Consider a pure exchange economy with a representative agent with additively separable utility receiving a stochastic endowment. This agent can choose to consume this endowment or invest in a portfolio of \( j \)-period bonds, \( \{B_{j,t} : j = 1, \ldots, k \} \). The bonds are assumed to be default free. Expectations at time \( t \) are conditioned on the information set \( \mathbf{F}_t \) – which contains all the information about the environment available at time \( t \). Consumption \( C_t \) is required to be measurable at \( t \) with respect to \( \mathbf{F}_t \). The consumer maximizes the following objective:

\[
\max_{\{C_t, B_{j,t}\}_{t=0}^\infty} \sum_{j=1}^k \delta^t E_0 U(C_t); \quad 0 < \delta < 1,
\]

subject to:

\[
C_t + \sum_{j=1}^k B_{j,t} \leq Y_t + \sum_{j=1}^k B_{j,t-j} (1 + R_{j,t-j}), \quad (A.1)
\]

where \( C_t \) is the agent’s real consumption, \( C_t^N \) represents nominal consumption, \( R_{j,t-j} \) is the nominal yield on a \( j \)-period bond bought at time \( t - j \), \( E_t \) is the expectation operator conditioned on information \( \mathbf{F}_t \), \( Y_t^N \) is the nominal endowment\(^1\) and \( \delta \) is the consumer’s constant time discount factor. Form the Lagrangian \( L \):

\[
L = E_0 \sum_{t=0}^\infty \delta^t \left\{ U(C_t) + \lambda_t (Y_t P_t + \sum_{j=1}^k B_{j,t-j} (1 + R_{j,t-j}) - C_t P_t - \sum_{j=1}^k B_{j,t}) \right\} \quad (A.2)
\]

The first-order conditions are:

\(^1\) The conditional expectation \( E_t[Y_{t+j}^N] \) is assumed to exist for all \( j \geq 1 \).
\[ C_t : \quad U'(C_t) - P_t \lambda_t = 0 \]

\[ B_{j,t} : \quad -\lambda_t + \delta^j E_t \lambda_{t+j}(1 + R_{j,t}^N) = 0 \quad j = 1, \ldots, k. \]

Solve the first condition for \( \lambda_t \) and substitute this into the second:

\[
\frac{U'(C_t)}{P_t} = \delta^j E_t \frac{U'(C_{t+j})}{P_{t+j}}(1 + R_{j,t}^N)
\]

\[
1 = \delta^j E_t \frac{U'(C_{t+j})}{U'(C_t)} \frac{P_t}{P_{t+j}}(1 + R_{j,t}^N)
\]

Substitute the real interest rate, \( R_{j,t} \) times the inflation rate on the RHS and the Euler equation (2.2) of chapter 2 obtains:

\[
E \left[ \delta^j \frac{U'(C_{t+j})}{U'(C_t)}(1 + R_{j,t}) - 1 \bigg| F_t \right] = 0, \quad \text{for} \quad j = 1, \ldots, k, \quad (A.3)
\]

The consumer’s problem does not restrict the agent from selling the bonds before maturity. \( B_{j,t} \) can be positive or negative. Other assets can also enter into the problem. They will not affect the basic first-order conditions. Note that to keep the consumer’s problem well behaved, the arbitrary rolling over of debt is ruled out. Essentially, a limitation on the terminal value of the debt is imposed.