Structure and Asymptotic Theory for Multivariate Asymmetric Volatility: Empirical Evidence for Country Risk Ratings[*]

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Abstract: Following the rapid growth in the international debt of less developed countries in the 1970s and the increasing incidence of debt rescheduling in the early 1980s, country risk has become a topic of major concern for the international financial community. A critical assessment of country risk is essential because it reflects the ability and willingness of a country to service its financial obligations. Various risk rating agencies employ different methods to determine country risk ratings, combining a range of qualitative and quantitative information regarding alternative measures of economic, financial and political risk into associated composite risk ratings. This paper provides an international comparison of country risk ratings compiled by the International Country Risk Guide (ICRG), which is the only international rating agency to provide detailed and consistent monthly data over an extended period for a large number of countries. As risk ratings can be treated as indexes, their rate of change, or returns, merits attention in the same manner as financial returns. For this reason, a constant correlation multivariate asymmetric ARMA-GARCH model is presented and its underlying structure is established, including the unique, strictly stationary and ergodic solution of the model, its causal expansion, and convenient sufficient conditions for the existence of moments. Alternative empirically verifiable sufficient conditions for the consistency and asymptotic normality of the quasi-maximum likelihood estimator are established under non-normality of the conditional (or standardized) shocks. The empirical results provide a comparative assessment of the conditional means and volatilities associated with international country risk returns across countries and over time, enable a validation of the regularity conditions underlying the models, highlight the importance of economic, financial and political risk ratings as components of a composite risk rating, evaluate the multivariate effects of alternative risk returns and different countries, and evaluate the usefulness of the ICRG risk ratings in modelling risk returns.

Keywords: Economic risk, financial risk, political risk, composite risk, risk ratings, risk returns, multivariate structure, asymmetric effects, regularity conditions, asymptotic theory, empirical validation.

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1. Introduction

1.1 Country Risk

The 1970s witnessed a lending boom by Western banks to Eastern bloc, Latin American, and other less developed countries. This boom was in response to demand for funds by these countries beyond those provided by the World Bank and the International Monetary Fund (IMF) to aid their development. Moreover, Western banks needed to recycle their large petrodollar funds from oil producing countries, such as Saudi Arabia and Kuwait. These banks plotted their lending course in pursuit of profits and to maintain their competitive positions in world financial markets. Lending decisions were frequently made with little judgment regarding the credit quality of the borrowing country. As a result, the debt repayment problems of Poland and other Eastern bloc countries in the beginning of the 1980s, and the debt moratoria announced by the Mexican and Brazilian governments in the fall of 1982, caused major and long-lasting effects on the balance sheets and profits of the commercial banks in some countries (Saunders and Lange, 1996).

In light of these events, the concept of country risk, or the likelihood that a sovereign state or borrower from a particular country may be unable and/or unwilling to fulfil their obligations towards one or more foreign lenders and/or investors (Krayenbuehl, 1985), has become a topic of major concern for the international financial community. A lending decision to a party residing in a foreign country is a two-step decision. Apart from assessing the underlying credit quality of the borrower, as would be done for a domestic loan, the lender must assess the risk associated with the country in which the borrower resides. Should the credit risk or quality of the borrower be assessed as good but the country risk assessed as bad, the loan should not be made. Thus, in international lending decisions, considerations of country risk dominate those of private credit risk (Saunders and Lange, 1996).

The three major components of country risk are economic, financial and political risk. This literature holds that the three risk components affect each other. Economic and financial risks include factors such as sudden deterioration in the country's terms of trade, rapid increases in production costs and/or energy prices, unproductively invested foreign funds, and unwise lending by foreign banks (Nagy, 1988). Other important factors, such as changes in the macroeconomic and financial management of the country, are also important as they interfere with the free flow of capital or arbitrarily alter the expected risk-return features for investment (Juttner, 1995). In general, political risk is viewed as a non-business risk introduced strictly by domestic and international political forces. Political risk has been identified by banks and other multinational corporations as a factor that could seriously affect the profitability of their international ventures (Shanmugam, 1990). Examples of political risk relate to the possibility that the sovereign government may impose foreign exchange and capital controls, additional taxes, and asset freezes or expropriations due to political changes (Juttner, 1995).

1.2 Country Risk Ratings

Following the international debt crisis in the early 1980s, leading risk rating agencies such as Moody's, Euromoney, S&P, Institutional Investor, Economist Intelligence Unit, International Country Risk Guide, and Political Risk Services, have compiled country risk ratings as measures of credit risk associated with sovereign countries. These rating agencies provide qualitative and quantitative country risk ratings, combining information regarding alternative measures of economic, financial and political risk ratings to obtain a composite risk rating. This paper provides an international comparison of country risk ratings and returns compiled by the International Country Risk Guide (ICRG). Although most risk rating agencies provide an independent analysis of country risk and a systematic method of risk assessment, the ICRG is the only international rating agency to provide detailed and consistent monthly data over an extended period for a large number of countries.

Time series data relating to risk ratings contain both conditional mean and conditional variance (or volatility) components, both of which may vary over time. Volatility is used in risk analysis for examining portfolio selection, asset management, valuation of warrants and options, modelling the premium in forward and futures prices, evaluation of risk spillovers across markets, designing optimal hedging strategies for options and futures markets, and measuring the announcement effects in event studies, among others. Moreover, derivative assets are used to hedge against commodity price risk and to hedge against issued bonds. As such, optimal hedging strategies and an evaluation of the risks underlying risk ratings require knowledge of the volatility of the underlying stochastic process. As volatility is generally unknown, it must be estimated. Estimated and predicted volatilities are fundamental to risk management in financial portfolio models that describe the trade-off between risk and returns. Estimating and testing the volatility associated with risk ratings would seem to be a first step in establishing a market for pricing risk ratings as a primary or derivative asset.

Conditional volatility has been used to evaluate risk, asymmetric shocks, and leverage effects in economics and finance. Volatility that is present in country risk ratings will naturally reflect risk considerations inherent in such ratings. For this reason, the rate of change in risk ratings, that is, their underlying returns, merits the same attention as has been bestowed on financial returns. If these vary over time, they can be modelled using time series methods. Engle (1982) developed the autoregressive conditional heteroskedasticity (ARCH(p)) model to capture time-varying volatility, and this was subsequently generalised to the GARCH(p,q) model by Bollerslev (1986). These time-varying models have several attractive features, such as the ability to capture persistence of volatility, volatility clusters, thick-tailed distributions, and even an infinite unconditional variance. In many cases in practice, positive and negative shocks can have asymmetric effects, with negative shocks having a greater effect on volatility than positive shocks. Glosten, Jagannathan and Runkle (1993) extended the univariate GARCH(p,q) model to the univariate GJR(p,q) model by introducing asymmetry into the conditional volatility process. However, an extension of the multivariate GARCH(p,q) model does not yet seem to have been developed to accommodate the multivariate asymmetric effects of shocks.

Several important structural and asymptotic results underlying a range of estimation methods have been established for a wide variety of GARCH models. Li, Ling and McAleer (2002) survey recent theoretical results regarding the structure and asymptotic theory for GARCH models, all of which provide a solid theoretical and statistical foundation for applying the various models in practice. Theoretical results underlying the structure and estimation of GARCH models include convenient sufficient conditions for strict stationarity and ergodicity, for the existence of moments, and for the appropriate estimators to be consistent and asymptotically normal. Although theoretical results regarding the structure have been established for some asymmetric models, the asymptotic theory for the GJR(p,q) model has not yet been developed, especially for multivariate processes. In this paper, the consistency and asymptotic normality of a multivariate GJR(p,q) model will be established under empirically verifiable conditions.

In addition to the structural and asymptotic results associated with the multivariate GJR(p,q) model, the main purpose of the paper is to estimate and test the multivariate GARCH and GJR models across alternative risk returns and countries, specifically:

(1) for a given country, estimate the multivariate effects of four different risk returns and test for asymmetric effects;

(2) for a given risk return, estimate the multivariate effects of four different countries and test for asymmetric effects.

The plan of the paper is as follows. Section 2 presents the monthly ICRG data on economic, financial, political and composite risk ratings for the period 1984(1)-2002(5), and analyses their trends and volatilities. A constant correlation multivariate asymmetric ARMA-GJR model is presented in Section 3, and its underlying structure is established, including the unique, strictly stationary and ergodic solution of the model, its causal expansion, and the sufficient conditions for the existence of moments. Alternative empirically verifiable sufficient conditions, specifically log-moment and moment conditions, for the consistency and asymptotic normality of the quasi-maximum likelihood estimator (QMLE) are established under non-normality of the conditional (or standardized) shocks. The univariate and multivariate empirical results in Section 4 provide a comparative assessment of the conditional means and volatilities associated with country risk returns for different risk returns and countries over time, enable a validation of the regularity conditions underlying the model, highlight the importance of economic, financial and political risk ratings as components of a composite risk rating, and evaluate the usefulness of the ICRG risk ratings. Univariate GARCH and GJR models are estimated for four risk returns for each of four countries, multivariate GARCH and GJR models are estimated for four risk returns for each of four countries, and multivariate GARCH and GJR models are estimated for four countries for each of four risk returns. Some concluding remarks are given in Section 5.

2. Trends and Volatilities in Country Risk Ratings

2.1 Data Definitions

International Country Risk Guide (ICRG) has compiled economic, financial, political and composite risk ratings for 93 countries on a monthly basis since January 1984. As of November 2002, the four risk ratings were available for a total of 140 countries. Structural changes are, in general, not accommodated in the risk ratings. The ICRG ratings system was adjusted in late-1997 to reflect the changing international climate created by the ending of the Cold War. By 1997, the risk assessments were made by the ICRG on the basis of independently generated data, such as from the IMF, which could be referenced consistently over time.

The ICRG rating system comprises 22 variables representing three major components of country risk, namely economic, financial and political. These variables essentially represent risk-free measures. The economic risk rating measures a country's current economic strengths. In general, when a country's strengths outweigh its weaknesses, it presents a low economic risk, and vice-versa. This permits an assessment of the ability to finance its official, commercial, and trade debt obligations. The 5 economic variables are:

(i) GDP per Head of Population;
(ii) Real Annual GDP Growth;
(iii) Annual Inflation Rate;
(iv) Budget Balance as a Percentage of GDP;
(v) Current Account Balance as a Percentage of GDP.

Financial risk rating is another measure of a country's ability to service its financial obligations. This rating assesses a country's financial environment based on the following 5 financial variables:

(i) Foreign Debt as a Percentage of GDP;
(ii) Foreign Debt Service as a Percentage of Export in Goods and Services;
(iii) Current Account as a Percentage of Export in Goods and Services;
(iv) Net Liquidity as Months of Import Cover;
(v) Exchange Rate Stability.

Political risk rating measures the political stability of a country, which affects the country's ability and willingness to service its financial obligations. The 12 political risk variables are:

(i) Government Stability;
(ii) Socio-economic Conditions;
(iii) Investment Profile;
(iv) Internal Conflict;
(v) External Conflict;
(vi) Corruption;
(vii) Military in Politics;
(viii) Religious Tensions;
(ix) Law and Order;
(x) Ethnic Tensions;
(xi) Democratic Accountability;
(xii) Bureaucracy Quality.

Using each set of variables, a separate risk rating is created for the three components, on a scale of 0-100. The economic and financial components account for 25% each, and the political component accounts for 50%, of the composite risk rating. The lower (higher) is a given risk rating, the higher (lower) is the associated risk. In essence, the country risk rating is a measure of country creditworthiness.

2.2 Four Selected Countries

The risk rating indexes and volatilities are discussed for four countries for which risk ratings data have been collected since January 1984, namely Australia, Canada, Japan and the USA. Of these countries, interdependence might be expected between Canada and the USA, and between Australia and Japan, while other combinations would be dependent on the data. Using data for these four countries, univariate GARCH and GJR models are estimated for four risk returns, multivariate GARCH and GJR models are estimated for four risk returns by country, and multivariate GARCH and GJR models are estimated for four countries by risk returns. A comparison across countries and risk returns enables an evaluation as to the significance of the multivariate short and long run effects of risk returns and countries on the conditional volatility of risk returns for each country.

2.2.1 Risk Rating Indexes and Volatilities

For each country, the risk rating indexes in Figures 1-4 are denoted ECO-R, FIN-R, POL-R and COM-R for the economic, financial, political and composite risk rating indexes, respectively. Defining volatility as the squared deviation of each observation from the respective sample mean risk rating index, the four volatility counterparts are denoted ECO-V, FIN-V, POL-V and COM-V, respectively. Information on the economic and political profiles for the four countries has been obtained from three sources, namely the BBC News: Country Profiles and Timeline [http://news.bbc.co.uk/1/shared/bsp/hi/country_profiles/html/default.stm], U.S. Department of State: Countries and Regions [http://www.state.gov/countries/], and The World Factbook 2002, prepared by the Central Intelligence Agency [http://www.odci.gov/ cia/publications/factbook/index.html].

In Figure 1, the economic risk rating index for Australia followed a generally increasing trend, with discernible clustering of volatilities until 1998. After a period of fast growth, the economic index followed a downward trend from 1998 to mid-1999 as a result of falling investments and rising debt. However, in 1999 the index started to increase as the economy grew faster than both the US and European Union economies. The index fell again in late 2000, following an economic slowdown caused by the implementation of the GST. After 2001 the Australian economy strengthened and the economic risk index increased. There is a noticeable structural change in the financial risk index in 1997 when the index decreased by almost 20 points, prior to which there was some variation but no trend. Consequently, while there is substantial volatility in the financial risk rating index after 1997, there is little volatility before 1997. Upon the introduction of the GST in late 2000, the financial risk rating index fell, after which it followed an increasing trend but remained relatively low. The political risk rating index decreased until 1991, when Australia sent troops to assist US forces in the Gulf conflict, and then increased, with an associated clustering of volatilities. When John Howard became Prime Minister at the 1996 elections, this led to an increased index until late 1997. The political index fell, but started to increase after the re-election of Howard in October 1998. The index fell again in 1999 when Australia led an international coalition force to restore order in East Timor. Not surprisingly, the terrorist attacks of 11 September 2001 had a negative impact on the political risk index for Australia. As a weighted sum of the three indexes, the composite risk rating index for Australia had an increasing trend in the middle of the sample, after which the index decreased and then increased. There is comparable volatility in the composite risk index relative to the volatilities in the economic and political risk indexes.

The risk rating indexes and volatilities for Canada are given in Figure 2. Until 1990 the economic risk index for Canada was generally flat, after which it followed a decreasing trend due to an economic recession. By the end of 1992, with the terms of NAFTA being finalised, the index increased and followed an increasing trend until 2000, with a discernible clustering of volatilities. The economic downturn in 2001, in response to the recession in the USA and the terrorist attacks of 11 September 2001, caused the economic risk index to fall, after which the index followed an increasing trend. Though there was little variation in the financial risk rating index before 1997, the structural change in 1997 is similar to that of Australia in that the index decreased by almost 20 points, after which there was some variation but no trend. There was, in general, little volatility in the financial risk rating index for Canada, especially before 1997. Until 1993, the political risk rating index decreased and then increased, a pattern which was repeated with associated clustering of volatilities. The index followed an increasing trend after the 1993 elections, which saw Jean Chretien elected as Prime Minister when the Conservatives were defeated by the Liberals. Re-election of the Liberal Party in 1997 led to an increase in the political index. The events of 11 September 2001 had a negative impact on the political risk index, which is associated with a peak in volatility. Prior to 1993, the composite risk rating index for Canada increased and then decreased, after which the index had a slightly increasing trend. There was greater apparent volatility in the composite risk rating index for Canada than in the three component risk rating indexes.

Risk rating indexes and volatilities for Japan are given in Figure 3. Japan has long been the second largest economy in the world, with one of the highest economic growth rates during the period 1960-1980. The economy slowed dramatically in the early 1990s and entered a severe recession in 1997. Consequently, the recession caused a sharp fall in the economic index in 1997, prior to which it decreased and then increased. The index continued to decrease until the end of the sample period, with an associated increase in volatilities. Unlike Australia and Canada, there was no structural change in the financial rating index for Japan. There was a peak in the associated volatility in 1998, prior to which there was a perverse clustering in volatilities, but was otherwise unchanged. The political risk rating index had a slightly decreasing trend until 1992, when the background of bribery scandals and economic decline led to a loss of power for the Liberal Democratic Party for the first time since 1955. In 1993 the elections brought a seven-party coalition to power, which subsequently collapsed in 1994, after which an administration supported by the LDP and the Socialists took over. During this period, the political index increased and then decreased, after which it followed a generally increasing trend until 1997, when the economy entered the severe recession. In 1998, when Keizo Obuchi of the LDP became Prime Minster, the political index started an increasing trend, which ended in 2001. Unlike Australia and Canada, the composite risk rating index for Japan generally had a lower apparent volatility than in two of the three component risk rating indexes.

The economic risk rating index in Figure 4 for the USA, the world’s largest economy, does not resemble those of the other three countries. There has generally been a slight upward trend, with a single sharp decrease in 1996 and a clear peak in the associated volatility. The strong economic performance from 1994 to 2000 ended in 2001, with the economic index starting a decreasing trend. Only a slight negative impact on the index was discernible from the terrorist attacks of 11 September 2001. A moderate increase in the index occurred in late 2001, after which the economic index remained flat. The financial risk index resembled that of Japan prior to 1997, in that there was virtually no change, although a structural change occurred in 1997. Consequently, the volatility was entirely flat before 1997, but mild thereafter. For the political risk rating index, a downward trend until 1992 was followed by an upward trend until 2000, and the volatilities are observed to be tri-modal. The election of the Democratic Party candidate, Bill Clinton, as President in 1992 caused a change in the direction of the trend in the political index. Perhaps coincidentally, the upward trend ended with the US elections in late 2000. After a long series of legal challenges in January 2001, George W Bush was elected President, thereby causing the political index to rise. With the events of 11 September 2001 and their aftermath, the index fell and remained at this level until the end of the sample period. Overall, there was a downward trend in the composite risk rating index, with greater volatility at the end of the sample. Somewhat surprisingly, the tragedy of 11 September 2001 seems to have had only a small impact on the economic risk index, no apparent impact on the financial risk index, and substantial impacts on both the political and composite risk rating indexes for the USA.

2.2.2 Risk Returns and Volatilities

Risk returns are defined as the monthly percentage change in the respective risk rating indexes. The descriptive statistics for risk returns by country are given in Table 1, the correlation coefficients for risk returns by country are given in Table 2, and the correlation coefficients for countries by risk returns are given in Table 3. For each country the risk returns in Figures 5-8 are denoted ECO-R, FIN-R, POL-R and COM-R for the economic, financial, political and composite risk returns, respectively. Defining volatility as the squared deviation of each observation from the respective sample mean risk returns, the four volatility counterparts are denoted ECO-V, FIN-V, POL-V and COM-V, respectively.

The means of the four risk returns for the four countries in Table 1 are all very close to zero, with standard deviations in the range (0.008, 0.021) for Australia, (0.006, 0.014) for Canada, (0.008, 0.016) for Japan, and (0.010, 0.031) for the USA. There is no general pattern of skewness for the four risk returns for the four countries, with negatively skewed economic risk returns for Australia and Japan, negatively skewed financial risk returns for Australia, Canada and the USA, positively skewed political risk returns for Australia, Canada and Japan, and negatively skewed composite risk returns for Japan and the USA.

Table 2 reports the correlation coefficients for the risk returns by country. The economic, financial and political risk returns seem to be highly correlated with the composite risk returns, but not with each other. For each country, the highest correlation coefficient is between the political and composite risk returns. In the case of Australia and the USA, the second highest correlation is between the financial and composite risk returns, while for Canada and Japan the second highest correlation coefficient is between the economic and composite risk returns.

In Table 3, the correlation coefficients are presented for the four countries by risk returns. The highest correlation coefficients are observed for financial risk returns. For all risk returns, the highest correlation coefficients hold between Canada and the USA. In the case of economic and financial risk returns, the second highest correlation coefficient is between Australia and Canada, while Australia and Japan have the second highest correlation coefficient for political and composite risk returns.

Risk returns and volatilities for the four countries are given in Figures 5-8. In keeping with the discussion of Figures 1-4, structural breaks are apparent in 1997 for financial risk returns for Australia, Canada and the USA. Outliers and/or extreme observations are apparent throughout the risk returns and volatilities, especially for economic and financial risk returns and their associated volatilities for Canada and the USA. For Australia in Figure 5, there are clusters of volatilities, as well as outliers and/or extreme observations for all four components. The absence of any noticeable volatility for the financial risk returns is striking. Canada has more obvious volatilities in Figure 6 for the political and composite risk returns, though there are no noticeable outliers and/or extreme observations, a similar volatility pattern to that of Australia for economic risk returns, and less volatility than Australia for financial risk returns. Japan's economic and composite risk returns and volatilities in Figure 7 are similar to those of Australia, but the financial and political risk returns and volatilities are different from both Australia and Canada. There seem to be outliers and/or extreme observations in all four components, especially in the economic risk returns. The risk returns and volatilities for the USA in Figure 8 most closely resemble those of Japan, especially for the political and composite risk returns and their associated volatilities. The economic risk returns and volatility are dominated by a single outlier, while the financial risk returns and volatility are dominated by three outliers and/or extreme observations.

3. The Constant Correlation Multivariate GJR(*p*,*q*) (CC-MGJR) Model: Theoretical Results

Multivariate extensions of some GARCH models are available in the literature; see, for example, Engle, Granger and Kraft (1984), Bollerslev, Engle and Wooldridge (1988), Engle and Rodrigues (1989), Bollerslev (1990), Ling and Deng (1993), Engle and Kroner (1995), Wong and Li (1997), and Ling and McAleer (2002a), among others. However, the primary purpose in each of these papers has been to examine the structure of the model rather than to derive the asymptotic properties of the estimators. Exceptions to this general rule are Ling and Deng (1993), Jeantheau (1998) and Ling and McAleer (2002a).

In this section, a constant correlation multivariate asymmetric ARMA-GJR model, or CC-MGJR, is proposed which includes the constant correlation multivariate GARCH model of Bollerslev (1990) and the constant correlation multivariate ARMA-GARCH model of Ling and McAleer (2002a) as special cases. The sufficient conditions for strict stationarity and ergodicity, a causal representation of the CC-MGJR model, a simple sufficient condition for the existence of the moments, and sufficient conditions for the consistency and asymptotic normality of the quasi-maximum likelihood estimator (QMLE) of the CC-MGJR model, are obtained as extensions of Ling and McAleer

(2002a). Consistency is obtained under both the weak log-moment condition and the computationally more straightforward second moment condition, and asymptotic normality of the local (global) QMLE is obtained under the fourth (sixth) moment condition. Extensions of some of these results for more general models, such as the asymmetric power GARCH model, have been examined for univariate processes in Ling and McAleer (2002d).

Throughout this paper, the following notation is used: $|\cdot|$ denotes the absolute value of a univariate variable or the determinant of a matrix; $\|\cdot\|$ denotes the Euclidean norm of a matrix or a vector; A' denotes the transpose of the matrix or a vector *A*; $\rightarrow_p$ (or $\rightarrow_L$) denotes convergence in probability (or in distribution); and $\rho(A)$ denotes the eigenvalue of the matrix *A* with largest absolute value.

Bollerslev (1990) presented an *m*-dimensional multivariate conditional covariance model, namely,

$$\begin{aligned} Y_t &= E(Y_t \mid F_{t-1}) + \varepsilon_{0t}, \\ \varepsilon_{0t} &= D_{0t}\eta_{0t} \\ Var(\varepsilon_{0t} \mid F_{t-1}) &= D_{0t}\Gamma_0 D_{0t} \end{aligned} \qquad (1)$$

where F_t is the past information available up to time *t*, $D_{0t} = diag\,(h_{0it}^{1/2})$, $i = 1,\dots,m$ and

$$\Gamma_0 = \begin{pmatrix} 1 & \rho_{012} & \cdots & \rho_{01m} \\ \rho_{021} & 1 & \rho_{023} & \cdots \\ & & \cdots & \\ \rho_{0m1} & \cdots & \rho_{0m,m-1} & 1 \end{pmatrix}$$

in which $\rho_{0ij} = \rho_{0ji}$ for $i, j = 1,\dots,m$. The main feature of this model is that the conditional correlation $E(\varepsilon_{0it}\varepsilon_{0jt} \mid F_{t-1}) \Big/ \sqrt{E(\varepsilon_{0it}^2 \mid F_{t-1})E(\varepsilon_{0jt}^2 \mid F_{t-1})} = \rho_{0ij}$ is constant over time, where $i \neq j$, $i,\ j = 1,\dots,m$, and ε_{0it} is the *i*th element of ε_{0t}. Bollerslev (1990) assumed that

$$h_{0it} = \omega_{0i} + \sum_{l=1}^{r} \alpha_{0il}\varepsilon_{0it-l}^2 + \sum_{l=1}^{s} \beta_{0il} h_{0it-l}, \qquad i = 1,\dots, m \qquad (2)$$

in which there is no interdependence between h_{0it} and $(\varepsilon_{0jt-k}, h_{0jt-l})$ for $i \neq j;\ i, j = 1,..., m;\ k = 1,..., r;$ and $l = 1,..., s,$ and hence no relationship of volatilities across different markets, stocks, risk ratings, risk returns or countries. Thus, the multivariate effects are determined solely through the conditional correlation matrix, Γ_0. The multivariate conditional correlation model based on equations (1)-(2) will be referred to as CC-MGARCH (Bollerslev).

An extension of (2) to accommodate asymmetries with respect to ε_{0it}, and hence η_{0it}, is given by

$$h_{0it} = \omega_{0i} + \sum_{l=1}^{r} \alpha_{0il}\varepsilon_{0it-l}^{2} + \sum_{l=1}^{r} \gamma_{0il} I(\eta_{0it-l})\varepsilon_{0it-l}^{2} + \sum_{l=1}^{s} \beta_{0il} h_{0it-l}, \qquad i = 1,..., m \qquad (3)$$

in which $\varepsilon_{0it} = \eta_{0it}\sqrt{h_{0it}}$ for all i and t, and $I(\eta_{0it})$ is an indicator variable such that

$$I(\eta_{0it}) = \begin{cases} 1, & \varepsilon_{0it} \leq 0 \\ 0, & \varepsilon_{0it} > 0. \end{cases}$$

Let $\eta_{0t} = (\eta_{01t},..., \eta_{0mt})'$ be a sequence of i.i.d. random vectors, with zero mean and covariance Γ_0, so that $\varepsilon_{0t} = D_{0t}\eta_{0t}$, in which D_{0t} depends only on $H_{0t} = (h_{01t},..., h_{0mt})'$. The multivariate effects are still determined through the conditional correlation matrix, Γ_0.

As an extension of (3) to incorporate multivariate effects across equations (such as in the case of alternative risk returns or alternative countries), it is necessary to define h_{0it} so as to contain past information from ε_{0it}, ε_{0jt}, h_{0it} and h_{0jt} for $i \neq j;\ i, j = 1,..., m$. Thus, the multivariate ARMA(*p*,*q*)-GJR(*r*,*s*) model to be developed is defined as follows:

$$\Phi_0(L)(Y_t - \mu_0) = \Psi_0(L)\varepsilon_{0t} \qquad (4)$$

$$\begin{aligned} \varepsilon_{0t} &= D_{0t}\eta_{0t} \\ H_{0t} &= W_0 + \sum_{l=1}^{r} A_{0l}\vec{\varepsilon}_{0t-l} + \sum_{l=1}^{r} C_{0l} I(\eta_{0t-l})\vec{\varepsilon}_{0t-l} + \sum_{l=1}^{s} B_{0l} H_{0t-l} \end{aligned} \qquad (5)$$

where $D_{0t} = diag(h_{0it}^{1/2})$, A_{0l}, C_{0l} and B_{0l} are $m \times m$ matrices with typical elements α_{0ij}, γ_{0ij} and β_{0ij}, respectively, for $i, j = 1,..., m$, $I(\eta_{0t}) = diag(I(\eta_{0it}))$ is an $m \times m$ matrix, $\Phi_0(L) = I_m - \Phi_{01}L - ... - \Phi_{0p}L^p$ and $\Psi_0(L) = I_m - \Psi_{01}L - ... - \Psi_{0q}L^q$ are polynomials in L, I_k is the $k \times k$ identity matrix, and $\vec{\varepsilon}_{0t} = (\varepsilon_{01t}^2,...,\varepsilon_{0mt}^2)'$. The true parameter vector is denoted by $\lambda_0 = (\varphi'_0, \delta'_0, \rho'_0)'$, where

$$\varphi_0 = vec(\mu_0, \Phi_{01},..., \Phi_{0p}, \Psi_{01},..., \Psi_{0q})$$

$$\delta_0 = vec(W_0, A_{01},..., A_{0r,} C_{01},..., C_{0r}, B_{01},..., B_{0s})$$

$$\rho_0 = (\rho_{021},..., \rho_{0m1}, \rho_{032},..., \rho_{0m2},..., \rho_{0m,m-1})'.$$

The univariate constant-mean GJR model is obtained from (4)-(5) either by setting $m = 1$ and $\Phi_0(L) = \Psi_0(L) = 1$, or by specifying A_{0l}, C_{0l} and B_{0l} as diagonal matrices. Bollerslev's (1990) multivariate model (2) is obtained from (4)-(5) by setting $A_{0l} = diag(\alpha_{0il})$, $B_{0l} = diag(\beta_{0il})$ and $C_{0l} = 0$ for $l = 1,..., r$, while Ling and McAleer's (2002a) multivariate model is obtained from (4)-(5) by setting $C_{0l} = 0$ for $l = 1,..., r$.

The model for the unknown parameter vector $\lambda = (\varphi', \delta', \rho')'$, with φ, δ, and ρ defined in a similar manner to φ_0, δ_0, and ρ_0, respectively, is

$$\Phi(L)(Y_t - \mu) = \Psi(L)\varepsilon_t \qquad (6)$$

$$\varepsilon_t = D_t \eta_t$$

$$H_t = W + \sum_{l=1}^{r} A_l \vec{\varepsilon}_{t-l} + \sum_{l=1}^{r} C_l I(\eta_{t-l}) \vec{\varepsilon}_{t-l} + \sum_{l=1}^{s} B_l H_{t-l} \qquad (7)$$

where $H_t = (h_{1t},..., h_{mt})'$, $D_t = diag(h_{it}^{1/2})$, $\eta_t = (\eta_{1t},...,\eta_{mt})'$, $\vec{\varepsilon}_t = (\varepsilon_{1t}^2,...,\varepsilon_{mt}^2)'$, and $\Phi(L)$ and $\Psi(L)$ are defined in a similar manner to $\Phi_0(L)$ and $\Psi_0(L)$, respectively. First, the ε_t are computed from the observations $Y_1,..., Y_n$ from (6), with initial value $\bar{Y}_0 = (Y_0,..., Y_{1-p}, \varepsilon_0,..., \varepsilon_{1-q})$. Then H_t can be calculated from (7), with initial values $\bar{\varepsilon}_0 = (\vec{\varepsilon}_0,..., \vec{\varepsilon}_{1-r}, H_0,..., H_{1-s})$. As an extension of the assumptions in Ling and McAleer (2002a), it is assumed that the parameter space Θ is a compact subspace of Euclidean space, such that λ_0 is an interior point in Θ and, for each $\lambda \in \Theta$, it is assumed that:

Assumption 1. All the roots of $|\Phi(L)| = 0$ and of $|\Psi(L)| = 0$ are outside the unit circle.

Assumption 2. $\Phi(L)$ and $\Psi(L)$ are left coprime (i.e., if $\Phi(L) = U(L)\Phi_1(L)$ and $\Psi(L) = U(L)\Psi_1(L)$, the $U(L)$ is unimodular with constant determinant), and satisfy other identifiability conditions given in Dunsmuir and Hannan (1976).

Assumption 3. Γ is a finite and positive definite symmetric matrix, with the elements on the diagonal being 1 and $\rho(\Gamma)$ having a positive lower bound over Θ; all the elements of A_l, C_l and B_k are non-negative, $l = 1,...,r$, $k = 1,...,s$; each element of W has positive lower and upper bounds over Θ; and all the roots of $| I_m - \sum_{l=1}^{r} A_l L^l - \sum_{l=1}^{r} C_l I(\eta_{t-l}) L^l - \sum_{k=1}^{s} B_k L^k | = 0$ are outside the unit circle.

Assumption 4. $I_m - \sum_{l=1}^{r} A_l L^l - \sum_{l=1}^{r} C_l I(\vec{\eta}_{t-l}) L^l$ and $\sum_{k=1}^{s} B_k L^k$ are left coprime; and satisfy other identifiability conditions given in Jeantheau (1998) (see also Dunsmuir and Hannan (1976)).

The following propositions relate to the structure of model (4)-(5), namely a unique, strictly stationary and ergodic solution, with a useful causal expansion and convenient sufficient conditions for the existence of moments.

Proposition 1. *Under Assumptions 1 and 3, model (4)-(5) possesses an* F_t*-measurable unique second-order stationary solution* $\{Y_t, \varepsilon_{0t}, H_{0t}\}$*, given the* η_{0t}*, where* F_t *is a* σ*-field generated by* $\{\eta_{0k} : k \le t\}$*. The solutions* $\{Y_t\}$ *and* $\{H_{0t}\}$ *have the following causal representations:*

$$Y_t = \sum_{k=0}^{\infty} \Lambda_{0k} \varepsilon_{0t-k}, \quad a.s. \tag{8}$$

$$H_{0t} = W_0 + \sum_{\tau=1}^{\infty} c'(\prod_{l=1}^{\tau} \tilde{A}_{0t-l}) \xi_{t-\tau-1}, \quad a.s. \tag{9}$$

where $\Phi_0^{-1}(L)\Psi_0(L)=\sum_{k=0}^{\infty}\Lambda_{0k}L^k$, $\xi_t=[(\tilde{\eta}_{0t}W_0)',0,\ldots,0,W_0',0,\ldots,0]'_{(r+s)m\times 1}$, *that is, the subvector consisting of the first m components is* $\tilde{\eta}_{0t}W_0$ *and the subvector consisting of the* $(rm+1)th$ *to* $(r+1)mth$ *components is* W_0*;* $\tilde{\eta}_{0t}=diag(\eta_{01t}^2,\ldots,\eta_{0mt}^2)$, $c'=(0,\ldots,0,I_m,0,\ldots 0)_{m\times(r+s)m}$, *with the subvector consisting of the* $(rm+1)th$ *to* $(r+1)mth$ *columns being* I_m, *and*

$$\tilde{A}_{0t}=\left[\begin{array}{ccc|ccc}
\tilde{\eta}_{0t}A_{01}^* & \cdots & \tilde{\eta}_{0t}A_{0r}^* & \tilde{\eta}_{0t}B_{01} & \cdots & \tilde{\eta}_{0t}B_{0s} \\
\multicolumn{3}{c|}{I_{m(r-1)} \quad O_{m(r-1)\times m}} & \multicolumn{3}{c}{O_{m(r-1)\times ms}} \\
\hline
A_{01}^* & \cdots & A_{0r}^* & B_{01} & \cdots & B_{0s} \\
\multicolumn{3}{c|}{O_{m(s-1)\times mr}} & \multicolumn{3}{c}{I_{m(s-1)} \quad O_{m(s-1)\times m}}
\end{array}\right]$$

where

$$A_{0l}^* = A_{0l} + C_{0l}I(\vec{\eta}_{0t-l}), \qquad l=1,\ldots,r. \tag{10}$$

Hence, $\{Y_t,\varepsilon_{0t},H_{0t}\}$ *are strictly stationary and ergodic.*

Proof: The proof is similar to that of Theorem 2.1 in Ling and McAleer (2002a), except that A_{0l} in $\tilde{A}_{0t}$ is replaced by A_{0l}^*, $l=1,\ldots,r$, as defined in (10). ■

Proposition 2. *Under the Assumptions of Proposition 1, if* $\rho[E(\tilde{A}_{0t}^{\otimes k})]<1$, *with k being a strictly positive integer, then the 2kth moments of* $\{Y_t,\varepsilon_{0t}\}$ *are finite, where* $\tilde{A}_{0t}$ *is defined in Proposition 1 and* $\tilde{A}_{0t}^{\otimes k}$ *is the Kronecker product of the k matrices* $\tilde{A}_{0t}$.

Proof: The proposition is similar to that of Theorem 2.2 in Ling and McAleer (2002a), except that A_{0l} in $\tilde{A}_{0t}$ is replaced by A_{0l}^*, $l=1,\ldots,r$, as defined in (10). ■

The estimators of the parameters in model (4)-(5) are obtained by maximizing, conditional on the true $(\bar{Y}_0,\bar{\varepsilon}_0)$,

$$\begin{aligned}
L_n(\lambda) &= \frac{1}{n}\sum_{t=1}^{n} l_t(\lambda) \\
l_t(\lambda) &= -\frac{1}{2}\log|D_t\Gamma D_t| - \frac{1}{2}\varepsilon_t'(D_t\Gamma D_t)^{-1}\varepsilon_t
\end{aligned} \tag{11}$$

where $L_n(\lambda)$ takes the form of the Gaussian log-likelihood, Γ is defined in (1), and $D_t = diag(h_{it}^{1/2})$. Since it is not assumed that η_{0t} is normal, the estimators from (11) are the QMLE. Note that the processes ε_τ and D_τ, $\tau \leq 0$, are unobserved, and hence they are only some chosen constant vectors. Thus, $L_n(\lambda)$ is the likelihood function which is not conditional on $(\bar{Y}_0, \bar{\varepsilon}_0)$, but is conditional on any initial values. Maximization of (11) leads to the following consistency result.

Proposition 3. *Denote* $\hat{\lambda}_n$ *as the solution to* $\max_{\lambda \in \Theta} L_n(\lambda)$. *Under Assumptions 1-4 and Lemma 4.2 in Ling and McAleer (2002a),* $\hat{\lambda}_n \to_p \lambda_0$.

Proof: The proof is similar to that of Theorem 4.1 in Ling and McAleer (2002a), except that A_{0l} is replaced by A_{0l}^*, $l = 1, \ldots r$, as defined in (10). ■

Remark 1. Only a second-order moment condition is required for the proof of Proposition 3. Jeantheau (1998) examined a special case of the CC-MGJR model, namely (4)-(5) with $p = q = 0$, so that the conditional mean was specified as a constant drift, and $C_{0l} = 0$ for $l = 1, \ldots, r$, that is, with no asymmetric effects. In order to prove strict stationarity and ergodicity, Jeantheau (1998) assumed the existence of second-order moments. However, the proof of strict stationarity and ergodicity in Proposition 1 does not assume the existence of second moments.

In Jeantheau's (1998) proof of consistency, the following finite log-moment condition was assumed.

Assumption 5. For all $\lambda_0 \in \Theta$,

$$E_{\lambda_0}[|\log(|H_{0t}|)|] < \infty \qquad (12)$$

where H_{0t} is given in (9).

This leads to the following consistency result.

Proposition 4. *Denote* $\hat{\lambda}_n$ *as the solution to* $\max_{\lambda \in \Theta} L_n(\lambda)$. *Under Assumptions 1-5,* $\hat{\lambda}_n \to_p \lambda_0$.

Proof: The proposition is similar to Theorems 2.1 and 3.2 in Jeantheau (1998), except that H_{0t} in (7), for which the causal expansion is given in (9), is more general than the constant drift mean with symmetric effects in Jeantheau (1998). ■

Remark 2. The multivariate log-moment condition in (12) is weaker than the second-order moment condition in Lemma 4.2 in Ling and McAleer (2002a), which was used in Proposition 3. Jeantheau (1998) showed that the multivariate log-moment condition could be verified under the additional assumption that the determinant of the unconditional variance of ε_{0t} in (1) was finite.

Remark 3. The system was assumed to extend infinitely far into the past in Jeantheau (1998), whereas Proposition 1 makes it clear that this is a consequence of the existence of the unique stationary solution, and is not an assumption.

In order to prove the asymptotic normality of the QMLE, the second derivative of (11) is required. For model (4)-(5), asymptotic normality of the local (global) QMLE requires the fourth-order (sixth-order) moment condition (refer to Lemma 5.4 in Ling and McAleer (2002a) for a distinction between local and global QMLE in the context of multivariate GARCH models).

The QMLE is efficient only if η_t is normal. When the standardised shock η_t is not normal, adaptive estimation can be used to obtain efficient estimators. Ling and McAleer (2002c) investigate the properties of adaptive estimators for univariate non-stationary ARMA models with GARCH(*p*,*q*) errors.

The existence of higher-order moments leads to the following asymptotic normality result.

Proposition 5. *Let* Y_t *be generated by (4)-(5) satisfying Assumptions (1)-(4) and* $E\|Y_t\|^6 < \infty$. *Define* $\Omega_0 = E[(\partial l_{0t}^{\varepsilon} / \partial\lambda)(\partial l_{0t}^{\varepsilon} / \partial\lambda')]$, *which is finite, where* l_{0t}^{ε} *is the unobserved log-likelihood conditional on the infinite past observations. If* $\Omega_0 > 0$ *and* $\Gamma_0^{-1} * \Gamma_0 \geq I_m$, *where* $*$ *denotes the Hadamard element-by-element product, then* $\sqrt{n}(\hat{\lambda}_n - \lambda_0) \rightarrow_L N(0, \Sigma_0^{-1}\,\Omega_0\,\Sigma_0^{-1})$. *Furthermore,* Σ_0 *and* Ω_0 *can be estimated consistently by* $\hat{\Sigma}_n$ *and* $\hat{\Omega}_n$, *respectively.*

Proof: The proposition is similar to Theorem 5.1 in Ling and McAleer (2002a), except that A_{0l} is replaced by A_{0l}^{*}, $l = 1,\ldots,r$, as defined in (10). ■

The QMLE in Proposition 5 is the global maximum over the whole parameter space. If the local QMLE is considered, the fourth moment condition is sufficient. Therefore, the structure and asymptotic theory of the multivariate constant correlation GJR(*p*,*q*) model is complete.

Remark 4. Boussama (2000) showed that the univariate version of the log-moment condition in (12) is sufficient for asymptotic normality of the QMLE for the GARCH(*p*,*q*) model. However, an extension of the result to the multivariate GARCH(*p*,*q*) model, and hence the GJR(*p*,*q*) model, is not yet available.

For the univariate GJR(1,1) process when $m = r = s = 1$, the log-moment condition (12) is given as

$$E(\log[(\alpha + \gamma I(\eta_t))\eta_t^2 + \beta]) < 0 . \qquad (13)$$

A special case of (13) is the well-known log-moment condition for GARCH(1,1) when $\gamma = 0$, namely

$$E(\log(\alpha\eta_t^2 + \beta)) < 0 \qquad (14)$$

(see Nelson (1990) and Lee and Hansen (1994)). The second-order moment condition for the GJR(1,1) model is given as

$$\alpha + \beta + \frac{1}{2}\gamma < 1 \qquad (15)$$

(see Ling and McAleer (2002b)). A special case of (15) when $\gamma = 0$ is the well-known second moment condition for GARCH(1,1), which is given as

$$\alpha + \beta < 1 . \qquad (16)$$

The conditions in (13)-(16) for the univariate case, $m = 1$, are straightforward to check, and hence provide useful diagnostic information regarding the regularity conditions. Bougerol and Picard (1992) examine a similar condition to (14) for the GARCH(*p*,*q*) model, and show that the appropriate condition is the negativity of an associated Lyapunov exponent.

It is clear from (13) and (14) that the log-moment conditions involve the expectation of a function of a random variable and unknown parameters. Although the log-moment conditions in (13) and (14) are sufficient for the QMLE of the GJR(1,1) and GARCH(1,1) models to be consistent and asymptotically normal, the stronger second moment conditions given in (15) and (16), respectively, are more straightforward to check in practice as they do not involve the mean of a function of a random variable. Moreover, the second moment condition can easily be used to verify consistency and asymptotic normality in the event that the log-moment condition cannot be computed because $(\alpha + \gamma I(\eta_t))\eta_t^2 + \beta < 0$ in (13) or $\alpha\eta_t^2 + \beta < 0$ in (14) for any $t = 1,...,n$ (this will be discussed in greater detail in Section 4).

For the GARCH(1,1) model, the ARCH (or α) effect indicates the short run persistence of shocks, while the GARCH (or β) effect indicates the contribution of shocks to long run persistence (namely, $\alpha + \beta$), as in (14). In the case of the GJR(1,1) model, the asymmetric effect, γ , measures the contribution of shocks to both short run persistence, $\alpha + \frac{\gamma}{2}$, and to long run persistence, $\alpha + \beta + \frac{\gamma}{2}$, as in (15). Sufficient conditions for $h_{it} > 0$ in GARCH(1,1) are $\omega_i > 0, \alpha_i \geq 0$ and $\beta_i \geq 0$ for $i = 1,...,m$, while GJR(1,1) requires $\omega_i > 0, \alpha_i \geq 0$, $\alpha_i + \gamma_i \geq 0$ and $\beta_i \geq 0$ for $h_{it} > 0$ for $i = 1,...,m$. However, in the finance literature, negative shocks increase risk so that γ_i is generally expected to be positive.

Although the multivariate correlations are specified as being constant over time, the CC-MGJR model discussed above has the advantage of multivariate asymmetry. Table 4 presents alternative multivariate models in the literature and examines their structure for $r = s = 1$ on the basis of: (i) the sufficient conditions for the univariate conditional variances to be positive; (ii) the sufficient conditions for the corresponding matrix of multivariate conditional variances to be positive definite; (iii) the modelling of the multivariate correlations; and (iv) the number of parameters in the model. The dynamic conditional correlation (DCC) model of Engle (2002) is equivalent to the varying correlation (VC)-MGARCH model of Tse and Tsui (2002). Although the CC-MGARCH, VC-MGARCH and CC-MGJR models can be specified with or without interdependence between h_{it} and $(\varepsilon_{jt-k}^2, h_{jt-l})$ for $i, j = 1,...,m$; $k = 1,...,r$; and $l = 1,...,s$; for purposes of Table 4, h_{it} depends only on $(\varepsilon_{it-1}^2, h_{it-1})$. With the exception of the CC-MGJR model, all the multivariate models in Table 4 display symmetry. The vech (or VAR) model of Engle and Kroner (1995) is a highly parameterised model, which does not guarantee that the conditional variances are positive and does not model the multivariate correlations. For $m = (2, 3, 4)$, the number of parameters is (21, 78, 210). The diagonal model of Bollerslev et al. (1988) has a similar structure to the vech model but with fewer parameters, namely (9, 18, 30) for $m = (2, 3, 4)$. Engle and Kroner's (1995) BEKK model guarantees that the univariate conditional variances are positive and that its multivariate counterpart is positive definite, with (11, 24, 42) parameters for $m = (2, 3, 4)$. None of the CC-MGARCH, VC-MGARCH and CC-MGJR models associated with Bollerslev (1990), Ling and McAleer (2002a), Engle (2002), Tse and Tsui (2002), and this paper guarantees that the univariate conditional variances are positive, but the structure of each of these models guarantees that their multivariate counterparts are positive definite. The correlations are also modelled in each case, with constant correlations for CC-MGARCH and CC-MGJR, and varying correlations for VC-MGARCH. For $m = (2, 3, 4)$, the numbers of parameters are (7, 12, 18), (9, 14, 20) and (9, 15, 22) for CC-MGARCH, VC-MGARCH and CC-MGJR, respectively, which demonstrates that these multivariate models are considerably more parsimonious than the first three multivariate models.

4. **Empirical Results**

All the estimates in this paper are obtained using EViews 4, unless otherwise stated. The Berndt, Hall, Hall and Hausman (BHHH) (1974) algorithm has been used in most cases, but the Marquardt algorithm is used when the BHHH algorithm does not converge. Several different sets of initial values have been used in each case, but do not lead to a substantial difference in the estimates.

4.1 Univariate Models

The univariate AR(1)-GARCH(1,1) and AR(1)-GJR(1,1) models are used to estimate risk returns and volatilities for Australia, Canada, Japan and the USA using monthly data for the period 1984(1)-2002(5). Tables 5 and 6 report the GARCH(1,1) and GJR(1,1) estimates, respectively. The log-moment and second moment conditions in Table 5 are the empirical versions of conditions (14) and (16), respectively, while the log-moment and second moment conditions in Table 6 are the empirical versions of conditions (13) and (15), respectively. In order to calculate the empirical counterparts of the log-moment conditions, the QMLE of the parameters are substituted into (13) and (14), together with the corresponding estimated standardised residuals from the respective models. The second moment conditions in (15) and (16) are evaluated at their respective QMLE. These empirical moment conditions provide practical diagnostic checks of the regularity conditions.

Asymptotic and robust t-ratios (see Bollerslev and Wooldridge (1992) for the derivation of the robust standard errors) are reported for the QMLE in Tables 5-6. Although there is no algebraic relationship between the asymptotic and robust t-ratios, it would be expected that the robust t-ratios are generally smaller in absolute value, especially in the presence of extreme observations and outliers. Of the 48 pairs of t-ratios reported in Table 5, in 44 cases the robust t-ratios are smaller in absolute value than their asymptotic counterparts. A similar outcome is observed in 53 of the 64 pairs of t-ratios reported in Table 6.

Estimates of the univariate GARCH(1,1) volatilities associated with the economic, financial, political and composite risk returns for the four countries are given in Table 5. The estimates for economic risk returns are plausible for all four countries, with all the α and β estimates being positive fractions, and the log-moment condition being satisfied in all cases. Although the second moment condition is not satisfied for Canada, the log-moment condition ensures that the QMLE are consistent and asymptotically normal in the presence of infinite second moments.

For the financial risk returns, the α and β estimates are positive fractions in all cases. Although the second moment condition is not satisfied for the USA, the log-moment condition is satisfied for all four countries. Hence, the QMLE are consistent and asymptotically normal.

For political risk returns, the log-moment condition could not be calculated for Australia or Japan, but the second moment conditions are satisfied in both cases. Although the α estimates are positive fractions for Australia and Japan, the β estimates are negative in both cases. The α and β estimates, as well as the log-moment and second moment conditions, are sensible for the USA. Although the α estimate is negative for Canada, the log-moment and second moment conditions are satisfied.

Apart from the negative α estimate for composite risk returns for Canada, the α and β estimates are otherwise satisfactory. The log-moment condition is satisfied for all countries, and the second moment condition is satisfied for Australia, Canada and Japan. For the USA, even though the second moment is infinite, the log-moment condition is satisfied, so that the QMLE are consistent and asymptotically normal.

The GJR(1,1) estimates for the four risk returns for the four countries in Table 6 are rarely superior to their GARCH(1,1) counterparts in Table 5. For economic risk returns, the α estimates are positive fractions except for Japan, the β estimates are all positive fractions, and the γ estimates are positive in all cases. Moreover, the sums of the α and γ estimates are positive for Australia, Canada and the USA, but the sum is negative for Japan. The second moment condition is satisfied only for Japan, but the log-moment condition is satisfied for all countries. Overall, the GARCH(1,1) and GJR(1,1) estimates are similar, especially as the γ estimates for economic risk returns are insignificant for all four countries.

Just as the GARCH(1,1) estimates are satisfactory for the financial risk returns, a similar comment generally applies to the GJR(1,1) estimates in Table 6. The α estimates are negative in three of four cases, but the β estimates are positive fractions in all cases. In two of four cases, the γ estimates are significant. Moreover, the sums of the α and γ estimates are positive for Australia and Japan. The log-moment and second moment conditions are satisfied in all cases. Overall, the GARCH(1,1) model is preferred for Canada and the USA, while the GJR(1,1) model is preferred for Australia and Japan for financial risk returns.

The GJR (1,1) estimates for political risk returns are somewhat mixed. The log-moment and second moment conditions are satisfied in all cases, just as the γ and β estimates are positive fractions in all cases. While all the α estimates are negative fractions, all the sums of the α and γ estimates are positive. Overall, as none of the γ estimates is significant, the GARCH(1,1) estimates are preferable to their GJR(1,1) counterparts for political risk returns.

Similar comments apply to the relative performance of GJR(1,1) and GARCH(1,1) for composite risk returns. Apart from the log-moment and second moment conditions for the USA, all the log-moment and second moment conditions are satisfied. All the β estimates are positive fractions and the α estimates for Australia and the USA are positive fractions. However, while three of the γ estimates are positive, none is significant. Moreover, the sums of the α and γ estimates are positive for Australia, Japan and the USA. Overall, the GARCH(1,1) estimates are preferable to their GJR(1,1) counterparts for composite risk returns.

Overall, at the univariate level, the GARCH(1,1) model is generally preferable to its GJR(1,1) counterpart.

4.2 Multivariate Models

Using the same data as for the univariate models in the previous sub-section, the multivariate AR(1)-GARCH(1) and AR(1)-GJR(1) models are used to provide estimates of the risk returns and volatilities for the four risk returns and four countries. Table 7 reports the multivariate GARCH(1,1) estimates for four risk returns by country, Table 8 reports the multivariate GJR(1,1) estimates for four risk returns by country, Table 9 reports the multivariate GARCH(1,1) estimates for four countries by risk returns, and Table 10 reports the multivariate GJR(1,1) estimates for four countries by risk returns. Asymptotic and robust t-ratios are reported in Tables 7-10. In general, the robust t-ratios are smaller in absolute value than their asymptotic counterparts.

The estimates reported in Tables 7-10 are for special cases of the following two multivariate GJR(1,1) models:

$$h_{it} = \omega_i + \sum_i \alpha_i \varepsilon_{it-1}^2 + \gamma_i I(\eta_{it-1})\varepsilon_{it-1}^2 + \sum_i \beta_i h_{it-1} \qquad \text{(given } j\text{)} \qquad (17)$$

$$h_{jt} = \omega_j + \sum_j \alpha_j \varepsilon_{jt-1}^2 + \gamma_j I(\eta_{jt-1})\varepsilon_{jt-1}^2 + \sum_j \beta_j h_{jt-1} \qquad \text{(given } i\text{)} \qquad (18)$$

where $i = E, F, P$ and C for economic, financial, political and composite risk returns, respectively, and $j = A, C, J$ and U for Australia, Canada, Japan and USA, respectively. Tables 7-8 give estimates of equation (17), while Tables 9-10 give estimates of equation (18).

Table 8 reports the estimates for the multivariate GJR(1,1) model for four risk returns by country, as given in equation (17). The estimates of the multivariate GARCH(1,1) model in Table 7 are obtained by imposing the following restrictions on equation (17):

$$H_0 : \gamma_i = 0, \qquad i = E, F, P, C. \qquad (19)$$

Proposition 5 in Section 3 can be used to test the null hypothesis in (19). Table 10 reports the estimates for the multivariate GJR(1,1) model for four countries by risk returns, as given in equation (18). The estimates of the multivariate GARCH(1,1) model in Table 9 are obtained by imposing the following restrictions on equation (18):

$$H_0 : \gamma_j = 0, \qquad j = A, C, J, U. \tag{20}$$

Proposition 5 in Section 3 can be used to test the null hypothesis in (20).

The estimates of the multivariate GJR(1,1) model in equation (17) are given in Table 8. For economic and financial risk returns (that is, for $i = E$ and $i = F$ in (19)), each of the γ_i estimates is insignificant, so that multivariate GARCH(1,1) is preferred to multivariate GJR(1,1). For political risk returns (that is for $i = P$ in (19)), the γ_i estimates are significant for all countries, so that multivariate GJR(1,1) is preferred to multivariate GARCH(1,1). The γ_i estimates are also insignificant for Australia, Canada and Japan in the case of composite risk returns (that is $i = C$ in (19)). Based on the results in Table 8, the political risk returns for Australia are affected by previous long run shocks in economic, financial and political risk returns; for Canada and Japan, by previous short and/or long run shocks in all four risk returns; and for the USA by previous short and/or long run shocks in financial, political and composite risk returns. The composite risk returns for the USA in Table 8 are affected by previous short and/or long run shocks in financial, political and composite risk returns.

Except for the five cases discussed in the previous paragraph, in which the restrictions in (19) were rejected, the multivariate GARCH(1,1) model in Table 7 is preferred to the multivariate GJR(1,1) model in Table 8. Multivariate short and/or long run effects are observed in virtually every case, with the sole exception of 16 cases being financial risk returns for Australia in Table 7. Overall, a comparison of the four risk returns across four countries at the multivariate level for both the GARCH(1,1) and GJR(1,1) models shows that economic and financial risk returns display symmetry for all four countries, and for three of four countries for composite risk returns, whereas there is asymmetry for political risk returns for all four countries.

The estimates of the multivariate GARCH(1,1) and GJR(1,1) model for four countries by risk returns are given in Tables 9 and 10. For economic risk returns in Table 10, the γ_j estimates are insignificant for Australia and the USA (that is, for $j = A$ and $j = U$ in (20)), so that multivariate GARCH(1,1) is preferred to the multivariate GJR(1,1) model. As the γ_j estimates are significant for Canada and Japan (that is, for $j = C$ and $j = J$ in (20)), the multivariate GJR(1,1) model is preferred. Regardless of the choice of model, the effects of multivariate short and/or long run shocks are significant for all four countries for economic risk returns.

The γ_j estimates in Table 10 for financial risk returns are significant for all four countries, so that multivariate GJR(1,1) is preferred to the multivariate GARCH(1,1) model. Multivariate short and/or long run shocks are significant for three of four countries, the exception being financial risk returns for the USA.

Unlike both economic and financial risk returns, in which there were significant γ_j estimates for two and four countries, respectively, there are significant γ_j estimates for political risk returns in three cases in Table 10, the exception being Canada (that is, $j = C$ in (20)). Whether or not the preferred model is GARCH(1,1) or GJR(1,1), multivariate short and/or long run shocks are significant for three of four countries, the exception being political risk returns for the USA.

Composite risk returns are different from each of its three components in that the multivariate GJR(1,1) model is preferred to its GARCH(1,1) counterpart in only one of four cases in Table 10, namely with a significant γ_j estimate only for Japan (that is, $j = J$ in (20)). Regardless of the choice of model, the effects of multivariate short and/or long run shocks are significant for all four countries for composite risk returns.

Significant multivariate short and/or long run shocks are observed in 14 of 16 cases, the two exceptions being financial and political risk returns for the USA in the multivariate GJR(1,1) model in Table 10. Moreover, as summarised in Tables 11 and 12, interdependence is detected for each of the three pairs (Australia, Canada), (Australia, USA) and (Canada, Japan) for economic risk returns, for the single pair (Australia, Canada) for financial risk returns, for the two pairs (Australia, Japan) and (Canada, Japan) for political risk returns, and for the five pairs (Australia, Canada), (Australia, Japan), (Australia, USA), (Canada, Japan) and (Canada, USA) for composite risk returns. Statistical independence is observed for the two pairs (Australia, USA) and (Canada, USA) for financial risk returns, and the single pair (Australia, USA) for political risk returns.

It is interesting to note that in three of four cases, the USA has a significant effect on Japan but not the reverse, the exception being economic risk returns, in which Japan affects the USA. Canada and the USA have a particularly interesting relationship in four separate effects in four cases, with Canada affecting the USA for economic risk returns, the two countries being independent of each other for financial risk returns, the USA affecting Canada for political risk returns, and the two countries being interdependent for composite risk returns.

Estimates of the CC-MGARCH conditional correlation coefficients for risk returns by country are given in Table 13. These estimates are based on equations (6)-(7) with $C_l = 0$ for $l = 1,...,r$, as in Ling and McAleer (2002a). It is clear that the conditional correlations are generally not zero, with the conditional correlation coefficients of the composite risk returns with each of the economic, financial and political risk returns being the highest for each country. The estimates in Table 13 are quantitatively similar to those obtained using the CC-MGARCH (Bollerslev) model based on equations (1)-(2), and on the CC-MGJR model based on equations (6)-(7). In virtually all cases, the conditional correlations are positive.

The CC-MGARCH, CC-MGARCH (Bollerslev) and CC-MGJR conditional correlation coefficients for countries by risk returns are given in Tables 14-16, respectively. As in Table 13, the conditional correlations are positive in virtually all cases. Although a few of the conditional correlations seem close to zero for some country pairs for the four risk returns, which is in contrast to the results in Table 13, several pairs differ from zero. This is particularly the case for financial risk returns for all four countries. For financial risk returns, the smallest conditional correlations are reported in Table 14 for the CC-MGARCH model, while the largest conditional correlations are reported in Table 15 for the CC-MGARCH (Bollerslev) model. The conditional correlation coefficients for financial risk returns for the CC-MGJR model, which are given in Table 16, lie in between those reported in Tables 14 and 15 for all six country pairs.

5. Concluding Remarks

This paper has provided an international comparison of country risk ratings compiled by the International Country Risk Guide (ICRG), which is the only international rating agency to provide detailed and consistent monthly data over an extended period for a large number of countries. As risk ratings can be treated as indexes, their rate of change, or returns, was analysed in the same manner as financial returns. Although there does not yet seem to be a viable market for pricing risk ratings as a primary or derivative asset, modelling the volatility associated with risk ratings is seen as a first step in this direction.

A constant correlation multivariate asymmetric ARMA-GARCH model was presented and its underlying structure was established, including the unique, strictly stationary and ergodic solution of the model, its causal expansion, and convenient sufficient conditions for the existence of moments. Alternative sufficient conditions for the consistency and asymptotic normality of the quasi-maximum likelihood estimator (QMLE) were established under non-normality of the conditional (or standardized) shocks.

The empirical results provided a comparative assessment of the conditional means and volatilities associated with international country risk ratings across alternative risk ratings and countries over time, enabled a validation of the regularity conditions underlying the model, highlighted the importance of economic, financial and political risk ratings as components of a composite risk rating, and evaluated the usefulness of the ICRG risk ratings. In particular, at the univariate level for both the symmetric GARCH and asymmetric GJR models, the sufficient parametric conditions for the estimated volatilities to be positive were generally satisfied, as were the log-moment and second moment conditions for the QMLE to be consistent and asymptotically normal.

A comparison of the four risk returns by country at the multivariate level for both the symmetric GARCH(1,1) and asymmetric GJR(1,1) models showed that economic and financial risk returns displayed symmetry for all four countries, political risk returns displayed asymmetry for all four countries, whereas there was asymmetry for composite risk returns for only one country. Multivariate effects were observed across all risk returns for all countries, with the exception of financial risk returns for Australia. Finally, estimation of the multivariate GARCH(1,1) and GJR(1,1) models for each of the four countries by risk returns indicated the presence of multivariate effects in virtually all cases. Moreover, significant asymmetric effects were observed in a majority of risk returns as well as countries.

Finally, the estimated conditional correlation coefficients for risk returns by country, and for countries by risk returns, were generally found to be different from zero, which argues against univariate modelling of the risk associated with risk returns. The estimates obtained from the CC-MGARCH (Bollerslev), CC-MGARCH and CC-MGJR models were generally found to be quantitatively similar, except for financial risk returns for each country.

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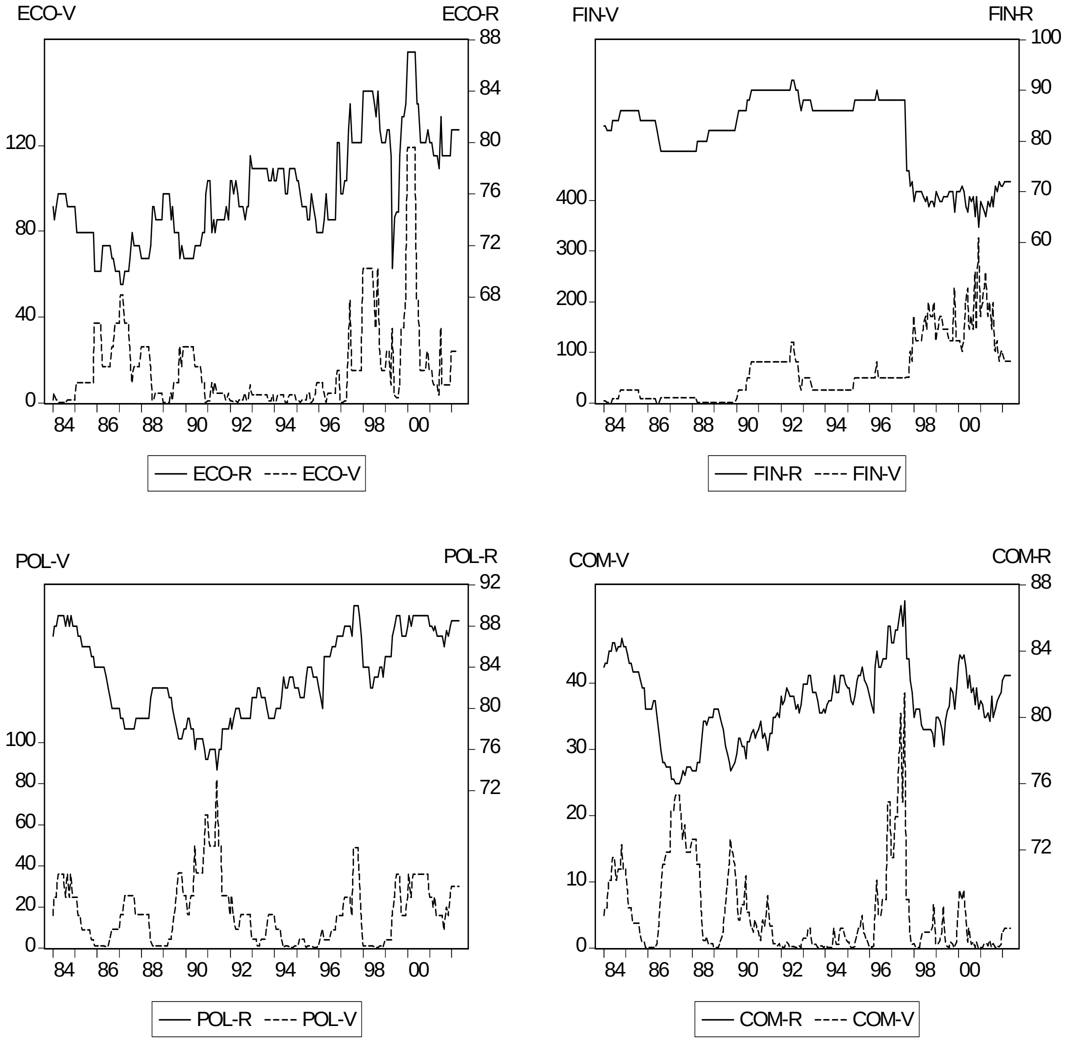
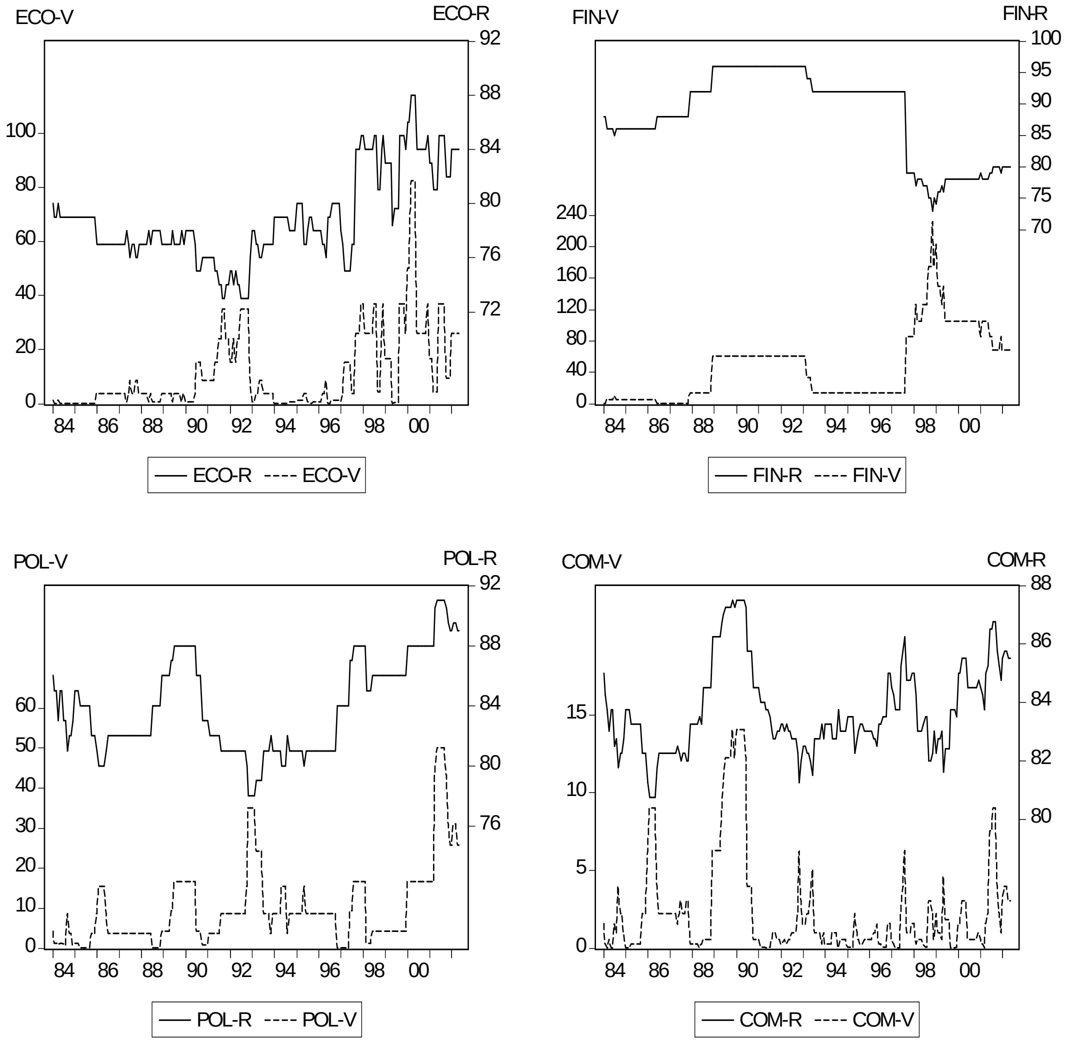
Figure 1: Risk Rating Indexes and Volatilities for Australia



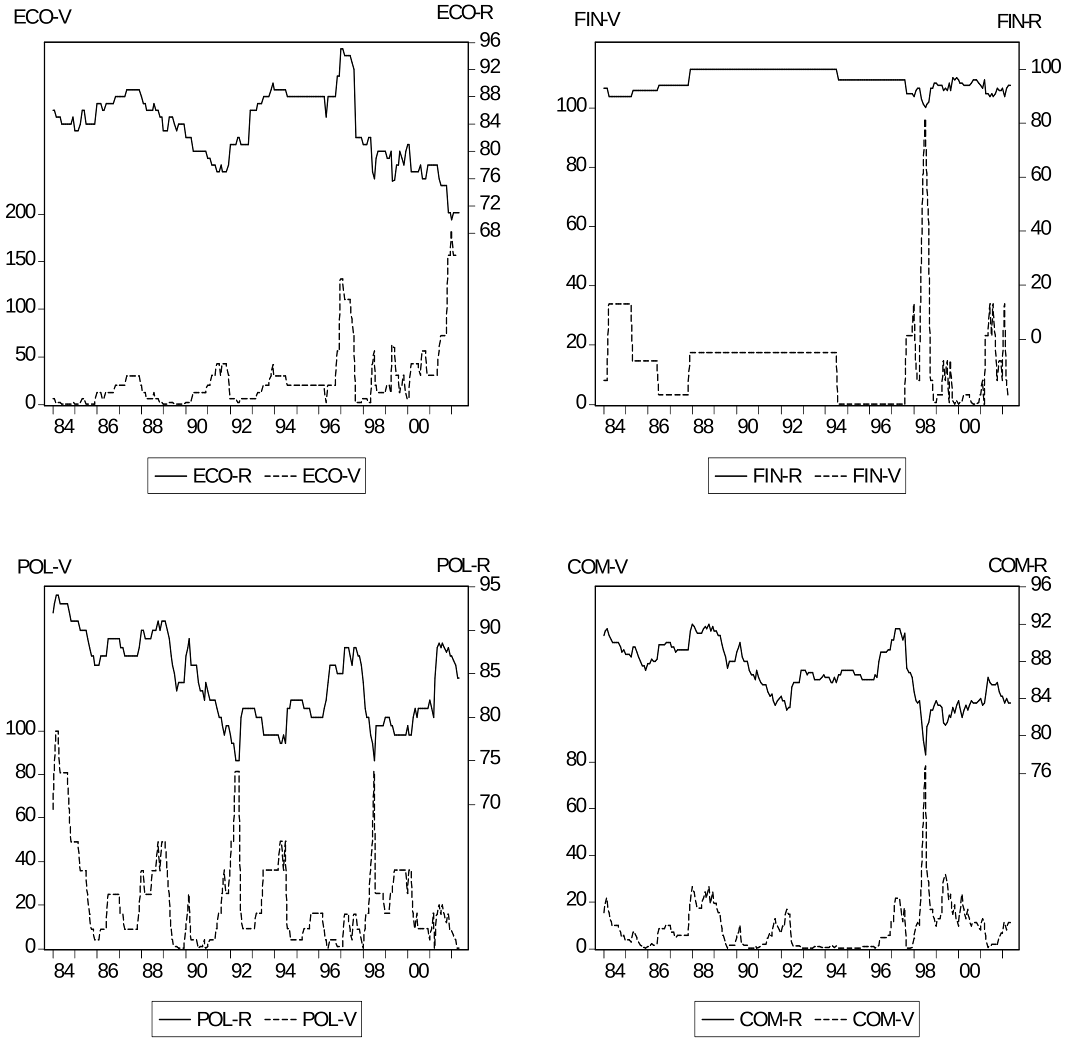
Note: Economic (ECO), Financial (FIN), Political (POL) and Composite (COM) risk rating indexes and their associated volatilities are denoted by R and V, respectively.

Figure 2: Risk Rating Indexes and Volatilities for Canada



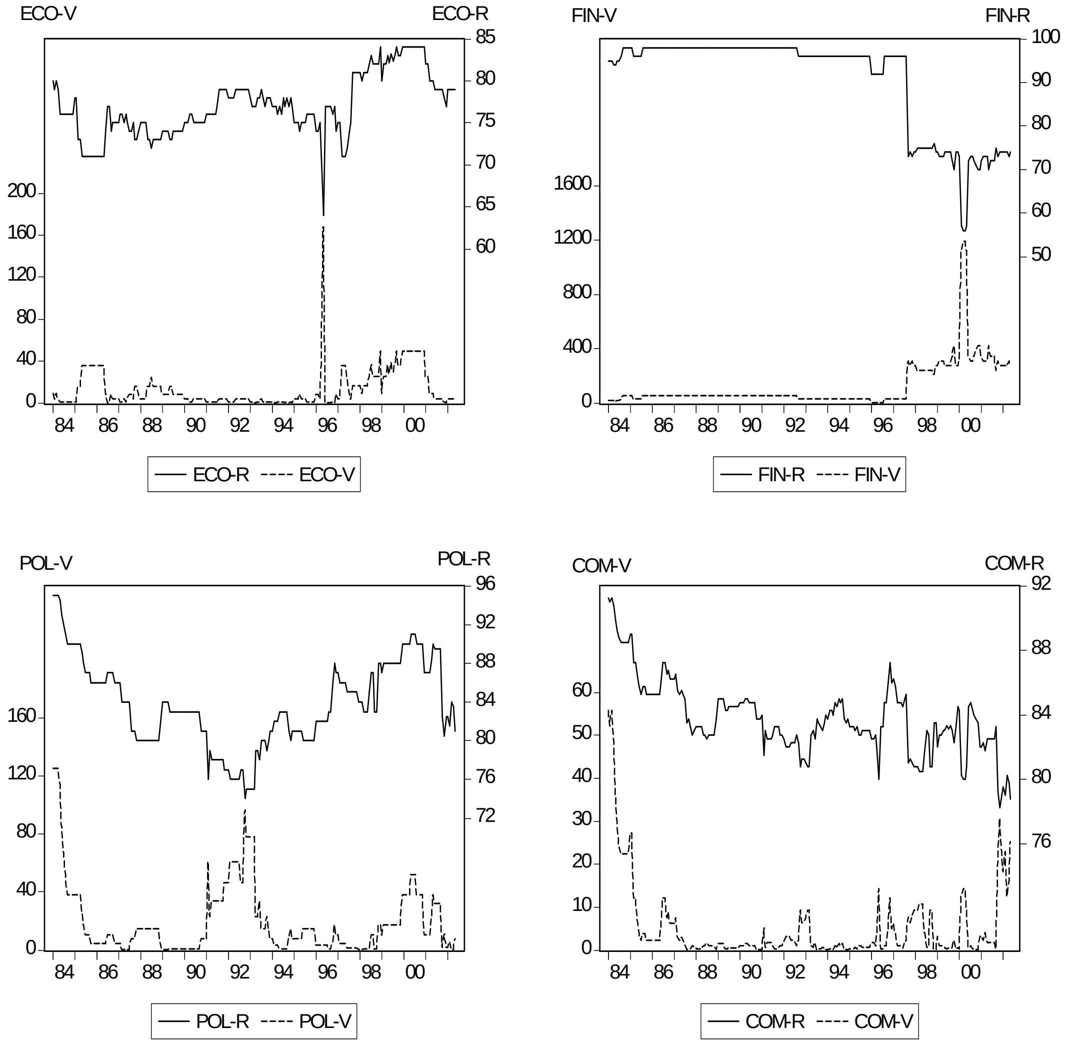
Note: Economic (ECO), Financial (FIN), Political (POL) and Composite (COM) risk rating indexes and their associated volatilities are denoted by R and V, respectively.

Figure 3: Risk Rating Indexes and Volatilities for Japan



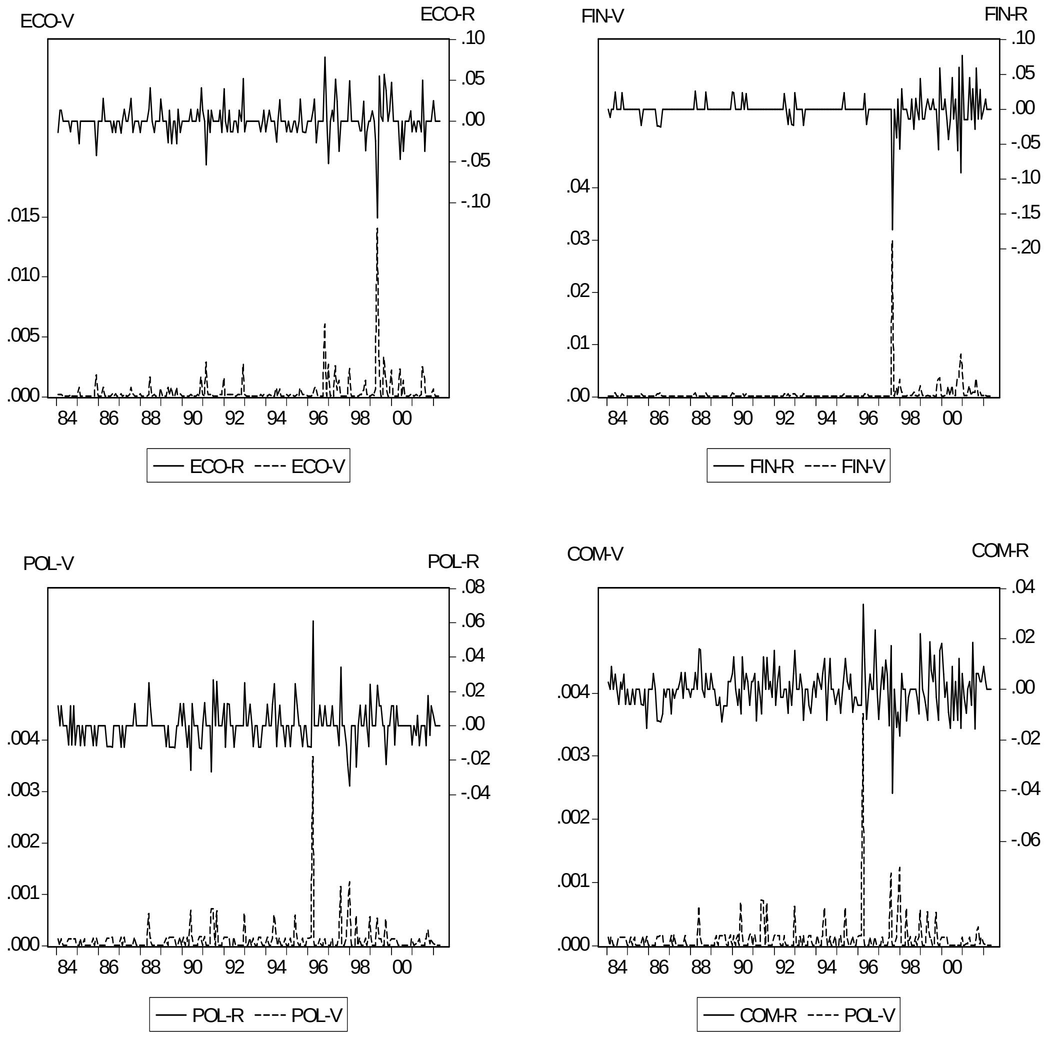
Note: Economic (ECO), Financial (FIN), Political (POL) and Composite (COM) risk rating indexes and their associated volatilities are denoted by R and V, respectively.

Figure 4: Risk Rating Indexes and Volatilities for USA



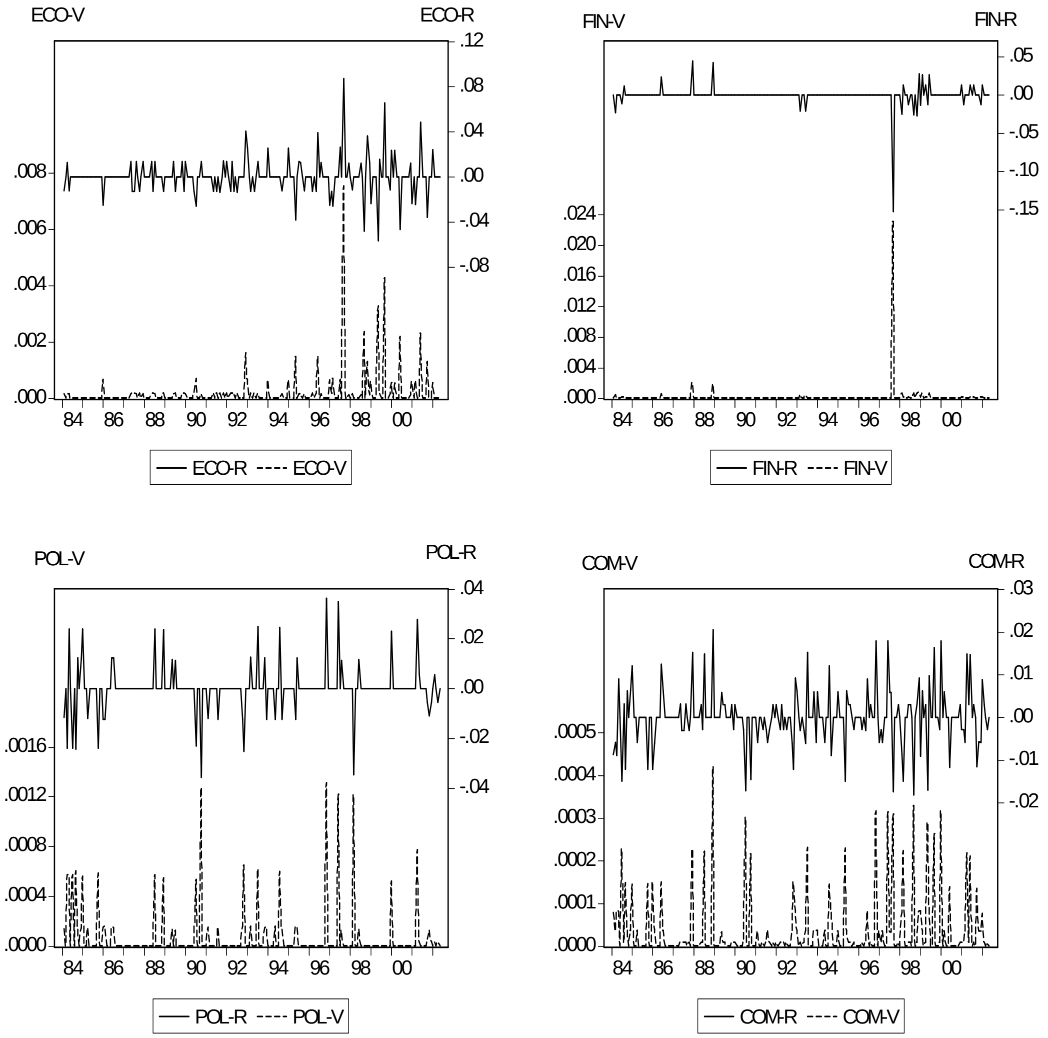
Note: Economic (ECO), Financial (FIN), Political (POL) and Composite (COM) risk rating indexes and their associated volatilities are denoted by R and V, respectively.

Figure 5: Risk Returns and Volatilities for Australia



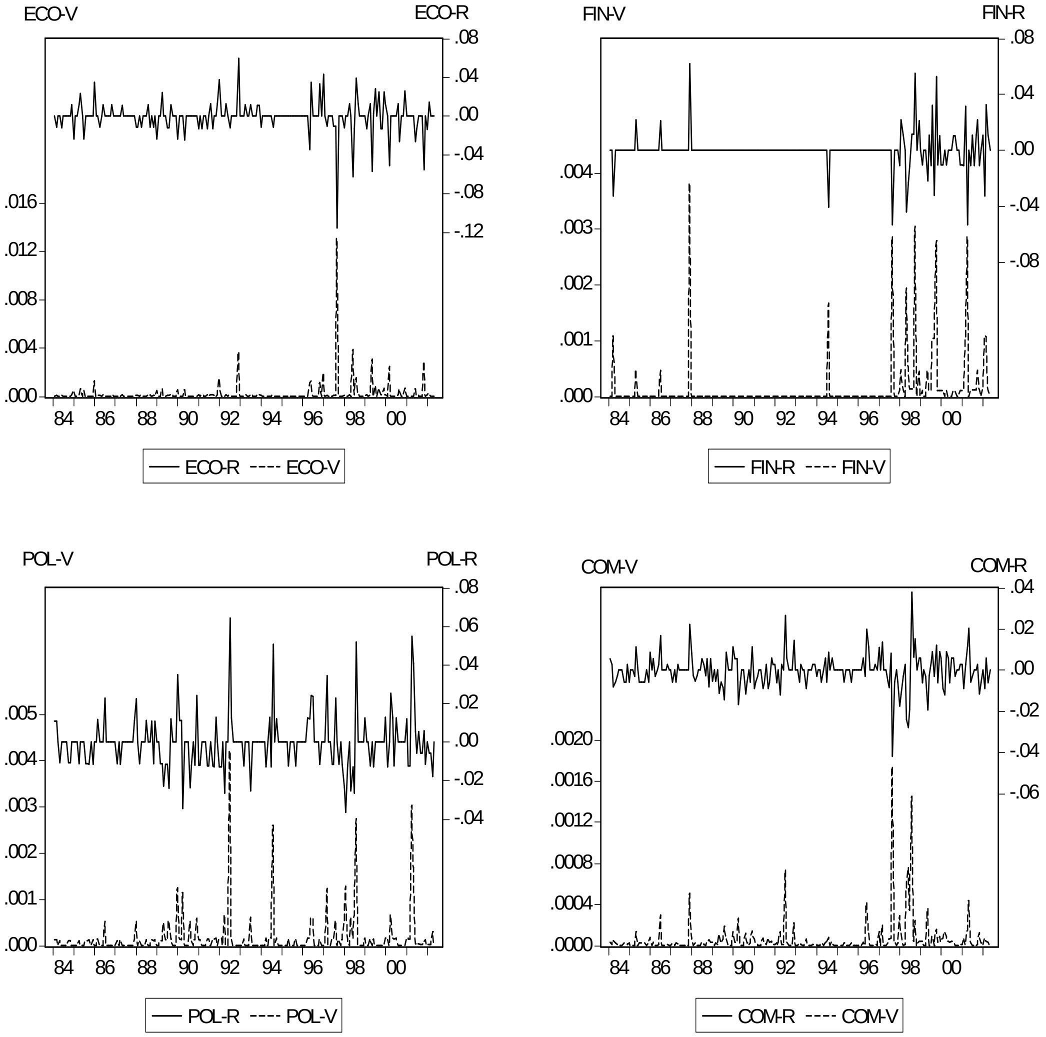
Note: Risk returns (R) and their associated volatilities (V) refer to the rates of change in the respective risk rating indexes.

Figure 6: Risk Returns and Volatilities for Canada



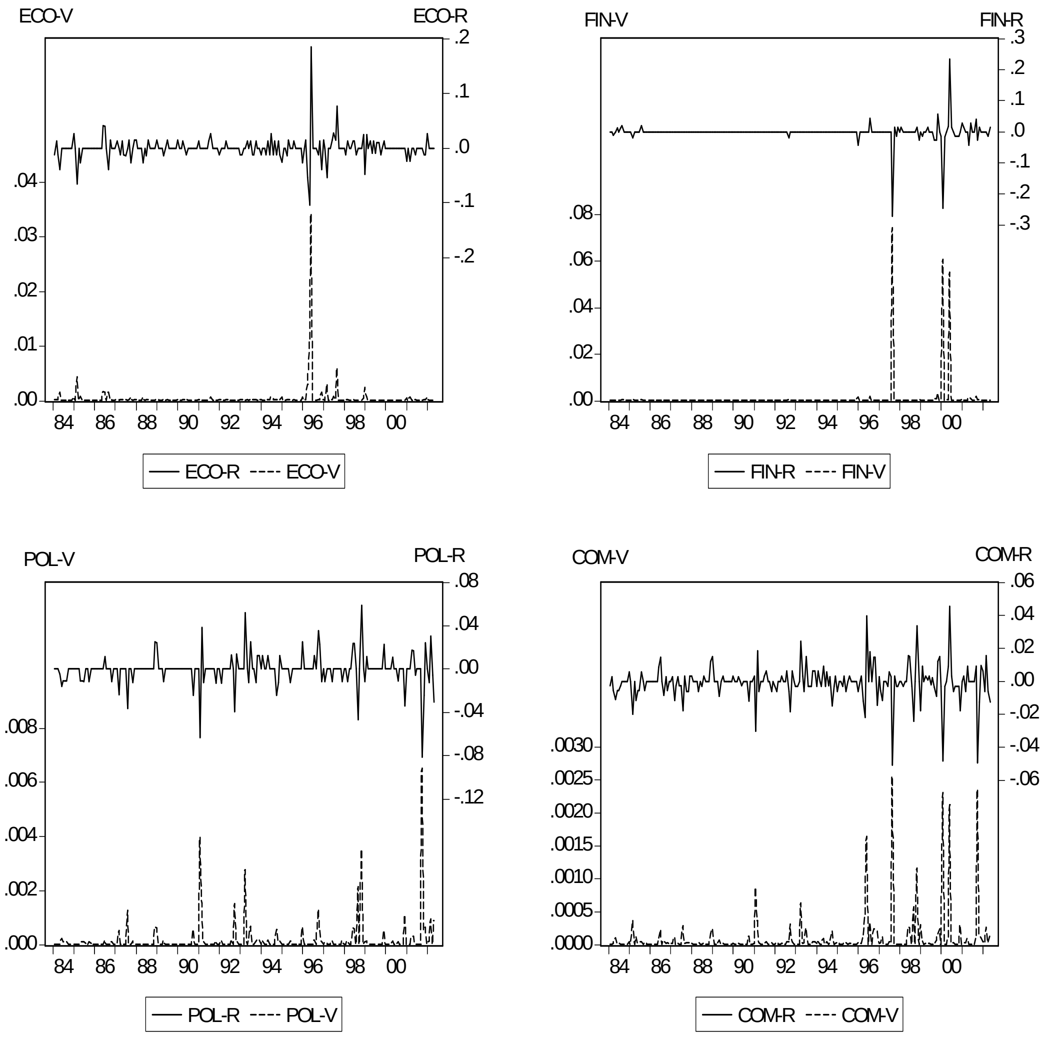
Note: Risk returns (R) and their associated volatilities (V) refer to the rates of change in the respective risk rating indexes.

Figure 7: Risk Returns and Volatilities for Japan



Note: Risk returns (R) and their associated volatilities (V) refer to the rates of change in the respective risk rating indexes.

Figure 8: Risk Returns and Volatilities for USA



Note: Risk returns (R) and their associated volatilities (V) refer to the rates of change in the respective risk rating indexes.

Table 1: Descriptive Statistics for Risk Returns by Country

Country	Risk Returns	Mean	SD	Skewness
Australia	Economic	0.000350	0.019158	-0.448578
	Financial	-0.000646	0.020811	-2.624942
	Political	0.000078	0.010578	0.951969
	Composite	-0.000028	0.008478	0.001765
Canada	Economic	0.000222	0.014450	1.038457
	Financial	-0.000433	0.012679	-7.502530
	Political	0.000156	0.008767	0.201160
	Composite	2.67E-05	0.006209	0.127021
Japan	Economic	-0.000871	0.015692	-1.937662
	Financial	4.86E-05	0.012003	0.299467
	Political	-0.000387	0.013118	1.491078
	Composite	-0.000378	0.008093	-0.090309
USA	Economic	-5.72E-05	0.020467	2.615394
	Financial	-0.001136	0.031031	-3.382983
	Political	-0.000725	0.013714	-0.832759
	Composite	-0.000670	0.010343	-0.703243

Table 2: Correlation Coefficients for Risk Returns by Country

Country	Risk Returns	Economic	Financial	Political	Composite
Australia	Economic	1.000	-0.037	-0.017	0.502
	Financial	-0.037	1.000	0.054	0.564
	Political	-0.017	0.054	1.000	0.657
	Composite	0.502	0.564	0.657	1.000
Canada	Economic	1.000	-0.248	0.050	0.464
	Financial	-0.248	1.000	0.032	0.400
	Political	0.050	0.032	1.000	0.754
	Composite	0.464	0.400	0.754	1.000
Japan	Economic	1.000	0.219	-0.004	0.549
	Financial	0.219	1.000	-0.104	0.430
	Political	-0.004	-0.104	1.000	0.732
	Composite	0.549	0.430	0.732	1.000
USA	Economic	1.000	-0.150	0.046	0.356
	Financial	-0.150	1.000	0.001	0.589
	Political	0.046	0.001	1.000	0.686
	Composite	0.356	0.589	0.686	1.000

Table 3: Correlation Coefficients for Countries by Risk Returns

Risk Returns	Country	Australia	Canada	Japan	USA
Economic	Australia	1.000	0.264	0.153	-0.080
	Canada	0.264	1.000	-0.111	0.306
	Japan	0.153	-0.111	1.000	0.016
	USA	-0.080	0.306	0.016	1.000
Financial	Australia	1.000	0.450	0.247	0.320
	Canada	0.450	1.000	0.336	0.468
	Japan	0.247	0.336	1.000	0.181
	USA	0.320	0.468	0.181	1.000
Political	Australia	1.000	-0.010	0.122	-0.014
	Canada	-0.010	1.000	0.117	0.215
	Japan	0.122	0.117	1.000	0.045
	USA	-0.014	0.215	0.045	1.000
Composite	Australia	1.000	0.110	0.237	-0.027
	Canada	0.110	1.000	0.158	0.251
	Japan	0.237	0.158	1.000	0.126
	USA	-0.027	0.251	0.126	1.000

Table 4: Multivariate Models and Their Parametric Structure for $r = s = 1$

Model	Authors	Guarantee of $h_{iit} > 0$?	Guarantee of Positive Definite H_t?	Correlations Modelled?	Number of Parameters
Vech (or VAR)	Engle and Kroner (1995)	No	No	No	$\frac{1}{2}m(m+1)[1+m(m+1)]$
Diagonal	Bollerslev, Engle and Wooldridge (1988)	No	No	No	$\frac{3}{2}m(m+1)$
BEKK	Engle and Kroner (1995)	Yes	Yes	No	$\frac{1}{2}m(5m+1)$
CC-MGARCH	Bollerslev (1990), Ling and McAleer (2002a)	No	Yes	Yes	$\frac{1}{2}m(m+5)$
DCC/ VC-MGARCH	Engle (2002), Tse and Tsui (2002)	No	Yes	Yes	$\frac{1}{2}m(m+5)+2$
CC-MGJR	This paper	No	Yes	Yes	$\frac{1}{2}m(m+7)$

Notes:

1. The dynamic conditional correlation (DCC) model of Engle (2002) is equivalent to the varying correlation (VC)-MGARCH model of Tse and Tsui (2002).
2. Although the CC-MGARCH, DCC/VC-MGARCH and CC-MGJR models can be specified with or without interdependence between h_{it} and $(\varepsilon^2_{jt-k}, h_{jt-l})$ for $i, j = 1,\dots,m;$ $k = 1,\dots,r;$ and $l = 1,\dots,s$; for purposes of Table 4, h_{it} depends only on $(\varepsilon^2_{it-1}, h_{it-1})$.

Table 5: Univariate GARCH(1,1) Estimates for Four Risk Returns

Economic Risk Returns

Country	ω	α	β	Log-moment	Second moment
Australia	8.67E-06	0.029	0.952	-0.024	0.981
	1.231	1.850	31.255		
	0.435	1.238	12.262		
Canada	8.67E-07	0.062	0.949	-0.002	1.012
	0.535	3.803	47.078		
	0.351	2.019	31.971		
Japan	-3.46E-06	0.002	0.915	-0.013	0.917
	-15.199	1.174	24.939		
	-2.918	0.094	8.115		
USA	6.61E-05	0.590	0.328	-0.568	0.918
	6.994	5.394	5.455		
	2.475	2.442	2.645		

Financial Risk Returns

Country	ω	α	β	Log-moment	Second moment
Australia	1.80E-04	0.138	0.400	-0.774	0.538
	2.606	1.430	1.732		
	0.822	1.145	0.982		
Canada	1.99E-09	0.003	0.951	-0.003	0.954
	0.001	1.949	58.712		
	0.001	0.117	30.254		
Japan	7.24E-05	0.057	0.909	-2.124	0.966
	14.685	12.051	4.497		
	2.189	3.498	2.256		
USA	2.60E-04	0.676	0.505	-0.539	1.181
	4.911	1.906	5.133		
	0.981	1.117	5.500		

Political Risk Returns

Country	ω	α	β	Log-moment	Second moment
Australia	1.02E-04	0.450	-0.143	N.C.	0.307
	9.203	4.003	-7.385		
	3.590	2.391	-2.917		
Canada	9.07E-06	-0.052	0.926	-0.151	0.875
	9.262	-3.539	6.457		
	2.820	-3.012	2.976		
Japan	1.68E-04	0.225	-0.174	N.C.	0.052
	12.939	6.444	-3.336		
	5.272	1.726	-1.918		
USA	5.49E-05	0.060	0.653	-0.362	0.712
	1.331	1.766	2.752		
	1.012	0.820	1.931		

Composite Risk Returns

Country	ω	α	β	Log-moment	Second moment
Australia	4.29E-05	0.299	0.130	-1.253	0.429
	2.591	3.178	0.487		
	3.156	1.925	0.857		
Canada	1.97E-05	-0.115	0.597	-0.773	0.482
	4.573	-25.739	5.226		
	5.781	-3.328	2.634		
Japan	6.73E-06	0.089	0.808	-0.130	0.897
	1.653	2.047	7.884		
	1.105	1.319	7.026		
USA	5.45E-07	0.037	0.968	-0.002	1.005
	0.643	3.218	55.072		
	0.171	1.350	18.746		

Notes:

1. N.C. denotes that the log-moment could not be calculated because $(\alpha\eta_t^2 + \beta)$ in (13) was negative for one observation.
2. The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.

Table 6: Univariate GJR(1,1) Estimates for Four Risk Returns

Economic Risk Returns

Country	ω	α	γ	β	Log-moment	Second moment
Australia	-1.93E-06	0.050	0.068	0.994	-0.002	1.079
	-0.565	2.362	2.559	59.522		
	-0.126	2.081	0.853	14.330		
Canada	4.10E-07	0.033	0.065	0.952	-0.002	1.018
	0.212	2.021	1.994	42.528		
	0.145	1.112	0.623	31.053		
Japan	1.30E-04	-0.094	0.066	0.642	-0.494	0.581
	1.693	-9.268	12.040	2.685		
	5.377	-4.265	1.305	7.904		
USA	7.02E-05	0.877	0.439	0.287	-0.649	1.383
	6.939	3.970	1.774	4.730		
	2.705	1.927	0.839	2.378		

Financial Risk Returns

Country	ω	α	γ	β	Log-moment	Second moment
Australia	2.54E-04	-0.217	0.385	0.489	-0.701	0.465
	1.661	-6.509	1.838	1.565		
	3.133	-1.959	2.419	2.583		
Canada	1.96E-05	-0.093	0.090	0.925	-0.084	0.876
	24.027	-8.001	7.422	25.693		
	10.019	-0.877	0.963	12.238		
Japan	1.70E-06	-0.025	0.100	0.971	-0.206	0.996
	11.533	-13.472	14.102	42.951		
	1.377	-0.462	1.881	21.201		
USA	2.09E-04	0.100	-0.779	0.594	-0.409	0.304
	3.171	2.103	-2.032	4.865		
	1.029	0.585	-0.443	5.641		

Political Risk Returns

Country	ω	α	γ	β	Log-moment	Second moment
Australia	6.51E-05	-0.043	0.524	0.388	-0.782	0.607
	3.864	-0.747	2.531	3.878		
	2.653	-4.064	1.672	1.876		
Canada	2.98E-05	-0.067	0.068	0.656	-0.151	0.624
	15.436	-11.638	2.163	15.884		
	5.485	-5.569	1.559	9.127		
Japan	3.12E-05	-0.004	0.582	0.634	-0.258	0.921
	3.387	-0.256	3.637	8.968		
	1.327	-0.095	1.517	2.862		
USA	1.67E-05	-0.059	0.109	0.918	-0.109	0.914
	3.178	-5.337	4.457	31.845		
	1.381	-2.160	1.545	16.289		

Composite Risk Returns

Country	ω	α	γ	β	Log-moment	Second moment
Australia	3.96E-05	0.224	0.178	0.170	-1.091	0.483
	2.250	2.180	0.789	0.569		
	2.959	1.118	0.759	1.155		
Canada	2.75E-05	-0.118	0.017	0.439	-0.773	0.330
	3.632	-6.253	0.452	2.281		
	2.001	-3.244	0.391	4.890		
Japan	1.09E-05	-0.046	0.214	0.766	-0.213	0.827
	2.941	-1.439	2.211	9.471		
	0.740	-1.617	0.986	2.500		
USA	2.35E-07	0.046	-0.029	0.977	0.003	1.009
	0.242	2.849	-1.156	50.994		
	0.106	1.090	-0.316	27.010		

Note: The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.

Table 7: Multivariate GARCH(1,1) Estimates for Four Risk Returns by Country

Economic Risk Returns

Country	ω_E	α_E	β_E	α_F	β_F	α_P	β_P	α_C	β_C
Australia	-5.84E-06	0.008	0.765	-0.104	0.144	-0.489	0.148	1.438	0.435
	-0.107	0.286	6.337	-2.215	0.807	-2.872	0.363	2.418	0.279
	-0.738	0.171	6.754	-1.789	0.865	-1.940	0.332	1.729	0.241
Canada	3.72E-05	0.099	0.919	-0.025	-0.140	0.224	0.859	-0.424	-0.525
	1.662	2.928	3.155	-2.211	-0.807	4.679	19.512	-3.630	-1.477
	0.631	1.444	2.481	-0.906	-0.636	2.231	1.953	-2.814	-1.464
Japan	3.85E-04	-0.266	0.988	0.000	0.791	0.098	-0.111	0.714	-2.686
	5.359	-1.195	1.592	0.019	6.593	3.373	-7.220	6.235	-5.757
	1.930	-0.968	1.421	0.021	6.507	1.369	-1.548	5.602	-1.956
USA	1.85E-04	0.408	0.389	-0.013	-0.005	-0.099	-0.219	0.230	0.155
	3.448	4.223	3.326	-1.734	-1.279	-2.322	-1.184	1.326	0.421
	2.307	1.595	3.230	-1.483	-3.936	-1.798	-0.742	1.075	0.743

Financial Risk Returns

Country	ω_F	α_E	β_E	α_F	β_F	α_P	β_P	α_C	β_C
Australia	9.63E-06	-0.042	0.520	-0.020	0.671	0.263	-1.016	0.485	0.403
	0.147	-1.334	17.899	-0.915	4.675	1.680	-2.138	3.664	0.337
	0.118	-1.539	1.347	-0.284	4.092	0.452	-0.791	0.772	0.264
Canada	3.81E-04	0.074	-0.042	0.006	0.692	-0.024	-3.724	-0.250	1.087
	4.102	2.699	-2.491	0.161	4.297	-0.869	-9.435	-2.050	1.528
	2.155	2.149	-1.562	0.301	3.294	-4.053	-6.062	-1.587	1.096
Japan	1.98E-04	-0.044	1.174	0.352	-0.117	-0.029	0.042	-0.026	0.890
	1.141	-3.172	1.758	2.754	-1.507	-0.731	0.244	-0.115	1.855
	4.384	-2.249	2.727	2.401	-1.903	-0.699	0.312	-0.133	0.602
USA	-8.77E-06	0.092	0.028	0.142	0.751	0.261	0.567	-2.379	3.691
	-0.145	1.147	0.336	1.992	4.861	1.993	0.885	-3.047	1.763
	-3.711	1.681	0.702	1.705	8.517	1.124	0.892	-2.000	1.907

Political Risk Returns

Country	ω_P	α_E	β_E	α_F	β_F	α_P	β_P	α_C	β_C
Australia	8.18E-05	0.052	-0.139	0.005	0.182	0.313	0.341	-0.142	-0.686
	31.835	2.605	-5.483	6.284	2.247	2.992	3.763	-0.862	-1.977
	2.912	2.166	-5.025	0.448	2.181	2.392	3.770	-0.736	-2.199
Canada	2.04E-04	-0.025	0.221	0.002	0.503	-0.076	0.408	-0.024	0.288
	7.484	-4.729	8.348	1.127	19.704	-2.061	2.300	-0.419	0.387
	5.447	-2.298	6.341	0.586	7.762	-2.266	2.811	-0.177	0.289
Japan	5.30E-05	-0.071	0.322	-0.020	-0.091	-0.002	0.697	0.639	-0.530
	2.573	-1.638	2.114	-0.429	-4.828	-0.026	6.313	2.074	-1.442
	5.693	-2.240	2.553	-0.366	-2.275	-0.037	10.606	2.122	-2.127
USA	4.43E-05	-0.004	0.007	0.000	-0.003	0.040	0.745	-0.084	0.279
	2.744	-0.272	0.594	-0.226	-2.539	1.008	9.577	-1.498	2.542
	1.735	-0.415	0.736	-0.079	-1.072	0.755	7.456	-0.488	0.876

Composite Risk Returns

Country	ω_C	α_E	β_E	α_F	β_F	α_P	β_P	α_C	β_C
Australia	7.05E-05	2.11E-04	-3.86E-04	0.023	-0.102	0.163	-0.427	-0.037	1.025
	1.821	2.870	-1.718	3.318	-6.773	12.306	-6.426	-0.515	5.514
	2.152	1.270	-0.160	3.108	-3.150	7.059	-5.514	-1.527	5.500
Canada	-4.06E-6	3.60E-5	-2.62E-5	0.003	0.017	-0.007	0.029	-0.040	1.022
	-6.555	0.368	-0.238	2.188	6.590	-2.158	6.799	-1.778	40.273
	-9.663	0.053	-0.036	1.166	4.297	-0.984	2.930	-1.005	22.168
Japan	3.06E-05	-0.001	0.000	-0.004	-0.020	0.027	-0.175	0.058	0.948
	8.568	-1.923	0.360	-0.209	-1.110	3.980	-11.783	2.599	3.281
	2.434	-0.942	0.124	-0.139	-0.731	1.984	-1.839	1.750	2.081
USA	4.15E-05	0.001	-0.001	-0.020	0.006	-0.124	0.069	0.619	0.427
	7.501	2.512	-0.688	-2.623	1.525	-2.540	0.312	2.901	2.278
	1.152	1.709	-0.637	-2.485	1.612	-1.764	0.580	2.281	2.574

Notes:

1. The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.
2. The parameters in equation (17) associated with Economic, Financial, Political and Composite Risk Returns are denoted by subscripts E, F, P and C, respectively.
3. The model is based on equations (6)-(7) with $C_l = 0$ for $l = 1,...,r$, as in Ling and McAleer (2002a).

Table 8: Multivariate GJR(1,1) Estimates for Four Risk Returns by Country

Economic Risk Returns

Country	ω_E	α_E	γ_E	β_E	α_F	β_F	α_P	β_P	α_C	β_C
Australia	3.36E-05	-0.066	0.133	0.759	-0.074	0.052	-0.441	-0.114	0.888	1.726
	0.370	-0.931	0.995	4.476	-8.397	0.462	-2.519	-0.170	6.639	0.885
	0.321	-0.712	1.100	5.900	-1.454	0.284	-1.982	-0.295	0.990	0.737
Canada	1.50E-06	0.022	0.101	0.965	-0.001	0.056	0.162	0.135	-0.500	-0.406
	0.014	0.663	3.814	3.702	-0.085	0.309	1.316	0.120	-2.233	-0.419
	2.286	0.499	0.836	2.333	-0.039	0.205	1.901	0.264	-1.561	-1.060
Japan	2.95E-04	-0.056	0.094	0.721	0.022	-0.079	0.608	-2.069	-0.469	1.771
	3.726	-1.281	1.807	5.112	0.187	-0.882	4.513	-3.995	-1.305	1.891
	2.921	-1.652	1.419	5.825	0.478	-1.400	4.149	-2.245	-1.415	2.179
USA	1.92E-04	0.516	-0.066	0.359	-0.015	-0.005	-0.110	-0.220	0.295	0.149
	3.108	2.223	-0.252	3.228	-1.827	-4.118	-0.789	-0.476	1.625	0.286
	2.572	1.864	-0.149	3.471	-1.622	-3.814	-1.809	-0.777	1.254	0.707

Financial Risk Returns

Country	ω_F	α_E	β_E	α_F	γ_F	β_F	α_P	β_P	α_C	β_C
Australia	8.52E-05	0.003	0.004	0.071	-0.083	0.857	0.383	-1.398	0.315	1.123
	1.583	0.130	0.079	1.003	-1.182	10.344	2.195	-5.304	1.527	1.799
	4.133	0.141	0.105	0.606	-1.151	5.351	0.638	-1.001	0.352	0.595
Canada	4.85E-4	3.82E-5	-0.407	-0.076	0.088	0.355	0.004	-0.623	-0.067	-5.852
	4.225	0.001	-1.966	-0.616	0.588	1.454	0.019	-1.012	-0.091	-25.782
	0.985	0.001	-1.286	-1.328	1.519	1.000	0.031	-1.590	-0.163	-0.740
Japan	1.32E-04	-0.074	0.763	0.057	0.075	0.686	-0.093	-0.093	0.294	0.274
	4.729	-2.359	3.945	0.608	0.436	4.633	-2.320	-3.706	1.316	0.700
	3.523	-2.182	3.494	0.569	0.752	4.196	-2.216	-2.144	1.200	0.465
USA	2.25E-04	0.127	0.020	0.025	0.151	0.784	0.499	-1.687	-2.875	4.218
	1.962	1.802	0.274	7.556	2.476	7.250	2.560	-3.938	-3.431	2.472
	1.872	1.329	0.488	0.292	0.932	2.166	1.146	-2.302	-1.388	1.680

Political Risk Returns

Country	ω_P	α_E	β_E	α_F	β_F	α_P	γ_P	β_P	α_C	β_C
Australia	6.84E-05	0.028	-0.108	-0.004	0.086	-0.064	0.581	0.471	0.074	-0.292
	3.596	1.760	-4.250	-0.408	5.077	-0.806	2.879	4.047	0.318	-0.803
	2.865	1.659	-6.762	-0.502	2.207	-1.267	2.392	3.964	0.274	-1.246
Canada	1.39E-04	-0.027	0.120	0.003	0.770	-0.135	0.134	0.753	0.169	0.706
	20.763	-2.624	10.064	1.595	9.105	-3.814	2.352	8.752	1.166	1.762
	4.802	-2.840	6.945	1.405	1.957	-3.296	2.305	7.669	1.300	2.086
Japan	1.21E-05	0.014	0.092	0.078	-0.060	-0.049	0.363	0.923	-0.166	-0.240
	3.260	1.258	5.197	1.682	-1.295	-3.426	3.844	42.701	-1.871	-2.577
	3.606	0.876	2.292	1.760	-1.249	-2.273	4.238	19.834	-1.478	-1.822
USA	1.38E-05	-0.008	0.008	-0.003	-0.001	-0.068	0.156	0.843	-0.047	0.376
	1.134	-2.224	1.616	-1.114	-0.808	-3.024	2.543	12.025	-0.907	3.919
	6.434	-0.969	1.309	-0.899	-1.748	-2.758	2.633	13.959	-0.468	2.143

Composite Risk Returns

Country	ω_C	α_E	β_E	α_F	β_F	α_P	β_P	α_C	γ_c	β_C
Australia	5.17E-05	0.000	0.000	0.015	-0.060	0.121	-0.344	0.007	-0.050	0.998
	17.184	0.406	-1.378	4.267	-5.480	5.408	-6.414	0.279	-1.534	34.170
	3.980	0.318	-0.585	1.697	-1.896	3.631	-3.716	0.131	-0.375	15.456
Canada	7.23E-7	3.55E-5	-1.87E-4	0.002	0.010	-0.014	-0.010	-0.039	0.002	1.019
	3.594	0.750	-2.490	2.026	4.981	-47.01	-2.322	-2.659	0.137	66.749
	1.683	0.038	-0.206	0.902	1.248	-1.312	-0.646	-0.854	0.035	12.206
Japan	4.52E-05	-3.69E-04	0.000	0.007	-0.015	0.054	-0.292	-0.008	0.065	0.963
	5.968	-3.488	3.463	2.599	-9.152	5.244	-10.421	-0.400	2.244	17.531
	4.095	-0.316	0.485	0.220	-0.556	2.865	-3.821	-0.719	1.010	6.680
USA	4.56E-05	0.001	-0.001	-0.020	0.007	-0.129	0.080	0.656	-0.034	0.409
	1.681	2.221	-0.530	-5.425	1.518	-1.717	1.387	3.891	-8.113	2.236
	0.800	0.779	-0.515	-3.044	1.740	-2.737	0.522	2.573	-3.427	2.019

Notes:

1. The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.
2. The parameters in equation (17) associated with Economic, Financial, Political and Composite Risk Returns are denoted by subscripts E, F, P and C, respectively.
3. The model is based on equations (6)-(7).

Table 9: Multivariate GARCH(1,1) Estimates for Four Countries by Risk Returns

Economic Risk Returns

Country	ω_E	α_A	β_A	α_C	β_C	α_J	β_J	α_U	β_U
Australia	2.51E-04	-0.020	0.853	0.059	0.171	-0.077	1.059	-0.021	0.038
	2.550	-0.570	13.212	8.973	1.643	-4.316	1.941	-6.117	1.547
	1.440	-0.900	5.910	2.408	1.186	-1.895	1.424	-5.342	2.047
Canada	5.84E-04	0.026	0.212	-0.130	0.757	0.025	2.187	0.007	-0.018
	2.700	1.641	2.015	-3.335	6.960	1.162	2.616	3.528	-6.043
	1.940	1.174	3.256	-2.952	5.284	0.795	2.189	1.024	-2.947
Japan	-1.54E-05	0.009	0.275	-0.109	0.371	0.011	0.687	-0.003	-0.010
	-0.332	0.281	43.161	-1.179	1.021	0.172	2.402	-0.187	-0.398
	-0.288	0.339	0.787	-1.927	1.076	0.458	2.403	-0.307	-0.528
USA	4.05E-04	0.048	-0.024	-0.086	-0.086	-0.043	2.265	0.425	0.284
	2.172	1.079	-0.121	-3.203	-0.495	-3.173	2.964	4.121	2.356
	0.385	2.429	-0.097	-5.286	-0.341	-3.064	0.572	2.200	2.078

Financial Risk Returns

Country	ω_F	α_A	β_A	α_C	β_C	α_J	β_J	α_U	β_U
Australia	5.23E-05	-0.132	1.030	0.114	0.274	8.69E-05	0.003	0.009	0.011
	6.086	-3.762	7.567	3.020	3.965	0.078	0.027	1.244	2.222
	4.978	-2.211	3.319	1.459	1.984	0.032	0.012	0.285	0.367
Canada	7.30E-06	-0.001	-0.001	-1.73E-03	0.986	-0.007	-0.006	-0.001	0.000
	2.889	-0.377	-1.795	-0.497	3.968	-0.658	-0.500	-1.367	1.500
	3.552	-0.199	-0.117	-0.012	2.730	-0.157	-0.415	-1.833	0.412
Japan	5.02E-04	-0.062	-0.022	0.111	-2.463	-0.079	0.454	-0.006	-0.002
	5.510	-2.606	-0.284	2.350	-6.450	-1.373	2.134	-3.951	-0.491
	2.304	-2.435	1.103	2.539	-1.879	-0.825	1.705	-2.583	-1.375
USA	8.44E-05	-0.217	1.165	0.005	-1.409	-0.048	-0.036	0.053	0.787
	2.371	-2.569	2.774	0.034	-2.468	-0.465	-0.494	1.291	7.748
	2.490	-1.936	2.466	0.029	-1.772	-0.635	-1.781	0.888	8.668

Political Risk Returns

Country	ω_P	α_A	β_A	α_C	β_C	α_J	β_J	α_U	β_U
Australia	1.35E-04	0.315	-0.101	0.079	-0.546	-3.27E-02	-0.036	-0.013	0.161
	1.937	3.627	-2.426	1.273	-1.173	-6.839	-0.544	-1.106	0.739
	1.786	2.341	-2.185	1.682	-1.103	-4.203	-0.559	-1.101	0.643
Canada	2.59E-06	-0.024	0.131	-6.45E-02	0.936	-2.43E-04	0.043	-0.007	-0.049
	0.925	-1.374	2.424	-5.434	20.493	-9.017	0.553	-1.096	-9.224
	0.173	-5.336	0.731	-2.544	7.260	-3.230	0.129	1.184	-6.165
Japan	9.09E-05	-0.046	0.313	-0.195	-0.621	0.090	0.796	-0.045	-0.064
	1.878	-0.467	1.036	-5.689	-1.268	2.057	13.824	-4.737	-1.782
	2.113	-0.179	0.319	-4.533	-0.494	1.360	6.171	-2.517	-0.091
USA	-7.09E-05	-0.030	-0.022	-0.086	-0.081	0.032	0.643	-0.020	0.880
	-6.661	-0.474	-0.141	-7.796	-0.758	1.214	9.036	-1.718	35.735
	-2.722	-2.701	-3.105	-1.954	-0.176	0.554	2.076	-0.601	7.422

Composite Risk Returns

Country	ω_C	α_A	β_A	α_C	β_C	α_J	β_J	α_U	β_U
Australia	2.75E-05	0.367	0.295	-0.031	1.172	-7.68E-02	0.311	-0.040	0.084
	1.284	3.355	1.295	-0.230	4.528	-3.616	19.720	-1.603	0.875
	3.631	2.360	2.424	-0.703	2.294	-1.982	3.003	-4.971	1.441
Canada	1.37E-06	0.049	-0.044	-1.08E-01	0.965	-6.75E-03	0.051	-0.022	0.029
	0.540	7.771	-1.052	-7.574	3.126	-0.613	1.725	-5.148	2.951
	1.257	3.072	-2.155	-5.563	2.472	-0.448	1.719	-4.975	4.581
Japan	1.07E-05	0.128	-0.132	-0.015	-0.159	-0.022	1.010	-0.051	0.023
	9.088	7.932	-5.691	-0.954	-5.714	-2.488	6.781	-5.523	2.832
	3.768	3.804	-1.671	-0.509	-1.769	-0.907	2.697	-3.318	2.383
USA	3.32E-05	0.245	-0.373	-0.025	-0.353	-0.005	-0.051	-0.062	1.016
	2.370	4.202	-2.268	-1.107	-3.010	-0.259	-1.184	-7.663	3.844
	2.172	4.043	-2.238	-0.581	-2.112	-0.204	-0.883	-2.838	2.898

Notes:

1. The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.
2. The parameters in equation (18) associated with Australia, Canada, Japan and the USA are denoted by subscripts A, C, J and U, respectively.
3. The model is based on equations (6)-(7) with $C_l = 0$ for $l = 1,\ldots, r$, as in Ling and McAleer (2002a).

Table 10: Multivariate GJR(1,1) Estimates for Four Countries by Risk Returns

Economic Risk Returns

Country	ω_E	α_A	γ_A	β_A	α_C	γ_C	β_C	α_J	γ_J	β_J	α_U	γ_U	β_U
Australia	2.74E-04	-0.051	0.050	0.897	0.023		0.133	-0.059		1.147	-0.022		0.035
	16.893	-1.177	0.892	2.512	0.243		1.602	-1.121		5.900	-3.521		2.346
	9.711	-0.889	0.769	2.045	0.307		2.003	-1.500		2.445	-2.892		1.584
Canada	3.55E-04	0.020		0.116	-0.129	0.072	0.871	0.030		1.333	0.001		-0.009
	2.401	1.613		3.119	-3.236	3.329	19.965	1.339		3.327	0.113		-0.682
	1.525	0.953		2.320	-2.767	3.933	11.777	1.094		2.375	0.210		-1.250
Japan	-2.01E-05	0.026		0.217	-0.119		0.268	-0.047	0.065	0.777	-0.007		2.81E-04
	-0.539	0.936		1.142	-1.478		6.742	-0.911	1.842	8.327	-0.648		0.016
	-0.498	0.907		0.962	-2.066		1.274	-2.049	2.593	5.458	-0.943		0.019
USA	3.77E-04	0.006		-0.116	-0.028		-0.080	-0.006		2.105	0.828	-0.408	0.196
	1.802	0.306		-0.901	-0.973		-0.762	-0.536		2.257	3.053	-1.387	2.308
	1.741	0.534		-0.374	-0.929		-0.303	-0.480		6.781	2.237	-0.922	1.737

Financial Risk Returns

Country	ω_F	α_A	γ_A	β_A	α_C	γ_C	β_C	α_J	γ_J	β_J	α_U	γ_U	β_U
Australia	1.19E-04	-0.231	0.278	0.938	-0.149		-0.327	0.142		-0.173	0.022		0.001
	2.703	-5.693	5.667	9.520	-3.761		-1.264	1.154		-1.525	1.671		0.108
	2.668	-4.019	3.735	7.277	-3.416		-1.605	0.711		-0.969	1.444		0.070
Canada	1.34E-04	-0.021		-0.102	-0.094	0.190	0.738	-0.011		-0.031	-3.13E-04		-0.001
	2.760	-1.073		-2.492	-1.100	2.455	6.179	-0.462		-1.130	-0.150		-0.274
	1.470	-2.690		-0.877	-1.310	1.834	4.767	-0.185		-0.623	-0.082		-0.168
Japan	4.05E-04	-0.063		-1.09E-04	0.108		-2.167	-0.089	-0.016	0.656	-0.006		-0.001
	3.680	-12.400		-0.002	14.465		-4.131	-3.569	-3.616	4.089	-3.480		-1.210
	2.440	-2.842		-1.082	1.439		-2.926	-1.471	-2.828	1.155	-2.603		-2.000
USA	6.09E-04	0.195		-0.271	-0.667		0.669	-0.171		0.175	-0.021	0.216	0.681
	1.339	0.539		-0.246	-1.649		0.691	-0.300		0.273	-0.198	2.991	2.659
	1.143	0.329		-0.388	-1.250		0.215	-0.796		0.662	-0.161	1.948	1.735

Political Risk Returns

Country	ω_P	α_A	γ_A	β_A	α_C	γ_C	β_C	α_J	γ_J	β_J	α_U	γ_U	β_U
Australia	4.75E-05	-0.054	0.593	0.392	0.050		0.202	-0.045		-0.068	-0.017		0.187
	0.959	-0.793	2.518	3.992	0.616		0.411	-3.183		-1.009	-0.929		1.012
	3.677	-1.068	2.530	3.502	0.995		0.988	-4.157		-1.226	-1.663		1.693
Canada	7.88E-06	-0.029		0.144	-0.063	0.006	0.930	-0.006		0.081	-8.21E-03		-0.055
	0.624	-2.263		3.356	-4.976	0.374	8.751	-0.717		5.989	-1.079		-1.099
	0.957	-2.948		2.379	-3.347	0.378	6.763	-0.425		1.341	-1.561		-1.896
Japan	-1.18E-05	-0.074		0.213	-0.121		0.119	-0.050	0.441	0.697	-0.061		0.382
	-0.260	-2.311		0.861	-1.426		0.243	-1.731	1.731	4.872	-22.887		2.723
	-0.327	-2.499		1.286	-2.478		0.338	-2.797	2.245	5.687	-4.377		2.458
USA	6.25E-06	-0.041		0.093	-0.064		-0.576	0.032		0.408	-0.063	0.083	0.892
	0.371	-2.800		0.865	-1.327		-4.739	0.435		0.980	-1.721	1.401	13.019
	0.102	-0.480		0.726	-0.943		-1.132	0.579		1.221	-1.224	1.829	7.789

Composite Risk Returns

Country	ω_C	α_A	γ_A	β_A	α_C	γ_C	β_C	α_J	γ_J	β_J	α_U	γ_U	β_U
Australia	4.20E-05	0.321	0.030	0.303	0.024		1.222	-0.083		0.411	-0.035		0.090
	1.864	1.263	0.139	2.316	0.371		5.835	-6.506		4.004	-2.683		1.754
	2.352	2.960	0.123	1.664	0.187		3.013	-5.470		1.490	-1.785		1.258
Canada	1.70E-06	0.043		-0.040	-0.103	-0.025	0.941	-0.005		0.058	-0.021		0.031
	0.785	2.499		-0.876	-6.972	-1.289	2.076	-0.745		2.189	-3.902		4.310
	1.843	3.073		-2.019	-4.247	-1.198	2.584	-0.382		2.067	-5.200		4.691
Japan	6.87E-06	0.157		-0.205	0.088		0.363	-0.073	0.121	0.941	-0.037		0.023
	1.596	13.824		-2.794	2.538		4.439	-5.095	3.974	20.663	-11.390		2.357
	2.679	2.883		-1.384	1.748		2.149	-1.481	1.944	11.342	-2.779		1.951
USA	2.99E-05	0.225		-0.033	-0.046		-0.721	0.017		-0.003	-0.082	0.043	0.924
	2.437	4.435		-0.395	-0.782		-2.339	0.380		-0.025	-2.982	1.054	26.938
	1.171	2.347		-0.145	-0.736		-3.368	0.330		-0.030	-2.751	1.156	18.505

Notes:

1. The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.
2. The parameters in equation (18) associated with Australia, Canada, Japan and the USA are denoted by subscripts A, C, J and U, respectively.
3. The model is based on equations (6)-(7).

Table 11: Multivariate Effects for Four Countries by Risk Returns

Economic	Financial	Political	Composite
$A \leftrightarrow C$	$A \leftrightarrow C$	$A \rightarrow C$	$A \leftrightarrow C$
$J \rightarrow A$	$A \rightarrow J$	$A \leftrightarrow J$	$A \leftrightarrow J$
$A \leftrightarrow U$	$A \cap U = \phi$	$A \cap U = \phi$	$A \leftrightarrow U$
$C \leftrightarrow J$	$C \rightarrow J$	$C \leftrightarrow J$	$C \leftrightarrow J$
$C \rightarrow U$	$C \cap U = \phi$	$U \rightarrow C$	$C \leftrightarrow U$
$J \rightarrow U$	$U \rightarrow J$	$U \rightarrow J$	$U \rightarrow J$

Notes:

1. ϕ denotes the empty set.
2. Australia, Canada, Japan and USA are denoted as A, C, J and U, respectively.

Table 12: Multivariate Effects Between Pairs of Countries for Four Risk Returns

Country Pairs	Outcomes
(Australia, Canada)	3 interdependent effects, one effect from Australia to Canada
(Australia, Japan)	2 interdependent effects, 2 separate uni-directional effects
(Australia, USA)	2 interdependent effects, 2 independent effects
(Canada, Japan)	3 interdependent effects, one effect from Canada to Japan
(Canada, USA)	4 separate effects
(Japan, USA)	3 effects from USA to Japan, 1 effect from Japan to USA

Table 13: CC-MGARCH Conditional Correlation Coefficients for Risk Returns by Country

Country	Risk Returns	Economic	Financial	Political	Composite
Australia	Economic	1.000	-0.005	0.024	0.528
	Financial	-0.005	1.000	0.115	0.496
	Political	0.024	0.115	1.000	0.644
	Composite	0.528	0.496	0.644	1.000
Canada	Economic	1.000	-0.165	0.041	0.424
	Financial	-0.165	1.000	0.051	0.417
	Political	0.041	0.051	1.000	0.745
	Composite	0.424	0.417	0.745	1.000
Japan	Economic	1.000	0.189	-0.032	0.473
	Financial	0.189	1.000	-0.021	0.380
	Political	-0.032	-0.021	1.000	0.718
	Composite	0.473	0.380	0.718	1.000
USA	Economic	1.000	-0.173	0.049	0.342
	Financial	-0.173	1.000	-0.010	0.486
	Political	0.049	-0.010	1.000	0.683
	Composite	0.342	0.486	0.683	1.000

Note: The CC-MGARCH conditional correlation coefficients are based on equations (6)-(7) with $C_l = 0$ for $l = 1,\ldots,r$, as in Ling and McAleer (2002a).

Table 14: CC-MGARCH Conditional Correlation Coefficients for Countries by Risk Returns

Risk Returns	Country	Australia	Canada	Japan	USA
Economic	Australia	1.000	0.240	0.137	-0.068
	Canada	0.240	1.000	-0.037	0.217
	Japan	0.137	-0.037	1.000	-0.045
	USA	-0.068	0.217	-0.045	1.000
Financial	Australia	1.000	0.326	0.183	0.161
	Canada	0.326	1.000	0.299	0.355
	Japan	0.183	0.299	1.000	0.237
	USA	0.161	0.355	0.237	1.000
Political	Australia	1.000	-0.007	0.101	0.026
	Canada	-0.007	1.000	0.138	0.210
	Japan	0.101	0.138	1.000	0.034
	USA	0.026	0.210	0.034	1.000
Composite	Australia	1.000	0.079	0.191	-0.047
	Canada	0.079	1.000	0.173	0.214
	Japan	0.191	0.173	1.000	0.107
	USA	-0.047	0.214	0.107	1.000

Note: The CC-MGARCH conditional correlation coefficients are based on equations (6)-(7) with $C_l = 0$ for $l = 1,\ldots,r$, as in Ling and McAleer (2002a).

Table 15: CC-MGARCH (Bollerslev) Conditional Correlation Coefficients for Countries by Risk Returns

Risk Returns	Country	Australia	Canada	Japan	USA
Financial	Australia	1.000	0.535	0.306	0.518
	Canada	0.535	1.000	0.416	0.596
	Japan	0.306	0.416	1.000	0.356
	USA	0.518	0.596	0.356	1.000

Notes:
1. The CC-MGARCH (Bollerslev) conditional correlation coefficients are based on equations (1)-(2).
2. The conditional correlation coefficients for Economic, Political and Composite risk returns were quantitatively similar to those obtained using the CC-MGARCH and CC-MGJR models in Tables 14 and 16, respectively.

Table 16: CC-MGJR Conditional Correlation Coefficients for Countries by Risk Returns

Risk Returns	Country	Australia	Canada	Japan	USA
Financial	Australia	1.000	0.351	0.231	0.314
	Canada	0.351	1.000	0.379	0.474
	Japan	0.231	0.379	1.000	0.290
	USA	0.314	0.474	0.290	1.000

Notes:
1. The CC-MGJR conditional correlation coefficients are based on equations (6)-(7).
2. The conditional correlation coefficients for Economic, Political and Composite risk returns were quantitatively similar to those obtained using the CC-MGARCH and CC-MGARCH (Bollerslev) models in Tables 14 and 15, respectively.