Allocation ADVISOR

And

The Black-Litterman Model

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Zephyr Associates, Inc.
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Please send questions to: Support@styleadvisor.com
Subject: Black-Litterman
Overview / Outline

• What is Allocation ADVISOR?
• Why is a new model important?
• What is the Black-Litterman model?

• A Demonstration of the model in AA
• A Look at other new features in AA
What is AllocationADVISOR?

Asset Allocation

Monte Carlo

Analytic Projections

Initial Value: 1,000,000 - 20 Year Time Horizon - 95% of Projected Return Distribution

Range of Returns

Best Case Return $12,186,652 $10,329,342 $14,671,306 $27,409,592

Expected Return $5,653,481 $5,832,404 $6,851,781 $7,713,900

Worst Case Return $2,602,431 $3,279,156 $3,175,620 $2,125,230

Portfolio Allocations

Emerging Markets 0.0% 11.2% 17.8% 33.7%

LB Aggregate Bond 40.0% 20.3% 0.0% 0.0%

LB Global Ex US Bond 0.0% 48.6% 44.4% 0.0%

MSCI EAFE Index 10.0% 0.0% 6.4% 41.3%

Russell 3000 Index 50.0% 19.9% 31.4% 25.0%
Why is a new model important?

Non-Diversified Portfolios

Efficient frontiers based on historical data lead to highly concentrated portfolios.

What is the Solution?
The Black-Litterman Model
Why is a new model important?

MV Frontier Allocations (Historical)
What is the Black-Litterman Model?

It is….

• A Bayesian Asset Allocation Approach
• A Strategic Asset Allocation Model
• A Tactical Asset Allocation Model
• A tool for creating a set of expected returns
• A tool for blending implied returns and investor returns

It is not…

• A source of Alpha
The Black-Litterman Model starts with Implied Returns

Other Names for Implied Returns...

- CAPM Returns
- Reverse Optimized Returns
- Market Returns
- Consensus Returns
- Imputed Returns
- Equilibrium Returns
Implied Returns

A rich theoretical background….


Implied Returns

• The return of any asset or asset class can be separated into three parts:
  – Risk-Free Return
  – Return Correlated with Benchmark
  – Return Not Correlated with Benchmark

• Returns that are correlated with the benchmark result in beta risk (systematic risk, benchmark risk, non-diversifiable risk, or market risk)

• Beta risk is the type of risk that is rewarded with a premium
Implied Returns: Using the CAPM

Expected returns are a function of beta risk….

\[ E(R_i) = R_f + \beta_{i,B} (R_B - R_f) \]

- \( R_f \) = Risk-Free Rate
- \( \beta_{i,B} \) = Beta of Asset Class i Relative to Benchmark
- \( R_B \) = Return of Benchmark
- \( R_B - R_f \) = Forward Looking Risk Premium of Benchmark*
  (Return of Benchmark over the Risk-Free Rate)

* In this case the benchmark is the market capitalization weighted portfolio
Implied Returns:
Using Reverse Optimization

The same excess returns result from reverse optimization.

\[ \Pi = \lambda \Sigma w_{mkt} \]

- \(\Pi\) = Implied Excess returns over the risk free rate \((N \times 1)\)
- \(\lambda\) = Risk aversion coefficient \((1 \times 1)\)
- \(\Sigma\) = Covariance matrix of returns \((N \times N)\)
- \(w_{mkt}\) = Market capitalization weight of the assets \((N \times 1)\)
Implied Returns: Risk Aversion Coefficient ($\lambda$)

What is the Risk Aversion Coefficient?

$$\lambda = \frac{R_B - r_f}{\sigma_B^2}$$

- Risk Aversion Coefficient
- Risk Premium

$$\sigma_B^2$$
- Variance of the Benchmark

- The Risk Aversion Coefficient characterizes the risk-return trade-off.
### Implied Returns: Market Capitalization Weights ($w_{mkt}$)

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Market Capitalization Estimate</th>
<th>Market Capitalization Weights ($w_{mkt}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Bonds</td>
<td>$8,360,741,000,000</td>
<td>20.16%</td>
</tr>
<tr>
<td>Global Bonds xUSD</td>
<td>$11,583,275,710,000</td>
<td>27.93%</td>
</tr>
<tr>
<td>World Equity xUS</td>
<td>$9,212,460,000,000</td>
<td>22.21%</td>
</tr>
<tr>
<td>Emerging Equity</td>
<td>$964,647,000,000</td>
<td>2.33%</td>
</tr>
<tr>
<td>US Large Cap Growth</td>
<td>$5,217,844,438,500</td>
<td>12.58%</td>
</tr>
<tr>
<td>US Large Cap Value</td>
<td>$5,217,844,438,500</td>
<td>12.58%</td>
</tr>
<tr>
<td>US Small Cap Growth</td>
<td>$459,897,061,500</td>
<td>1.11%</td>
</tr>
<tr>
<td>US Small Cap Value</td>
<td>$459,897,061,500</td>
<td>1.11%</td>
</tr>
<tr>
<td>Total</td>
<td>$41,476,606,710,000</td>
<td>100.00%</td>
</tr>
</tbody>
</table>
**Implied Returns: Example**

\[
\Pi = \lambda \sum w_{mkt}
\]

<table>
<thead>
<tr>
<th>Implied Return (%)</th>
<th>Weight Matrix</th>
<th>Implied Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08%</td>
<td>1.00000000</td>
<td>20.16%</td>
</tr>
<tr>
<td>0.95%</td>
<td>1.00000000</td>
<td>27.93%</td>
</tr>
<tr>
<td>3.95%</td>
<td>1.00000000</td>
<td>22.21%</td>
</tr>
<tr>
<td>5.37%</td>
<td>1.00000000</td>
<td>2.33%</td>
</tr>
<tr>
<td>5.14%</td>
<td>1.00000000</td>
<td>12.58%</td>
</tr>
<tr>
<td>3.68%</td>
<td>1.00000000</td>
<td>12.58%</td>
</tr>
<tr>
<td>6.12%</td>
<td>1.00000000</td>
<td>1.11%</td>
</tr>
<tr>
<td>3.50%</td>
<td>1.00000000</td>
<td>1.11%</td>
</tr>
</tbody>
</table>
## Implied Returns: Example

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Implied Excess Return</th>
<th>Risk-Free Rate</th>
<th>Total Implied Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Bonds</td>
<td>0.08%</td>
<td>+ 4.00%</td>
<td>= 4.08%</td>
</tr>
<tr>
<td>Global Bonds xUSD</td>
<td>0.95%</td>
<td>+ 4.00%</td>
<td>= 4.95%</td>
</tr>
<tr>
<td>World Equity xUS</td>
<td>3.95%</td>
<td>+ 4.00%</td>
<td>= 7.95%</td>
</tr>
<tr>
<td>Emerging Equity</td>
<td>5.37%</td>
<td>+ 4.00%</td>
<td>= 9.37%</td>
</tr>
<tr>
<td>US Large Cap Growth</td>
<td>5.13%</td>
<td>+ 4.00%</td>
<td>= 9.13%</td>
</tr>
<tr>
<td>US Large Cap Value</td>
<td>3.68%</td>
<td>+ 4.00%</td>
<td>= 7.68%</td>
</tr>
<tr>
<td>US Small Cap Growth</td>
<td>6.12%</td>
<td>+ 4.00%</td>
<td>= 10.12%</td>
</tr>
<tr>
<td>US Small Cap Value</td>
<td>3.50%</td>
<td>+ 4.00%</td>
<td>= 7.50%</td>
</tr>
</tbody>
</table>
Implied Returns Lead to Diversified Portfolios

MV Frontier Allocations (Implied Returns)
Asset Allocation Analysis

Efficient Frontier
Return vs. Risk (Standard Deviation)

Asset Allocations
- US Bonds
- Global Bonds xUSD
- World Equity xUS
- Emerging Equity
- US Large Cap Growth
- US Large Cap Value
- US Small Cap Growth
- US Small Cap Value

Risk (Standard Deviation) vs. Return (Standard Deviation) chart shows the efficient frontier with various asset allocations represented.
Historical Returns **DO NOT** Lead to Diversified Portfolios
Why do historical returns lead to concentrated portfolios while implied returns lead to diversified portfolios?
Excess Return vs. Beta

Circles = Historical Returns
Diamonds = Implied Returns

MV Frontier Allocations (Historical)

US Bonds and US Small Value have the highest historical excess return to beta ratio

Risk (Standard Deviation)

% Allocation per Asset Class

- US Bonds
- Global Bonds xUSD
- World Equity xUS
- Emerging Equity
- US Large Cap Growth
- US Large Cap Value
- US Small Cap Growth
- US Small Cap Value

Diamonds = Implied Returns
Circles = Historical Returns

Beta Relative to Market Capitalization Benchmark
Implied Returns: Key Observations

1. The point on the efficient frontier with the highest Sharpe ratio is the presumed efficient benchmark.

2. The implied returns are the starting point for the Black-Litterman model.

3. Most investors should stop now!
Who shouldn’t stop now?
The Black-Litterman Model

- You may or may not agree with the implied returns.
- If you don’t agree with the implied returns, the Black-Litterman model provides an elegant framework for combining the implied returns with your unique views that results in well-diversified portfolios that reflect your views.
The Black-Litterman Model: Two Types of Views

- Absolute Views
  Asset A will have a return of X%

- Relative Views
  Asset A will outperform Asset B by X%
The Black-Litterman Model: Sample View

- US Equity will outperform World Equity xUS by 150 basis points

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Total Implied Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Bonds</td>
<td>4.08%</td>
</tr>
<tr>
<td>Global Bonds xUSD</td>
<td>4.95%</td>
</tr>
<tr>
<td>World Equity xUS</td>
<td>7.95%</td>
</tr>
<tr>
<td>Emerging Equity</td>
<td>9.37%</td>
</tr>
<tr>
<td>US Large Cap Growth</td>
<td>9.13%</td>
</tr>
<tr>
<td>US Large Cap Value</td>
<td>7.68%</td>
</tr>
<tr>
<td>US Small Cap Growth</td>
<td>10.12%</td>
</tr>
<tr>
<td>US Small Cap Value</td>
<td>7.50%</td>
</tr>
</tbody>
</table>

- Implied Return World Equity x US = 7.95%
- What is the Implied Return of US Equity?
  \[
  (9.13\% \times 12.58\%)+(7.68\% \times 12.58\%)+(10.12\% \times 1.11\%)+(7.5\% \times 1.11\%) \\
  12.58\%+12.58\%+1.11\%+1.11\% \\
  = 8.44\%
  \]
- The Implied Return of World Equity x US is 49 basis points below the return of US Equity
- This is a bullish view on US Equity
The Black-Litterman Model: Sample View

• The final aspect of the view is a user-specified confidence level (0% to 100%) indicating the certainty in the view

  95% confidence indicates very high certainty in the view

  5% confidence indicates very low certainty in the view

• Sample View
US Equity will outperform World Equity xUS by 150 basis points with 75% confidence
The Black-Litterman Model:  
Formula

\[ E[R] = \left[ (\tau \Sigma)^{-1} + P' \Omega^{-1} P \right]^{-1} \left[ (\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q \right] \]

\[ E[R] = \text{New Combined Return Vector (N x 1 column vector)} \]
\[ \tau = \text{Scalar} \]
\[ \Sigma = \text{Covariance Matrix (N x N matrix)} \]
\[ P = \text{View Participation Matrix (K x N matrix that identifies the assets involved in the views)} \]
\[ \Omega = \text{Diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view (K x K matrix)} \]
\[ \Pi = \text{Implied Excess returns over the risk free rate (N x 1 column vector)} \]
\[ Q = \text{View Vector (K x 1 column vector)} \]

For more details, email support@styleadvisor.com for a copy of our white paper, “A Step-By-Step Guide to the Black-Litterman Model: Incorporating User-Specified Confidence Levels”
Does this seem too complex?

Don’t worry – it is extremely easy in Allocation ADVISOR!
Using the BL Model in AA

1. Select a portfolio (or palette) of asset classes

2a. Use our estimated inputs
2b. Override our estimated inputs
   – Risk-Free Rate
   – Risk Premium of the presumed efficient portfolio
   – Market Capitalizations*
   – Standard Deviations
   – Correlations

3. Specify Views (optional)

* Estimates of the market capitalization of the major asset classes and the asset class index proxies are updated monthly.
Select assets for this case. Enter values for palette risk premium and risk-free rate.

Reverse Optimization:
- Palette Risk Premium: 25
- Risk-Free Rate: 4.0
- Ten year:

<table>
<thead>
<tr>
<th>Use</th>
<th>Assets</th>
<th>Market Cap</th>
<th>Date</th>
<th>Weight</th>
<th>Forecast Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>US Bonds</td>
<td>8,360,741</td>
<td>Dec 2003</td>
<td>20.16</td>
<td>4.08</td>
</tr>
<tr>
<td>2</td>
<td>Global Bonds xUSD</td>
<td>11,583,276</td>
<td>Dec 2003</td>
<td>27.93</td>
<td>4.83</td>
</tr>
<tr>
<td>3</td>
<td>World Equity xUS</td>
<td>9,212,460</td>
<td>Dec 2003</td>
<td>22.21</td>
<td>7.70</td>
</tr>
<tr>
<td>4</td>
<td>Emerging Equity</td>
<td>964,647</td>
<td>Dec 2003</td>
<td>2.33</td>
<td>9.43</td>
</tr>
<tr>
<td>5</td>
<td>US Large Cap Growth</td>
<td>5,217,844</td>
<td>Dec 2003</td>
<td>12.58</td>
<td>9.72</td>
</tr>
<tr>
<td>6</td>
<td>US Large Cap Value</td>
<td>5,217,844</td>
<td>Dec 2003</td>
<td>12.58</td>
<td>8.10</td>
</tr>
<tr>
<td>7</td>
<td>US Small Cap Growth</td>
<td>459,897</td>
<td>Dec 2003</td>
<td>1.11</td>
<td>10.77</td>
</tr>
<tr>
<td>8</td>
<td>US Small Cap Value</td>
<td>459,897</td>
<td>Dec 2003</td>
<td>1.11</td>
<td>7.69</td>
</tr>
</tbody>
</table>
Asset Allocation Analysis

Efficient Frontier
Return vs. Risk (Standard Deviation)

Asset Allocations

Copy of Black-Litterman Case
- Black-Litterman Case

- US Bonds
- Global Bonds xUSD
- World Equity xUS
- Emerging Equity
- US Large Cap Growth
- US Large Cap Value
- US Small Cap Growth
- US Small Cap Value
- GSMI Global Securities
- Global Bonds xUSD
- US Bonds

Risk (Standard Deviation)

Return
Implied Returns Lead to Diversified Portfolios

MV Frontier Allocations (Implied Returns)
Black-Litterman Returns Lead to Diversified Portfolios that Reflect Your Views

MV Frontier Allocations (Black-Litterman)

Risk (Standard Deviation)

% Allocation per Asset Class

Highest Sharpe Ratio Portfolio

- US Bonds
- Global Bonds xUSD
- World Equity xUS
- Emerging Equity
- US Large Cap Growth
- US Large Cap Value
- US Small Cap Growth
- US Small Cap Value
Allocation Analysis

View: US Equity outperforms World Equity xUS by 150 Basis Points

Assets not named in the views are unaffected - no active risk!

All else equal, higher confidence in a view results in greater departures away from the presumed efficient benchmark weights.
What’s new in AllocationADVISOR?

- Multiple Allocation Cases
- Plot Two Efficient Frontiers on the same graph
- Plot the benchmark on the efficient frontier
- Select blends as assets
- Multiple custom portfolios
Questions?
Misc. Items
What is Resampling?

• Resampling is a Monte Carlo technique for estimating the inputs for mean-variance optimization and eventually the resampled efficient frontier. It results in well diversified portfolios.

• Patented, licensed, and promoted by Richard Michaud
How to Resample

1. Estimate returns, standard deviations, and correlations
2. Run a multivariate simulation that results in a new set of returns, standard deviations, and correlations.
3. From the resulting “efficient frontier” record the weights and the returns of the efficient portfolios at predetermined standard deviation intervals (i.e. 5%, 6%, 7%, etc.)
4. Repeats Steps 2 and 3 1000+ times.
5. Calculate the average allocation to the assets for each predetermined interval and the average return, and then graph them in return – standard deviation space to create the resampled frontier.
Shortcomings of Resampling I

1. Portfolios inherit the estimation error in the original inputs – Scherer 2002
2. Lack of theory – No reason why resampled portfolios should be optimal – Scherer 2002
3. Resampling can result in frontiers with upward sloping sections – Scherer 2002
4. In the absence of views, resampling results in active risk relative to a policy benchmark – why take bets with out a reason?
5. No framework for incorporating views
Shortcomings of Resampling II

6. Surprising large amount of variation in recommended portfolio overtime

Allocations from the Maximum Sharpe Ratio Point of the Resampled Frontier
### Shortcomings of Resampling III

#### 7. Underperformed in Historical Back Test

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Returns</th>
<th>Implied Returns</th>
<th>Buy and Hold</th>
<th>Resampling</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Return Statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual Geometric Return</td>
<td>11.76%</td>
<td>11.76%</td>
<td>11.40%</td>
<td>10.67%</td>
</tr>
<tr>
<td>Annualized Standard Deviation</td>
<td>10.55%</td>
<td>10.55%</td>
<td>12.07%</td>
<td>14.98%</td>
</tr>
<tr>
<td>Realized Sharpe Ratio</td>
<td>0.7667</td>
<td>0.7667</td>
<td>0.6408</td>
<td>0.4676</td>
</tr>
<tr>
<td><strong>Beginning Value (1/1/1983)</strong></td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
</tr>
<tr>
<td><strong>Ending Value (12/31/2003)</strong></td>
<td>$1133</td>
<td>$1133</td>
<td>$1066</td>
<td>$941</td>
</tr>
<tr>
<td><strong>Benchmark Relative Statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Historical Alpha</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.98%</td>
<td>-1.74%</td>
</tr>
<tr>
<td>Residual Risk</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.99%</td>
<td>7.83%</td>
</tr>
<tr>
<td>Active Risk</td>
<td>0.00%</td>
<td>0.00%</td>
<td>2.41%</td>
<td>8.14%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>0</td>
<td>0</td>
<td>-0.4926</td>
<td>-0.2219</td>
</tr>
</tbody>
</table>
Fund-of-Funds Optimization

Approach:
Manager Selection is a portfolio construction problem. The methodology is analogous to the construction of portfolios of individual securities. It attempts to maximize active return for a given level of active risk.
Fund-of-Funds Optimization: Key Benefits

1. Controls active risk introduced by active managers

2. Determines the optimum allocation of funds between active and passive managers based on the investor’s active risk tolerance / aversion level.

3. Allows the use of off-benchmark managers and balanced fund managers by including completion analysis (Completion Analysis attempts to minimize style bets i.e. the difference between the aggregate style exposures and the policy benchmark style exposures).
Fund of Funds Optimization Vs. Optimize Managers to Track A Benchmark:

• Optimizing managers to track a benchmark attempts to minimize misfit risk – the difference between the aggregate style exposures and the policy benchmark style exposures.

• Fund of funds optimization attempts to maximize alpha for a given level of total active risk. Total active risk includes “Selection” active risk and misfit active risk.
Fund-of-Funds Optimization

Theoretical Background:

- Optimizing residual returns against residual risk
- Returns-based style analysis