Aspirational preferences and their representation by risk measures

(Online Appendix)

David B. Brown, Enrico de Giorgi, and Melvyn Sim

This version: February 2, 2012

Stochastic dominance properties of aspiration measures

Though the general setup does not require a particular probability measure, there may be situations in which the decision maker makes choices according to some subjective, probabilistic beliefs. For such decision makers, it is useful to understand the implied stochastic ordering properties of their choices relative to the underlying probability measure \( \mathbb{P} \in \mathcal{P} \) that they are using. In this section, we characterize these properties for aspiration measures. We show that aspiration measures share the stochastic dominance properties of their underlying risk family. Moreover, we show that under the mild assumption that the aspiration measure is indifferent to all acts with the same distribution under \( \mathbb{P} \), then the aspiration measure preserves first-order stochastic dominance (FSD) for all acts, second-order stochastic dominance (SSD) for all acts in the diversification favoring set, and risk-seeking stochastic dominance (RSSD) for all acts in the concentration favoring set.

We first recall the definition of the stochastic orders just mentioned. Note that in this section, if not specified explicitly, expectations are taken with respect to the probability measure \( \mathbb{P} \). We say that \( f \) dominates \( g \) by FSD if and only if \( \mathbb{E} [u(f)] \geq \mathbb{E} [u(g)] \) for all nondecreasing functions \( u \); in this case we write \( f \succeq_{(1)} g \). Similarly, \( f \) dominates \( g \) by SSD (respectively RSSD) if and only if \( \mathbb{E} [u(f)] \geq \mathbb{E} [u(g)] \) for all \( u \) nondecreasing and concave (respectively convex); in this case we write \( f \succeq_{(2)} g \) (respectively \( f \succeq_{(-2)} g \)). Equivalent definitions of first order, second order and risk-seeking stochastic dominance can be found in Levy (2006)\(^1\).

We first note the following.

**Proposition 4.** Let \( \mu \) be a risk measure and suppose that \( \mu \) preserves FSD, i.e., if \( f \succeq_{(1)} g \) then \( \mu(f) \leq \mu(g) \). Then the risk measure \( \bar{\mu}(f) = -\mu(-f) \) also preserves FSD. Moreover, if \( \mu \) preserves SSD, then \( \bar{\mu} \) preserves RSSD.

Since we can always express a concave risk measure \( \bar{\mu}(f) \) equivalently as \(-\mu(-f)\), this proposition shows that the property of FSD carries over from convex risk measures to concave risk measures, and SSD of a convex risk measure implies RSSD of its concave counterpart. Given that convex risk measures are well-studied, Proposition 4 will allow us to use known results on stochastic dominance of convex risk measures to establish analogous such results for aspiration measures. To do this, we first need to show that stochastic dominance properties for aspiration measures are implied by those of the associated family of risk measures. We now show this.

**Proposition 5.** Let \( \{\mu_k\}_{k \in \mathbb{R}} \) be a family of risk measures inducing the AM \( \rho \) with target function \( k \to \tau(k) \). Suppose that \( \mu_k \) preserves FSD for all \( k \). Then \( \rho \) preserves FSD, i.e.,

\[
\forall f \in F, g \in F_{++} \text{ such that } f \succeq_{(1)} g \Rightarrow \rho(f) \geq \rho(g).
\]

Moreover, if \( \mu_k \) preserves SSD for \( k > \hat{k} \) and RSSD for \( k < \hat{k} \), then

\[
\forall f \in F, g \in F_{++} \text{ such that } f \succeq_{(2)} g \Rightarrow \rho(f) \geq \rho(g)
\]

\[
\forall f \in F, g \in F_{--} \text{ such that } f \succeq_{(-2)} g \Rightarrow \rho(f) \geq \rho(g).
\]

In general, convex risk measures do not preserve FSD or SSD, as shown by De Giorgi (2005)\(^2\). For the case of coherent risk measures, therefore, Proposition 5 is of little help if we do not specify conditions on the family of risk measures \( \mu_k \) such that stochastic dominance is preserved. It is well known that stochastic dominance orders are fully characterized by an act’s cumulative distribution function under the specified probability measure \( \mathbb{P} \) (see Levy (2006)). When a risk measure \( \mu \) does not only depend on the distribution function of the act, we can find two acts \( f \) and \( g \) that only differ on zero-probability events, but possess different values for the risk measure, e.g., \( \mu(f) > \mu(g) \). In this case, we can define a

---


third act $h = f + \epsilon$, $0 < \epsilon < \mu(f) - \mu(g)$, which obviously dominates $f$ by FSD (and thus also dominates $g$ by FSD), but $\mu(f) > \mu(f) - \epsilon = \mu(h)$ and $\mu(h) = \mu(f) - \epsilon > \mu(g)$. This shows that a necessary property on risk measures in order to have preservation of stochastic dominance orders is that they only depend on the probability distribution of the act. We thus introduce the following.

**Definition 5.** Let $\mathbb{P}$ be a probability measure on $(S, \Sigma)$. A function $r : F \to \mathbb{R}$ is called law-invariant (with respect to $\mathbb{P}$) if and only if $r(f) = r(g)$ whenever $f$ and $g$ have the same cumulative distribution function under $\mathbb{P}$, i.e.,

$$\mathbb{P}(\{s \in S : f(s) \leq x\}) = \mathbb{P}(\{s \in S : g(s) \leq x\})$$

for all $x \in \mathbb{R}$.

Law-invariance\(^3\) means the underlying mapping between the event space and the consequence space is irrelevant; all that matters is the distribution of the acts under $\mathbb{P}$. It also means that zero-probability events do not matter, i.e., it might be that two acts differ on events $A \in \Sigma$, but as long as $\mathbb{P}(A) = 0$, this does not have any impact on the function $r$. This seems like an eminently reasonable property, common to many models of decision making under uncertainty.

In our context, law-invariance is important because it is closely linked to stochastic dominance properties.

**Proposition 6.** Let $(S, \Sigma, \mathbb{P})$ be a probability space. The following hold:

1. If $\rho$ is law-invariant on $F$ and $(S, \Sigma, \mathbb{P})$ is atomless, then $\rho$ preserves FSD on $F$, SSD on $F_{++}$ and RSSD on $F_{--}$.

2. If $\rho$ preserves FSD (resp. SSD/RSSD) on $F$ (resp. $F_{++}$/$F_{--}$), then $\rho$ is law-invariant on $F$ (resp. $F_{++}$/$F_{--}$).

---

\(^3\)Law-invariance is also referred to as “probabilistic sophistication”; see, for instance: Machina, M., and D. Schmeidler (1992). A more robust definition of subjective probability. *Econometrica*, 60, pp. 745-780.
Henceforth, we assume $\tau > 0$. For the entropic risk measure, we have, by Jensen’s inequality,
\[
\mu_k(f) = \frac{1}{k} \log \mathbb{E}\left[\exp(-k f)\right] \geq \frac{1}{k} \log \exp(\mathbb{E}[-k f]) = -\mathbb{E}[f].
\]
For the CVaR risk measure, we have
\[
\mu_k(f) = \inf_{\nu \in \mathbb{R}} \left\{ \nu + e^k \mathbb{E}\left[(-f - \nu)^+\right] \right\} \\
\geq \inf_{\nu \in \mathbb{R}} \left\{ \nu + \mathbb{E}\left[(-f - \nu)^+\right] \right\} \\
\geq \inf_{\nu \in \mathbb{R}} \left\{ \nu + \mathbb{E}[-f - \nu] \right\} = \mathbb{E}[-f].
\]

**Proof of Theorem 2**

The boundedness properties for the family $\{\mu_k\}$ imply $\mu_k(f) \geq \mathbb{E}[-f]$ for $k > 0$ and $\mu_k(f) \leq \mathbb{E}[-f]$ for $k < 0$. It follows that when $\mathbb{E}[f] < \tau$, then for all $k > 0$
\[
\mu_k(f - \tau) = \mu_k(f) + \tau(k) \geq \mathbb{E}[-f] + \tau > 0.
\]
Since $\mu_k(f - \tau(k)) > 0$ for all $k > 0$, it follows from Theorem 1 that $\rho(f) \leq 0$. Likewise, if $\mathbb{E}[f] \geq \tau_u$, then for all $k < 0$,
\[
\mu_k(f - \tau(k)) = \mu_k(f) + \tau(k) \leq \mathbb{E}[-f] + \tau_u \leq 0.
\]
Since $\mu_k(f) \leq 0$ for all $k < 0$, from Theorem 1, we have $\rho(f) \geq 0.

**Proof of Proposition 1**

It suffices to show that $\mu_k(f) \geq \mathbb{E}[-f]$ for $k > 0$. For $k < 0$ we have
\[
\mu_k(f) = -\mu_{-k}(-f) \leq \mathbb{E}[-f].
\]

Henceforth, we assume $k > 0$. For the entropic risk measure, we have, by Jensen’s inequality,
\[
\mu_k(f) = \frac{1}{k} \log \mathbb{E}[\exp(-k f)] \geq \frac{1}{k} \log \exp(\mathbb{E}[-k f]) = -\mathbb{E}[f].
\]

**Proof of Proposition 2**

First, it is clear that $\rho_e(f_A) = \infty$ and $\rho_r(f_B) < \infty$ for any $\tau \in (0, y]$. We thus focus on comparing $C$ to $D$ over the range $\tau \in (0, y]$.

There is a one-to-one mapping between target levels and SAM levels. In particular, for a particular SAM level $\rho$, let $\tau_C(\rho)$ and $\tau_D(\rho)$ be the corresponding target levels that induce the SAM level $\rho$ for gambles $C$ and $D$, respectively. For $\rho \neq 0$ we have
\[
\tau_C(\rho) = -\frac{1}{\rho} \log \left[ 1 + p(e^{-x\rho} - 1) \right] \\
\tau_D(\rho) = -\frac{1}{\rho} \log \left[ 1 + q(e^{-y\rho} - 1) \right],
\]
and for $\rho = 0$
\[
\tau_C(0) = px, \\
\tau_D(0) = qy.
\]

Note that $\tau_C(\rho)$ and $\tau_D(\rho)$ are both decreasing and continuous in $\rho$. To see this, compute
\[
\tau'_C(\rho) = \frac{\log \left[ 1 + p(e^{-x\rho} - 1) \right] + pxe^{-x\rho}}{\rho^2} \\
\text{and let } t = p(e^{-x\rho} - 1) > -1, \text{ then}
\]
\[
\tau'_C(\rho) \leq 0 \iff (1 + t) \log(1 + t) - (p + t) \log \left( \frac{p + t}{p} \right) \leq 0.
\]
Since \( p \to (p+t) \log ((p+t)/p) \) decreases on \((0,1]\) for all \( t \), the latter inequality is satisfied and thus \( \tau_C(\rho) \) decreases in \( \rho \). Similarly for \( \tau_D(\rho) \).

We will compare \( \tau_C(\rho) \) and \( \tau_D(\rho) \) as \( \rho \) varies and will show that there exists a unique \( \rho^* > 0 \) such that \( \tau_C(\rho^*) = \tau_D(\rho^*) \), and that \( \tau_D(\rho) > \tau_C(\rho) \) for all \( \rho > \rho^* \), and \( \tau_D(\rho) < \tau_C(\rho) \) for all \( \rho < \rho^* \). This shows that \( \rho_*(f_C) > \rho_*(f_D) \) if and only if \( \tau > \tau_C(\rho^*) = \tau_D(\rho^*) \).

First, consider \( \rho < 0 \). Over this range, we have
\[
\tau_C(\rho) > \tau_D(\rho) \iff -\frac{1}{\rho} \log \left[ 1 + p(e^{-x\rho} - 1) \right] > -\frac{1}{\rho} \log \left[ 1 + q(e^{-y\rho} - 1) \right] 
\iff p(e^{-x\rho} - 1) - q(e^{-y\rho} - 1) > 0.
\]

Let \( v(\rho) = p(e^{-x\rho} - 1) - q(e^{-y\rho} - 1) \), the left hand side of the latter inequality. Over \( \rho < 0 \), we have
\[
v'(\rho) = -p x e^{-x\rho} + q y e^{-y\rho} < q y (e^{-y\rho} - e^{-x\rho}) < 0,
\]
where in the first line we use the fact that \( px > qy \) and in the second line we use \( \rho < 0 \) and \( y > x \). In addition,
\[
\lim_{\rho \to 0} v(\rho) = 0 \quad \lim_{\rho \to -\infty} v(\rho) = +\infty.
\]

In sum, \( v(\rho) \) is a strictly decreasing function from \(+\infty\) to 0 as \( \rho \uparrow 0 \) and therefore must be strictly positive over the range, which implies that \( v(\rho) > 0 \) over \( \rho < 0 \), and thus \( \tau_C(\rho) > \tau_D(\rho) \) over this range.

For \( \rho = 0 \), the target levels reduce to the expected values; thus, \( \tau_C(0) = px > qy = \tau_D(0) \).

Finally, consider \( \rho > 0 \). Similar to the first case, we have over this range
\[
\tau_C(\rho) > \tau_D(\rho) \iff p(e^{-x\rho} - 1) - q(e^{-y\rho} - 1) < 0.
\]

Let \( v(\rho) \) be as before. We have \( \lim_{\rho \to 0} v(\rho) = 0 \) and \( \lim_{\rho \to \infty} v(\rho) = q - p > 0 \). Moreover, \( v'(\rho) = -p x e^{-x\rho} + q y e^{-y\rho} \), so
\[
v'(\rho) \leq 0 \iff \rho \leq \left( \frac{1}{x-y} \right) \log \left[ \frac{px}{qy} \right] = \bar{\rho} > 0.
\]

Thus, \( v(\rho) \) over \( \rho \geq 0 \) has a left limit of zero, a right limit of the positive value \( q - p \), and is nonincreasing for \( \rho \leq \bar{\rho} \) and increasing otherwise. This implies that there exists a unique \( \rho^* \geq \bar{\rho} > 0 \) when \( v(\rho) \) crosses zero. Note furthermore that \( v(\rho^*) = 0 \) is equivalent to \( \tau_C(\rho^*) = \tau_D(\rho^*) \), i.e., \( \mu_{\rho^*}(f_C) = \mu_{\rho^*}(f_D) \). Also, since \( \rho^* > 0 \), we must have \( \tau_C(\rho^*) = \tau_D(\rho^*) < E[f_D] = qy \) as claimed.

In summary, we have shown that there is a single target level \( \tau^* \) with the desired construction such that the SAM levels of C and D coincide at \( \tau^* \); below \( \tau^* \), D is preferred to C and vice versa for above \( \tau^* \). This completes the proof.

\[\square\]

**Proof of Theorem 3**

Suppose there exists a \( Q^* \in \Omega \) such that \( E_{Q^*}[f] < \tau_i \). From Theorem 2, we have, for \( k > 0 \),
\[
\mu_k(f - \tau(k)) = \mu_k(f) + \tau(k) = \sup_{Q \in \Omega} \mu_{Q,k}(f) + \tau(k) \geq \mu_{Q^*,k}(f) + \tau(k) \geq E_{Q^*}[f] + \tau_i > 0,
\]
where the strict inequality follows by \( E_{Q^*}[f] < \tau_i \). Hence, \( \rho(f) \leq 0 \). Likewise, suppose there exists a \( Q^* \in \Omega \) such that \( E_{Q^*}[f] \geq \tau_u \); we have, for \( k < 0 \),
\[
\mu_k(f - \tau(k)) = \mu_k(f) + \tau(k) = \inf_{Q \in \Omega} \mu_{Q,k}(f) + \tau(k) \leq \mu_{Q^*,k}(f) + \tau(k) \leq E_{Q^*}[f] + \tau_i \leq 0,
\]
and again invoking the representation theorem for SAM, it must be that \( \rho(f) \geq 0 \).

\[\square\]
Proof of Proposition 3

The worst case expectation can be obtained by solving the following optimization problem:

\[
\begin{align*}
\sup_q & \quad \mathbb{E}_q[\exp(-af)] \\
\text{s.t.} & \quad \mu \leq \mathbb{E}_q[f] \leq \overline{\mu}, \\
& \quad \mathbb{E}_q[1] = 1, \\
& \quad q(y) \geq 0, \quad \forall y \in [\underline{f}, \overline{f}].
\end{align*}
\]

We can consider \( q \) to be an infinite dimensional vector indexed by \( y \in [\underline{f}, \overline{f}] \). By weak duality, an upper bound to the above problem is

\[
\begin{align*}
\inf_{r,s,t} & \quad \{ r + \overline{\mu} s - \underline{\mu} t : r + ys - yt \geq \exp(-ay) \forall y \in [\underline{f}, \overline{f}], s, t \geq 0 \} \\
& \quad \inf_{r,s,t} \left\{ r + \overline{\mu} s - \underline{\mu} t : r \geq \max_{y \in [\underline{f}, \overline{f}]} \{\exp(-ay) - ys + yt\}, s, t \geq 0 \right\} \\
& \quad \inf_{s,t} \left\{ \max\{\exp(-a\overline{f}) - \overline{f}s + \overline{f}t, \exp(-a\underline{f}) - \underline{f}s + \underline{f}t\} : s, t \geq 0 \right\}.
\end{align*}
\]

By inspection, when \( a \geq 0 \), strong duality is obtained by a two point distribution \( \mathbb{P} \) with \( \mathbb{P}(f = \underline{f}) = \overline{p} \) and \( \mathbb{P}(f = \overline{f}) = \underline{q} \) and dual variables \( s = 0, t = (\exp(-a\overline{f}) - \exp(-a\underline{f}))/ (\overline{f} - \underline{f}) \geq 0 \). Likewise, when \( a < 0 \), strong duality is achieved by a two point distribution with \( \mathbb{P}(f = \underline{f}) = \overline{p} \) and \( \mathbb{P}(f = \overline{f}) = \underline{q} \) and dual variables \( s = (\exp(-a\underline{f}) - \exp(-a\overline{f}))/ (\overline{f} - \underline{f}) \geq 0, \)

\( t = 0 \).

Proof of Proposition 4

Suppose that \( f \geq_{(1)} g \). Then \( \mathbb{E}[u(f)] \geq \mathbb{E}[u(g)] \) for all \( u \) nondecreasing. Since \( u(x) \) is nondecreasing if and only if \( -u(-x) \) is also nondecreasing, we have \( -\mathbb{E}[u(-f)] \geq -\mathbb{E}[u(-g)] \), or, equivalently, \( \mathbb{E}[u(-f)] \leq \mathbb{E}[u(-g)] \) for \( u \) nondecreasing. This implies that \( -g \geq_{(1)} -f \). Therefore,

\[
\mu(f) = -\mu(-f) \leq -\mu(-g) = \overline{\mu}(g).
\]

For SSD, we observe that a function \( u(x) \) is nondecreasing and concave if and only if \( -u(-x) \) is nondecreasing and convex. Hence, \( f \geq_{(2)} g \) if and only if \( -g \geq_{(-2)} -g \). Similarly to above, the result follows.

Proof of Proposition 5

Note that if \( f \geq_{(1)} g \), then \( \mu_k(f) \leq \mu_k(g) \) for all \( k \in \mathbb{R} \) since \( \mu_k \) preserves FSD. It follows that

\[
\mu_k(f - \tau(k)) = \mu_k(f) + \tau(k) \leq \mu_k(g) + \tau(k) = \mu_k(g - \tau(k))
\]

for all \( k \in \mathbb{R} \) and by the definition of \( \rho \), it follows immediately that \( \rho(f) \geq \rho(g) \), i.e., \( \rho \) also preserves FSD.

For the next claim, note that \( g \in F_{++} \) implies that \( \rho(g) > \hat{k} \). Since \( f \geq_{(2)} g \) and \( \mu_{\rho(g)} \) preserves SSD, we have \( \mu_{\rho(g)}(f) \leq \mu_{\rho(g)}(g) \)
and

\[
\mu_{\rho(g)}(f - \tau(\rho(g))) = \mu_{\rho(g)}(f) + \tau(\rho(g)) \leq \mu_{\rho(g)}(g) + \tau(\rho(g)) = \mu_{\rho(g)}(g - \tau(\rho(g)) \leq 0.
\]

Therefore, \( \rho(f) \geq \rho(g) \).

Likewise, \( g \in F_{--} \) implies that \( \rho(g) \leq \hat{k} \). Since \( f \geq_{(-2)} g \) and \( \mu_{\rho(g)} \) preserves RSSD, we have \( \mu_{\rho(g)}(f) \leq \mu_{\rho(g)}(g) \)
and

\[
\mu_{\rho(g)}(f - \tau(\rho(g))) = \mu_{\rho(g)}(f) + \tau(\rho(g)) \leq \mu_{\rho(g)}(g) + \tau(\rho(g)) = \mu_{\rho(g)}(g - \tau(\rho(g)) \leq 0.
\]

Therefore, \( \rho(f) \geq \rho(g) \). 

\( \square \)
Proof of Proposition 6

We first show 1. First, it is easy to see that law-invariance of the aspiration measure implies law-invariance of the underlying family of risk measures (see Equation (3)). Föllmer and Schied (2004) show that on atomless probability spaces any law-invariant risk measure preserves FSD, and any convex, law-invariant risk measure preserves SSD; the claim now follows by Propositions 4 and 5.

For 2, we consider the case when \( \rho \) preserves FSD on \( F \); the other two cases follow by similar arguments. Now consider \( f \in F \) and \( g \in F \) with the same law under \( \mathbb{P} \). Note that \( f \geq_{(1)} g \) and \( g \geq_{(1)} f \). These together with \( \rho \) preserving FSD on \( F \) imply \( \rho(f) = \rho(g) \). □

---

Additional tables

<table>
<thead>
<tr>
<th>Asset</th>
<th>( \nu_i )</th>
<th>( \pi_i )</th>
<th>( \nu_i )</th>
<th>( \pi_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>-30.0</td>
<td>6.0</td>
<td>4.0</td>
<td>5.0</td>
</tr>
<tr>
<td>3</td>
<td>-40.0</td>
<td>8.0</td>
<td>5.0</td>
<td>6.0</td>
</tr>
<tr>
<td>4</td>
<td>-50.0</td>
<td>10.0</td>
<td>8.0</td>
<td>9.0</td>
</tr>
<tr>
<td>5</td>
<td>-60.0</td>
<td>15.0</td>
<td>11.0</td>
<td>12.0</td>
</tr>
<tr>
<td>6</td>
<td>-100.0</td>
<td>20.0</td>
<td>15.0</td>
<td>16.0</td>
</tr>
</tbody>
</table>

Table 3: Supports \([\nu_i, \pi_i]\) of the distributions of assets’ percentage returns \(V_i\) and the corresponding ranges \([\nu_i, \pi_i]\) for the expected returns for the portfolio choice example in Section 5.

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>(\rho)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1.000 0.000 0.000 0.000 0.000 0.000 ∞</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>0.718 0.065 0.049 0.077 0.054 0.036 0.3220</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>0.435 0.130 0.099 0.155 0.109 0.073 0.1610</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>0.153 0.195 0.148 0.232 0.163 0.109 0.1073</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>0.000 0.192 0.164 0.292 0.210 0.142 0.0795</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>0.000 0.069 0.138 0.348 0.261 0.183 0.0585</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.0</td>
<td>0.000 0.000 0.000 0.350 0.361 0.289 0.0315</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.0</td>
<td>0.000 0.000 0.000 0.488 0.512 0.0146</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.0</td>
<td>0.000 0.000 0.000 0.000 1.000 0.0031</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.0</td>
<td>0.000 0.000 0.000 0.000 0.000 1.000 -0.0136</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Optimal asset allocation under the ESAM for various values of the target \(\tau\) for the portfolio choice example in Section 5.