Is Mean-Variance Analysis Applicable to Hedge Funds?

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Abstract

This paper shows that the mean-variance analysis of hedge funds approximately preserves the ranking of preferences in standard utility functions. This extends the results of Levy and Markowitz (1979) and Hlawitschka (1994) for individual stocks and portfolios of stocks.

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1. Introduction

Mean-variance analysis is appropriate when returns are normally distributed or investors' preferences are quadratic. In actual applications, returns are typically not normally distributed and utilities are typically non-quadratic. Levy and Markowitz (1979) justify the practice of using mean-variance analysis by showing that mean-variance analysis can be regarded as a second order Taylor-series approximation of standard utility functions (such as the power utility and the exponential utility). The reliability of mean-variance analysis therefore depends jointly on the degree of non-normality of the returns data and the nature of the (non-quadratic) utility function. Levy and Markowitz (1979) show that the second order approximations are highly correlated to actual values of power and exponential utility functions over a wide range of parameter values for mutual funds. Hlawitschka (1994) extends the Levy and Markowitz result to show that the mean-variance ranking of funds is highly correlated to the ranking based on the true utility function, and that third or even higher order approximations do not necessarily improve the rank correlation.

This paper deals with the question: does the Levy and Markowitz (1979) and Hlawitschka (1994) results on mutual funds extend to hedge funds? In addition, we also pose the question for other criterion which do not depend on the parameter of the true utility function: the growth optimal criterion and the Sharpe ratio criterion. Hedge funds are unregistered, unregulated investment pools for high net worth individuals and institutions. Fung and Hsieh (1997) find that hedge fund returns are dramatically different from those of mutual funds and standard asset classes. This indicates that hedge funds can potentially add diversification. Thus, it is of interest to these investors whether mean-variance analysis is applicable to hedge funds.

2. Quadratic Approximation

Given a utility function U(R), defined over the gross return, R. The actual utility function and its quadratic approximation are calculated as follows. Suppose we have gross returns for T periods: R_1, ..., R_T. The expected value of the utility function U( ), E[U(R)], is estimated by:

$$(1/T) \sum U(R_t). \quad (1)$$

The second order Taylor series approximation of the utility around the population mean μ=E[R_t] is given by:

$$U(R_t) \approx U(\mu) + U'(\mu)\ (R_t-\mu) + \quad U''(\mu)\ (R_t-\mu)^2.$$

Taking expected value for both sides, we have:

$$E[U(R_t)] \approx U(\mu) + \quad U''(\mu)\ \sigma^2,$$

where σ^2=Var(R_t) is the population variance. As μ and σ^2 are unknown, we replace them with the sample mean and the sample variance:

$$R \quad = (1/T) \sum U(R_t),$$

$$s^2 = (1/T) \sum U(R_t-R)^2,$$

to arrive at the approximation:

$$U(R) + \quad U''(R)\ s^2. \quad (2)$$

To assess the quality of the quadratic approximation, we follow Levy and Markowitz (1979) and Hlawitschka (1994) and assume that investors have either power utility or exponential utility function over R, the gross 1-period return.

The power utility is given by:

$$U(R) = R^{(1-\gamma)}/(1-\gamma), \quad \gamma > 0,$$

where γ is the Arrow-Pratt coefficient of relative risk aversion. Note that when γ=1, U(R) = log(R).

The exponential utility is given by:

$$U(R) = -\ e^{-\gamma R}, \quad \gamma > 0,$$

where γ is the Arrow-Pratt coefficient of absolute risk aversion. In this paper, we confine γ between 0 and 30 for both cases.

The actual values and the second order approximation of these utility functions are applied to the hedge fund database in Fung and Hsieh (1997), which has 410 hedge funds with at least 3 years of monthly returns. In this paper, we select a subsample of 233 hedge funds having at least 5 years of monthly returns. [The results are even better if we use 3 years.]

We rank the 233 hedge funds according to the actual utility in (1) and the quadratic approximation in (2). Following Hlawitschka (1994), we use the correlation between these two rankings to measure the quality of the quadratic approximation. A high rank correlation means that the ranking based on the quadratic approximation is close to the truth. A low rank correlation means that the ranking based on the quadratic approximation is not good.

The resulting rank correlations are given in the fourth column in Table 1. For all values of γ between 0 and 30, the rank correlations between the quadratic approximation and the actual utility are uniformly higher than 0.95, which means that the ranking using the quadratic approximation is very good.

For comparison, we use 2,111 mutual funds in the Morningstar database with at least 5 years of monthly returns. The results are given in the third column in Table 1. The rank correlations are uniformly higher than 0.98.

As a second comparison, we draw 10,000 samples of 60 random monthly returns from the Ibbotson large cap stock returns from January 1926 to December 1995. The results are given in the second column of Table 1. The rank correlations are all equal to 1.00.

3. Growth Optimal Criterion

Next we examine the appropriateness of using the logarithmic utility function. The justification for this is that the logarithmic utility corresponds to the growth optimal portfolio, as shown by Bansal and Lehman (1997).

For each fund, we compute the actual utility as in (1), and the logarithmic utility as follows:

$$(1/T) \sum \log(R_t). \tag{3}$$

The rank correlation between the actual utility and the log utility are given in Table 1. The results for hedge funds are in column 7. They show that the log utility is generally good when the risk aversion is low, but the quality deteriorates rapidly when risk aversion is high. Similar results are evident for mutual funds (in column 6) and for the simulation (in column 5).

4. Sharpe Ratio

Finally, we examine the appropriateness of using the Sharpe ratio to rank funds. This is not the traditional Sharpe ratio, as we do not subtract the risk free rate from the numerator.

The rank correlation between the actual utility and the Sharpe ratio for hedge funds are in column 10. This criterion works poorly when the risk aversion is low, but works reasonable well when risk aversion is high.

In comparison, the quadratic approximation works the best. For low risk aversion, the growth optimal portfolio is the next best. For high risk aversion, the Sharpe ratio is the next best.

5. Conclusions

The results suggest that using a mean-variance criterion to rank hedge funds and mutual funds will produce rankings which are nearly correct. They also suggest that the growth optimal criterion and the Sharpe ratio criterion for ranking funds are less useful, depending on the degree of investor risk aversion.

It is important to point out that there are circumstances when mean-variance analysis is not appropriate. In particular, risk assessments cannot be done accurately using a second order (i.e. mean-variance) approach.

In assessing risk, we are interested in the probability that a given fund has a large negative return in the next period. If the fund's returns are normally distributed, that this probability is determined by the mean and the standard deviations. But when returns are not normally distributed (as is true for hedge funds), the first two moments (i.e. mean and standard deviation) are not sufficient to give an accurate probability. [We may be able to use Chebychev's inequality to bound the probabilities. But these bounds are very loose and not very useful.]

The situation becomes even more complex if we need to access the probability of a large loss over multiple time periods. Take the Epstein-Zin (1989, 1991) type of non-expected utility functions. Here, we may have to look at multi-period returns, rather than single period returns, to access risk. The distribution of returns over 1-period may not be informative about the distribution of returns over multiple periods, if the 1-period returns are not independent and identically distributed.

Table 1
Rank Correlation Between Actual and Second Order Approximations
for Power and Exponential Utilities

	Second Order Approx			Log			Sharpe		
	Sim. Portf	Mut. Funds	Hedge Funds	Sim. Portf	Mut. Funds	Hedge Funds	Sim. Portf	Mut. Funds	Hedge Funds
Power Utility:									
γ									
0.1	1.00	1.00	1.00	1.00	0.99	0.97	1.00	-0.13	0.49
0.2	1.00	1.00	1.00	1.00	0.99	0.97	1.00	-0.12	0.50
0.3	1.00	1.00	1.00	1.00	1.00	0.98	1.00	-0.12	0.52
0.4	1.00	1.00	1.00	1.00	1.00	0.99	1.00	-0.11	0.53
0.5	1.00	1.00	1.00	1.00	1.00	0.99	1.00	-0.10	0.55
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.07	0.62
1.5	1.00	1.00	1.00	1.00	1.00	0.99	1.00	-0.04	0.68
2.0	1.00	1.00	1.00	1.00	1.00	0.98	1.00	-0.01	0.73
2.5	1.00	1.00	1.00	1.00	0.99	0.95	1.00	0.01	0.77
3.0	1.00	1.00	1.00	1.00	0.99	0.92	1.00	0.04	0.81
3.5	1.00	1.00	0.99	1.00	0.98	0.87	1.00	0.07	0.84
4.0	1.00	1.00	0.99	1.00	0.97	0.83	1.00	0.10	0.85
4.5	1.00	1.00	0.99	1.00	0.96	0.79	1.00	0.13	0.87
5.0	1.00	1.00	0.99	1.00	0.95	0.75	1.00	0.16	0.89
10.0	1.00	1.00	0.99	1.00	0.71	0.48	1.00	0.49	0.89
15.0	1.00	0.99	0.98	1.00	0.37	0.37	1.00	0.75	0.87
20.0	1.00	0.98	0.97	1.00	0.07	0.29	1.00	0.89	0.85
25.0	1.00	0.97	0.96	1.00	-0.15	0.24	1.00	0.92	0.83
30.0	1.00	0.98	0.95	1.00	-0.26	0.20	1.00	0.91	0.81
Exponential Utility:									
γ									
0.1	1.00	1.00	1.00	1.00	0.99	0.97	1.00	-0.13	0.49
0.2	1.00	1.00	1.00	1.00	0.99	0.98	1.00	-0.12	0.50
0.3	1.00	1.00	1.00	1.00	1.00	0.98	1.00	-0.11	0.52
0.4	1.00	1.00	1.00	1.00	1.00	0.99	1.00	-0.11	0.54
0.5	1.00	1.00	1.00	1.00	1.00	0.99	1.00	-0.10	0.55
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-0.07	0.63
1.5	1.00	1.00	1.00	1.00	1.00	0.99	1.00	-0.04	0.69
2.0	1.00	1.00	1.00	1.00	1.00	0.97	1.00	-0.01	0.73
2.5	1.00	1.00	1.00	1.00	0.99	0.95	1.00	0.02	0.77
3.0	1.00	1.00	1.00	1.00	0.99	0.91	1.00	0.04	0.81
3.5	1.00	1.00	0.99	1.00	0.98	0.87	1.00	0.07	0.84
4.0	1.00	1.00	0.99	1.00	0.97	0.83	1.00	0.10	0.86
4.5	1.00	1.00	0.99	1.00	0.96	0.78	1.00	0.13	0.87
5.0	1.00	1.00	0.99	1.00	0.94	0.75	1.00	0.16	0.89
10.0	1.00	1.00	0.99	1.00	0.71	0.48	1.00	0.49	0.90
15.0	1.00	0.99	0.98	1.00	0.37	0.37	1.00	0.75	0.87
20.0	1.00	0.98	0.98	1.00	0.07	0.30	1.00	0.89	0.85
25.0	1.00	0.98	0.97	1.00	-0.15	0.25	1.00	0.92	0.83
30.0	1.00	0.98	0.96	1.00	-0.26	0.21	1.00	0.91	0.82

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