Nonlinear Dynamics in Financial Markets: Evidence and Implications

by

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Abstract

Daily asset returns exhibit two key statistical properties. Returns are not autocorrelated. But the absolute value of returns are strongly autocorrelated. Nonlinear processes can generate this type of behavior, while linear processes cannot. This paper investigates two types of nonlinear processes. Additively nonlinear processes are consistent with the view that expected returns are time varying. While much effort has been applied to modeling expected returns, there has been little evidence to support the view that time varying expected returns can account for the strong nonlinearity in the observed returns data. Multiplicatively nonlinear models are consistent with the view that expected volatilities are time varying. Evidence from price changes as well as options implied volatilities show that volatility is time varying and mean reverting. In fact, multiplicatively nonlinear models have been able to explain a great deal of the nonlinearity in asset returns. Thus, it is possible to forecast future volatility, even though it is difficult to forecast the direction of price changes. This has important implications for short term financial risk management.

Daily price changes for a large number of assets exhibit two key statistical features. There is very little autocorrelation in price changes, but there is strong autocorrelation in the absolute value of price changes. Evidence is provided in Table 1 (1a through 1h). Table 1a uses stock index futures prices for the S&P, Nikkei, DAX, FTSE, and CAC.

Notationally, let P_t be the price of an asset at date t. Define

$$x_t = \ln[P_t/P_{t-1}]$$

as the continuous rate of change between the price at dates t-1 and t. The top panel of Table 1a provides the first ten autocorrelation coefficients of x_t and the Box-Pierce test for all these coefficients to be zero. It is clear that the autocorrelation coefficients of x_t are not different from zero. This is in agreement with the evidence in the literature. What is striking, however, is that the autocorrelation coefficients of $|x_t|$ are much larger in the bottom panel of Table 1a. Most of the first order autocorrelation coefficients are larger than 0.10, and quite a few are larger than 0.20. These magnitudes are substantial, indicating that the log price differences are definitely not random. Specifically, a time series is 'random' if each number is not predictable based on preceding numbers and all numbers have the same statistical distribution. The precise statistical term for a random time series is that its numbers are 'independent and identically distribution.'

The lack of autocorrelation in x_t and the large autocorrelation in $|x_t|$ are characteristic of high frequency (e.g. weekly, daily, hourly) data in asset markets. They show up strongly in daily price changes of government bond futures (Table 1b), short interest rate futures (Table 1c), currency futures (Table 1d), and commodity futures (Table 1e through 1h). This finding is not sensitive to the sampling period. This paper focuses on the characterization of these two features, and ignores other well known characteristics of high frequency data, such as the leptokurtic distribution of returns.

Explanations of Non-Random Behavior

There are several competing explanations for the non-random behavior of asset price changes. In the first place, structural changes or regime changes in the economy can affect prices behavior in asset markets. A frequently cited example is the increase in the variance of interest rates between 1979 and 1981, which has been attributed to the change in the Federal Reserve policy from targeting interest rates to money supplies. This nonstationarity hypothesis is most persuasive for data spanning long periods of time (e.g. annual data over many decades), since the underlying structure of the economy is unlikely to remain constant. But it is not an appealing explanation of the non-random behavior of asset returns in Table 1. The reason is that non-random behavior of asset returns is most pronounced in high frequency (i.e. weekly, daily, or hourly frequencies) data. As the sampling interval is lengthened to a monthly or quarterly frequency, the autocorrelation of the absolute value of price changes declines. This is not consistent with structural change hypothesis. Furthermore, if the structure of the economy indeed changes at the rate of daily frequencies, it would be impossible to study the economy statistically, since most economic time series are available only at monthly or quarterly frequencies.

An alternative to the nonstationarity hypothesis is that the non-random behavior is an intrinsic part of the dynamics of asset prices. Most of the past literature on the empirical behavior of price changes have focused on linear time series models, such as autoregressive-moving average (ARMA) models. The evidence in Table 1 show that x_t is not linear. No linear model can produce x_t which is not autocorrelated but $|x_t|$ is autocorrelated. This leads naturally to nonlinear time series models for x_t.

Theoretically, there is good reason for believing that x_t is nonlinear. Modern finance theory suggests that the current price of an asset, P_t, is the expected discounted value of future payoffs:

$$P_t = E[\, m_{t,t+1}\,(D_{t+1} + P_{t+1}) \mid I_t].$$

In this fundamental pricing equation, D_{t+1} is the net cash flow generated by holding the asset between period t and t+1, and $m_{t,t+1}$ is a discount factor. The expectation E[] is taken conditional on the information available at period t, I_t. Any asset pricing theory must specify the information set I_t and the discount factor $m_{t,t+1}$. In a typical asset pricing model, such as the consumption capital asset pricing model, $m_{t,t+1}$ is the ratio of the marginal utility of consumption between time t+1 and time t. The asset payoff $(D_{t+1}+P_{t+1})$ affects the amount of consumption at time t+1, and therefore the discount factor $m_{t,t+1}$, so that the pricing equation is a nonlinear stochastic difference equation, for which there is no general solution available. One thing, however, is clear. It is very likely that the asset price P_t which solves the pricing equation will be a nonlinear rather than a linear stochastic process. Thus the logarithm of price changes ($x_t=\ln[P_t/P_{t-1}]$) is also likely to be a nonlinear stochastic process.

Empirically, the world of nonlinear processes is vastly richer than the world of linear processes. Nonlinear processes can generate much more interesting dynamics than linear processes. Specifically, quite a few nonlinear processes can generate x_t which has no autocorrelation but $|x_t|$ has strong autocorrelation. But a strict discipline must be followed when fitting nonlinear models to data. There are so many nonlinear processes that it is very easy to overfit the data. This paper will adhere to the principle of parsimony --- simple models are preferred to complex models. Two classes of simple nonlinear stochastic models will be examined: additive and multiplicative models.

Additively Nonlinear Models

Suppose x_t is generated by the following model:

$$x_t = F(I_{t-1}) + e_t,$$

where e_t is random, with mean zero and finite variance, and I_{t-1} is the information available at time t-1. For the purposes of this paper, I_{t-1} consists of the past history of x_{t-1} and e_{t-1}. The function F() is the conditional mean function, which gives expected return of the asset between time t-1 and t using the information in I_{t-1}. F() cannot be linear. If it were, x_t would exhibit serial correlation. Such a model is called an additively nonlinear model, because the error term e_t is added to the nonlinear function F(). If price changes are generated by such a model, it means that most of the nonlinear dynamics are coming from changes in expected returns.

In the time series literature, there are many examples of additively nonlinear models: the nonlinear moving average model of Robinson (1977), the bilinear model of Granger and Anderson (1978), and the threshold autoregressive model of Tong and Lim (1980). In addition, deterministic chaos (e.g. the pseudo-random number generators used in most computer simulations) is a special case in which the noise term e_t vanishes.

If F() is known, the direction of price change is forecastable. This has led to much excitement in the finance community. Since F() is unknown, nonparametric methods (e.g. kernels, neural nets, nearest neighbors, and series expansions) have been used to estimate F(). The results thus far have proved disappointing. White (1988), Diebold and Nason (1990), Hsieh (1991, 1993a and 1993b), among others, have used various nonparametric methods to estimate the conditional mean function for stocks and foreign currencies. The out-of-sample forecasts perform uniformly worse than the naive model that prices follow a random walk.

The failure to find a statistically significant conditional mean function implies that the size and variation of the expected return of holding assets over one trading day is quite small relative to the magnitude of the observed price changes. This is not a surprising result. Over long periods of time, stock returns have averaged on the order of 10% per annum with a volatility of 20% per annum, which translates to an average return of 0.04% and a volatility of 1.26% per trading day. The average returns of the bonds, currencies, and commodities, are typically even small. The strong evidence of non-random behavior of asset returns is unlikely to be caused by the variation of expected returns.

Multiplicatively Nonlinear Models

Suppose x_t is obtained by the following model:

$$x_t = G(I_{t-1})\ e_t,$$

where e_t is random, with mean zero and finite variance σ^2. The function G() is known as the conditional variance function, and must be positive. The quantity $G(I_{t-1})\sigma^2$ can be interpreted as the expected variance of x_t based on information at time t-1. The expected value of x_t is zero. Such a model is called a multiplicatively nonlinear model, because the error term e_t is multiplied to the function G(). If price changes are generated by such a model, it means that most of the nonlinear dynamics are coming from changes in expected variance.

Examples of multiplicatively nonlinear models are the autoregressive conditional heteroskedasticity (ARCH) model of Engle (1982) and its generalized version (GARCH) of Bollerslev (1986). The ARCH-type models have gained great popularity in the empirical finance literature, as measured by the number of articles surveyed by Bollerslev, Chow, and Kroner (1992). This is due to the fact that ARCH-type models have been able to fit the nonlinearity in asset returns, in the following sense. After an ARCH-type model is estimated, it provides an estimate of the daily volatility, which can be used to standardize the return series. The resulting series typical exhibit little remaining nonlinearity.

In the past, researchers have discovered evidence of volatility changes at annual and monthly frequencies. See, for example, Officer (1973) and French, Schwert, and Stambaugh (1987). The evidence now indicate that volatility changes occur even at daily frequencies. The time-varying nature of volatility is corroborated by the implied volatilities of options, which can fluctuate quite a bit from day to day.

The Dynamics of Volatility

In order to assess the implication of volatility changes on the application of finance theory, it is important to document the time series properties of volatility. As early as Mandelbrot (1963), researchers have known that asset returns exhibit volatility clustering. If the volatility is high one period, it tends to remain high the next period. If volatility is low one day, it tends to remain low the next day.

A second key feature of volatility is that it is strongly mean reverting. This is confirmed in Hsieh (1994), who uses both price based and option based information to measure volatility, and finds that it is mean reverting. This observation is further confirmed by the behavior of the term structure of implied volatilities. Figures 1 and 2 are the implied volatilities of over-the-counter 1- and 6- month options on the US Dollar/German Mark exchange rate from 1985 to 1992. The 1-month implied volatility is much more volatile than the 6-month implied volatility, and both revert to a long run mean around 12%. Moreover, these figures indicate that volatility tends to revert fairly rapidly back to its long run average.

This last observation directly contradicts the results of most GARCH models, which have found very high persistence in volatility, to the point that volatility appears to follow a random walk process. Whether or not volatility follows a random walk or stationary process is not particularly relevant for one-day ahead forecasts of volatility, but it is critically important for multi-day ahead forecasts. This difference is dramatic in the application to follow.

To capture the mean-reverting behavior of volatility, Hsieh (1993b) proposed the autoregressive volatility (AV) model:

$$x_t = \sigma_t e_t.$$

$$\log \sigma_t = \alpha + \Sigma \beta_i \log \sigma_{t-i} + \nu_t.$$

Here, (e_t, ν_t) is iid with zero mean (0, 0); e_t has finite variance σ, ν_t has finite variance σ, and e_t and ν_t has correlation ρ.

There are two important differences between the AV model and the popular

GARCH model. In the first place, the AV model has found much less volatility persistence than the GARCH model. In the second place, the GARCH model have been estimated using the maximum likelihood method, which requires a specific distributional assumption on the error terms e_t. The AV model does not require any distributional assumptions.

Applications to Financial Risk Management

Once the conditional variance function G() has been estimated, whether it be the popular GARCH model or the AV model, the conditional distribution of future values of x_t can be obtained using simulation methods. The conditional distribution can be more informative than the unconditional distribution, which uses the histogram of x_t, thus pretending that returns are random. As volatility is strongly mean reverting, the conditional distribution should converge to the unconditional distribution over time. Thus, the conditional distribution is most useful for assessing the distribution of short term price changes, probably up to a few weeks.

The conditional distribution of returns can provide useful information on the market risk of asset and liability positions, as demonstrated in Hsieh (1993b). Table 2 provides estimates of the capital requirements for 90% coverage probabilities of one German Mark (DM) futures contract traded on the Chicago International Money Market over different holding horizons, from 1 to 180 trading days. Suppose the DM futures is trading at $0.40 per DM. One futures contract is for the delivery of 125,000 DM, or a total value of $50,000. Based on the AV model, a trader holding a long position for one day needs 0.72% of the value of the contract, or $360, to cover 90% of all potential losses the next day. The capital requirement increases to 11.38%, or $5,690, if the trader wants to cover 90% of all possible losses in the next 180 trading days.

This table compares the difference in capital requirements using the AV model, the unconditional distribution (which pretends that asset returns are iid), and a special variant of the GARCH model, called the exponential GARCH

(EGARCH) model. When the holding period is short, the three models give reasonably similar results. However, as the holding period increases, the AV and the unconditional distribution model yield similar results, while the EGARCH model gives dramatically different results. This is due to the strong volatility persistence in the EGARCH model.

It is straight forward to extend this risk management analysis from a single asset to a portfolio of assets, provided that one is willing to make some assumptions regarding the correlation between asset returns. Such a portfolio risk management system is discussed in Hsieh (1993c) using the unconditional distribution for monthly returns. A similar system using the conditional distribution from the AV model for daily returns can be quite easily implemented. This portfolio risk assessment system can also serve as an asset allocation model for short holding periods.

Summary and Conclusion

This paper documents two interesting observations in asset markets, that daily price changes are not autocorrelated, yet they are non-random. While it is possible that expected returns are changing over time, they are not able to explain the strong evidence of non-random behavior. A much more successful explanation is that volatility is time varying. Volatility tends to cluster, but it is strongly mean reverting. Conditional variance models, such as ARCH-type models, can explain a great deal of the non-random behavior in asset returns. While GARCH and EGARCH models have a tendency to put too much persistence in volatility, the autoregressive volatility (AV) model is much better able to capture mean reversion in volatility. These conditional variance models provide a way to simulate the future distribution of asset returns, and yield some interesting applications to pricing of options and financial risk management.

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Table 1a
Autocorrelation of x_t and $|x_t|$
Stock Index Futures

	S&P	Nikkei	DAX	FTSE	CAC
Autocorrelation of x_t:					
r(1)	-0.014	-0.010	-0.049	0.018	-0.031
r(2)	-0.155	-0.048	-0.050	-0.028	0.017
r(3)	-0.026	-0.020	-0.049	0.026	-0.049
r(4)	-0.032	0.050	0.038	0.050	0.030
r(5)	0.070	-0.030	0.027	-0.002	-0.020
r(6)	0.006	-0.009	-0.027	-0.007	-0.017
r(7)	0.012	0.005	-0.013	0.015	-0.012
r(8)	-0.055	-0.002	-0.081	0.009	-0.034
r(9)	-0.026	0.069	0.020	0.022	0.043
r(10)	0.002	0.043	-0.026	0.023	0.020
$Q_x(10)$	3.97	12.46	13.23	4.20	9.30
Autocorrelation of $|x_t|$:					
r(1)	0.272	0.223	0.139	0.268	0.130
r(2)	0.311	0.234	0.123	0.256	0.107
r(3)	0.252	0.214	0.117	0.200	0.161
r(4)	0.147	0.226	0.079	0.143	0.098
r(5)	0.206	0.244	0.062	0.166	0.181
r(6)	0.167	0.189	0.086	0.158	0.098
r(7)	0.119	0.230	0.069	0.148	0.060
r(8)	0.146	0.165	0.090	0.144	0.139
r(9)	0.169	0.231	0.053	0.169	0.069
r(10)	0.067	0.165	0.059	0.121	0.145
$Q_{|x|}(10)$	1041.97	878.91	97.29	886.69	259.84

Notes:
r(k) is the autocorrelation coefficient at lag k.
$Q_x(10)$ is the Box-Pierce statistic for n lags, adjusted for heteroskedasticity.
$Q_{|x|}(10)$ is the Box-Pierce statistic for n lags, without adjustment for heteroskedasticity.

S&P: Standard & Poors 500 stock index futures, Mar 1985-Mar 1995.
Nikkei: Nikkei stock index futures, Dec 1987-Mar 1995.
DAX: DAX stock index futures, Nov 1990-Mar 1995.
FTSE: FTSE stock index futures, Mar 1985-Mar 1995.
CAC: CAC stock index futures, Nov 1988-Mar 1995.

Table 1b
Autocorrelation of x_t and $|x_t|$
Government Bond Futures

	USF	JGB	Bund	Gilt	Notional
Autocorrelation of x_t:					
r(1)	0.024	0.004	0.014	-0.018	-0.019
r(2)	0.010	-0.001	-0.001	-0.008	0.027
r(3)	-0.030	-0.021	0.003	-0.004	-0.015
r(4)	-0.020	0.007	0.048	0.008	-0.016
r(5)	-0.020	0.056	-0.003	0.044	0.018
r(6)	-0.001	-0.007	-0.053	-0.020	-0.080
r(7)	0.023	0.033	-0.011	-0.011	0.014
r(8)	0.002	0.001	0.032	0.029	0.018
r(9)	-0.025	0.022	-0.022	0.049	0.025
r(10)	0.026	-0.018	-0.032	-0.005	-0.031
$Q_x(10)$	7.46	6.43	6.87	11.10	4.67
Autocorrelation of $|x_t|$:					
r(1)	0.076	0.180	0.189	0.126	0.295
r(2)	0.104	0.159	0.230	0.114	0.298
r(3)	0.113	0.155	0.187	0.146	0.363
r(4)	0.139	0.157	0.190	0.154	0.283
r(5)	0.167	0.171	0.207	0.122	0.261
r(6)	0.137	0.134	0.197	0.137	0.285
r(7)	0.096	0.152	0.177	0.138	0.267
r(8)	0.115	0.146	0.214	0.132	0.246
r(9)	0.117	0.131	0.191	0.113	0.174
r(10)	0.094	0.101	0.187	0.141	0.214
$Q_{|x|}(10)$	369.95	428.78	664.88	466.24	1776.47

Notes:
r(k) is the autocorrelation coefficient at lag k.
$Q_x(10)$ is the Box-Pierce statistic for n lags, adjusted for heteroskedasticity.
$Q_{|x|}(10)$ is the Box-Pierce statistic for n lags, without adjustment for heteroskedasticity.

USF: US treasury bond futures, Mar 1985-Mar 1995.
JGB: Japanese government bond futures, Jan 1988-Mar 1995.
Bund: German government bond futures, Oct 1988-Mar 1995.
Gilt: UK government bond futures, Mar 1985-Mar 1995.
Notional: French government bond futures, Feb 1986-Mar 1995.

Table 1c
Autocorrelation of x_t and $|x_t|$
Short Interest Rate Futures

	LIBOR	SYN	LIP	LID	PIBOR
Autocorrelation of x_t:					
r(1)	0.066	0.036	0.018	0.004	0.074
r(2)	0.022	0.023	-0.040	0.058	0.009
r(3)	-0.011	0.004	0.055	-0.039	-0.044
r(4)	-0.009	0.066	-0.024	0.034	0.027
r(5)	-0.014	0.044	0.046	-0.015	-0.011
r(6)	0.002	0.013	-0.015	0.010	-0.030
r(7)	0.053	0.044	0.010	-0.023	0.002
r(8)	-0.007	-0.010	0.014	0.009	0.007
r(9)	-0.042	-0.024	0.005	0.045	-0.009
r(10)	-0.007	-0.026	0.021	0.006	0.053
$Q_x(10)$	11.42	11.87	7.38	7.59	5.89
Autocorrelation of $|x_t|$:					
r(1)	0.139	0.142	0.244	0.143	0.357
r(2)	0.207	0.100	0.183	0.240	0.303
r(3)	0.191	0.159	0.171	0.135	0.283
r(4)	0.173	0.084	0.169	0.119	0.298
r(5)	0.216	0.091	0.119	0.147	0.286
r(6)	0.116	0.128	0.111	0.101	0.291
r(7)	0.133	0.143	0.100	0.096	0.278
r(8)	0.162	0.099	0.099	0.150	0.267
r(9)	0.157	0.105	0.088	0.061	0.245
r(10)	0.128	0.118	0.136	0.103	0.261
$Q_{|x|}(10)$	723.52	202.52	590.85	294.26	1429.08

Notes:
r(k) is the autocorrelation coefficient at lag k.
$Q_x(10)$ is the Box-Pierce statistic for n lags, adjusted for heteroskedasticity.
$Q_{|x|}(10)$ is the Box-Pierce statistic for n lags, without adjustment for heteroskedasticity.

LIBOR: 90-day Eurodollar futures, Mar 1985-Mar 1995.
SYN: 90-day Euroyen futures, Nov 1989-Mar 1995.
LIP: 90-day Euromark futures, Mar 1985-Mar 1995.
LID: 90-day Eurosterling futures, Apr 1989-Mar 1995.
PIBOR: 90-day Euro-French Franc futures, Sep 1988-Mar 1995.

Table 1d
Autocorrelation of x_t and $|x_t|$
Currency Futures

	BPF	CDF	DMF	JPF	SFF
Autocorrelation of x_t:					
r(1)	0.014	0.043	-0.002	-0.013	-0.002
r(2)	0.012	-0.034	-0.006	-0.008	-0.014
r(3)	-0.011	-0.030	0.005	0.003	0.000
r(4)	0.006	0.001	0.000	0.004	0.000
r(5)	0.011	0.013	-0.006	0.011	-0.007
r(6)	-0.003	-0.015	-0.004	-0.020	-0.014
r(7)	-0.031	0.021	-0.020	-0.012	-0.015
r(8)	0.028	-0.021	0.034	0.028	0.040
r(9)	-0.001	-0.018	0.001	0.014	-0.020
r(10)	0.005	0.000	0.013	0.047	0.030
$Q_x(10)$	4.83	10.93	3.91	9.31	7.80
Autocorrelation of $|x_t|$:					
r(1)	0.081	0.090	0.052	0.116	0.042
r(2)	0.094	0.076	0.038	0.058	0.009
r(3)	0.100	0.091	0.064	0.093	0.060
r(4)	0.114	0.106	0.060	0.036	0.051
r(5)	0.084	0.136	0.050	0.080	0.027
r(6)	0.128	0.086	0.115	0.106	0.104
r(7)	0.064	0.078	0.064	0.045	0.080
r(8)	0.058	0.080	0.058	0.052	0.048
r(9)	0.090	0.103	0.040	0.066	0.007
r(10)	0.111	0.073	0.106	0.025	0.087
$Q_{|x|}(10)$	237.57	231.55	125.94	142.85	94.47

Notes:
r(k) is the autocorrelation coefficient at lag k.
$Q_x(10)$ is the Box-Pierce statistic for n lags, adjusted for heteroskedasticity.
$Q_{|x|}(10)$ is the Box-Pierce statistic for n lags, without adjustment for heteroskedasticity.

BPF: British Pound futures, Mar 1985-Mar 1995.
CDF: Canadian Dollar futures, Mar 1985-Mar 1995.
DMF: Deutschemark futures, Mar 1985-Mar 1995.
JYF: Japanese Yen futures, Mar 1985-Mar 1995.
SFF: Swiss Franc futures, Mar 1985-Mar 1995.

Table 1e
Autocorrelation of x_t and $|x_t|$
Commodity Futures

	CR	CC	JO	KC	PB
Autocorrelation of x_t:					
r(1)	-0.071	0.000	-0.007	0.012	0.053
r(2)	-0.020	-0.046	-0.027	0.014	0.031
r(3)	-0.048	-0.005	0.036	0.018	0.001
r(4)	0.043	-0.015	0.051	0.008	0.015
r(5)	0.002	0.014	0.021	-0.030	-0.005
r(6)	-0.030	-0.009	0.004	-0.037	0.011
r(7)	0.007	-0.014	0.000	0.000	0.006
r(8)	-0.040	0.005	-0.001	0.040	0.026
r(9)	0.033	-0.014	0.043	0.010	-0.007
r(10)	-0.046	0.017	0.000	0.068	0.031
$Q_x(10)$	17.37	7.47	11.26	8.51	12.81
Autocorrelation of $|x_t|$:					
r(1)	0.164	0.053	0.150	0.191	0.089
r(2)	0.149	0.030	0.141	0.180	0.050
r(3)	0.146	0.068	0.139	0.174	0.049
r(4)	0.172	0.074	0.128	0.150	0.079
r(5)	0.200	0.074	0.094	0.170	0.067
r(6)	0.135	0.069	0.089	0.155	0.093
r(7)	0.129	0.077	0.111	0.173	0.082
r(8)	0.161	0.072	0.110	0.139	0.063
r(9)	0.137	0.066	0.128	0.149	0.069
r(10)	0.146	0.109	0.072	0.216	0.043
$Q_{|x|}(10)$	555.80	135.95	373.24	773.86	130.58

Notes:
r(k) is the autocorrelation coefficient at lag k.
$Q_x(10)$ is the Box-Pierce statistic for n lags, adjusted for heteroskedasticity.
$Q_{|x|}(10)$ is the Box-Pierce statistic for n lags, without adjustment for heteroskedasticity.

CR: CRB index futures, Jun 1986-Mar 1995.
CC: Cocoa futures, Mar 1985-Mar 1995.
JO: Orange juice futures, Mar 1985-Mar 1995.
KC: Coffee futures, Mar 1985-Mar 1995.
PB: Pork belly futures, Mar 1985-Mar 1995.

Table 1f
Autocorrelation of x_t and $|x_t|$
Commodity Futures

	SB	SY	KW	CL	HO	NG
Autocorrelation of x_t:						
r(1)	-0.078	0.019	0.047	0.004	0.044	0.050
r(2)	-0.050	-0.022	-0.107	0.005	0.001	-0.028
r(3)	-0.005	-0.012	-0.005	-0.094	-0.087	0.057
r(4)	0.016	0.013	0.022	0.033	-0.018	0.028
r(5)	0.028	-0.023	-0.015	-0.010	-0.030	-0.002
r(6)	-0.026	-0.042	-0.001	-0.008	-0.038	-0.060
r(7)	0.040	0.036	0.005	0.026	0.009	-0.007
r(8)	0.026	-0.040	0.036	-0.064	-0.068	0.016
r(9)	-0.030	0.038	0.056	0.015	0.017	0.041
r(10)	-0.034	-0.024	-0.013	-0.007	0.027	0.009
$Q_x(10)$	11.50	10.02	17.68	8.13	11.77	12.84
Autocorrelation of $\|x_t\|$:						
r(1)	0.233	0.208	0.243	0.283	0.234	0.098
r(2)	0.197	0.189	0.192	0.266	0.240	0.065
r(3)	0.189	0.203	0.149	0.331	0.268	0.121
r(4)	0.203	0.225	0.129	0.214	0.170	0.094
r(5)	0.150	0.246	0.205	0.260	0.196	0.086
r(6)	0.171	0.205	0.190	0.215	0.156	0.078
r(7)	0.155	0.222	0.166	0.216	0.169	0.118
r(8)	0.134	0.184	0.199	0.273	0.195	0.038
r(9)	0.130	0.159	0.188	0.212	0.157	0.079
r(10)	0.085	0.167	0.156	0.249	0.190	0.071
$Q_{\|x\|}(10)$	759.81	1082.74	896.71	1713.07	1063.78	91.98

Notes:
r(k) is the autocorrelation coefficient at lag k.
$Q_x(10)$ is the Box-Pierce statistic for n lags, adjusted for heteroskedasticity.
$Q_{|x|}(10)$ is the Box-Pierce statistic for n lags, without adjustment for heteroskedasticity.

SB: World sugar futures, Mar 1985-Mar 1995.
SY: Soybean futures, Mar 1985-Mar 1995.
KW: Wheat futures, Mar 1985-Mar 1995.
CL: Crude oil futures, Mar 1985-Mar 1995.
HL: Heating oil futures, Mar 1985-Mar 1995.
NG: Natural gas futures, Oct 1990-Mar 1995.

Table 1g
Autocorrelation of x_t and $|x_t|$
Commodity Futures

	HG	PL	GC	SI	ALU
Autocorrelation of x_t:					
r(1)	-0.038	-0.005	-0.065	-0.068	-0.130
r(2)	0.008	-0.011	-0.022	-0.004	-0.101
r(3)	-0.029	-0.024	-0.021	-0.013	0.021
r(4)	-0.019	-0.007	0.026	0.008	-0.002
r(5)	-0.004	-0.010	0.021	0.003	0.009
r(6)	0.027	-0.010	-0.044	-0.047	-0.008
r(7)	-0.011	-0.021	0.007	-0.005	0.029
r(8)	-0.030	0.011	0.004	-0.007	-0.003
r(9)	0.062	-0.015	0.027	0.006	0.010
r(10)	0.034	0.052	0.012	-0.026	-0.031
$Q_x(10)$	10.23	7.61	13.73	5.28	21.89
Autocorrelation of $|x_t|$:					
r(1)	0.123	0.156	0.166	0.197	0.195
r(2)	0.075	0.123	0.102	0.161	0.196
r(3)	0.070	0.120	0.123	0.179	0.111
r(4)	0.077	0.111	0.119	0.137	0.095
r(5)	0.091	0.144	0.154	0.137	0.069
r(6)	0.134	0.107	0.117	0.136	0.076
r(7)	0.088	0.156	0.113	0.108	0.072
r(8)	0.087	0.153	0.097	0.040	0.084
r(9)	0.130	0.122	0.099	0.037	0.094
r(10)	0.064	0.146	0.127	0.129	0.141
$Q_{|x|}(10)$	132.03	480.18	404.65	486.40	394.32

Notes:
r(k) is the autocorrelation coefficient at lag k.
$Q_x(10)$ is the Box-Pierce statistic for n lags, adjusted for heteroskedasticity.
$Q_{|x|}(10)$ is the Box-Pierce statistic for n lags, without adjustment for heteroskedasticity.

HG: Copper futures, Nov 1989-Mar 1995.
PL: Platinum futures, Mar 1985-Mar 1995.
GC: Gold, London afternoon fixing, Mar 1985-Mar 1995.
SI: Silver, Handy Harmon, Mar 1985-Mar 1995.
ALU: Aluminum, New York, Mar 1985-Mar 1995.

Table 1h
Autocorrelation of x_t and $|x_t|$
Commodity Futures

	LEAD	NICKEL	TIN	ZINC
Autocorrelation of x_t:				
r(1)	-0.061	0.062	0.102	0.120
r(2)	-0.028	0.026	-0.045	-0.048
r(3)	-0.049	-0.029	-0.005	-0.098
r(4)	0.016	-0.016	-0.004	0.017
r(5)	0.048	0.007	0.027	0.020
r(6)	-0.001	0.023	0.047	0.023
r(7)	0.013	0.033	-0.009	-0.018
r(8)	0.029	0.005	0.035	0.023
r(9)	-0.004	0.016	0.021	0.076
r(10)	0.058	0.028	0.025	0.045
$Q_x(10)$	18.63	7.93	20.30	40.61
Autocorrelation of $|x_t|$:				
r(1)	0.189	0.295	0.183	0.173
r(2)	0.154	0.280	0.133	0.127
r(3)	0.136	0.215	0.167	0.105
r(4)	0.141	0.178	0.164	0.130
r(5)	0.125	0.198	0.173	0.119
r(6)	0.121	0.170	0.094	0.119
r(7)	0.162	0.179	0.139	0.120
r(8)	0.119	0.163	0.134	0.124
r(9)	0.094	0.169	0.138	0.121
r(10)	0.094	0.181	0.114	0.137
$Q_{|x|}(10)$	491.14	1138.82	413.43	275.59

Notes:
r(k) is the autocorrelation coefficient at lag k.
$Q_x(10)$ is the Box-Pierce statistic for n lags, adjusted for heteroskedasticity.
$Q_{|x|}(10)$ is the Box-Pierce statistic for n lags, without adjustment for heteroskedasticity.

LEAD: Lead, cash, Mar 1985-Mar 1995.
NICKEL: Nickel, cash, Mar 1985-Mar 1995.
TIN: Tin, cash, Mar 1985-Mar 1995.
ZINC: Zinc, cash, Mar 1985-Mar 1995.

Table 2
Capital Requirement for 90% Coverage Probability
As a Percentage of the Value of One DM Futures Contract

No. of Days	Long Position			Short Position		
	AV	Uncond	EGARCH	AV	Uncond	EGARCH
1	0.72	0.87	0.83	0.89	1.00	0.95
5	1.89	2.18	2.34	2.23	2.70	2.91
10	2.77	3.14	3.93	3.40	4.12	5.03
15	3.52	3.86	5.37	4.36	5.30	6.92
20	4.05	4.45	6.54	5.19	6.14	8.91
25	4.55	4.90	7.86	6.14	7.21	10.69
30	4.93	5.37	8.75	7.02	7.88	12.36
60	7.16	7.24	13.14	11.36	12.38	20.86
90	8.87	8.39	16.06	14.68	16.16	27.75
180	11.38	10.35	21.69	24.25	26.25	45.68

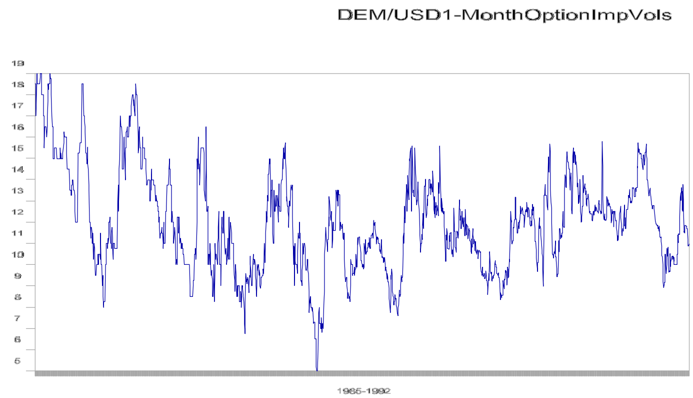

Figure 1

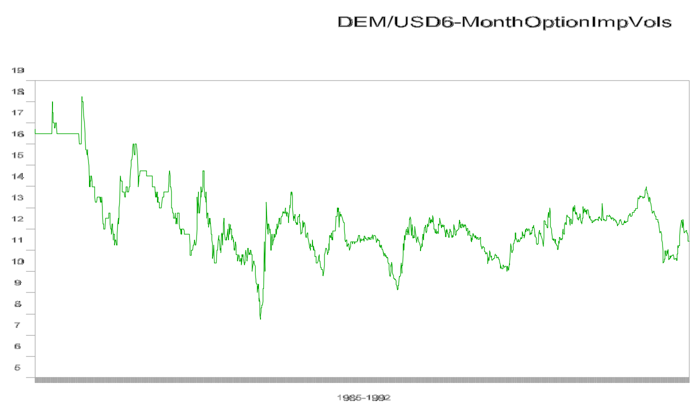

Figure 2