Allocation**ADVISOR**
And
The Black-Litterman Model

Tom Idzorek, CFA
Zephyr Associates, Inc.
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Please send questions to: Support@styleadvisor.com
Subject: Black-Litterman

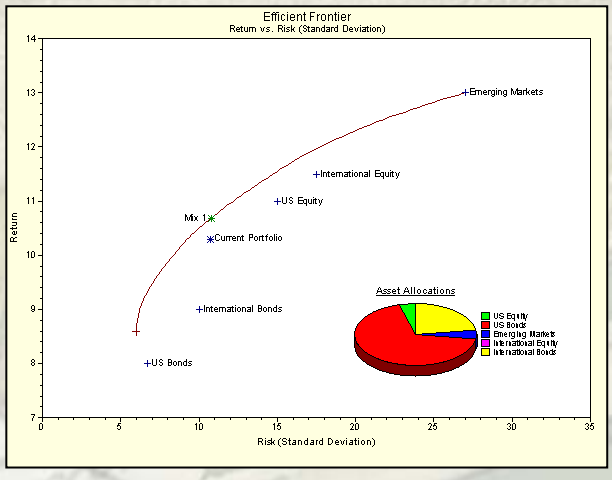
Overview / Outline

- What is Allocation**ADVISOR**?
- Why is a new model important?
- What is the Black-Litterman model?

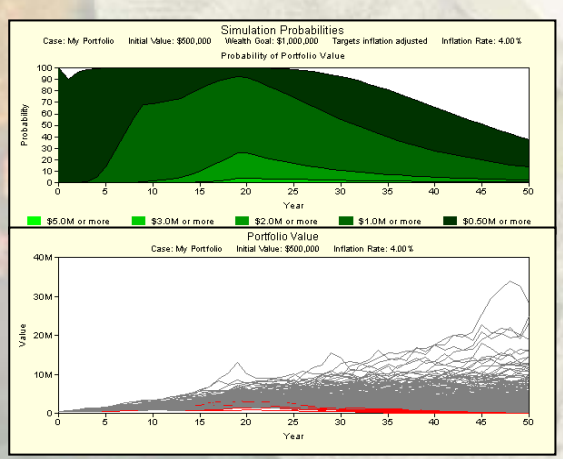
- A Demonstration of the model in AA
- A Look at other new features in AA

What is Allocation**ADVISOR**?

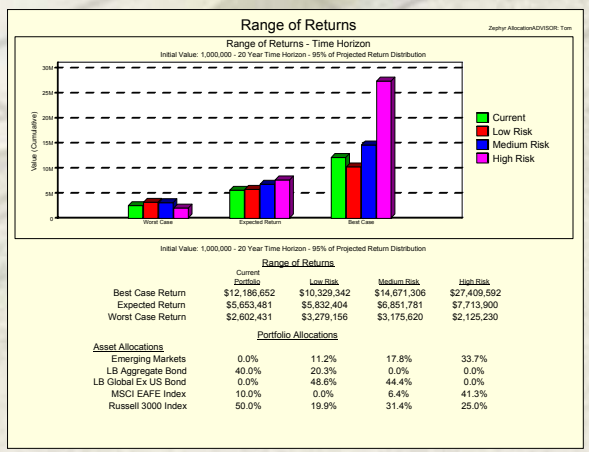
Asset Allocation



Monte Carlo



Analytic Projections



Why is a new model important?

Non-Diversified Portfolios

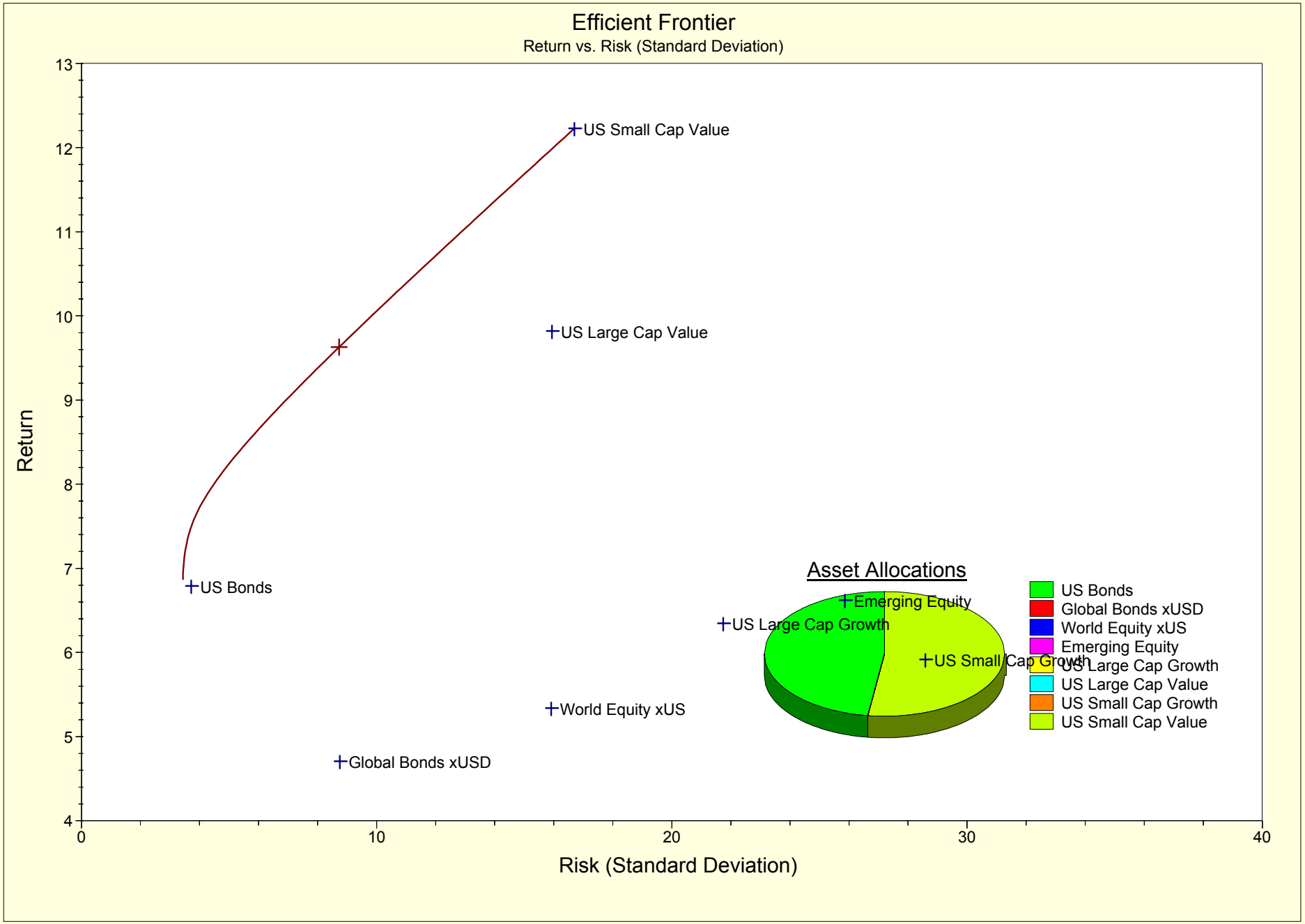
Efficient frontiers based on historical data lead to highly concentrated portfolios.

What is the Solution?

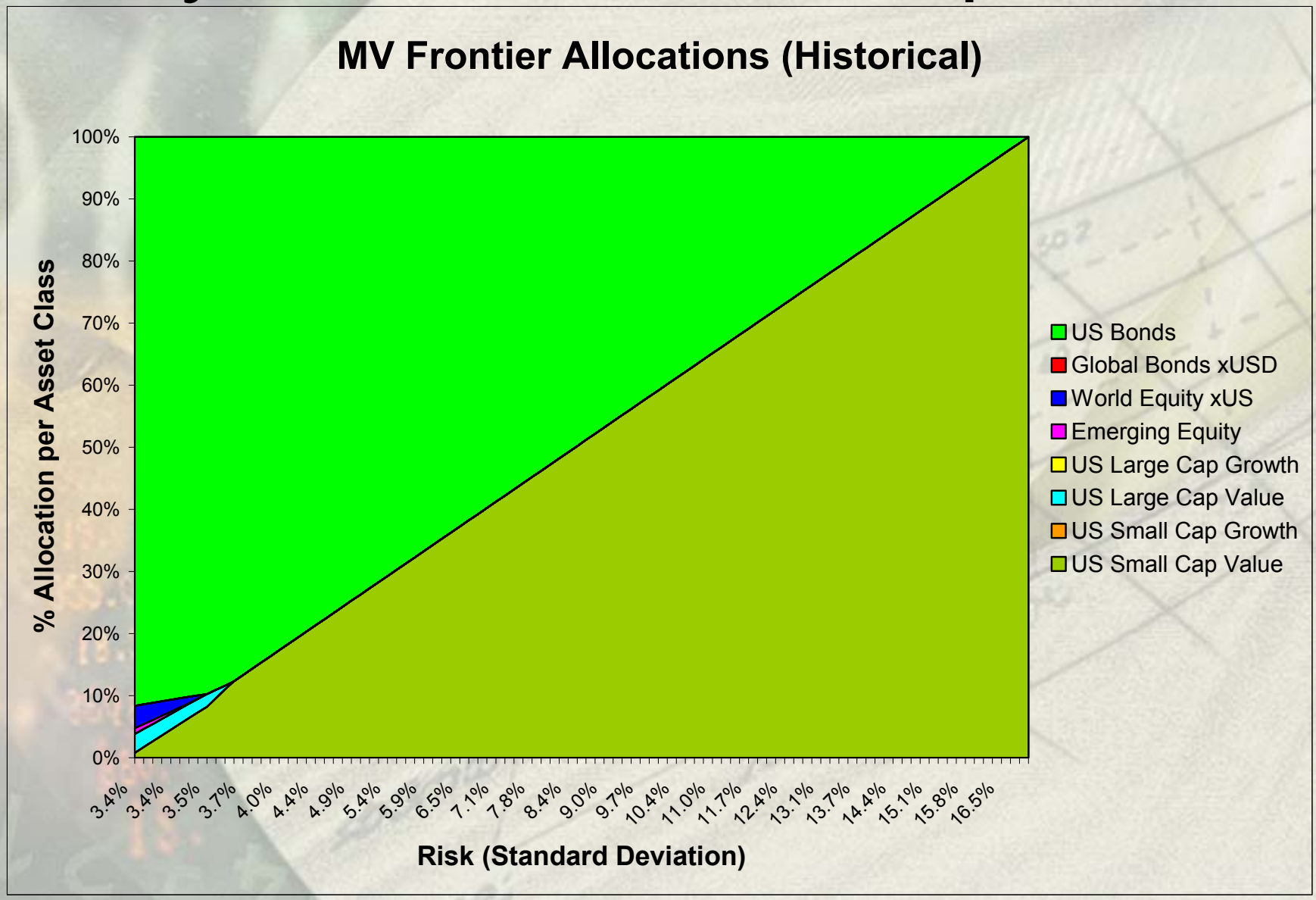
The Black-Litterman Model

Asset Allocation Analysis

Zephyr AllocationADVISOR: Tom



Why is a new model important?



What is the Black-Litterman Model?

It is….

- A Bayesian Asset Allocation Approach
- A Strategic Asset Allocation Model
- A Tactical Asset Allocation Model
- A tool for creating a set of expected returns
- A tool for blending implied returns and investor returns

It is not…

- A source of Alpha

The Black-Litterman Model starts with Implied Returns

Other Names for Implied Returns….

- CAPM Returns
- Reverse Optimized Returns
- Market Returns
- Consensus Returns
- Imputed Returns
- Equilibrium Returns

Implied Returns

A rich theoretical background….

- Sharpe, William F. (1964). “Capital Asset Prices: A Theory of Market Equilibrium.” *Journal of Finance*, September, 425-442.

- Sharpe, William F. (1974). “Imputing Expected Security Returns from Portfolio Composition.” *Journal of Financial and Quantitative Analysis*, June, 463-472.

Implied Returns

- The return of any asset or asset class can be separated into three parts:
  - Risk-Free Return
  - Return Correlated with Benchmark
  - Return Not Correlated with Benchmark
- Returns that are correlated with the benchmark result in beta risk (systematic risk, benchmark risk, non-diversifiable risk, or market risk)
- Beta risk is the type of risk that is rewarded with a premium

Implied Returns: Using the CAPM

Expected returns are a function of beta risk….

$$E(R_i) = R_f + \beta_{i,B}(R_B - R_f)$$

R_f = Risk-Free Rate

$\beta_{i,B}$ = Beta of Asset Class i Relative to Benchmark

R_B = Return of Benchmark

$R_B - R_f$ = Forward Looking Risk Premium of Benchmark*
(Return of Benchmark over the Risk-Free Rate)

* In this case the benchmark is the market capitalization weighted portfolio

Implied Returns: Using Reverse Optimization

The same *excess* returns result from reverse optimization….

$$\Pi = \lambda \Sigma w_{mkt}$$

Π = Implied Excess returns over the risk free rate (*N x 1*)

λ = Risk aversion coefficient (*1 x 1*)

Σ = Covariance matrix of returns (*N x N*)

w_{mkt} = Market capitalization weight of the assets (*N x 1*)

Implied Returns: Risk Aversion Coefficient (λ)

What is the Risk Aversion Coefficient?

$$\lambda = \frac{R_B - r_f}{\sigma_B^2} = \frac{\text{Risk Premium}}{\text{Variance}}$$

σ_B^2 = Variance of the Benchmark

- The Risk Aversion Coefficient characterizes the risk-return trade-off.

Implied Returns: Market Capitalization Weights (w_{mkt})

Asset Class	Market Capitalization Estimate	Market Capitalization Weights w_{mkt}
US Bonds	$8,360,741,000,000	20.16%
Global Bonds xUSD	$11,583,275,710,000	27.93%
World Equity xUS	$9,212,460,000,000	22.21%
Emerging Equity	$964,647,000,000	2.33%
US Large Cap Growth	$5,217,844,438,500	12.58%
US Large Cap Value	$5,217,844,438,500	12.58%
US Small Cap Growth	$459,897,061,500	1.11%
US Small Cap Value	$459,897,061,500	1.11%
Total	$41,476,606,710,000	100.00%

Implied Returns: Example

$$\Pi = \lambda \Sigma w_{mkt}$$

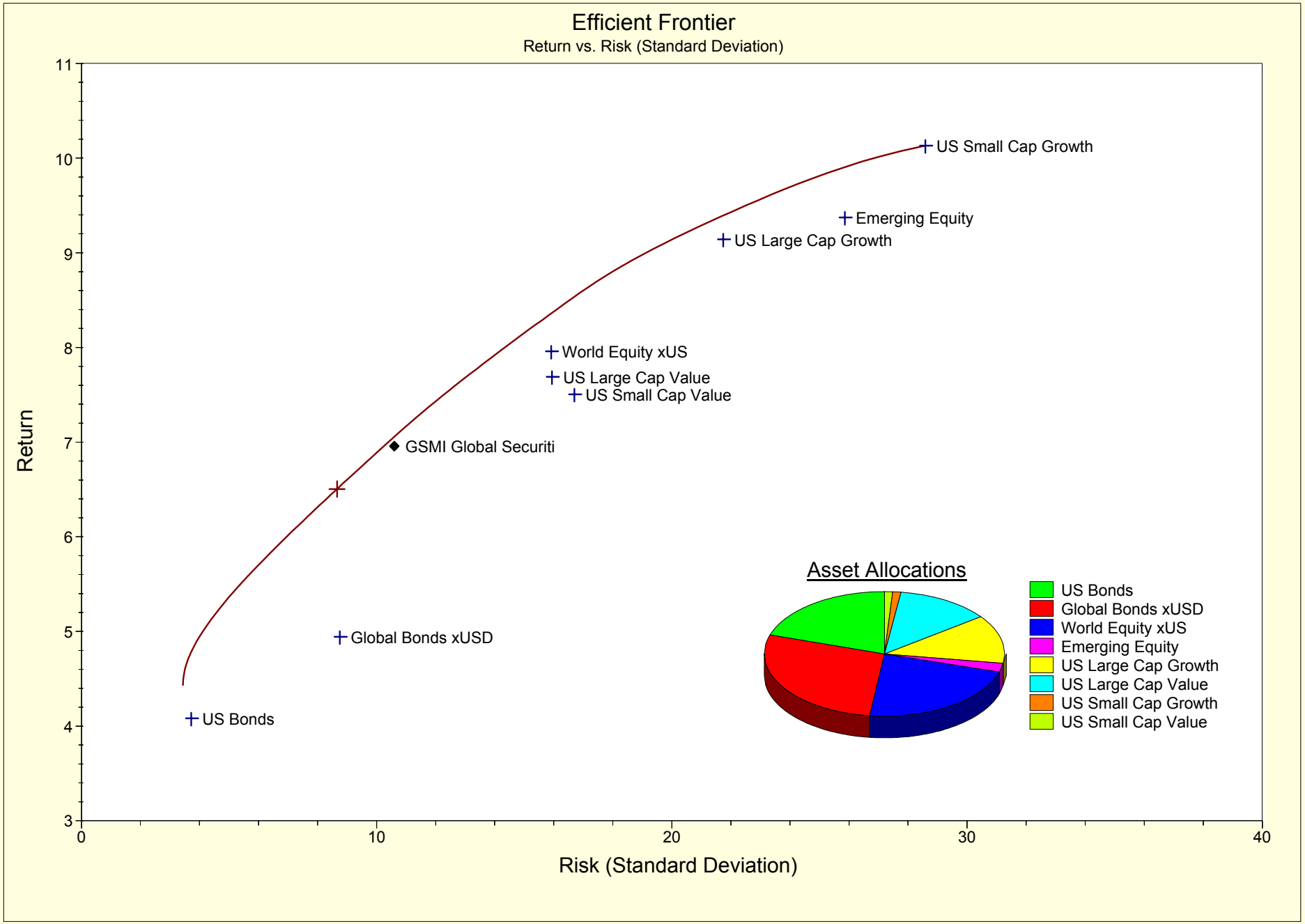
$$\begin{bmatrix} 0.08\% \\ 0.95\% \\ 3.95\% \\ 5.37\% \\ 5.14\% \\ 3.68\% \\ 6.12\% \\ 3.50\% \end{bmatrix} = 3.37 \begin{bmatrix} .0014 & .0015 & -.0008 & -.0017 & -.0010 & -.0007 & -.0015 & -.0006 \\ .0015 & .0076 & .0026 & -.0006 & -.0013 & -.0003 & -.0002 & .0005 \\ -.0008 & .0026 & .0251 & .0292 & .0208 & .0147 & .0248 & .0134 \\ -.0017 & -.0006 & .0292 & .0663 & .0359 & .0244 & .0490 & .0268 \\ -.0010 & -.0013 & .0208 & .0359 & .0468 & .0283 & .0520 & .0260 \\ -.0007 & -.0003 & .0147 & .0244 & .0283 & .0252 & .0314 & .0215 \\ -.0015 & -.0002 & .0248 & .0490 & .0520 & .0314 & .0809 & .0411 \\ -.0006 & .0005 & .0134 & .0268 & .0260 & .0215 & .0411 & .0276 \end{bmatrix} \begin{bmatrix} 20.16\% \\ 27.93\% \\ 22.21\% \\ 2.33\% \\ 12.58\% \\ 12.58\% \\ 1.11\% \\ 1.11\% \end{bmatrix}$$

Implied Returns: Example

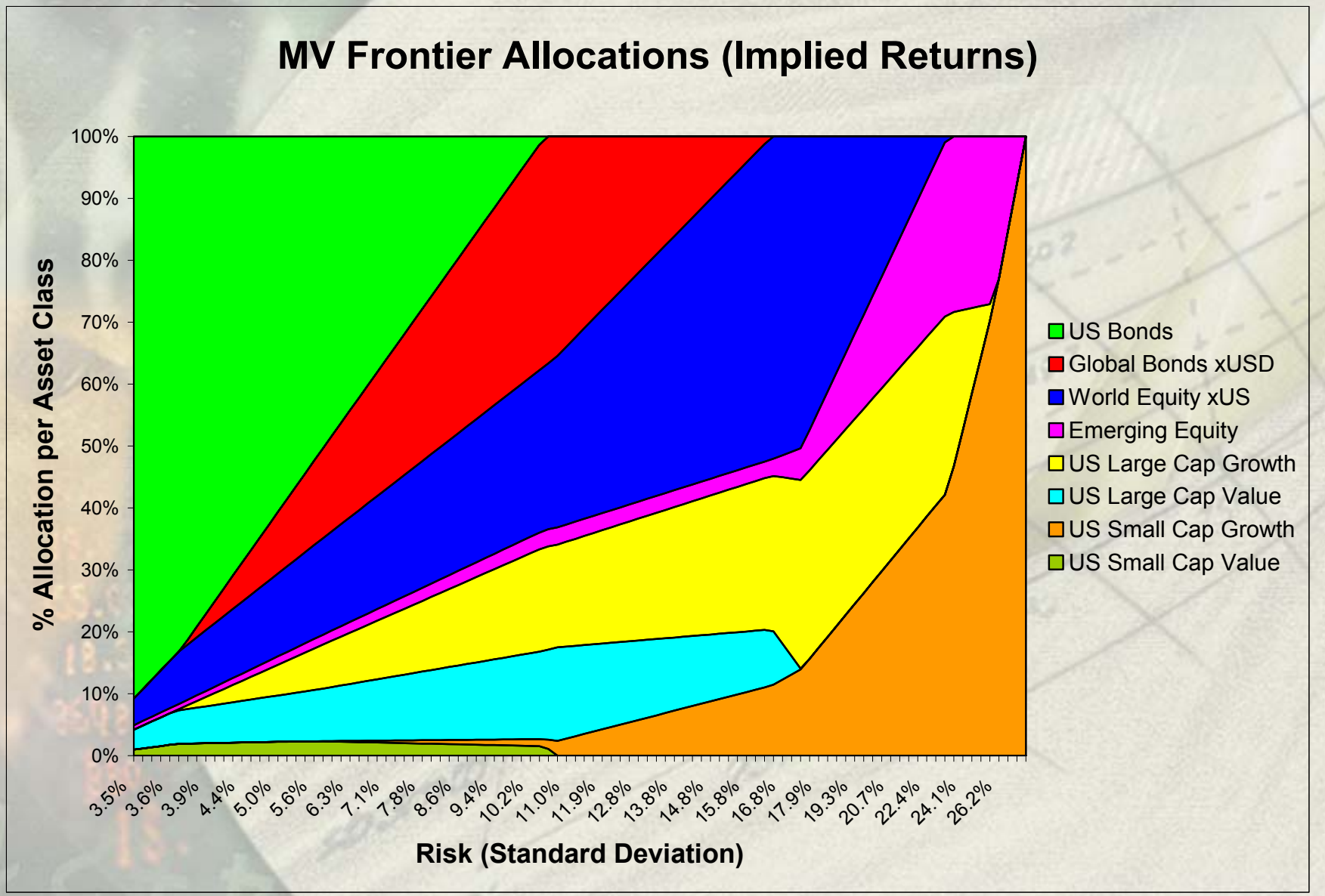
Asset Class	Implied Excess Return		Risk-Free Rate		Total Implied Return
US Bonds	0.08%	+	4.00%	=	4.08%
Global Bonds xUSD	0.95%	+	4.00%	=	4.95%
World Equity xUS	3.95%	+	4.00%	=	7.95%
Emerging Equity	5.37%	+	4.00%	=	9.37%
US Large Cap Growth	5.13%	+	4.00%	=	9.13%
US Large Cap Value	3.68%	+	4.00%	=	7.68%
US Small Cap Growth	6.12%	+	4.00%	=	10.12%
US Small Cap Value	3.50%	+	4.00%	=	7.50%

Asset Allocation Analysis

Zephyr AllocationADVISOR: Tom

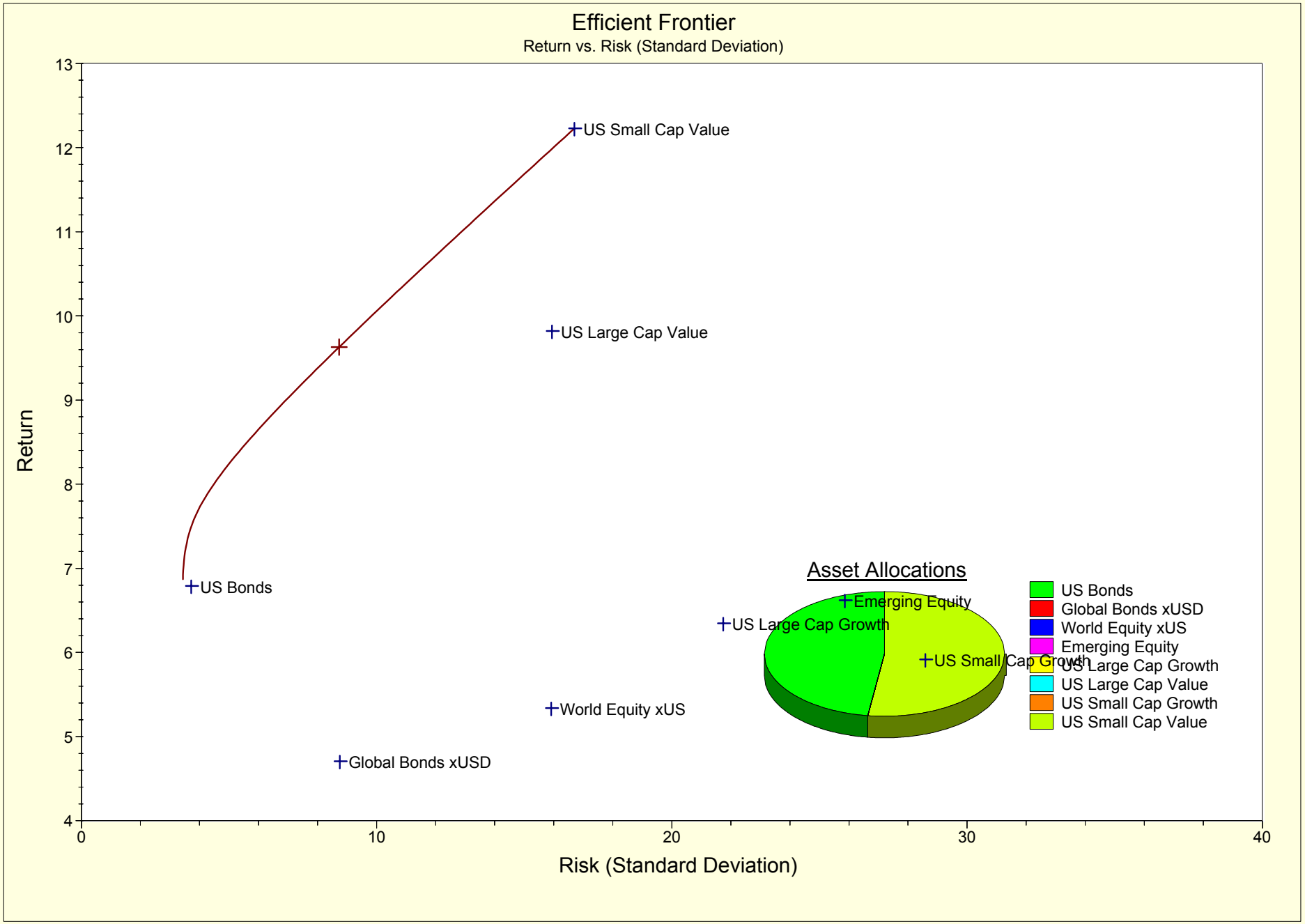


Implied Returns Lead to Diversified Portfolios

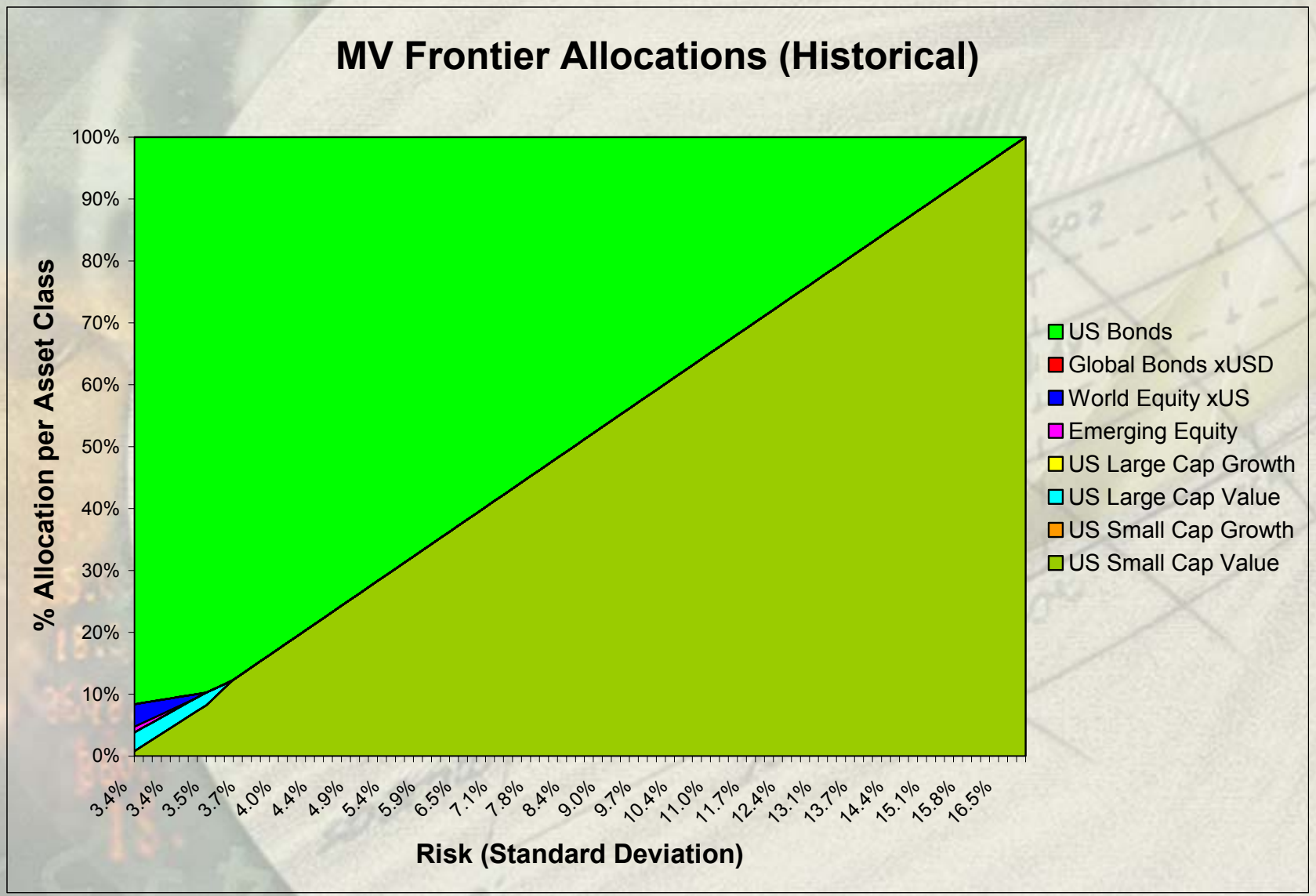


Asset Allocation Analysis

Zephyr AllocationADVISOR: Tom

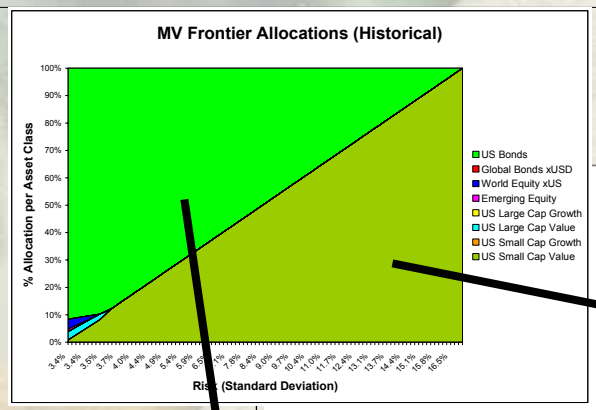


Historical Returns DO NOT Lead to Diversified Portfolios



Why do historical returns lead to concentrated portfolios while implied returns lead to diversified portfolios?

Excess Return vs. Beta



Excess Return

8.00%
6.00%
4.00%
2.00%
0.00%

US Bonds and US Small Value have the highest historical excess return to beta ratio

0.00 0.50 1.00 1.50 2.00 2.50

Beta Relative to Market Capitalization Benchmark

- US Bonds
- Global Bonds xUSD
- World Equity xUS
- Emerging Equity
- US Large Cap Growth
- US Large Cap Value
- US Small Growth
- US Small Value

Diamonds = Implied Returns
Circles = Historical Returns

Implied Returns: Key Observations

1. The point on the efficient frontier with the highest Sharpe ratio is the presumed efficient benchmark

2. The implied returns are the starting point for the Black-Litterman model

3. Most investors should stop now!

Who shouldn’t stop now?

The Black-Litterman Model

- You may or may not agree with the implied returns
- If you don't agree with the implied returns, the Black-Litterman model provides an elegant framework for combining the implied returns with your unique views that results in well-diversified portfolios that reflect your views.

The Black-Litterman Model: Two Types of Views

- Absolute Views

  Asset A will have a return of X%

- Relative Views

  Asset A will outperform Asset B by X%

The Black-Litterman Model: Sample View

- US Equity will outperform World Equity xUS by 150 basis points

Asset Class	Total Implied Return
US Bonds	4.08%
Global Bonds xUSD	4.95%
World Equity xUS	7.95%
Emerging Equity	9.37%
US Large Cap Growth	9.13%
US Large Cap Value	7.68%
US Small Cap Growth	10.12%
US Small Cap Value	7.50%

- Implied Return World Equity x US = 7.95%
- What is the Implied Return of US Equity?

$$\frac{(9.13\%*12.58\%)+(7.68\%*12.58\%)+(10.12\%*1.11\%)+(7.5\%*1.11\%)}{12.58\%+12.58\%+1.11\%+1.11\%}$$

$$= 8.44\%$$

- The Implied Return of World Equity xUS is 49 basis points below the return of US Equity
- This is a bullish view on US Equity

The Black-Litterman Model: Sample View

- The final aspect of the view is a user-specified confidence level (0% to 100%) indicating the certainty in the view

  95% confidence indicates very high certainty in the view

  5% confidence indicates very low certainty in the view

- Sample View

US Equity will outperform World Equity xUS by 150 basis points with 75% confidence

The Black-Litterman Model: Formula

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

$E[R]$ = New Combined Return Vector (*N x 1* column vector)

τ = Scalar

Σ = Covariance Matrix (*N x N* matrix);

P = View Participation Matrix (*K x N* matrix that identifies the assets involved in the views)

Ω = Diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view (*K x K* matrix);

Π = Implied Excess returns over the risk free rate (*N x 1* column vector)

Q = View Vector (*K x 1* column vector)

For more details, email support@styleadvisor.com for a copy of our white paper, "A Step-By-Step Guide to the Black-Litterman Model: Incorporating User-Specified Confidence Levels"

Does this seem too complex?

Don’t worry – it is extremely easy in Allocation**ADVISOR**!

Using the BL Model in AA

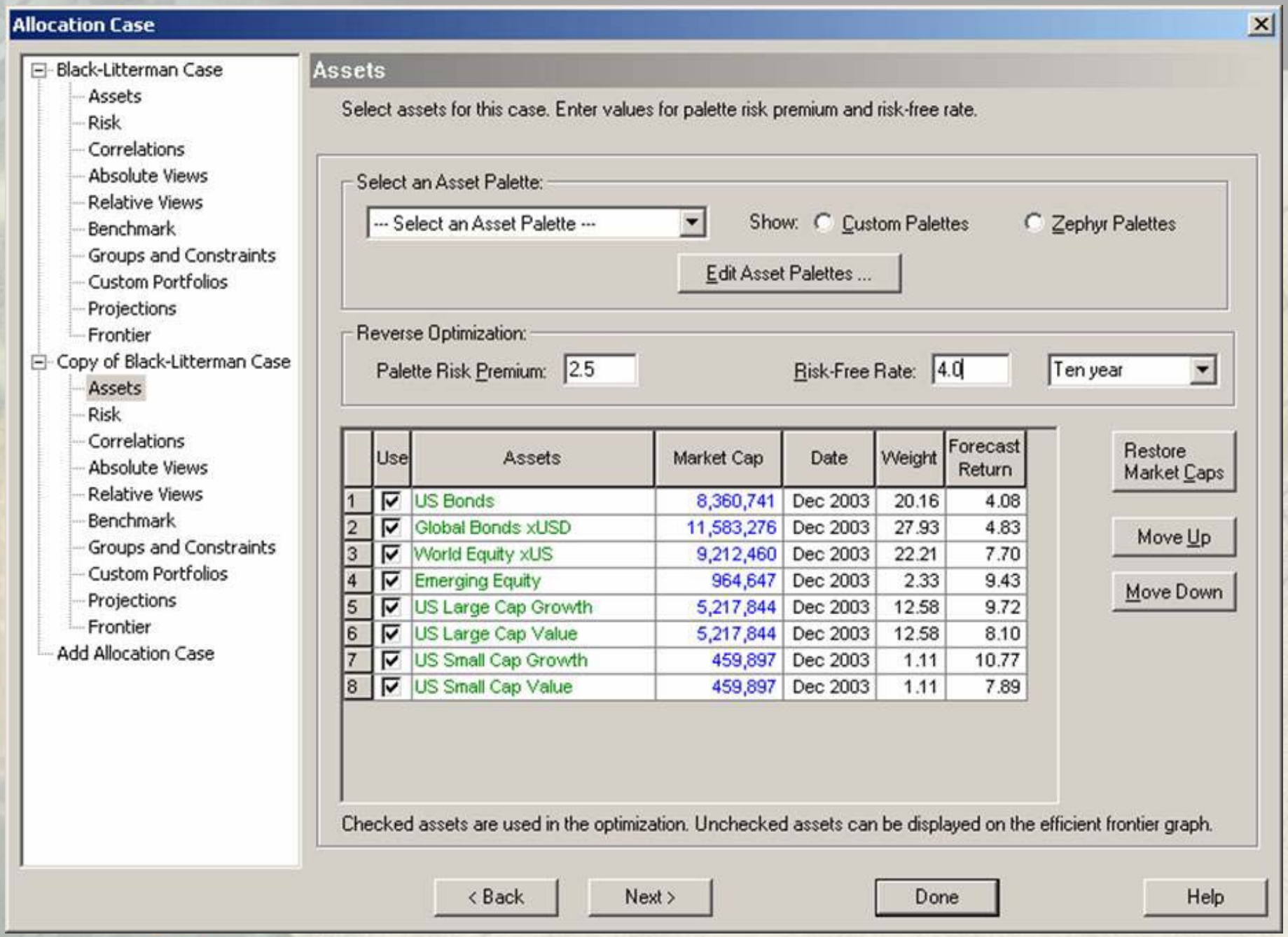
1. Select a portfolio (or palette) of asset classes

2a. Use our estimated inputs

2b. Override our estimated inputs
   - Risk-Free Rate
   - Risk Premium of the presumed efficient portfolio
   - Market Capitalizations*
   - Standard Deviations
   - Correlations

3. Specify Views (optional)

* Estimates of the market capitalization of the major asset classes and the asset class index proxies are updated monthly.


Allocation Case
Black-Litterman Case
Assets
Risk
Correlations
Absolute Views
Relative Views
Benchmark
Groups and Constraints
Custom Portfolios
Projections
Frontier
Copy of Black-Litterman Case
Assets
Risk
Correlations
Absolute Views
Relative Views
Benchmark
Groups and Constraints
Custom Portfolios
Projections
Frontier
Add Allocation Case
Assets
Select assets for this case. Enter values for palette risk premium and risk-free rate.
Select an Asset Palette:
--- Select an Asset Palette ---
Show: Custom Palettes Zephyr Palettes
Edit Asset Palettes ...
Reverse Optimization:
Palette Risk Premium: 2.5
Risk-Free Rate: 4.0
Ten year
Use
Assets
Market Cap
Date
Weight
Forecast Return
1 US Bonds 8,360,741 Dec 2003 20.16 4.08
2 Global Bonds xUSD 11,583,276 Dec 2003 27.93 4.83
3 World Equity xUS 9,212,460 Dec 2003 22.21 7.70
4 Emerging Equity 964,647 Dec 2003 2.33 9.43
5 US Large Cap Growth 5,217,844 Dec 2003 12.58 9.72
6 US Large Cap Value 5,217,844 Dec 2003 12.58 8.10
7 US Small Cap Growth 459,897 Dec 2003 1.11 10.77
8 US Small Cap Value 459,897 Dec 2003 1.11 7.89
Restore Market Caps
Move Up
Move Down
Checked assets are used in the optimization. Unchecked assets can be displayed on the efficient frontier graph.
< Back
Next >
Done
Help

Asset Allocation Analysis

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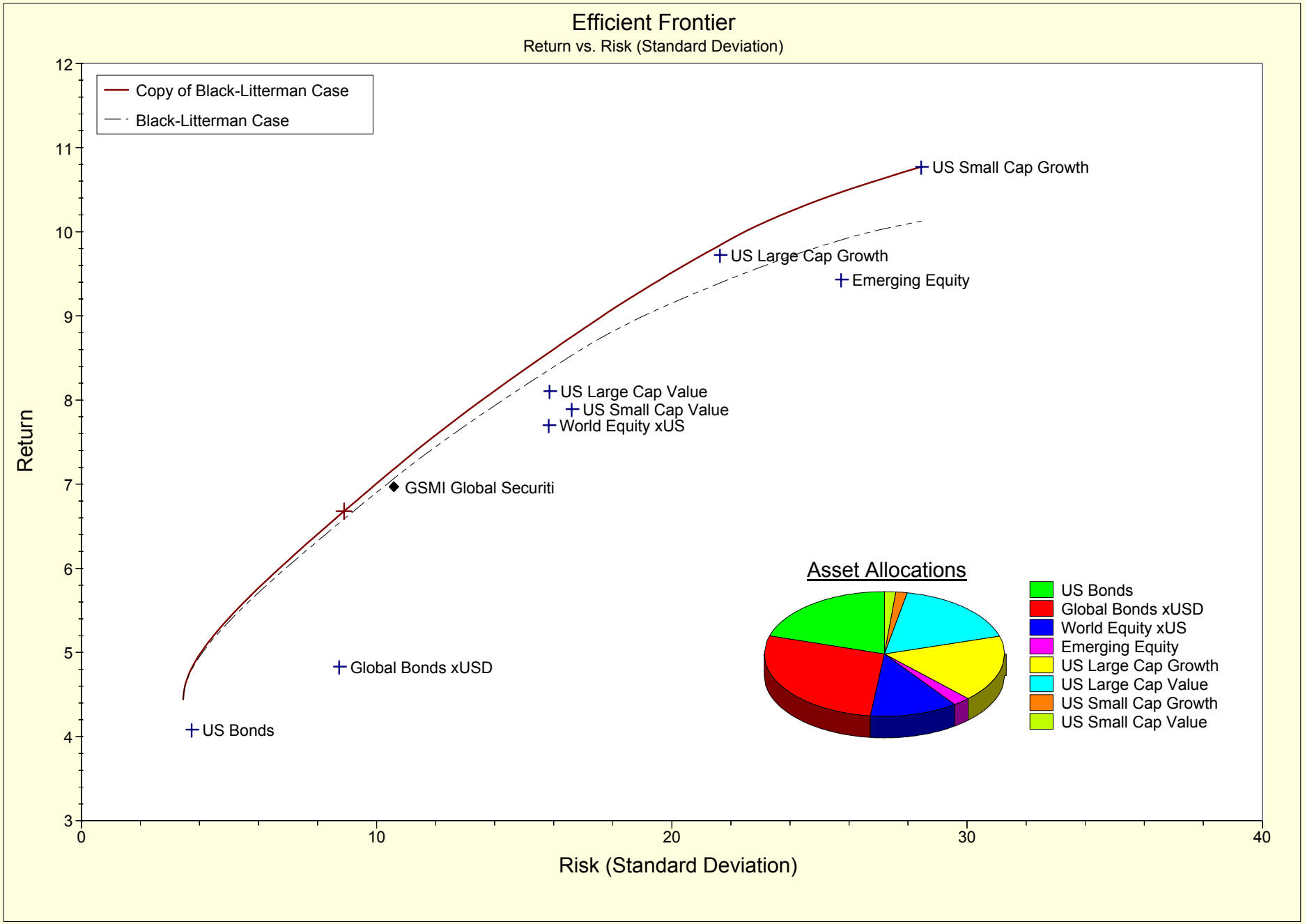
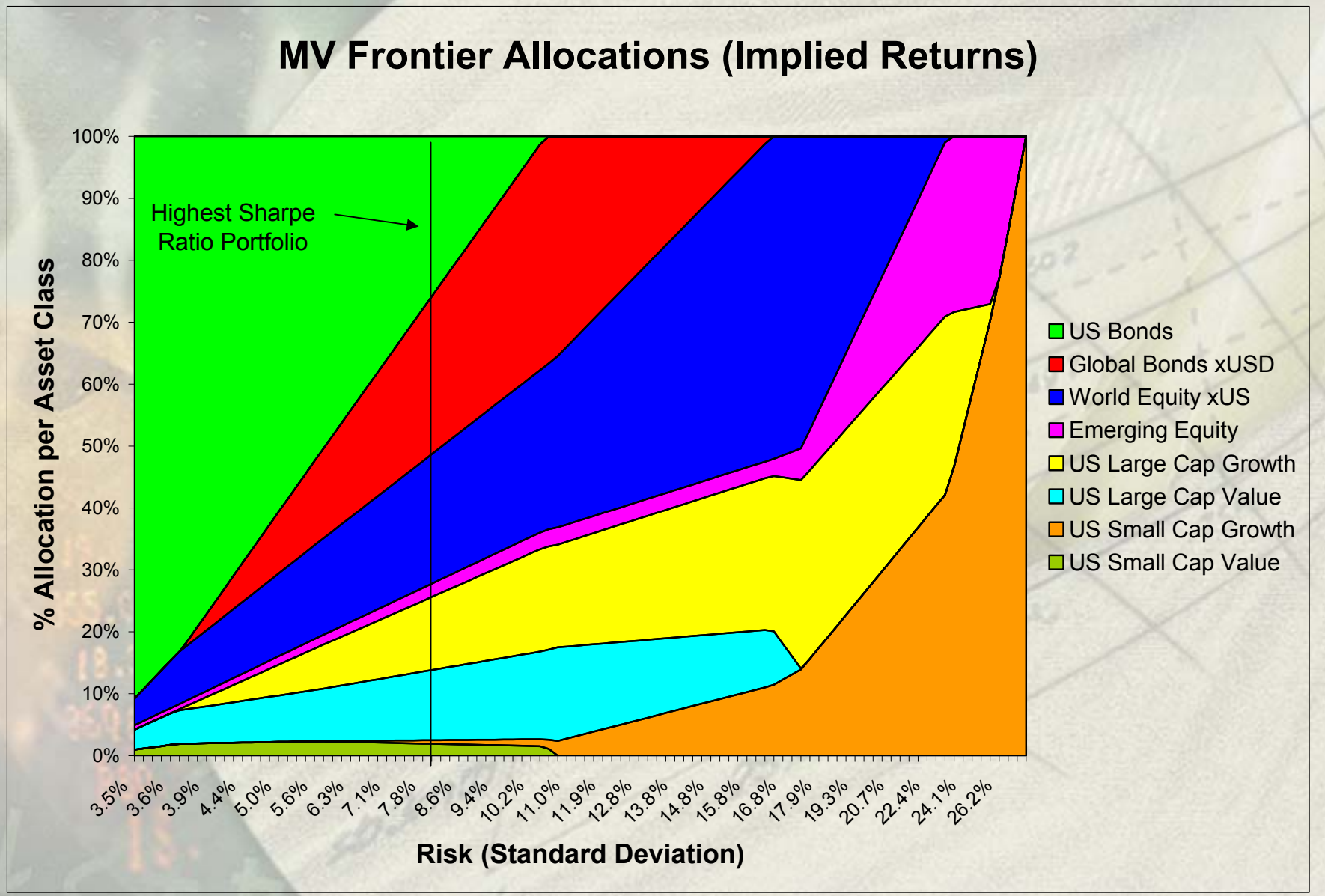
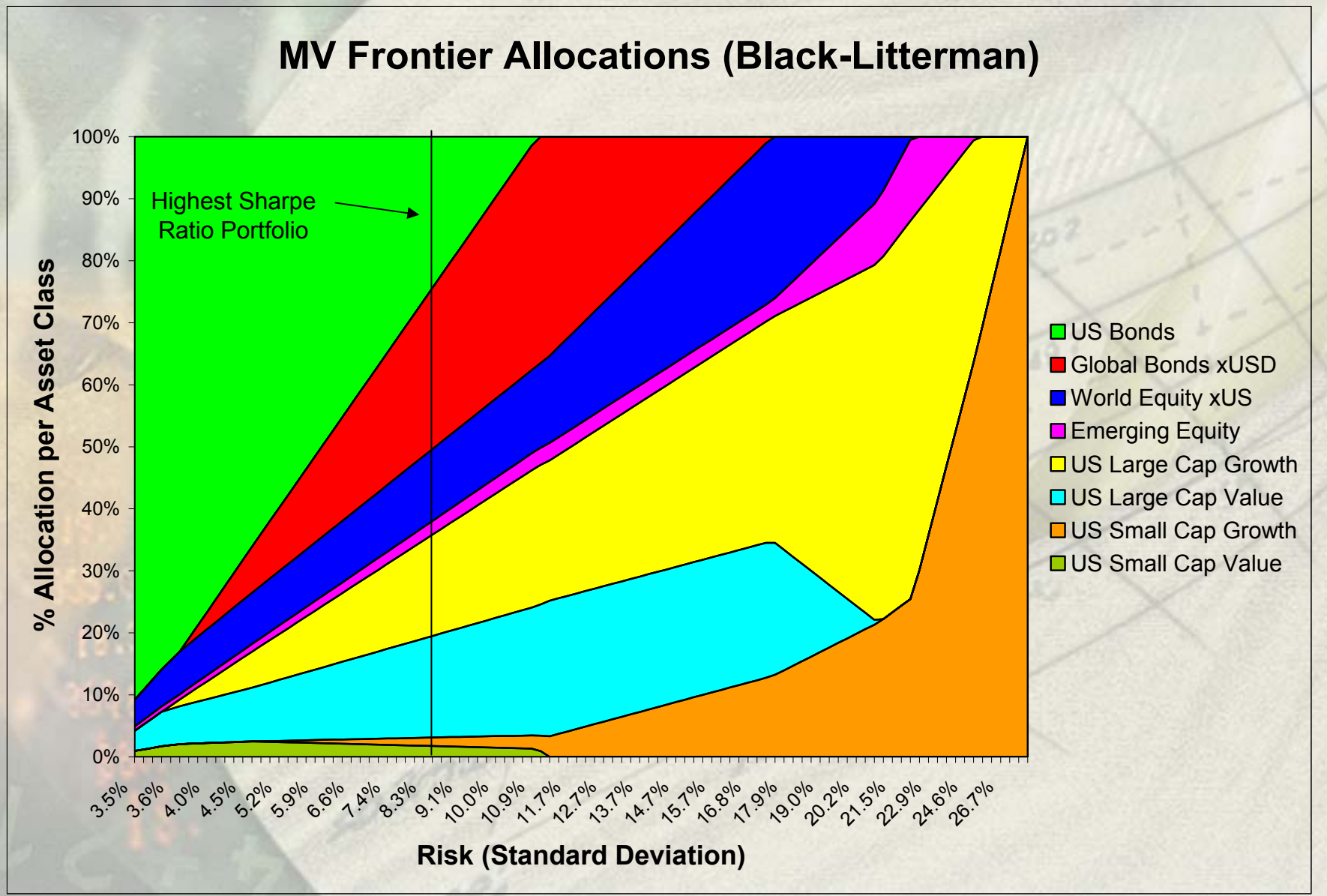


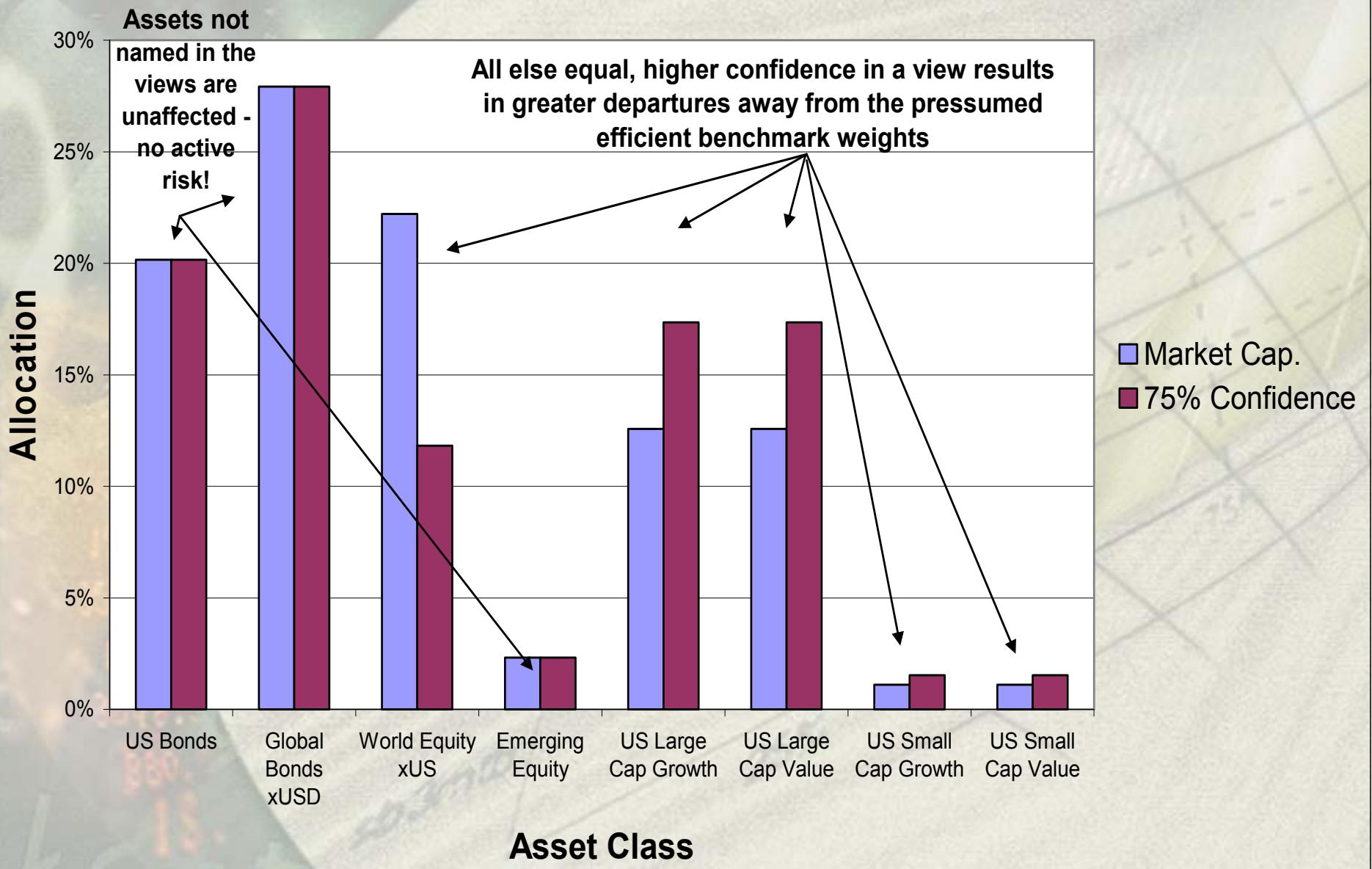
Implied Returns Lead to Diversified Portfolios



Black-Litterman Returns Lead to Diversified Portfolios that Reflect Your Views



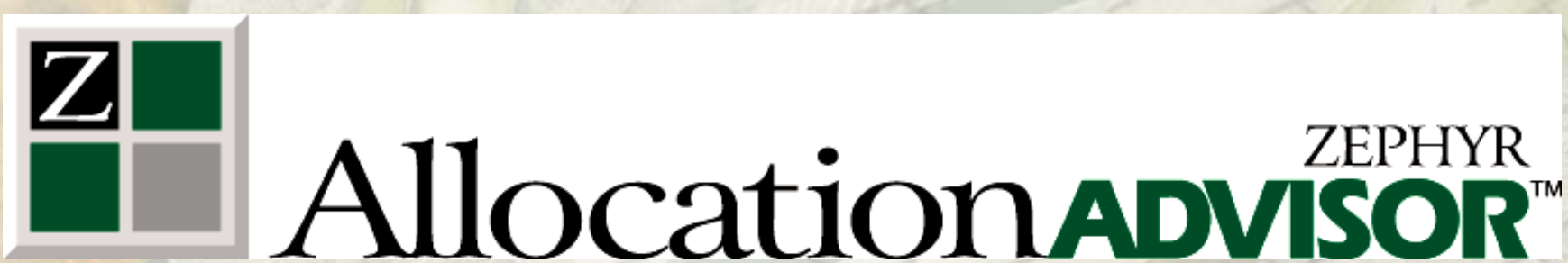
Allocation Analysis
View: US Equity outperforms World Equity xUS by 150 Basis Points



What's new in AllocationADVISOR?

- Multiple Allocation Cases
- Plot Two Efficient Frontiers on the same graph
- Plot the benchmark on the efficient frontier
- Select blends as assets
- Multiple custom portfolios

Questions?



Misc. Items

What is Resampling?

- Resampling is a Monte Carlo technique for estimating the inputs for mean-variance optimization and eventually the resampled efficient frontier. It results in well diversified portfolios.

- Patented, licensed, and promoted by Richard Michaud

How to Resample

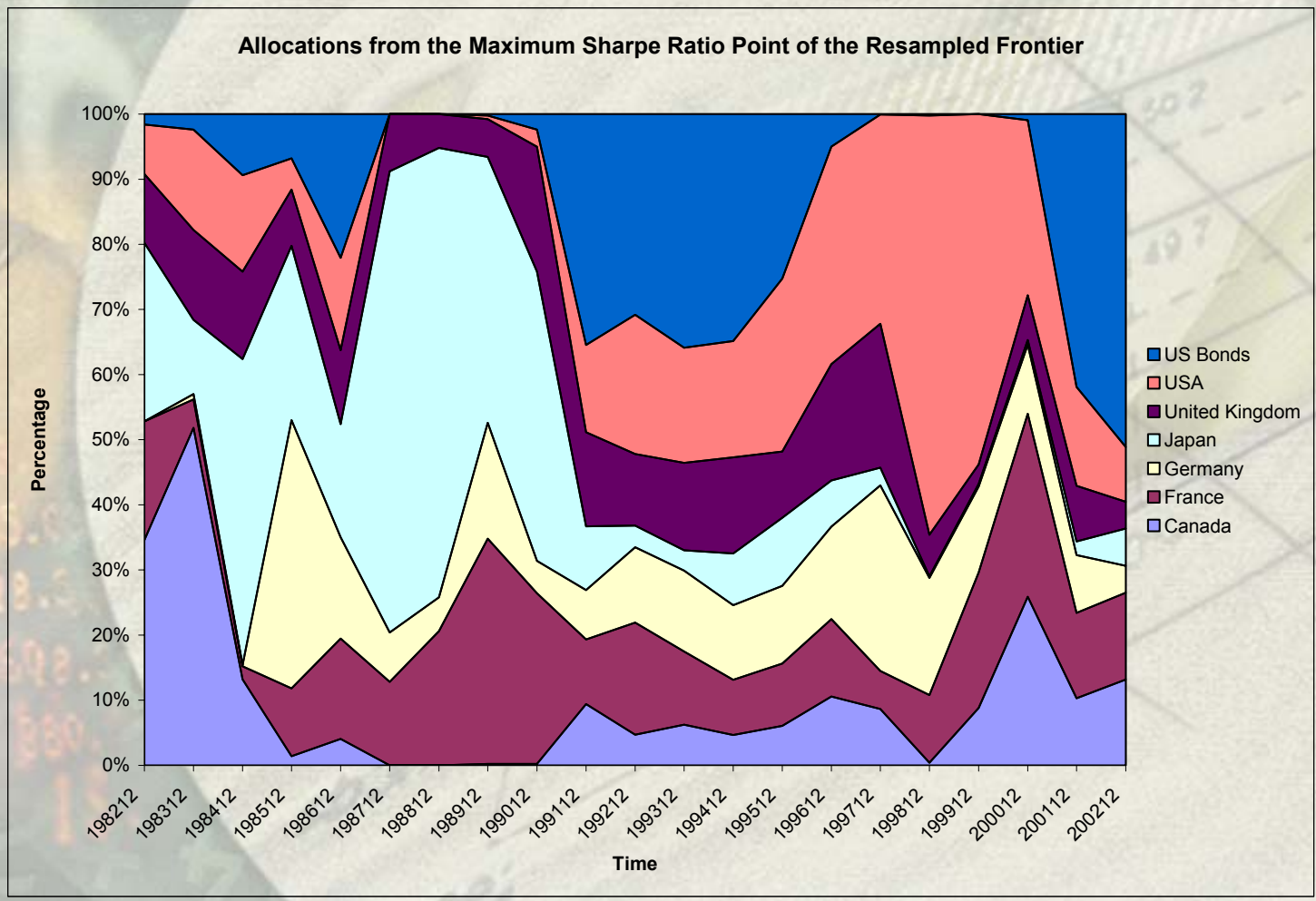
1. Estimate returns, standard deviations, and correlations
2. Run a multivariate simulation that results in a new set of returns, standard deviations, and correlations.
3. From the resulting “efficient frontier” record the weights and the returns of the efficient portfolios at predetermined standard deviation intervals (i.e. 5%, 6%, 7%, etc.)
4. Repeats Steps 2 and 3 1000+ times.
5. Calculate the average allocation to the assets for each predetermined interval and the average return, and then graph them in return – standard deviation space to create the resampled frontier.

Shortcomings of Resampling I

1. Portfolios inherit the estimation error in the original inputs – *Scherer 2002*
2. Lack of theory – No reason why resampled portfolios should be optimal – *Scherer 2002*
3. Resampling can result in frontiers with upward sloping sections – *Scherer 2002*
4. In the absence of views, resampling results in active risk relative to a policy benchmark – why take bets with out a reason?
5. No framework for incorporating views

Shortcomings of Resampling II

6. Surprising large amount of variation in recommended portfolio overtime



Shortcomings of Resampling III

7. Underperformed in Historical Back Test

	Benchmark Returns	Implied Returns	Buy and Hold	Resampling
Total Return Statistics				
Annual Geometric Return	11.76%	11.76%	11.40%	10.67%
Annualized Standard Deviation	10.55%	10.55%	12.07%	14.98%
Realized Sharpe Ratio	0.7667	0.7667	0.6408	0.4676
Beginning Value (1/1/1983)	**$100**	**$100**	**$100**	**$100**
Ending Value (12/31/2003)	**$1133**	**$1133**	**$1066**	**$941**
Benchmark Relative Statistics				
Historical Alpha	0.00%	0.00%	-0.98%	-1.74%
Residual Risk	0.00%	0.00%	1.99%	7.83%
Active Risk	0.00%	0.00%	2.41%	8.14%
Information Ratio	0	0	-0.4926	-0.2219

Fund-of-Funds Optimization

Approach:

Manager Selection is a portfolio construction problem. The methodology is analogous to the construction of portfolios of individual securities. It attempts to maximize active return for a given level of active risk

Fund-of-Funds Optimization: Key Benefits

1. Controls active risk introduced by active managers
2. Determines the optimum allocation of funds between active and passive managers based on the investor's active risk tolerance / aversion level.
3. Allows the use of off-benchmark managers and balanced fund managers by including completion analysis (Completion Analysis attempts to minimize style bets i.e. the difference between the aggregate style exposures and the policy benchmark style exposures).

Fund of Funds Optimization Vs. Optimize Managers to Track A Benchmark:

- Optimizing managers to track a benchmark attempts to minimize misfit risk – the difference between the aggregate style exposures and the policy benchmark style exposures.
- Fund of funds optimization attempts to maximize alpha for a given level of total active risk. Total active risk includes "Selection" active risk and misfit active risk.

Fund-of-Funds Optimization

Theoretical Background:

- Optimizing residual returns against residual risk
- Waring, B., D. Whitney, J. Pirone, and C. Castille. “Optimizing Manager Structure and Budgeting Manager Risk.” Journal of Portfolio Management, (Spring 2000): 90-104.
- Grinold, R. and R. Kahn. 1999. *Active Portfolio Management*. 2nd ed. New York: McGraw-Hill.
- Returns-based style analysis
- Sharpe, W. “Determining a Fund’s Effective Asset Mix.” Investment Management Review, (December 1988): 7-19.
- Sharpe, W. “Asset Allocation: Management Style and Performance Measurement.” The Journal of Portfolio Management, (Winter 1992): 7-19.