

# Consistent Return Estimates - The Black-Litterman Approach

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COMINVEST

PM High Alpha / Portfolio Construction

- April 2005 -

## Abstract

# The Black-Litterman Approach

Consistent asset return estimates - saving classical mean/variance...

In asset management, the forecast of asset returns is essential within the investment process. In this context, the Black-Litterman approach (1992) yields consistent asset return forecasts as a weighted combination of (strategic) market equilibrium returns and (tactical) subjective forecasts ("views"). The Black-Litterman formalism allows to implement both absolute views (return levels) and relative views (outperforming vs. underperforming assets) for selected assets investigated under „core competence“. For any particular view, individual confidence levels for the return estimates have to be specified. The formalism spreads these informations consistently across all assets in the portfolio. The BL-revised returns then serve as a consistent input for mean-variance portfolio optimisation procedures, thus allowing for the implementation of additional constraints. It turns out that BL-optimized portfolios overcome some well-known Markowitz insufficiencies as unrealistic sensitivity to input factors or extreme portfolio weights. The BL process will be introduced both from its theoretical background and its implementation in practice.

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Frankfurt, March 2005

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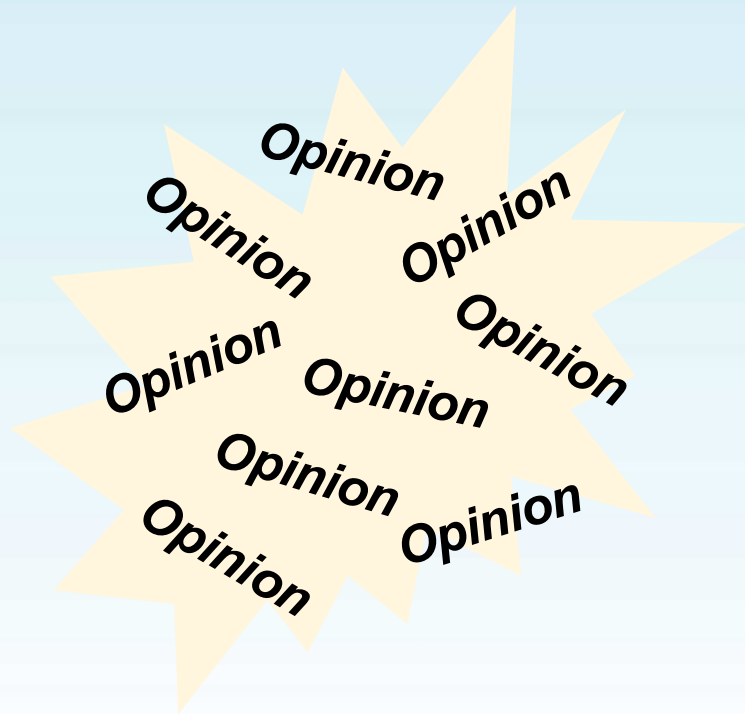
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Data source: DataStream

# Investment Process: Opinions & Portfolio Context

## RESEARCH

- *The c.p. world of core competences -*

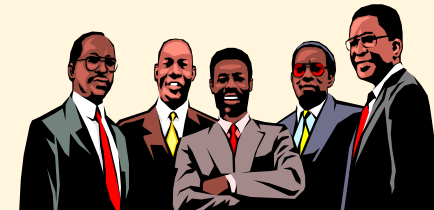


## PORTFOLIO CONSTRUCTION

- *The portfolio context: Thinking in terms of correlations -*

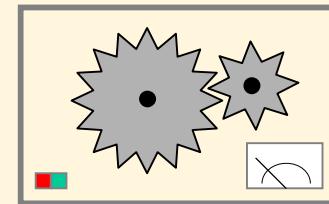


Committee



*or*

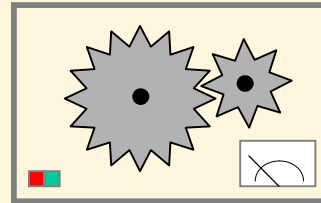
Quant tool



c.p. = ceteribus paribus

# Defining the subjectives... $\Rightarrow$ Black & Litterman

Let's talk about the ... Quant tool



Some  
nice-  
to-  
haves

*"Consistent" (non-c.p.) input  
for portfolio construction*

*Transparent*

*Managable*

*Intuitive results*

*Tactical deviations from  
"some" strategic allocation*

*Overcome some of the  
problems of plain MV  
(Markowitz)*

*Reliable output*

*Weighting estimates  
according to confidence*

***Black-Litterman***

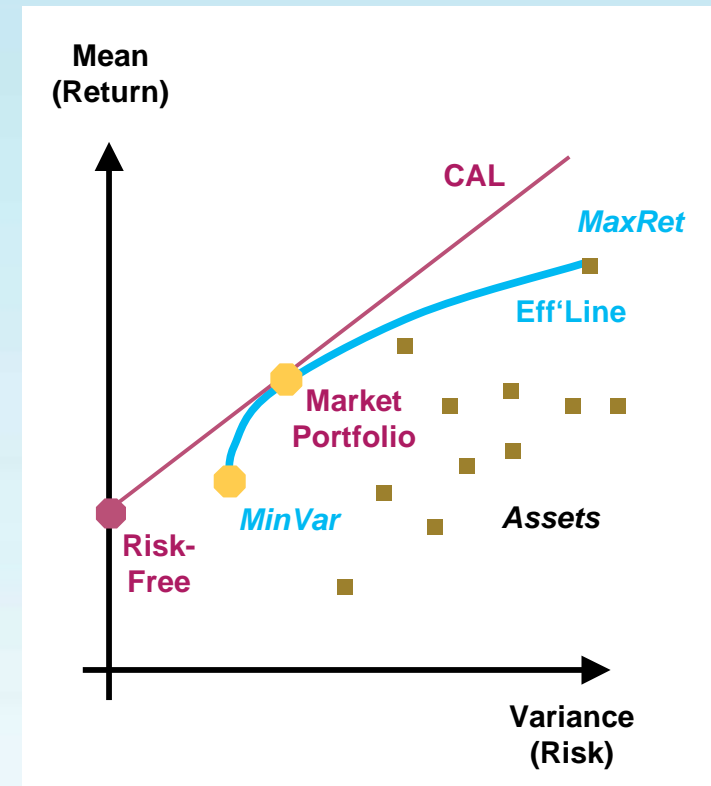
# Topics to be discussed...

- Classical Markowitz - the straight way
- MV-optimized portfolios
- BL - the role within the investment process
- BL - implementation
- BL - example

# The Markowitz Approach - in short

## Efficient portfolios in the mean-variance approach

- **Starting point** in a world of normally distributed returns: The assets are described by the first two moments of return - mean and variance.
- In an **efficient portfolio** the assets are weighted such that for any given level of risk a maximum return is achieved. (equivalently: for a given return the risk is minimized). Diversification reduces risk.
- All efficient portfolios form the **efficiency line**. It starts in the minimum variance portfolio and ends in the maximum return portfolio (which is the asset of maximum return).
- If a risk-free asset exists, all efficient portfolios are located on the **Capital Allocation Line** (CAL), starting at the risk-free asset and tangentially touching the efficiency line at the market portfolio. Efficient portfolios are then a combination of the risky **market portfolio** and the **risk-free asset** (with risk-free *long* or *short*), a.k.a. *Tobin Separation (1958)*.



# The Markowitz Approach - dealing with its problems

Deficits of the mean-variance (MV) concept, suggestions for solutions...

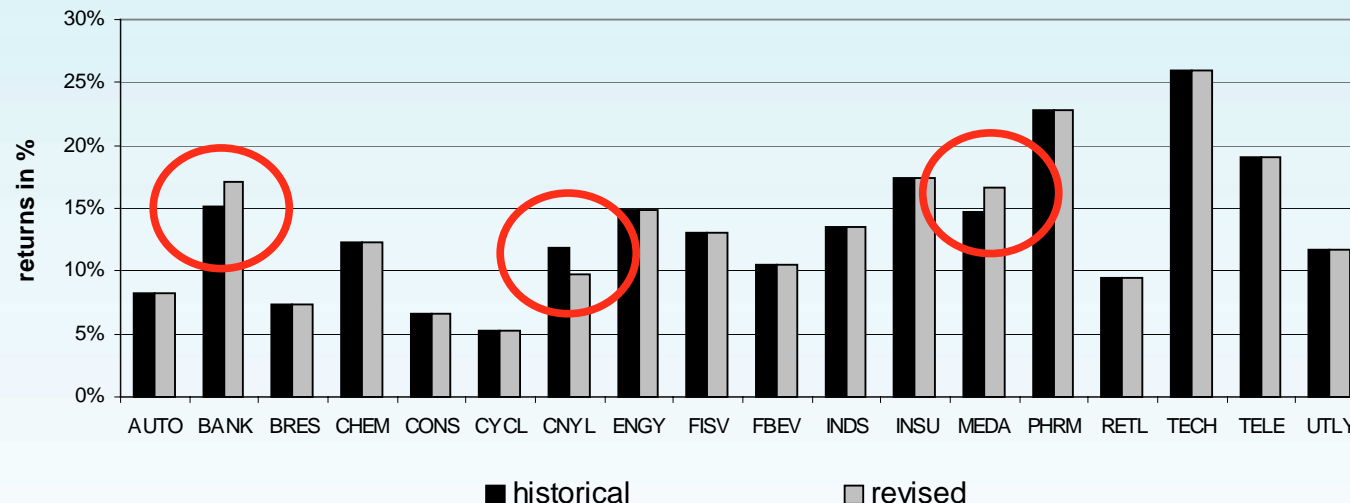
Deficit	Possible Solution
■ <u>High sensitivity on inputs</u> (return estimates!) leads to large weight fluctuations in the optimal portfolio.	<i>Black-Litterman</i>
■ „Corner solutions“ : <u>Extreme portfolio weights</u> (also in the case of optimization algorithms using constraints)	<i>Black-Litterman</i>
■ <u>Aggregation</u> : Consistent aggregation of huge number of estimated returns overburdens the investment process	<i>Black-Litterman</i>
■ No quantification of <u>confidence</u> in estimated returns	<i>Black-Litterman</i>
■ One-periodical approach	Multi-period approaches, ...
■ „Variance“ = restricting risk to symmetric return volatility	VaR, ...
■ Requires ex-ante-estimates of covariance matrix	Vola-modeling, ...

# The Markowitz Approach - straight is not enough

When return estimates change...

- Let the investment universe be the 18 STOXX sectors in Euroland.
- At time 0:** Returns : Historical returns  
Weights : To be determined via MV
- At time 1:** Returns : +2%pts for BANK and MEDA, -2%pts for CNYL, others unchanged  
Weights : To be determined via MV

Historical returns and revised expected returns



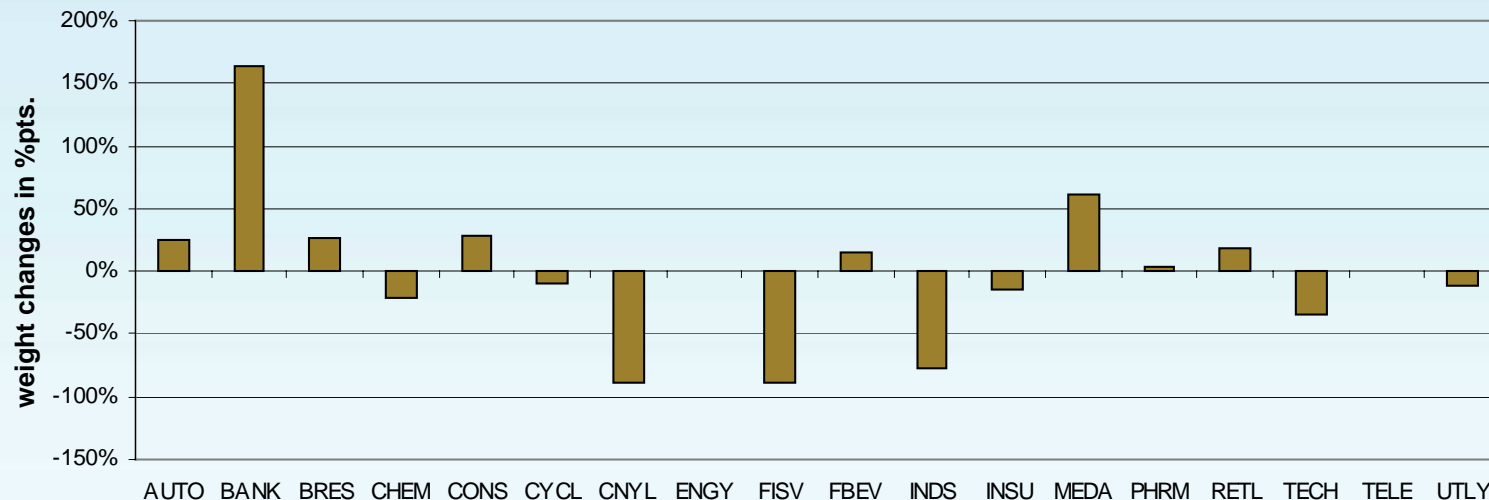


# The Markowitz Approach - straight is not enough

Fluctuation of asset weights due to revised return estimates

- Even small and selected changes in expected returns lead to huge unrealistic shifts in asset weights! ( $\gamma=3$ , historical covariances)

Changes in portfolio weights due to changes of expected returns



- Problems: Communication of results, (re-) allocation in real portfolios, acceptance of method.

# The Markowitz Approach - straight

The formal optimization approach for risky assets, basic outline.

- Markowitz Theory relates risk & return
- MV optimization problem:

$$w^T R - \frac{\gamma}{2} \cdot w^T \Omega w \rightarrow \max_w$$

$R$  = vector of returns  
 $\Omega$  = covariance matrix  
 $\gamma$  = risk aversion parameter  
 $w$  = vector of weights

- Solution: Optimal portfolio weights  $w^*$  (no constraints):

$$w^* = (\gamma \Omega)^{-1} R$$

- Markowitz provides a mechanism to achieve optimal (efficient) portfolios.  
What about the input factors ???

# Extending the Markowitz Approach

## Equilibrium returns

### Supply & demand

- Traditional approach of maximum return & minimum risk is demand-side perspective.
- Need to balance with supply-side...

### Concept of equilibrium returns:

- The market portfolio exists in market equilibrium, i.e. supply & demand are in equilibrium.
- Therefore, equilibrium returns reflect neutral „fair“ reference returns  $\Pi$  :
- Inverse optimization yields:

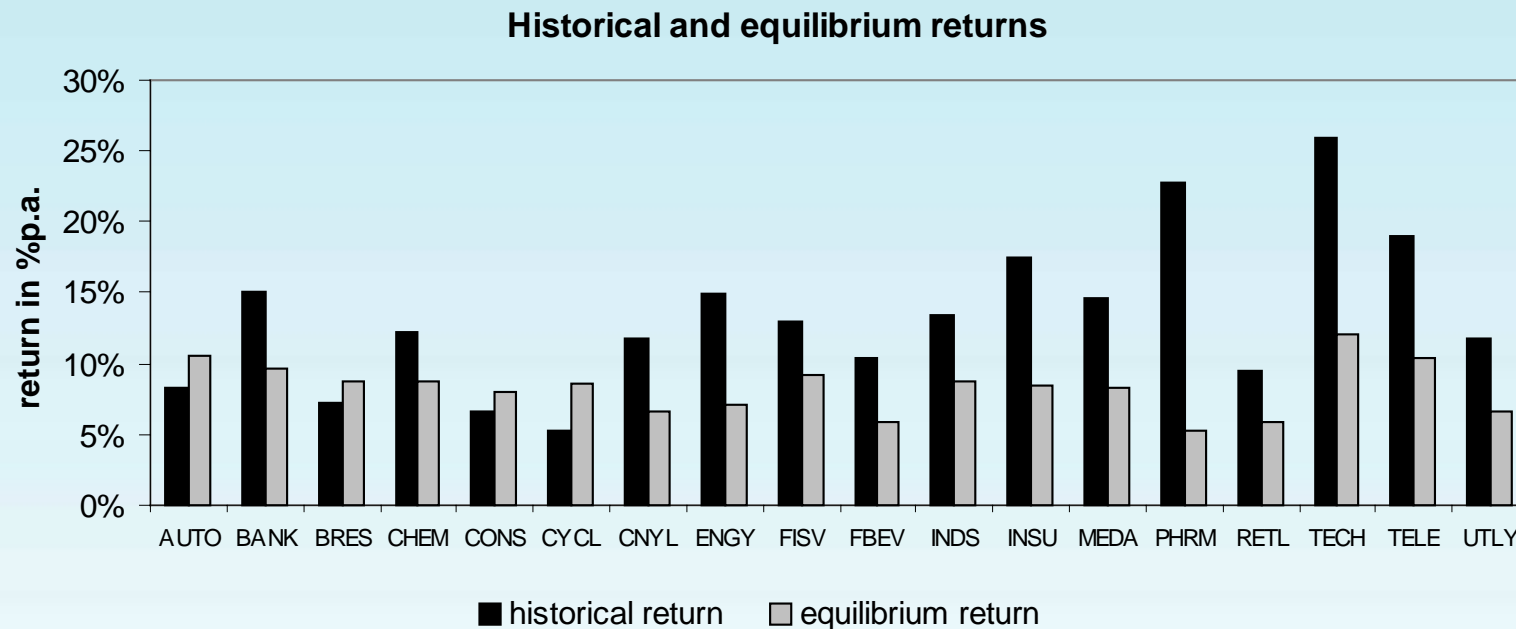
$$\Pi = (\gamma \Omega) w_{MCap} \quad w_{MCap} = \text{market capitalization}$$

### Conclusion

- Use of equilibrium returns as a long term strategic reference for any return estimate („market neutral starting point“).

# Extending the Markowitz Approach

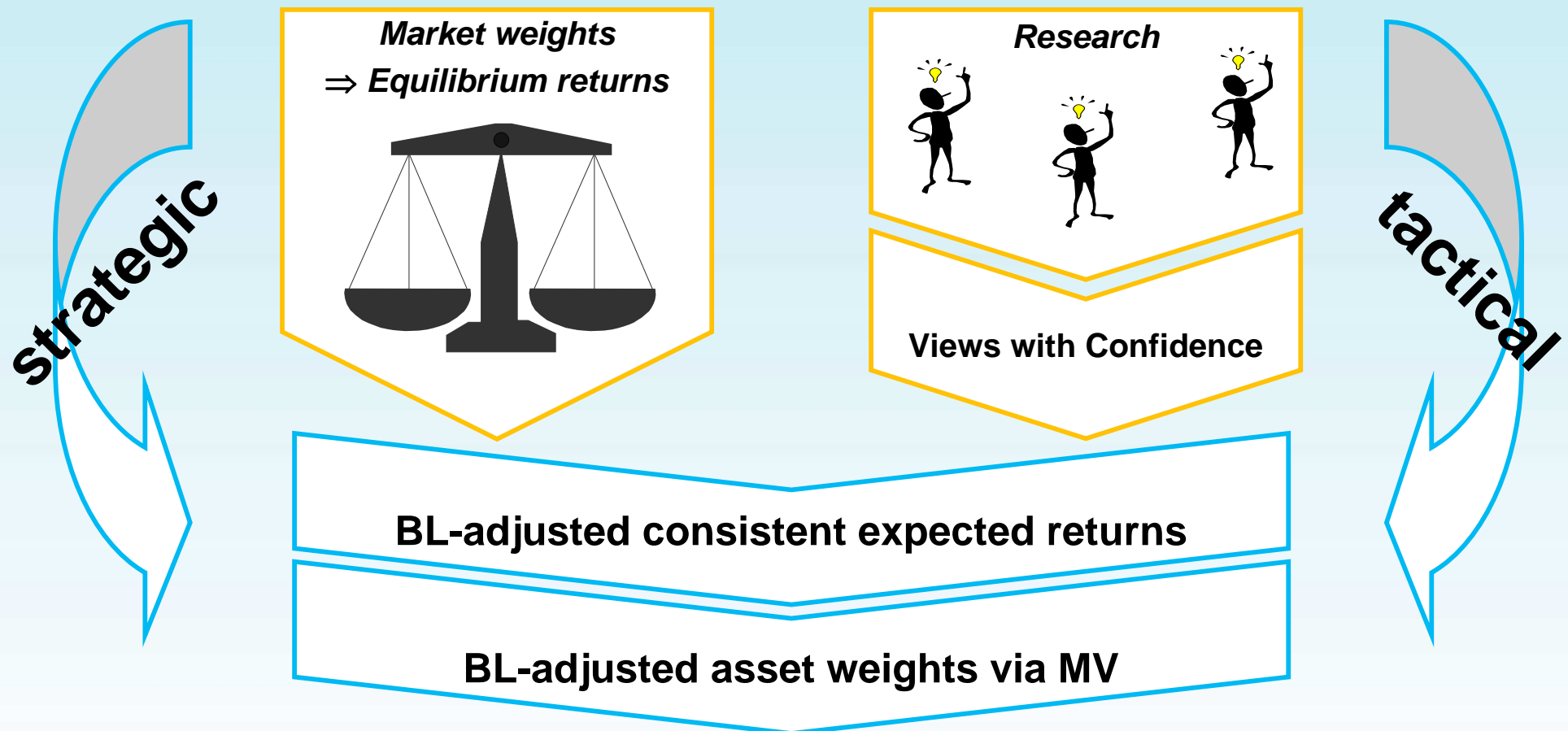
Equilibrium (or implicit) returns for the STOXX sectors



- These implicit returns serve as reference returns for all further investigations
- Note that equilibrium returns are *calculated*; they do not require any estimation procedure.

# Black-Litterman Approach - basic outline

BL provides a concept to combine long term equilibrium returns with short term return estimates as a consistent input for MV-optimization.



# Black-Litterman Approach - going math

Optimization s.t. constraints

- Determine the optimal estimate  $E(R)$ , which minimizes the variance of  $E(R)$  w.r.t. equilibrium returns  $\Pi$ : (minimizing the Mahalanobis distance):

$$\left[ E(R) - \Pi \right]^T \cdot (\tau \Omega)^{-1} \cdot \left[ E(R) - \Pi \right] \rightarrow \min_{E(R)}$$

where:  $E(R) = \Pi + v$  with  $v \sim N(0, \tau \Omega)$ .

s.t.

$$P \cdot E(R) = \begin{cases} V & \text{certain Views} \\ V + e & \text{uncertain Views} \end{cases}$$

where:  $P \cdot E(R) \sim N(V, \Sigma)$ ,  $\Sigma_{ii} = e_i$

# Black-Litterman Approach - the formulas

Master equations for the BL-return estimates

- Solution in the case of **certain estimates** ( $\Sigma \equiv$  zero matrix):

$$\bar{E}(R) = \Pi + (\tau \Omega) P^T \cdot \left( P(\tau \Omega) P^T \right)^{-1} \cdot (V - P \Pi)$$

- Solution in the case of **uncertain estimates** ( $\Sigma =$  diagonal matrix):

$$\bar{E}(R) = \left[ (\tau \Omega)^{-1} + P^T \Sigma^{-1} P \right]^{-1} \cdot \left[ (\tau \Omega)^{-1} \Pi + P^T \Sigma^{-1} V \right]$$

- The **constraints**  $P \cdot E(R) = V + e$  are implicitly fulfilled.

# Black-Litterman Approach - more math I

Formal proof for the case „certain estimates“

**Proposition:** The optimization problem  $[E(R) - \Pi]^T \cdot (\tau \Omega)^{-1} \cdot [E(R) - \Pi] \rightarrow \min_{E(R)}$  s.t.  $P \cdot E(R) = V$  yields variance-minimum returns  $\bar{E}(R) = \Pi + (\tau \Omega) P^T \cdot (P(\tau \Omega) P^T)^{-1} \cdot (V - P \Pi)$

**Proof:**

Lagrangian:  $L := [E - \Pi]^T \cdot (\tau \Omega)^{-1} \cdot [E - \Pi] - \lambda \cdot (PE - V)$

f.o.c.'s: (1)  $\frac{\partial L}{\partial E} = 0$  and (2)  $\frac{\partial L}{\partial \lambda} = 0$

"scalarizing":  $\frac{\partial L}{\partial E_i} = \frac{\partial}{\partial E_i} \left\{ \tau^{-1} \sum_{j,k} E_j \Omega_{jk}^{-1} E_k - \sum_k \lambda_k \left( \sum_j P_{kj} E_j - V_k \right) \right\} = 2\tau^{-1} \sum_k \Omega_{ik}^{-1} E_k - \sum_k P_{ki} \lambda_k$

$$\frac{\partial L}{\partial \lambda_i} = -\frac{\partial}{\partial \lambda_i} \sum_k \lambda_k \left( \sum_j P_{kj} E_j - V_k \right) = -\left( \sum_j P_{ij} E_j - V_i \right)$$

"revectorizing": (1)  $\frac{\partial L}{\partial E} = 2(\tau \Omega)^{-1} E - 2(\tau \Omega)^{-1} \Pi - P\lambda = 0$  and (2)  $\frac{\partial L}{\partial \lambda} = PE - V = 0$

Solve Eq. (1) for  $E$ , insert in Eq.(2), thereof expression for  $\lambda$ , result follows with Eq. (1).  $\diamond$



# Black-Litterman Approach - more math II

Formal proof for the case „uncertain estimates“ (BL-Master Formula)

**Proposition:**  $[E(R) - \Pi]^T \cdot (\tau \Omega)^{-1} \cdot [E(R) - \Pi] \rightarrow \min_{E(R)}$  with constraints  $P \cdot E(R) = V + e$

yields variance-minimum returns  $\bar{E}(R) = [(\tau \Omega)^{-1} + P^T \Sigma^{-1} P]^{-1} \cdot [(\tau \Omega)^{-1} \Pi + P^T \Sigma^{-1} V]$

**Proof:**

Given:  $\Pi = E(R) + v$  and  $V = P \cdot E(R) + e$

Setting  $Y := \begin{pmatrix} \Pi \\ V \end{pmatrix}$ ,  $X := \begin{pmatrix} I \\ P^T \end{pmatrix}$ ,  $W := \begin{pmatrix} \tau \Omega & 0 \\ 0 & \Sigma \end{pmatrix}$  and  $u \sim N(0, W)$

so that  $Y = X \cdot E(R) + u$  and using generalized least square  $E(R) = (X^T W^{-1} X)^{-1} X^T W^{-1} Y$  we get

$$\begin{aligned} E(R) &= \left[ \begin{pmatrix} I & P^T \end{pmatrix} \begin{pmatrix} \tau \Omega & 0 \\ 0 & \Sigma \end{pmatrix}^{-1} \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} \times \left[ \begin{pmatrix} I & P^T \end{pmatrix} \begin{pmatrix} \tau \Omega & 0 \\ 0 & \Sigma \end{pmatrix}^{-1} \begin{pmatrix} \Pi \\ V \end{pmatrix} \right] \\ &= \left[ \begin{pmatrix} (\tau \Omega)^{-1} & P^T \Sigma^{-1} \end{pmatrix} \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} \times \left[ \begin{pmatrix} (\tau \Omega)^{-1} & P^T \Sigma^{-1} \end{pmatrix} \begin{pmatrix} \Pi \\ V \end{pmatrix} \right] = [(\tau \Omega)^{-1} + P^T \Sigma^{-1} P]^{-1} \times [(\tau \Omega)^{-1} \Pi + P^T \Sigma^{-1} V] \diamond \end{aligned}$$

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Non-Bayesian proof, taken from „Asset Allocation Model“, Daniel Blamont, Global Markets Research, Dt.Bank, July 30 2003

For Bayesian proof see, e.g., Fusai and Meucci

# Black-Littermann - the master formula

$$E(R) = \left[ (\tau \Omega)^{-1} + P^T \Sigma^{-1} P \right]^{-1} \cdot \left[ (\tau \Omega)^{-1} \cdot \Pi + P^T \Sigma^{-1} \underbrace{P \cdot P^{-1}}_{= \mathbb{1}} V \right]$$

- Complex interaction between equilibrium returns and subjective return expectations
- First factor („Denominator“) : Normalisation
- Second factor („Numerator“) : Balance between  $\Pi$  (= equilibrium returns) and  $V$  (= Views). Inverse of covariance  $(\tau \Omega)^{-1}$  and confidence  $P^T \Sigma^{-1} P$  serve as weighting factors.
- Constituents:
  - Matrix  $\tau \Omega$  : covariance of historical returns,  $\tau$  = parameter
  - Matrix  $P$  : formal aggregation of Views
  - Matrix  $\Sigma^{-1}$  : confidence in Views ( $\Sigma$  = „covariance of estimated Views“)
  - $\Sigma$  assumed to be diagonal, i.e. no cross-informations on Views.
- Limiting case “no estimates“  $\Leftrightarrow P=0$ :  $E(R) = \Pi$  i.e. BL-returns = equil. returns.
- Limiting case “no estimation errors“  $\Leftrightarrow \Sigma^{-1} \rightarrow \infty$ :  $E(R) = P^{-1}V$  i.e. BL-returns = View returns.

# Black-Litterman Approach - remarks

## Use of CAPM to determine equilibrium returns

- Alternative to inverse optimization: Equilibrium returns  $\Pi_{Eq}$  from CAPM
- Additional input for CAPM: Risk-free rate  $r_f$ , risk premium vs market (M), Beta coefficients

- Evaluation:

(fair return for asset  $i$ )

$$\Pi_{i,Eq} = r_f + \beta_i \cdot (r_M - r_f) \quad \text{with} \quad \beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

## BL returns as a Bayesian a-posteriori estimator

- Bayes Theorem (or "Law" or "Rule") states how to determine conditional expectations.
- Given an a-priori known distribution of a random variable. Adding new information leads to a revised conditional distribution, the so-called a-posteriori distribution (result of „learning“).
- BL-return estimates are a-posteriori (multivariate) normally distributed return expectations.

# Black-Litterman Approach - $\gamma$

Remark: Risk aversion parameter  $\gamma$

- How does risk change for an additional bp of return?
- **Suggestions:**

Satchell & Scowcroft and Best & Grauer:

$$\text{Let } \gamma = (r_M - r_f) / \sigma_M^2$$

where  $\sigma_M^2 = w^T \Omega w$ ,  $w$  = market cap.

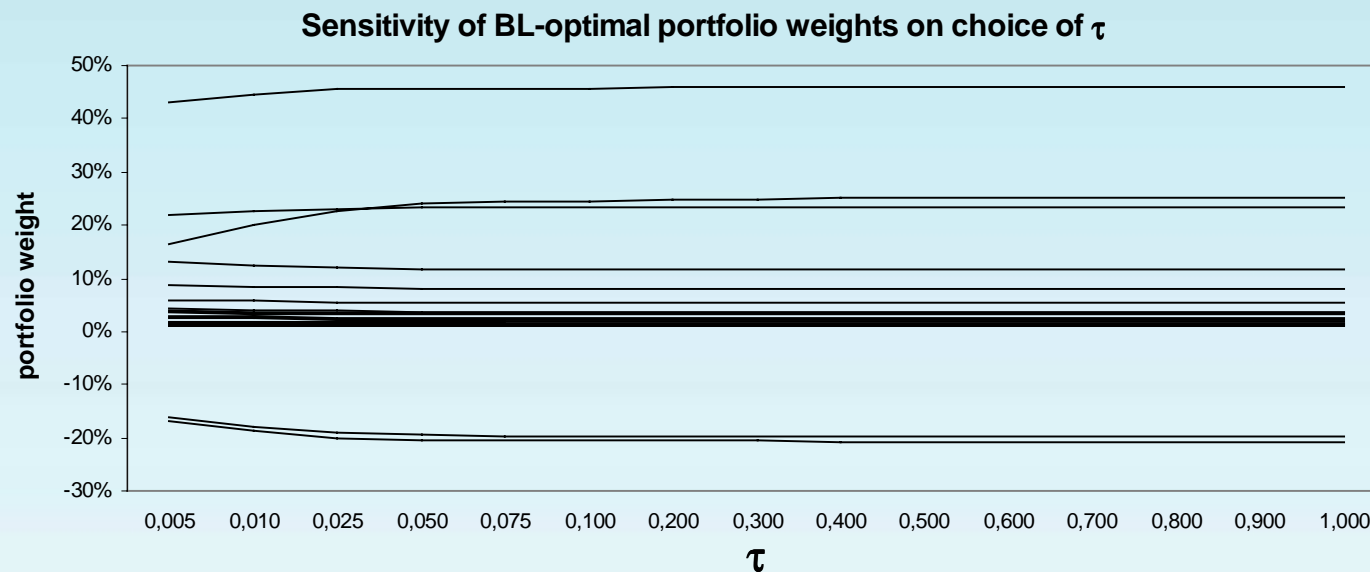
Zimmermann et al.:  $\sigma_M = 16.9\%$  p.a. for STOXX-data (own calculation)  
Chose  $\gamma = 3$ , which corresponds to a risk premium of 8.6%.

Idzorek: (DJIA, USA): Risk premium=7.5%:  $\gamma = 2.25$ .

## Parameters in BL

# Black-Litterman Approach - $\tau$

Remark: Scaling parameter  $\tau$  for the covariance matrix



## Results/Setting

- Covariances of expected returns are proportional to historical covariances:  $\tau \Omega$ .
- $\tau$  measures confidence in benchmark, i.e. overall balance between BM and Views.
- $\tau$  small:  $VAR[E(R)] \ll VAR[historical\ returns]$
- $\tau = 0.3$  "plausible" (used for numerical evaluations throughout).

# Black-Litterman Approach - some real problems

Additional remarks on the recent remarks

- Calibration problems with parameter  $\tau$  (“plausible”, “adjusted to IR=1”, ...)
- Calibration problems with parameter  $\gamma$  (“world wide risk aversion”, ...)
- Calibration problems with expressing the degree of confidence (“1..3”, “0..100%”)

# Black-Litterman Approach - Views

Implementing Views on expected returns deviating from equilibrium figures

## Views

- Return estimates differing from the (strategic) equilibrium returns are the essential input to the BL estimation process.

## Specification of Views

- ... as absolute return expectations for individual assets  
*and / or*
- ... as relative return expectations relating assets or aggregates of assets.  
Formal constraint:  $\#Views \leq \#Assets$ .

## Confidence

- Each View has to be assigned the level of confidence for an interval of uncertainty.

## Selective Views

- Views can be restricted to selected assets for which in-depth analysis is available.

# Black-Litterman Approach - Views

- A **relative View** can be stated as follows: „The sectors Pharmacy and Industry will outperform Telecom and Technology by 3% ± 1% with a confidence of 90%“:

$$\begin{aligned} & \left[ w_{PHRM} \cdot E(R_{PHRM}) + w_{INDU} \cdot E(R_{INDU}) \right] \\ & - \left[ w_{TELE} \cdot E(R_{TELE}) + w_{TECH} \cdot E(R_{TECH}) \right] = 3\% + (0.61\%)^2 \end{aligned}$$

- Basically: A *long*-portfolio with outperformers, a *short*-portfolio with underperformers.

- An **absolute View** can be stated as follows: „The sector of Non-Cyclical Goods (CNYL) will perform better than stated by the equilibrium return of 6.66%. Our new target return is 7.5% with 90% of confidence within a range of ±1.5%“:

$$1 \cdot E(R_{CNYL}) = 7.5\% + (0.91\%)^2$$



# Black-Litterman Approach - combining Views I

## Formal aggregation of Views

- Relative and absolute Views are aggregated in a system of linear equations:

$$P \cdot E(R) = V + e$$

where ( $k = \#Views$  and  $n = \#Assets$ , with  $k \leq n$ ):

$E(R)$  =  $n \times 1$  vector of expected asset returns, unknown

$P$  =  $k \times n$  matrix, weighting the assets

$V$  =  $k \times 1$  vector, absolute / relative return expectations (i.e., levels or over-/underperforming)

$e$  =  $k \times 1$  vector of squared StDev's

(note that  $\Sigma^{-1}$  is a  $k \times k$  diagonal matrix expressing confidence (assuming independent estimation errors) with  $\Sigma_{ii}^{-1} = e_i$ )

- This relation is incorporated in the BL master equation.

BL-Views

# Black-Litterman Approach - combining Views II

Result shown from the example mentioned on page 24.

- Combining the aforementioned Views using  $P \cdot E = V + e$ , we get:

$$P \cdot \begin{pmatrix} E(R_{AUTO}) \\ \vdots \\ E(R_{UTLY}) \end{pmatrix} = \begin{pmatrix} 3\% \\ \vdots \\ 7.5\% \end{pmatrix} + \begin{pmatrix} (0.61\%)^2 \\ \vdots \\ (0.91\%)^2 \end{pmatrix}$$

- The View-related weights  $P$  are given by:

$$P = \begin{pmatrix} \text{View 1, rel.} \\ \text{View 2, abs.} \end{pmatrix} = \begin{pmatrix} 0 & \dots & 0 & \dots & 0 & 0.34 & 0 & 0 & 0.66 & 0 & -0.51 & -0.49 & 0 \\ 0 & \dots & 1 & 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \end{pmatrix}$$

AUTO

⋮

↑

asset projector

GNVL

ENGY

FISV

FBEV

INDS

INSU

MEDA

PHRM

RETL

TECH

TELE

UTLY

long positions

↓

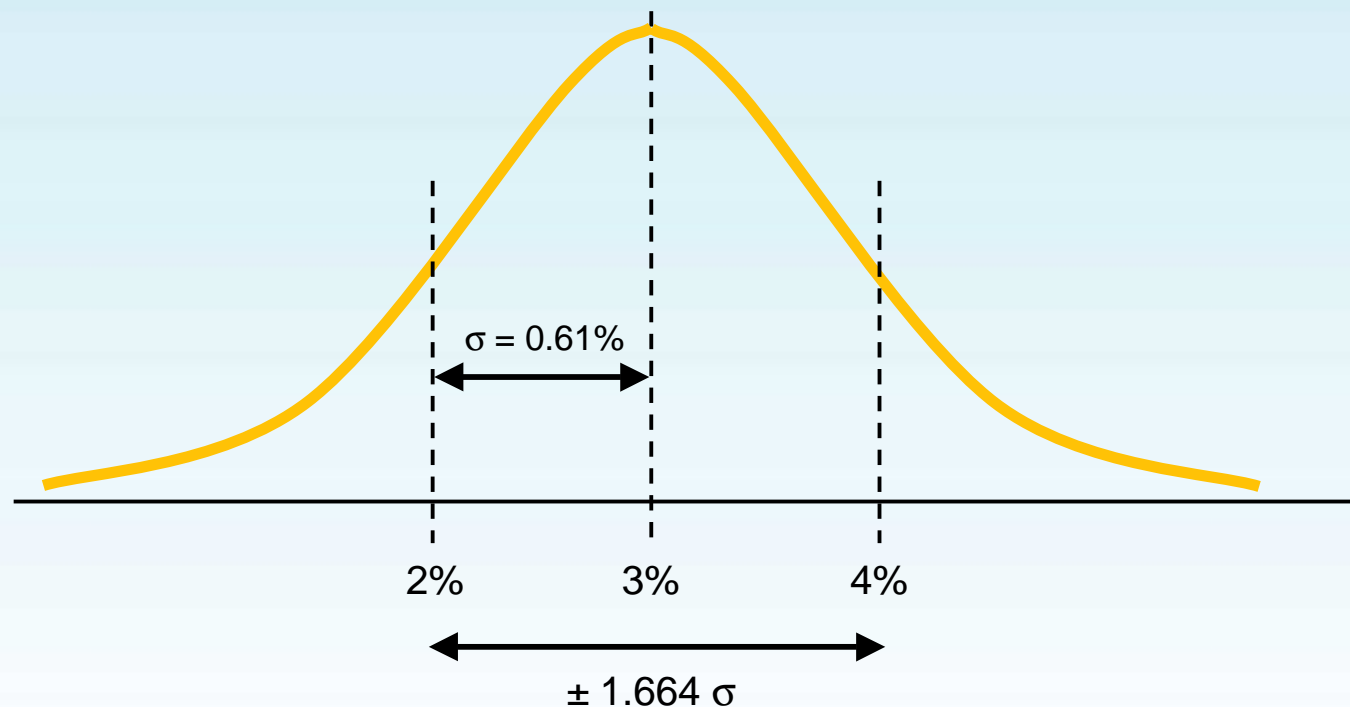
short positions

↓

# Black-Litterman Approach - confidence

Technical note on „confidence“

- Comment on determination of  $e$ : The fact that the amount of, e.g., *relative* outperformance (View 1) of  $3\% \pm 1\%$  is assigned a 90% probability is interpreted within a normal distribution.
- **mean** = 3% and **variance** =  $VAR = \sigma^2 = (0.61\%)^2 = e_1 \equiv \Sigma_{11}$ .



Example: DJ STOXX

# Black-Litterman Approach - example in detail

„Sector allocation, Dow Jones STOXX“

- Example follows the lines of  
„Einsatz des Black-Litterman-Verfahrens in der Asset Allocation“, *H.Zimmermann et al.*  
publ. in „Handbuch Asset Allocation“  
(Editors: Dichtl, Schlenger u. Kleeberg, publ. by Uhlenbruch-Verlag, 2002).
- Notation, scenarios and data therein have been used, some data were missing.
- Missing data - volatilities and covariances - had to be calculated from scratch, thus causing some deviations in the numerical results between this presentation and cited literature.  
Nevertheless, all relevant results are reproduced.
- All calculations can be (have been) implemented and performed in Excel (TM).

Example: DJ STOXX

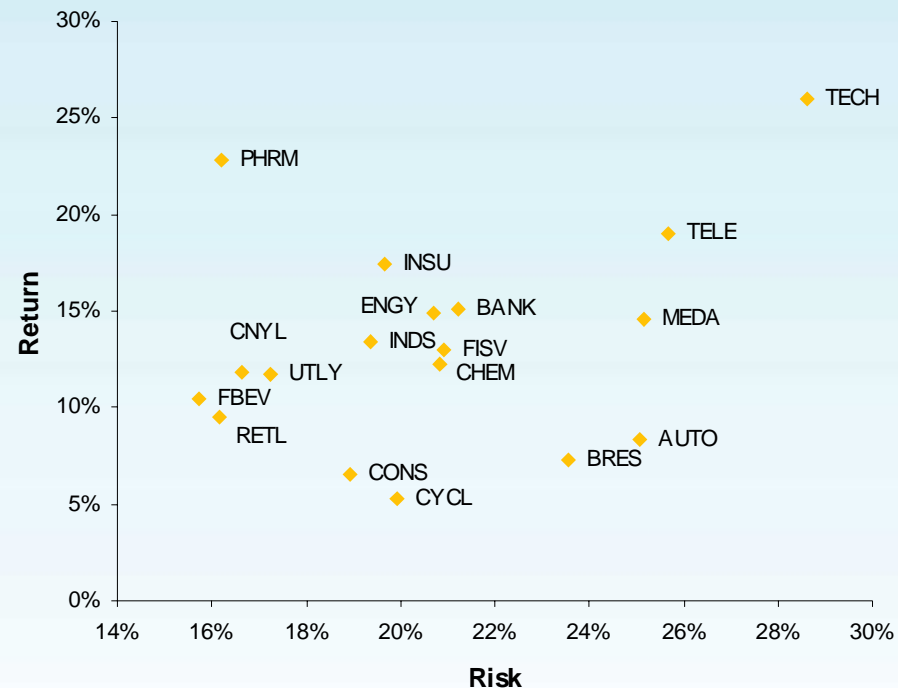
# Black-Litterman Approach - the data

Sectors in the Dow Jones STOXX index

- Monthly returns in Sfr (Swiss francs), period: 06/1993 - 11/2000, annualized data.

Sector	hist.Return	hist.Volatility	MarketCap
<b>total:</b>	<b>average:</b>		<b>total:</b>
18	16,22%		100,01%
<b>AUTO</b>	8,32%	25,09%	1,65%
<b>BANK</b>	15,14%	21,21%	15,04%
<b>BRES</b>	7,31%	23,56%	1,22%
<b>CHEM</b>	12,25%	20,81%	1,80%
<b>CONS</b>	6,56%	18,92%	1,26%
<b>CYCL</b>	5,24%	19,94%	2,85%
<b>CNYL</b>	11,80%	16,66%	2,90%
<b>ENGY</b>	14,92%	20,72%	10,30%
<b>FISV</b>	13,01%	20,91%	4,12%
<b>FBEV</b>	10,47%	15,72%	4,59%
<b>INDS</b>	13,45%	19,35%	5,19%
<b>INSU</b>	17,43%	19,68%	6,89%
<b>MEDA</b>	14,63%	25,17%	3,27%
<b>PHRM</b>	22,83%	16,20%	10,24%
<b>RETL</b>	9,49%	16,16%	2,27%
<b>TECH</b>	25,95%	28,60%	11,03%
<b>TELE</b>	18,99%	25,69%	10,56%
<b>UTLY</b>	11,77%	17,25%	4,83%

Historical Data - Risk/Return Characteristics



Example: DJ STOXX

# Black-Litterman Approach - the correlations

Correlation matrix of Dow Jones STOXX sectors

- Calculation based on monthly returns

	AUTO	BANK	BRES	CHEM	CONS	CYCL	CNYL	ENGY	FISV	FBEV	INDS	INSU	MEDA	PHRM	RETL	TECH	TELE	UTLY
AUTO	100%	74%	73%	83%	78%	75%	73%	55%	73%	71%	79%	72%	46%	43%	68%	69%	65%	64%
BANK	74%	100%	63%	73%	74%	75%	71%	59%	92%	75%	75%	87%	39%	63%	62%	67%	59%	64%
BRES	73%	63%	100%	83%	81%	78%	60%	66%	69%	56%	78%	56%	44%	31%	59%	62%	52%	41%
CHEM	83%	73%	83%	100%	85%	82%	72%	67%	72%	74%	82%	70%	51%	45%	69%	65%	57%	57%
CONS	78%	74%	81%	85%	100%	90%	72%	66%	75%	76%	89%	64%	54%	39%	67%	66%	63%	64%
CYCL	75%	75%	78%	82%	90%	100%	67%	64%	79%	70%	87%	63%	58%	43%	67%	70%	63%	56%
CNYL	73%	71%	60%	72%	72%	67%	100%	55%	69%	75%	69%	74%	41%	58%	73%	53%	59%	71%
ENGY	55%	59%	66%	67%	66%	64%	55%	100%	59%	59%	59%	54%	28%	43%	55%	43%	32%	46%
FISV	73%	92%	69%	72%	75%	79%	69%	59%	100%	75%	73%	85%	39%	61%	58%	64%	57%	58%
FBEV	71%	75%	56%	74%	76%	70%	75%	59%	75%	100%	62%	74%	27%	63%	61%	40%	41%	66%
INDS	79%	75%	78%	82%	89%	87%	69%	59%	73%	62%	100%	65%	72%	38%	68%	82%	77%	67%
INSU	72%	87%	56%	70%	64%	63%	74%	54%	85%	74%	65%	100%	36%	67%	61%	60%	56%	68%
MEDA	46%	39%	44%	51%	54%	58%	41%	28%	39%	27%	72%	36%	100%	21%	42%	75%	77%	57%
PHRM	43%	63%	31%	45%	39%	43%	58%	43%	61%	63%	38%	67%	21%	100%	43%	35%	37%	58%
RETL	68%	62%	59%	69%	67%	67%	73%	55%	58%	61%	68%	61%	42%	43%	100%	52%	53%	57%
TECH	69%	67%	62%	65%	66%	70%	53%	43%	64%	40%	82%	60%	75%	35%	52%	100%	81%	55%
TELE	65%	59%	52%	57%	63%	63%	59%	32%	57%	41%	77%	56%	77%	37%	53%	81%	100%	70%
UTLY	64%	64%	41%	57%	64%	56%	71%	46%	58%	66%	67%	68%	57%	58%	57%	55%	70%	100%

- Covariance matrix via  $\Omega_{ij} = \sigma_i \sigma_j \rho_{ij}$ .

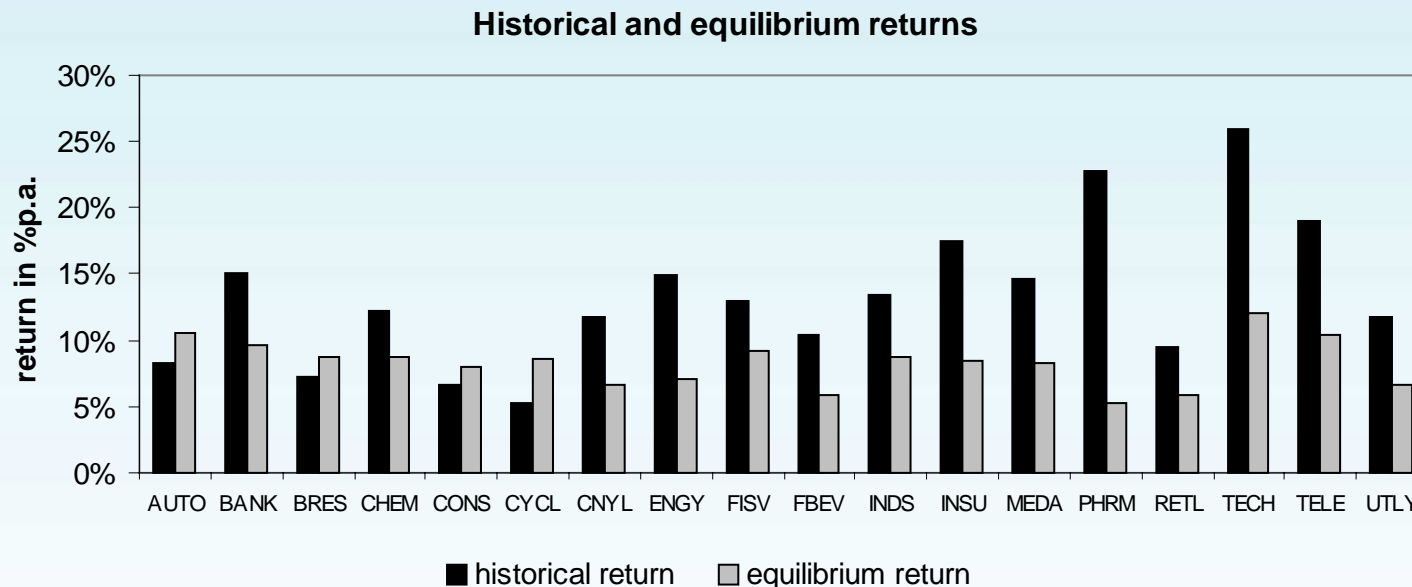
Example: DJ STOXX

# Black-Litterman Approach - the equilibrium returns

BL-starting point: Equilibrium - implicit - returns of market portfolio

- “inverse optimization” yields equilibrium returns

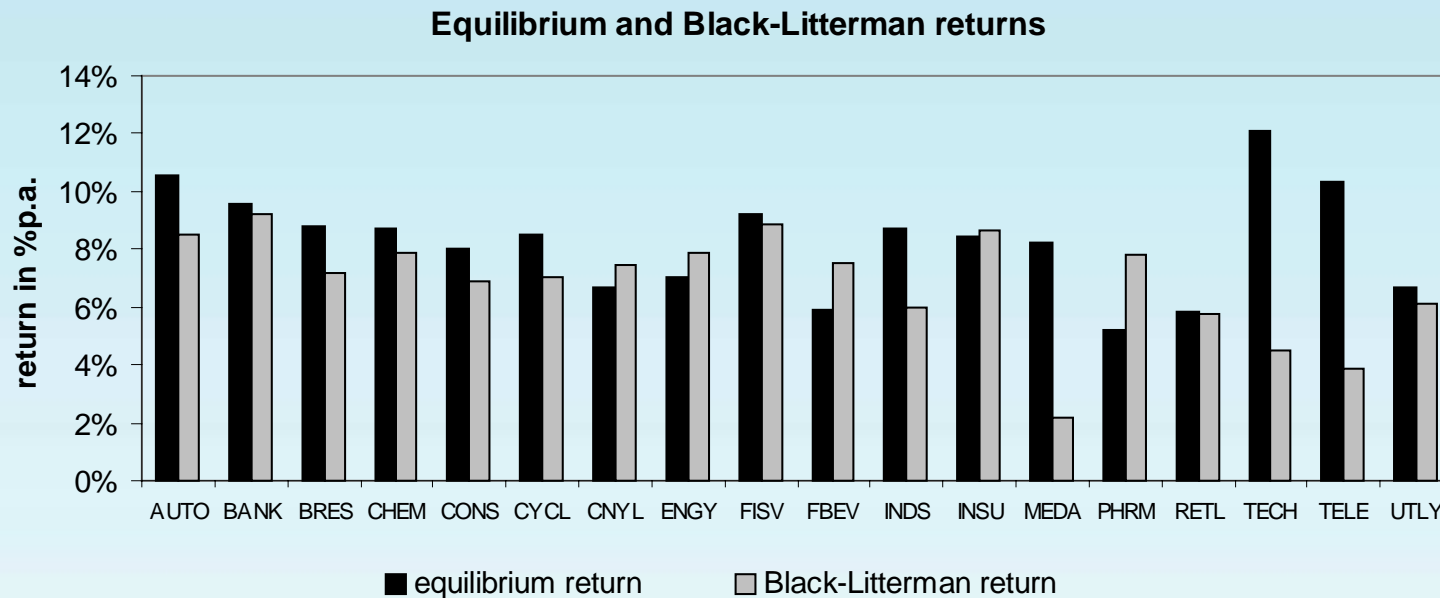
$$R_{Equilibrium} = (\tau \Omega) \cdot w_{Equilibrium} \quad \text{where} \quad w_{Equilibrium} = w_{Market Portfolio}$$



Example: DJ STOXX

# Black-Litterman Approach - the BL returns

From equilibrium returns to Black-Litterman returns (Views as given)



## Having implemented the Views... :

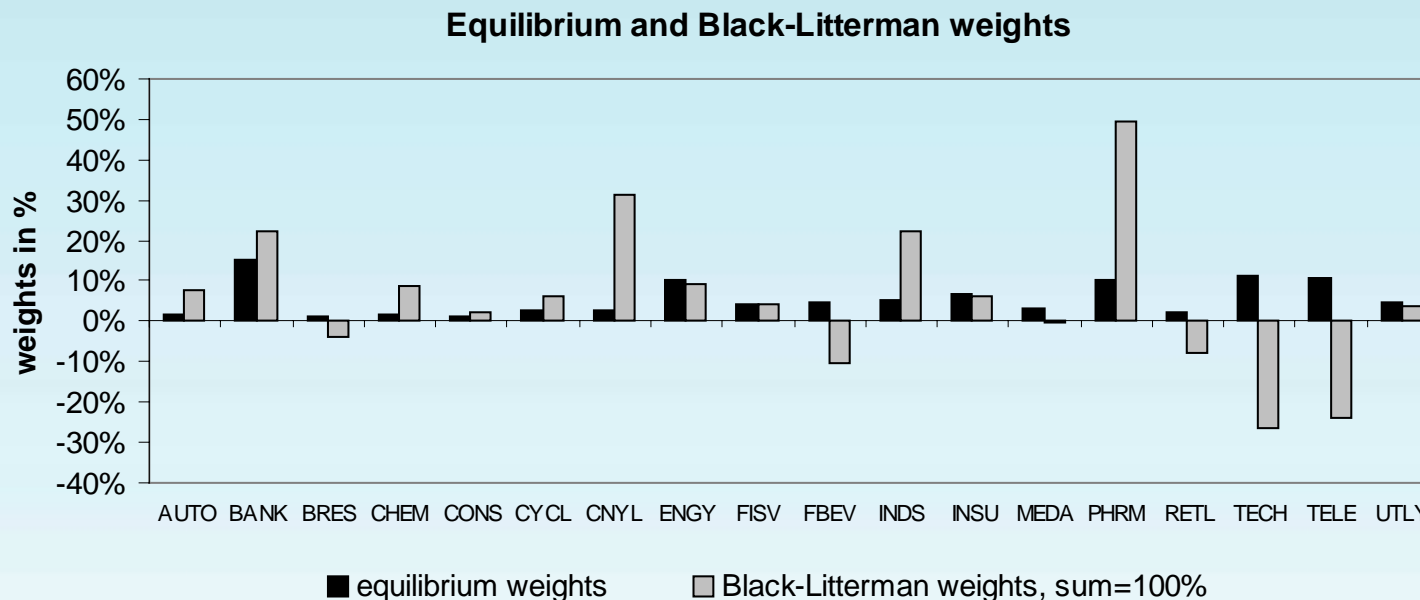
- BL return expectations significantly lowered for TECH und TELE.
- BL return expectation higher in PHRM but lower in INDS (still okay because the *relative* View „... better than TELE und TECH“ remains intact !)
- For CNYL, the expected return shifts from 6.66% to 7.48% (90% confidence in View 7,5%).
- Example: MEDA (correlated by 75% to TECH, 77% to TELE) has significantly lower return.



Example: DJ STOXX

# Black-Litterman Approach - the BL weights

Comparing equilibrium weights (market cap.) and Black-Litterman weights



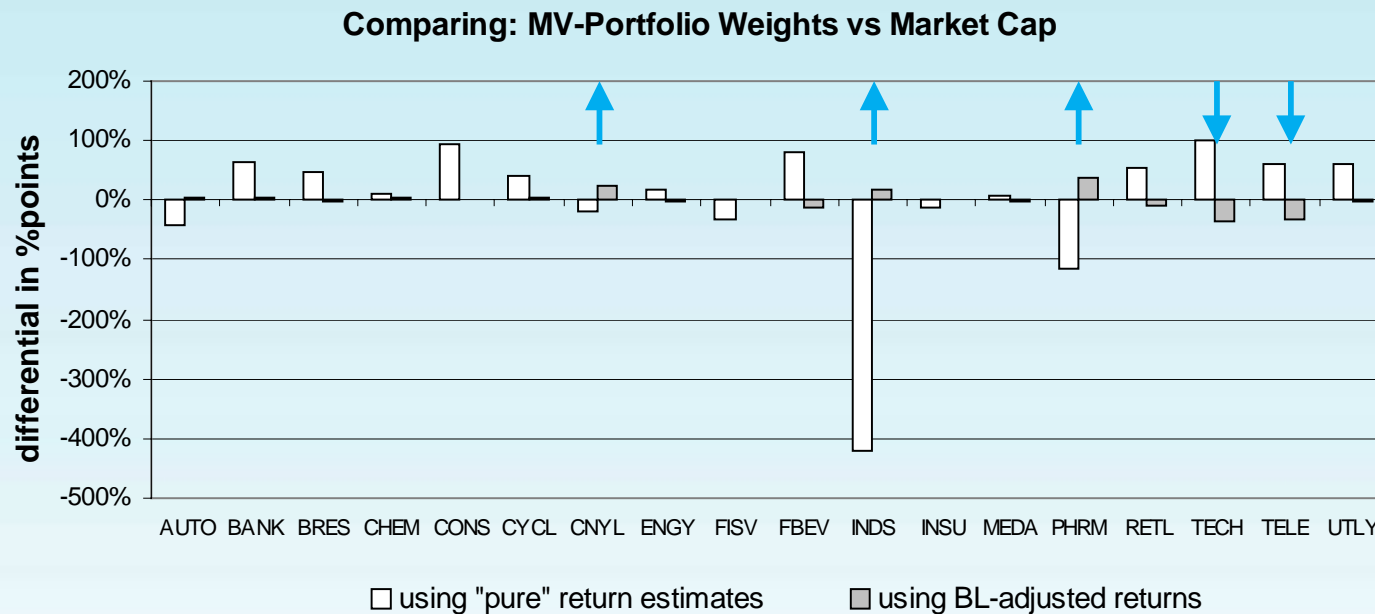
Using the BL returns (note: strong confidence in Views of 90%):

- Significant weight reduction for TELE and TECH (also, e.g., for MEDA)
- Significant weight increase for INDS and PHRM
- Significantly increased weight for CNYL due to higher return expectation (+0.84%pts)

Key Features

# Black-Litterman Approach vs straight MV

Given the return scenario (Views up or down,  $\uparrow\downarrow$ ), the revised portfolio structure clearly benefits from the BL-enhanced optimization process.



- **Straight MV optimization** - which is a naive approach in terms of „c.p.“ return estimates - yields extreme and unreliable changes in portfolio weights
- **The BL-adjusted return input for MV** stabilizes the weights, leading to a reliable and intuitively sound new portfolio structure.

Example: DJ STOXX

# Black-Litterman Approach - constraints I

Calculation of weights s.t. constraints

- In general: Use mean/variance optimizer with constraints
- No constraints:

$$w = \frac{1}{\gamma} \Omega^{-1} \cdot R$$

- “Budget constraint”, i.e. sum of weights = 100% ( $I = 1$ -Vector):

$$w = \frac{\Omega^{-1} I}{I^T \Omega^{-1} I} + \frac{1}{\gamma} \Omega^{-1} \cdot \left( R - \frac{I^T \Omega^{-1} R}{I^T \Omega^{-1} I} I \right)$$

Example: DJ STOXX

# Black-Litterman Approach - constraints II

Calculation of weights s.t. constraints

- Additional constraint: **Tracking Error** ( $w_{act}$  = active weights)

$$TE^2 = w_{act}^T \cdot \Omega \cdot w_{act}$$

- Additional constraint: **Portfolio-BETA** („directional risk in the portfolio“)

$$\beta_P = \sum_{i=1}^{\#Assets} w_i \beta_i$$

- Additional constraint: „**No short positions**“

$$w_i \geq 0 \quad \forall i = 1.. \#Assets$$

---

(see, e.g., paper of K. Iordanidis); in Excel: requires additional calculations & solver constraints

Example: DJ STOXX

# Black-Litterman Approach - weights

Remark on treating weights w.r.t. absolute / relative Views

- The sum of portfolio weights has to add up to **100%**.
- **Purely *absolute Views*** are translated into independent *long* and *short* portfolios, thus causing portfolio weights to deviate from 100%. Therefore, normalization of weights is recommended.
- **Purely *relative Views*** are translated into weight-balanced *long* and *short* portfolios, so that portfolio weights still sum up to 100%.
- The use of ***absolute and relative Views*** again leads to portfolio weights deviating from 100%. Therefore, again, normalization of weights is recommended.
- The normalization to 100% has to be included in the optimization process as a constraint.
- *Note that - unfortunately - the use of constraints is contra-intuitive for BL-adjusted allocations.*

Example: DJ STOXX

# Black-Litterman Approach

Numerical example on treating weights w.r.t. absolute / relative Views

- The example is based on the View scenario given on slide 24.
- **Purely *absolute* View:** Weights add up to 110%, with all weights unchanged except for the asset (sector) CNYL under view (weight increases by 10%pts.).  
Normalization to 100% leads to weight changes in *all* positions - as it should be!  
(Note: Lowering the expected return of CNYL to, e.g., 5,5% yields a total portfolio weight of only 86%. Again, weight normalization is recommended, spreading for the -14%pts across *all* weights.)
- **Purely *relative* Views:** Weights add up to 100%, with the weights of the viewed assets just offsetting the *long* and *short* positions.
- ***Absolute and relative* Views:** Weights add up to 129%, with *long* and *short* positions for the viewed assets as intuitively expected and the unviewed assets' weights remaining unchanged.  
Normalization to 100% consequently leads to weight changes in *all* positions - as it should be!

Example: DJ STOXX

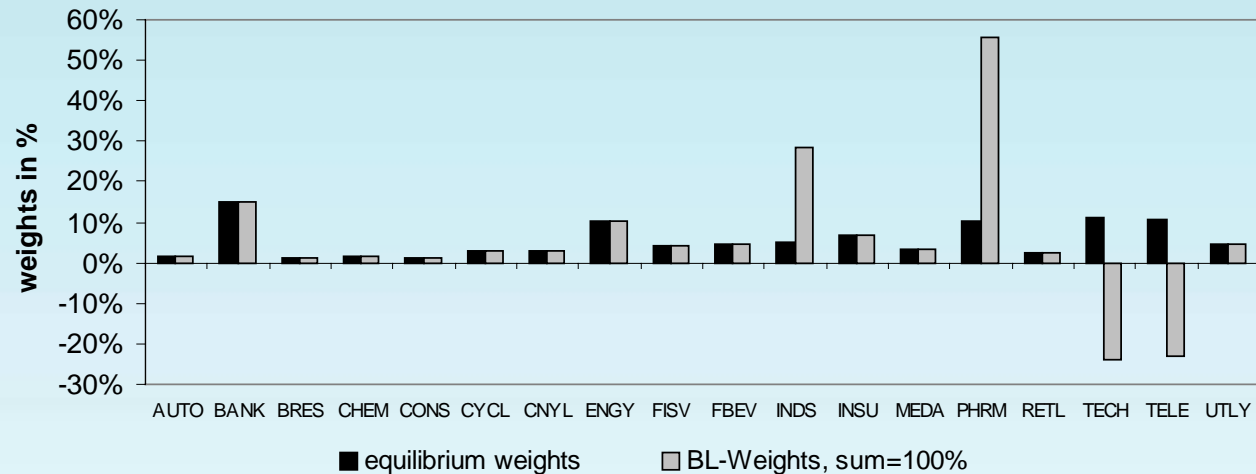
# Black-Litterman Approach

(cont.) non-normalized weights

**Purely relative Views**

$\Sigma$  weights = 100%,  
no normalization required

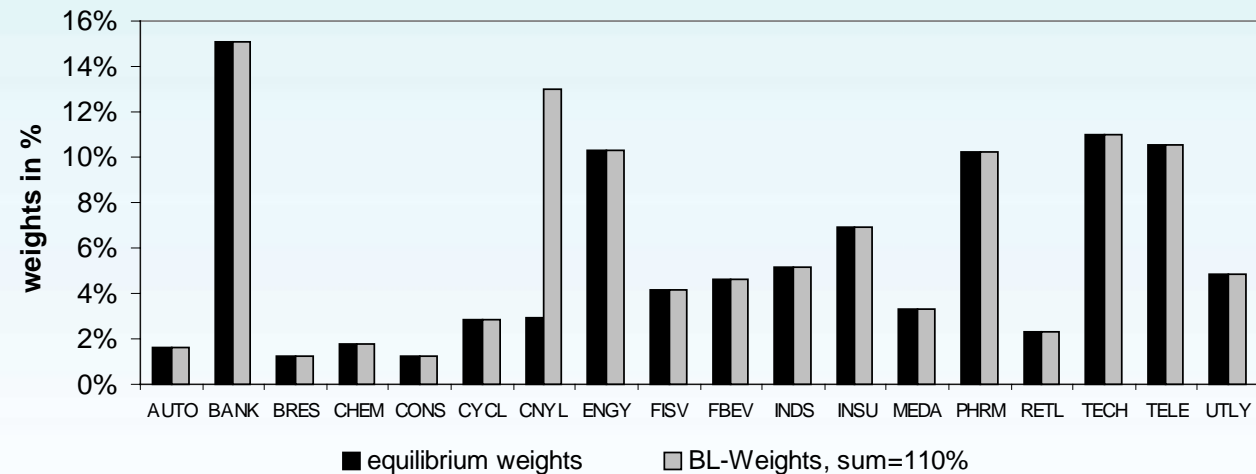
Equilibrium and Black-Litterman Weights, not normalized



**Purely absolute Views**

$\Sigma$  weights = 110%,  
normalization recommended

Equilibrium and Black-Litterman Weights, not normalized



Example: DJ STOXX

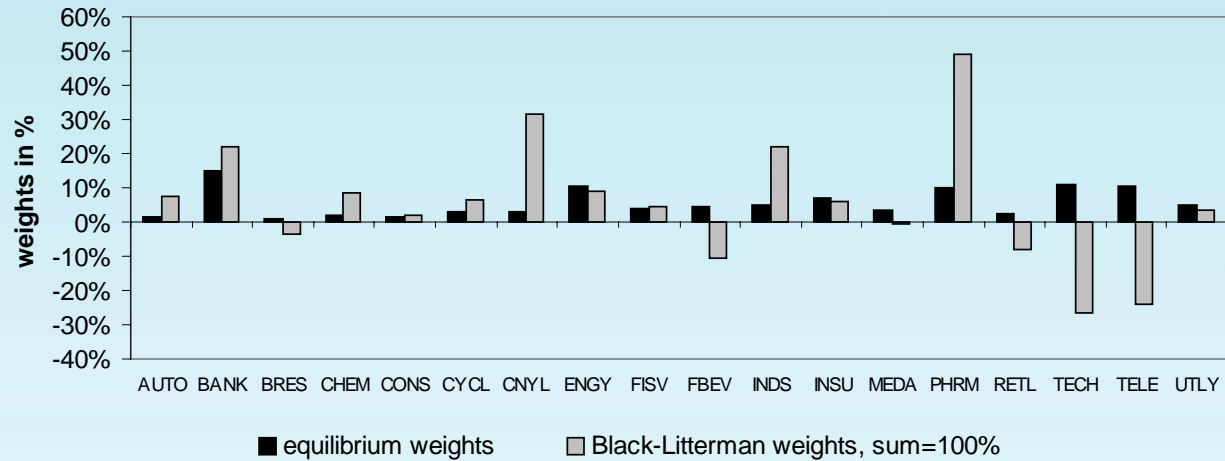
# Black-Litterman Approach

Numerical example (cont.)

## Relative and absolute Views

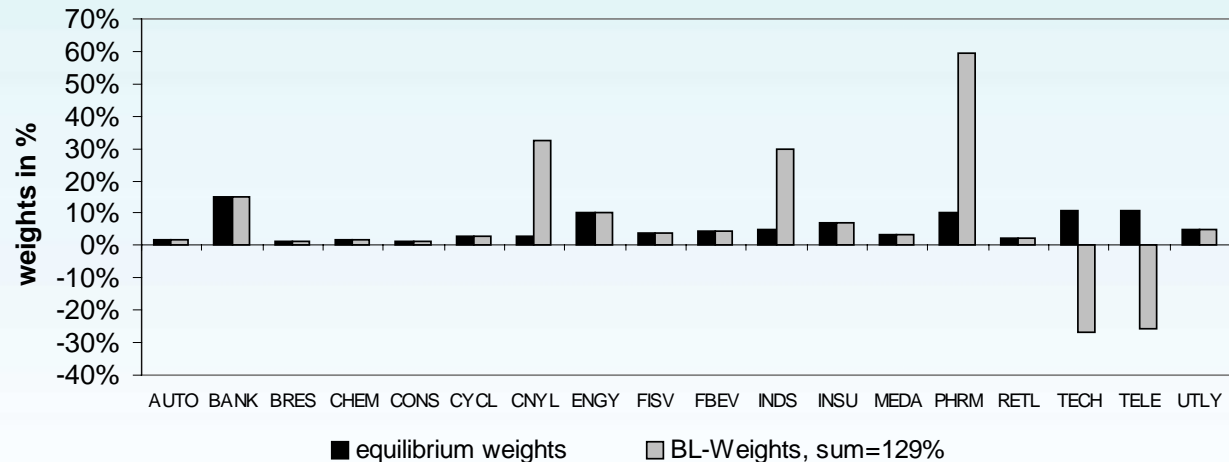
- weights normalized  
 $\Sigma$  weights = 100%

Equilibrium and Black-Litterman weights



- weights not normalized  
 $\Sigma$  weights = 129%

Equilibrium and Black-Litterman Weights, not normalized





Example: DJ STOXX

# Black-Litterman Approach - confidence and CNYL

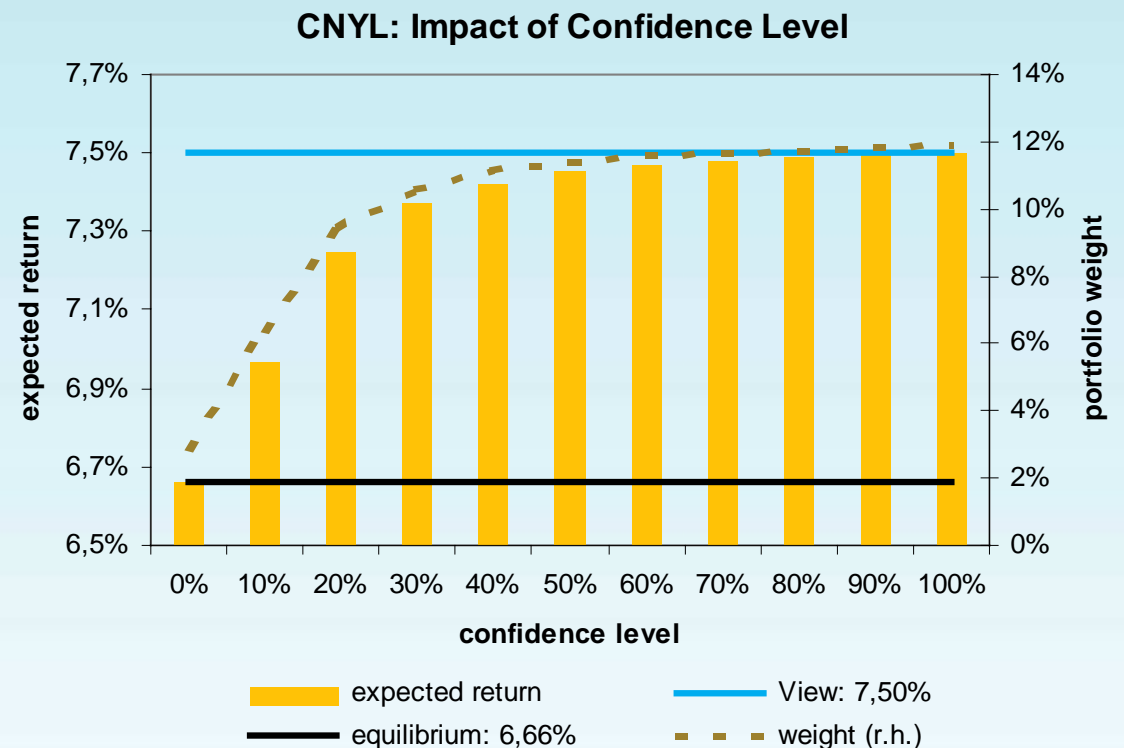
Influence of degree of confidence on BL-returns and BL-weights, I

Focus on asset „CNYL“ only:

- Equilibrium return = 6.66%,
- Equilibrium weight = 2.90%
- View on return: 7.5% ± 1.5%  
(equivalent to a range of 6 - 9%)

Observations:

- Low confidence: → equilibrium return
- High confidence: Asymptotic approach to the View value of 7.5%.
- Limit: At a confidence level of 100%, BL fully accepts the strong view of 7.5%.
- Weights: from 2.9% (= market cap, due to confidence of 0% a *no view*-case), up to 12% (overweighting due to the *strong view* confidence of 100%).



Example: DJ STOXX

# Black-Litterman Approach - weights of CNYL I

BL compared to straight MV

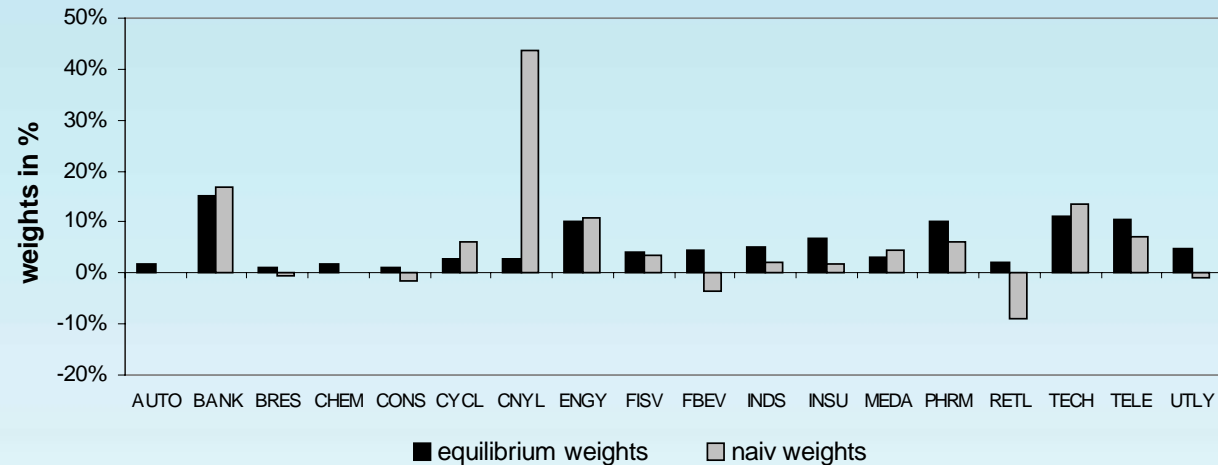


View on Return:  
from 6.6% up to 7.5%  
(with strong confidence)

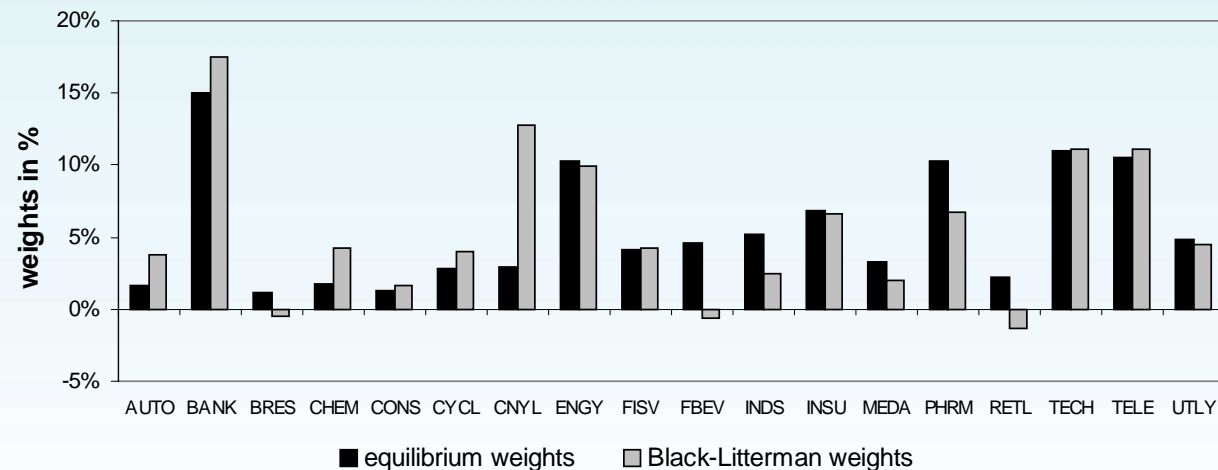
## Result

- Realistic weight changes in BL
- Volatile weight scenario in straight MV approach

Equilibrium and Naiv Weights



Equilibrium and Black-Litterman Weights



Example: DJ STOXX

# Black-Litterman Approach - weights of CNYL II

BL compared to straight MV

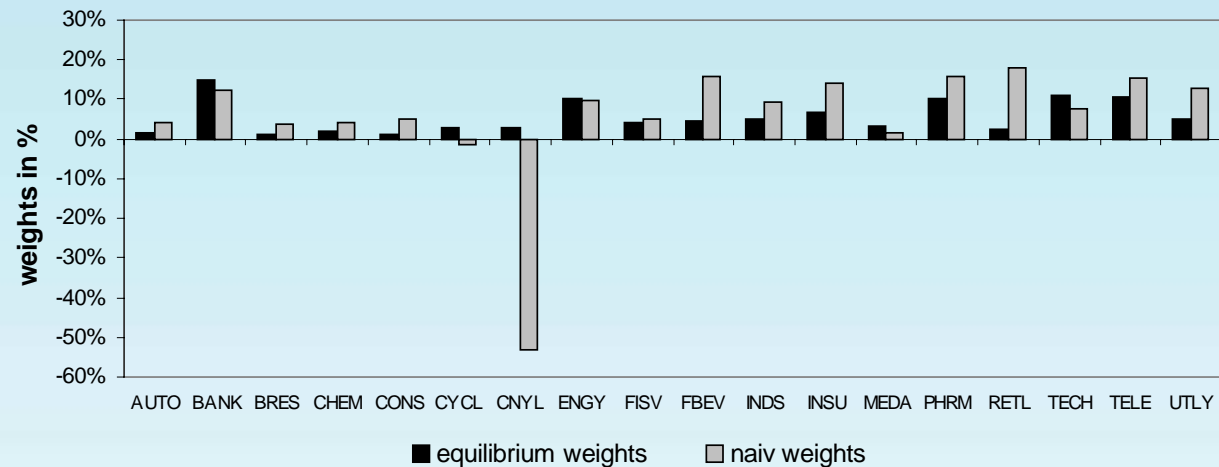


View on Return:  
from 6.6% down to 5.5%  
(with strong confidence)

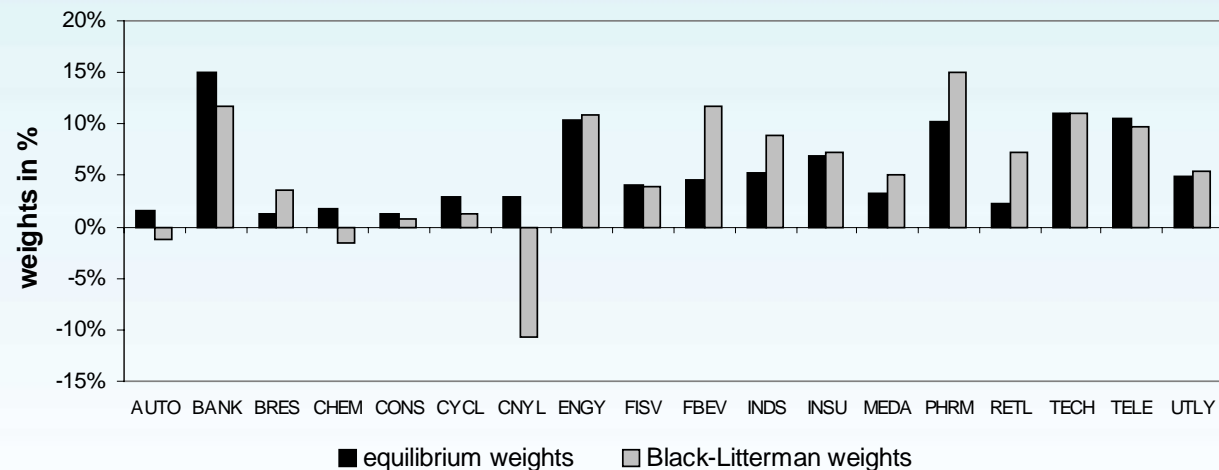
## Result

- Realistic weight changes in BL
- Volatile weight scenario in straight MV approach

Equilibrium and Naiv Weights



Equilibrium and Black-Litterman Weights



Example: DJ STOXX

# Black-Litterman Approach - confidence and weights

Influence of degree of confidence on BL-returns and BL-weights , II

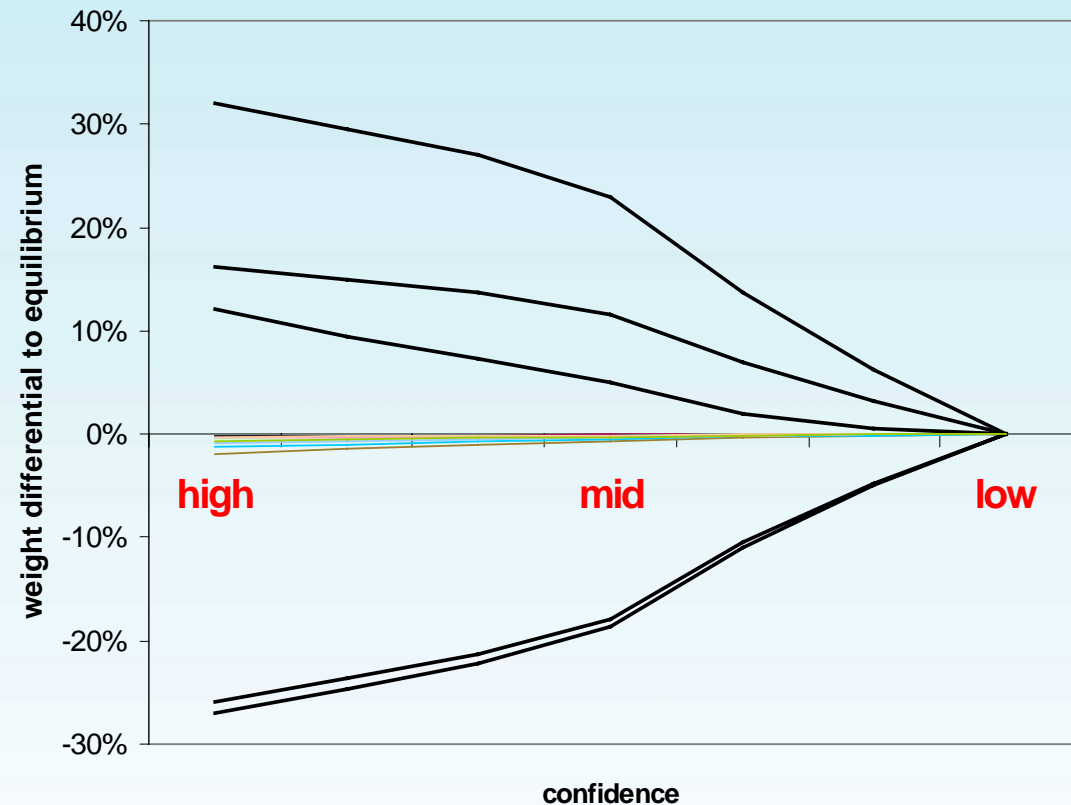
## Behavior of asset weights:

- Complete portfolio, 18 sectors

## Observations:

- Low degrees of confidence: BL-weights are close to weights in equilibrium (=market cap's).
- Higher degree of confidence: Weights approach equilibrium values on either underweighting (*short*) or overweighting (*long*) path.
- Most significant weight changes for the assets under View.

Sensitivity of weights on degree of confidence



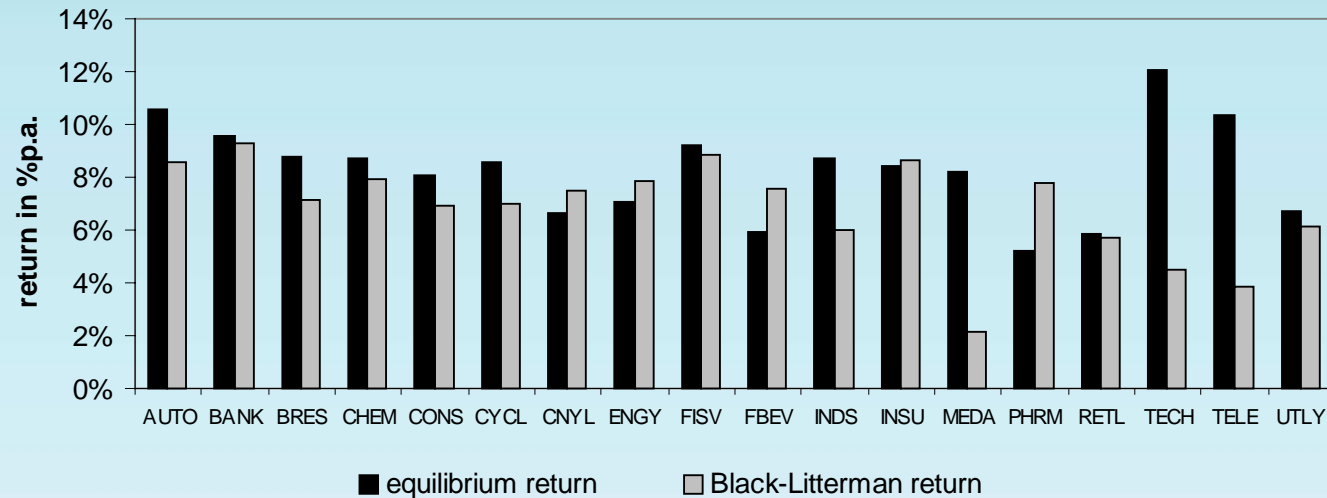
Example: DJ STOXX

# Black-Litterman Approach

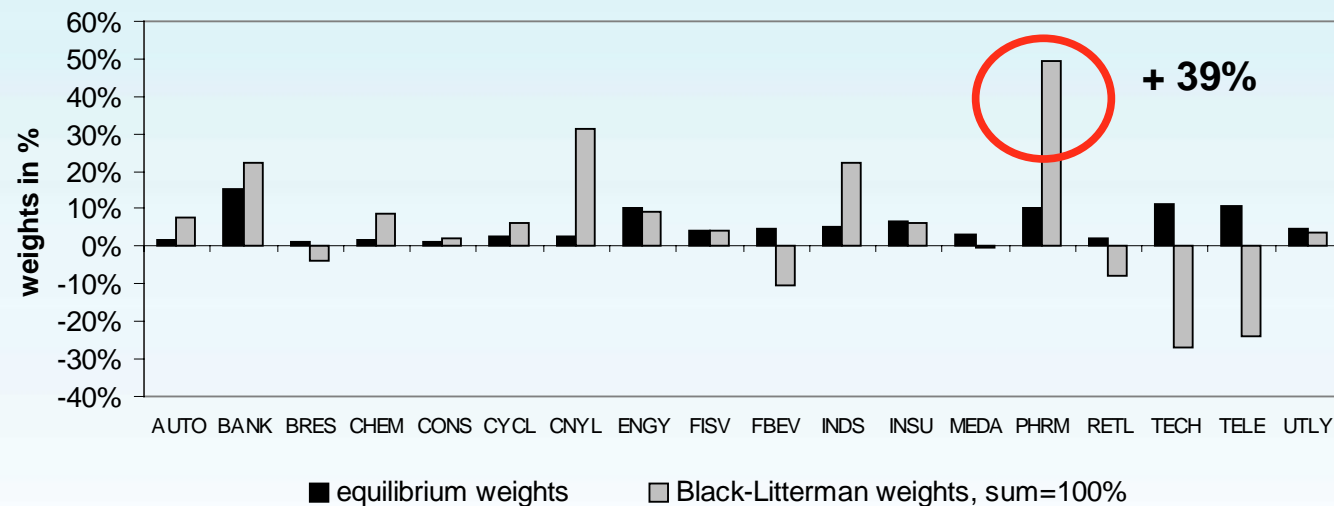
Strong confidence

- Large changes in weights due to the "strong views"

Equilibrium and Black-Litterman returns



Equilibrium and Black-Litterman weights



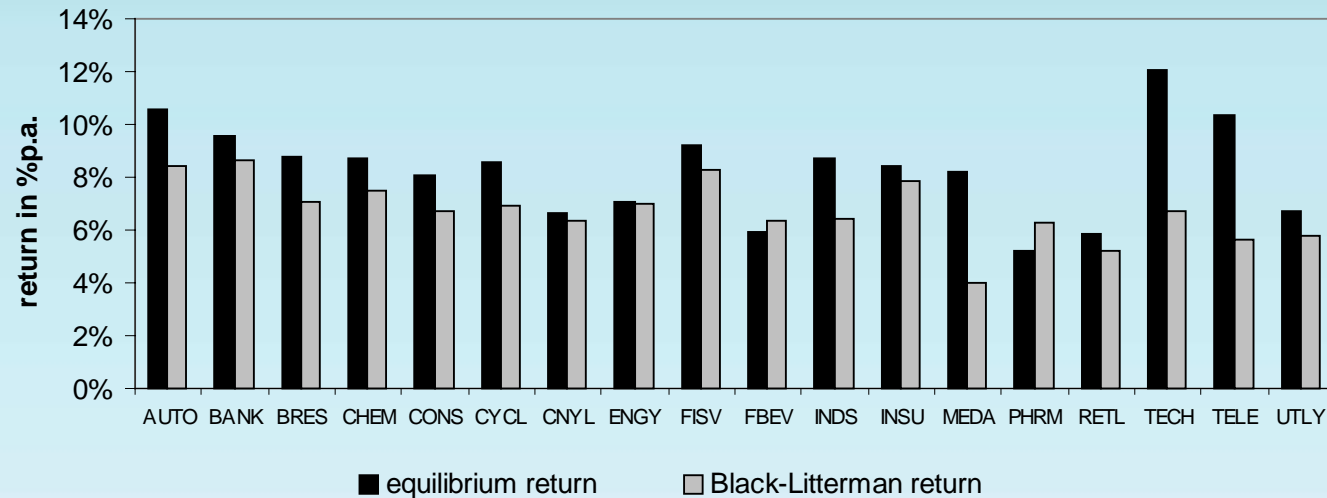
Example: DJ STOXX

# Black-Litterman Approach

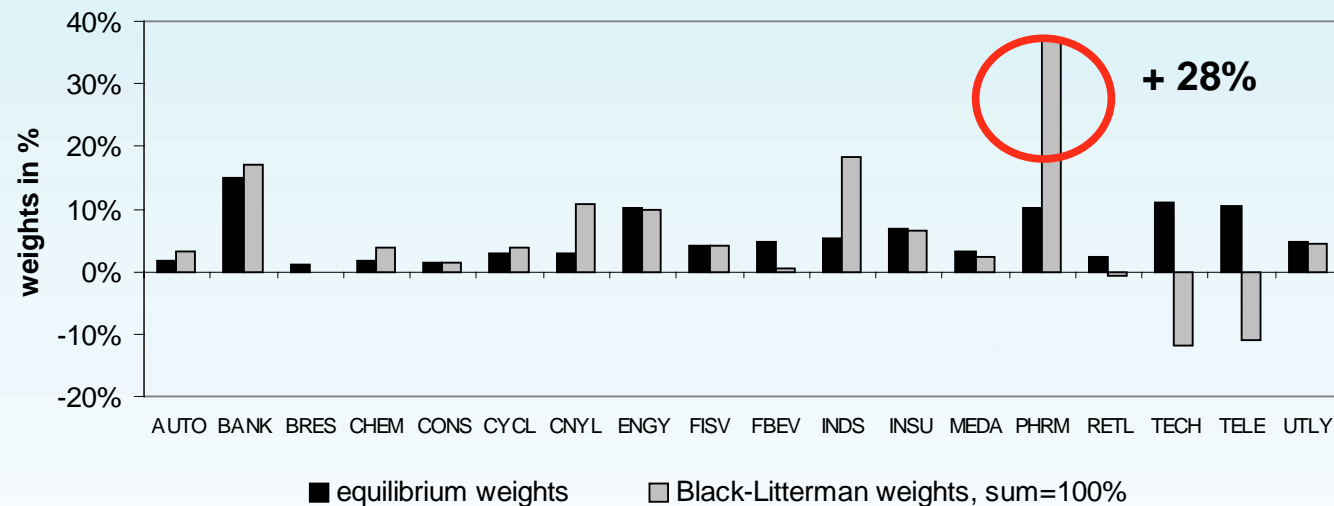
Mid confidence

- Moderate changes in weights

Equilibrium and Black-Litterman returns



Equilibrium and Black-Litterman weights



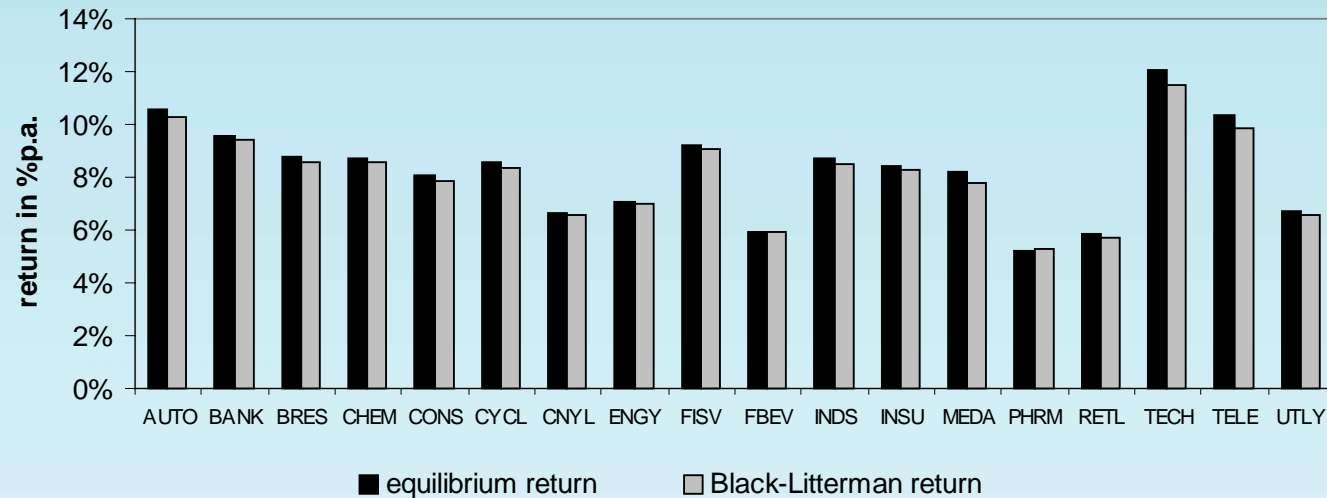
Example: DJ STOXX

# Black-Litterman Approach

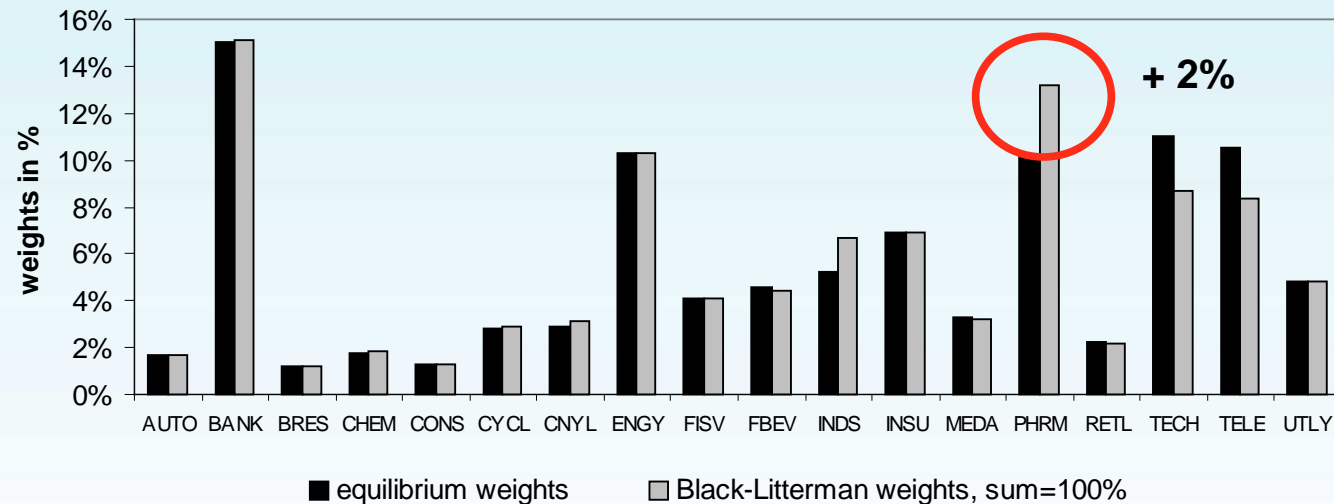
Poor confidence

- Weights stay close to equilibrium weights

Equilibrium and Black-Litterman returns



Equilibrium and Black-Litterman weights



## Key Features

# Black-Litterman Approach - Conclusion I

Traditional „Straight MV“ vs „BL plus MV“ approach

### straight MV

### Black-Litterman → MV

#### Return estimates:

- o required for each asset
- o assumed as certain
- o absolute return figures
- o c.p.

required only for selected assets  
 degree of confidence  
absolute or relative Views  
 consistent

#### Reference return:

- o none

equilibrium returns



## Key Features

# Black-Litterman Approach - Conclusion II

Traditional „Straight MV“ vs „BL plus MV“ approach

### straight MV

### Black-Litterman → MV

#### ■ MV-optimized Portfolios:

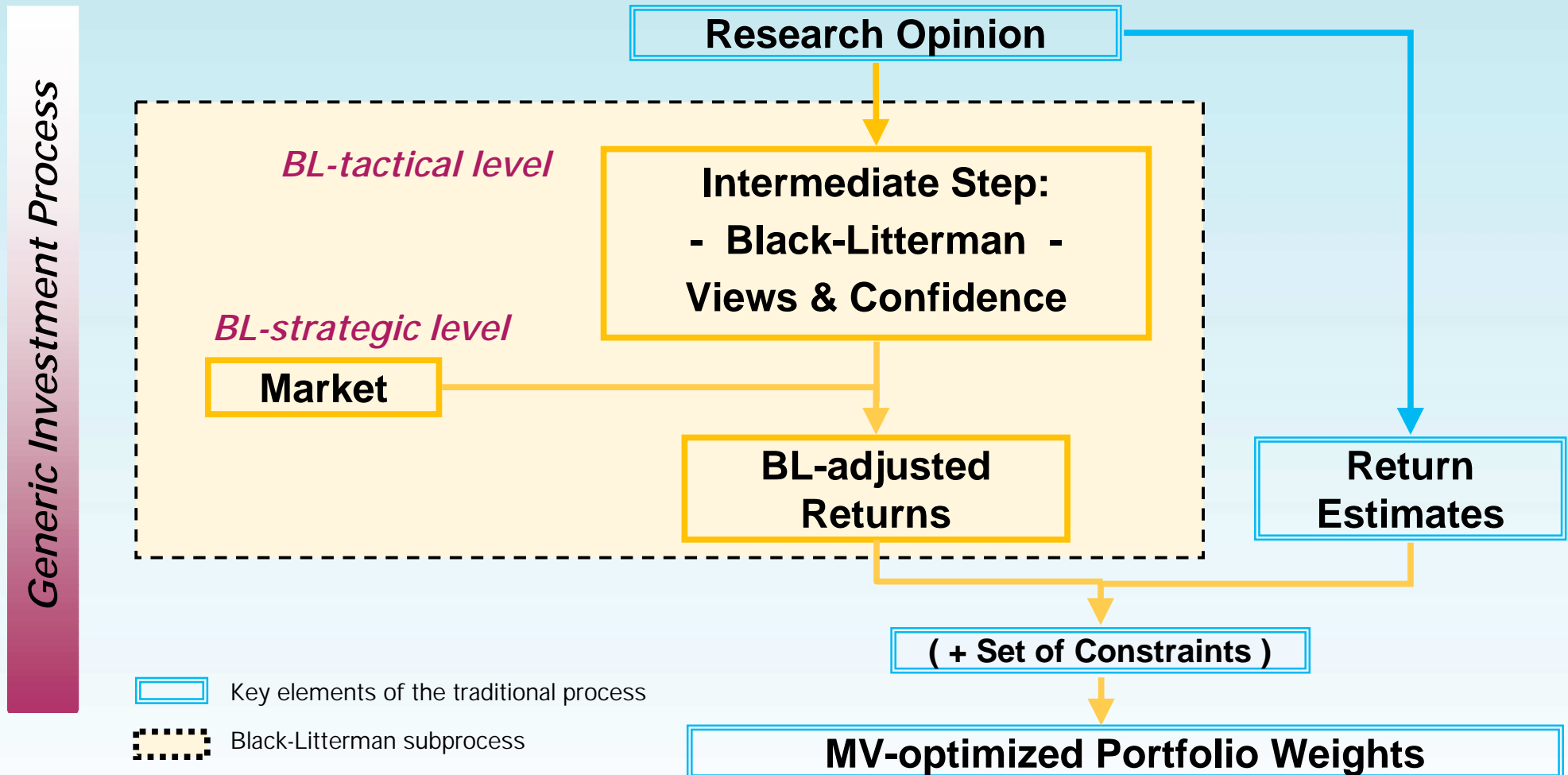
- extreme asset weights
- changes in return estimates  
⇒ huge weight fluctuations
- portfolios unreliable
- MV-results hardly accepted
- reflecting c.p. opinions

- reliable asset weights**
- ⇒ moderate weight changes**
- consistent structure**  
**„intuitively reasonable“**
- higher acceptance**
- „correlated Views“**

Key Features

# Black-Litterman Approach - Conclusion III

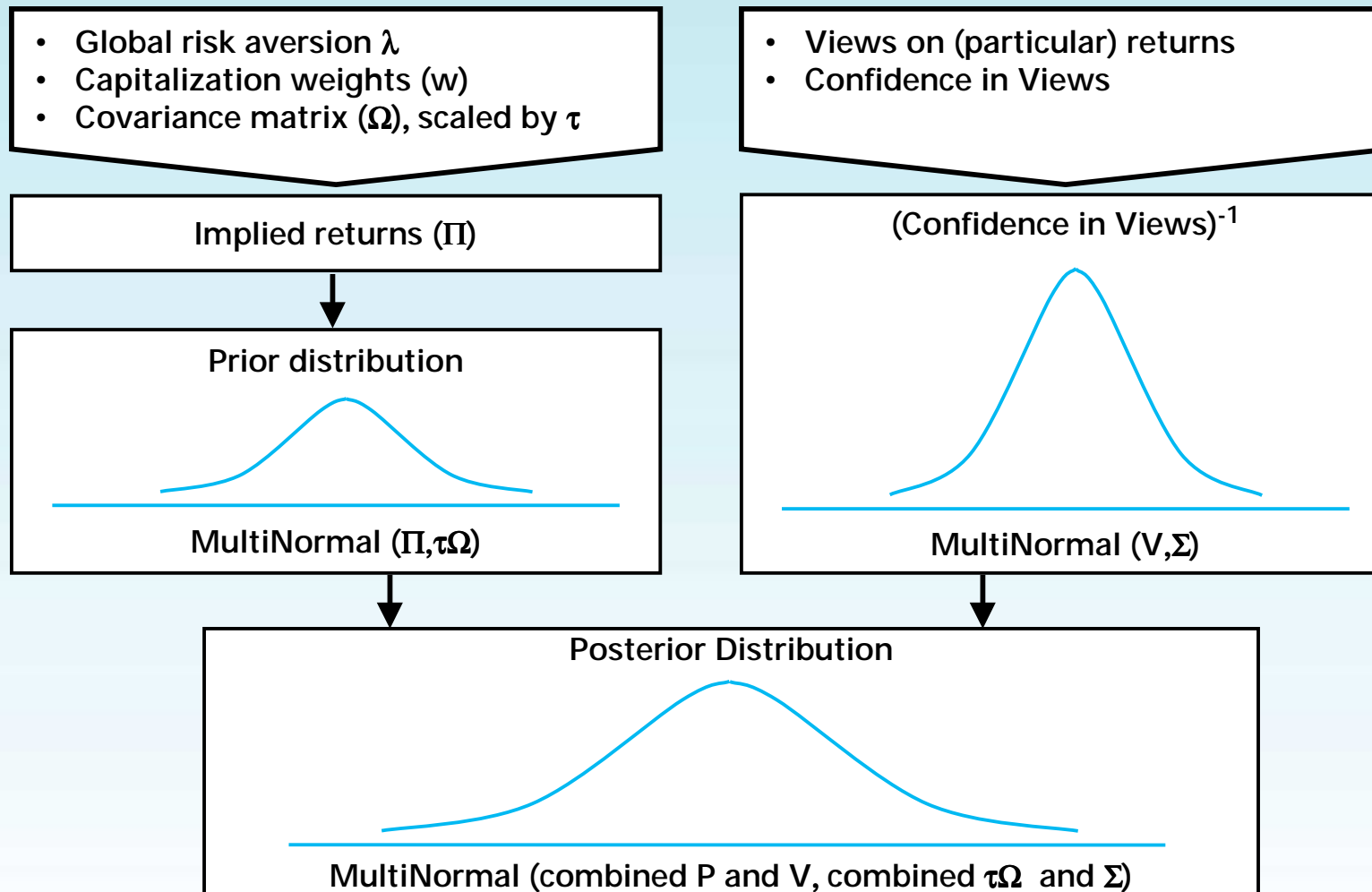
BL as a building-block of an enhanced asset allocation process



Key Features

# Black-Litterman Approach - Conclusion IV

... in a more formal way



Layout inspired by  
K.Iordanidis

## Some literature

# Black-Litterman Approach - more insights

## Suggestions for further reading...

- Black F. and Litterman R.: *Global Portfolio Optimization*, Fin.Analysts Journal, Sep.1992
- Black F. and Litterman R.: *Asset Allocation: Combining Investor Views with Market Equilibrium*, Goldman-Sachs, Fixed Income Research, Sep.1990
- Zimmermann H., Drobetz W. and Oertmann P.: *Global Asset Allocation: New Methods and Applications*, publ. by Wiley & Sons, Nov.2002
- Christodoulakis G.A.: *Bayesian Optimal Portfolio Selection: The BL Approach*, Nov.2002
- He Q. and Litterman R.: *The Intuition behind BL-Model Portfolios*, Dec.1999
- Idzorek T.: *Step-by-Step Guide to the BL-Model*, Feb.2002
- Iordanidis K.: *Global Asset Allocation: Portfolio Construction & Risk Management*, Jan.2002
- Fusai G. and Meucci A.: *Assessing views*, Risk, Mar.2003
- Bevan A. and Winkelmann K.: *Using the BL Global Asset Allocation Model: Three Years of Practical Experience*, Goldman-Sachs, Fixed Income Research, Jun.1998

in German:

- Drobetz T.: *Einsatz des BL-Verfahrens in der Asset Allocation*, Working paper, Mar.2002
- Zwahlen St.: *Kritische Analyse des BL-Ansatzes*, Seminararbeit, Uni St.Gallen-HSG, Jul.2004
- Zimmermann H. et al.: *Einsatz des Black-Litterman-Verfahrens in der Asset Allocation*, in „Handbuch Asset Allocation“, Editors: Dichtl, Schlenger u. Kleeberg, Uhlenbruch-Verlag, 2002.