Consistent Return Estimates
- The Black-Litterman Approach

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Abstract

The Black-Litterman Approach
Consistent asset return estimates - saving classical mean/variance...

In asset management, the forecast of asset returns is essential within the investment process. In this context, the Black-Litterman approach (1992) yields consistent asset return forecasts as a weighted combination of (strategic) market equilibrium returns and (tactical) subjective forecasts ("views"). The Black-Litterman formalism allows to implement both absolute views (return levels) and relative views (outperforming vs. underperforming assets) for selected assets investigated under "core competence". For any particular view, individual confidence levels for the return estimates have to be specified. The formalism spreads these informations consistently across all assets in the portfolio. The BL-revised returns then serve as a consistent input for mean-variance portfolio optimisation procedures, thus allowing for the implementation of additional constraints. It turns out that BL-optimized portfolios overcome some well-known Markowitz insufficiencies as unrealistic sensitivity to input factors or extreme portfolio weights. The BL process will be introduced both from its theoretical background and its implementation in practice.

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Data source: DataStream
Investment Process: Opinions & Portfolio Context

**RESEARCH**
- The c.p. world of core competences

**PORTFOLIO CONSTRUCTION**
- The portfolio context: Thinking in terms of correlations

[Image of Committee vs Quant tool]

c.p. = ceteribus paribus
Defining the subjectives... \( \Rightarrow \) Black & Litterman

Let's talk about the...

Some nice-to-haves

"Consistent" (non-c.p.) input for portfolio construction

Transparent

Overcome some of the problems of plain MV (Markowitz)

Manageable

Intuitive results

Reliable output

Tactical deviations from "some" strategic allocation

Weighting estimates according to confidence
Topics to be discussed...

- Classical Markowitz - the straight way
- MV-optimized portfolios
- BL - the role within the investment process
- BL - implementation
- BL - example
The Markowitz Approach - in short

Efficient portfolios in the mean-variance approach

- **Starting point** in a world of normally distributed returns: The assets are described by the first two moments of return - mean and variance.
- In an **efficient portfolio** the assets are weighted such that for any given level of risk a maximum return is achieved. (equivalently: for a given return the risk is minimized). Diversification reduces risk.
- All efficient portfolios form the **efficiency line**. It starts in the minimum variance portfolio and ends in the maximum return portfolio (which is the asset of maximum return).
- If a risk-free asset exists, all efficient portfolios are located on the **Capital Allocation Line (CAL)**, starting at the risk-free asset and tangentially touching the efficiency line at the market portfolio. Efficient portfolios are then a combination of the risky **market portfolio and the risk-free asset** (with risk-free long or short), a.k.a. Tobin Separation (1958).
## The Markowitz Approach - dealing with its problems

Deficits of the mean-variance (MV) concept, suggestions for solutions...

<table>
<thead>
<tr>
<th>Deficit</th>
<th>Possible Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>High sensitivity on inputs (return estimates!) leads to large weight fluctuations in the optimal portfolio.</td>
<td>Black-Litterman</td>
</tr>
<tr>
<td>„Corner solutions“: Extreme portfolio weights (also in the case of optimization algorithms using constraints)</td>
<td>Black-Litterman</td>
</tr>
<tr>
<td>Aggregation: Consistent aggregation of huge number of estimated returns overburdens the investment process</td>
<td>Black-Litterman</td>
</tr>
<tr>
<td>No quantification of confidence in estimated returns</td>
<td>Black-Litterman</td>
</tr>
<tr>
<td>One-periodical approach</td>
<td>Multi-period approaches, ...</td>
</tr>
<tr>
<td>„Variance“ = restricting risk to symmetric return volatility</td>
<td>VaR, ...</td>
</tr>
<tr>
<td>Requires ex-ante-estimates of covariance matrix</td>
<td>Vola-modeling, ...</td>
</tr>
</tbody>
</table>
The Markowitz Approach - straight is not enough
When return estimates change...

- Let the investment universe be the 18 STOXX sectors in Euroland.
- **At time 0**: Returns: Historical returns
  Weights: To be determined via MV
- **At time 1**: Returns: +2%pts for BANK and MEDA, -2%pts for CNYL, others unchanged
  Weights: To be determined via MV
The Markowitz Approach - straight is not enough

Fluctuation of asset weights due to revised return estimates

- Even small and selected changes in expected returns lead to huge unrealistic shifts in asset weights! ($\gamma=3$, historical covariances)

Problems: Communication of results, (re-) allocation in real portfolios, acceptance of method.
The Markowitz Approach - straight

The formal optimization approach for risky assets, basic outline.

- Markowitz Theory relates risk & return

- MV optimization problem:
  \[
  w^T R - \frac{\gamma}{2} \cdot w^T \Omega w \rightarrow \max_w
  \]

  \( R \) = vector of returns
  \( \Omega \) = covariance matrix
  \( \gamma \) = risk aversion parameter
  \( w \) = vector of weights

- Solution: Optimal portfolio weights \( w^* \) (no constraints):
  \[
  w^* = \left( \gamma \Omega \right)^{-1} R
  \]

- Markowitz provides a mechanism to achieve optimal (efficient) portfolios. What about the input factors ???
Extending the Markowitz Approach

Equilibrium returns

Supply & demand
- Traditional approach of maximum return & minimum risk is demand-side perspective.
- Need to balance with supply-side...

Concept of equilibrium returns:
- The market portfolio exists in market equilibrium, i.e. supply & demand are in equilibrium.
- Therefore, equilibrium returns reflect neutral “fair” reference returns \( \Pi \):
- Inverse optimization yields:

\[
\Pi = (\gamma \Omega) w_{MCap}
\]

\( w_{MCap} \) = market capitalization

Conclusion
- Use of equilibrium returns as a long term strategic reference for any return estimate („market neutral starting point“).
Extending the Markowitz Approach

Equilibrium (or implicit) returns for the STOXX sectors

These implicit returns serve as reference returns for all further investigations.

Note that equilibrium returns are calculated; they do not require any estimation procedure.
Black-Litterman Process

Black-Litterman Approach - basic outline
BL provides a concept to combine long term equilibrium returns with short term return estimates as a consistent input for MV-optimization.

- Market weights \( \Rightarrow \) Equilibrium returns
- BL-adjusted consistent expected returns
- BL-adjusted asset weights via MV

Research - Views with Confidence

strategic

tactical
Black-Litterman Approach - going math

Optimization s.t. contraints

- Determine the optimal estimate $E(R)$, which minimizes the variance of $E(R)$ w.r.t. equilibrium returns $\Pi$: (minimizing the Mahalanobis distance):

$$\left[E(R) - \Pi\right]^T \cdot (\tau \Omega)^{-1} \cdot \left[E(R) - \Pi\right] \rightarrow \min_{E(R)}$$

where: $E(R) = \Pi + \nu$ with $\nu \sim N(0, \tau \Omega)$.

s.t.

$$P \cdot E(R) = \begin{cases} V \text{ certain Views} \\ V + e \text{ uncertain Views} \end{cases}$$

where: $P \cdot E(R) \sim N(V, \Sigma)$, $\Sigma_{ii} = e_i$
Black-Litterman Approach - the formulas

Master equations for the BL-return estimates

- Solution in the case of **certain estimates** \((\Sigma \equiv \text{zero matrix})\):

\[
\bar{E}(R) = \Pi + (\tau \, \Omega) \cdot P^T \cdot \left( P(\tau \, \Omega) \cdot P^T \right)^{-1} \cdot (V - P \, \Pi)
\]

- Solution in the case of **uncertain estimates** \((\Sigma = \text{diagonal matrix})\):

\[
\bar{E}(R) = \left[ (\tau \, \Omega)^{-1} + P^T \Sigma^{-1} P \right]^{-1} \cdot \left[ (\tau \, \Omega)^{-1} \Pi + P^T \Sigma^{-1} V \right]
\]

- The **constraints** \(P \cdot E(R) = V + e\) are implicitly fulfilled.
BL Optimization Problem

Black-Litterman Approach - more math I

Formal proof for the case „certain estimates“

Proposition: The optimization problem \( [E(R) - \Pi]^T \cdot (\tau \Omega)^{-1} \cdot [E(R) - \Pi] \rightarrow \min_{E(R)} \) s.t. \( P \cdot E(R) = V \)

yields variance-minimum returns \( \overline{E}(R) = \Pi + (\tau \Omega) P^T \cdot \left(P(\tau \Omega) P^T\right)^{-1} \cdot (V - P \Pi) \)

Proof:

Lagrangian: \( L := [E - \Pi]^T \cdot (\tau \Omega)^{-1} \cdot [E - \Pi] - \lambda \cdot (P \cdot E - V) \)

f.o.c.'s: \( \frac{\partial L}{\partial E} = 0 \) and \( \frac{\partial L}{\partial \lambda} = 0 \)

"scalarizing":

\[
\frac{\partial L}{\partial E_i} = \frac{\partial}{\partial E_i} \left\{ \tau^{-1} \sum_{j,k} E_j \Omega^{-1}_{jk} E_k - \sum_k \lambda_k \left( \sum_j P_{kj} E_j - V_k \right) \right\} = 2 \tau^{-1} \sum_k \Omega^{-1}_{ik} E_k - \sum_k P_{ki} \lambda_k
\]

\[
\frac{\partial L}{\partial \lambda_i} = - \frac{\partial}{\partial \lambda_i} \sum_k \lambda_k \left( \sum_j P_{kj} E_j - V_k \right) = \left( \sum_j P_{ij} E_j - V_i \right)
\]

"revectorizing": \( \frac{\partial L}{\partial E} = 2(\tau \Omega)^{-1} E - 2(\tau \Omega)^{-1} \Pi - P \lambda = 0 \) and \( \frac{\partial L}{\partial \lambda} = PE - V = 0 \)

Solve Eq. (1) for \( E \), insert in Eq. (2), thereof expression for \( \lambda \), result follows with Eq. (1). ◇
Black-Litterman Approach - more math II

Formal proof for the case „uncertain estimates“ (BL-Master Formula)

**Proposition:** \[ (E(R) - \Pi)^T \cdot (\tau \Omega)^{-1} \cdot (E(R) - \Pi) \rightarrow \min_{E(R)} \] with constraints \[ P \cdot E(R) = V + e \]
yields variance-minimum returns \[ \bar{E}(R) = \left( (\tau \Omega)^{-1} + P^T \Sigma^{-1} P \right)^{-1} \cdot \left( (\tau \Omega)^{-1} \Pi + P^T \Sigma^{-1} V \right) \]

**Proof:**

Given: \[ \Pi = E(R) + \nu \quad \text{and} \quad V = P \cdot E(R) + e \]

Setting \( Y := \begin{pmatrix} \Pi \\ V \end{pmatrix}, \quad X := \begin{pmatrix} I \\ P^T \end{pmatrix}, \quad W := \begin{pmatrix} \tau \Omega & 0 \\ 0 & \Sigma \end{pmatrix} \) and \( u \sim N(0,W) \)

so that \( Y = X \cdot E(R) + u \) and using generalized least square \( E(R) = (X^T W^{-1} X)^{-1} X^T W^{-1} Y \) we get

\[
E(R) = \left[ \begin{pmatrix} I & P^T \end{pmatrix} \begin{pmatrix} \tau \Omega & 0 \\ 0 & \Sigma \end{pmatrix}^{-1} \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} \times \left[ \begin{pmatrix} I & P^T \end{pmatrix} \begin{pmatrix} \tau \Omega & 0 \\ 0 & \Sigma \end{pmatrix}^{-1} \begin{pmatrix} \Pi \\ V \end{pmatrix} \right] \\
= \left[ \begin{pmatrix} (\tau \Omega)^{-1} & P^T \Sigma^{-1} \end{pmatrix} \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} \times \left[ \begin{pmatrix} (\tau \Omega)^{-1} & P^T \Sigma^{-1} \end{pmatrix} \begin{pmatrix} \Pi \\ V \end{pmatrix} \right] = \left[ (\tau \Omega)^{-1} + P^T \Sigma^{-1} P \right]^{-1} \times \left[ (\tau \Omega)^{-1} \Pi + P^T \Sigma^{-1} V \right]
\]

Non-Bayesian proof, taken from „Asset Allocation Model“, Daniel Blamont, Global Markets Research, Dt.Bank, July 30 2003
For Bayesian proof see, e.g., Fusai and Meucci
BL Optimization Problem

Black-Littermann - the master formula

\[
E(R) = \left[ (\tau \Omega)^{-1} + P^T \Sigma^{-1} P \right]^{-1} \cdot \left[ (\tau \Omega)^{-1} \right] \cdot \Pi + P^T \Sigma^{-1} P \cdot P^{-1} V
\]

- Complex interaction between equilibrium returns and subjective return expectations
- First factor („Denominator“) : Normalisation
- Second factor („Numerator“) : Balance between \( \Pi \) (= equilibrium returns) and \( V \) (= Views). Inverse of covariance \((\tau \Omega)^{-1}\) and confidence \(P^T \Sigma^{-1} P\) serve as weighting factors.
- Constituents:
  - Matrix \( \tau \Omega \) : covariance of historical returns, \( \tau \) = parameter
  - Matrix \( P \) : formal aggregation of Views
  - Matrix \( \Sigma^{-1} \) : confidence in Views (\( \Sigma \) = „covariance of estimated Views“)
  - \( \Sigma \) assumed to be diagonal, i.e. no cross-informations on Views.
- Limiting case “no estimates“ \( \Leftrightarrow P = 0 \): \( E(R) = \Pi \) i.e. BL-returns = equil. returns.
- Limiting case “no estimation errors“ \( \Leftrightarrow \Sigma^{-1} \to \infty \): \( E(R) = P^{-1} V \) i.e. BL-returns = View returns.
Black-Litterman Approach - remarks

Use of CAPM to determine equilibrium returns

- Alternative to inverse optimization: Equilibrium returns $\Pi_{Eq}$ from CAPM
- Additional input for CAPM: Risk-free rate $r_f$, risk premium vs market ($M$), Beta coefficients
- Evaluation:
  $$\Pi_{i,Eq} = r_f + \beta_i \cdot (r_M - r_f)$$
  with
  $$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

BL returns as a Bayesian a-posteriori estimator

- Bayes Theorem (or “Law” or “Rule”) states how to determine conditional expectations.
- Given an a-priori known distribution of a random variable. Adding new information leads to a revised conditional distribution, the so-called a-posteriori distribution (result of „learning“).
- BL-return estimates are a-posteriori (multivariate) normally distributed return expectations.
Parameters in BL

**Black-Litterman Approach - \( \gamma \)**

Remark: Risk aversion parameter \( \gamma \)

- How does risk change for an additional bp of return?

**Suggestions:**

Satchell & Scowcroft and Best & Grauer:

Let \( \gamma = (r_M - r_f) / \sigma_M^2 \)

where \( \sigma_M^2 = w^T \Omega w \), \( w = \) market cap.

Zimmermann et al.: \( \sigma_M = 16.9\% \) p.a. for STOXX-data (own calculation)

Chose \( \gamma = 3 \), which corresponds to a risk premium of 8.6%.

Idzorek: (DJIA, USA): Risk premium=7.5%: \( \gamma = 2.25 \).
Black-Litterman Approach - $\tau$

Remark: Scaling parameter $\tau$ for the covariance matrix.

Results/Setting

- Covariances of expected returns are proportional to historical covariances: $\tau \Omega$.
- $\tau$ measures confidence in benchmark, i.e. overall balance between BM and Views.
- $\tau$ small: $\text{VAR}[E(R)] \ll \text{VAR}[\text{historical returns}]$
- $\tau = 0.3$ “plausible” (used for numerical evaluations throughout).
Black-Litterman Approach - some real problems

Additional remarks on the recent remarks

- Calibration problems with parameter $\tau$ ("plausible", "adjusted to IR=1", ... )
- Calibration problems with parameter $\gamma$ ("world wide risk aversion", ... )
- Calibration problems with expressing the degree of confidence ("1..3", "0..100%")
BL-Views

Black-Litterman Approach - Views
Implementing Views on expected returns deviating from equilibrium figures

Views
- Return estimates differing from the (strategic) equilibrium returns are the essential input to the BL estimation process.

Specification of Views
- ... as absolute return expectations for individual assets and / or
- ... as relative return expectations relating assets or aggregates of assets.
  Formal constraint: \#Views \leq \#Assets.

Confidence
- Each View has to be assigned the level of confidence for an interval of uncertainty.

Selective Views
- Views can be restricted to selected assets for which in-depth analysis is available.
BL-Views

**Black-Litterman Approach - Views**

- **A relative View** can be stated as follows: „The sectors Pharmacy and Industry will outperform Telecom and Technology by 3% ± 1% with a confidence of 90%“:

\[
\left[ w_{PHRM} \cdot E(R_{PHRM}) + w_{INDU} \cdot E(R_{INDU}) \right] - \left[ w_{TELE} \cdot E(R_{TELE}) + w_{TECH} \cdot E(R_{TECH}) \right] = 3\% + (0.61\%)^2
\]

- Basically: A long-portfolio with outperformers, a short-portfolio with underperformers.

- **An absolute View** can be stated as follows: „The sector of Non-Cyclical Goods (CNYL) will perform better than stated by the equilibrium return of 6.66%. Our new target return is 7.5% with 90% of confidence within a range of ±1.5%“:

\[ 1 \cdot E(R_{CNYL}) = 7.5\% + (0.91\%)^2 \]
BL-Views

Black-Litterman Approach - combining Views I

Formal aggregation of Views

- Relative and absolute Views are aggregated in a system of linear equations:

\[ P \cdot E(R) = V + e \]

where (\( k = \#\text{Views} \) and \( n = \#\text{Assets}, \) with \( k \leq n \)):

- \( E(R) \) = \( n \times 1 \) vector of expected asset returns, unknown
- \( P \) = \( k \times n \) matrix, weighting the assets
- \( V \) = \( k \times 1 \) vector, absolute / relative return expectations (i.e., levels or over-/underperforming)
- \( e \) = \( k \times 1 \) vector of squared StDev’s
  (note that \( \Sigma^{-1} \) is a \( k \times k \) diagonal matrix expressing confidence (assuming independent estimation errors) with \( \Sigma^{-1}_{ii} = e_i \))

- This relation is incorporated in the BL master equation.
Black-Litterman Approach - combining Views II

Result shown from the example mentioned on page 24.

- Combining the aforementioned Views using \( P \cdot E = V + e \), we get:

\[
P \cdot \begin{pmatrix} E(R_{AUTO}) \\ \vdots \\ E(R_{UTLY}) \end{pmatrix} = \begin{pmatrix} 3\% \\ 7.5\% \end{pmatrix} + \begin{pmatrix} (0.61\%)^2 \\ (0.91\%)^2 \end{pmatrix}
\]

- The View-related weights \( P \) are given by:

\[
P = \begin{pmatrix} View1, rel. \\ View2, abs. \end{pmatrix} = \begin{pmatrix} 0 & \cdots & 0 & 0.34 & 0 & 0 & 0.66 & 0 & -0.51 & -0.49 & 0 \\ 0 & \cdots & 1 & 0 & & & & & & & \end{pmatrix}
\]
Black-Litterman Approach - confidence

Technical note on „confidence“

- Comment on determination of $e$: The fact that the amount of, e.g., relative outperformance (View 1) of 3% ±1% is assigned a 90% probability is interpreted within a normal distribution.

- $\text{mean} = 3\%$ and $\text{variance} = \text{VAR} = \sigma^2 = (0.61\%)^2 = e_1 \equiv \Sigma_{11}$. 

\[
\sigma = 0.61\% \\
\pm 1.664 \sigma
\]
Black-Litterman Approach - example in detail
„Sector allocation, Dow Jones STOXX“

- Example follows the lines of
  „Einsatz des Black-Litterman-Verfahrens in der Asset Allocation“, H.Zimmermann et al. 
  publ. in „Handbuch Asset Allocation“
  (Editors: Dichtl, Schlenger u. Kleeberg, publ. by Uhlenbruch-Verlag, 2002).
- Notation, scenarios and data therein have been used, some data were missing.

- Missing data - volatilities and covariances - had to be calculated from scratch, thus causing 
  some deviations in the numerical results between this presentation and cited literature.
  Nevertheless, all relevant results are reproduced.

- All calculations can be (have been) implemented and performed in Excel (TM).
Black-Litterman Approach - the data

Sectors in the Dow Jones STOXX index


<table>
<thead>
<tr>
<th>Sector</th>
<th>hist.Return</th>
<th>hist.Volatility</th>
<th>MarketCap</th>
</tr>
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<tbody>
<tr>
<td>total:</td>
<td>18</td>
<td>16.22%</td>
<td>100.01%</td>
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<td>AUTO</td>
<td>8.32%</td>
<td>25.09%</td>
<td>1.65%</td>
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<td>BANK</td>
<td>15.14%</td>
<td>21.21%</td>
<td>15.04%</td>
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<tr>
<td>BRES</td>
<td>7.31%</td>
<td>23.56%</td>
<td>1.22%</td>
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<tr>
<td>CHEM</td>
<td>12.25%</td>
<td>20.81%</td>
<td>1.80%</td>
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<tr>
<td>CONS</td>
<td>6.56%</td>
<td>18.92%</td>
<td>1.26%</td>
</tr>
<tr>
<td>CYCL</td>
<td>5.24%</td>
<td>19.94%</td>
<td>2.85%</td>
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<tr>
<td>CNYL</td>
<td>11.80%</td>
<td>16.66%</td>
<td>2.90%</td>
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<tr>
<td>ENGY</td>
<td>14.92%</td>
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<td>15.72%</td>
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<tr>
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<td>11.77%</td>
<td>17.25%</td>
<td>4.83%</td>
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</tbody>
</table>

Historical Data - Risk/Return Characteristics
**Black-Litterman Approach - the correlations**

Correlation matrix of Dow Jones STOXX sectors

- Calculation based on monthly returns

<table>
<thead>
<tr>
<th>AUTO</th>
<th>BANK</th>
<th>BRES</th>
<th>CHEM</th>
<th>CONS</th>
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- Covariance matrix via  \( \Omega_{ij} = \sigma_i \sigma_j \rho_{ij} \).
Black-Litterman Approach - the equilibrium returns

BL-starting point: Equilibrium - implicit - returns of market portfolio

- “inverse optimization“ yields equilibrium returns

\[ R_{Equilibrium} = (\tau \Omega) \cdot w_{Equilibrium} \]

where \( w_{Equilibrium} = w_{Market\ Portfolio} \)

Historical and equilibrium returns
Black-Litterman Approach - the BL returns

From equilibrium returns to Black-Litterman returns (Views as given)

Equilibrium and Black-Litterman returns

- BL return expectations significantly lowered for TECH und TELE.
- BL return expectation higher in PHRM but lower in INDS (still okay because the relative View „... better than TELE und TECH“ remains intact !)
- For CNYL, the expected return shifts from 6.66% to 7.48% (90% confidence in View 7,5%).
- Example: MEDA (correlated by 75% to TECH, 77% to TELE) has significantly lower return.
Black-Litterman Approach - the BL weights
Comparing equilibrium weights (market cap.) and Black-Litterman weights

Using the BL returns (note: strong confidence in Views of 90%):
- Significant weight reduction for TELE and TECH (also, e.g., for MEDA)
- Significant weight increase for INDS and PHRM
- Significantly increased weight for CNYL due to higher return expectation (+0.84% pts)
Key Features

Black-Litterman Approach vs straight MV

Given the return scenario (Views up or down, ΔΔ), the revised portfolio structure clearly benefits from the BL-enhanced optimization process.

- **Straight MV optimization** - which is a naive approach in terms of „c.p.“ return estimates - yields extreme and unreliable changes in portfolio weights
- **The BL-adjusted return input for MV** stabilizes the weights, leading to a reliable and intuitively sound new portfolio structure.
Example: DJ STOXX

**Black-Litterman Approach - constraints I**
Calculation of weights s.t. constraints

- **In general:** Use mean/variance optimizer with constraints

- **No constraints:**

\[ w = \frac{1}{\gamma} \Omega^{-1} \cdot R \]

- **“Budget constraint”, i.e. sum of weights = 100% (I = 1-Vector):**

\[ w = \frac{\Omega^{-1} I}{I^T \Omega^{-1} I} + \frac{1}{\gamma} \Omega^{-1} \left( R - \frac{I^T \Omega^{-1} R}{I^T \Omega^{-1} I} I \right) \]
Example: DJ STOXX

**Black-Litterman Approach - constraints II**

Calculation of weights s.t. constraints

- **Additional constraint: Tracking Error** ($w_{act} = \text{active weights}$)

\[
TE^2 = w_{act}^T \cdot \Omega \cdot w_{act}
\]

- **Additional constraint: Portfolio-BETA** („directional risk in the portfolio“)

\[
\beta_p = \sum_{i=1}^{\#\text{Assets}} w_i \beta_i
\]

- **Additional constraint: „No short positions“**

\[
w_i \geq 0 \quad \forall \ i = 1..\#\text{Assets}
\]

(see, e.g., paper of K. Iordanidis); in Excel: requires additional calculations & solver constraints
Black-Litterman Approach - weights

Remark on treating weights w.r.t. absolute / relative Views

- The sum of portfolio weights has to add up to \(100\%\).

- **Purely absolute Views** are translated into independent long and short portfolios, thus causing portfolio weights to deviate from 100%. Therefore, normalization of weights is recommended.

- **Purely relative Views** are translated into weight-balanced long and short portfolios, so that portfolio weights still sum up to 100%.

- The use of **absolute and relative Views** again leads to portfolio weights deviating from 100%. Therefore, again, normalization of weights is recommended.

- The normalization to 100% has to be included in the optimization process as a constraint.

- Note that - unfortunately - the use of constraints is contra-intuitive for BL-adjusted allocations.
Black-Litterman Approach
Numerical example on treating weights w.r.t. absolute / relative Views

- The example is based on the View scenario given on slide 24.

  **Purely absolute View:** Weights add up to 110%, with all weights unchanged except for the asset (sector) CNYL under view (weight increases by 10% pts.). Normalization to 100% leads to weight changes in all positions - as it should be!
  (Note: Lowering the expected return of CNYL to, e.g., 5.5% yields a total portfolio weight of only 86%. Again, weight normalization is recommended, spreading for the -14% pts across all weights.)

  **Purely relative Views:** Weights add up to 100%, with the weights of the viewed assets just offsetting the long and short positions.

  **Absolute and relative Views:** Weights add up to 129%, with long and short positions for the viewed assets as intuitively expected and the unviewed assets’ weights remaining unchanged. Normalization to 100% consequently leads to weight changes in all positions - as it should be!
Black-Litterman Approach (cont.) non-normalized weights

**Purely relative Views**
Σ weights = 100%, no normalization required

**Purely absolute Views**
Σ weights = 110%, normalization recommended
Black-Litterman Approach

Numerical example (cont.)

Relative and absolute Views

- weights normalized
  \( \Sigma \) weights = 100%

- weights not normalized
  \( \Sigma \) weights = 129%
Black-Litterman Approach - confidence and CNYL
Influence of degree of confidence on BL-returns and BL-weights, 1

Focus on asset „CNYL“ only:
- Equilibrium return = 6.66%,
- Equilibrium weight = 2.90%
- View on return: 7.5% ± 1.5% (equivalent to a range of 6 - 9%)

Observations:
- Low confidence: → equilibrium return
- High confidence: Asymptotic approach to the View value of 7.5%.
- Limit: At a confidence level of 100%, BL fully accepts the strong view of 7.5%.
- Weights: from 2.9% (= market cap, due to confidence of 0% a no view-case), up to 12% (overweighting due to the strong view confidence of 100%).
Black-Litterman Approach - weights of CNYL I

BL compared to straight MV

View on Return:
from 6.6% up to 7.5%
(with strong confidence)

Result
- Realistic weight changes in BL
- Volatile weight scenario in straight MV approach
Example: DJ STOXX

Black-Litterman Approach - weights of CNYL II
BL compared to straight MV

View on Return:
from 6.6% down to 5.5%
(with strong confidence)

Result
- Realistic weight changes in BL
- Volatile weight scenario in straight MV approach
Black-Litterman Approach - confidence and weights

Influence of degree of confidence on BL-returns and BL-weights, II

Behavior of asset weights:
- Complete portfolio, 18 sectors

Observations:
- Low degrees of confidence:
  BL-weights are close to weights in equilibrium (=market cap’s).
- Higher degree of confidence:
  Weights approach equilibrium values on either underweighting (short) or overweighting (long) path.
- Most significant weight changes for the assets under View.

Sensitivity of weights on degree of confidence
Black-Litterman Approach

Strong confidence

- Large changes in weights due to the "strong views"
Example: DJ STOXX

Black-Litterman Approach
Mid confidence

- Moderate changes in weights

Equilibrium and Black-Litterman returns

Equilibrium and Black-Litterman weights

+ 28%
Example: DJ STOXX

Black-Litterman Approach

Poor confidence

- Weights stay close to equilibrium weights
### Black-Litterman Approach - Conclusion I

**Traditional „Straight MV“ vs „BL plus MV“ approach**

<table>
<thead>
<tr>
<th></th>
<th>straight MV</th>
<th>Black-Litterman → MV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Return estimates:</strong></td>
<td>o required for each asset</td>
<td>required only for selected assets</td>
</tr>
<tr>
<td></td>
<td>o assumed as certain</td>
<td>degree of confidence</td>
</tr>
<tr>
<td></td>
<td>o absolute return figures</td>
<td>absolute or relative Views</td>
</tr>
<tr>
<td></td>
<td>o c.p.</td>
<td>consistent</td>
</tr>
<tr>
<td><strong>Reference return:</strong></td>
<td>o none</td>
<td>equilibrium returns</td>
</tr>
</tbody>
</table>

- **Key Features**
  - Return estimates:
  - o required for each asset
  - o assumed as certain
  - o absolute return figures
  - o c.p.
  - o none
  - equilibrium returns
# Black-Litterman Approach - Conclusion II

Traditional „Straight MV“ vs „BL plus MV“ approach

<table>
<thead>
<tr>
<th>straight MV</th>
<th>Black-Litterman → MV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MV-optimized Portfolios:</strong></td>
<td></td>
</tr>
<tr>
<td>o extreme asset weights</td>
<td><strong>reliable asset weights</strong></td>
</tr>
<tr>
<td>o changes in return estimates</td>
<td><strong>moderate weight changes</strong></td>
</tr>
<tr>
<td>⇒ huge weight fluctuations</td>
<td>⇒ moderate weight changes</td>
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<tr>
<td>o portfolios unreliable</td>
<td><strong>consistent structure</strong></td>
</tr>
<tr>
<td>o MV-results hardly accepted</td>
<td>„intuitively reasonable“</td>
</tr>
<tr>
<td>o reflecting c.p. opinions</td>
<td><strong>higher acceptance</strong></td>
</tr>
<tr>
<td></td>
<td>„correlated Views“</td>
</tr>
</tbody>
</table>
Black-Litterman Approach - Conclusion III
BL as a building-block of an enhanced asset allocation process

Key Features

BL as a building-block of an enhanced asset allocation process

Research Opinion

Intermediate Step:
- Black-Litterman
- Views & Confidence

BL-adjusted Returns

Return Estimates

( + Set of Constraints )

MV-optimized Portfolio Weights

Generic Investment Process

BL-tactical level

BL-strategic level

Market

Key elements of the traditional process
Black-Litterman subprocess
Black-Litterman Approach - Conclusion IV

... in a more formal way

- Global risk aversion $\lambda$
- Capitalization weights ($w$)
- Covariance matrix ($\Omega$), scaled by $\tau$

Implied returns ($\Pi$) → Prior distribution → Posterior Distribution

Prior distribution: MultiNormal ($\Pi, \tau\Omega$)

Posterior Distribution: MultiNormal (combined $P$ and $V$, combined $\tau\Omega$ and $\Sigma$)

- Views on (particular) returns
- Confidence in Views

(Confidence in Views)$^{-1}$

MultiNormal ($V, \Sigma$)

Layout inspired by K.Iordanidis
Black-Litterman Approach - more insights

Suggestions for further reading...


in German:
- Zwahlen St.: Kritische Analyse des BL-Ansatzes, Seminararbeit, Uni St.Gallen-HSG, Jul.2004