Skewness and Leverage

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I. Introduction

Researchers in financial markets have been greatly interested in distributional properties of equity returns and determinants of these properties. An important component of this research is the dynamics of volatilities and betas. Temporal variations in both volatilities and betas are widely accepted in the empirical asset pricing literature. However, our understanding of why and how volatilities and betas evolve over time is still in its infancy. The third moment, skewness, also appears to change over time as shown in Harvey and Siddique (1999). Leverage, resulting from the firm using debt financing, is considered a determinant of the properties of equity returns, see, for example, Schwert (1989) and Bhandari (1988). Leverage also affects the higher moments of equity returns and ignoring leverage’s role in determining the higher moments can result in erroneous conclusion regarding its impact on the lower moments.

In this article, our objective is to understand how leverage affects skewness of equity returns and then aggregate the impact of leverage and skewness across individual equities. We first examine how skewness arises in individual equity returns from option-like features that equities possess because of limited liability. Merton (1974), Smith (1979) and Leland (1994) adapt the Black-Scholes option pricing methodology to value the capital structure of a leveraged firm. In this model, the value of the assets of the firm is viewed as the state variable. Issuance of debt can be viewed as giving the stockholders the option to purchase the assets of the firm at an exercise price equal to the face value of the debt. If equity were a call option, stock returns should be positively skewed even if the underlying returns on the firm value have a symmetric distribution.

Individual equities do in fact have in general have positive skewness at the monthly level. However, the returns on market indices uniformly have negative skewness. This implies that aggregation of skewness across equities is quite complex. So we posit models that can explain this incongruity. Next, we attempt to understand how our explanations of the individual equity/index incongruity bear upon systematic skewness in equity returns. Finally, we examine what happens to measurements of beta when systematic skewness and its relation to leverage is ignored.
Harvey and Siddique (2000) examine what determines coskewness at the individual firm level. Leverage is one of the variables they examine. However, they do not examine how leverage aggregates across individual equities to produce the properties seen at the portfolio and index levels. Relation between leverage and skewness at the individual equity level can differ dramatically from that at the index level because of coskewness.

II. Model of the firm

Merton (1974) proposed the original valuation model of the equity of a firm as an equity call option. This model assumes that changes in firm value follows a geometric brownian motion:

\[
\frac{dV}{V} = \mu dt + \sigma dZ.
\]

where \( \mu \) is the instantaneous standard deviation and \( \sigma \) is the instantaneous standard deviation of the value process. Assuming no bankruptcy costs or tax-advantages of debt-financing, Merton (1974) (with modifications for example in Smith (1979)) the equity of the firm is valued as the following:

\[
E(V) = VN \left( \frac{\ln \left( \frac{V}{D} \right) + (r + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} \right) - e^{-\sigma^2 T} D N \left( \frac{\ln \left( \frac{V}{D} \right) + (r - \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} \right)
\]

In this model the firm has one zero-coupon bond of face-value \( D \) and maturity \( T \), \( r \) is the riskfree interest rate and bankruptcy is determined exogenously in that bankruptcy occurs if \( V \) falls below \( D \) at maturity. Diba, Guo, and Schwartz (1995) and Eberhart (2000) have used this model to value equities. Perspectives from this option-pricing model has been widely used in the corporate finance literature to understand managerial or shareholder decisions.

With the assumption of a geometric brownian motion for the firm value change, the distribution of logarithmic returns on the firm value is symmetric. However, if equity is determined by (2), the equity return (stock return) does not have a symmetric distribution because of the call option feature. As leverage increases (defined using either the face value or the market value of debt), value of equity falls. With falling equity value, expected stock return increases. In effect, increasing leverage in the presence of costless bankruptcy transfers wealth from bondholders to shareholders by increasing the moneyness of the option. Thus, increasing leverage in such a model of equity value, increases the skewness of the return
distributions. Increases in the volatility of the underlying assets has a similar effect on the skewness. The Merton model is used in Charoenrook (2000) to explain the properties of equity returns. Charoenrook (2000) finds that expected returns, volatility and betas are increasing functions of leverage.

However, assumptions that the firm has zero coupon bonds with no protection and zero bankruptcy costs are untenable. Bondholders use mechanisms such as issuing convertible bonds or including protective covenants to reduce the likelihood of the wealth-transfer problem. Additionally, the Merton (1974) model also ignores the tax-shield benefits of debt financing on the firm. Therefore, observed equity returns are unlikely to behave as the Merton (1974) model would predict.

Leland (1994) substantially modifies the Merton model to incorporate bankruptcy costs, tax advantages of debt-financing as well as endogenous determination of bankruptcy. The disadvantages are that such a model no longer has simple analytic solutions. Leland (1994) does analyze the statics of his model numerically. The underlying intuition of the Leland (1994) model is that there are positive bankruptcy costs that may be offset by the tax-shield benefits of debt-financing. In this setting, distribution of equity returns is no longer guaranteed to become more skewed as leverage increases.

To analyze how distribution of equity returns changes as leverage changes in this environment, we adapt the Leland (1994) model of valuation with proportional and time-independent bankruptcy costs and value of the tax-shield. We assume that bankruptcy is triggered if the total value of the firm’s assets falls below the initial market value of the debt. This may be viewed as a protective covenant rather than an endogenous determination of bankruptcy.

In this framework, the value of the equity is given by

$$ E(V) = V + TB(V) - BC(V) - D(V) $$

(3)

where, $TB(V) = \frac{\tau C}{r} - \frac{\tau C}{r} \left( \frac{V}{D_0(V_0)} \right)^{\frac{2\alpha}{\sigma^2}}$

(4)

$BC(V) = \alpha D_0(V_0) \left( \frac{V}{D_0(V_0)} \right)^{\frac{2\alpha}{\sigma^2}}$

(5)

$D(V) = \frac{C}{r} + \left[ (1 - \alpha) D_0(V_0) - \frac{C}{r} \right] \left( \frac{V}{D_0(V_0)} \right)^{\frac{2\alpha}{\sigma^2}}$

(6)
\[ D_0(V_0) = \frac{C}{r} + \left[(1 - \alpha)D_0(V_0) - \frac{C}{r}\right]\left(\frac{V_0}{D_0(V_0)}\right)^{-\frac{\tau}{r}} \]  

(7)

In this model, \( V \) is the total asset value of the firm that is governed by the law of motion in (1). \( TB(V) \) is the value of the tax-shield that is generated from the deductibility of interest payments. \( BC(V) \) is the market value of the claim to the bankruptcy costs should it occur. \( D(V) \) is the value of debt at a given time. Bankruptcy occurs if value of the assets falls below \( D_0(V_0) \). This may be viewed as a protective covenant and bankruptcy, thereby, occurs exogenously. The parameters of the model are: \( \alpha \), the proportional loss in bankruptcy, \( \tau \), the tax rate, and, \( C \), the coupon per instant of time that the firm pays as long it is solvent. \( D_0(V_0) \) is the initial market value of the debt that needs to be solved numerically in (7).

### III. Changes in leverage and equity returns

The Leland (1994) model as presented in (3)-(7) suggests that changes in leverage will impact the value of equity and thereby affect equity returns.

We first carry out simulations that illustrate how skewness of the equity-as-call-option returns varies as leverage is changed. We assume that the initial value of the assets of the firm is 100, use a \( \tau \) of 30\%, \( \alpha \) of .04, and assume that there are 700 months of observations per firm. For the coupon, \( C \), we use 0.20.

Our estimate of \( \sigma^2 \) is taken from the equity returns on the Dow 30 firms and we examine various \( \sigma^2 \) values. For \( r \) we use both a fixed rate as well as \( r \) that increases with leverage (defined as \( \frac{D}{V} \)).

In general, with increasing leverage equity returns become more skewed if the interest rate \( r \) is assumed to be constant. However, at very high leverage levels and with a high underlying asset volatility, \( \sigma^2 \), skewness can actually decrease. This can be understood as bankruptcy resulting a sample of equity returns where the negative 100bankruptcy dominates the higher positive returns. The decreasing skewness is even more pronounced when the interest rate increases with leverage.
IV. Portfolios and Skewness

One of the puzzling empirical observation regarding skewness is that even though individual equities have positive skewness, index returns are negatively skewed. This indicates that as a portfolio is formed, diversification along with reduction in volatility, also decreases skewness and can actually make it negative.

We first form a portfolio using the model of individual equity returns in the previous section, with zero correlation across stocks. We select ten stocks out of the Dow 30 and use their means and variances to generate the individual equity returns. This portfolio has positive skewness at all levels of leverage since even though some individual equities do have negative skewness. We then generate a portfolio using the observed historical correlations across the ten stocks. This portfolio still has positive skewness.

This indicates the existence of a common component in individual equities that has a negative skewness. In other words, the positive returns of the individual equities diversify away but diversification is less applicable for the negative outcomes. This is consistent with the findings in Solnik, Boucrelle, and LeFur (1996) with greater correlation in times of higher volatility (which are also times of more negative returns).

Therefore, we carry out simulations where we assume that correlation differs across negative and positive individual equity returns. As correlation for the negative returns increases, the portfolio returns themselves more negatively skewed even though the individual stock returns still remain positively skewed.

V. Coskewness of the stocks

Harvey and Siddique (2000) suggests that there is a substantial common component in skewness across equity returns that can explain cross-sectional variation in those returns. Our simulations suggest that the common component is much greater for the negative returns than for the positive returns.
VI. Implications for Beta

The implications of changing leverage for beta and systematic coskewness are rather different. Increase of leverage should increase the beta and thereby increase the expected return. Thus, a more levered firm should have higher price than a less levered firm. However, the systematic component of skewness will also increase and since the risk premium for systematic skewness is negative, a stock with greater skewness should have lower expected return. Thus, the impact of leverage on the expected return of a firm can have quite different inferences than including beta alone.

VII. Skewness and Coskewness for a set of US Securities

VIII. Conclusions
References


Table I
Estimation Results