Banks as Producers of Financial Services

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2018-12-30

Abstract

This paper documents that a rise in government debt is associated with a fall in shadow banking and a rise in traditional banking. This is explained in a model where banks are valued for the financial services they offer. Government debt does not compete directly with banks in providing financial services, but the demand for government debt by banks imparts a liquidity premium to government debt. A rise in government debt is estimated to disproportionately benefit traditional banks so they expand at the expense of shadow banks. An optimal debt policy leads both types of banks to become default free.

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1 Introduction

We begin this paper by documenting that a rise in government debt is associated with a fall in shadow banking assets and a rise in traditional/commercial banking assets. The curious positive association between government debt and traditional banking poses a challenge to a variety of banking models that posit banks’ value stem primarily from their production of safe, liquid assets. If safety per se receives special value in an economy, and banks are primarily involved in the production of safe assets, then government debt should be a substitute for all types of banking, including traditional banking. Models of banking that primarily stress the role of banks in producing safe, liquid assets include Gorton and Pennacchi (1990), Holmström and Tirole (1998), Krishnamurthy and Vissing-Jorgenson (2012, 2015), Stein (2012), Sunderam (2014), Greenwood, et. al. (2015), and Diamond (2017) among many others. In this paper we argue that banks’ value stem primarily from their production of a menu of financial services, albeit services whose expected value depends to a significant extent on the safety of banks. As it regards government debt, this distinction is important, as government debt does not per se produce financial services. Focusing on financial services also provides a natural distinction between traditional and shadow banks in terms of the menu of financial services each produces. Traditional banks offer a full menu of check-clearing and electronic transaction services and are meant to accommodate high-volume transaction accounts, whereas shadow banks offer more limited free check-writing features and are meant to satisfy liquidity needs. We show that such a model is able to explain the different response of traditional and shadow banks to a change in the supply of government debt. This model is also consistent with a variety of other features of banking, including how traditional and shadow banks respond in different ways to a rise in uncertainty. Specifically, with deposit insurance the model predicts that a rise in uncertainty leads to a shift towards traditional banks and away from shadow banks, but without deposit insurance the model predicts that a rise in uncertainty leads to a shift away from both traditional and shadow banks. We validate this prediction by comparing the behavior of banking during the Great Depression to the behavior during the 2007/08 Financial Crisis.

In our model, shadow banks emerge in a competitive banking system to compete with traditional banks to provide financial services. In defining shadow banks, the model builds on the unique role of banking in offering financial services backed up by

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1To be sure, Diamond and Dybvig (1983) stress the safe, liquid aspect of bank deposits, but in their setup banks offer risk sharing that differentiates their deposits from government debt from the perspective of depositors.
holdings of safe assets. Due to their more limited role in providing financial services, shadow banks find it optimal to hold a somewhat riskier portfolio of assets in comparison to traditional banks. Traditional banks, in contrast, offer financial services such as checkable deposits that depend more on safety and therefore choose to hold a safer portfolio of assets. In this setup, government debt does not provide financial services and hence does not compete directly with financial institutions. Rather, financial institutions purchase government debt to manage their exposure to risk, which is especially important for financial institutions as the service they provide depends to an important degree of the safety of the assets they create. Indeed, if government debt is in limited supply, the demand for government debt by financial institutions leads to a liquidity premium in the price of government debt. This liquidity premium is not a reflection of a financial service offered by government debt or any other riskless asset, but rather is a reflection of the demand for government debt by financial institutions that offer financial services that depend to a significant extent on the expectation of the fulfillment of this service. This distinction is important to understand the value created by various types of financial institutions and to understand how this value depends on the supply of government debt.

A rise in the supply of government debt leads to a fall in its liquidity premium that encourages all financial institutions to hold more of this debt, to become safer, and to expand. A key insight of this paper is that with a sufficient supply of government debt to eliminate the liquidity premium on this debt, all financial institutions should become riskless. Indeed, we show that this is the optimal government debt policy. The relative size of shadow to traditional banking would then depend entirely on the relative cost and value of the financial service they offer. Any substitution of traditional banking for shadow banking as government debt expands from relatively low levels thus depends on a comparison of how each is able to compete in an environment where they must absorb risk versus an environment where each becomes riskless. Of course, the situation is quite different with deposit insurance for traditional banks. With deposit insurance traditional banks continue to benefit from an expansion of government debt, as the liquidity premium on government debt acts like a tax, and hence a lower tax due to an expansion of government debt encourages traditional banks to expand. Shadow banks will have to accommodate this expansion insofar as they compete in offering financial services. Here too we show that the optimal government debt policy is to expand the supply of government debt to eliminate the liquidity premium that then encourages all banks to become free of default risk, obviating the need for deposit insurance.

This model addresses a number of issues raised in recent literature. Gorton and
Metrick (2012) argue that securitized lending through shadow banks emerged to compete with traditional banks due to the limits of deposit insurance. Surely the securitized lending feature of shadow banks is an important part of their development, but the emergence of shadow banking, defined generally as financial institutions that compete with traditional financial institutions of the day, predates deposit insurance, so their continual emergence seems to be more fundamental to banking than just a reaction to deposit insurance. As argued here, banking fundamentally involves a variety of services that differ by cost and value, so if traditional banks are constrained in one way or another, alternative forms of banking emerge to offer a limited menu of financial services at lower cost. In modern parlance, institutional investors may not require high-cost checkable deposits, and may be better served by lower cost Money Market Mutual Funds that mostly offer some form of liquidity. Part of the technological innovation spurring the expansion of Fintech today may be thought of in a similar manner. Also, since the cost of default may not be as drastic as with traditional banking, in pursuit of higher returns shadow banks may also choose to expose themselves more to bankruptcy risk.

Krishnamurthy and Vissing-Jorgenson (2015) provide empirical support for the argument that government debt competes with the financial sector in the production of safe assets, hence an increase in the supply of government debt crowds out financial sector lending. Their model supports this finding by including all safe assets, whether produced by banks or supplied by the government, directly in preferences. In our model traditional banks offer an important service that essentially enhances the value of government debt, so in this sense rather than competing with government debt in the provision of liquidity, government debt is an important input into their production of financial services. Traditional banks are thus constrained by the supply of safe government debt, by which is meant that a limited supply of government debt leads to a high liquidity premium that makes government debt expensive to hold, so an expansion of government debt increases the size of the traditional banking sector. As shadow banks compete with traditional banks, a rise in the supply of government debt tends to shrink the size of shadow banking. We re-examine the empirical evidence presented by Krishnamurthy and Vissing-Jorgenson (2015) and find indeed that a rise in the supply of government debt is associated with a fall in the size of shadow banking.

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2 As discussed below, prior to the creation of the Federal Reserve or federal deposit insurance, in the late 19th and early 20th century state-chartered Trust Companies emerged to compete with traditional national banks. To be sure, though, some states offered state-level insurance.

3 In a related approach, Nagel (2016) places bank deposits and government debt in preferences as imperfect substitutes and estimates a high degree of substitutability between money and near money assets.
but, as our model predicts, a rise in the size of traditional banking.

The next section of this paper highlights the characteristics of Bank Trusts and Money Market Mutual Funds that will be the foundation for thinking of them as shadow banks in this paper. The section after that summarizes U.S. time series data on shadow and traditional banking, which in particular distinguishes between the period prior to deposit insurance and period during deposit insurance. The next part develops a model of shadow and traditional banking in a competitive market without deposit insurance. Following the presentation of the model we show, first through a simple example and then via simulations of a more general specification, that the model exhibits key features that characterize the data. We then extend the model to include a discussion of deposit insurance and contrast the behavior of the model with and without deposit insurance. We show that the model without deposit insurance can explain key aspects of the behavior of shadow and traditional banks during the Great Depression, and the model with deposit insurance can explain key aspects of the behavior of shadow and traditional banks during the 2007/2008 Financial Crisis. We then conclude. All proofs are in the appendix.

2 Trusts and Money Market Funds as Shadow Banks

In the model below, we provide a fundamental, economic motivation for the existence of two types of financial intermediaries, one we call a shadow bank and one we call a traditional bank. By shadow bank we do not mean a bank-like financial intermediary that operates outside a regulatory environment imposed on traditional banks (although this could certainly be a feature of a particular banking environment). By shadow bank we mean a financial intermediary that offers a menu of financial services that share some characteristics of a traditional bank but are otherwise more limited in scope. As a consequence, these shadow banks face different costs of providing their service, different costs of default on this service and as well face different incentives for investing in risky portfolios. With our focus more on financial services than maturity transformation, we have a somewhat more narrow perspective on shadow banking that what is described in the Financial Stability Board’s (2018) Global Shadow Banking Monitoring Report, which includes insurance corporations and pension funds in their measure of shadow banking. Here we provide some discussion along these lines of two types of shadow

banks, trust companies that were prevalent in the late 19th and early 20th century, and Money Market Mutual Funds along with certain aspects of investment banks beginning in the second half of the 20th century.

2.1 Trust Companies

As described by Moen and Tallman (1992), trust companies are state-chartered institutions that emerged out of the Free Banking Era to compete with national banks, so that by 1907 trust companies in New York State had about 75 percent of the total assets of national banks. Both trust companies and national banks accepted deposits and cleared checks, so in this sense they were similar. However, as summarized by Moen and Tallman (1992, p.613):

Although trusts could perform many of the particular functions of banks, the general role of trusts in the New York financial market was different from that of national banks. While the volume of deposits at trust companies in New York City was comparable to that at national banks, trusts did much less clearing activity than national banks. The trusts had only 7 percent of the clearings of national banks, so were not like commercial banks that provided transactions services to the average depositor. According to George Barnett, trust deposit accounts served as "surplus funds of individuals and corporations deposited for income and pending investment," and thus were not widely used as transactions or checking accounts, despite the fact that the deposits were demand deposits subject to check.

There is thus a strong sense in which trust companies were not in the business of creating high-volume transaction services, as would a national bank. The use of a trust company is highlighted in the following quote from Herrick (1908, p. 378) that is cited in Moen and Tallman (1992):

Banks and trust companies are not identified with each other in the popular mind. Banks are ancient, trust companies are modern. Banks deal primarily with merchants, trust companies with all classes, without distinction. Banks lend on personal credit, trust companies on the security of pledged collaterals. Banks take on the risk of the business success of mercantile enterprises, while trust companies incur only the risk of a decline in investment values. Banks actively promote commerce, while trust companies manage investments. What they have in common is that they both receive deposits....

Regarding investments by trust companies, Moen and Tallman (1992, p. 613-614) note: “... trusts were an important source of short-term business financing, actively
underwriting and distributing securities. ... in contrast to national banks, trusts could own stock equity directly. They could also own real estate, although no more than 15 percent of their total assets could be in this form.”

Thus, it seems that trust companies offered limited financial services to a clientele that did not require full-service checkable accounts. Presumably, trusts offered these serves at a lower cost than the full menu of services of a national bank, and as stated by Moen and Tallman (1992, p. 613), “... they generally offered higher interest rates on deposits than did national banks.” Trusts were also freed to invest in a broader set of asset classes than traditional banks, and they chose to do so.

2.2 Money Market Mutual Funds

In contemporary banking, limited bank-like services would seem to correspond to services offered by Money Market Mutual Funds (MMMF). These funds typically offer limited free check-writing, automated electronic exchange services, telephone exchange and redemption, and are highly liquid and easily converted into cash on demand. Firms use them as part of their cash flow management system. Indeed, the initial MMMFs were created to be substitutes to commercial bank accounts that were limited in interest payments by Regulation Q. At their peak in 2008, MMMFs had over $3.9 trillion in assets, which was 26 percent of GDP. MMMFs are also intimately connected to what is broadly thought of as the shadow banking sector. MMMFs are the largest holder of commercial paper issued or underwritten by investment banks, at their peak holding almost 50 percent of all open-market paper. In this sense, MMMFs are a key component in the demand for maturity transformation fueling the shadow banking system. Indeed, conceptually we will think of the shadow banking sector as reflected in an integrated balance sheet, with MMMFs offering limited financial services on the one end, and investment banks issuing commercial paper backed up by risky assets (such as mortgages) on the other end (to be sure, a significant fraction of the activity—M&A, IPOs, market making, etc.—of investment banks seems unrelated to what would reasonable be thought of as shadow banking activity).

Corresponding to our association of MMMFs with shadow banking, we will associate M2 minus the currency component of M2 minus retail money funds with traditional banking. Retail money funds are incorporated into M2 because they are considered cash equivalent and held in small amounts (institutional money funds are held in large amounts). However, retail money funds are not issued by commercial banks, rather they are issued by asset managers. Along these lines, they are not regulated by the
FDIC or the Fed, but are regulated by the SEC. For these reasons, we move retail money funds out of M2 and keep them in our measure of MMMFs as we separate measures of traditional banking from shadow banking.

3 The Data

Here we describe data in two time periods. This first time period is prior to 1933, which is a historical period without deposit insurance, and the second period is following 1980, a period with deposit insurance. For the latter time period we focus on data since 1980 because MMMFs, our measure of the size of shadow banking, only came into existence in 1971 and did not exist in substantial quantity prior to 1980. For the period prior to 1933 we are similarly constrained by the emergence of trust companies, which did not become important financial intermediaries until the early 1900s (Moen and Tallman, 1992). The longer time series after 1980 also allows us to more thoroughly examine some issues, such as the relation between bank size and government debt, which is difficult to do for the earlier time period. The first time period includes the Great Depression and the latter time period includes the Financial Crisis of 2007/2008, which will allow us to compare the behavior of shadow and commercial bank size with and without deposit insurance during a financial panic.

3.1 Bank Assets and the Supply of Government Debt

Here we describe the relation between the size of traditional and shadow banks and the supply of government debt using U. S. quarterly data from 1980 to 2017. For the supply of safe government debt we use the supply of U.S. Treasury securities held by the public (net of any foreign holdings) obtained from the Federal Reserve’s Flow of Funds accounts. For a measure of traditional bank size we use M2 minus the currency held by the public component of M1 minus retail money funds, all obtained from St. Louis FRED. For a measure of the size of liquidity provided by shadow banks we use financial assets held by Money Market Mutual Funds, which includes retail and institutional money funds, also available from St. Louis FRED. All monetary aggregates are measured relative to nominal GDP, obtained from St. Louis FRED.

Fig. 1 displays the data on traditional bank size, shadow bank size, and government debt, all relative to GDP. Table 1 displays results of regressing bank size on the supply of government debt and real per-capita GDP growth volatility (5-year smoothed). Due to the trend in MMMF, all regressions involving MMMF include a time trend. The
first regression shows results regressing the ratio of traditional bank size to shadow bank size on government debt and GDP volatility. Here we see very clearly that a rise in government debt leads to a rise in traditional banking relative to shadow banking. We also see that a rise in uncertainty as captured in the volatility of GDP also leads to a substitution towards traditional banking and away from shadow banking. The second regression shows results of regressing traditional bank size on government debt and GDP volatility and the third regression shows results of regressing shadow bank size on government debt and GDP volatility. Government debt enters significant and positive in the traditional bank regression and significant and negative in the shadow bank regression, reflecting the complementary nature of government debt and traditional banking and the substitutability between traditional and shadow banking. Also, the coefficient on GDP volatility is positive and significant in the traditional bank regression and negative and significant in the shadow bank regression, reflecting a flight to quality during times of heightened uncertainty.

3.2 Bank Assets, Financial Crises, and Deposit Insurance

Here we contrast the behavior of shadow and traditional banks with and without deposit insurance during a financial panic. Historical data on bank trust assets is obtained from various edition of the Annual Report of the Comptroller of the Currency. Historical data on M2 and Currency Held by the Public is obtained from Friedman and Schwartz (1963). For the period 1928 to 1935, Fig. 2 plots for value of M2 net of
Table 1: Shadow and Traditional Bank Regressions  

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<td>(1)</td>
<td>(2)</td>
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<td>Constant</td>
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<td>-3.92***</td>
<td>-0.45***</td>
<td>-0.55***</td>
<td>-5.25***</td>
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<td>(0.31)</td>
<td>(0.39)</td>
<td>(0.11)</td>
<td>(0.13)</td>
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<tr>
<td>Time</td>
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<td>0.01***</td>
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<tr>
<td>Treas (Dom)/GDP</td>
<td>-0.76***</td>
<td>-0.78***</td>
<td>0.27***</td>
<td>0.31***</td>
<td>-0.49***</td>
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<td>(0.20)</td>
<td>(0.19)</td>
<td>(0.08)</td>
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<td>(0.15)</td>
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<tr>
<td>GDP Vol</td>
<td>-61.04***</td>
<td>23.54***</td>
<td>-26.70**</td>
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<td>(16.49)</td>
<td>(5.36)</td>
<td>(11.32)</td>
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<tr>
<td>$R^2$</td>
<td>0.65</td>
<td>0.71</td>
<td>0.25</td>
<td>0.43</td>
<td>0.79</td>
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<td>0.80</td>
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All ratios are in log. HAC-adjusted standard errors in parentheses.  
** denotes significance at 5 percent. *** denotes significance at 1 percent.

currency held by the public as well as the value of trust assets. For the period 2006 to 2013, Fig. 3 plots the value of M2 minus currency held by the public minus retail money funds as well as the value of MMMF deposits. As can be seen from these figures, during the period without federal deposit insurance (up to 1933), the financial crisis (the Great Depression) was associated with a collapse of commercial banking and trust assets. During the period with deposit insurance, the financial crises (the 2007-2008 Financial Crisis) was associated with a continued rise in traditional banking and a fall in shadow banking. The model developed in this paper will account for the different behavior of shadow and traditional banking during financial crises with and without deposit insurance.

5Recall as well that commercial banking and trust companies collapsed during the 1907 Financial Panic. However, as argued by Tallman and Moen, 1995, the collapse of trust companies was due largely to the loss of clearinghouse privileges by trusts in New York, which is not a feature captured in the paper. Note, though, that trusts in Chicago maintained their clearinghouse privileges and as reported by Tallman and Moen, in Chicago during the financial panic deposits at trusts fell 6 percent while deposits at national banks fell 7 percent.
4 The Model

The model consists of firms, a government, financial intermediaries, and households in a discrete-time, infinite-horizon, risk-neutral setting subject to uncertainty.

4.1 Firms

A large number of identical firms arise each period and exist for only one period. Production is an exogenous amount and is fully perishable at the end of the period. Denote the aggregate output of all firms in the current period by $y \geq 0$. Aggregate output $y'$ in the next period is drawn from a conditional distribution $F(y', y)$.

**Assumption 1.** $F(y', y) > 0$ is a distribution function for every $y \geq 0$ with compact support $0 < y_{min} \leq y' \leq y_{max}$.

As we discuss the equilibrium, we will make further assumptions on $F$. Denote the expectation operator over next period’s realizations of $y'$ conditional on the current realization $y$ as $E_y$ and denote $\tilde{y} = E_y[y']$. Denote also the gross growth rate of aggregate output by $g' = y'/y$.

For simplicity, the only claim on firms is a claim on a portfolio of all firms that trades one period prior to production. That is, owning $z'$ claims in the current period entitles the owner to $z'y'$ units of aggregate output $y'$ in the next period. All claims
are initially owned by households and are traded one period prior to the production of output. Denote the price of a unit of such a claim in terms of current consumption by \( p \).

### 4.2 Government

The government fully honors its existing debt obligations, issues new one-period debt, and levies a lump-sum tax or pays a lump-sum distribution. Denote the face value in units of current output of outstanding debt in the current period as \( B \). The government issues new one-period debt \( B' \) at a per-unit price \( q_b \). The difference \( T = B - q_b B' \) is financed by a lump-sum tax (positive or negative) on households. Denote debt per unit of aggregate output as \( b' = B'/y \). Assume the government implements a stationary policy to achieve an amount of debt per unit of aggregate output \( b' = G(y) \) based on realizations of \( y \).

**Assumption 2.** \( G(y) > 0 \) for every \( y \geq 0 \).

### 4.3 Financial Intermediaries

Households create and manage financial intermediaries. Financial intermediaries accept deposits from households and invest in a portfolio of government bonds and equities.
There are two types of financial intermediaries, labeled as type $i$ for $i \in \{0, 1\}$, which differ by the service value of their deposits to households and the cost of offering this service. The value to households will be more fully described in the section that describes households. We will think of and refer to financial intermediaries of type 0 as traditional banks and financial intermediaries of type 1 as shadow banks. It costs $\theta_i$ per unit of deposit in the current period to operate a financial intermediary (new or continuing) of type $i$.

**Assumption 3.** $0 < \theta_i < 1$, $i \in \{0, 1\}$.

Designate the fraction of a financial intermediary’s portfolio invested in equities by $\alpha$. In the current period, each financial intermediary of type $i$ with portfolio $\alpha$ issues one-period debt (deposits) denoted by $\hat{d}_i'(\alpha) > 0$ at a per unit price $q_i(\alpha)$. Each financial intermediary is of fixed size normalized to one, by which is meant

$$q_i(\alpha)\hat{d}_i'(\alpha) = 1.$$  

The overall size of the financial intermediation sector, and the relative size of shadow banking to traditional banking, will thus be entirely along an external margin that determines the number of banks and not the size of banks.

Financial intermediary $(i, \alpha)$ purchases government debt denoted by $\hat{b}_i'(\alpha)$ and equity denoted by $\hat{z}_i'(\alpha)$, so

$$q_b\hat{b}_i'(\alpha) + p\hat{z}_i'(\alpha) = 1,$$

where

$$p\hat{z}_i'(\alpha) = \alpha$$

and

$$q_b\hat{b}_i'(\alpha) = 1 - \alpha.$$  

In the next period financial intermediary $(i, \alpha)$ will receive a cash flow of $\hat{b}_i'(\alpha) + \hat{z}_i'(\alpha)y'$, which is distributed to its debt holders and owners. Debt holders receive either the face value of their debt or the entire cash flow, whichever is less. Let $x_i'(\alpha)$ denote the payout on their debt during the next period, so that every unit of face value gets paid $x_i'(\alpha)$. From the perspective of the current period the payout ratio $x_i'(\alpha)$ is a random variable, which is given by the relation $x_i'(\alpha)\hat{d}_i'(\alpha) = \min\left\{\hat{d}_i'(\alpha), \hat{b}_i'(\alpha) + \hat{z}_i'(\alpha)y'\right\}$, and
can be rewritten as

\[ x_i'(\alpha) = \min \left\{ 1, \left( \frac{\alpha y'}{p} + \left( 1 - \alpha \right) \frac{1}{q_b} \right) q_i(\alpha) \right\}. \]

The next-period payoff to owners of one financial intermediary can similarly be written as

\[ \omega_i'(\alpha) = \max \left\{ 0, \alpha y' + \left( 1 - \alpha \right) \frac{1}{q_b} - \frac{1}{q_i(\alpha)} \right\}. \]

Households begin a period with \( N_i(\alpha) \) financial intermediaries of type \( i \in \{0, 1\} \) holding portfolio \( \alpha \in [0, 1] \) for various values of \( \alpha \) and choose \( N_i'(\alpha) \) financial intermediaries for the current period for various values of \( \alpha \). To clarify with an example, households may begin a period with \( N_1(\alpha) = 0 \) except for \( N_1(.5) > 0 \) and choose \( N_1'(\alpha) = 0 \) except for \( N_1'(.75) > 0 \), which means that households begin a period with all financial intermediaries of type 1 investing 50 percent of their portfolio in equities and for the next period choose all financial intermediaries of type 1 investing 75 percent of their portfolio in equities. This notation will be suitable for characterizing an equilibrium in which, for each \( i \), there is an optimal portfolio mix \( \alpha \) that all households choose, but in which this optimal portfolio mix may change over time.

### 4.4 Households

Risk-neutral households value an infinite sequence of consumption \( \{\hat{c}, \hat{c}', \hat{c}'', \ldots\} \) via the expected utility function with constant discount factors

\[ \hat{c} + \beta E[\hat{c}'] + \beta^2 E[\hat{c}''] + \ldots \] (1)

where \( 0 < \beta < 1 \).

Transactions involving goods cost resources and financial intermediaries offer a service that reduces this transaction cost. This service is embodied in the debt or account \( \hat{d}'_i \), such as various levels of check writing features (e.g., a traditional bank would allow essentially an unlimited number of checks to be written against the account whereas a money money mutual fund would allow limited check writing), instant liquidity, or some accounting/bookkeeping features. To simplify the notation, from the outset assume that for each type \( i \) financial intermediary, households choose to purchase debt from one portfolio type \( \alpha'_i \), but choose the portfolio type optimally. Later we will show that households will never have an incentive to deviate from such an equilibrium (that is, at any point in time there will never be two types of traditional
banks or two types of shadow banks, although these types of banks can differ from each other and over time). Financial services are derived from the CES composite \( \sum_{j=0}^{1} \gamma_{j} (q_{j}(\alpha_{j})d_{j}^{i}(\alpha_{j}))^{\gamma} \), where \( q_{i}(\alpha_{i})d_{i}^{i}(\alpha_{i}) \) for \( i \in \{0, 1\} \) is the value of debt holdings or deposits that a household holds of financial intermediaries of type \( i \) with portfolio \( \alpha_{i}^{t} \) (throughout the remainder of this paper, we will simply use \( \Sigma \) to denote the summation over \( j \) from 0 to 1). With this composite, transaction costs are given by
\[
\nu \hat{c} + \frac{\psi}{\eta-1} \hat{y}^{\eta} \left( \sum_{j=0}^{1} \gamma_{j} (q_{j}(\alpha_{j})d_{j}^{i}(\alpha_{j}))^{\gamma} \right)^{\frac{1-\eta}{\eta}},
\]
where \( \nu > 0, \psi > 0, \eta > 1, \epsilon \leq 1, \gamma_{0} = 1 \) (a normalization) and \( \hat{y} \) is per-capita firm output (which is exogenous to household decisions).

Regarding the form of the transaction-cost function, note that a rise in consumption and liquid assets leads to a less than proportionate rise in transaction costs, reflecting an economy of scale in transaction costs for wealthier households. Over time the rise in \( \hat{y} \) will ensure that as consumption and liquid assets rise, transaction costs will rise in proportion in equilibrium.

A key assumption underlying this model concerns the cost of default of a financial intermediary for a household. We have in mind that if a financial intermediary is relied on to provide a service based on a credible promise to maintain the value of deposits, such as the clearing of checks or providing other forms of liquidity, then to households there is a cost in addition to the lost value of the defaulted asset if the financial intermediary is not able to honor its debt (Bansal and Coleman, 1996, essentially assume this cost is infinite). Specifically, we assume that this additional cost of default is given by
\[
\Sigma \xi_{j} (1 - x_{j}(\alpha_{i}))d_{j}(\alpha_{j}).
\]
Note that \( x_{i}(\alpha_{i}) \) is the payout rate on debt from financial intermediary \((i, \alpha_{i})\) that matures in the current period, so \( (1 - x_{i}(\alpha_{i}))d_{i}(\alpha_{i}) \) is the face value of debt that is not paid. The cost of default is the key mechanism by which the riskiness of an asset limits its usefulness in providing transaction or liquidity services. While this is an ex-post feature, certainly ex-ante assets that have a relatively high likelihood of default will have a lower expected value in terms of financial services. Note that the cost of default can differ by financial intermediary type, capturing the idea that different levels of service may be associated with different levels of default cost.

Households own all the financial intermediaries and begin each period directly owning all the firms that yield output in the next period. Denote by \( \hat{z}_{h} \) the amount of equity directly owned by each household that is a claim to current output (a remaining
amount, say $\hat{z}_b$, is owned by financial intermediaries). Denote by $\hat{z}'$ the amount of equity owned by each household at the beginning of a period that is a claim to next period’s output (which in aggregate equals one). During the period households sell claims on firms (to financial intermediaries), keeping equity $\hat{z}'_h$. Households can also purchase government debt directly, denoted by $\hat{b}'_h$, but in doing so do not receive any financial services (all financial services are provided by financial intermediaries). At the beginning of each period households also make or receive lump-sum tax payments to the government, denoted by $\hat{T}$. To distinguish between a choice of financial intermediary for deposits (as a consumer of financial services) from one for investment (as a producer of financial services), let $\alpha'_i$ denote a choice of a financial intermediary for deposits and $\tilde{\alpha}'_i$ denote a choice of financial intermediary for investment.

The flow budget constraint for households in the current period is given by

$$
\hat{c} + \nu\hat{c} + \frac{\psi}{\eta - 1} \hat{y}^{\eta} \left( \sum \gamma_j (q_j(\alpha'_j)\hat{d}'_j(\alpha'_j))^t \right)^{\frac{1-\eta}{\eta}} + \sum \xi_j (1 - x_j(\alpha_j))\hat{d}_j(\alpha_j) + \hat{b}'_h + \sum q_j(\alpha'_j)\hat{d}'_j(\alpha'_j) + p(\hat{z}'_h - \hat{z}') + \sum \hat{n}'_j(\tilde{\alpha}'_j)\theta_j + \hat{T} = \hat{b}_h + \sum x_j(\alpha_j)\hat{d}_j(\alpha_j) + \hat{z}_h y + \sum \hat{n}_j(\tilde{\alpha}_j)\omega_j(\tilde{\alpha}_j).$$

Households must also satisfy various non-negativity conditions, such as $\hat{n}'_i \geq 0$, $\hat{d}'_i \geq 0$, $\hat{b}'_h \geq 0$ and $\hat{z}'_h \geq 0$.

### 4.5 Equilibrium Conditions

With free entry, if financial intermediary $(i, \alpha)$ is in operation, then owners should expect discounted profits to equal zero, which leads to the following relation under risk neutrality

$$
\theta_i = \beta E y \max \left\{ 0, \alpha \frac{q'_i}{p} + (1 - \alpha) \frac{1}{q_b} - \frac{1}{q_i(\alpha)} \right\}.
$$

In general, the value $q_i(\alpha)$ is the lowest price a financial intermediary of type $i$ with portfolio $\alpha$ will accept for its debt. It remains to determine if households will choose to hold debt from a financial intermediary of type $i$ with portfolio $\alpha$ at the price $q_i(\alpha)$.

Households choose state-contingent sequences for $\hat{c}$, $\hat{b}'_h$, $\hat{d}'_i$, $\hat{n}'_i$, $\alpha'_i$, $\tilde{\alpha}'_i$ and $\hat{z}'_h$ to maximize preferences given by (1), subject to their flow budget constraints given by (2). Denote the number of next-period financial intermediaries of type $i$ with portfolio $\alpha$ per unit of aggregate output by $n'_i(\alpha) = N'_i(\alpha)/y$, and denote aggregate financial intermediary debt per unit of output by $d'_i(\alpha) = n'_i(\alpha)\hat{d}'_i(\alpha)$. Denote aggregate household consumption by $c$. Denote household aggregate debt holdings by $B'_h$ and household
aggregate debt holdings per unit of aggregate output by \( b_h' = B_h'/y \). Note that initial aggregate household equity holdings equal 1. The first-order conditions, after imposing market-clearing conditions and transforming to aggregate variables, can be written as:

\[
y = c + \nu c + \frac{\psi}{\eta - 1} \left( \Sigma \gamma_j (q_j d_j^*)^e \right)^{\frac{1 - \nu}{\epsilon}} y + \frac{y}{y} \Sigma \xi_j (1 - x_j) d_j + y \Sigma n_j^i \theta_j, \quad (3)
\]

\[
q_b \geq \beta, \text{ w/eq. if } b_h' > 0, \quad (4)
\]

\[
q_i \geq \beta E_y [x_i^*] + \psi (\Sigma \gamma_j (q_j d_j^*)^e)^{\frac{1 - \nu}{\epsilon}} \gamma_i (q_i d_i^*)^{\epsilon - 1} q_i - \beta \xi_i (1 - E_y [x_i^*]), \quad \text{w/eq. if } d_i^* > 0,
\]

\[
q_i b_i' = q_b b_h' + \Sigma (1 - \alpha_i^j) q_j d_j', \quad (5)
\]

\[
p = \beta \tilde{y}, \quad (6)
\]

\[
q_i d_i^* = n_i^i, \quad (7)
\]

where “w/eq.” denotes “with equality,” where

\[
E_y [x_i^*] = E_y \left[ \min \left\{ 1, \left( \alpha_i^j \frac{y}{y} + (1 - \alpha_i^j) \frac{\beta}{q_b} \right) \frac{q_i}{\beta} \right\} \right], \quad (9)
\]

\[
\theta_i = E_y \left[ \max \left\{ 0, \alpha_i^j \frac{y}{y} + (1 - \alpha_i^j) \frac{\beta}{q_b} - \frac{\beta}{q_i} \right\} \right], \quad (10)
\]

and the non-negativity constraints \( d_i^* \geq 0 \) and \( b_h' \geq 0 \) hold. Further, as shown by considering deposits at financial intermediaries with alternative strategies for \( \alpha_i^j \), optimality with respect to \( \alpha_i^j \) requires that

\[
\hat{q}_i \geq \beta E_y [\hat{x}_i^*] + \psi (\Sigma \gamma_j (q_j d_j^*)^e)^{\frac{1 - \nu}{\epsilon}} \gamma_i (q_i d_i^*)^{\epsilon - 1} \hat{q}_i - \beta \xi_i (1 - E_y [\hat{x}_i^*]), \ \text{for any } 0 \leq \alpha_i^j \leq 1, \quad (11)
\]

for \( E_y [\hat{x}_i^*] \) and \( \hat{q}_i \) defined by

\[
E_y [\hat{x}_i^*] = E_y \left[ \min \left\{ 1, \left( \alpha_i^j \frac{y}{y} + (1 - \alpha_i^j) \frac{\beta}{q_b} \right) \frac{\hat{q}_i}{\beta} \right\} \right], \quad (12)
\]

\[
\theta_i = E_y \left[ \max \left\{ 0, \alpha_i^j \frac{y}{y} + (1 - \alpha_i^j) \frac{\beta}{q_b} - \frac{\beta}{\hat{q}_i} \right\} \right]. \quad (13)
\]

Eq. (3) is the aggregate flow budget constraint. Eq. (4) stipulates that if some government debt is not held to generate a liquidity value, then its rate of return must equal the rate of time preference. Eq. (5), the key equation in this model, stipulates that the return on financial intermediary debt/deposits must reflect both the liquidity value of this debt as well as the costs of not fulfilling the promise of delivery on any promised financial service. Eq. (6) accounts for the various holdings of government.
debt. Eq. (7) relates the price of equity to its expected payoff in a risk-neutral setting. Eq. (8) relates the number of financial intermediaries (and the total cost of financial intermediation) to the overall volume of financial intermediation. Eqs. (9)-(10) summarize the expected payout on financial intermediary debt as well as the zero expected profit condition for financial intermediaries. Conditions (11)-(13) are conditions for optimality for the type of financial intermediaries and thereby rule out any excess profits for other types of financial intermediaries to enter. To simplify the notation, define \( \theta = (\theta_0, \theta_1) \), and similarly for \( \xi, q, d', n' \), and \( \alpha \). For given parameters \( (\theta, \psi, \xi, \eta) \), the system defined by eqs. (3)-(13) determines a recursive, stationary equilibrium comprised of endogenous variables \( (c, q_b, q, p, d', n', \alpha') \) conditional on the state \( y \).

To solve for the equilibrium, motivated by eq. (10), for fixed \( y \) define \( H(a, e) \), \( 0 \leq a \leq 1, 0 < e \leq 1 \), such that

\[
0 = E_y \left[ \max \left\{ -e, a \left( \frac{y'}{y} - 1 \right) - H(a, e) \right\} \right].
\]

(14)

**Lemma 1:** Under Assumptions 1 and 3, a unique function \( H(a, e) \) exists such that eq. (14) holds for every \( 0 \leq a \leq 1, 0 < e \leq 1 \). The solution \( H(a, e) \) is a continuous function of \( a \) and \( e \). If \( a \leq e \) then \( H(a, e) = 0 \), else \( H(a, e) \geq 0 \). If \( e \leq \hat{e} \) then \( H(a, e) \geq H(a, \hat{e}) \) for every \( 0 \leq a \leq 1 \).

For ease of notation, define \( h_i(a) = H(a, \theta_i) \). In some sense \( h_i(\alpha'_i) \) is a measure of the exposure to default of a financial intermediary of type \( i \) with investor capital \( \theta_i \) and portfolio \( \alpha'_i \), as it is zero when there is no risk of default and positive if there is risk of default. From the definition of \( h_i \) and eq. (10) it follows that

\[
\alpha'_i + (1 - \alpha'_i) \frac{\beta}{q_b} - \frac{\beta}{q_i} = \theta_i - h_i(\alpha'_i).
\]

(15)

Combine eqs. (9) and (10) to derive

\[
E_y[x'_i] = \left( \alpha'_i + (1 - \alpha'_i) \frac{\beta}{q_b} - \theta_i \right) \frac{q_i}{\beta}.
\]

(16)

Substitute eqs. (15) and (16) into eq. (5) to derive the following relation that must hold in equilibrium

\[
(1 - \alpha'_i) \left( 1 - \frac{\beta}{q_b} \right) + \xi_i h_i(\alpha'_i) + \theta_i \geq \psi \left( \Sigma \gamma_j(q_j d'_j)^e \right)^{1 - \eta - 1} \gamma_i(q_i d'_i)^{\epsilon - 1}, \text{ w/eq. if } d'_i > 0.
\]

(17)
Similarly, the condition of optimality with respect to $\alpha'_i$ can be summarized as

$$(1 - \hat{\alpha}'_i) \left(1 - \frac{\beta}{q_b}\right) + \xi_i h_i(\hat{\alpha}'_i) + \theta_i \geq \psi \left(\Sigma \gamma_j (q_j d'_j) \right)^{\frac{1-n}{\theta}} - 1 \gamma_i (q_i d'_i)^{\theta - 1}$, for any $\theta_i \leq \hat{\alpha}'_i \leq 1$. \hspace{1cm} (18)

To summarize, an equilibrium can now be characterized as a $(q_b, q, b'_h, d', \alpha')$ that solve eqs. (4), (6), (15), (17) and (18), along with $(c, n', p)$ that solve eqs. (3), (7) and (8), and in which the non-negativity constraints $d' \geq 0$ and $b'_h \geq 0$ hold.

The equilibrium can be characterized more sharply by assuming the conditional distribution $F(y', y)$ is continuous with density $f(y', y)$.

Assumption 4.

$F(y', y)$ is continuous with probability density function $f(y', y)$ for every $y \geq 0$.

Define $\Omega_i(a), a > 0$, defined as

$$\Omega_i(a) = 1 + \frac{h_i(a) - \theta_i}{a}.$$  

In addition to the results of Lemma 1, we can now establish the following properties of $h_i$.

**Lemma 2:** Under Assumptions 1-4, the function $h_i(a)$ that solves eq. (14) is a twice continuously-differentiable, concave function with derivatives given by

$$h'_i(a) = \frac{\int_{y\Omega_i(a)}^{\infty} \left(\frac{y'}{\bar{y}} - 1\right) f(y', y) dy'}{\int_{y\Omega_i(a)}^{\infty} f(y', y) dy'} \geq 0,$$  \hspace{1cm} (19)

$$h''_i(a) = \left(\frac{ah'_i(a) - h_i(a) + \theta_i}{a}\right)^2 \frac{f(y\Omega_i(a), y') y}{\int_{y\Omega_i(a)}^{\infty} f(y', y) dy'} \geq 0.$$  \hspace{1cm} (20)

Eq. (18) can now be rewritten using the following Lemma.

**Lemma 3:** Under Assumptions 1-4, eq. (18) holds if and only if

$$\xi_i h'_i(\alpha'_i) \leq 1 - \frac{\beta}{q_b}, \text{ w/eq. if } \alpha'_i < 1.$$  \hspace{1cm} (21)
To summarize, an equilibrium is \((q_b, q, b_h', d', \alpha')\) that solve eqs. (4), (6), (15), (17), and (21), along with \((c, n', p)\) that solve eqs. (3), (7) and (8), and in which the non-negativity constraints \(d' \geq 0\) and \(b_h' \geq 0\) hold. It turns out that the existence of an equilibrium is remarkably easy to prove.

**Theorem 4:** Under Assumptions 1-4, there exists an equilibrium \((q_b, q, b_h', d', \alpha')\) that solves eqs. (4), (6), (15), (17), and (21), along with \((c, n', p)\) that solve eqs. (3), (7) and (8), and in which the non-negativity constraints \(d' \geq 0\) and \(b_h' \geq 0\) hold.

### 4.6 Equilibrium for a Simple Example

Suppose aggregate output can take on only two values, 0 with probability \(1 - \pi\) and \(\hat{y} > 0\) with probability \(\pi\), \(0 < \pi < 1\), and that the liquidity aggregate is linear in traditional and shadow bank deposits, achieved by setting \(\epsilon = 1\). With this two-state distribution, the function \(h_i\) is given by

\[
h_i(a) = \max \left\{ 0, (a - \theta_i) \frac{1 - \pi}{\pi} \right\}. \tag{22}
\]

Note that this discrete distribution violates Assumption 4, hence we need to ensure eq. (18) holds instead of the condition in Lemma 3. Substitution of eq. (22) into (18) reveals that

\[
\alpha_i' = \begin{cases} 
\theta_i, & \text{if } \frac{1 - \pi}{\pi} \xi_1 \geq 1 - \frac{\theta}{q_b} \\
1, & \text{otherwise}
\end{cases}. \tag{23}
\]

Hence, a financial intermediary either chooses \(\alpha_i' = \theta_i\) and will thereby be risk-free, or \(\alpha_i' = 1\) and thereby invests all of its assets in equities.

To simplify the exposition, we will make the following additional assumptions for this example.

**Assumption 5.**

\[
\begin{align*}
(i) : \quad & \frac{\gamma_1}{\theta_1} \leq \frac{1}{\theta_0}, \quad \tag{24} \\
(ii) : \quad & \gamma_1 < 1, \quad \tag{25} \\
(iii) : \quad & \frac{1 - \pi}{\pi} \xi_1 < 1 \leq \frac{1 - \pi}{\pi} \xi_0, \quad \tag{26}
\end{align*}
\]

Assumption 5(i) is a key assumption that will lead to traditional banks dominating
in the provision of liquidity if they are not constrained by the supply of safe assets. Assumptions 5(ii) and 5(iii) narrow down the equilibria to one in which traditional banks always choose to be safe and shadow banks, if they choose to enter and compete with traditional banks, choose a risky portfolio of assets.

To assist in the characterization of the equilibrium, define the function \( w = W(D) \), \( D > 0 \) such that

\[
1 - (1 - \theta_0)w = \psi \left( \frac{D}{(1 - \theta_0)w} \right)^{-\eta}.
\]  \hspace{1cm} (28)

**Lemma 5:** For any \( D > 0 \) there exists a unique \( w \) that solves eq. (28), \( 0 < w < 1/(1 - \theta_0) \), and both \( W(D) \) and \( D/W(D) \) are increasing functions of \( D \).

Use \( W \) to define \( D^*_0 \) such that

\[
1 = W(D^*_0),
\]

and to define \( D^*_1 \) such that \( D^*_1 = 0 \) if

\[
\xi_1(1 - \theta_1)\frac{1 - \pi}{\pi} + \theta_1 \geq \gamma_1,
\]

otherwise \( D^*_1 \) solves

\[
\xi_1(1 - \theta_1)\frac{1 - \pi}{\pi} + \theta_1 = [(1 - \theta_0)(1 - W(D^*_1)) + \theta_0]\gamma_1.
\]

Note that with Lemma 5 and Assumption 5(i) it follows that

\[
0 \leq D^*_1 < D^*_0.
\]

**Proposition 6:** Under Assumptions 1-3 and 5, the equilibrium is comprised of the following three regions:
(Region 1): If $\beta' > D^*_0$, then the equilibrium is given by
\[
\alpha_i' = \theta_i, \quad i \in \{0, 1\} \quad (29)
\]
\[
\frac{\beta}{q_b} = 1, \quad (30)
\]
\[
\frac{\beta}{q_i} = 1 - \theta_i, \quad i \in \{0, 1\} \quad (31)
\]
\[
q_0d'_0 = \frac{D^*_0}{1 - \theta_0}, \quad (32)
\]
\[
q_1d'_1 = 0, \quad (33)
\]
\[
q_bh'_{d_b} = \beta' - D^*_0. \quad (34)
\]

(Region 2): If $D^*_1 < \beta' \leq D^*_0$, then the equilibrium is given by
\[
\alpha_0' = \theta_0, \quad (35)
\]
\[
\alpha_1' = \begin{cases} 
\theta_1, & \text{if } \frac{1 - \pi}{\pi} \xi_1 \geq 1 - W(\beta'), \\
1, & \text{otherwise}.
\end{cases} \quad (36)
\]
\[
\frac{\beta}{q_b} = W(\beta'), \quad (37)
\]
\[
\frac{\beta}{q_0} = (1 - \theta_0)W(\beta'), \quad (38)
\]
\[
\frac{\beta}{q_1} = \begin{cases} 
(1 - \theta_1)W(\beta'), & \text{if } \frac{1 - \pi}{\pi} \xi_1 \geq 1 - W(\beta'), \\
\frac{1 - \theta_1}{\pi}, & \text{otherwise}.
\end{cases} \quad (39)
\]
\[
q_0d'_0 = \frac{\beta'W(\beta')}{(1 - \theta_0)W(\beta')}, \quad (40)
\]
\[
q_1d'_1 = 0, \quad (41)
\]
\[
q_bh'_{d_b} = 0. \quad (42)
\]
(Region 3): If $0 < \beta b' \leq D_1^*$, then the equilibrium is given by

\[
\begin{align*}
\alpha'_0 &= \theta_0, \\
\alpha'_1 &= 1, \\
\frac{\beta}{q_b} &= W(D_1^*), \\
\frac{\beta}{q_0} &= (1 - \theta_0)W(D_1^*), \\
\frac{\beta}{q_1} &= \frac{1 - \theta_1}{\pi}, \\
q_0d'_0 &= \frac{\beta b'}{(1 - \theta_0)W(D_1^*)}, \\
q_1d'_1 &= \frac{1}{\gamma_1 (1 - \theta_0)W(D_1^*)}, \\
q_b' &= 0.
\end{align*}
\]

When there is a large amount of government debt outstanding, specifically $\beta b' > D_0^*$, then the economy is in Region 1, which means that traditional banks completely crowd out shadow banks. With such high levels of government debt, the price of government debt does not reflect a liquidity premium. Hence, shadow banks cannot take advantage of a higher risk-adjusted return from shifting their portfolio to riskier securities (whose price never reflects a liquidity premium). Traditional and shadow banks thus compete only on cost $\theta_i$ and return $\gamma_i$ characteristics, and by assumption traditional banks dominate in this comparison in the provision of liquidity.

As the supply of government debt falls, the economy enters Region 2 and then later Region 3. In Region 2 the price of government debt begins to reflect a liquidity premium, but still the advantage to shadow banks of shifting their portfolio to riskier securities does not encourage them to enter. As the economy enters Region 3 the price of government debt reflects a sufficient premium that both lowers the return on safe assets held by traditional banks and provides a sufficient advantage to shadow banks that can absorb the risk of investing in riskier assets, so that shadow banks begin to enter. In Region 3, a region in which both traditional and shadow banks enter, a rise in government securities $b'$ leads to a rise in traditional banking and a fall in shadow banking. This matches a key feature of the data. In the model, a rise in government securities leads traditional banks to expand as they earn a higher return on their safe portfolio and leads shadow banks to contract as they are less able to exploit a liquidity
premium by investing in risky securities.

4.7 Some General Results

From eq. (17) it follows that if \( q_0 d_0^p > 0 \) and \( q_1 d_1^p > 0 \) (which will occur if \( \epsilon < 1 \)), then

\[
\frac{q_1 d_1^p}{q_0 d_0^p} = \left( \gamma_1 \frac{(1 - \alpha_0') (1 - \frac{\beta}{\gamma_0}) + \xi_0 h_0 (\alpha_0') + \theta_0}{(1 - \alpha_1') (1 - \frac{\beta}{\gamma_0}) + \xi_1 h_1 (\alpha_1') + \theta_1} \right)^{\frac{1}{1-\epsilon}}.
\]

This equation captures a number of different features that determine the relative size of shadow to traditional banking. A higher value of shadow banking as reflected in \( \gamma_1 \) tends to lead to more shadow banking. Similarly, a lower cost of shadow banking as reflected in a low value of \( \theta_1 \) (or a higher cost of traditional banking as reflected in a high value of \( \theta_0 \)) tends to lead to more shadow banking. Holding government debt in some sense acts like a tax due to its low return. The total effect of this tax is captured by the amount of government debt held, \( 1 - \alpha_1' \) for shadow banks and \( 1 - \alpha_0' \) for traditional banks, times the departure of the return on government debt from its fundamentals (the rate of time preference), as captured by \( 1 - \beta/q_b \). Lastly, the cost of default times the exposure to default risk, as captured by \( \xi_1 h_1 (\alpha_1') \) for shadow banks and \( \xi_0 h_0 (\alpha_0') \) for traditional banks, determines the relative size of shadow banking.

To further explore the size of shadow to traditional banking, consider two extremes, one in which government debt is so high that there is no liquidity premium on government debt, and one in which no government debt is issued. Suppose also that there is imperfect substitutability between traditional and shadow bank deposits (\( \epsilon < 1 \)) and that there is a continuous distribution of shocks. Define \( \ell_0 \) and \( \ell_1 \) such that

\[
\theta_i = \psi \left( \Sigma_j \ell_j^e \right)^{\frac{1-\eta}{\epsilon} - 1} \gamma_i \ell_i^{\epsilon-1}, \tag{51}
\]

which means that

\[
\ell_0 = \left( \frac{\theta_0}{\psi (1 + \gamma_1 \varphi^e)^{\frac{1-\eta}{\epsilon} - 1}} \right)^{-\frac{1}{\eta}}, \tag{52}
\]

\[
\ell_1 = \left( \frac{\theta_0}{\psi (1 + \gamma_1 \varphi^e)^{\frac{1-\eta}{\epsilon} - 1}} \right)^{-\frac{1}{\eta}} \varphi, \tag{53}
\]

23
for

$$\varphi = \left( \frac{\gamma_1 \theta_0}{\theta_1} \right)^{\frac{1}{1-\varepsilon}}.$$ 

**Proposition 7:** (a) If

$$\beta b' \geq \Sigma(1 - \theta_j)\ell_j$$

then the equilibrium is given by

$$\alpha'_i = \theta_i, \quad q_b = \beta, \quad \frac{\beta}{q_i} = 1 - \theta_i, \quad q_b'b_h = \beta' - \Sigma(1 - \theta_j)\ell_j, \quad q_id'_i = \ell_i,$$

(b) If $b' = 0$, then the equilibrium is given by

$$\alpha'_i = 1, \quad \frac{\beta}{q_i} = 1 - \theta_i + h_i(1), \quad b'_h = 0,$$

and $q_id'_i$ for $i \in \{0, 1\}$ solve

$$\xi_i h_i(1) + \theta_i = \psi \left( \sum_j \gamma_j (q_jd'_j)^\varepsilon \right)^{\frac{1-\varepsilon}{\varepsilon}-1} \gamma_i (q_id'_i)^{\varepsilon-1}.$$

From Proposition 7(a) it follows that as $q_b'b'$ becomes very large, the size of shadow banking relative to traditional banking is determined by

$$\frac{q_1d'_1}{q_0d'_0} = \left( \frac{\gamma_1 \theta_0}{\theta_1} \right)^{\frac{1}{1-\varepsilon}}. \quad (54)$$

Large values of $q_b'b'$ thus do not completely crowd out any particular type of financial institution, but rather leads the relative size of financial institutions to be determined by their relative value as reflected in $\gamma_1$ and relative cost as reflection in $\theta_1/\theta_0$. With large values of $q_b'b'$, shadow banking is relatively larger when it has a higher relative value ($\gamma_1$) and a lower relative cost ($\theta_1/\theta_0$). As well, large values of $q_b'b'$ make all types
of financial institutions safer by raising the return on safe, government assets which in turn encourages financial intermediaries to invest in these types of safe assets. With sufficiently large values of $q_b b'$ financial intermediaries invest sufficiently in government securities so that they become free of default risk.

From Proposition 7(b) it follows that as $q_b b'$ becomes very small, the size of shadow banking relative to traditional banking is determined by

$$
\frac{q_1 d_1'}{q_0 d_0'} = \left( \frac{\gamma_1 (\xi_0 h_0(1) + \theta_0)}{\xi_1 h_1(1) + \theta_1} \right)^{\frac{1}{\eta}}.
$$

Since both types of financial intermediaries are now exposed to risk of default, the cost of default as captured by $\xi_0$ and $\xi_1$ is now important in determining their relative size. Shadow banking is relatively larger when the cost of default of shadow banking is relatively low ($\xi_1$) or the cost of default of traditional banking is relatively high ($\xi_0$). Their relative size is also determined by how each type of financial intermediary is exposed to default risk, as captured by $h_0(1)$ and $h_1(1)$. A high value of $\theta_0$ relative to $\theta_1$ will lead to low value of $h_0(1)$ relative to $h_1(1)$ as a higher value of $\theta_0$ leads to a larger capital investment in traditional banks that makes them safer for depositors. Note that these effects tend to work in opposite directions, as a traditional bank may have a higher value of $\xi_0$, but due potentially to a higher value of $\theta_0$, may have a lower value of $h_0(1)$.

In comparing the two extreme cases, $(q_1 b')/(q_0 d_0')$ falls as $q_b b'$ rises from 0 to a very large value if

$$
\frac{\xi_1 h_1(1)}{\xi_0 h_0(1)} < \frac{\theta_1}{\theta_0}.
$$

The right side can be thought of as capturing the cost advantage of traditional banking relative to shadow banking and the left side can be thought of as the risk advantage of traditional banking relative to shadow banking. If the cost advantage exceeds the risk advantage, then a rise in government debt will be associated with a rise in traditional banking relative to shadow banking. The reason is that a rise in government debt makes risk less important as a feature distinguishing all types of banking. In this range of parameter values traditional banking has a cost advantage, so as the equilibrium moves from one where relative size is determined by a risk advantage to one where relative size is determined by a cost advantage, traditional banking will rise relative to shadow banking. It is of course not at all clear how this comparison works out in actual banking environments, as it will depend on values of parameters that need to be estimated. It is thus not clear that a rise in government debt should be expected to
lead to an expansion or contraction of traditional banking relative to shadow banking in an environment without deposit insurance.

4.8 Simulation: Lognormal Distribution for Output

To study the performance of the model with a continuous distribution for shocks to aggregate output and for all possible values of government debt, consider simulations of the model when the log of aggregate output is drawn from a Normal distribution with mean $\mu$ and variance $\sigma^2$: $\ln y' \sim \mathcal{N}(\mu, \sigma^2)$. Let $\Phi$ denote the CDF for $\mathcal{N}(0, 1)$. Recall the definition of $\Omega_i(\alpha)$ and note that $\Omega_i(\alpha)$ solves

$$
\Omega_i(\alpha) = \Omega_i(\alpha) \Phi \left( \frac{\ln (\Omega_i(\alpha))}{\sigma} + \frac{\sigma}{2} \right) - \Phi \left( \frac{\ln (\Omega_i(\alpha))}{\sigma} - \frac{\sigma}{2} \right) + \frac{\alpha - \theta_i}{\alpha}.
$$

The solution $\Omega_i(\alpha)$ (and thereby $h_i(\alpha)$) will have to be computed numerically, which will be facilitated by the result that $\Phi$ has good approximations that are closed form. Note as well that

$$
h'_i(\alpha) = \frac{1}{\alpha} \left( \frac{\theta_i}{1 - \Phi \left( \frac{\ln (\Omega_i(\alpha))}{\sigma} + \frac{\sigma}{2} \right)} \right) + \Omega_i(\alpha) - 1.
$$

We will choose parameter values so that the model matches various moments listed in Table 2. Since this version of the model does not include deposit insurance (deposit insurance will be included in the next section), the data in Table 2 largely covers the period 1900 - 1933, which is prior to the Banking Act of 1933 that created federal deposit insurance. Specifically, we will choose parameters so that the model matches the conditional variance of log real per-capita GDP, the real return to the stock market, real interest rates on short-term government debt, commercial paper, and traditional bank deposits, traditional bank assets as measured by M2 minus currency held by the public as a ratio to GDP, trust bank assets as a ratio to GDP, as well as the fraction of asset portfolios that commercial and trust banks invest in government debt or cash. Note that one moment we will not match is the ratio of government debt to GDP, but rather by construction we will match the ratio of government debt held by financial intermediaries to GDP. In the model they are the same (at least for the equilibria in which government debt offers a liquidity premium), but in reality these two ratios can be quite different. There are surely a variety of reasons for holding government debt that are not captured by this model, and here we focus mainly on the demand for

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6Due to the difficulty of obtaining historical data, all of the variables do not begin in the same year.
government debt by financial intermediaries.

The model is assumed to operate at a quarterly frequency. The variance $\sigma^2$ of the conditional distribution of log income $y$ is chosen to match the variance of the first difference of log per-capita quarterly real GDP, estimated as the annual variance divided by 4. The subjective discount factor $\beta$ is chosen to match the real return on the stock market. The parameter $\theta_0$ is chosen so that eq. (15) for $i = 0$ holds at values of $\alpha_0$, $q_b$, and $q_0$ from the data, and similarly for $\theta_1$. Given $\theta_0$, the value of $\xi_0$ is chosen to be consistent with eq. (21) for $i = 0$, and similarly for $\xi_1$. Eq. (17) represents two equations in the remaining four unknown parameters $\eta$, $\epsilon$, $\psi$, and $\gamma_1$. Our approach will be to use eq. (17) to determine $\psi$ and $\gamma_1$ for various choices of $\eta$ and $\epsilon$. In some sense the more interesting parameter is $\epsilon$, which determines the substitutability between traditional and shadow bank deposits, so essentially we will be asking how the predictions of the model depend on the substitutability between these two types of assets (we will always assume $\eta = 2$). The values of the parameters are in Table 3. The solution to the model with these parameter values is presented in Table 4. Note that the solution to the model does not depend on the particular choice of $\epsilon$, as $\psi$ and $\gamma_1$ are chosen to maintain the calibration, but the response of the model to a change as the quantity of government debt or uncertainty may depend on the value of $\epsilon$.

Consider how each type of bank responds to a rise in government debt. Fig. 4 shows that a rise in government debt leads to a fall in traditional banking and a rise in shadow banking. The possibility of this result is already explained above. For the parameter values in Table 3, due to the larger initial capital investment required of a traditional bank ($\theta_0 > \theta_1$), at low levels of government debt traditional banks have a risk advantage over shadow banks that is larger than the cost advantage of traditional banks. Thus, traditional banks dominate at low levels of government debt. As government debt expands, traditional banks lose this advantage and, at the estimated parameter values, shadow banks offer sufficient value so that they begin to replace traditional banks. As we will see, though, the situation is very different with deposit insurance.

Fig. 5 documents that the size of traditional and shadow banking both fall with a rise in uncertainty. Both types of banks choose to be exposed to risk, and the liquidity value of their deposits depends importantly on the safety of these deposits, so a rise in uncertainty leads to a fall in the expected value of both types of bank deposits. This behavior mimics the observed behavior of both types of banks during the Great Recession.

---

7In the simulation $h_0(1) = 0.0116$ and $h_1(1) = 0.0147$, so that $(\xi_1 h_1(1))/(\xi_0 h_0(1)) = 1.07$ is higher than $\theta_1/\theta_0 = .79$, which leads to $\left(\frac{\gamma_1 \theta_0}{\theta_1}\right)^{1.07} = .91$ and $\left(\frac{\gamma_1 (\xi_0 h_0(1)+\theta_0)}{\xi_1 h_1(1)+\theta_1}\right)^{1.07} = 0.35$.
Table 2: U. S. Annual Data, 1900 - 1933.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation diff. log real per-capita GDP</td>
<td>0.032</td>
</tr>
<tr>
<td>Real return value-weighted U. S. Equity</td>
<td>0.066</td>
</tr>
<tr>
<td>Real rate 30 day Treasury bill (1920 - 1933)</td>
<td>0.049</td>
</tr>
<tr>
<td>Real rate AA Commercial Paper (1920 - 1933)</td>
<td>0.062</td>
</tr>
<tr>
<td>Real rate Checking Accounts (1920 - 1933)</td>
<td>0.039</td>
</tr>
<tr>
<td>Gross Federal Debt / GDP</td>
<td>0.162</td>
</tr>
<tr>
<td>(M2 - Currency) / GDP (1914-1933)</td>
<td>0.409</td>
</tr>
<tr>
<td>Trust Assets / GDP (1914-1933)</td>
<td>0.145</td>
</tr>
<tr>
<td>Gov’t Assets + Cash + Reserves / Assets for Commercial Banks (1928-1933)</td>
<td>0.150</td>
</tr>
<tr>
<td>Gov’t assets + Cash + Reserves / Assets for Trust Banks (1928-1933)</td>
<td>0.149</td>
</tr>
</tbody>
</table>


Depression, which is a period of no deposit insurance.

5 Deposit Insurance

In response to bank failures during the Great Depression, the U. S. passed the Banking Act of 1933 that established federal deposit insurance which protected depositors from loss of value should a bank enter bankruptcy. This section considers the consequence of deposit insurance for the competition between traditional and shadow banks.

5.1 Modifying the Model to Include Deposit Insurance

By deposit insurance we simply mean that the government guarantees the value of deposits at traditional banks. In the event of bankruptcy of a traditional bank, the government will make up any shortfall so that depositors receive the full value of their deposits. This transfer is financed by a lump-sum tax on the population. The owners of the bank receive no value once their bank enters bankruptcy. In return for deposit insurance, the regulators limit the risk exposure of banks by choosing a value of $\alpha_0$. 

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Table 3: Parameter Values for Model Without Deposit Insurance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>income std. dev.</td>
<td>$0.0320$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>time discount</td>
<td>$0.9841$</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>unit cost trad. bnk.</td>
<td>$0.0140$</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>unit cost shad. bnk.</td>
<td>$0.0110$</td>
</tr>
<tr>
<td>$\xi_0$</td>
<td>default cost trad. bnk.</td>
<td>$0.1884$</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>default cost shad. bnk.</td>
<td>$0.1588$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>transaction cost curv.</td>
<td>$2.0$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>transaction cost coef.</td>
<td>$0.0047$</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>liq. value shad. bnk.</td>
<td>$0.7075$</td>
</tr>
</tbody>
</table>

Table 4: Model Solution Without Deposit Insurance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_b$</td>
<td>interest gov’t debt</td>
<td>$0.049$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>interest trad. bnk.</td>
<td>$0.039$</td>
</tr>
<tr>
<td>$r_1$</td>
<td>interest shad. bnk.</td>
<td>$0.062$</td>
</tr>
<tr>
<td>$q_0b'$</td>
<td>value gov’t debt</td>
<td>$0.083$</td>
</tr>
<tr>
<td>$q_0d_0'$</td>
<td>value trad. bnk.</td>
<td>$0.400$</td>
</tr>
<tr>
<td>$q_1d_1'$</td>
<td>value shad. bnk.</td>
<td>$0.145$</td>
</tr>
<tr>
<td>$q_0b'_h$</td>
<td>value gov’t debt holding pvt.</td>
<td>$0.0$</td>
</tr>
<tr>
<td>$\alpha_0'$</td>
<td>frac. equity debt holding trad. bnk.</td>
<td>$0.85$</td>
</tr>
<tr>
<td>$\alpha_1'$</td>
<td>frac. equity holding shad. bnk.</td>
<td>$0.849$</td>
</tr>
</tbody>
</table>

Note: All rates are annualized.

(the fraction of assets invested in risky securities). Since a choice of $\alpha_0'$ limits the risk exposure of banks, it is meant to capture the effects of both reserve requirements and capital adequacy requirements. The set-up of the problem is as before, except with $x_0' = 1$ (reflecting deposit insurance), and $\alpha_0'$ given by the regulator. The Euler eq. (5) for $i = 0$ now becomes

$$q_0 \geq \beta + \psi \left( \Sigma \gamma_j (q_jd_j')^\epsilon \right)^{1-\epsilon} \gamma_0 (q_0d_0')^{-1}q_0, \text{ w/eq. if } d_0' > 0, \quad (55)$$
This leads to replacing eq. (17) for $i = 0$ by

$$(1 - \alpha'_0) \left(1 - \frac{\beta}{q_0}\right) - h_0(\alpha'_0) + \theta_0 \geq \psi \left(\sum \gamma_j (q_j d'_j)^{\epsilon}\right)^{\frac{1-r}{r}-1} \gamma_0 (q_0 d'_0)^{\epsilon-1}, \ w/eq. \ if \ d'_0 > 0.$$  

(56)

**Proposition 8:** Under Assumptions 1-4, and for any $\theta_0 \leq \alpha'_0 \leq 1$, there exists an equilibrium $(q_b, q, b'_h, d', \alpha'_1)$ that solves eqs. (4), (6), (15), (17) for $i = 1$, (21) for $i = 1$ and (56), along with $(c, n', p)$ that solves eqs. (3), (7) and (8), and in which the non-negativity constraints $d'_0 \geq 0$ and $b'_h \geq 0$ hold.

### 5.2 Some General Results

Using eq. (56) instead of eq. (17) for $i = 0$, it follows that if $q_0 d'_0 > 0$ and $q_1 d'_1 > 0$, then

$$\frac{q_1 d'_1}{q_0 d'_0} = \left(\frac{(1 - \alpha'_0) \left(1 - \frac{\beta}{q_0}\right) - h_0(\alpha'_0) + \theta_0}{\left(1 - \alpha'_1\right) \left(1 - \frac{\beta}{q_0}\right) + \xi h_1(\alpha'_1) + \theta_1}\right)^{\frac{1}{\epsilon - 1}}.$$  

(57)

With deposit insurance the parameter $\xi_0$ is no longer relevant and the implicit subsidy to risk-taking by traditional banks introduces the term “$-h_0(\alpha'_0)$” that tends lower the supply of shadow banks relative to traditional banks. For a supply of government debt that is so high that there is no liquidity premium on government debt, the equilibrium
is the same as before since both banks acquire sufficient government debt to become default free (provided $\alpha_0' = \theta_0$). With a fixed value of $\alpha_0' > 0$ it is no longer possible to run the thought experiment of letting the supply of government debt go to zero. However, from eq. (57) we can see that a low required value of $\alpha_0'$ acts like a tax in forcing traditional banks to hold low interest-bearing government debt, and thus a rise in the supply of government debt that raises the interest rate may disproportionately benefit traditional banks.

5.3 Simulation of the Model With Deposit Insurance

We again assume a log-normal distribution of output and calibrate the model in the similar manner as before, which leads to using the same method for choosing values of parameters for $\sigma$, $\beta$, $\theta_0$, $\theta_1$, $\xi_1$, $\alpha_0$ and $\alpha_1$, except based on data in Table 5 instead of Table 2. Table 5 is based on more recent data since 1980 that includes deposit insurance. We will choose parameters to match similar moments as before, although now traditional bank assets are measured by M2 minus currency held by the public minus retail money funds, and shadow bank assets are measured by financial assets held by money market mutual funds. Since depositors never lose any value of their deposits, the parameter $\xi_0$ is no longer relevant. The only parameters that required a new method for calibration are $\psi$ and $\gamma_1$, which are now based on eq. (56) for $i = 0$ and eq. (17) for $i = 1$. The new parameter values are listed in Table 6 and the new

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation diff. log real per-capita GDP (1950-2017)</td>
<td>0.009</td>
</tr>
<tr>
<td>Real return value-weighted CRSP (1950 - 2017)</td>
<td>0.072</td>
</tr>
<tr>
<td>Real rate 30 day Treasury bill</td>
<td>0.022</td>
</tr>
<tr>
<td>Real rate AA Commercial Paper</td>
<td>0.029</td>
</tr>
<tr>
<td>Real rate Checking Accounts (6/2009-4/2018)</td>
<td>-0.015</td>
</tr>
<tr>
<td>Marketable U. S. Treasuries (domestic) / GDP</td>
<td>0.255</td>
</tr>
<tr>
<td>(M2 - Currency - RMF) / GDP</td>
<td>0.442</td>
</tr>
<tr>
<td>MMMF / GDP</td>
<td>0.118</td>
</tr>
<tr>
<td>Treasury bills plus Reserves / Assets for Commercial Banks</td>
<td>0.25</td>
</tr>
<tr>
<td>Treasury bills / Assets for Shadow Banks</td>
<td>0.10</td>
</tr>
</tbody>
</table>

*Note:* All rates are annualized based on quarterly data. *Source:* Equity return is value-weighted total return from CRSP. All other data is from St. Louis Federal Reserve Bank Fred Database.

The model solution is in Table 7. The essential difference is that the calibration leads to a significantly higher value of $\xi_1$ and a lower value of $\gamma_1$, which suggests that shadow banks today offer a different value than shadow banks prior to the Great Depression.

Table 6: Parameter Values for Model With Deposit Insurance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>income std. dev.</td>
<td>$\epsilon = .85$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>time discount</td>
<td>0.9832</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>unit cost trad. bnk.</td>
<td>0.0177</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>unit cost shad. bnk.</td>
<td>0.0098</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>default cost shad. bnk.</td>
<td>4.8919</td>
</tr>
<tr>
<td>$\eta$</td>
<td>transaction costs curv.</td>
<td>2.0</td>
</tr>
<tr>
<td>$\psi$</td>
<td>transaction cost coef.</td>
<td>0.0057</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>liq. value shad. bnk.</td>
<td>0.5335</td>
</tr>
</tbody>
</table>

Fig. 6 shows that with deposit insurance, a rise in government debt leads to a rise in traditional bank assets and a fall in shadow bank assets. This is supported in the data in Table 1 and is explained by the model. Government debt does not directly offer financial services and hence does not compete with banks in the provision of financial services. The only effect of changing government debt is thus through directly
Table 7: Model Solution With Deposit Insurance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_b$</td>
<td>interest gov’t debt</td>
<td>0.022</td>
</tr>
<tr>
<td>$r_0$</td>
<td>interest trad. bnk.</td>
<td>-0.015</td>
</tr>
<tr>
<td>$r_1$</td>
<td>interest shad. bnk.</td>
<td>0.029</td>
</tr>
<tr>
<td>$q_b b'$</td>
<td>value gov’t debt</td>
<td>0.1164</td>
</tr>
<tr>
<td>$q_0 d_0'$</td>
<td>value trad. bnk.</td>
<td>0.4420</td>
</tr>
<tr>
<td>$q_1 d_1'$</td>
<td>value shad. bnk.</td>
<td>0.1180</td>
</tr>
<tr>
<td>$q_b b'_h$</td>
<td>value gov’t debt holding pvt.</td>
<td>0.0</td>
</tr>
<tr>
<td>$\alpha'_0$</td>
<td>frac. equity holding trad. bnk.</td>
<td>0.75</td>
</tr>
<tr>
<td>$\alpha'_1$</td>
<td>frac. equity holding shad. bnk.</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Note: All rates are annualized.

changing the profitability of banks. The demand for government debt by banks, either by regulation of traditional banks or by optimal risk management by shadow banks, leads to a liquidity premium in government debt that lowers the return on government debt. An expansion of government debt leads to a fall in its liquidity premium and thus a rise in its rate of return. Both types of banks benefit from this higher rate of return, but traditional banks benefit more so because they are required to hold more government debt than what shadow banks choose to hold, hence a rise in the return on government debt raises the relative profitability of traditional banks over shadow banks and thereby the size of traditional banks over shadow banks.

An important difference in the behavior of this model also regards the flight to safety in response to a rise in uncertainty. With deposit insurance, traditional bank assets are entirely safe assets, whereas shadow bank assets remain a risky form of liquidity. Hence, a rise in uncertainty leads to a rise in traditional bank assets and a fall in shadow bank assets, which is shown in Fig. 7. As shown in Fig. 3, this feature is also documented during the financial panic of 2007/08.

6 Welfare and Optimal Debt Policy

An optimal government policy is an amount of government debt outstanding to maximize consumption each period. From eq. (3) this amounts to a policy that minimizes the sum of transaction costs and banking costs. From Proposition 6(a) any debt level that satisfies

$$\beta b' \geq \sum (1 - \theta_j) \ell_j$$

(58)
will lead to the term $\sum \xi_j (1 - x_j) d_j$ achieving its minimum value of zero. Also according to Proposition 6(a), as long as debt exceeds the bound in (58) it follows that $q_i d'_i = \ell_i$. Since $(\ell_0, \ell_1)$ is also the solution to

$$\min_{\ell_0, \ell_1} \frac{\psi}{\eta - 1} \left( \sum \gamma_j \ell'_j \right)^{\frac{1}{\eta}} + \sum \ell_j \theta_j,$$

it follows that any value of government debt that exceeds the bound in (58) achieves the maximum welfare for the model without deposit insurance. Since there is no default in this equilibrium, along with $\alpha'_0 = \theta_0$ this is also the optimal government policy with deposit insurance.

Optimal government policy is thus one with sufficient debt so that interest rates do not reflect a liquidity premium and all types of banks hold sufficient amount of government debt to be free of default risk. Both shadow and traditional banks would thus be free of default risk and would compete solely on value ($\gamma$) and cost ($\theta$) considerations. Since both offer a different value/cost bundle and thus offer a value in such an equilibrium, neither is completely crowded out by the other.
7 Summary

This paper presents a model in which banks’ value stem from the services they offer. Safety is an important aspect of banking, but not because banks are unique in offering safety, but rather that safety is a key dimension of the services they offer. An efficient payment mechanism, which is the raison d’être of banking, requires an assurance that payments will be honored and hence requires that banks are safe institutions. Government debt does not per se produce financial services, and hence does not directly compete with banks. Rather, banks view government debt as a source of safety and their demand for these safe assets imparts a liquidity premium to government debt.

The view of banks as offering financial services that rely on safety opens the door to different types of financial institutions offering different bundles of financial services and safety. Traditional banks are perhaps more full-service financial institutions that put a high premium on safety, whereas shadow banks are perhaps more limited-service financial institutions that put less of a premium on safety. The relationship between the supply of government debt and the size of traditional and shadow banks is thus a complex one. Since both value safety, with a sufficient supply of government debt that eliminates the liquidity premium on government debt, both should hold sufficient government debt to be free of default risk, and thus the size of traditional versus shadow banks should be entirely determined by the value and cost of their respective financial services. With a very limited supply of government debt, the size of traditional versus
shadow banks will depend on how each one is able to accommodate a higher exposure to risk. The parameter values determined in this paper suggests that traditional banks are better able to accommodate risk, which also implies that a rise in government debt should lead to a fall in the relative size of traditional to shadow banks. However, in a world with deposit insurance, the size of traditional banking depends more on the implicit tax of being required to hold low-interest government debt, so a rise in government debt and consequent rise in yield on government debt leads to a rise in traditional banking relative to shadow banking.

An important implication of this paper is that a rise in government debt that eliminates the liquidity premium on government debt will encourage all financial institutions, in particular traditional and shadow banks, to hold more government debt and thereby be less subject to default risk. With the absence of a liquidity premium that would otherwise hamper the development of either traditional or shadow banks, they would have to compete based on their fundamental value/cost bundles. It should not be expected or even desirable that shadow banking vanish due to a rise in the supply of government debt, as shadow banking may offer a value/cost bundle of services distinct from traditional banks. A rise in the supply of government debt that eliminated the liquidity premium would encourage all financial institutions to become safer, thereby achieving important welfare gains, whatever happens to the relative size of traditional and shadow banks.

8 Appendix

Proof of Lemma 1: Define $Z(a, e, h) = E_y[\max\{-e, a(\bar{y}/\bar{y} - 1) - h\}]$. Note that $Z(a, e, 0) \geq E[a(\bar{y}/\bar{y} - 1)] = 0$, $Z(a, e, a(\bar{y}/\bar{y} - 1)) \leq 0$ since all terms inside the expectations operator are non-positive, and $Z(a, e, h)$ is a strictly-decreasing function of $h$ in the range $0 \leq h \leq a(\bar{y}/\bar{y} - 1)$. There thus exists a unique solution $H(a, e) \geq 0$ such that $Z(a, e, H(a, e)) = 0$. $Z(a, e, h)$ is a continuous function of $a$, $e$ and $h$, so $H(a, e)$ is a continuous function of $a$ and $e$. For $a \leq e$, note that $E_y[\max\{-e, a(\bar{y}/\bar{y} - 1)\}] = E[a(\bar{y}/\bar{y} - 1)] = 0$ hence $H(a, e) = 0$ for $a \leq e$. The monotonicity of $H$ with respect to $e$ follows from the monotonicity of $Z$ with respect to $e$ and $h$. Q.E.D.

Proof of Lemma 2: The solution $h_i(a)$ can be written as

$$0 = -\theta_i F(\bar{y} \Omega_i(a), y) + \int_{\bar{y} \Omega_i(a)}^{\infty} \left( a \left( \frac{y'}{\bar{y}} - 1 \right) - h_i(a) \right) f(y', y) dy'.$$
The results follow from repeated use of the Fundamental Theorem of Calculus. Note that \( h_i'(\theta_i) = 0 \) is well defined, as the left and right limits agree. Q.E.D.

**Proof of Lemma 3:** Follows from the concavity of the left side of inequality (18) with respect to \( \alpha'_i \). Q.E.D.

**Proof of Theorem 4:** Define the function \( T(q_b) \) as

\[
T(q_b) = q_b b' - \Sigma(1 - \alpha'_j)q_j d'_j,
\]

where the dependence of \( \alpha'_i \) on \( q_b \) is given by eq. (21) and the dependence of \( q_id'_i \) on \( q_b \) is given by eq. (17) along with the dependence of \( \alpha'_i \) on \( q_b \) just described. Note that \( T \) is a well-defined, continuous function. If \( T(\beta) \geq 0 \) then the solution is \( q_b = \beta \), \( q_b b' = T(\beta) \), \( \alpha_i = \theta_i \), \( q_i = \beta/(1 - \theta_i) \), and \( q_id'_i \) solves eq. (17) where the left side equals \( \theta_i \). Suppose \( T(\beta) < 0 \). Note that since \( \theta_i > 0 \), it follows that \( \Sigma(1 - \alpha'_j)q_j d'_j \) as a function of \( q_b \) is bounded above, hence there exists a \( q^*_b \) such that \( T(q_b) \geq 0 \) for any \( q_b \geq q^*_b \). There thus exists a value of \( q_b \) between \( \beta \) and \( q^*_b \) such that \( T(q_b) = 0 \). The associated value of \( (q, b', d', \alpha') \) defines an equilibrium. Q.E.D.

**Proof of Lemma 5:** These results are straightforward to verify. Q.E.D.

**Proof of Proposition 6:** (Region 1): Eq. (4) holds since \( q_b = \beta \). Eq. (6) can be shown to hold just by substitution of the solution into this equation. To show that eq. (15) holds, note that \( h_i(\theta_i) = 0 \). Eq. (23) holds since \( \alpha'_i = \theta_i \) and again since \( q_b = \beta \). Eq. (17) holds with equality for \( i = 0 \) by construction of \( D^*_0 \), and holds with inequality for \( i = 1 \) as \( \theta_1 \geq \theta_0 \gamma_1 \) holds by assumption.\(^8\)

(Region 2): To show that eq. (4) holds, note that \( W(\beta b') \leq W(D^*_0) = 1 \). Eq. (6) can again be shown to hold just by substitution of the solution into this equation. Through straightforward substitution, eq. (15) can be shown to hold for \( i = 0 \) and for \( i = 1 \) when \( \frac{1 - \pi}{\pi} \xi_1 \geq 1 - W(\beta b') \). When \( \frac{1 - \pi}{\pi} \xi_1 < 1 - W(\beta b') \) use the result that \( h_1(1) = (1 - \theta_1)\frac{1 - \pi}{\pi} \) to show that eq. (15) holds for \( i = 1 \). Eq. (17) for \( i = 0 \) holds by definition of \( W \). Eq. (17) for \( i = 1 \) and \( \frac{1 - \pi}{\pi} \xi_1 \geq 1 - W(\beta b') \) holds if

\[
1 - (1 - \theta_1) W(\beta b') \geq (1 - (1 - \theta_0) W(\beta b')) \gamma_1,
\]

which holds since \( 0 \leq W(\beta b') \leq W(D^*_0) = 1 \) and \( \gamma_1 < 1 \) and \( \theta_1 \geq \theta_0 \gamma_1 \) by assumption.

\(^8\)Equilibria also exist for \( \alpha'_0 < \theta_0 \), but all prices and the supply of liquidity are the same for these equilibria.
Eq. (17) for \( i = 1 \) and \( \frac{1-\pi}{\pi} \xi_1 < 1 - W(\beta b') \) holds if

\[
(1 - \theta_1)\xi_1 \frac{1 - \pi}{\pi} + \theta_1 \geq ((1 - \theta_0)(1 - W(\beta b')) + \theta_0)\gamma_1,
\]

which holds since in this region \( W(D_1^*) \leq W(\beta b') \) and this inequality holds for \( W(D_1^*) \) by construction of \( D_1^* \). Eq. (23) holds for \( i = 0 \) since \( \frac{1-\pi}{\pi} \xi_0 > 1 \) by assumption (and thus \( \frac{1-\pi}{\pi} \xi_0 > 1 - W(\beta b') \)). Eq. (23) holds for \( i = 1 \) by construction, given that \( \frac{\beta}{q_0} = W(\beta b') \).

**Region 3:** Eq. (4) holds since \( W(D_1^*) \leq 1 \). Eq. (6) and (15) can be shown to hold by substitution of the solution into these equations. Eq. (17) for \( i = 0 \) holds by definition of \( W \) and eq. (17) for \( i = 1 \) holds by construction of \( D_1^* \). Eq. (23) holds for \( i = 0 \) for the same reason as in Region 2. Eq. (23) holds for \( i = 1 \) if \( \frac{1-\pi}{\pi} \xi_1 \leq 1 - W(D_1^*) \). From the definition of \( D_1^* \), this inequality holds if

\[
\frac{1-\pi}{\pi} \xi_1 (1 - \gamma_1) + \left( 1 - \frac{1-\pi}{\pi} \xi_1 \right) (\theta_1 - \theta_0 \gamma_1) \geq 0,
\]

which holds since \( 1 - \gamma_1 > 0, \theta_1 - \theta_0 \gamma_1 > 0 \), and \( 1 - \frac{1-\pi}{\pi} \xi_1 > 0 \) by assumption. **Q.E.D.**

**Proof of Proposition 7:** These results are straightforward to verify. **Q.E.D.**

**Proof of Proposition 8:** The proof is similar to that of Theorem 4. **Q.E.D.**

**Proof of Proposition 9:** These results are straightforward to verify. **Q.E.D.**
9 References


