+1}$ have a noncentral conditional-t distribution. In contrast to the normal or central-t distributions, a conditional noncentral t distribution allows us to estimate time-varying skewness of either sign. A noncentral conditional-t distribution also allows us to write the moments using simple and familiar functional forms. The conditional noncentral-t distribution is defined by two time-varying parameters, ν_{t+1}, the degrees of freedom, and δ_{t+1}, the noncentrality parameter. The conditional variance is the scale parameter controlling the dispersion of the data. We use the conditional variance to standardize the returns to have unit variance (with non-zero mean, however,) and then use the conditional mean and skewness to compute ν_{t+1} and δ_{t+1}.

The mean and skewness are respectively the location and shape parameters. A noncentral-t distribution scaled to have a unit variance is a generalization of the central-t distribution. The noncentrality parameter controls the shape. If it is negative, the distribution has a tail to the left implying that the median is greater than the mean. For a positive noncentrality parameter, the tail is to the right and the median is less than the mean. For the noncentral-t with unit variance, the sample likelihood function can be written as:

$$L(\epsilon_{t+1}|\mathbf{Z}_t,\mathbf{\Theta}) = \prod_{t=1}^{T} \frac{\nu_{t+1}^{\frac{\nu_{t+1}}{2}}}{,\left(\frac{\nu_{t+1}}{2}\right)} \frac{\exp^{\frac{-\delta_{t+1}^2}{2}}}{\sqrt{\pi}(\nu_{t+1}+\frac{\epsilon_{t+1}^2}{1})^{\frac{\nu_{t+1}+1}{2}}} \times \sum_{i=0}^{\infty} ,\left(\frac{\nu_{t+1}+i+1}{2}\right)\left(\frac{\delta_{t+1}^i}{i!}\right)\left(\frac{2\frac{\epsilon_{t+1}^2}{1}}{\nu_{t+1}+\frac{\epsilon_{t+1}^2}{1}}\right)^{\frac{i}{2}}, \tag{2}$$

where , is the *gamma* function and ν_{t+1} is the degrees of freedom of the t distribution. The likelihood function has two time-dependent terms, ν_{t+1} and δ_{t+1}, the degrees of freedom and the noncentrality parameter.[2] The noncentrality parameter determines the shape (and therefore skewness) of the distribution.

The GARCH(1,1) specification for the conditional variance and skewness is autoregressive.[3] Define $h_t = \text{Var}_{t-1}[r_{M,t}]$, and $s_t = \text{Skew}_{t-1}[r_{M,t}]$, thus:

$$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 \epsilon_{t-1}^2 \tag{3}$$

$$s_t \quad = \quad \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 \epsilon_{t-1}^3 \tag{4}$$

We call this specification of variance and skewness as the GARCHS(1,1,1) (GARCH with Skewness) model. Variance and skewness need to be constrained so that they are stationary, and in the case of variance, positive. To ensure that conditional variances and skewnesses are nonexplosive, we need to impose the constraints that $0 < \beta_1 < 1$, $0 < \beta_2 < 1$, $-1 < \gamma_1 < 1$, $-1 < \gamma_2 < 1$ and $\beta_1 + \beta_2 < 1$ and $-1 < \gamma_1 + \gamma_2 < 1$. We have used penalty functions as well as the logistic, $(1 + \exp^{-x})^{-1}$, and $\tanh^{-1}(x)$ functional forms to operationally impose these constraints on β_1, β_2, γ_1 and γ_2.

To operationalize the estimation, we estimate the central conditional variance and skewness and use the recurrence relation in Kendall, Stuart and Ord (1991) to obtain the noncentral skewness and variance from the central moments:

$$\begin{aligned} \mu_3 &= \mu_3' - 3\mu_2'\mu_1 + 2\mu_1^3 \\ \mu_2 &= \mu_2' - \mu_1^2 \end{aligned}$$

where μ_2 and μ_3 are the central moments (about the mean) and μ_2' and μ_3' are the noncentral moments (about 0). The noncentrality parameter δ_{t+1} and the degrees of freedom, ν_{t+1}, (after the returns are normalized to have unit variance) can be computed by solving the following system of nonlinear equations:

$$\mu_1 \quad = \quad (\frac{1}{2}\nu)^{\frac{1}{2}} \frac{,\ (\frac{1}{2}(\nu - 1))}{,\ (\frac{1}{2}\nu)} \delta \tag{5}$$

$$\mu_3 \quad = \quad \mu_1 \left\{ \frac{\nu(2\nu - 3 + \delta^2)}{(\nu - 2)(\nu - 3)} - 2 \right\} \tag{6}$$

We compute the noncentrality parameter and degrees of freedom implied by the conditional mean and skewness at each observation, and compute the log-likelihood.

The noncentral skewness can be expressed in terms of the noncentrality parameter, δ_t as in Bain (1969).

$$s_t = \sum_{i=1}^{2} \frac{3!\delta_t^3}{(2i-1)!(2-i)!2^{2-i}} \frac{,\ \left(\frac{\nu_t - 3}{2}\right) \nu_{t+1}^{\frac{3}{2}}}{2^{\frac{3}{2}},\ \left(\frac{\nu_t}{2}\right)} \tag{7}$$

We set the initial conditional variance and skewness, h_1 and s_1, to the unconditional variance and skewness. The parameters to estimate are:

$$\boldsymbol{\Theta} = [\boldsymbol{\alpha} \quad \boldsymbol{\beta} \quad \boldsymbol{\gamma} \quad]'$$

We maximize the log-likelihood function in (3) to obtain the parameter estimates.

An important consideration is whether the conditional skewness dynamics implied by the GARCH(1,1) model are consistent with the dynamics for conditional skewness. Lee and Hansen (1994) show that if rescaled innovations have a bounded fourth moment, then the QMLE parameter estimates are consistent. The noncentral t distribution we assume for the innovations in our model does have bounded fourth moments. Therefore, parameter estimates from our likelihood estimation are consistent. With the GARCH(1,1) model, assuming that the errors have a conditional Normal or t distribution, there should be no skewness left. However, the residuals from fitting GARCH(1,1) models do have skewness. For example, when a GARCH(1,1)-normal model is fit to daily S&P500 returns, the residuals have an unconditional skewness of -2.08 and kurtosis of 54.94, excluding the week of October 21, 1987. Standardized (i.e. divided by standard deviation) residuals have a negative skewness of -0.27 and excess kurtosis of 4.74. In contrast, when we fit the noncentral t distribution, the standardized residuals have a skewness of -0.17 and kurtosis of 0.90. This implies that noncentral-t distribution is a better description of the data-generating process. Additionally, Newey and Steigerwald (1997) show that in case of non-Gaussian likelihood estimates, unless there is an adjustment for the location shift in the estimation of the mean, there is an asymptotic bias.

The likelihood function is highly nonlinear and to obtain the global maximum, good starting parameter values are essential. For this purpose, we estimate the parameters in stages moving from simpler models to more complex specifications. The stages of the estimation also serve diagnostic purposes, since the simpler models are nested in the complex models. We compute the standard errors on the parameter estimates using the quasi-maximum likelihood approach in Bollerslev and Wooldridge (1992).

The final specification we propose makes the conditional variance and skewness of an asset, i, dependent on covariance and coskewness with another asset, M. We can view asset, M, as the market. The specifications for mean, variance, and skewness in this bivariate GARCHS(1,1,1)-M model are:

$$\begin{aligned} r_{i,t} &= \alpha_{0,i} + \alpha_{1,i} h_{i,t-1} + \Phi_i r_{\mathrm{M},t} + \eta_{i,t} \\ r_{\mathrm{M},t} &= \alpha_{0,\mathrm{M}} + \alpha_{1,\mathrm{M}} h_{\mathrm{M},t-1} + \eta_{\mathrm{M},t} \\ h_{i,t} &= \beta_{0,i} + \beta_{1,i} h_{i,t-1} + \beta_{2,i} \eta_{i,t-1}^2 + \delta_1 h_{\mathrm{M},t} + \delta_2 \eta_{i,t-1} \eta_{\mathrm{M},t-1} \end{aligned} \tag{8}$$

$$h_{\mathrm{M},t} = \beta_{0,\mathrm{M}} + \beta_{1,\mathrm{M}} h_{\mathrm{M},t-1} + \beta_{2,\mathrm{M}} \eta^2_{\mathrm{M},t-1}$$
$$s_{i,t} = \gamma_{0,i} + \gamma_{1,i} s_{i,t-1} + \gamma_{2,i} \eta^3_{i,t-1} + \omega_1 s_{\mathrm{M},t} + \omega_2 \eta_{i,t-1} \eta^2_{\mathrm{M},t-1}$$
$$s_{\mathrm{M},t} = \gamma_{0,\mathrm{M}} + \gamma_{1,\mathrm{M}} s_{\mathrm{M},t-1} + \gamma_{2,\mathrm{M}} \eta^3_{\mathrm{M},t-1}$$

where we assume that the returns $r_{i,t}$ and $r_{\mathrm{M},t}$ are distributed as noncentral-t variables.[4] The advantage of the specification is that the return on asset i depends on its own lagged variance, $h_{i,t-1}$ and the *beta* with the market, Φ_i. However, variance of asset i, $h_{i,t}$ depends on lagged own variance $h_{i,t-1}$ as well as two terms that come from the market. These are contemporaneous market variance, $h_{M,t}$ as well as product of lagged innovations in asset i and the market M. The specification of skewness for asset i is also similar.

A. Models for asymmetric variance

The basic GARCH specification for conditional variance in (3) assumes that the innovations do not have differential impacts based on the sign of the innovation. To accommodate the possibility of asymmetric variance and seasonalities modifications of the GARCH model have been proposed. We use two specifications for asymmetric variance. The first is the Glosten, Jagannathan, and Runkle (1993) specification for returns on NYSE value-weighted index. They also find significant seasonal effects in variance. Their most successful specification is is:

$$\begin{aligned} r_t &= \alpha_0 + \alpha_1 h_{t-1} + \epsilon_t \\ \epsilon_t &= (1 + \lambda_1 OCT - \lambda_2 JAN)\eta_t \\ h_{t-1} &= \beta_0 + \beta_1 h_{t-2} + \beta_2 \eta^2_{t-1} + \beta_3 R^2_{f,t-1} + \kappa \eta^2_{t-1} I_{t-1} \\ & I_{t-1} = 1 \text{ if } \eta_{t-1} > 0 \text{ and } 0 \text{ otherwise} \end{aligned} \tag{9}$$

To understand how skewness interacts with asymmetric variance, we use these specifications for conditional mean and variance for the monthly returns. There are 9 parameters in this specification.

An alternative specification for capturing asymmetry in conditional variance is the EGARCH model introduced by Nelson (1991). The EGARCH models parameterize the logarithm of the conditional variance and permits an asymmetric relation. In the context of the specifications above, the EGARCH-M model has the following specification for conditional variance.

$$(10) \qquad H_t = \beta_0 + \beta_1 H_{t-1} + \beta_2 \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} + \beta_3 R_{f,t} + \kappa \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} I_t \quad \text{where } H_{t-1} = \log(h_{t-1})$$

The different specifications for conditional variance have found rather different relations between expected return and conditional variance on one hand as well as very different forms of variance asymmetry on the other.

In modeling daily returns, we adapt the Glosten, Jagannathan and Runkle (1993) and Nelson (1991) specifications for monthly conditional mean and variance. We use a Monday dummy in variance specification instead of riskfree rate as an instrument. This is motivated by the observation that Mondays appear to be characterized by substantially greater volatility than other days of the week. This choice is also supported by the finding in Foster and Viswanathan (1993) that Mondays are characterized by high volatility and trading costs. The specifications are then:

$$\begin{aligned} \text{GARCH Specification: } h_t &= \beta_0 + \beta_1 h_{t-1} + \beta_2 \eta_{t-1}^2 + \beta_3 MON_t + \kappa \eta_{t-1}^2 I_t \\ \text{EGARCH Specification: } H_t &= \beta_0 + \beta_1 H_{t-1} + \beta_2 \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} + \beta_3 MON_t + \kappa \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} I_t \\ & \quad H_{t-1} = \log(h_{t-1}) \quad MON = 1 \text{ if } t \text{ is a Monday and 0 else.} \end{aligned}$$

Additionally, we also use specifications that allow different GARCH-M coefficient α_1 for Mondays and the other days of the week.

III. Empirical Results

A. Data

We examine several data sets on daily and monthly frequency in our empirical work. For the daily returns we use S&P500 (U.S.) over the period January 4, 1969 to December 31, 1997, DAX 30 (Germany) over the period over January 4, 1975 to December 31, 1997, and Nikkei 225 (Japan) over the period January 4, 1980 to December 31, 1997. Table 1 presents average return, variance and skewness of continuously compounded daily returns on S&P500, DAX 30, and Nikkei 225 index by year. These are unconditional moments over the periods. The summary statistics show that mean, variance and skewness vary substantially by year with skewness for U.S. varying from -5.081 in 1987 to 0.805 in 1984, for Germany from

-4.125 in 1989 to 0.186 in 1981, and for Japan from -2.940 in 1987 to 1.542 in 1982. We use bootstrapped standard errors for skewness and test using the multivariate t-test to test whether skewness is equal across the years. This is rejected for all three markets. Figure 1 illustrates the variation in skewness month by month for the S&P500 index. For the bivariate GARCHS model we use daily returns on IBM and S&P500 over the period January 4, 1969 to December 31, 1997.

We then examine how the moments vary by month and day of the week. These summary statistics are presented in panels A and B of Table 2. The seasonal variations in skewness for all three markets by month as well as day of the week are very strong. In particular, Mondays are characterized by high volatility and negative skewness for both U.S. and Germany, even if the crash of 1987 is excluded.

We also examine the monthly CRSP value-weighted index return for NYSE over the period January 1951 to December 1995. This is the updated version of the data used by Glosten, Jagannathan, and Runkle (1993). We use the total return on the 30-day U.S. Treasury bill reported by Ibbotson Associates as the riskfree rate. The returns are continuously compounded. The unconditional skewness for this series is -0.729 (-0.314 excluding September, October, and November of 1987.)

B. Results

In our empirical work, we estimate the different models for conditional variance using GARCH, EGARCH and noncentral-t specifications. Our estimation procedure is multi-step. First, we estimate the conditional normal GARCH(1,1)-M model of Glosten, Jagannathan and Runkle (1993). There are a total of seven parameters in this specification for daily returns. with two parameters in the mean. Conditional variance has three GARCH(1,1) parameters, the asymmetric variance parameter, and the parameter for the Monday dummy. For monthly returns, this specification has nine parameters with two additional seasonal dummies and the coefficient for Monday replaced by the coefficient on riskfree rate of return.

Next we estimate an EGARCH(1,1)-M model, i.e. assuming that the conditional variance follow the EGARCH process of Nelson (1991). This model has the same number of parameters as the GARCH specification above.

Finally, we estimate the model with time-varying conditional skewness.[5] There are three additional parameters in the conditional skewness equation. Degrees of freedom and noncentrality are jointly determined by the mean and conditional skewness. Thus, we have a total of ten parameters for the daily returns and twelve parameters for monthly returns.

The three classes of models we estimate, nest most other models of conditional variance. We also estimate the nested models, for example without the asymmetric variance parameter or seasonals.

Table 3 presents the results for U.S. daily returns. In the basic GARCH-M and EGARCH models, the relation between returns and conditional variance, as measured by α_1, is positive and insignificant. Conditional variance shows as very high level of persistence as shown by the β_1 estimates. The parameter for asymmetric variance, κ is significant in both GARCH and EGARCH models but has opposite signs. In this specification, the coefficient on conditional variance is negative and insignificant on Mondays but positive and significant for other days of the week. The finding that Mondays are different and return and conditional variance are negatively related is consistent with Foster and Viswanathan (1993) and may be explained by their result that trading costs are higher on Mondays. Finally, we estimate the model with conditional noncentral-t. The coefficients on conditional skewness are significant and negative for γ_1. The coefficients for conditional variance decline substantially. The parameters for GARCH-M and asymmetric variance are negative and insignificant. The coefficient for the Monday dummy in variance, β_3, shows up as positive for GARCH and EGARCH models but not for noncentral-t. This may indicate that skewness in returns is somehow linked to Mondays, and including skewness obviates the need for including a Monday dummy in conditional variance.

Table 4 presents the results for Germany. German index returns show less persistence in variance than U.S. returns. Additionally, α_1, the parameter for the relation between returns and conditional variance is positive for GARCH, EGARCH, and noncentral-t models. Use of a noncentral-t model causes a substantial decline in persistence. The sign of κ, the coefficient for asymmetric variance, appears to be linked to the Monday dummy, β_3, in variance. For noncentral-t, β_3 comes out as negative. Conditional skewness parameters, γ_1 and γ_2 are significant.

Table 5 presents the results for Japanese daily compounded returns. As seen in the case for U.S. and Germany, the inclusion of skewness causes a substantial decline in persistence in variance as well as a decline in κ, the parameter for asymmetric variance. The parameter for the relation between returns and conditional variance, α_1 is positive but insignificant for noncentral-t whereas it is positive for the GARCH specification and negative and insignificant for the EGARCH specification.

Table 6 presents the results for monthly returns on value-weighted NYSE index. We have estimated all the models with and without seasonals and with and without an asymmetric variance component. The models without skewness show fairly high persistence levels in conditional variance. However, the addition of skewness to the GARCH equation causes the persistence to decline somewhat with β_2, the coefficient for lagged variance declining from 0.57 to 0.45. Interestingly, the addition of seasonal dummies causes the persistence to increase.

The coefficient for lagged skewness is significant and negative (-.28) implying that periods of high skewness are followed by low skewness. This coefficient increases if an asymmetric variance component is permitted.

As Glosten, Jagannathan and Runkle (1993) and Engle and Ng (1993) have found, asymmetry in variance depends on the specification used. In particular, EGARCH shows an insignificant positive asymmetric coefficient, κ whereas it appears significant and negative with a GARCH specification. However, if skewness is added to the specification, κ declines substantially.

C. Diagnostic tests

For diagnostics, we focus on the properties of the standardized residuals. For daily returns on the S&P500, the standardized (i.e. divided by standard deviation) residuals from GARCH(1,1)-M model have a skewness of -0.27 and excess kurtosis of 4.74. In comparison, standardized residuals from the GARCHS(1,1,1)-M model have a skewness of -0.17 and excess kurtosis of 0.90.

We also graph the behavior conditional variance and skewness for the U.S. returns. Figure 1 shows the conditional skewness for the daily S&P500 returns. Panels A, B, and C of Figure 2 plot conditional variances for the monthly U.S. returns. Panel A shows the very noisy variations in volatility caused by the inclusion of monthly seasonals in the GARCH model specification of Glosten, Jagannathan, and Runkle (1993.) In contrast, the EGARCH model gives us a much smoother plot in panel B. However, even in the GARCH model including seasonals, inclusion of skewness through a noncentral-t distribution smoothes out the conditional variance function as seen in panel B. The noncentral-t model also captures the substantial increase in conditional variance in October 1987.

Conditional moment tests were introduced in Newey (1987), Engle, Lilien and Robbins (1987) and Nelson (1991) for testing the specification of a model. Using the standardized residuals from the estimated models, a set of orthogonality conditions are constructed that should be satisfied if the model is correctly specified. These orthogonality conditions, with a proper covariance matrix, can then be used to construct a Wald statistic distributed as a χ^2 with degrees of freedom equal to the number of orthogonality conditions tested. We use the standardized residuals, $\hat{Z}_t$ from both the GARCH(1,1)-M and the GARCHS(1,1,1)-M model applied to the monthly returns on the value-weighted NYSE/AMEX index to construct the following sixteen orthogonality conditions:

$$
(11) \qquad \begin{aligned}
E\left[\hat{Z}_t\right] &= 0 \\
E\left[\hat{Z}_t^2 - 1\right] &= 0 \\
E\left[\frac{\hat{Z}_t^3}{\hat{Z}_t^{\frac{3}{2}}} - 3\right] &= 0 \\
E\left[\frac{\hat{Z}_t^4}{\hat{Z}_t^2} - 0\right] &= 0 \\
E\left[\hat{Z}_t \hat{Z}_{t-j}\right] &= 0 \;\text{ for }\; j = 1, \ldots, 4 \\
E\left[(\hat{Z}_t^2 - 1)(\hat{Z}_{t-j}^2 - 1)\right] &= 0 \;\text{ for }\; j = 1, \ldots, 4 \\
E\left[(\hat{Z}_t^3)(\hat{Z}_{t-j}^3)\right] &= 0 \;\text{ for }\; j = 1, \ldots, 4
\end{aligned}
$$

We test the individual conditions as well as conduct a χ^2 test with sixteen degrees of freedom.

The evidence in Table 7 suggest that the GARCH(1,1)-M model is misspecified. There are two moment conditions with t-ratios that exceed 2.00. The overall test-statistic rejects the model at 5% level of confidence. In contrast, for the GARCHS(1,1,1)-M model, The individual t-statistics are all essentially zero. In addition, we do not reject the model at the conventional significance levels of 5% and 10%. When the three months around October, 1987 are dropped from the sample, the significance level increases to 0.115.[6] As a benchmark we also carry out the tests using the GARCH(1,1,1)-M model. The results show that the individual t-statistics that relate to the skewness in the residuals are somewhat significant. Additionally, the χ^2-statistic also rejects that the residuals are all 0.

Another diagnostic test can be constructed based on Newey and Steigerwald (1997). They show that the quasi-maximum likelihood estimators are not consistent in the presence of asymmetric distributions. To produce consistent estimators, we use the following specification for mean:

$$
(12) \qquad \begin{aligned}
r_t &= \alpha_0 + \alpha_1 h_{t-1} + \delta\sqrt{h_{t-1}} + \zeta_t \\
\epsilon_t &= \zeta_t - \delta\sqrt{h_{t-1}}
\end{aligned}
$$

where α_1 and δ are identified since the residuals used to compute conditional variance are constructed excluding δ.[7] Our estimate of δ in the GARCHS(1,1,1) model for daily returns is 0.003 and insignificant whereas for the GARCH(1,1)-M model it is -0.23 and significant. This implies that the GARCHS(1,1,1) specification captures the asymmetry in distribution successfully.

As an alternative diagnostic for our estimation method, we also carry out simulations. In doing the simulations, we are confronted with the problem that the specifications we are interested in display time-varying means, variances and skewnesses, i.e. they come from a stochastic process rather than a single distribution.

We generate a data sample of 718 observations assuming that the conditional variance and skewness have the same coefficients as the weekly U.S. data set. We use a noncentral t distribution with 8.00 degrees of freedom and the noncentrality parameter is -1.07. We then estimate the parameters for a noncentral t model for the sample. We repeat this procedure a 1000 times. Thus, we have a 1000 estimates of noncentrality parameter, δ_t, and and the degrees of freedom ν_t, We use these estimates to see how powerful our model is in detecting skewness in data. We find that the 10th percentile point for the noncentrality parameter is -3.50 and the 90th percentile is -0.10. Therefore, the coverage in detecting skewness is quite high.

A final diagnostic is to examine how well the various models perform in explaining the squared residuals of the returns. Table 8 presents the actual squared residuals and the conditional variance predictions for three models including the GARCHS(1,1,1) model for 12 months starting with April 1987. None of the GARCH-type models, including the GARCHS(1,1,1) model, appears able to predict October 1987. However, the substantial increase in conditional variance after October 1987 is picked up by the GARCHS(1,1,1) model.

D. Other financial time-series with skewness

We also examine other time-series to understand how conditional skewness affects their properties. We focus on Mexico, Chile, Thailand, and Taiwan. The returns cover the period 1/6/89 to 1/16/98, a total of 472 weeks. Bekaert and Harvey (1997) present a model of volatility using weekly world and local information that impact local volatility in emerging markets. They find that the relative importance of world versus local information changes through time. The sample we investigated is, by and large, after the financial integration of these markets had commenced. Our GARCHS(1,1,1) model is only meant to illustrate potential applications in these markets. A more complete volatility model would likely account for some of the features of the Bekaert and Harvey model.

The unconditional skewness over this period has been -1.343 for Mexico, -0.138 for Taiwan, -0.656 for Thailand and 0.110 for Chile. Annualized standard deviations have ranged from 68.10% for Chile to 114.89% for Taiwan. However, these numbers mask substantial variation over time. Figure 3 plots the variations in volatility and skewness for the four countries. When variance and skewness are computed quarterly, substantial serial correlation in the quarterly variances and skewnesses also exist. The serial correlations in variance are 0.54, 0.56 and 0.58 for Mexico, Taiwan and Thailand but -0.11 for Chile. The serial correlations in the quarterly skewness are 0.17 for Chile, -0.02 for Mexico, -0.16 for Taiwan and -0.25 for Thailand.

These summary statistics suggest that an autoregressive model may be successful in explaining the time-series variations in the variances and skewnesses of the emerging market returns. Therefore, we estimated GARCHS(1,1,1)-M models with 10 parameters for each of the four countries. We find that the coefficient on lagged skewness, γ_1, the coefficient for lagged skewness is positive for Mexico and negative for the other three. γ_2 is rather insignificant. The estimates of γ_1 are -0.11 for Chile, 0.31 for Mexico, -0.04 for Taiwan and -0.21 for Thailand. The parameter estimates for conditional variance are quite high for all four countries in GARCH(1,1)-M estimation, with the sum ranging from 0.91 for Mexico to 0.98 for Taiwan. With the inclusion of conditional skewness in the model, the parameters decline though not substantially. These results suggest that conditional skewness should likely be incorporated in the models for estimating the volatility dynamics in the emerging market returns.

IV. Conclusions

We present a new methodology for simultaneously modeling and estimating conditional mean, variance and conditional skewness in a maximum likelihood framework assuming a noncentral conditional t-distribution. Our application of this methodology daily, weekly and monthly returns on the U.S., German, Japanese, Mexican, Chilean, Taiwanese and Thailand stock index returns confirms that autoregressive conditional skewness is important. Additionally, the inclusion of skewness impacts the persistence in variance.

We also explore the relation between conditional skewness and the asymmetric variance models proposed by Nelson (1991), Glosten, Jagannathan, and Runkle (1993) and others. We find that the inclusion of skewness can cause asymmetry in variance to disappear. However, we find that the relation between return, variance, and skewness of equity returns is intimately linked to the seasonal variations in the conditional moments.

With the significant presence of skewness in returns and the impact of skewness on returns and volatility, the importance of conditional skewness in portfolio analysis is an important extension of this paper. In particular, energy markets, small size stocks, and distressed firms' stock returns display substantial skewness and understanding performance of returns of such assets needs recognition of this feature. Additionally, use of the autoregressive conditional variance and skewness in option pricing is another possible extension of this paper.

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Table 1

Summary statistics for daily returns by year

This table presents summary statistics for continuously compounded value-weighted returns on S&P500 index (U.S.) from January 4, 1969 to December 31, 1997, a total of 7566 observations, on DAX 30 Index (Germany) from January 4, 1975 to December 31, 1997, a total of 6000 observations, and on Nikkei 225 (Japan) from January 4, 1980 to December 31, 1997, a total of 4694 observations, The returns are broken down by year.

	U.S.			Germany			Japan		
Year	Mean $\times 10^4$	Var$\times 10^4$	Skew	Mean $\times 10^4$	Var$\times 10^4$	Skew	Mean $\times 10^4$	Var$\times 10^4$	Skew
1969	-4.636	0.418	0.398						
1970	0.037	1.010	0.767						
1971	3.925	0.601	0.609						
1972	5.587	0.309	-0.250						
1973	-7.308	0.969	0.020						
1974	-13.512	1.908	0.453						
1975	10.506	0.943	0.072	12.942	0.771	0.114			
1976	6.687	0.481	0.065	-3.864	0.582	-0.013			
1977	-4.700	0.319	-0.104	2.932	0.332	0.059			
1978	0.406	0.613	0.409	1.766	0.275	-0.292			
1979	4.448	0.458	-0.272	-5.535	0.388	0.140			
1980	8.752	1.065	-0.240	-1.316	0.499	0.146	2.764	0.220	-0.616
1981	-3.922	0.706	-0.039	0.747	0.591	0.186	3.217	0.854	0.554
1982	5.275	1.270	0.610	4.588	0.566	-0.325	1.635	0.629	1.542
1983	6.128	0.692	-0.009	12.945	0.803	0.090	8.092	0.391	0.732
1984	0.533	0.625	0.805	2.257	0.566	-0.228	5.906	0.546	-0.386
1985	8.956	0.394	0.453	19.517	0.885	-0.244	4.800	0.338	-1.048
1986	5.228	0.838	-0.982	1.808	1.674	0.133	13.932	0.829	-0.423
1987	0.769	4.362	-5.081	-13.764	3.445	-0.969	5.213	2.887	-2.940
1987*	7.930	1.824	-1.138	-9.108	2.819	-0.560	8.427	1.572	-0.403
1988	4.479	1.131	-1.054	10.865	1.388	0.174	12.853	0.540	1.176
1989	9.269	0.665	-1.840	11.494	1.688	-4.125	9.805	0.290	0.179
1990	-2.599	0.980	-0.171	-9.472	2.371	0.148	-18.761	3.971	0.836
1991	8.948	0.784	0.190	4.634	1.566	-1.087	-1.415	1.676	-0.056
1992	1.667	0.360	0.055	-0.805	0.779	0.115	-11.679	3.358	0.443
1993	2.612	0.285	-0.178	14.684	0.719	0.174	1.099	1.569	0.257
1994	-0.597	0.373	-0.298	-2.817	1.087	-0.201	4.782	1.198	0.896
1995	11.288	0.234	-0.051	2.600	0.661	-0.455	0.282	1.954	0.111
1996	7.043	0.536	-0.616	9.471	0.624	-0.852	-0.986	0.920	0.031
1997	10.348	1.271	-0.682	14.791	2.228	-0.968	-9.124	2.905	0.028
All	2.954	0.849	-2.084	3.931	1.068	-0.834	1.997	1.309	-0.202
All*	3.198	0.761	-0.197	4.145	1.038	-0.700	2.156	1.242	0.279

* Excluding the week October 15, 1987 to October 22, 1987.

Table 2

A. Summary statistics for daily returns by day of the week

This table presents summary statistics for continuously compounded value-weighted returns on S&P500 index (U.S.) from January 4, 1969 to December 31, 1997, a total of 7566 observations, on DAX 30 Index (Germany) from January 4, 1975 to December 31, 1997, a total of 6000 observations, and on Nikkei 225 (Japan) from January 4, 1980 to December 31, 1997, a total of 4694 observations, The returns are broken down by day of the week.

	U.S.			Germany			Japan		
Day	Mean $\times 10^4$	Var$\times 10^4$	Skew	Mean $\times 10^4$	Var$\times 10^4$	Skew	Mean $\times 10^4$	Var$\times 10^4$	Skew
Mon	-5.940	1.263	-6.052	-6.692	1.512	-2.242	-6.434	1.770	-0.132
Mon*	-4.434	0.920	-0.915	-5.875	1.433	-1.995	-5.712	1.630	-0.149
Tue	4.972	0.783	0.543	2.589	1.101	-0.557	-0.957	1.514	-1.307
Wed	8.651	0.743	0.987	8.706	1.003	0.178	10.071	1.329	0.367
Thu	1.204	0.707	-0.005	5.476	0.874	0.311	5.452	1.132	0.179
Fri	5.875	0.738	-0.705	9.570	0.834	-0.041	0.833	1.220	0.358

B. Summary statistics for daily returns by month

This table presents summary statistics for continuously compounded value-weighted returns on S&P500 index (U.S.) from January 4, 1969 to December 31, 1997, a total of 7566 observations, on DAX 30 Index (Germany) from January 4, 1975 to December 31, 1997, a total of 6000 observations, and on Nikkei 225 (Japan) from January 4, 1980 to December 31, 1997, a total of 4694 observations, The returns are broken down by month.

	U.S.			Germany			Japan		
Month	Mean $\times 10^4$	Var$\times 10^4$	Skew	Mean $\times 10^4$	Var$\times 10^4$	Skew	Mean $\times 10^4$	Var$\times 10^4$	Skew
Jan	8.057	0.850	-0.609	4.232	1.320	0.391	8.556	1.508	0.404
Feb	2.037	0.622	-0.117	10.189	0.953	0.013	-0.435	0.826	-0.574
Mar	2.822	0.620	-0.270	5.827	0.867	0.093	3.996	1.254	0.049
Apr	4.048	0.691	-0.075	5.809	0.689	-0.22	10.644	1.558	-0.142
May	2.940	0.690	0.308	-0.877	0.711	-0.057	4.752	0.888	-0.096
Jun	1.866	0.576	0.058	6.848	0.661	-0.342	-6.074	0.889	-0.455
Jul	1.650	0.608	-0.066	10.305	0.875	-0.147	1.292	1.191	0.314
Aug	3.166	0.859	0.571	0.061	1.288	-1.497	1.323	1.706	-0.103
Sep	-3.273	0.801	-0.038	-4.413	0.931	-0.178	-6.287	1.239	-0.15
Oct	0.137	2.280	-5.459	-2.792	2.494	-2.055	-0.235	2.626	-0.993
Oct*	3.005	1.249	-0.872	-0.317	2.163	-1.939	1.079	1.616	2.011
Nov	4.617	0.861	-0.294	4.739	1.209	-0.154	-0.717	1.669	0.362
Dec	7.143	0.695	-0.036	7.714	0.774	-0.172	4.182	1.337	-0.108

* Excluding the week October 15, 1987 to October 22, 1987.

Table 3

Model for U.S. daily returns

This table presents the results for three models for the conditional mean, conditional variance and conditional skewness for U.S. daily returns. The sample includes continuously compounded value-weighted returns on S&P500 index from January 4, 1969 to December 31, 1997, a total of 7566 observations.

$$r_t = \alpha_0 + \alpha_1 \mathrm{Var}_{t-1}(\eta_t) + \eta_t$$

$$\text{GARCH Specification:} \quad h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 \eta_{t-1}^2 + \beta_3 MON_t + \kappa \eta_{t-1}^2 I_t$$

$$\text{EGARCH Specification:} \quad H_t = \beta_0 + \beta_1 H_{t-1} + \beta_2 \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} + \beta_3 MON_t + \kappa \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} I_t \quad H_{t-1} = \log(h_{t-1})$$

$I_t = 1$ if $\eta_{t-1} > 0$ and 0 otherwise.

MON= 1 if t is a Monday and 0 else.

$$s_t = \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 \eta_{t-1}^3$$

Model		GARCH	EGARCH	Noncentral t
	Parameter	Estimate t-statistic	Estimate t-statistic	Estimate t-statistic
Mean	α_0	0.000*** 2.88	0.000* 1.77	0.000 0.00
Equation	α_1	0.882 0.64	0.787 0.39	-0.092 -0.87
	β_0	0.000 1.44	-0.291*** -9.84	0.000 0.00
	β_1	0.924*** 364.37	0.984*** 375.66	0.517*** 2.52
Variance Equation	β_2	0.094*** 18.54	-0.176*** -15.89	0.215 0.22
	β_3	0.000 1.58	0.242*** 4.55	1.98 0.000
	κ	-0.063*** -9.34	0.246*** 14.18	-0.012 -1.01
	γ_0			0.000 0.00
Skewness Equation	γ_1			-0.652*** -3.65
	γ_2			-0.015 -0.06
Likelihood		25696	25710	27687

The t-statistics are reported with * denoting significance at 10%, ** denoting significance at 5%, and *** denoting significance at 1%.

Table 4

Model for German daily returns

This table presents the results for three models for the conditional mean, conditional variance and conditional skewness for German daily returns. The sample includes continuously compounded value-weighted returns on DAX 30 index from January 4, 1975 to December 31, 1997, a total of 6000 observations.

$$r_t = \alpha_0 + \alpha_1 \mathrm{Var}_{t-1}(\eta_t) + \eta_t$$

GARCH Specification: $$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 \eta_{t-1}^2 + \beta_3 MON_t + \kappa \eta_{t-1}^2 I_t$$

EGARCH Specification: $$H_t = \beta_0 + \beta_1 H_{t-1} + \beta_2 \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} + \beta_3 MON_t + \kappa \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} I_t \quad H_{t-1} = \log(h_{t-1})$$

$I_t = 1$ if $\eta_{t-1} > 0$ and 0 otherwise.

MON= 1 if t is a Monday and 0 else.

$$s_t = \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 \eta_{t-1}^3$$

Model		GARCH	EGARCH	Noncentral t
	Parameter	Estimate t-statistic	Estimate t-statistic	Estimate t-statistic
Mean Equation	$\alpha_0 \times 1000$	-0.163* 0.00	0.336 1.87	-21.721*** -166.55
	α_1	7.270*** 7.28	0.884 0.42	0.138*** 143.21
Variance Equation	$\beta_0 \times 1000$	0.000 0.00	-470.297*** -10.23	70.673*** 101.11
	β_1	0.844 0.85	0.977*** 239.00	0.662*** 149.25
	β_2	0.146 0.15	-0.208*** -13.34	0.000 0.00
	β_3	0.000 0.00	0.643*** 10.95	-0.004*** -9.43
	κ	-0.060 -0.06	0.339*** 12.53	0.740*** 2.52
Skewness Equation	γ_0			-0.329*** -31.81
	γ_1			-0.627*** -10.01
	γ_2			0.003 0.00
Likelihood		18456	18678	19753

The t-statistics are reported with * denoting significance at 10%, ** denoting significance at 5%, and *** denoting significance at 1%.

Table 5

Model for Japanese daily returns

This table presents the results for three models for the conditional mean, conditional variance and conditional skewness for Japanese daily returns. The sample includes continuously compounded value-weighted returns on Nikkei 225 index from January 4, 1980 to December 31, 1997, a total of 4694 observations.

$$r_t = \alpha_0 + \alpha_1 \mathrm{Var}_{t-1}(\eta_t) + \eta_t$$

GARCH Specification: $$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 \eta_{t-1}^2 + \beta_3 MON_t + \kappa \eta_{t-1}^2 I_t$$

EGARCH Specification: $$H_t = \beta_0 + \beta_1 H_{t-1} + \beta_2 \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} + \beta_3 MON_t + \kappa \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} I_t \quad H_{t-1} = \log(h_{t-1})$$

$I_t = 1$ if $\eta_{t-1} > 0$ and 0 otherwise.

MON= 1 if t is a Monday and 0 else.

$$s_t = \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 \eta_{t-1}^3$$

Model		GARCH	EGARCH	Noncentral t
	Parameter	Estimate t-statistic	Estimate t-statistic	Estimate t-statistic
Mean Equation	$\alpha_0 \times 1000$	0.280* 1.92	0.430*** 6.39	-0.052 0.00
	α_1	2.052* 1.76	-0.508*** -5.01	0.246 1.25
Variance Equation	$\beta_0 \times 1000$	0.000 0.00	-514.540*** -18.34	0.888 0.00
	β_1	0.756*** 67.40	0.974*** 380.02	0.669*** 11.99
	β_2	0.238*** 17.51	-0.355*** -20.11	0.340*** 3.40
	β_3	0.000*** 9.25	0.499*** 8.33	-0.001 -.08
	κ	-0.243*** -14.34	0.479*** 17.95	1.159 1.59
Skewness Equation	γ_0			0.000*** -11.77
	γ_1			-0.384*** -2.82
	γ_2			0.277** 2.17
Likelihood		18767	18656	19453

The t-statistics are reported with * denoting significance at 10%, ** denoting significance at 5%, and *** denoting significance at 1%.

Table 6

Model for U.S. monthly returns

This table presents the results for three models for the conditional mean, conditional variance and conditional skewness for U.S. returns. The sample includes value-weighted returns on the NYSE index from 1951 April to 1995 December, a total of 537 observations.

$$r_t = \alpha_0 + \alpha_1 \mathrm{Var}_{t-1}(\epsilon_t) + \epsilon_t$$
$$\epsilon_t = (1 + \lambda_1 OCT - \lambda_2 JAN)\eta_t$$
$$\text{GARCH Specification: } h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 \eta_{t-1}^2 + \beta_3 R_{f,t} + \kappa \eta_{t-1}^2 I_t$$
$$\text{EGARCH Specification: } H_t = \beta_0 + \beta_1 H_{t-1} + \beta_2 \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} + \beta_3 R_{f,t} + \kappa \frac{\eta_{t-1}}{\sqrt{h_{t-1}}} I_t \quad H_{t-1} = \log(h_{t-1})$$
$$I_t = 1 \text{ if } \eta_{t-1} > 0 \text{ and } 0 \text{ otherwise}$$
$$s_t = \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 \eta_{t-1}^3$$

Model		GARCH	GARCH	EGARCH	Noncentral t	Noncentral t
	Parameter	Estimate t-statistic	Estimate t-statistic	Estimate t-statistic	Estimate t-statistic	Estimate t-statistic
Mean Equation	$\alpha_0 \times 1000$	14.781*** 3.83	3.577** 2.27	3.336** 2.10	7.693*** 611.30	7.313*** 2.59
	α_1	-7.459*** -2.82	0.000 -0.02	0.024 0.11	-0.239*** -11.66	1.174 0.56
	λ_1		141.920 0.05	16.586 0.39	-0.125*** -84.42	-0.126*** -8.86
	λ_2		-368.607 -0.09	-34.624 -0.22	0.263*** 31.43	0.266*** 19.97
Variance Equation	$\beta_0 \times 1000$	0.285** 2.14	0.414*** 2.52	-4.733*** -3.93	0.515*** 27.14	0.985* 1.92
	β_1	0.371 1.40	0.574** 2.08	0.547** 2.00	0.466*** 17.28	0.381*** 3.45
	β_2	0.173** 2.22	0.198*** 2.65	-0.324*** -3.21	0.069*** 7.22	0.005 0.23
	β_3	0.157** 2.07	0.053*** 2.37	58.567*** 2.74	0.050*** 11.55	0.115*** 2.95
	κ	-0.239*** -2.94	-0.262*** -3.22	0.097 0.48	-0.013 -1.14	
Skewness Equation	$\gamma_0 \times 1000$				-0.028*** -18.02	-0.033*** -2.81
	γ_1				-0.432*** -27.46	-0.283* -1.76
	γ_2				-0.005 * -1.82	-0.006 -1.00
Likelihood		1178	1256	1231	1301	1298

The t-statistics are reported with * denoting significance at 10%, ** denoting significance at 5%, and *** denoting significance at 1%.

Table 7

Specification tests for the GARCHS(1,1,1)-M model

We carry out conditional moment tests of the GARCH(1,1)-M and GARCHS(1,1,1)-M model using the methodology presented in Nelson (1991). Using the standardized residuals from the estimated model we construct 16 orthogonality conditions. The first four should hold for the first four moments. The conditions following test the serial correlations in the mean, variance and skewness. We apply this model to the monthly returns on the value-weighted NYSE/AMEX index. The t-statistic are computed using the sample averages. We also compute a χ^2 statistic with all 16 orthogonality conditions.

	Orthogonality Condition	GARCH(1,1)-M		GARCHS(1,1,1)-M	
		Sample Average	t-statistic	Sample Average	t-statistic
1	$E\left[\hat{Z}_t\right] = 0$	0.003	-0.006	0.002	0.045
2	$E\left[\hat{Z}_t^2 - 1\right] = 0$	0.004	0.584	0.009	-0.996
3	$E\left[\frac{\hat{Z}_t^3}{\hat{Z}_t^{\frac{3}{2}}} - 3\right] = 0$	-1.321	-2.560	0.121	0.121
4	$E\left[\frac{\hat{Z}_t^4}{\hat{Z}_t^2} - 0\right] = 0$	0.993	0.996	0.993	0.996
5	$E\left[\hat{Z}_t\hat{Z}_{t-1}\right] = 0$	0.000	-0.005	0.000	0.065
6	$E\left[\hat{Z}_t\hat{Z}_{t-2}\right] = 0$	0.000	-0.059	0.000	-0.028
7	$E\left[\hat{Z}_t\hat{Z}_{t-3}\right] = 0$	0.000	-0.008	0.000	0.011
8	$E\left[\hat{Z}_t\hat{Z}_{t-4}\right] = 0$	0.000	0.019	0.000	0.017
9	$E\left[(\hat{Z}_t^2 - 1)(\hat{Z}_{t-1}^2 - 1)\right] = 0$	0.989	0.996	0.989	0.996
10	$E\left[(\hat{Z}_t^2 - 1)(\hat{Z}_{t-2}^2 - 1)\right] = 0$	0.989	0.996	0.989	0.996
11	$E\left[(\hat{Z}_t^2 - 1)(\hat{Z}_{t-3}^2 - 1)\right] = 0$	0.989	0.996	0.989	0.996
12	$E\left[(\hat{Z}_t^2 - 1)(\hat{Z}_{t-4}^2 - 1)\right] = 0$	0.989	0.996	0.989	0.996
13	$E\left[(\hat{Z}_t^3)(\hat{Z}_{t-1}^3)\right] = 0$	-0.312	-2.089	0.000	0.026
14	$E\left[(\hat{Z}_t^3)(\hat{Z}_{t-2}^3)\right] = 0$	0.003	-0.054	0.000	-0.058
15	$E\left[(\hat{Z}_t^3)(\hat{Z}_{t-3}^3)\right] = 0$	0.000	0.022	0.000	0.029
16	$E\left[(\hat{Z}_t^3)(\hat{Z}_{t-4}^3)\right] = 0$	0.000	-0.068	0.000	-0.060
		χ^2 * = 27.12 (p-value=0.040)		χ^2 * = 23.00 (p-value=0.113)	

*Excluding the months of September, October and November 1987, the χ^2 statistic for GARCHS(1,1,1)-M has a value of 22.93 and a p-value of 0.115.

Table 8

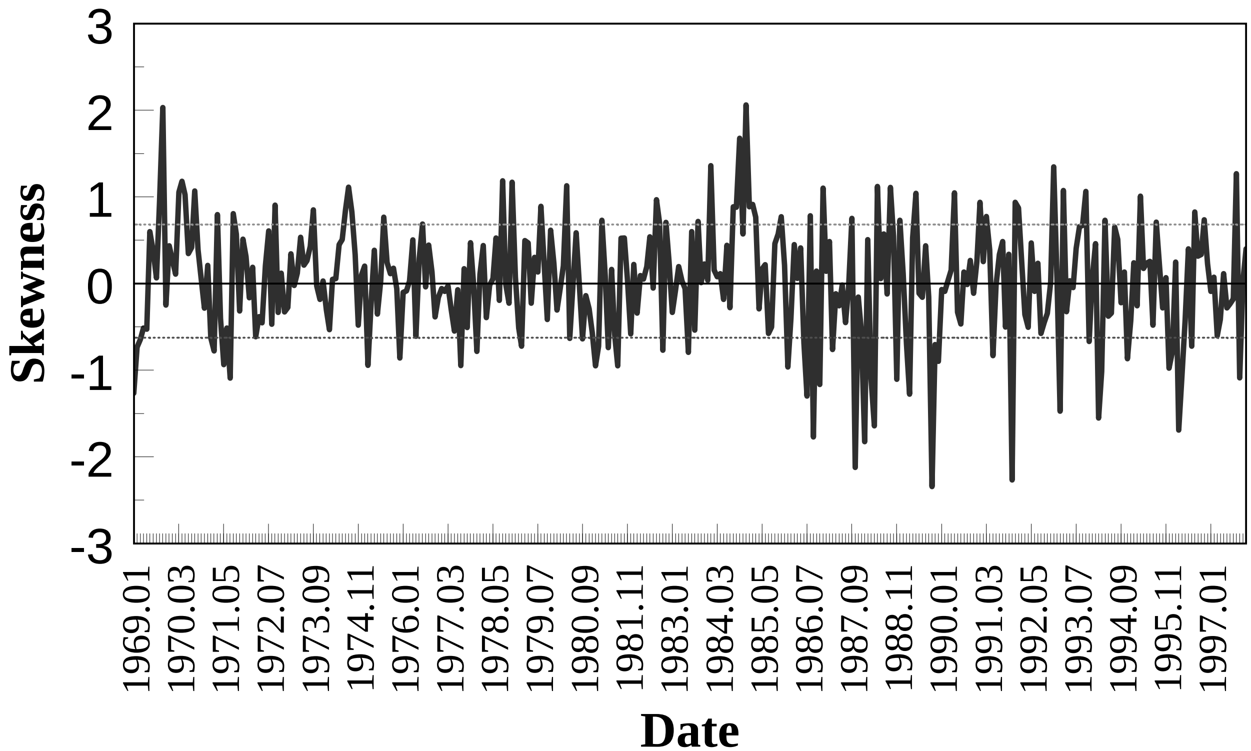
Actual versus prediced variances: the Crash of 1987

The following table compares the predicted variances from three models to the actual squared residuals for the period April 1987 to March 1988 for the value-weighted NYSE index returns. $RESID^2$ is the squared residual computed using the unconditional mean of the returns. NCT is the conditional variance prediction using the GARCHS(1,1,1) model. GJR is the GARCH(1,1)-M specification in Glosten, Jagannathan, and Runkle (1993) with dummies for January and October. EGARCH is a EGARCH(1,1)-M specification with dummies for January and October.

Month	$RESID^2$	NCT	GJR	EGARCH
April	0.90	1.31	0.35	1.08
May	0.00	1.54	0.40	1.55
June	1.20	1.63	0.38	1.46
July	1.30	1.60	0.35	1.09
August	0.80	1.58	0.34	0.95
September	1.20	1.59	0.34	0.94
October	85.50	1.40	0.45	1.80
November	9.20	9.56	0.38	1.45
December	4.00	5.97	0.88	2.91
January	1.60	1.82	0.45	1.17
February	1.60	2.32	0.40	1.25
March	0.90	1.89	0.34	0.97

Figure 1: Monthly skewness of S&P500 returns

computed using daily returns



The two dotted lines represent one standard deviation above and
one standard deviation below average monthly skewness over the period

Figure 2A: Conditional Variance from GJR Model
For Value-weighted NYSE with Seasonals.

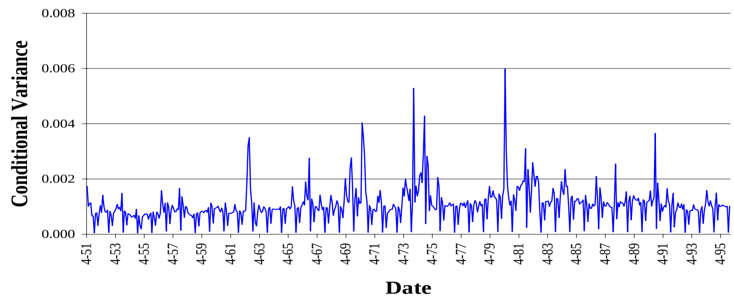


Figure 2B: Conditional Variance from E-GARCH model
Based on value-weighted NYSE returns

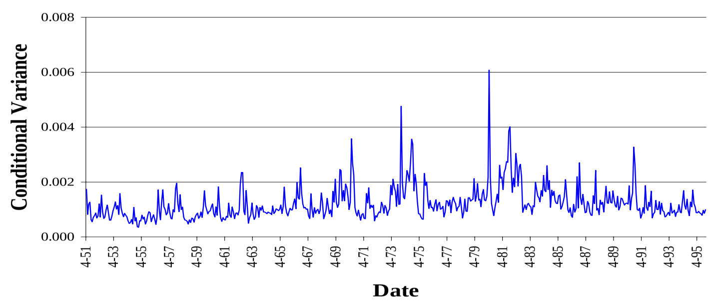


Figure 2C: Conditional Variance from Noncentral-t
NYSE Value-weighted returns

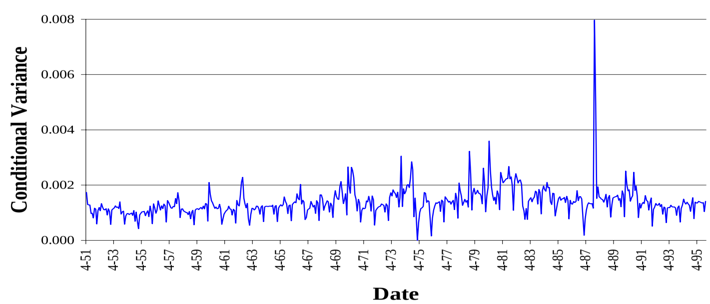
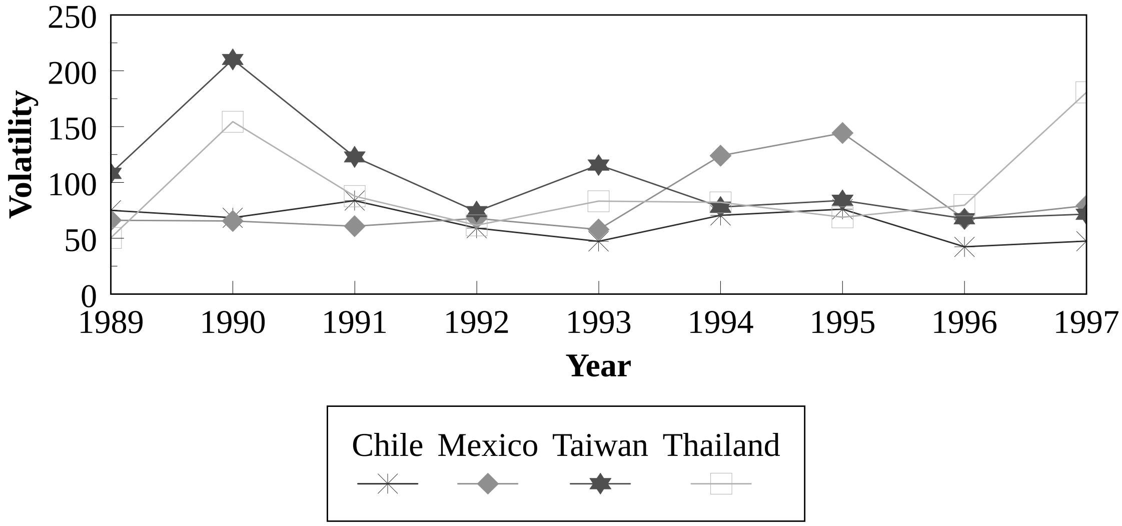


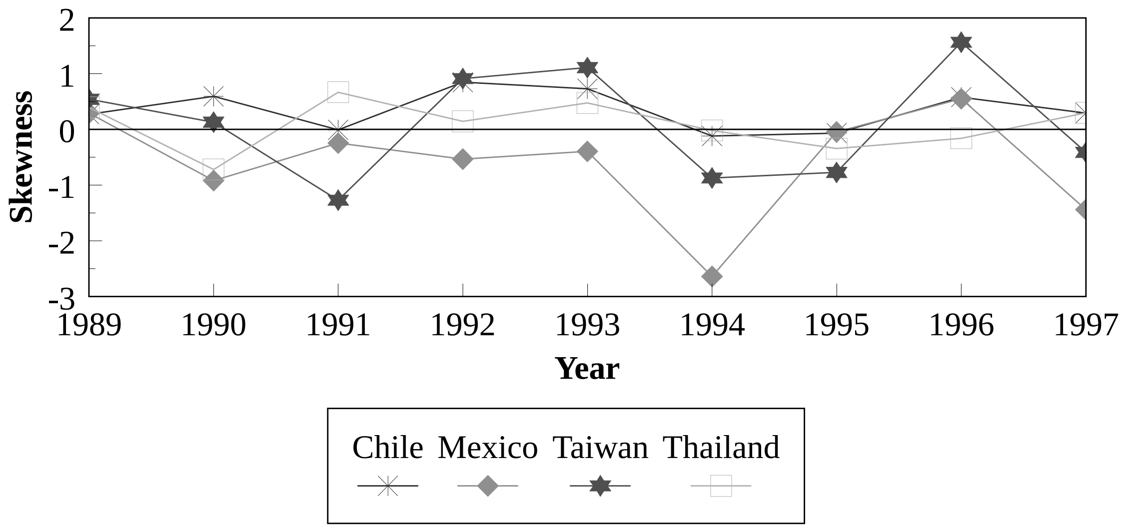
Figure 3

A. Plot of annualized volatilities



The volatilities are computed from weekly IFC index returns.

B. Plot of annual skewnesses



The skewnesses are computed from weekly IFC index returns.

Notes

[1]Our choice of a simple specification is also consistent with the results in Nelson (1992) that misspecification has little influence on the estimated conditional variance. We have used more instruments as well, and our conclusions about persistence and asymmetry are not affected. However, the inclusion of instruments does affect the relation between return and variance. Engle and Gonzalez-Rivera (1991) and Gray (1996) use semi-parametric ARCH specifications for conditional mean and variance.

[2]An alternative method of introducing skewness in the distribution of returns would be to use a mixture of two normal distributions or mixture of two t distributions. These are the "SPARCH" models. The major drawback in using a SPARCH model is that the choice of a weighting scheme between the two distributions is somewhat arbitrary. An alternative parameterization of the noncentral-t distribution was proposed by Hansen (1994) where it was applied in modeling the term-structure of interest rates.

[3]We have also used an alternative specification for conditional skewness without an intercept, in effect forcing γ_1 to be the same as unconditional skewness.

[4]We can also accommodate kurtosis by incorporating a dynamic equation for time-varying kurtosis

[5] We have also estimated a GARCH(1,1)-M-t model, i.e. assuming that the returns are from a conditional t distribution. The degrees of freedom is the additional parameter in this model. The t-distribution accommodates the thick tails of the data but does not permit skewness.

[6]An alternative test of an autoregressive conditional skewness model can be constructed in the Newey (1987) generalized method of moments framework. For this we specify the following models for conditional mean, variance and skewness:

$$\begin{aligned} r_t &= \alpha_0 + \alpha_1 h_{t-1} \\ h_t &= \beta_0 + \beta_1 e_{t-1}^2 + \beta_2 e_{t-2}^2 + \beta_3 e_{t-3}^2 + \beta_4 e_{t-4}^2 + \beta_5 e_{t-5}^2 \qquad \text{(i)} \\ s_t &= \gamma_0 + \gamma_1 e_{t-1}^3 + \gamma_2 e_{t-2}^3 + \gamma_3 e_{t-3}^3 + \gamma_4 e_{t-4}^3 + \gamma_5 e_{t-5}^3 \end{aligned}$$

These specifications are ARCH(5) for variance and skewness. We estimate this model for daily returns on the S&P500 in a method of moments framework using the lagged score functions upto lag 4 as instruments. The system is exactly identified and Hansen's J test gives us a χ^2 statistic of 0.192 which fails to reject the null at a p-value of 0.67. The conditional moments test-statistic is distributed as a χ^2 with 6 degrees of freedom, using three instruments to multiply the residuals in (i). We fail to reject that the model is correctly specified.

[7] We are grateful for the detailed suggestions Doug Steigerwald on this point.