



**Yale ICF Working Paper No. 00-60**

February 22, 2002

## **LONG-TERM GLOBAL MARKET CORRELATIONS**

**William N. Goetzmann**

Yale School of Management

**Lingfeng Li**

Yale University

**K. Geert Rouwenhorst**

Yale School of Management

This paper can be downloaded without charge from the  
Social Science Research Network Electronic Paper Collection:

<http://papers.ssrn.com/abstract=288421>

# Long-Term Global Market Correlations

William N. Goetzmann, *Yale School of Management*

Lingfeng Li, *Yale University*

K. Geert Rouwenhorst, *Yale School of Management*

February 22, 2002

## Abstract:

In this paper we examine the correlation structure of the major world equity markets over 150 years. We find that correlations vary considerably through time and are highest during periods of economic and financial integration such as the late 19<sup>th</sup> and 20<sup>th</sup> centuries. Our analysis suggests that the diversification benefits to global investing are not constant, and that they are currently low compared to the rest of capital market history. We decompose the diversification benefits into two parts: a component that is due to variation in the average correlation across markets, and a component that is due to the variation in the investment opportunity set. There are periods, like the last two decades, in which the opportunity set expands dramatically, and the benefits to diversification are driven primarily by the existence of marginal markets. For other periods, such as the two decades following World War II, risk reduction is due to low correlations among the major national markets. From this, we infer that periods of globalization have both benefits and drawbacks for international investors. They expand the opportunity set, but diversification relies increasingly on investment in emerging markets.

JEL classification code: G10 G15

Acknowledgements: We thank Ibbotson Associates for providing data. We thank George Hall for suggesting sources of WWI data. We thank Geert Bekaert, Campbell Harvey, Ricardo Leal, and seminar participants at the Stockholm School of Economics and Yale School of Management for suggestions. We thank the International Center for Finance at Yale for support.

Author contact [william.goetzmann@yale.edu](mailto:william.goetzmann@yale.edu).

# Long-Term Global Market Correlations

Abstract:

In this paper we examine the correlation structure of the major world equity markets over 150 years. We find that correlations vary considerably through time and are highest during periods of economic and financial integration such as the late 19<sup>th</sup> and 20<sup>th</sup> centuries. Our analysis suggests that the diversification benefits to global investing are not constant, and that they are currently low compared to the rest of capital market history. We decompose the diversification benefits into two parts: a component that is due to variation in the average correlation across markets, and a component that is due to the variation in the investment opportunity set. There are periods, like the last two decades, in which the opportunity set expands dramatically, and the benefits to diversification are driven primarily by the existence of marginal markets. For other periods, such as the two decades following World War II, risk reduction is due to low correlations among the major national markets. From this, we infer that periods of globalization have both benefits and drawbacks for international investors. They expand the opportunity set, but diversification relies increasingly on investment in emerging markets.

JEL classification code: G10 G15

## I. Introduction

There is considerable academic research that documents the benefits of international diversification. Grubel (1968) finds that between 1959 and 1966, U.S. investors could have achieved better risk and return opportunities by investing part of their portfolio in foreign equity markets. Levy and Sarnat (1970) analyze international correlations in the 1951-1967 period, and show the diversification benefits from investing in both developed and developing equity markets. Grubel and Fadner (1971) show that between 1965 and 1967 industry correlations within countries exceed industry correlations across countries. These early studies marked the beginning of an extensive literature in financial economics on international diversification. However, the benefits to international diversification have actually been well-known in the investment community for much longer. The 18<sup>th</sup> century development of mutual funds in Holland was predicated on the benefits of diversification through holding equal proportions of international securities.<sup>1</sup> The quantitative analysis of international diversification dates at least to Henry Lowenfeld's (1909) study of equal-weighted, industry-neutral, risk-adjusted, international diversification strategies, using price data from the global securities trading on the London Exchange around the turn of the century. His book, *The Geographical Distribution of Capital* is illustrated with graphs documenting the imperfect co-movement of securities from various countries. Based on these, he argues that superior investment performance can be obtained by spreading capital in equal proportion across a number of geographical sectors and carefully re-balancing back to these proportions on a regular basis.

It is significant to see how entirely all the rest of the Geographically Distributed stocks differ in their price movements from the British stock. It is this individuality of movement on the part of each security, included in a well-distributed Investment List, which ensures the first great essential of successful investment, namely, Capital Stability.<sup>2</sup>

---

<sup>1</sup> For example, the 1774 "Negotiatie onder de Zinspreuk *EENDRAGT MAAKT MAGT*" organized by Abraham van Ketwich, obliged the manager to hold as close as possible an equal-weight portfolio of bonds from the Bank of Vienna, Russian government bonds, government loans from Mecklenburg and Saxony, Spanish Canal loans, English colonial securities, South American Plantation loans and securities from various Danish American ventures, all of which were traded in the Amsterdam market at the time.

<sup>2</sup> Lowenfeld, Henry, 1909, *Investment an Exact Science*, The Financial Review of Reviews, London. p. 49.

Considering the widespread belief in the benefits to international diversification over the past 100 to 200 years, and the current importance of diversification for research and practice in international finance, we believe that it is important to examine how international diversification has actually fared -- not just over the last 30 years since the beginnings of academic research, but over much longer intervals of world market history. In this paper, we use long-term historical data to ask whether the global diversification strategies developed by Henry Lowenfeld and his predecessors actually served investors well over the last century and a half. In addition, we consider the long-term lessons of capital market history with regard to the potential for international diversification looking forward.

The first contribution of our paper is to document the correlation structure of world equity market over the period from 1850 to the present using the largest available sample of time-series data we can assemble. Stock market data over such long stretches are inevitably messy and incomplete. Despite the limitations of our data, we find that international equity correlations change dramatically through time, with peaks in the late 19<sup>th</sup> century, the Great Depression and the late 20<sup>th</sup> Century. Thus, the diversification benefits to global investing are not constant. Perhaps most important to the investor of the early 21<sup>st</sup> Century is that the international diversification potential today is very low compared to the rest of capital market history. Recent history bears a close resemblance to the turn of the 19th century, when capital was relatively free to flow across international borders. While capital market integration does embed a prediction about the correlations between markets, we find that periods of free capital flows are associated with high correlations.

One important question to ask of this data is whether diversification works when it is most needed. This issue has been of interest in recent years due to the high correlations in global markets conditional upon negative shocks. Evidence from capital market history suggests that periods of poor market performance, most notably the Great Depression, were associated with high correlations, rather than low correlations. Wars were associated with high benefits to diversification, however these are precisely the periods in which international ownership claims may be abrogated, and international

investing in general may be difficult. Indeed, investors in the past who have apparently relied upon diversification to protect them against extreme swings of the market have been occasionally disappointed. In 1929, the chairman of Alliance Trust Company, whose value proposition plainly relied upon providing diversification to the average investor lamented:

“Trust companies...have reckoned that by a wide spreading of their investment risk, a stable revenue position could be maintained, as it was not to be expected that all the world would go wrong at the same time. But the unexpected has happened, and every part of the civilized world is in trouble...”<sup>3</sup>

The Crash of 1929 thus not only surprised investors by its magnitude, but also by its international breadth. As we show in this paper, the Great Crash was associated with a structural change in not only the volatility of world markets, but in the international correlations as well. Average correlations went up and reached a peak in the 1930’s that has been unequaled until the modern era. Although global investing in the pre-War era, as now, was facilitated by relatively open capital markets and cross-border listing of securities, the ability to spread risk across many different markets was less of a benefit than it might have at first appeared.

The second contribution of our paper is that we provide a decomposition of the benefits of international diversification. To examine the interplay between global market liberalization and co-movement, we focus our analysis on two related sources of the benefits to diversification, both of which have affected investor risk throughout the last 150 years. The first source is the variation in the average correlation in equity markets through time. The second source is the variation in the investment opportunity set. This decomposition is a useful framework for understanding the benefits of global investing – markets come and go in the world economy, and the menu of investment choices at any given time may have an important effect on diversification. For example, in the last two decades, the opportunity set expanded dramatically at the same time correlations of the major markets has increased. As a result, the benefits to international diversification have

---

<sup>3</sup> Quoted in Bullock, Hugh, 1959, The Stork of Investment Companies, Columbia University Press, New York.

recently been driven by the existence of emerging capital markets – smaller markets on the margin of the world economy where the costs and risks of international investing are potentially high. For other periods, such as the two decades following the era of World War II, risk reduction derived from low correlations among the major national markets. From this, we infer that periods of globalization have both benefits and drawbacks for the international investor. They expand the opportunity set, but the diversification benefits of cross-border investing during these periods relies increasingly on investment in emerging markets.

A third contribution of this paper is the development of a new econometric test for hypotheses about shifts in the correlation among markets through time. We construct tests about not only the change in the correlation matrix between time periods, but about the change in the average correlation across markets. Bootstrap studies of the robustness of these tests show they work well as a basis for distinguishing among periods of differing asset correlations. The results of our tests show that we convincingly reject the constancy of the global correlation structure between various periods in world economic history in our sample.

The remainder of the paper is organized as follows. The next section reviews the literature on capital market correlations. Section III contains a description of our data. The fourth section presents our empirical results including our decomposition of the benefits of diversification, followed by our conclusions in Section V.

## **II. History and Prior Research**

The theoretical and statistical evidence on international diversification, market integration, and the correlation between markets, beginning with the early empirical studies cited above, is legion. Because we take a longer temporal perspective, however, the historical framework is important as well. Recent contributions in economic history have been useful in comparing and contrasting the recent period of international cross-border investing to earlier periods in world history. Bordo, Eichengreen and Kim (1998), for example, use historical data to argue for a period of market integration in the

pre-1914 era. Prakash and Taylor (1999) uses the experience of the pre-World War I era as a guide to understand current global financial flows and crises. Obstfeld and Taylor (2001) further document the relation between the integration of global capital flows and relate this directly to the temporal variation in the average correlation of world equity markets. Goetzmann, Ukhov and Zhu (2001) document parallels to China's encounter with the global markets at the turn of the 19<sup>th</sup> and the turn of the 20<sup>th</sup> centuries. The broad implication of these and related studies is that the modern era of global investing has parallels to the pre-WWI era. Indeed, the period from 1870 to 1913 was, in some ways, the golden era of global capitalism. As Rajan and Zingales (2001) convincingly show, the sheer magnitude of the equity capital listed on the world's stock markets in 1913 rivaled the equity listings today in per capital terms. Following this peak, the only constant is change. The sequence of World War I, hyperinflation, Great Depression, World War II, the rise of Stalinist socialism and the de-colonialization of much of the world had various and combined effects on global investing, affecting not only the structural relationship across the major markets such as U.S., U.K., France, Germany and Japan, but also affecting the access by less developed countries to world capital. While world market correlations of the major markets affect the volatility of a balanced international equity portfolio, at least as important to the international investor of the twentieth century is the number, range and variety of markets that emerged or re-emerged in the last quarter of the 20<sup>th</sup> Century following the reconstruction of global capitalism on post-colonial foundations.

In addition to studies in economic and financial history, there is considerable literature attempting to understand shifts in the correlation structure of world equity markets and the reasons for their low correlations in the late 20<sup>th</sup> century. Longin and Solnik (1995) study the shifts in global correlations from the 1960 to 1990. This analysis leads to the rejection of the hypothesis of a constant conditional correlation structure. Their more recent study Longin and Solnik (2001), focusing on the correlation during extreme months, find evidence of positive international equity market correlation shifts conditional upon market drops over the past 38 years.

To address the underlying reason for international market correlations, Roll (1992), proposes a compelling Ricardian explanation based upon country specialization.



However Heston and Rouwenhorst (1994) show that industry differences and country specializations by industry cannot explain the degree to which country stock markets co-move. They find that country effects -- whether due to fiscal, monetary, legal, cultural or language differences -- dominate industrial explanations. Other authors have investigated the possibility that international market co-movements are due to co-variation in fundamental economic variables such as interest rates and dividend yields. Campbell and Hamao (1992) show how these fundamentals drive co-movement between Japan and the U.S. Bracker, Docking and Koch (1999) propose bi-lateral trade and its macro-economic and linguistic determinants as a cause of international stock market co-movement. These studies, however, are limited to the most recent time-period of capital market history.

A lack of market integration has also been proposed as an explanation for low co-movement. Chen and Knez (1995) and Korajczyk (1996), for example, tested international market integration using asset-pricing tests that consider the international variation in the price of risk. This approach is based upon the presumption that the market price of systematic risk factors may differ across markets due to informational barriers, transactions barriers and costs of trade, but it is silent on the root cause of risk-price differences and the determinants of market co-movement.

Yet another strand of literature about the co-movement of equity markets focuses on the econometric estimation of parametric models of markets that allow first, second and third moments to co-vary depending upon institutional structures that facilitate international investor access to markets. Bekaert and Harvey (1995, 2000) provide direct evidence that market integration and financial liberalization change the co-movement of emerging markets stock returns with the global market factor. The implication is that evolution from a segmented to an integrated market fundamentally changes the co-movement with other markets as well.

### III. Historical Data

Because the benefits of diversification depend critically on both the number and the performance of international capital markets, our analysis uses cross-sectional, time series information about the returns to the world's stock markets. In this study, we draw from four data sources: Global Financial Data [GFD]<sup>4</sup>, the Jorion and Goetzmann (1999) [JG] sample of equity markets, The Ibbotson Associates database of international markets [IA] and the IFC database of emerging markets. These recent efforts to assemble global financial data have vastly improved the information available for research. Nevertheless, our analysis is still hampered by an incomplete measure of the international investor opportunity set over the 150-year period. Our combined sample includes markets from Eastern and Western Europe, North and South America, South and East Asia, Africa and Australasia, however there are notable holes. In particular, no index exists for the Russian market over the one hundred years of its existence, nor are continuous data available for such potentially interesting markets such as Shanghai Stock Exchange from the 1890's to the 1940's and The Teheran Stock Exchange in the 1970's. As a consequence, our analysis is confined to a sub-set of the markets that were available, and in all probability, to sub-periods of the duration over which one might trade in them. There are essentially two general data problems confronting our analysis. The first is that there may have been markets that existed and were investable in past periods that we have no record of, and are thus not a part of this study. For example, the origins of the Dutch market date back to the early 1600's, but we do not have a market index for The Netherlands until 1919. The second is that we have historical time-series data from markets that existed, but were not investable, or that the surviving data provides a misleading measurement of the returns to measurement. To get a better sense of these two classes of problems, we collected what information we could on the known equity markets of the world. This data is represented

---

<sup>4</sup> Questions have been raised about the quality of early data series in GFD. GFD provides credible sources for most of the data series. We also compare its early series with other independent sources. US data and capital appreciation series turn to have better quality. For instance, GFD US stock price index is very close to an early NYSE index documented in Goetzmann, Ibbotson and Peng (2001). GFD US total return index is exactly the same as the total return index in Schwert (1990). Nonetheless, early dividend yield series are often problematic. For instance, early UK dividend yield only contains Bank of England shares until 1917. We carefully conduct analysis on both capital appreciation series and total return series, and compare how

in tabular form in Figure 1. There are more than eighty markets that appear to have existed at some time currently or in the past. As a guide to future potential data collection, we have represented what we believe to be the periods in which these markets operated and for which printed price data might be available. Colored bars indicate periods in which markets were open and closed, and periods described as crises. The broad coverage of this data table suggests the surprising age of equity markets of the world, as well as the extent to which markets closed as well as opened. Finally it suggests that the current empirical work in finance relies on just a very small sample of markets.

Table 1 reports the dates and summary information for the data we actually used in the analysis. It contains substantially less than the larger potential data set shown in Figure 1. As such we believe our analysis may be a conservative picture of the potential for international diversification, if indeed all-extant markets were available to all the world's investors at each point in time. Of course, constraints – particularly in times of war – likely hampered global diversification. Table 1 also lists current historical stock markets of the world with information about their founding dates. This provides some measure of the time-series and cross sectional coverage of our data. Table 2 provides annualized summary statistics for a set of eight representative countries that extended through most of the period of the sample.

The Global Financial Data compiled by Bryan Taylor contains monthly financial and economic data series from about 100 countries, covering equity markets, bonds markets and industrial sectors. Nevertheless, since the historical development of capital markets varies a great deal among these countries, they are not comparable in quality to the international equity index data we now enjoy. Some country indices such as U.K., U.S, France and Germany extend from the early 19<sup>th</sup> century, but their composition varies with the availability of securities data in different historical time periods. Beginning in the 1920's however, the League of Nations began to compile international equity indices with some standardization across countries, and these indices form the basis for the GFD series, as well as the JG analysis. The United Nations and then IFC apparently maintained

---

much our results vary with different choices of returns. Fortunately, from the perspective of investigating correlations, the low frequency dividend yield data do not impose a serious problem.

the basic methodology of international index construction through the middle of the 20th Century. International data from the last three decades has become more widely available, via the Ibbotson database that provides MSCI and FTSE equity series' as well as the IFC emerging markets data.

In order to maintain comparability across countries, we have converted monthly return series' to monthly dollar-returns. Where possible, we calculate both capital appreciation returns and total return series, but for many countries dividend data are unavailable. We have found that correlation estimates using total returns vary little from those using capital appreciation series, due to the fact that variations in dividend yields are much smaller compared with those in stock capital appreciation returns. Thus for several countries we use only capital appreciation converted to dollar returns. In total, we have been able to identify 50 total return or capital appreciation series', which we are then able to convert into dollar-valued returns.

In many countries, there are short periods during which markets were closed or data were simply not recorded. For instance, during World War I, the U.S. and U.K. markets closed briefly, and for other countries, there were even larger data gaps. The GFD is missing a block of returns for several European markets during the First World War. Fortunately, we are able to fill this gap with data collected by the Young Commission, which was formed by the US Congress to study the possibility of returning to Gold standard during the post WWI era. In some cases it is necessary for us to interpolate the months of closure for our correlation calculations.

#### **IV. The Benefits of International Diversification**

One of the most well known results in finance is the decrease in portfolio risk that occurs with the sequential addition of stocks to a portfolio. Initially, the portfolio variance decreases rapidly as the number of the securities increases, but levels off when the number of securities becomes large. Statman (1987) argues that most of the variance reduction is achieved when the number of stocks in a portfolio reaches 30. The intuition is that, while individual security variance matters for portfolios with few stocks, portfolio

variance is driven primarily by the average covariance when the number of securities becomes large. The lower the covariance between securities, the smaller the variance of a diversified portfolio becomes, relative to the variance of the securities that make up the portfolio. The primary motive for international diversification has been to take advantage of the low correlation between stocks in different national markets. Solnik (1976), for example, shows that an internationally diversified portfolio has only half the risk of a diversified portfolio of U.S. Stocks. In his study, the variance of a diversified portfolio of U.S. stocks approaches 27 percent of the variance of a typical security, as compared to 11.7 percent for a globally diversified portfolio. More recently, Ang and Bekaert (2002) show that, despite the risk of time varying correlations, under most circumstances, the benefits of international diversification are still significant.

Unfortunately, the lack of individual stock return data for more than the last few decades precludes us from studying the benefits of international diversification at the asset level. But given that these benefits are largely driven by the correlation across markets, a simple analogue can be constructed by comparing the variance of a portfolio of country indices relative to the variance of portfolios that invest only in a single country. This will provide a gauge to compare the incremental benefits of diversifying internationally rather than investing in a single domestic market. Figure 2 shows the variance of a portfolio of country returns as a fraction of the variance of individual markets. The full sample is divided into seven sub-periods following Basu and Taylor (1999): *I. early integration (1875-1889)*; *II. Turn of the century (1890-1914)*; *III. WWI (1915-1918)*; *IV. Between the wars (1919-1939)*; *IV. WWII (1940-1945)*; *VI. The Bretton Woods Era (1946-1971)*; *VII. Present (1972-2000)*. Across sub-samples, the variance of an internationally diversified portfolio ranges from less than 10% to more than 30% of the variance of an individual market. Countries are equal-weighted in these portfolios and all returns are measured in dollars at the monthly frequency.

Figure 2 illustrates the two main factors that drive the benefits of international diversification. The first factor is the average covariance – or correlation – between markets. A lower covariance rotates the diversification-curve downwards. The second important factor is the number of markets that are available to investors. An increase in the available markets allows investors to move down along a given diversification-curve.

This factor is important for a study of the long-term benefits of international diversification, because the number of available markets has varied a great deal over the past 150 years. The steady increase in the number of equity markets over the past century has provided additional diversification opportunities to investors. In the next two sections we attempt to separately measure the effects of changes in correlation and changes in the investment opportunity set. Most studies in the literature have concentrated only on the first effect and argued that globalization of equity markets has led to increased correlations among markets, thereby reducing the benefits of diversification. In addition to studying these correlations over the past 150 years, we pose the question in this paper to what extent a gradual increase in the investment opportunity set has been an offsetting force.

#### *IV.1. Equity market correlations over the last 150 years.*

Table 3 gives the correlations of the four major markets for which we have total return data available since 1872 – France, Germany, the United Kingdom, and the United States, organized by the sub-periods suggested by Basu and Taylor (1999). The average pair-wise correlation among the four major markets ranges from  $-0.073$  during WWI to  $0.475$  to the most recent period between 1972 and 2000. The correlation between the U.S. and the U.K. varies from near zero to over 50 percent, and the correlation between Germany and France ranges from  $-0.175$  during WWI – correlations are expected to be negative among battling neighbors – to  $0.62$  during the most recent period. The Table does not provide standard errors, however in the Appendix, we outline and implement a formal test that shows that these differences are indeed statistically significant. For now it is sufficient to conclude that there seems to be important variation in the correlation structure of major markets.

Figure 3 plots the average cross-country rolling correlations of the capital appreciation return series for the entire set of countries available at each period of time. Rolling correlations are calculated over a backward-looking window of 60 months. This figure illustrates a similar pattern to Table 3, namely that correlations have changed dramatically over the last 150 years. Peaks in the correlations occur during the period

following the 1929 Crash and the period leading up to the present. The period beginning in the late 19<sup>th</sup> and up to WWII, which marked the beginning of renewed segmentation, and the postwar period up to the present, both show gradual increases in the average correlations between countries. During the latter period, the increase in correlation appears initially less pronounced due to the fact that many submerged markets re-emerged, and “new” markets emerged for the first time.

This “U” shape in the correlation structure is noted by Obstfeld and Taylor (2001) for its close analogue to the pattern of global capital market flows over the same time period. For example, the authors present compelling evidence on the scale of cross-border capital flows during the height of the European colonial era, suggesting that global economic and financial integration around the beginning of 20<sup>th</sup> century achieved a level comparable to what we have today. Bordo, Eichengreen and Irwin (1999) examine this hypothesis and claim that, despite these comparable integration levels, today’s integration is much deeper and broader than what had happened in history. The implication of this clear historical structure is that the liberalization of global capital flows cuts two ways. It allows investors to diversify across borders, but it also reduces the attractiveness of doing so. In Section IV.3, we will separate the effects of correlation and variation in the investment opportunity set. The next section provides a formal test of changes in the correlation between markets over time.

#### *IV.2 Testing constancy of equity market correlations*

Are the temporal variations in the correlation structure statistically significant? This question has intrigued many researchers. Tests for constancy of correlations generally fall into two categories: testing unconditional correlations using multivariate theory, e.g., Kaplanis (1988); and testing conditional correlations using multivariate GARCH models, e.g., Engle (2000), Tse (2000). Longin and Solnik (1995) apply both tests. Our test falls into the first category.

The test is based on the asymptotic distribution of correlation matrix derived in Browne and Shapiro (1986) and Neudecker and Wesselman (1990). They show that, under certain regularity conditions, a vectorized correlation matrix is asymptotically

normally distributed. Based on this property, we compute the asymptotic distributions for any two correlation matrices of interest, and then derive test statistics in the spirit of the classical Wald test<sup>5</sup>. Utilizing the asymptotic distribution of correlation matrix, which is unavailable until recently, our test shows two improvements over other tests within this category. First, the tests that Kaplanis (1988) and Longin and Solnik (1995) employ require the returns to be normally distributed, and they only test the constancy hypothesis indirectly through a transformation. To the contrary, we are able to relax restrictive assumption of normal distribution of underlying return series, construct the test in a simpler framework, and test the hypothesis directly. Second, we validate our test results with bootstrapping procedure, and compute the power of the test, which the early tests do not provide. In addition, our test also has certain advantages over covariance-matrix based alternatives such as multivariate GARCH tests, in that it works directly with the correlation matrices and can be easily modified according to different hypotheses. Unlike GARCH-based tests it is not computationally intensive and is less prone to model misspecifications. Our test can be used to test cross sectional equality in correlations while GARCH-based test cannot.

We test two null hypotheses. The first is that the correlation matrices from two periods are equal element-by-element. This is equivalent to a joint hypothesis that the correlation coefficients of any two countries are the same in the two periods of interest. The second hypothesis is that the average of the cross-country correlation coefficients are the same in two periods. In most cases, the second hypothesis is a weaker version of the first. In the Appendix we discuss the details of the test and address issues of the size and power.

One major issue in the literature on the tests about correlations is the problem of conditioning bias. Boyer, Gibson and Loretan (1997) first show that if the measured (conditional) variance is different from the true (unconditional) variance, then the measured (conditional) correlation will also be different from the true (unconditional) correlation. Using a simple example, Longin and Solnik (2001) show that two series' with same unconditional correlation coefficient will have a greater sample correlation

---

<sup>5</sup> See the Appendix for details.



coefficient, conditional upon large observations. Therefore, an interesting question arises whether the observation variation in correlations is due to changes in underlying correlation or simply due to occurrence of large observations or time varying volatility. Our test is not subject to this criticism because we choose the periods strictly according to the existing literature and historical events – had we focused on high-variance vs. low variance periods, our tests could be biased towards rejection. Besides, a sufficient condition for conditioning bias to occur is that volatilities of the stock returns used to calculate correlations change unproportionally. However, this is not a dominant phenomenon in our long-term data<sup>6</sup>.

We conduct the tests on correlation matrices of dollar-valued capital appreciation<sup>7</sup> returns to the equity markets of four “core” countries: U.K., U.S., France and Germany. We show the p-value of the test statistics in Table 4. The first two sub-tables report results for the entire correlation matrix and on the average level of correlation respectively, using the asymptotic test. Because we have little guidance about the performance of the test in small sample, the other two sub-tables summarize the test statistics based on bootstrapping, in which the bootstrap randomly assigns dates to the respective time periods being tested. We also calculate the sample means and variances of the bootstrapped empirical distributions. If the asymptotic Chi-squared distribution worked perfectly in this case, then the mean of the test mean would be 6, which is the number of upper off-diagonal elements in a 4 by 4 matrix, and the variance would equal 12. In the second test, the mean and variance should equal 1 and 2 respectively. The close match between the asymptotic test results and those from the bootstrapped values suggests that the asymptotic test performs well in small sample and can be relied upon for tests of structural changes in correlations.

---

<sup>6</sup> Corsetti, Pericoli and Sbracia (2001) show that the discrepancy between conditional and unconditional correlation coefficients actually occurs only if the ratio of the conditional variances of two series is different from that of the unconditional variances. A similar point is made in Ang and Chen (2002). Intuitively, as long as the relative dispersion of two time series across periods is stable, the correlation coefficient computed for a given period should be close to its population value. In our dataset, we find that only Germany and, to a lesser extent, France, have ill-shaped return distributions during WWI and WWII. We include these two periods in the test only as reference.

<sup>7</sup> We only provide test results on capital appreciation returns because, for early periods, quality of dividend yield series is not as good as that of price series. Nonetheless, test results on total returns are very similar to those presented in Table 4.

The results in Table 4 suggest that the historical definition of eras in global finance also define significant differences in correlation structure. Starred values indicate rejection at the 5% level and double-starred values represent rejection at the 1% level. The 1972-2000 period stands out as the most unusual -- all tests reject element-by-element equality and means equality with other time periods. Thus, while historically the current era has many features in common with the golden age of finance around the turn of the last century, we are able to reject the hypothesis that the modern correlation structure and correlation average of the capital markets resembles that of a century ago. This supports the findings of Bordo, Eichengreen and Kim (1998) and Bordo, Eichengreen and Irwin (1999). These authors argue that, due to less information asymmetry, reduced transaction costs, better institutional arrangements, and more complete international standards, “integration today is deeper and broader than 100 years ago.” The period 1919-1939 is the second most unusual, with pair-wise rejections of equality with respect to five other periods. This is not surprising, given that this era encompasses the period of hyperinflation in Germany, and Great Depression – the latter being, by most accounts, the most significant global economic event in the sample period.

As pointed out earlier, financial theory does not predict changes in correlations based on integration or segmentation of markets. However, if we compare the average correlation during periods of relatively high integration (1870-1913 and 1972-2000) to the periods of relatively low integration (1914-1971), we overwhelmingly reject equality.

Similarities as well as differences are interesting in the table. It is tempting, for example, to interpret the rejection failure for the correlation matrices of the two pre-WWI periods – during which the gold standard prevailed – and the Bretton Woods period (1946-1971) as evidence that the gold standard and the Bretton Woods exchange rate system effectively achieved similar goals and resulted in a similar correlation structure for equity markets. This must remain only a conjecture, however, given that we have not proposed an economic mechanism by which such similarity would be achieved.

In sum, our tests indicate that stock returns for these four key countries were once closely correlated around the beginning of 20th century, during the Great Depression, under the Bretton Woods system and at Present period. However, except for two brief periods – *Early Integration* and *World War I* -- the correlation structures differ a great

deal. In fact, the era from 1972 to the present is virtually unique in terms of structure and level of market co-movements.

#### *IV.3 Decomposition of the Benefits of International Diversification*

Important though it may be, the correlation among markets is only one variable determining the benefits of international diversification. Another important factor is the number of investable markets that are available to foreign investors. Having said this, it is difficult to precisely measure which markets were accessible to U.S. investors at each point in time during the past 150 years, or the costs that were associated with cross-border investing, for that matter. While we have been able to collect considerable time-series information on returns, it is almost certainly incomplete. Figure 4 plots the number of markets for which we have return data available. The bottom line in the Figure plots the availability of the return data for the four countries for which we have the longest return histories – France, Germany, the United Kingdom and the United States. Occasionally the line drops below four, because of the closing of these markets during war. The top line presents the total number of countries that are included in our sample at each point of time. The figure shows the dramatic increase in the investment opportunity set during the last century. At the beginning of the 20<sup>th</sup> century we have only 5 markets, and at the end of the century around 50. Not all countries that enter the sample have a complete return history. For example, Czechoslovakia drops out of the sample shortly before WWII, but re-emerges towards the end of the century. Of course it then splits into two countries – only one of which is represented in our data. This submergence and re-emergence of markets is captured by the middle line in Figure 4, which represents the number of countries that are in the sample, for which we also have return data available. The important message of Figures 3 and 4 is that the past century has experienced large variation in both the number of markets around the world, as well as the return correlations among these markets, and the middle of the 20<sup>th</sup> century was, in some ways, a reversal of the trends at the beginning and the end of the sample period.

Contemporary investment manuals give us some sense of the number and range of international markets we are missing. Lowenfeld (1909) lists forty countries with stock

markets open to British investors, however many of these securities were investable via the London Stock Exchange listings, and thus may reflect strategies open only to U.K. investors.<sup>8</sup> Rudolph Taüber's 1911 survey of the world's stock markets provides a useful overview of the world of international investing before the First World War. He describes bourses in more than thirty countries around the world available to the German investor.<sup>9</sup> These two surveys, written to provide concrete advice to British and German investors in the first and second decade of the 20th Century suggest that if anything, our analysis vastly understates the international diversification possibilities of European investors a century ago. Of course, other investors at that time might have had considerably reduced access. Because of this issue, it is important to be able to separate the effects of average correlation from the effects of increasing numbers of markets.

We attempt to measure the separate influence of these two components by returning to our earlier graphs, which we used to illustrate the benefits of international diversification. Algebraically, the ratio of the variance of an equally-weighted portfolio to average variance of a single market is given by:

$$\frac{\text{Var}\left(\sum_{i=1}^n x_i / n\right)}{\frac{1}{n} \sum_{i=1}^n \text{Var}(x_i)} = \frac{\frac{1}{n^2} \sum_{i=1}^n \text{Var}(x_i)}{\frac{1}{n} \sum_{i=1}^n \text{Var}(x_i)} + \frac{\frac{1}{n^2} \sum_{i \neq j}^n \text{Cov}(x_i, x_j)}{\frac{1}{n} \sum_{i=1}^n \text{Var}(x_i)}$$

Using upper bars to indicate averages, this can be written as:

---

<sup>8</sup> Great Britain, India, Canada, Australia, Tasmania, New Zealand, Straits Settlements (Singapore), Belgium, Denmark, Germany, Holland, Norway, Russia, Sweden, Switzerland, Austria, Bulgaria, France, Greece, Italy, Hungary, Portugal, Romania, Spain, Serbia, Turkey, Japan (Tokyo and Yokohama), China (Shanghai and Hong Kong), Cape Colony, Natal, Transvaal, Egypt, New York, Mexico, Argentine, Brazil, Chile, Peru and Uruguay.

<sup>9</sup> These include Germany, Austria, Switzerland, the Netherlands, Norway, Sweden, Denmark, Russia, Serbia, Greece, Rumania, Turkey, Italy, Spain, Portugal, Belgium, France, Great Britain, Ireland, New York, Haiti, Dominican Republic, Ecuador, Brazil, Peru, Argentina, Uruguay, Chile, Columbia, Venezuela, Japan, South Africa, Natal, Egypt and Australia.

$$\frac{1}{n} + \left( \frac{n-1}{n} \right) \times \frac{\overline{Cov}(x_i, x_j)}{\overline{Var}(x_i)}$$

As the number of markets ( $n$ ) becomes large, this simply converges to the ratio of the average covariance among markets to the average variance. If the correlations among individual markets were zero, virtually all risk would be diversifiable by holding a portfolio that combined a large number of countries. By contrast, in times of high correlations, even a large portfolio of country indices would experience considerable volatility. With a limited number of international markets in which to invest, however,  $n$  may be small.

In order to separate the effects of changes in correlations and the secular increase of the investment opportunity set, we compute the above equation using 5-year rolling windows under three different scenarios.

1. Our base case is the 4 major markets with the longest return history (France, Germany, UK, US).
2. Next, we evaluate the equation for  $n=4$ , averaged over all combinations of 4 countries that are available at a given time.
3. Finally, we evaluate the expression using all countries that have available return histories in at a given point in our sample ( $n$ =maximum available).

The first scenario isolates the effect of changes in correlations if the investment opportunity set were limited to these four countries. In the second scenario we track the evolution of the diversification benefits of the “average” portfolio of four countries over time, not only those for which we have the longest return history. Because new markets have a different covariance structure than our base set, the difference between scenario (1) and (2) measures the influence that additional markets have on the covariance structure. Note that this influence can either be positive or negative, depending on whether the additional markets increase or decrease the average among markets. The final scenario gives the benefits of diversification for the full set of available countries at each point in time. Since the same variances and covariances are used to evaluate the equation in scenarios (2) and (3), the effect of increasing the number of markets always leads to an improvement of the diversification opportunities. The decomposition

therefore isolates the effect that additional markets have on the correlation structure and the effect on the investment opportunity set.

Figure 5 illustrates the results of our decomposition. The top line labeled “Four with Limited Diversification” gives the diversification ratio driven by the correlation between the four base countries – France, Germany, UK and the US. The line reaches a peak at the end of our sample period, which indicates that the diversification opportunities among these major markets have reached a 150-year low. Even during the Crash of 1929 and the ensuing Great Depression these markets provided better opportunities for risk spreading than they do today. Fortunately for investors, additional markets have become available to offset this increase. First, the deterioration of the benefits of diversification has been unusually pronounced relative to the other markets that have been available. The portfolio representing the average across all combinations of four randomly markets, labeled “Four With Unlimited Diversification,” has also seen a recent deterioration in diversification opportunities, but to a level that does not exceed levels that were common during the early part of the 19<sup>th</sup> century. Compared to the major four markets, which currently provide risk reduction of only 30 per cent, the average four-country portfolio will eliminate about half of the variance that investors experience by concentrating on a single market. A second way in which the development of new markets has helped investors to alleviate the increase in correlations among the major markets is through their number. The bottom line, labeled “All With Unlimited Diversification,” shows that a portfolio that is equally diversified across all available markets can currently reduce portfolio risk to about 35 percent of the volatility of a single market. We conclude that about half of the total contribution of emerging markets to the current benefits of international diversification occurs through offering lower correlations, and half through expansion of the investment opportunity set.

Figure 5 also shows how the emergence of new markets has allowed investors to enjoy the benefits of international diversification during much of the post-War era -- even more so than in the era of capital market integration of a century ago. The gradual increase of the bottom-most line in Figure 5 suggests that good times may be coming to an end for modern investors. While a portfolio of country indices could achieve a 90

percent risk reduction in 1950, this has now fallen to about 65 percent at the turn of the new millennium.

#### *IV.4 The Benefits of International Diversification in Equilibrium*

One serious concern about the analysis thus far is that it cannot reflect equilibrium conditions. Although the benefits to an equal-weighted portfolio of international equity markets reduced risk historically (albeit less so in recent years) it is not possible for all investors in the economy to hold that portfolio. Since all assets need to be held in equilibrium, the average investor will have to hold the value-weighted world market portfolio. Therefore, in an equilibrium framework, the relevant benchmark for diversification is the capital-weighted portfolio. Given that the U.S. – or any of four of our core markets – represents a large proportion of the capitalization of the world equity markets, it is immediately clear that the capital-weighted portfolio will provide less diversification than an equal-weighted portfolio. And because many emerging markets are small, their contribution to the diversification benefits is likely to be overstated on an equally-weighted analysis.

In order to address this issue, we collected market capitalization for the equity indices of 45 countries, from 1973 to the present. Unfortunately, long-term data on market capitalization is unavailable, so our analysis is necessarily limited to the last decades of our sample. As is well known, some countries have cross-holdings that may cause market capitalization to be overstated, and our analysis makes no correction for this issue. In our sample period, the U.S. ranged from roughly 60% to roughly 30% of the world market.

Figure 7 compares the diversification ratio on the core 4 markets to the ratio computed from all entire markets since the 1970's, where each market is weighted by its relative capitalization. The figure confirms our previous intuition that, from a value-weight perspective, the benefits of diversification are generally lower. At the turn of the century a value-weighted portfolio of our core markets achieved a 20% risk reduction relative to the volatility of individual markets. This is somewhat less than in our equally-weighted analysis where we reported a risk reduction of 30%. A value-weighted portfolio of all markets achieved a risk reduction of 45%, compared to 70% found in our equal-

weight analysis. What is similar in both weighting schemes is that the risk reduction from diversifying across all markets is more than double the risk reduction that can be achieved by diversifying across the core markets only.

One striking feature of Figure 7 is that, in contrast to our previous results, the diversification benefits are *not* dramatically less in the 1990's than they are in the 1970s. This suggests that, while the average correlations among the average markets has increased over the past decade, many of these correlations are only marginally important in equilibrium. This evidence is consistent with the trends documented by Bekaert and Harvey (2000) of increasing global market integration who find that the “marginal” markets have been coming into the fold of the global financial system and increasing their correlations as a result. The figure suggests that this affects capital-weighted investors less than might be expected.

## **V. Conclusion**

Long-term investing depends upon meaningful long-term inputs to the asset allocation decisions. One approach to developing such inputs is to collect data from historical time periods. In this paper, we collect information from 150 years of global equity market history in order to evaluate the stationarity of the equity correlation matrix through time. Our tests suggest that the structure of global correlations shifts considerably through time. It is currently near an historical high – approaching levels of correlation last experienced during the Great Depression. Unlike the 1930's however, the late 1990's were a period of prosperity for world markets. The time-series of average correlations show a pattern consistent with the “U” shaped hypothesis about the globalization at the two ends of the 20<sup>th</sup> century. Decomposing the pattern of correlation through time, however, we find that roughly half the benefits of diversification available today to the international investor are due to the increasing number of world markets and available to the investor, and half is due lower average correlation among the available markets. An analysis of the capital-weighted portfolio suggests that benefits are less than the equal-weighted strategy, but that the proportionate risk reduction by adding in emerging markets has actually been roughly the same over the past 25 years.





## Appendix: Testing for Changes in Correlation

In this appendix, we describe our test for a structural change in the correlation matrix, and in the mean of the off-diagonal elements of the correlation matrix. In this section, we introduce an asymptotic test of the null of no structural change in the correlation matrix. This test provides a statistical framework under which structural changes in correlation matrices can be tested with a fairly general class of data generating processes.

Jennrich (1970) derive a  $\chi^2$  test for the equality of two correlation matrices, assuming observation vectors are normally distributed. Since Jennrich does not derive the asymptotic distribution of the correlation matrix, the consistency of his test statistics crucially relies on the assumption of the normal distribution of the data. To construct our test, we utilize the asymptotic distribution of the correlation matrix developed in Browne and Shapiro (1986) and Neudecker and Wesselman (1990). Let  $P$  be the true correlation matrix, then the sample correlation matrix  $\hat{P}$  has the following asymptotic distribution:

$$\sqrt{n} \bullet \text{vec}(\hat{P} - P) \xrightarrow{d} N(0, \Omega) \quad (\text{B1})$$

Where  $n$  is sample size and

$$\Omega = [I - M_s(I \otimes P)M_d] \left[ (\Lambda^{-1/2} \otimes \Lambda^{-1/2}) V (\Lambda^{-1/2} \otimes \Lambda^{-1/2}) \right] [I - M_d(I \otimes P)M_s] \quad (\text{B2})$$

This validity of the asymptotic distribution requires that the observation vectors are independently and identically distributed according to a multivariate distribution with finite fourth moments.

Suppose we want to test whether the correlations structure of two periods are different. Period I has  $n_1$  observations and period II has  $n_2$  observations, which are assumed to be independent. According to (A1), their sample correlation matrices,  $\hat{P}_1$  and  $\hat{P}_2$  should have the following asymptotic distributions for certain  $P_1, P_2, \Omega_1, \Omega_2$ :

$$\sqrt{n_1} \bullet \text{vec}(\hat{P}_1 - P_1) \xrightarrow{d} N(0, \Omega_1) \quad (\text{B1}')$$

$$\sqrt{n_2} \bullet \text{vec}(\hat{P}_2 - P_2) \xrightarrow{d} N(0, \Omega_2) \quad (\text{B1}'')$$

*Test 1 -- An element by element test*

To test whether these two correlation matrices are statistically different, we can impose the following hypothesis:

$$H_0 : P_1 = P_2 = P \text{ and } \Omega_1 = \Omega_2 = \Omega$$

$$H_1 : P_1 \neq P_2 \text{ or } \Omega_1 \neq \Omega_2$$

Under  $H_0$ , the difference between two sample correlation matrices has the following asymptotic distribution:

$$vec(\hat{P}_1 - \hat{P}_2) \xrightarrow{d} N\left(0, \left(\frac{1}{n_1} + \frac{1}{n_2}\right) \Omega\right) \quad (B3)$$

Then we can derive the following  $\chi^2$  test:

$$\Rightarrow [vec(\hat{P}_1 - \hat{P}_2)]^T \left[ \left(\frac{1}{n_1} + \frac{1}{n_2}\right) \Omega \right]^{-1} [vec(\hat{P}_1 - \hat{P}_2)] \xrightarrow{d} \chi^2(rk(\Omega)) \quad (B4)$$

Since  $P$  is a symmetric matrix with 1 on the diagonal, we perform the test on the upper off-diagonal part of  $P$  rather than on the entire matrix. This way, we not only significantly reduce computation but also avoid the singularity problem arising from inverting such matrices. From this point on,  $vec(P)$  is interpreted as the vector of upper off diagonal elements of the correlation matrix.

*Test 2: Test about the average correlation*

The test can be easily modified to allow more general restrictions. For instance, we can test changes in the average correlation as opposed to the element-by-element correlation shown above:

$$H_0 : \bar{P}_1 = \bar{P}_2 = \bar{P} \text{ and } \Omega_1 = \Omega_2 = \Omega$$

$$H_1 : \bar{P}_1 \neq \bar{P}_2 \text{ or } \Omega_1 \neq \Omega_2$$

Suppose  $vec(P)$  has  $k$  elements and  $i = (1,1,\dots,1)_{1 \times k}$  vector. Then the test statistic is:

$$\left[ \frac{i}{k} vec(\hat{P}_1 - \hat{P}_2) \right]^T \left[ \left(\frac{1}{n_1} + \frac{1}{n_2}\right) \frac{i}{k} \Omega \frac{i'}{k} \right]^{-1} \left[ \frac{i}{k} vec(\hat{P}_1 - \hat{P}_2) \right] \xrightarrow{d} \chi^2(1) \quad (B4')$$

One may think that hypothesis 2 is a more lenient version of hypothesis 1 and therefore will be more difficult to reject. This, though generally true, may not always be the case. If correlation coefficients change, but in opposite different directions, then test 1 fails to reject more frequently than does test 2. However, if correlation coefficients move in the same direction, then, due to Jensen's Inequality, the reverse will be the case.

#### *Heteroskedasticity and serial correlation issues*

The heteroskedasticity and serial correlation of stock market returns are well documented. However, heteroskedasticity does not necessarily pose a problem to our tests because we are only interested in correlation, which is scale-free. We simply treat the correlation matrices as if they were computed from returns series' with unit variance.

On the other hand, serial correlation potentially poses a more serious challenge, and is not necessarily susceptible to a closed-form solution. As an empirical matter, the unit root hypothesis is strongly rejected for the returns series' used in this paper, and the monthly autoregression coefficients are mostly insignificant. Unfortunately, this does not mean that others using this test on different data may ignore the effects of serial correlation.

#### *Bootstrap validation*

The bootstrap allows us to study the crucial issue of test statistic performance in small sample. The idea of the bootstrap is to bootstrapping under the null hypothesis that the correlation matrix is no different between the periods. To do this, we pool the standardized observations from the two periods and randomly draw  $n_1 + n_2$  cross-sectional return vectors with replacement from the combined dates in the pooled sample. We then divide them into two samples of appropriate size and perform the test. After repeating this process a number of times, we have an empirical distribution for test statistics under the joint null hypotheses of equality of correlation, homoskedasticity, and i.i.d. returns in time-series

#### *Power of the test*

We performed simulations to examine the stability of asymptotic distribution and the power of the proposed test. Simulation results show that our test is invariant to sample size, difference in mean return and variance, and non-normality in the data. We examine the power the test by looking at how the test is able to differentiate the samples generated by the following two correlation matrices:

$$A = \begin{bmatrix} 1 & a_{12} & a_{13} & a_{14} \\ & 1 & a_{23} & a_{24} \\ & & 1 & a_{34} \\ & & & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & b_{12} & b_{13} & b_{14} \\ & 1 & b_{23} & b_{24} \\ & & 1 & b_{34} \\ & & & 1 \end{bmatrix}$$

$b_{ij} = a_{ij} * \text{factor}$  for  $i \neq j$

Here we allow A and B to differ by a factor of .99, .95, .9, .7, .5 respectively and then compute the power function, which is shown in Figure 8. Ideally, we would hope that, if A and B are the same, i.e., under  $H_0$ , the power function is a 45 degree line so that the probability of statistically accepting the equality hypothesis perfectly reflects our confidence level. Alternatively, if A differs from B, we would hope that the power function is as flat as possible so that the probability of falsely accepting the null hypothesis is minimal. From Figure 8 we can see that our test is relatively powerful. If A and B are very close, with a factor of .95 and .99, the power function is almost a 45 degree straight line. It starts to deviate significantly when A and B differ by a factor of .9, and the null hypothesis is less likely to be accepted. If A and B differ by a factor of .5, our test almost completely rejects the null hypothesis of equality in all simulations. Although in this example, the setup of the alternative hypothesis is arbitrary, it still indicates that this test has decent power.

#### *Asymptotic distribution of the correlation matrix*

Let  $x$  be the  $p \times 1$  random vector of interest. Suppose that moments up to the fourth are finite. The first moment and the second centralized moment, i.e., the mean and the variance of  $x$  are:

$$\mu = E(x) \quad \Sigma = E(x - \mu)(x - \mu)'$$

Let  $\Lambda = \text{diag}(\Sigma)$ , then the correlation matrix corresponding to  $\Sigma$  is

$$P = \Lambda^{-\frac{1}{2}} \Sigma \Lambda^{-\frac{1}{2}}$$

With a sample  $n$  independently and identically distributed observations  $\{x_i, i = 1 \dots n\}$ , we can obtain the following set of sample analogues:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n x_i \quad \hat{\Sigma} = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{\mu})(x_i - \hat{\mu})'$$

$$\hat{\Lambda} = \text{diag}(\hat{\Sigma}) \quad \hat{P} = \hat{\Lambda}^{-\frac{1}{2}} \hat{\Sigma} \hat{\Lambda}^{-\frac{1}{2}}$$

Let,  $M_d = \sum_{i=1}^p (E_{ii} \otimes E_{ii})$ ,  $E_{ij}$  is a  $p \times p$  matrix with 1 on  $(i, j)$  and 0 elsewhere.

$$K = \sum_{i=1}^p \sum_{j=1}^p (E_{ij} \otimes E_{ij}') \text{ and } M_s = \frac{1}{2} (I_{p^2 \times p^2} + K)$$

$$V = E((x - \mu)(x - \mu)' \otimes (x - \mu)(x - \mu)') - (\text{vec}(\Sigma))(\text{vec}(\Sigma))'$$

Browne and Shapiro (1986) and Neudecker and Wesselman (1990) prove the following asymptotic distributions:

$$\sqrt{n} \text{vec}(\hat{\Sigma} - \Sigma) \xrightarrow{D} N(0, V)$$

$$\sqrt{n} \text{vec}(\hat{P} - P) \xrightarrow{D} N \left( 0, [I - M_s (I \otimes P)] M_d \left[ \Lambda^{-\frac{1}{2}} \otimes \Lambda^{-\frac{1}{2}} \right] V \left[ \Lambda^{-\frac{1}{2}} \otimes \Lambda^{-\frac{1}{2}} \right] [I - M_d (I \otimes P) M_s] \right)$$

## References:

Ang, Andrew, and Geert Bekaert, 2002, International Asset Allocation with Regime Shifts, *Review of Financial Studies*, forthcoming

Ang, Andrew, and Joseph Chen, 2002, Asymmetric Correlations of Equity Portfolios, *Journal of Financial Economics*, forthcoming

Basu, Susanto, and Alan M. Taylor, 1999, Business Cycles in International Historical Perspective, *Journal of Economic Perspectives* 13(2), 45-68.

Bekaert, Geert, and Campbell R. Harvey, 2000, Foreign Speculators and Emerging Equity Markets, *Journal of Finance* 55(2), 565-613.

Bekaert, Geert, and Campbell R. Harvey, 1995, Time-Varying World Market Integration, *Journal of Finance* 50(2), 403-44.

Bordo, Michael D., Barry Eichengreen and Jongwoo Kim, 1998, Was There Really an Earlier Period Of International Financial Integration Comparable to Today? *NBER Working Paper* No.W6738.

Bordo, Michael D., Barry Eichengreen and Douglas Irwin, 1999, Is Globalization Today Really Different than Globalization a Hundred Years Ago? *NBER Working Paper* No. W7195.

Boyer, Brian H., Michael S. Gibson and Mico Loretan, 1997, Pitfalls in Tests for Changes in Correlations, *Board of Governors of the Federal Reserve System, International Finance Discussion Papers*: 597

Bracker, Kevin, Diane Docking and Paul D. Koch, 1999, Economic Determinants of Evolution in International Stock Market Integration, *Journal of Empirical Finance* 6(1), 1-27.

Browne, M.W., and A. Shapiro, 1986, The Asymptotic Covariance Matrix of Sample Correlation Coefficients under General Conditions, *Linear Algebra and Its Applications* 82, 169-76.

Bullock, Hugh, 1959, *The Stork of Investment Companies*, Columbia University Press, New York

Campbell, John Y. and Yasushi Hamao, 1992, Predictable Stock Returns in the United States and Japan: A Study of Long-Term Capital Market Integration, *Journal of Finance* 47(1), 43-69

Chen, Zhiwu. and Peter J. Knez, 1995, Measurement of Market Integration and Arbitrage, *Review Of Financial Studies* 8(2), 287-325.

Corsetti, Giancarlo, Marcello Pericoli and Massimo Sbracia, 2001, Correlation Analysis of Contagion: What One Should Know before Running A Test, *Working Paper*.

Engle, Robert, 2000, Dynamic Conditional Correlation - A Simple Class of Multivariate GARCH Models, *Working Paper*, UCSD Department of Economics

Grubel, Herbert G., 1968, Internationally Diversified Portfolios, *American Economic Review* 68, 1299-1314

Grubel, Herbert G. and Kenneth Fadner, 1971, The Interdependence of International Equity Markets, *Journal of Finance* 26(1), 89-94.

Goetzmann, William N., Andrey Ukhov and Ning Zhu, 2001, "China and the World Financial Markets 1870-1930: Modern Lessons from Historical Globalization," Yale School of Management Working Paper.

Goetzmann, William N., Roger G. Ibbotson and Liang Peng, 2001, A New Historical Database for The NYSE 1815-1925: Performance and Predictability," *Journal of Financial Markets*, 4(1), 1-32

Heston, Steven L., K. Geert Rouwenhorst, 1994, Does Industrial Structure Explain the Benefits of International Diversification? *Journal of Financial Economics* 36(1), 3-27.

Jennrich, Robert I., 1970, An Asymptotic Chi-square Test for the Equity of Two Correlation Matrices, *Journal of the American Statistical Association* 65, 904-12

Jorion, Philippe and William N. Goetzmann, 1999, Global Stock Markets in the Twentieth Century, *Journal of Finance* 54(3), pages 953-80.

Kaplanis, Evi C., 1988, Stability and Forecasting of the Comovement Measures of International Stock Market Returns, *Journal of International Money and Finance* 7(1), 63-75.

Korajczyk, Roert A., 1996, Measure Of Stock Market Integration for Developed and Emerging Markets, *World Bank Economic Review* 10(2), 267-89.

Levy, Haim and Marshall Sarnat, 1970, International Diversification of Investment Portfolios, *American Economic Review* 60(4), 668-75.

Longin, Francois and Bruno Solnik, 1995, Is the Correlation in International Equity Returns Constant: 1960 – 1990? *Journal of International Money and Finance* 14 (February), 3-26.

Longin, Francois and Bruno Solnik, 2001, Extreme Correlation of International Equity Markets, *Journal of Finance* 56(2), 649-675.



Lowenfeld, Henry, 1909, Investment an Exact Science, *The Financial Review of Reviews*, London

Neudecker, H. and A.M. Wesselman, 1990, The Asymptotic Covariance of Sample Correlation Matrix, *Linear Algebra and Its Applications* 127, 589-99.

Obstfeld, Maurice and Alan M. Taylor, 2001, Globalization and Capital Markets, *NBER Working Paper*.

Prakash, Gauri and Alan M. Taylor, 1997, Measuring Market Integration: A Model of Arbitrage with an Econometric Application to the Gold Standard, 1879-1913, *NBER Working Paper*, No. W6073.

Rajan, Raghuram and Luigi Zingales, 2001, The Great Reversals: The Politics of Financial Development in the 20<sup>th</sup> Century, *NBER Working Paper*, No. W8178.

Roll, Richard, 1992, Industrial Structure and the Comparative Behavior of International Stock Market Indices, *Journal of Finance* 47(1), 3-41.

Schwert, G. William, 1990, "Indexes of United States Stock Prices from 1802 to 1987," *Journal of Business*, 63, 399-426

Solnik, Bruno, 1976, Why not diversify internationally rather than domestically?, *Financial Analyst Journal*, July.

Statman, Meir, 1987, How many stocks make a diversified portfolio?, *Journal of Financial and Quantitative Analysis*, September

Tse, Yiu Kuen, 2000, A test for constant correlations in a multivariate GARCH model, *Journal of Econometrics*, 98, 107 - 127

**Table 1: Summary of Global Equity Markets**

This table summarizes information about the indices used in the analysis. Indices are monthly total return or capital appreciation return series' for the country, or leading market in the country, as reported in secondary sources. All returns are adjusted to dollar terms at prevailing rates of exchange. Secondary sources for the data are noted in the text. Primary sources for the data are various. Data availability depends not only on the availability of the equity series' but also on the availability of exchange rate data. Information about founding dates and trading dates is from Conner & Smith (1991), Park and Agtmael (1993) or from self-reported historical information by the exchange itself, on the web. Two portals to world stock exchanges used to access this information are: <http://www.minemarket.com/stock.htm> and [http://dmoz.org/Business/Investing/Stocks\\_and\\_Bonds/Exchanges/](http://dmoz.org/Business/Investing/Stocks_and_Bonds/Exchanges/). Countries with no historical information are simply those with a current web-site for a stock exchange.

Country	Date by which equity trading is known or date of founding of exchange or both.	Beginning date for data used in study	Ending date for data used in study	Sub-period geometric mean (% per annum)	Sub-period arithmetic mean (% per annum)	Sub-period standard deviation (% per annum)	Correlation to equal-weighted portfolio	Number of Months
Argentina (1)	1872 founding	Apr-1947	Jul-1965	-23.4	-17.1	41.4	0.409	220
Argentina (2)		Dec-1975	Dec-2000	17.3	48.4	86.9	0.298	301
Australia	1828,1871	Feb-1875	Dec-2000	4.1	5.3	15.9	0.506	1511
Austria	1771 founding	Feb-1925	Dec-2000	4.4	6.6	21.0	0.368	911
Bahrain	1987 founding							
Belgium	1723,1771	Feb-1919	Dec-2000	1.1	4.6	25.0	0.455	983
Brazil	1845 Rio, 1890 San Paul	Feb-1961	Dec-2000	3.8	17.4	52.7	0.297	479
Botswana	1989 founding							
Bulgaria								
Canada	1817, 1874	Feb-1914	Dec-2000	4.8	6.2	17.0	0.550	1043
Chile	1892 founding	Feb-1927	Dec-2000	5.7	13.5	37.3	0.231	887
China	1891,1904							
China	1991 founding							
Colombia	1929 trading	Nov-1936	Dec-2000	-2.2	0.6	24.6	0.207	770
Croatia								
Cuba	1861 trading							
Czech (1)	1871 founding	Aug-1919	Apr-1945	1.6	4.2	22.6	0.502	309
Czech (2)		Jan-1995	Dec-2000	-3.7	1.7	33.0	0.460	72
Cyprus								
Denmark	1808 founding	Aug-1914	Dec-2000	3.9	5.6	19.3	0.411	1037

<b>Country</b>	<b>Date by which equity trading is known or date of founding of exchange or both.</b>	<b>Beginning date for data used in study</b>	<b>Ending date for data used in study</b>	<b>Sub-period geometric mean (% per annum)</b>	<b>Sub-period arithmetic mean (% per annum)</b>	<b>Sub-period standard deviation (% per annum)</b>	<b>Correlation to equal-weighted portfolio</b>	<b>Number of Months</b>
Ecuador	1970							
Egypt (1)	1883 founding	Aug-1950	Sep-1962	-1.6	-0.2	17.3	0.402	146
Egypt (2)	1993 trading	Jan-1995	Dec-2000	-3.7	-0.0	28.4	0.218	72
Estonia								
Finland	1865, 1912	Feb-1922	Dec-2000	8.4	10.9	23.0	0.362	947
France	1720 founding	Feb-1856	Dec-2000	2.4	4.9	21.2	0.467	1739
Germany (1)	1750 trading	Feb-1856	Aug-1914	0.4	0.9	9.9	0.588	703
Germany (2)		Dec-1917	Dec-1943	-1.4	6.5	42.0	0.417	313
Germany (3)		Jan-1946	Dec-2000	10.3	14.6	31.2	0.362	660
Ghana	1990 founding							
Greece (1)	1892 founding	Aug-1929	Sep-1940	-8.8	-6.1	25.2	0.369	134
Greece (2)		Jan-1988	Dec-2000	12.8	19.1	39.5	0.421	156
Hong Kong	1866, 1891	Jan-1970	Dec-2000	14.1	20.8	39.4	0.538	372
Hungary (1)	1864 founding	Feb-1925	Jun-1941	9.2	12.3	26.7	0.430	197
Hungary (2)		Jan-1995	Dec-2000	14.9	23.1	43.2	0.670	72
Iceland	1985 founding							
India	1830, 1877	Aug-1920	Dec-2000	1.6	3.9	21.8	0.287	965
Indonesia	1912 founding	Jan-1988	Dec-2000	-1.9	14.1	60.4	0.510	156
Iran	1968 founding							
Ireland	1790, 1799 founding	Feb-1934	Dec-2000	5.0	6.3	16.8	0.376	803
Israel	1953 founding	Mar-1957	Dec-2000	8.3	11.2	24.8	0.304	526
Italy	1808 founding	Oct-1905	Dec-2000	0.2	5.1	33.8	0.407	1143
Jamaica	1968 founding							
Japan (1)	1878 founding	Aug-1914	Aug-1945	-0.6	0.8	16.5	0.261	373
Japan (2)		May-1946	Dec-2000	10.3	14.8	28.9	0.335	656
Jordan	1978 founding	Jan-1988	Dec-2000	-4.7	-3.5	16.0	0.129	156
Kenya	1954 founding							
Korea	1911 founding	Jan-1976	Dec-2000	8.9	15.7	39.0	0.304	300
Kuwait	1962 founding							
Latvia								
Lebanon	1920 founding							
Lituania								

<b>Country</b>	<b>Date by which equity trading is known or date of founding of exchange or both.</b>	<b>Beginning date for data used in study</b>	<b>Ending date for data used in study</b>	<b>Sub-period geometric mean (% per annum)</b>	<b>Sub-period arithmetic mean (% per annum)</b>	<b>Sub-period standard deviation (% per annum)</b>	<b>Correlation to equal-weighted portfolio</b>	<b>Number of Months</b>
Luxembourg	1929 founding	Jan-1988	Dec-2000	13.3	15.5	24.7	0.456	156
Macedonia								
Malawi	1995 founding							
Malaysia	1930 founding	Jan-1988	Dec-2000	3.7	10.0	36.2	0.613	156
Malta	1996 trading							
Mauritius	1988 founding							
Mexico	1894 founding	Dec-1934	Dec-2000	6.3	10.8	29.6	0.444	793
Morocco	1929 founding							
Namibia	1992 founding							
Netherlands (1)	1611 founding	Feb-1919	Aug-1944	0.2	1.7	17.4	0.649	307
Netherlands (2)		Jan-1946	Dec-2000	7.7	9.1	18.1	0.562	660
New Zealand	1872 founding	Feb-1931	Dec-2000	2.4	3.7	16.2	0.529	839
Nicaragua								
Nigeria	1960 founding							
Norway	1881 founding	Feb-1918	Dec-2000	2.3	3.9	17.7	0.563	995
Pakistan	1934 founding	Aug-1960	Dec-2000	-0.8	2.1	22.8	0.231	485
Panama								
Peru (1)	1861 founding, 1890 equities	Apr-1941	Jan-1953	-5.5	-2.9	20.9	0.082	142
Peru (2)		Jan-1957	Dec-1977	-7.4	-6.6	13.6	-0.018	252
Peru (3)		Dec-1988	Dec-2000	25.2	44.3	73.7	0.213	145
Philippines	1927 founding	Aug-1954	Dec-2000	-3.0	2.9	39.2	0.392	557
Poland (1)	1811 founding, 1938 equities	Feb-1921	Jun-1939	-4.3	16.7	71.5	0.466	221
Poland (2)		Dec-1992	Dec-2000	21.5	36.6	64.4	0.542	96
Portugal (1)	1901 founding	Jan-1931	Apr-1974	5.0	9.3	44.0	0.231	520
Portugal (2)		Apr-1977	Dec-2000	11.5	18.9	44.1	0.444	285
Roumania	1929 founding							
Russia	1836 trading							
Singapore	1890 founding	Jan-1970	Dec-2000	10.4	14.6	30.6	0.595	372
Slovakia	1991 founding							

<b>Country</b>	<b>Date by which equity trading is known or date of founding of exchange or both.</b>	<b>Beginning date for data used in study</b>	<b>Ending date for data used in study</b>	<b>Sub-period geometric mean (% per annum)</b>	<b>Sub-period arithmetic mean (% per annum)</b>	<b>Sub-period standard deviation (% per annum)</b>	<b>Correlation to equal-weighted portfolio</b>	<b>Number of Months</b>
Slovenia	1924 founding							
South Africa	1887 founding	Feb-1910	Dec-2000	4.3	6.6	21.8	0.391	1091
Spain	1729, 1860	Jan-1915	Dec-2000	2.1	5.3	28.4	0.404	1032
Sri Lanka	1900 founding	Jan-1993	Dec-2000	-11.6	-6.9	32.8	0.515	96
Swaziland	1990 founding							
Sweden	1776 , 1863	Feb-1913	Dec-2000	2.3	5.1	25.6	0.480	1055
Switzerland	17 <sup>th</sup> century, 1850	Feb-1910	Dec-2000	4.8	6.0	16.4	0.511	1091
Taiwan	1960 founding	Jan-1985	Dec-2000	12.6	21.9	45.8	0.437	192
Thailand	1975 founding	Jan-1976	Dec-2000	6.7	12.7	35.3	0.515	300
Tanzania	1998 founding							
Trinidad-Tobago	1981 founding							
Turkey	1866 founding	Jan-1987	Dec-2000	18.8	38.6	68.6	0.397	168
Tunisia	1969 founding							
UK	1698, 1773 exchange	Jan-1800	Dec-2000	2.0	3.1	15.2	0.623	2411
Uruguay	1895, 1926							
USA	1790 founding	Jan-1800	Dec-2000	3.2	4.3	15.0	0.489	2411
Venezuela	1805, 1893	Nov-1937	Dec-2000	-0.1	4.6	30.2	0.147	758
Yugoslavian states	1894 founding							
Zambia	1994 founding							
Zimbabwe	1896 founding	Jan-1976	Dec-2000	5.2	11.9	36.7	0.269	300

**Table 2. Sample Statistics of Stock Market Total Returns**

This table provides mean and standard deviation of stock total returns of major countries. All returns are converted into dollar-denominated.

	<b>UK</b>	<b>US</b>	<b>France</b>	<b>Germany</b>	<b>Australia</b>	<b>Switzerland</b>	<b>Japan</b>	<b>Italy</b>
<b>1872-1889</b>								
Mean	5.3%	7.0%	7.1%	6.9%				
SD	5.2%	13.0%	7.2%	12.5%				
<b>1890-1914</b>								
Mean	2.0%	6.7%	4.7%	4.6%				
SD	6.1%	15.6%	6.9%	7.4%				
<b>1915-1918</b>								
Mean	1.2%	10.0%	10.8%	-23.5%	6.0%			
SD	8.0%	14.9%	13.7%	30.6%	9.1%			
<b>1919-1939</b>								
Mean	4.7%	10.4%	0.4%	-56.0%	11.3%	6.3%		
SD	14.5%	26.9%	24.0%	74.2%	14.2%	16.4%		
<b>1940-1945</b>								
Mean	5.4%	15.1%	15.9%	-1.1%	3.0%	16.1%	-9.1%	16.6%
SD	24.2%	15.9%	57.4%	42.8%	18.6%	16.6%	42.7%	96.0%
<b>1946-1971</b>								
Mean	13.3%	11.6%	14.3%	16.4%	13.3%	8.5%	25.6%	14.9%
SD	15.5%	13.4%	23.3%	32.6%	14.2%	14.5%	35.9%	25.4%
<b>1972-2000</b>								
Mean	14.8%	13.8%	16.4%	14.7%	13.4%	14.2%	10.9%	11.6%
SD	24.4%	15.6%	20.9%	20.3%	23.7%	18.9%	22.1%	26.2%

**Table 3. Correlation Matrices of Core Markets in Sub-Periods**

This table provides the correlation matrices of monthly equity returns (in US Dollars) of the four core countries (UK, US, France, and Germany) during seven sub-periods, as well as the correlation matrices during periods of integration and segmentation.

	US	France	Germany
<b>1872-1889</b>	<b>Average correlation = 0.102</b>		
UK	0.103	0.140	0.030
US		0.166	0.161
FRANCE			0.012
<b>1890-1914</b>	<b>Average correlation = 0.155</b>		
UK	0.078	0.1878	0.084
US		0.141	0.204
FRANCE			0.235
<b>1915-1918</b>	<b>Average correlation = -0.073</b>		
UK	-0.009	0.140	-0.166
US		-0.284	0.057
FRANCE			-0.175
<b>1919-1939</b>	<b>Average correlation = 0.228</b>		
UK	0.289	0.431	0.188
US		0.260	0.020
FRANCE			0.183
<b>1940-1945</b>	<b>Average correlation = 0.0460</b>		
UK	0.049	0.453	-0.075
US		0.017	-0.281
FRANCE			0.113
<b>1946-1971</b>	<b>Average correlation = 0.111</b>		
UK	0.182	0.112	0.039
US		-0.020	0.222
FR			0.132
<b>1972-2000</b>	<b>Average correlation =0.475</b>		
UK	0.508	0.499	0.429
US		0.414	0.378
FRANCE			0.620

**Table 3 (Continued)**

**Correlation in Integration vs. Segmentation Periods**

Note that the Integration and Segmentation periods are not endogenously defined, but specified as indicated in the text by historical events.

	<b>US</b>	<b>France</b>	<b>Germany</b>
<b>Full Sample: 1872-2000</b>	<b>Average correlation = 0.199</b>		
<b>UK</b>	0.265	0.351	0.143
<b>US</b>		0.163	0.083
<b>FRANCE</b>			0.189
<b>Integration: 1872-1913 1972-2000</b>	<b>Average correlation = 0.381</b>		
<b>UK</b>	0.345	0.467	0.369
<b>US</b>		0.301	0.284
<b>FRANCE</b>			0.520
<b>Segmentation: 1914-1971</b>	<b>Average correlation = 0.146</b>		
<b>UK</b>	0.193	0.311	0.097
<b>US</b>		0.101	0.041
<b>FRANCE</b>			0.135



**Table 4. Testing Equality of Correlation Structure**

This table provides probability values for test statistics for the null hypothesis that the corresponding two periods have the same correlation matrices. Correlation matrices are computed using stock returns of US, UK, France and Germany. Tests are performed on the entire correlation matrix (Panel A and B) and mean correlation coefficients (Panel C and D). Asymptotics-based tests (Panel A and C) are validated with bootstrapping-based tests (Panel B and D). Single stars indicate rejection at the 5% level, double stars indicate rejection at the 1% level.

**Panel A: Asymptotics Based Test of Correlation Matrices**

	<b>1890-1914</b>	<b>1915-1918</b>	<b>1919-1939</b>	<b>1940-1945</b>	<b>1946-1971</b>	<b>1972-2000</b>
<b>1870-1889</b>	0.246	0.000**	0.027*	0.023*	0.082	0.000**
<b>1890-1914</b>		0.000**	0.000**	0.008**	0.006**	0.000**
<b>1915-1918</b>			0.000**	0.012*	0.003**	0.000**
<b>1919-1939</b>				0.062	0.001**	0.000**
<b>1940-1945</b>					0.075	0.002**
<b>1946-1971</b>						0.000**

**Panel B: Asymptotics Based Test of Mean Correlation Coefficients**

	<b>1890-1914</b>	<b>1915-1918</b>	<b>1919-1939</b>	<b>1940-1945</b>	<b>1946-1971</b>	<b>1972-2000</b>
<b>1870-1889</b>	0.075	0.050*	0.554	0.060	0.218	0.000**
<b>1890-1914</b>		0.002**	0.189	0.002**	0.002**	0.001**
<b>1915-1918</b>			0.008**	0.483	0.227	0.000**
<b>1919-1939</b>				0.010*	0.054	0.000**
<b>1940-1945</b>					0.323	0.000**
<b>1946-1971</b>						0.000**

**Table 4. Testing Equality of Correlation Structure --- Continued**

This table provides probability levels of test statistics for the null hypothesis that the corresponding two periods have the same correlation matrices. Correlation matrices are computed using stock returns of US, UK, France and Germany. Tests are performed on the entire correlation matrix (Panel A and B) and mean correlation coefficients (Panel C and D). Asymptotics-based tests (Panel A and C) are validated with Bootstrapping based tests (Panel B and D). Single stars indicate rejection at the 5% level, double stars indicate rejection at the 1% level.

**Panel C: Bootstrapping Based Test of Correlation Matrices**

	<b>1890-1914</b>	<b>1915-1918</b>	<b>1919-1939</b>	<b>1940-1945</b>	<b>1946-1971</b>	<b>1972-2000</b>
<b>1870-1889</b>	0.261	0.001**	0.036**	0.081	0.088	0.000**
<b>1890-1914</b>		0.001*	0.000**	0.027*	0.008**	0.000**
<b>1915-1918</b>			0.000**	0.232	0.013*	0.000**
<b>1919-1939</b>				0.150	0.003**	0.000**
<b>1940-1945</b>					0.114	0.008**
<b>1946-1971</b>						0.000**
	<b>Mean<sup>10</sup></b>	<b>Variance</b>				
	6.213	12.043				

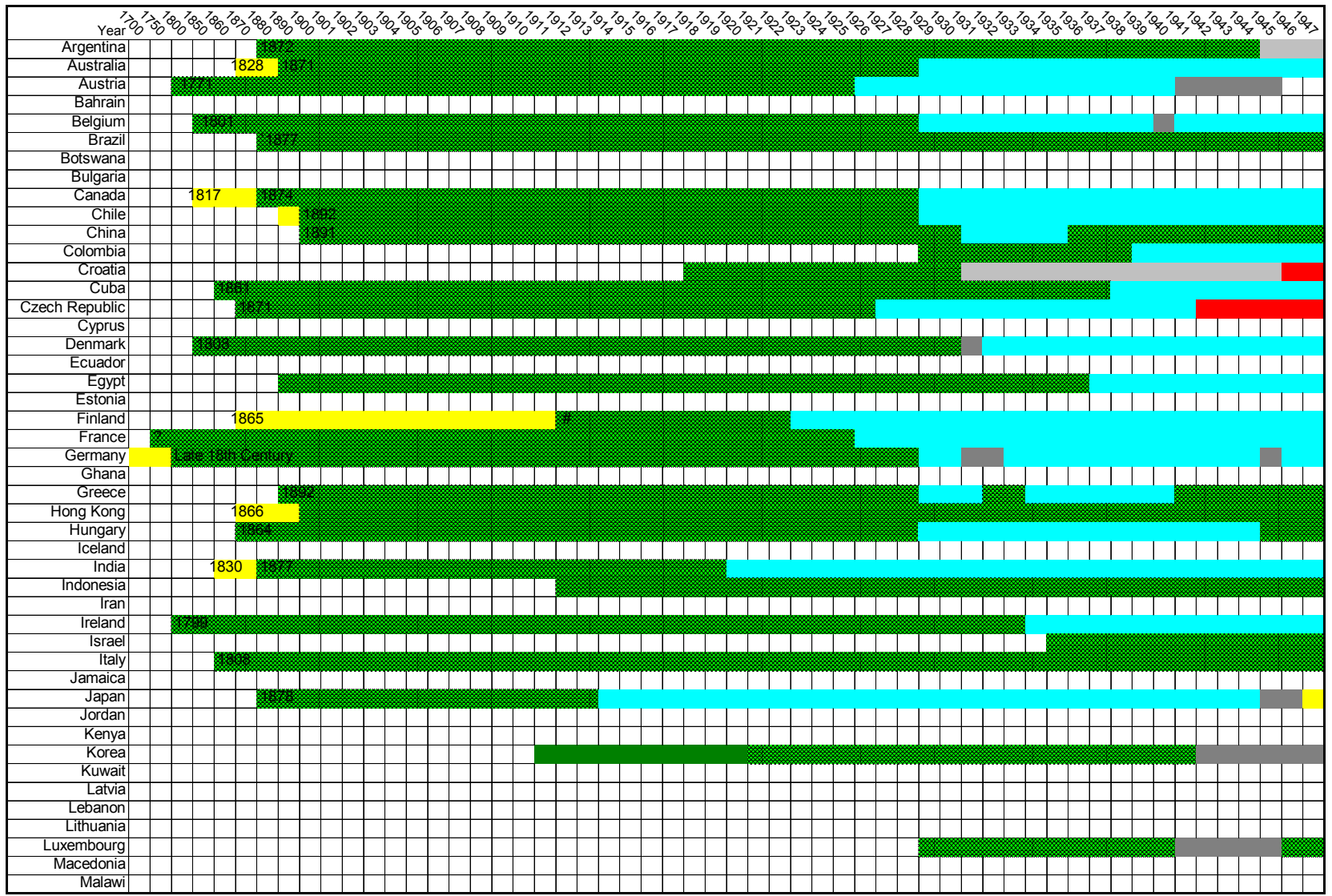
**Panel D: Bootstrapping Based Test of Mean Correlation Coefficients**

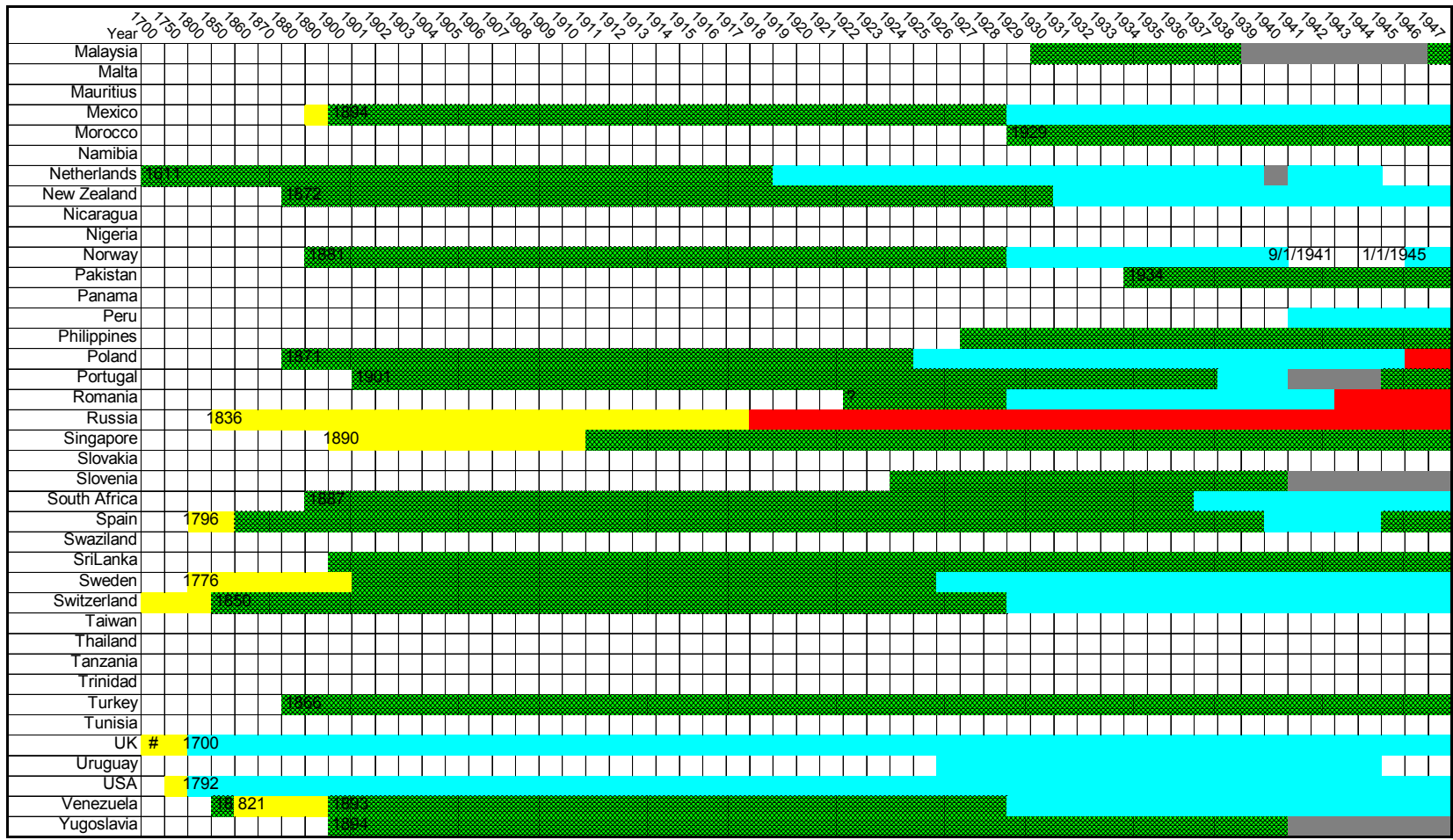
	<b>1890-1914</b>	<b>1915-1918</b>	<b>1919-1939</b>	<b>1940-1945</b>	<b>1946-1971</b>	<b>1972-2000</b>
<b>1870-1889</b>	0.070	0.040*	0.566	0.083	0.212	0.000**
<b>1890-1914</b>		0.002**	0.198	0.002**	0.002**	0.001**
<b>1915-1918</b>			0.012*	0.594	0.241	0.000**
<b>1919-1939</b>				0.018*	0.057	0.000**
<b>1940-1945</b>					0.298	0.000**
<b>1946-1971</b>						0.000**
	<b>Mean</b>	<b>Variance</b>				
	1.044	2.020				

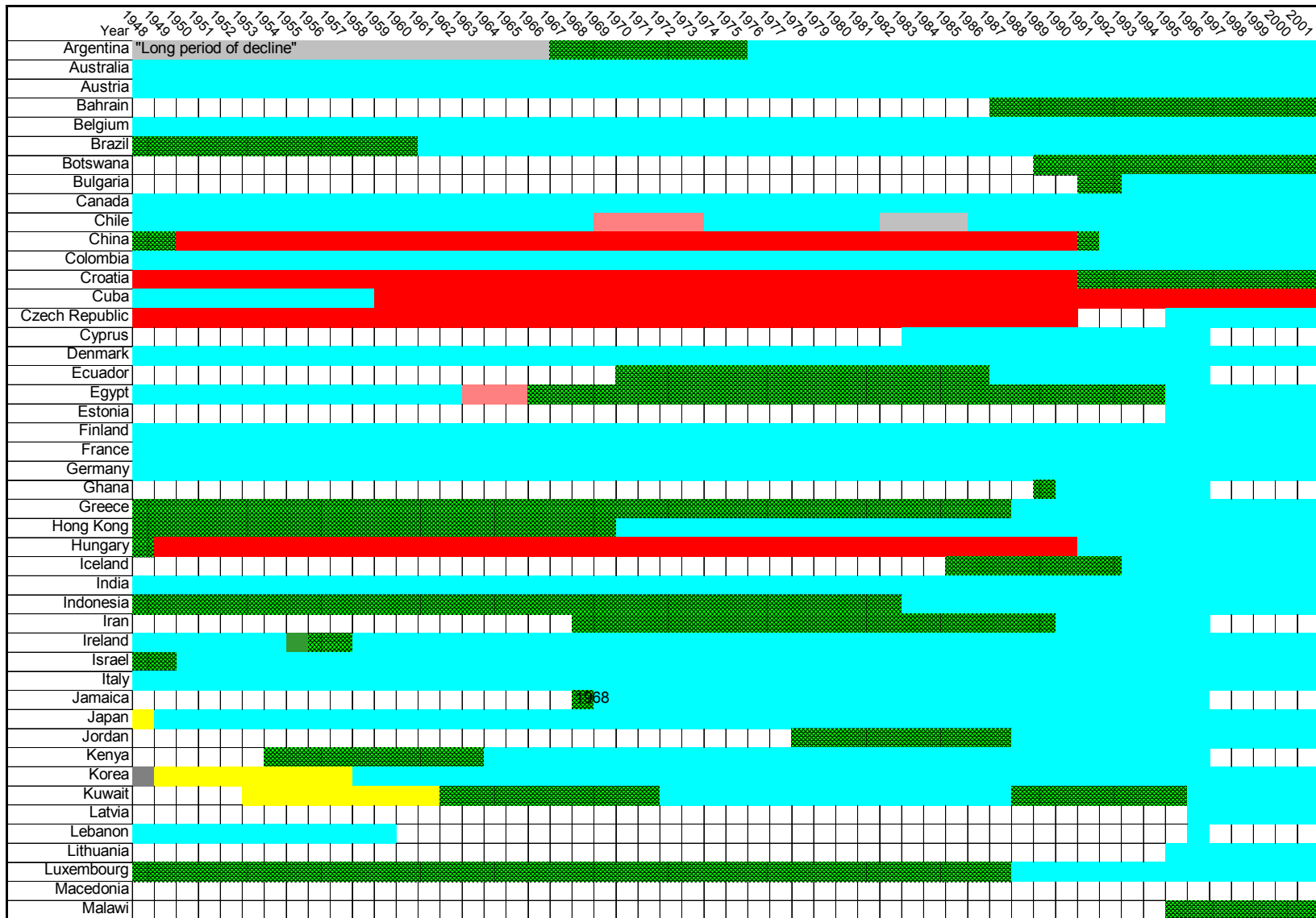
<sup>10</sup> These are the sample mean and variance of bootstrapped test statistics. If bootstrapped test statistics perfectly conform to Chi-squared distribution, as suggested by asymptotics theory, then test on correlation matrices should have mean of 6 and variance of 12, while test on mean correlation coefficients should have mean of 1 and variance of 2.

**Figure 1: Table of Market Openings, Failures and Data Availability**

This chart reports the best information available to the authors on 84 different equity markets of the world. The columns represent years, with the first few columns compressing fifty-year time periods. If the date of the founding of the market is documented on its web-site or in one of the sources cited in Table 1, that date is recorded at the beginning of the colored bar indicating the market starting date. Cells are coded by color. Yellow indicates that the market was founded, but we have no historical information confirming that equity securities were traded. Green indicates that there is some evidence that equity securities were traded after that date in the market. Blue indicates that price data exists in paper and/or electronic form. Red indicates the presumption of a market closure. Gray indicates market suspension or closure. Pink indicates a market crisis noted in the sources.









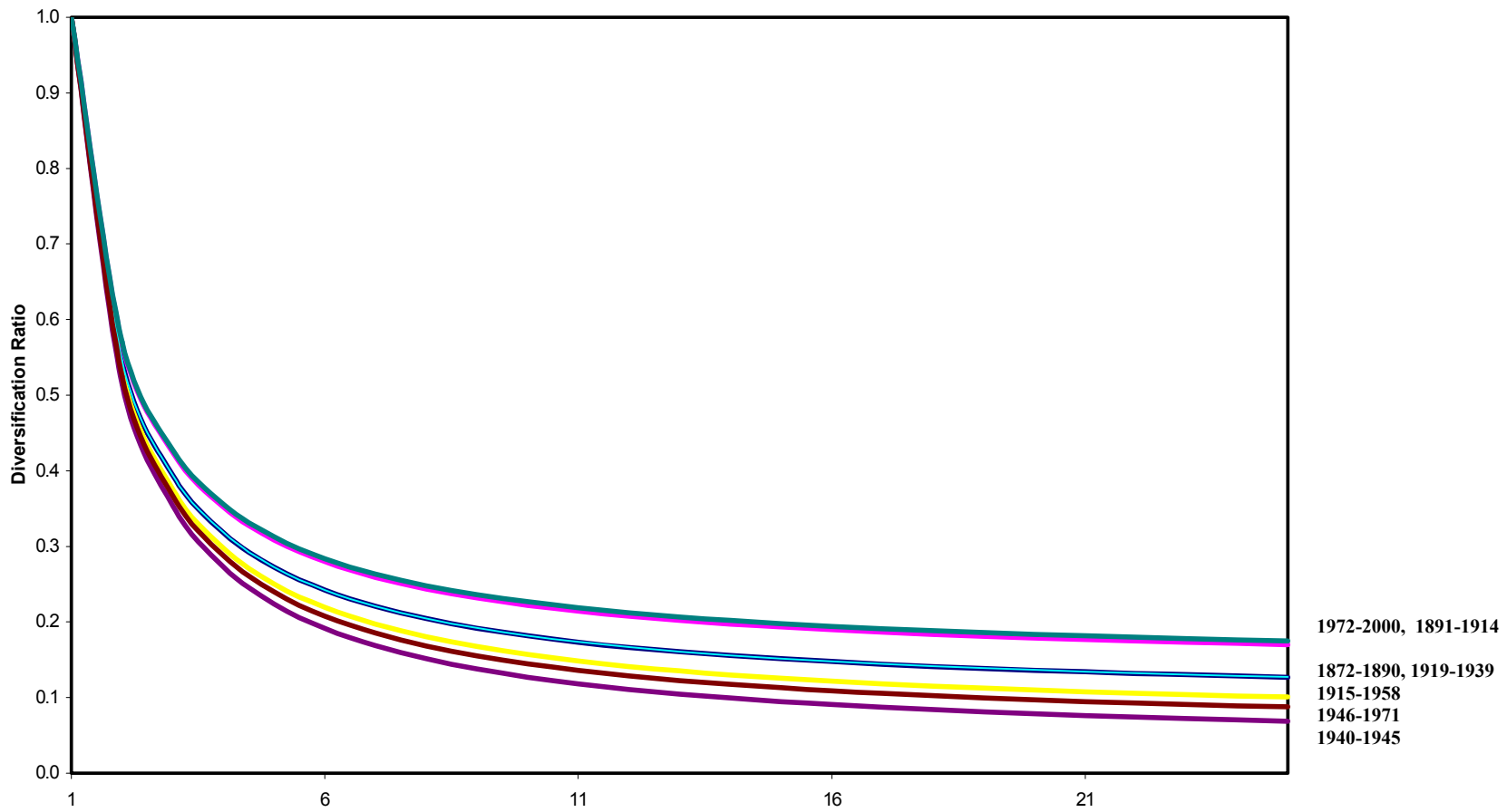
**Figure 2: Risk Reduction from International Diversification: Selected Periods**

This figure shows the ratio of variance of the equally weighted portfolio of country indices scaled by the average variance of the country indices, as a function of the number of countries in the portfolio. The ratio is computed as:

$$\text{Var}\left(\sum_{i=1}^n x_i / n\right) / \left(\frac{1}{n} \sum_{i=1}^n \text{Var}(x_i)\right) = \frac{1}{n} + \left(\frac{n-1}{n}\right) \cdot \frac{\overline{\text{Cov}(x_i, x_j)}}{\overline{\text{Var}(x_i)}}.$$

All returns are measure capital appreciation and exclude dividends, converted to US

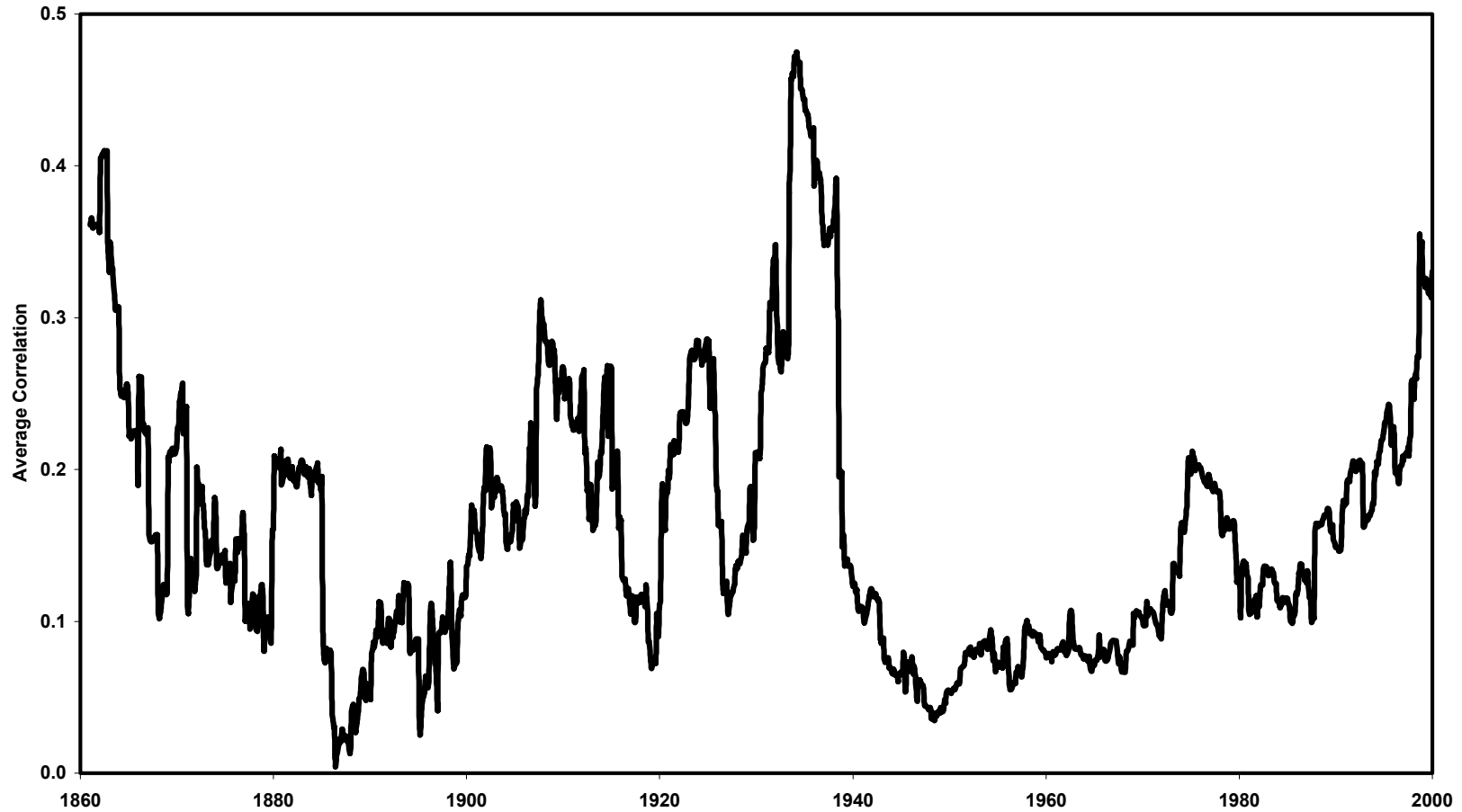
dollars.





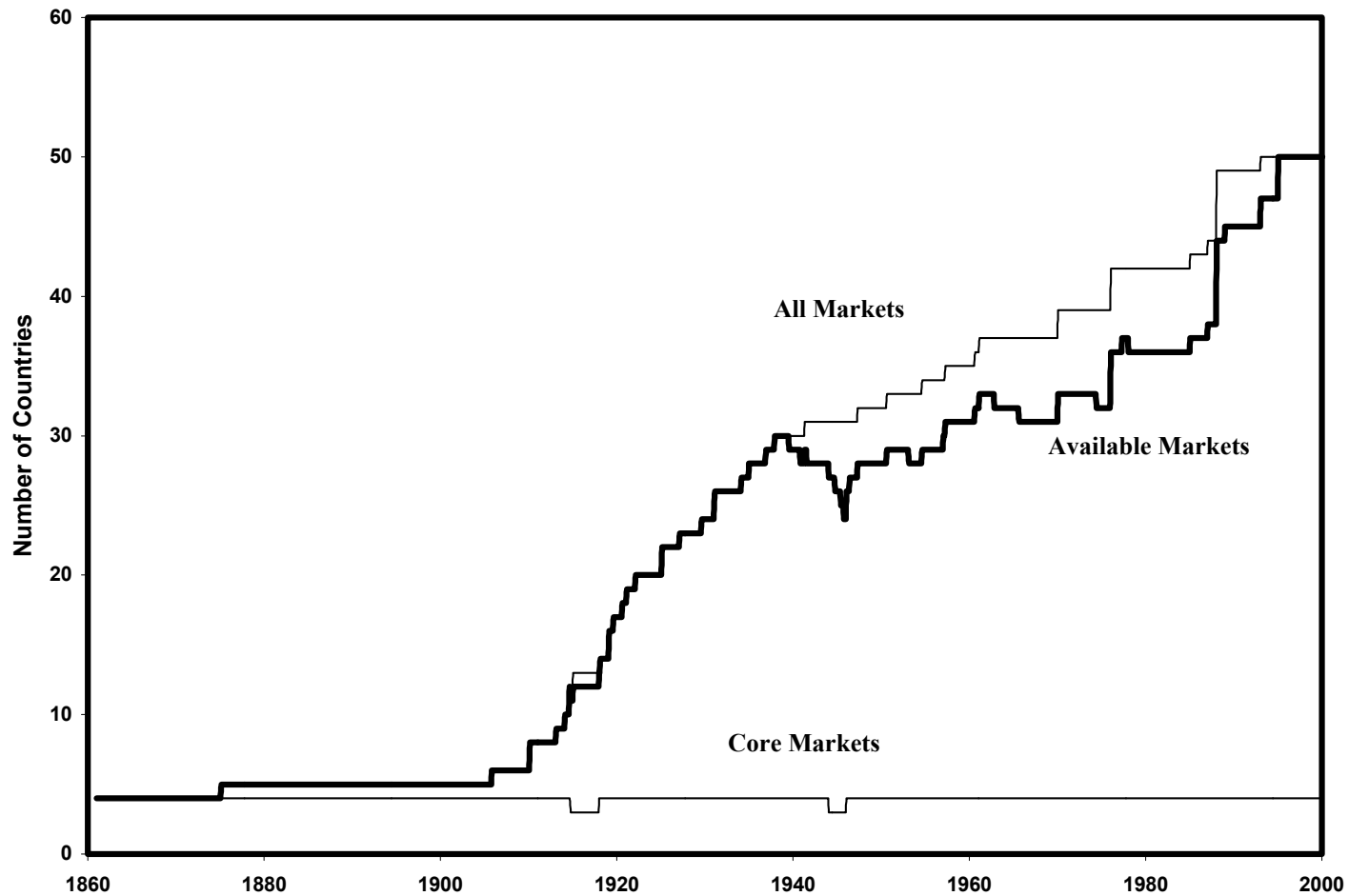
**Figure 3: Average Correlation of Capital Appreciation Returns for all Available Markets**

This figure shows the time-series of the average off-diagonal correlation of dollar-valued capital appreciation returns for all available markets. A rolling window of 60 months is used.



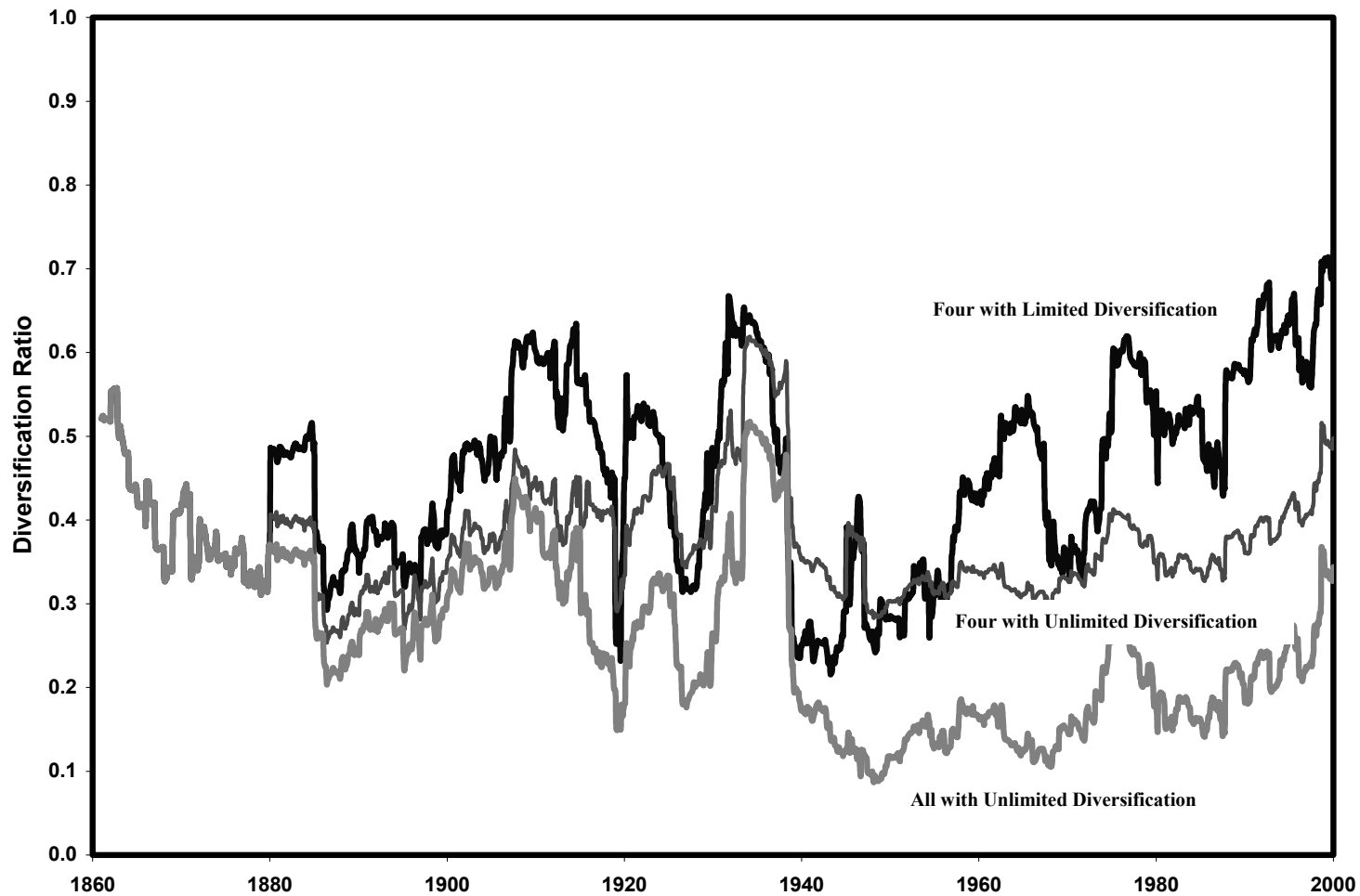
**Figure 4: Number of Countries**

This figure shows total number of countries that have appeared in our sample, the surviving countries, and the surviving core countries at each point in time. Core countries are: Germany, France, U.S. and U.K. Germany and Japan dropped out the global markets for short periods due to wartime. Some Eastern European countries dropped out of the global markets during the war and then re-joined as emerging markets in the early 1990s.



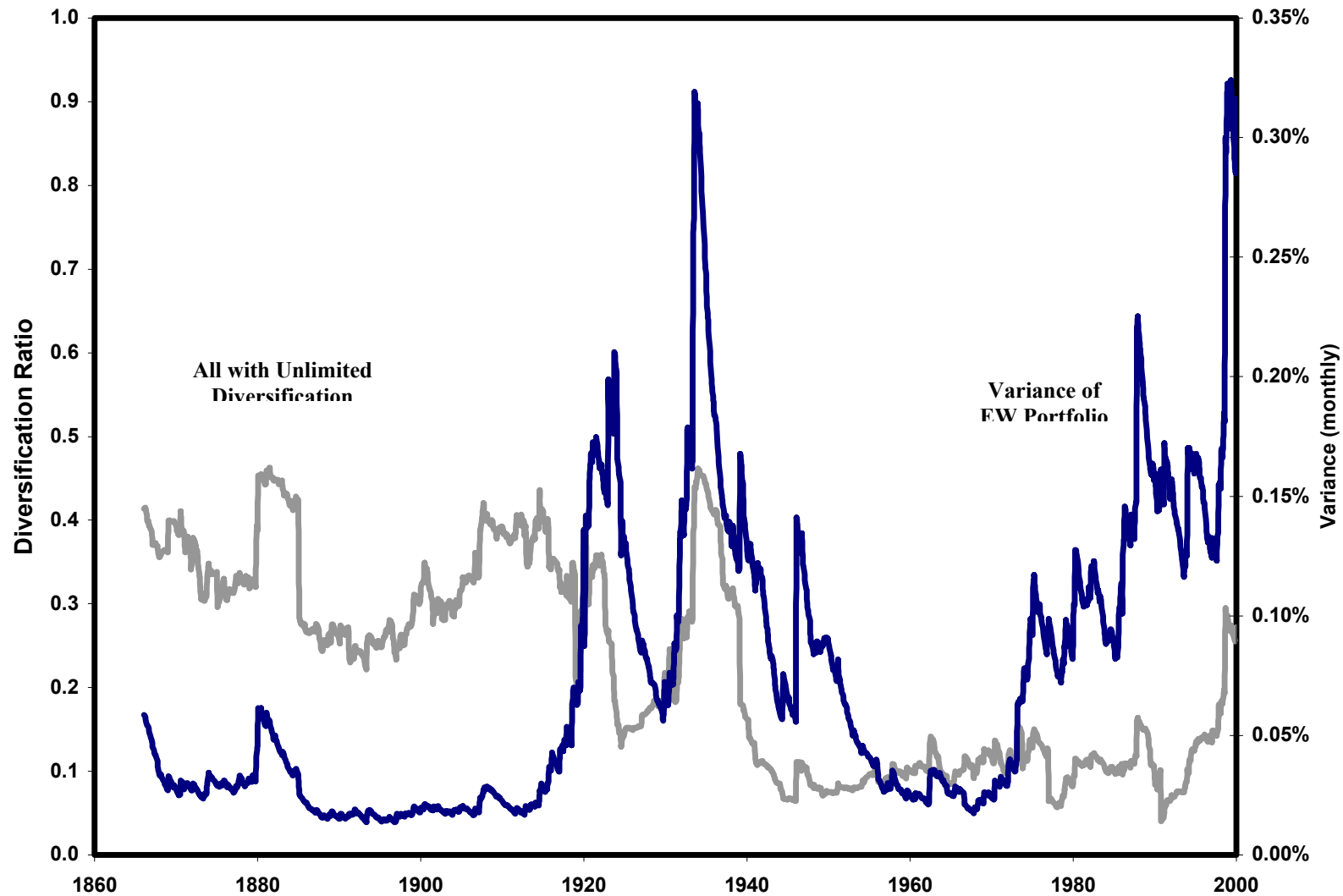
**Figure 5: Decomposition of the Diversification Effects Due to Average Correlation and the Number of Markets**

This figure shows the diversification benefits to hypothetical portfolios of country indices, under three sets of assumptions.. The first portfolio is defined by the constraint that the investor hold an equal-weighted portfolio of four countries, Germany, France, U.S. and U.K. It is labeled “Four With Limited Diversification.” The second portfolio relaxes the constraint that there are only four markets with average correlations of the core countries. In this sense it is entirely hypothetical – it assumes that an there are an unlimited number of country indices available so that all idiosyncratic risk can be diversified away. It is labeled “Four With Unlimited Diversification.” The third portfolio assumes an investor holds an equal-weighted portfolio across all countries in the sample at any given point in time. It is labeled “All with Unlimited Diversification.”



**Figure 6: Diversification Benefits and the Variance of the Equal-Weight Portfolio**

This figure shows the diversification ratio and the variance of the equal-weighted portfolio of all available markets. The diversification ratio is explained in Figure 5. A rolling window of 120 months is used. Returns are exponentially weighted with a half-life of 60 months so that more recent observations receive higher weights.



**Figure 7: Diversification with Capital Market Weights**

This figure shows the diversification ratio of for the capital-weighted portfolios of 45 country indices and the diversification ratio of the four core countries . A rolling window of 120 months is used. Returns are exponentially weighted with a half time of 60 months.



**Figure 8: Power of the Test For Differences in Correlation Matrices.**

This figure presents simulated power function of test for correlation matrix equality. It shows, for each significant level on the X axis, how likely the equality hypothesis will fail to be rejected by the test. The two simulated correlation matrices that are tested differ by a factor of .5, .7, .9, .95, .99, 1.0 respectively, with 1 being identical and .5 being furthest apart.

