Bidding for Labor

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We present a competing-auction theory of the labor market, where job candidates auction their labor services to employers. An equilibrium matching function emerges which has many of the features commonly assumed, including constant returns to scale in large economies. The auction mechanism also generates equilibrium wage dispersion among homogeneous workers and constrained-efficiency...
cient entry of vacancies in large economies. In a dynamic version of the model, we generate implied numerical values for equilibrium unemployment and wage dispersion. The theory makes the novel prediction that wage dispersion is a decreasing function of the discount factor and labor market tightness. *Journal of Economic Literature* Classification Numbers: E24, J31, J41, J64, D44. © 2000 Academic Press

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1. INTRODUCTION

The matching process between productive activities and human resources is central to economic prosperity but is still not well understood. Many questions remain about its basic properties. For example, to what extent do different matching frictions contribute to frictional and structural unemployment? That is, how do capacity constraints, coordination problems, externalities, heterogeneities, informational asymmetries, and other problems affect matching? Why do we observe significant wage dispersion among workers with similar characteristics? When can we expect the matching process to be efficient, in some sense? Do agents in the economy face the right incentives to invest in the appropriate levels and types of physical and human capital? What roles do the legal and institutional environments play? To answer these questions, we need a consistent theoretical framework that is flexible enough to accommodate the various frictions, yet simple and intuitive enough to yield understandable results.

In this paper, we approach this problem by considering a theory of the labor market with certain key features that, we believe, reflect the institutional environment that most employers and job candidates face. In particular, we assume that job offers from employers carry the following commitment: if a candidate accepts an employer’s offer, then the employer is committed to giving the candidate the job. Also, employers are capacity-constrained in the sense that, at any moment in time, they have a fixed number of vacancies to fill. Together, these restrictions imply that, at any moment in time, any employer can make no more offers than she has existing vacancies. Thus, each vacancy can be offered to at most one candidate at a time. On the other side of the market, however, candidates may apply to multiple vacancies since applying for a job does not carry a commitment to accept if it is offered and may consider multiple offers from employers simultaneously. We take this basic institutional asymmetry as given and explore its implications.

The theoretical structure that we use in this exploration is that of auctions. We view candidates as, in effect, *auctioning* their labor services to employers. The structure of auction theory is the closest that we can find to the institutional structure just outlined. Auction theory also has the
advantage of generating clear, simple, results in complex environments. Recent theoretical developments (McAfee, 1993; Peters, 1994; Julien, 1995, 1997; Peters and Severinov, 1997) now allow us to use auction theory to study equilibrium market outcomes in settings where multiple sellers compete with each other through the reserve prices that they announce ("competing auction theory").

We construct a simple competing-auction theoretic model of the labor market, based on the framework of Julien (1995, 1997). The advantage of this particular framework is that it allows for an analysis of the effects of market size, since it is defined for a small number of players. The results in "large" markets can then be found by taking limits as the number of players increases. This distinction is important in the welfare analysis that we consider.

We focus on three basic frictions in this paper: capacity constraints, coordination problems, and the externalities associated with new vacancy creation. The model abstracts from all other frictions listed above: candidates are homogeneous, employers are homogeneous, information is symmetric, all agents have rational expectations, and, in the dynamic analysis, all agents can commit to long-run contracts. We first study a static model with a fixed number of agents on each side of the market. We then allow entry of vacancies and, finally, make the model dynamic and study the steady state equilibrium. In all cases, candidates commit to using the auction structure. Each candidate announces a reserve wage: the lowest wage that he would be willing to work for. If only one employer approaches him, this is his equilibrium wage. However, if more than one employer approaches a worker, he then sells his labor services to the highest bidder. Once candidates have announced their reserve wages to all employers, employers choose which candidates to approach.

In this game, there exist multiple asymmetric pure strategy equilibria in which each employer approaches a different candidate. In any of these equilibria, if the number of employers and candidates is the same then no unemployment exists. However, since employers are modeled as moving simultaneously, coordinating on any of these equilibria is extremely unlikely in a market of any significant size. We therefore focus our attention on the unique symmetric mixed strategy equilibrium in which employers randomize when choosing candidates. The mixed strategies generate an equilibrium matching function and unemployment rate. This matching function has interesting properties—it has many of the features that are usually imposed, based on the work by Diamond (1982), Mortensen (1982), and Pissarides (1984). For example, the number of matches is an increasing function of both vacancies and candidates; also, in large markets, the

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3 See Bernhardt and Scoones (1993) for an auction-based analysis of a single worker's problem.
matching function has constant returns to scale. The equilibrium unemployment rate is, naturally, decreasing in the number of vacancies and increasing in the number of candidates. When the number of vacancies and candidates is equal, equilibrium unemployment exists solely due to the underlying frictions (capacity constraints and coordination problems). For the dynamic version of the model, we generate a table of values for the steady state equilibrium unemployment rate, as a function of the vacancy/candidate ratio and the separation rate.

Wage dispersion among homogeneous workers in homogeneous jobs is another feature of the mixed strategy equilibrium. This is obtained because, when employers play mixed strategies, some workers have more than one employer approach them, while others do not. Those workers who are lucky enough to have more than one employer approach them enjoy a wage premium. In the dynamic game, the size of the wage premium is limited by the fact that employers who find themselves in the position of bidding may choose to, instead, break off and approach another candidate; but this costs some time. For the dynamic model, we generate a table of mean wages and values of the coefficient of variation, for different values of the discount factor and the measure of labor market tightness. We find that wage dispersion decreases significantly as both the discount factor and tightness increase.

Having employers randomize over candidates, in the way they do in the mixed strategy equilibrium, is clearly inefficient because it generates a nonzero expected unemployment rate even when the numbers of employers and candidates are equal. That is, the existence of the coordination friction reduces aggregate expected surplus in an obvious way. Given this friction, however, one may ask a different question: When employers can create vacancies, would they choose to create the same number of vacancies as a planner who faces the same friction?4 This question is quite analogous to the problem of constrained efficiency considered in, for example, Hosios (1990) in the context of the standard matching-bargaining model.5 In Hosios’ framework, however, the matching function is viewed as a technological constraint facing both private agents and the planner (rather than as an equilibrium outcome). Also, bilateral bargaining is used as the wage-determination mechanism, rather than auctioning.

In Hosios’ environment, equilibrium entry will not generally be constrained-efficient, due to the arbitrariness of the surplus-shares in the Nash bargaining solution. By way of contrast, the auction mechanism in

4 This is equivalent to considering the problem of a constrained planner who can influence vacancy creation but not the equilibrium strategies that employers and candidates choose, once the number of candidates and vacancies is established.

5 Also, see Kennes (1997).
this paper generates constrained-efficient entry in large markets but not in small markets. In small markets, not enough vacancies are created. However, it is also true that markets do not have to be very large to approximate the “large” market outcome. Thus, for most of the labor markets that this theory might be applied to, equilibrium entry is (approximately) constrained-efficient.

The basic environment studied in this paper is very similar to the one studied in independent work by Montgomery (1991) and Burdett, Shi, and Wright (1997). Both of those papers have analogous capacity constraints and coordination problems similar to the one considered here. In those studies, however, employers post wages, and candidates are restricted to applying to only one job in any time period. They focus on the symmetric mixed strategy equilibrium in which candidates randomize their applications over employers. (In our framework, candidates apply to all employers; employers are restricted to offering each vacancy to only one worker, and employers randomize over candidates.) In both settings, randomization generates matching functions. Work by Shimer (1996) and Moen (1997) has explored the constrained efficiency of wage-posting equilibria in this type of setting under certain conditions. Other work by Lu and McAfee (1996) and Kultti (1999) has shown that, when sellers can choose selling mechanisms, both price-posting and auctioning will “drive out” bargaining in the sense that no evolutionary-stable equilibrium with any sellers bargaining exists in large markets. Kultti (1999) has also shown the revenue equivalence of posted prices and auctions in large markets. In small markets, however, when sellers can choose transactions mechanisms, all sellers choosing to auction is the dominant strategy equilibrium (Julien, Kennes, and King, 1999).

The paper is organized as follows. We first consider, in Section 2, a static version of the model, in which only one round of auctions occurs. We characterize its unique symmetric mixed strategy equilibrium and demonstrate the constrained-efficiency result. Section 3 of the paper then characterizes stationary symmetric mixed strategy equilibria in dynamic versions of the model in which a constant fraction of existing matches dissolves in every period. Tables of the equilibrium unemployment rate and the degree of wage dispersion are also presented. Section 4 then concludes and considers possible directions for future research.

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6 See, also, Shi (1997) and Shimer (1998).

7 Pissarides (1979) also considers the random assignment of candidates to vacancies, deriving an equivalent function.
2. THE STATIC MODEL

In this economy, \( N \) identical candidates and \( M \) identical employers are to be matched, where \( M \) and \( N \) are natural numbers. In this section of the paper, \( M \) and \( N \) are exogenous. Candidates have one indivisible unit of labor services to sell to employers. Each employer has access to a project worth a fixed amount \( \theta > 0 \), which is common knowledge to all agents and which requires one worker to operate. All agents are risk neutral and seek to maximize their expected income. Candidates may entertain offers from many employers, but employers can only offer their single job to (at most) one candidate (for the reasons described above). The sequence of events within the period is as follows. First, each candidate announces a reserve wage to induce offers from employers. Employers then decide which candidates to approach. Candidates then auction their labor services to the highest bidder. Equilibria in this model are solved backwards.

We start with the bidding game for candidates, once employers have decided who to bid for, and given each candidate’s announced reserve wage \( \sigma_i \), where \( i \in \{1, 2, \ldots, N\} \) is used to index candidates. Let \( m_i \in \{0, 1, 2, \ldots, M\} \) denote the number of employers bidding for candidate \( i \), let \( w(\sigma_i, m_i) \) denote the equilibrium wage obtained by candidate \( i \), and let \( \alpha \) and \( \gamma \), respectively, denote the candidates’ and employers’ outside options payments they would receive if they are not matched. We restrict \( \alpha + \gamma < \theta \), so that a match leads to a net gain in total surplus.

2.1. The Bidding Game

As is standard in an ascending-bid auction (McAfee and McMillan, 1987) with homogeneous buyers and complete information, the candidate’s wage (the seller in this model) is given by

\[
w(\sigma_i, m_i) = \begin{cases} 
0 & \text{if } m_i = 0 \\
\sigma_i & \text{if } m_i = 1 \\
\theta - \gamma & \text{if } m_i > 1.
\end{cases}
\]  

(1)

Clearly, if no employers approach the candidate, his wage will be zero (although his payoff will be his outside option, \( \alpha \)). If only one employer approaches, then the candidate receives his reserve wage \( \sigma_i \). If more than one employer approaches then Bertrand competition between the employers drives the wage up to the point where the candidate receives all the gains that employers make from operating the project, \( \theta - \gamma \).

2.2. Employers’ Choice of Candidate to Bid for

Having observed the candidates’ reserve wage announcement vector, employers decide which candidate to bid for. Since we will be focussing on
symmetric equilibria, for notational convenience we will assume that all candidates other than \( i \) choose the same reserve wage \( \sigma \). Let \( p_i(\sigma_i, \sigma) \) denote the probability that a particular employer bids for candidate \( i \). Thus, for any employer, the probabilities of offers to candidates must sum to one,

\[
\sum_{i=1}^{N} p_i(\sigma_i, \sigma) = 1. \tag{2}
\]

Also, given \( m \) identical offers, with symmetry, the probability that candidate \( i \) will accept any particular offer is given by

\[
\Pr\{i \text{ accepts}\} = \frac{1}{m_i}. \tag{3}
\]

Once an employer decides to make an offer to candidate \( i \), given \( m_i \), the expected payoff to this employer is

\[
R_i = (\theta - w(\sigma_i, m_i))\Pr\{i \text{ accepts } w(\sigma_i, m_i)\} \\
+ \gamma[1 - \Pr\{i \text{ accepts } w(\sigma_i, m_i)\}].
\]

Using (1) and (3), the above equation becomes

\[
R_i(\sigma_i, m_i) = \begin{cases} 
\theta - \sigma_i, & \text{if } m_i = 1 \\
\gamma & \text{if } m_i > 1.
\end{cases} \tag{4}
\]

In a symmetric equilibrium, \((1 - p_i(\sigma_i, \sigma))^{M-1}\) is the probability that the employer will be alone in his offer to candidate \( i \), and \([1 - (1 - p_i(\sigma_i, \sigma))^{M-1}]\) is the probability that at least one other employer will make this candidate an offer. Hence, before knowing \( m_i \), an employer’s expected payoff if she makes an offer to candidate \( i \) is

\[
\Pi_i(\sigma_i, \sigma) = (1 - p_i(\sigma_i, \sigma))^{M-1}(\theta - \sigma_i) + [1 - (1 - p_i(\sigma_i, \sigma))^{M-1}]\gamma. \tag{5}
\]

In a symmetric mixed strategy equilibrium, each employer chooses \( p_i(\sigma_i, \sigma), i = 1, 2, \ldots, N \), so that \( \Pi_i = \Pi, \forall i \). Let \( p(\sigma_i, \sigma) \) denote the symmetric mixed strategy probability assigned to all other candidates; then constraint (2) becomes

\[
p(\sigma_i, \sigma) = \frac{1 - p_i(\sigma_i, \sigma)}{N - 1}.
\]

\(^8\) Since all employers are homogenous, we do not use subscripts for employers.
Using this constraint and Eq. (5), one obtains
\[ p_i(\sigma_i, \sigma) = 1 - \frac{N - 1}{1 + (N - 1)((\theta - \gamma - \sigma_i)/(\theta - \gamma - \sigma))^1/(M-1)}. \] (6)

Notice that when \( \sigma_i = \sigma \) (as occurs in the symmetric equilibrium, below) then Eq. (6) collapses to \( p_i(\sigma, \sigma) = p(\sigma, \sigma) = 1/N \). However, we must keep \( p_i(\sigma, \sigma) \) general, at this stage, until the equilibrium choice of \( \sigma_i \) has been determined. We now turn to consider this choice.

2.3. The Candidates’ Reserve Wage Choice

Candidates choose their reserve wages to maximize their expected payoffs in a simultaneous move game with other candidates. Let \( q_{i0} \) and \( q_{i1} \) denote the probabilities that candidate \( i \) will receive zero offers and one offer, respectively. Thus
\[ q_{i0}(\sigma_i, \sigma) = (1 - p_i(\sigma_i, \sigma))^M \]
and
\[ q_{i1}(\sigma_i, \sigma) = Mp_i(\sigma_i, \sigma)(1 - p_i(\sigma_i, \sigma))^{M-1}. \]
The expected payoff function for candidate \( i \) is therefore given by
\[ V_i(\sigma_i, \sigma) = q_{i0}(\sigma_i, \sigma)\sigma_i + q_{i1}(\sigma_i, \sigma)(\theta - \gamma). \] (7)

Since candidates choose their reserve wages simultaneously, an equilibrium array of reserve wages is found by a standard Nash argument,
\[ \sigma_i^* = \text{arg max}_{\sigma_i} V_i(\sigma_i, \sigma^*). \]

Proposition 1. The unique symmetric equilibrium reserve wage is
\[ \sigma_i^* = \sigma^* = \frac{(M - 1)(\theta - \gamma) + (N - 1)^2 \alpha}{(M - 1) + (N - 1)^2} \quad \forall i = 1, 2, \ldots, N. \] (8)

Proof. See the Appendix.

2.4. Characterization of the Symmetric Mixed Strategy Equilibrium

Equation (8) gives the reserve wage that each candidate announces in the symmetric mixed strategy equilibrium. Notice that this is a weighted average of the wage he would receive if more than one employer ap-
proaches him \((\theta - \gamma)\) and his outside option \((\alpha)\). Notice also that in this
equilibrium, since \(\sigma_i^* = \sigma^*\) for all \(i\) then, by Eq. (6), employers assign
equal probabilities to making offers to each candidate. That is,

\[
p_i^*(\sigma^*, \sigma^*) = p^* = 1/N.
\] (9)

It is now straightforward to calculate the expected number of matches
\((x)\) in equilibrium: since \((1 - p^*)^M\) is candidate \(i\)'s probability of receiving
no offers, then \(1 - (1 - p^*)^M\) is the probability that candidate \(i\) will
receive at least one offer. In the symmetric equilibrium, the expected
number of matches is given by \(N\) times this number. Using the fact that \(p^* = 1/N,\)

\[
x^*(N, M) = N \cdot \left(1 - \left(\frac{N - 1}{N}\right)^M\right). \tag{10}
\]

The expected rate of equilibrium unemployment \((U^*(N, M))\) is simply the
expected number of candidates who receive no offers, divided by the size
of the labor force (in this section, the size of the labor force is the same as
the number of candidates), and is given by

\[
U^*(N, M) = \left(\frac{N - 1}{N}\right)^M. \tag{11}
\]

Notice that these functions have the standard properties

\[
x^*_N (N, M) > 0, \quad x^*_M (N, M) > 0, \\
U^*_N (N, M) > 0, \quad U^*_M (N, M) < 0. \tag{12}
\]

If we define \(\phi = M/N\) (a measure of labor market tightness), then Eqs.
(10) and (11) become

\[
x^*(N, \phi) = N \cdot \left(1 - \left(\frac{N - 1}{N}\right)^{\phi N}\right) \tag{10'}
\]

\[
U^*(N, \phi) = \left(\frac{N - 1}{N}\right)^{\phi N}. \tag{11'}
\]

Notice that the expected rate of unemployment, given in (11'), is positive
for all \(N > 1\). This remains true even in the special case where the
number of candidates is equal to the number of employers (i.e., when
In this special case, unemployment exists solely due to the fact that employers play mixed strategies. If, instead, employers are able to coordinate on a pure strategy equilibrium in which each candidate is assigned to a unique employer, then equilibrium unemployment is zero in this case.

Substitution of (9) into (5) and (7) yields the equilibrium expected payoff functions,

$$\Pi^* (\sigma^*; N, \phi) = \left( \frac{N-1}{N} \right)^{\phi N - 1} (\theta - \sigma^*) + \left( 1 - \left( \frac{N-1}{N} \right)^{\phi N - 1} \right) \gamma$$

(13)

$$V^* (\sigma^*; N, \phi) = \left( \frac{N-1}{N} \right)^{\phi N} \alpha + \phi \left( \frac{N-1}{N} \right)^{\phi N - 1} \sigma^*$$

$$+ \left( 1 - \left( \frac{N-1}{N} \right)^{\phi N} - \phi \left( \frac{N-1}{N} \right)^{\phi N - 1} \right) (\theta - \gamma),$$

(14)

where $\sigma^*$ is given in (8).

Intuitively, according to Eq. (13), the expected payoff to a representative employer from approaching any candidate is a weighted sum of the two possible ex post payoffs: $(\theta - \sigma^*)$ if the employer is alone in her offer to the candidate, and $\gamma$ if not, where the weights are the probabilities of each event. Similarly, Eq. (14) states that the expected payoff to a candidate is the weighted sum of the three possible ex post payoffs, $\alpha$, $\sigma^*$, and $(\theta - \gamma)$, which correspond to the events that zero, one, and more than one employers approach the candidate, respectively.

Notice also that wage dispersion exists among homogeneous workers in this equilibrium. By Eq. (1), candidates who have more than one employer approach them receive the wage $(\theta - \gamma)$, while candidates who have only one employer approach them receive $\sigma^*$. Since $\sigma^*$ is a weighted average of $(\theta - \gamma)$ and $\alpha$, and $\alpha < (\theta - \gamma)$, then workers that have only one employer approach them receive a lower wage (due simply to bad luck).

2.5. The Large Economy Case

We now consider the properties of this equilibrium, as the scale of the market becomes large. To do this, we hold $\phi$ constant and examine the case where $N$ is a very large, but finite, number. In this type of environment the economy can be closely approximated by the limit economy where $N \to \infty$. We maintain the assumption that $N$ is finite to keep aggregate surplus finite in our efficiency comparisons, but we use the limit
results because of their simplicity. The results in this section (and in later sections where we consider "large" economies) should be taken as approximations.

To avoid confusion, we denote the equilibrium values of these variables in the large economy by using a \( \sim \) over the variables, rather than an asterisk.

In the large economy, \( \sigma^* \) and \( p^* \) become, respectively,\(^9\)

\[
\tilde{\sigma} = \alpha \quad (15)
\]
\[
\tilde{\rho} = 0. \quad (16)
\]

Consider Eq. (16) first. This states the intuitively obvious result that as the size of the market gets large, the probability that any one employer will approach any particular candidate goes to zero. Thus, even though we hold the ratio of candidates to employers (\( \phi \)) constant as the scale of the market is increased, each individual worker becomes less significant as market size increases.

Equation (15) presents the less obvious result that, in this limit, workers’ reserve wage announcements are driven down to their outside option (\( \alpha \)). This result follows from the particular sequence of events assumed here (i.e., candidates move first in this game, by choosing their reserve wages) and the assumption that candidates can costlessly apply to all employers, while employers are restricted to making offers to only one candidate. With randomization, employers place less weight on approaching each candidate than each candidate places on approaching each employer. (Each candidate approaches each firm with certainty, but each employer approaches any candidate with some probability less than one.) Thus, as the market gets large, keeping the ratio of employers to candidates constant, there are asymmetric influences of increasing \( M \) and \( N \): for each candidate, the probability of being approached erodes more quickly (as \( N \) increases) than the analogous probability for employers (as \( M \) increases). This then erodes the first-mover advantage that candidates enjoy when announcing their reserve wages. Notice, however, that wage dispersion is still a feature of the equilibrium. In fact, the wage premium enjoyed by workers who are approached by more than one worker is greater in the large economy. These workers still receive \((\theta - \gamma)\), while candidates who only had one employer approach them receive only \( \tilde{\sigma} = \alpha < \sigma^* \).

\(^9\) The derivation of Eq. (15) uses L'Hôpital's rule.
Using the fact that \( \lim_{N \to \infty} (1 + x/N)^N = e^x \), the equilibrium matching function and unemployment rate in the large economy become, respectively,

\[
\tilde{x}(N, \phi) = N(1 - e^{-\phi}) \quad (17)
\]

\[
\tilde{U}(\phi) = e^{-\phi}. \quad (18)
\]

These have several noteworthy properties. Notice first that, in large economies, the matching function has constant returns to scale. This model is therefore consistent with the standard assumption of the constant returns matching function in macroeconomic models. Second, although these results are, strictly speaking, limit results, both \( x^*(N, \phi) \) and \( U^*(N, \phi) \) approach their limit values quite quickly. For example, if \( \phi = 1 \) then, in the limit, \( \tilde{U}(1) = 1/e = 0.3679 \), but \( U^*(10, 1) = 0.3487 \), \( U^*(50, 1) = 0.3642 \), and \( U^*(100, 1) = 0.3660 \). Thus, for any “large” labor market, \( \tilde{U}(\phi) = e^{-\phi} \) and \( \tilde{x}(N, \phi) = N(1 - e^{-\phi}) \) are good approximations.

Taking limits of the functions in (13) and (14), using (15), approximates the equilibrium payoff functions for employers and candidates in large economies,

\[
\tilde{I}(\phi) = e^{-\phi}(\theta - \alpha) + (1 - e^{-\phi})\gamma \quad (19)
\]

\[
\tilde{V}(\phi) = (e^{-\phi} + \phi e^{-\phi})\alpha + \left[1 - (e^{-\phi} + \phi e^{-\phi})\right](\theta - \gamma). \quad (20)
\]

Examination of Eqs. (19) and (20) reveals that, in the large economy, the expression \( e^{-\phi} \) represents not only the equilibrium unemployment rate and the probability that a representative employer will be alone when approaching a particular candidate, but also the probability that a representative candidate will have no offers. In Eq. (20), the probability that either zero or one employer approaches a candidate is given by \( (e^{-\phi} + \phi e^{-\phi}) \). In either case, the candidate will receive his outside option \( \alpha \). The probability that the candidate is approached by more than one employer is \( 1 - (e^{-\phi} + \phi e^{-\phi}) \). In this case, the candidate receives the payoff \( \theta - \gamma \).

It is worth noting, and straightforward to show from (19) and (20), that \( \tilde{I}'(\phi) < 0 \) and \( \tilde{V}'(\phi) > 0 \). That is, we have the intuitive result that the higher the ratio of vacancies to candidates is, the smaller will be the payoff to each vacancy, and the larger the payoff to candidates.

2.6. Entry and Constrained Efficiency

In this section we consider the question posed by Hosios (1990): If vacancies are costly to create, would the equilibrium number of vacancies be equal to the number picked by a surplus-maximizing planner who faces
the same matching friction as private agents? It turns out that the answer to this question depends on the size of the market. To demonstrate this, we first consider the equilibrium entry conditions, when each vacancy costs $\kappa$ to create (where $\kappa \geq \gamma$), in both the small and large economy cases. In both cases, given a fixed number of candidates $N$, employers will create vacancies until the expected marginal benefit from doing so is driven down to its marginal cost, $\kappa$. Using Eqs. (13) and (19), this yields the equilibrium entry conditions in the small and large economies respectively as

$$
\left( \frac{N - 1}{N} \right)^{\delta N} (\theta - \gamma - \sigma^*) = \kappa - \gamma 
$$

(21a)

$$
e^{-\phi}(\theta - \gamma - \alpha) = \kappa - \gamma,
$$

(21b)

where $\sigma^*$ is given in Eq. (8) in the small economy case.

We now consider the planner’s problem. The planner is constrained in the sense that she cannot avoid the matching friction. As in Hosios (1990), the planner views the matching friction as a technological constraint and (given $N$) chooses $M$ to maximize expected surplus. Expected surplus, in the small and large economies, respectively, is given by

$$
S_1 = N \left( 1 - \left( \frac{N - 1}{N} \right)^M \right) \theta + N \left( \frac{N - 1}{N} \right)^M \alpha
$$

$$+ \left[ M - N \left( 1 - \left( \frac{N - 1}{N} \right)^M \right) \right] \gamma - M \kappa
$$

(22a)

$$
S_2 = N(1 - e^{-\phi}) \theta + Ne^{-\phi} \alpha + (M - N(1 - e^{-\phi})) \gamma - M \kappa.
$$

(22b)

The first term on the right side of these equations is the expected surplus generated by the matches, where the production from each match is $\theta$. The second terms represent the surplus generated by unemployed candidates when each one produces $\alpha$ (his outside option). The third term shows the surplus generated by unfilled vacancies ($M$ minus the number of matches), when each one produces $\gamma$ (the outside option). The total costs of vacancy creation are $M \kappa$. The results are summarized in the following proposition.

**Proposition 2.** In small markets, given $N$, the equilibrium number of vacancies is smaller than the constrained efficient number. In large markets, these numbers coincide.
Proof. Let $M = \phi N$ in Eqs. (22a) and (22b). Given $N$, the first order conditions for maxima of $S_1$ and $S_2$ respectively are

$$N \ln \left( \frac{N-1}{N} \right) (\gamma + \alpha) = \kappa - \gamma$$

$$e^{-\phi}(\theta - \gamma - \alpha) = \kappa - \gamma. \quad (23a)$$

Concavity of $S_1$ and $S_2$ ensures that these conditions are sufficient for maxima. Comparing Eq. (21a) (the equilibrium entry condition in the small market case) with Eq. (23a), the right sides of both equations are identical. Since $\sigma^* > \alpha$ and $-N \ln((N-1)/N) > 1$, then the left side of (23a) is greater than the left side of (21a). Since the left sides of (21a) and (23a) are both decreasing in $\phi$, this gives us the first result. Comparing Eqs. (21b) and (23b), which are identical, gives us the second result.

The reasoning behind this result is as follows. Consider the choice of whether or not to add one more vacancy. With some probability, the employer with this new vacancy will approach a candidate who is also approached by another employer. In this case, with some probability, the entering vacancy will hire the worker, so the gains to the match with the other employer will be lost. This is an external cost associated with the new vacancy. However, there is also a benefit created: the match of the entering vacancy and the worker. Clearly, this cost and this benefit exactly cancel each other. Thus, the social return from such a new vacancy is zero. Due to the auction mechanism, this is precisely the private return that a new vacancy gets in this case. If, however, the entering vacancy approaches a worker whom otherwise would not be matched, then a social benefit is generated: the value of the match less the outside opportunities $(\theta - \gamma - \alpha)$. The expected marginal social benefit of the new vacancy is therefore the probability that the new vacancy will be alone when it approaches a worker, multiplied by $(\theta - \gamma - \alpha)$. The marginal social cost of generating a new vacancy is simply the cost of creating the vacancy minus the outside opportunity that it generates, $(\kappa - \gamma)$. A social planner equates these two (Eqs. (23a) and (23b)).

The equilibrium entry conditions (21a) and (21b) state that vacancies will be created until the expected payoffs from doing so (which equals the probability of being alone when approaching a worker, multiplied by the ex post payoff in that case $(\theta - \gamma - \alpha)$) equal the cost of vacancy creation minus the value of the outside opportunity created $(\kappa - \gamma)$. In the large economy, $\theta = \alpha$, so the equilibrium and planner solutions coincide. When

\(^{10}\) We are grateful to John Hillas for suggesting this line of reasoning.
N is small, however, \( \sigma^* > \alpha \), and the private expected payoff from vacancy creation is less than the social. In this case, not enough vacancies are created.

3. MAKING THE MODEL DYNAMIC

In this section we examine some of the implications of dynamic versions of this theory. In dynamic settings, several issues arise that are beyond the scope of the current paper. Here, our main objective is to get a sense of the magnitude of unemployment rates and wage dispersion that dynamic models of this sort imply in a steady state. Several different dynamic specifications are consistent with the same equilibrium unemployment rate, since this rate is generated simply by random matching and flow identities. These different specifications (in general) do imply different equilibrium payoffs, however. For this reason, we first focus our attention on this unemployment rate, then consider a structure that supports it.

3.1. Steady State Equilibrium Unemployment Rates

Let \( N_0 \) and \( M_0 \) denote the numbers of workers and employers in this economy, respectively, and let \( N_t \) and \( M_t \) denote the expected numbers of candidates and vacancies that is, unmatched workers and employers in periods \( t = 0, 1, 2, 3, \ldots \). We assume that \( N_0 \) and \( M_0 \) are exogenously determined. Also, at the beginning of every period, a fraction \( \rho \) of existing matches dissolves. Thus, in all periods \( t = 1, 2, 3, \ldots \),

\[
N_t = N_{t-1} - x_{t-1}(N_{t-1}, M_{t-1}) + \rho [N_0 - N_{t-1} + x_{t-1}(N_{t-1}, M_{t-1})],
\]

\( t = 1, 2, 3, \ldots \) (24a)

\[
M_t = M_{t-1} - x_{t-1}(N_{t-1}, M_{t-1}) + \rho [M_0 - M_{t-1} + x_{t-1}(N_{t-1}, M_{t-1})],
\]

\( t = 1, 2, 3, \ldots \) (24b)

where \( x_{t-1}(N_{t-1}, M_{t-1}) \) denotes the expected number of matches made in period \( t-1 \), and \( (N_0 - N_{t-1}) \) and \( (M_0 - M_{t-1}) \) respectively denote the expected numbers of matched workers and employers at the end of period \( t-1 \).

We restrict our attention to stationary symmetric equilibria in large economies with homogeneous candidates and employers, where employers play mixed strategies assigning equal weights to all candidates. Let \( \bar{N} \) and \( \bar{M} \) denote the number of candidates and vacancies in the steady state. Thus, the within-period matching function (17) becomes, in the steady state,

\[
\tilde{x}(\bar{N}, \bar{M}) = \bar{N}(1 - e^{-\bar{M}}),
\]

(25)
where $\bar{\phi} = \bar{M}/\bar{N}$. Also, the expected proportion of candidates that will be unemployed at the end of each period in the steady state, from (18), becomes

$$\bar{U}(\bar{\phi}) = e^{-\bar{\phi}}. \quad (26)$$

Imposing the matching function (25) on Eqs. (24a) and (24b) in the steady state, respectively, implies

$$\bar{N} = \frac{\rho}{1 - (1 - \rho)e^{-\bar{\phi}}} N_0 \quad (27a)$$

$$\bar{M} = \frac{\rho \bar{\phi}}{\rho \bar{\phi} + (1 - e^{-\bar{\phi}})(1 - \rho)} M_0. \quad (27b)$$

Now, dividing Eq. (27b) by Eq. (27a), one obtains an equation that pins down $\phi$,

$$\rho \bar{\phi} = - (1 - \rho)(1 - e^{-\bar{\phi}}) + \left[ 1 - (1 - \rho)e^{-\bar{\phi}} \right] \frac{M_0}{N_0}. \quad (27c)$$

Thus, given Eq. (27c), Eqs. (27a) and (27b) are reduced form expressions for the numbers of candidates and vacancies, respectively, in the steady state.

Next, we calculate the unemployment rate at the end of each period in the steady state. This is found by multiplying $\bar{N}$ by the rate given in Eq. (26), which gives us the number of candidates unemployed at the end of each period in the steady state, and dividing this amount by the size of the labor force, $N_0$. Using Eq. (27a), this yields

$$\bar{U} = \frac{\rho}{e^\bar{\phi} - 1 + \rho}. \quad (28)$$

Table I illustrates a wide range of rates of unemployment from 0.4% when $\rho$ is small and $\bar{\phi}$ is large, to 15.3% when $\rho$ is large and $\bar{\phi}$ is small. Four different values of $\rho$ and four different values of $\bar{\phi}$ are represented in this table. The appropriate value of $\rho$ depends on the length of time each round of auctions takes (that is, the length of a time period in our model). For example, in Canada, the separation rate for prime-aged workers is approximately 4% per month (Kuhn and Sweetman, 1998). If each round of auctions takes one month, then this is the appropriate separation rate (i.e., $\rho = 0.04$). Alternatively, if each round of auctions takes only one week, then $\rho = 0.01$ is more appropriate. Unfortunately, no data on time lengths of rounds in the labor market are currently available. However, we believe that the range of $\rho$ is covered in Table I.
We also consider a range for the value of $\theta$, the ratio of vacancies to candidates in the steady state. Estimates of this ratio, based on advertised positions, range from 0.2 to 1.0. (See, for e.g., Blanchard and Diamond, 1989.) However, lower values of this ratio are typically seen as underestimates of vacancies, since many positions are not advertised. Notice that unemployment exists in this model even if $\theta = 1$. For example, if $\theta = 1$ and $\rho = 0.04$, then Table I states that $U = 0.023$. That is, in this case, the rate of unemployment is approximately 2.3%.

Qualitatively, it is easy to show from Eq. (28) that the unemployment rate is increasing in the separation rate $\rho$ and decreasing in labor market tightness ($\overline{\theta}$). Neither of these effects is any surprise, and both are common to most labor matching models, since they follow directly from random matching and flow identities.

3.2. An Equilibrium Payoff Structure

As mentioned above, this equilibrium matching process is consistent with more than one payoff structure. The key feature required to generate this matching process is that employers randomize over candidates. This can be rationalized by the mixed strategy equilibria that we consider. In this section we present a particular structure that has this property and, in the next, we briefly discuss others.

Consider an economy as in Subsection 3.1, but where risk neutral agents face an infinite horizon, perfect capital markets, and a common discount factor $\beta$. Employers have access to a project that produces a value $\theta$ in every period if and only if it has a worker to operate it. However, any match in the current period may dissolve in the subsequent period with probability $\rho$. If an employer is matched with a candidate in period $t$, then the expected present value of the payments accruing from the project is

$$
\Theta = \theta + \beta (1 - \rho) \theta + \beta^2 (1 - \rho)^2 \theta + \cdots = \frac{\theta}{1 - \beta (1 - \rho)}. \quad (29)
$$
Let $\Pi^*$ and $V^*$ now denote the stationary symmetric mixed strategy equilibrium payoffs accruing to employers and candidates (in present value terms), respectively. We assume that playing the game in the following period is the only outside option that either type of agent has. Thus, $\gamma = \beta \Pi^*$ and $\alpha = \beta V^*$. Because we will be focusing attention mainly on large economies, we assume that agents take $\Pi^*$ and $V^*$ as parametric, then solve for the rational expectations equilibrium. Let $\Lambda$ denote the expected value accruing to an employer if she is matched with a candidate, but the candidate is paid nothing. This is given by

$$\Lambda = \theta + \beta [\rho \Pi^* + (1 - \rho) \theta] + \beta^2 (1 - \rho) [\rho \Pi^* + (1 - \rho) \theta] + \cdots.$$ 

The first term represents the payoff in the current period. The second term represents the expected payoff in the next period: with probability $(1-\rho)$, the match will survive, and the employer will receive payoff $\theta$. With probability $\rho$, the match will dissolve and the employer will receive the equilibrium value of a vacancy, $\Pi^*$. Subsequent terms represent expected payoffs in subsequent periods. Collecting terms and using Eq. (29), we have

$$\Lambda = \Theta + \frac{\rho \beta \Pi^*}{1 - \beta (1 - \rho)}. \quad (30)$$

The second term on the right side of this equation represents the expected present value that accrues to employers in this situation due to the fact that they can play the game again whenever their match dissolves.

The most surplus that any candidate can expect to extract from an employer in a new match in the current period is given by $\Lambda - \beta \Pi^*$. Using Eq. (30), this amount can be written as

$$\Theta - \hat{\rho} \beta \Pi^*,$$

where

$$\hat{\rho} = \frac{(1 - \beta) (1 - \rho)}{1 - \beta (1 - \rho)}. \quad (31)$$

For this reason, we can think of the term $\hat{\rho} \beta \Pi^*$ as representing the implied outside option for employers. It indicates the present value of the probability that a matched pair will not separate at the end of the period, multiplied by the average payoff that an employer will get if she waits one period and enters the matching process again.

---

11 This phrase will be abbreviated simply as “equilibrium” hereafter in this section.
Let $\Sigma_i$ denote the equilibrium expected present value of a contract that candidate $i$ would be willing to work for, in period $t$, if only one employer approached the candidate. That is, $\Sigma_i$ denotes the *reserve contract*. To keep things as simple as possible, we consider the case where both the candidate and the employer can commit to honoring lifetime contracts: they stay together until separated exogenously. Let $W(\Sigma_i, m_i)$ denote the value of candidate $i$'s equilibrium contract (in present value terms), determined in the bidding game,

$$W(\Sigma_i, m_i) = \begin{cases} 
0 & \text{if } m_i = 0 \\
\Sigma_i & \text{if } m_i = 1 \\
\Theta - \hat{\rho} \beta \Pi^* & \text{if } m_i > 1.
\end{cases} \quad (32)$$

Given this bidding game, before knowing $m_i$, an employer's equilibrium expected payoff from making an offer to candidate $i$ is

$$\Pi_i(\Sigma_i, \Sigma) = (1 - p_i(\Sigma_i, \Sigma))^{\sigma-1}(\Lambda - \Sigma_i)$$

$$+ \left[1 - (1 - p_i(\Sigma_i, \Sigma))^{\sigma-1}\right] \beta \Pi^*, \quad (33)$$

where $p_i(\Sigma_i, \Sigma)$ denotes the probability that the employer approaches candidate $i$, assuming that all other candidates announce the same reserve contract $\Sigma$. Equation (33) is the analogue of Eq. (5) in the static model. Exactly as in the static model, each employer chooses $p_i(\Sigma_i, \Sigma)$, $i = 1, 2, \ldots, N$, so that $\Pi_i = \Pi$, $\forall i$. This yields

$$p_i(\Sigma_i, \Sigma) = 1 - \frac{N - 1}{1 + (N - 1)((\Lambda - \beta \Pi^* - \Sigma_i)/((\Lambda - \beta \Pi^* - \Sigma_i))^{1/\sigma-1}. \quad (34)$$

Given Eq. (34), candidate $i$ chooses $\Sigma_i$ to maximize the expected payoff function,

$$V_i(\Sigma_i, \Sigma) = q_{i0} \beta V^* + q_{i1} \Sigma_i + (1 - q_{i0} - q_{i1})(\Theta - \hat{\rho} \beta \Pi^*)$$

$$+ \frac{(1 - q_{i0}) \rho \beta V^*}{1 - \beta(1 - \rho)}, \quad (35)$$
where
\[ q_{i0} = q_{i0}(\Sigma_i, \Sigma) = (1 - p_i(\Sigma_i, \Sigma))^{\overline{M}} \]
and
\[ q_{i1} = q_{i1}(\Sigma_i, \Sigma) = M p_i(\Sigma_i, \Sigma) (1 - p_i(\Sigma_i, \Sigma))^{\overline{M} - 1} \]
denote the probabilities that candidate \( i \) will receive zero offers and one offer, respectively. The first three terms in (35) are completely analogous to those in Eq. (7) in the static model. They correspond to the payoffs accruing to candidates in the cases where zero, one, and more than one employer approaches the candidate, respectively. The last term is analogous to the second term in Eq. (30) faced by employers. It represents the expected present value of returns to a candidate when matches dissolve. Candidates can have access to this only under the condition that they are matched in the first place, which occurs with probability \( (1 - q_{i0}(\Sigma_i, \Sigma)) \).

As in the static model, in each period, candidates choose their reserve contracts simultaneously. The following proposition summarizes the unique (symmetric, stationary) equilibrium array of reserve contracts.

**Proposition 3.** The unique stationary symmetric equilibrium reserve contract is
\[
\Sigma_i = \Sigma^* = \frac{(M - 1)(\Theta - \hat{\rho}_i \Pi^*) + (N - 1)^2 \hat{\rho}_i V^*}{(M - 1) + (N - 1)^2}. \tag{36}
\]

**Proof.** See the Appendix.

In equilibrium, as in the static model, \( p_i^*(\Sigma^*, \Sigma^*) = 1/N \). Using this in Eqs. (33) and (35), we obtain
\[
\Pi^* = \left( \frac{N - 1}{N} \right)^{\frac{\overline{M}}{\overline{N} - 1}} (\Lambda - \Sigma^*) + \left( 1 - \left( \frac{N - 1}{N} \right)^{\frac{\overline{M}}{\overline{N} - 1}} \right) \beta \Pi^* \tag{37}
\]
\[
V^* = \left( \frac{N - 1}{N} \right)^{\frac{\overline{M}}{\overline{N}} + \phi \left( \frac{N - 1}{N} \right)^{\frac{\overline{M}}{\overline{N} - 1}} \Sigma^*} + \left( 1 - \left( \frac{N - 1}{N} \right)^{\frac{\overline{M}}{\overline{N}} + \phi \left( \frac{N - 1}{N} \right)^{\frac{\overline{M}}{\overline{N} - 1}} } \right) (\Theta - \hat{\rho}_i \Pi^*) + \left( 1 - \left( \frac{N - 1}{N} \right)^{\frac{\overline{M}}{\overline{N}}} \right) \frac{\rho \beta V^*}{1 - \beta (1 - \rho)}. \tag{38}
\]
Given \( \bar{N} \) and \( \bar{\phi} \) from Eqs. (27a), (27c), Eqs. (36), (37), and (38) can now be solved simultaneously for the equilibrium values of \( \Sigma^*, \Pi^*, \) and \( V^* \).

**The large economy.** Here, we restrict ourselves to considering the large economy. Once again, to avoid confusion, we denote equilibrium values in the large economy by using tildes. Equation (36) becomes

\[
\hat{\Sigma} = \hat{\rho} \hat{\beta} \hat{V}
\]

and, using Eq. (36'), Eqs. (37) and (38) become

\[
\hat{\Pi} = e^{-\hat{\phi}} \left( \frac{\theta + \rho \hat{\beta} \hat{\Pi} - (1 - \beta)(1 - \rho) \beta \hat{V}}{1 - \beta(1 - \rho)} \right) + (1 - e^{-\hat{\phi}}) \hat{\beta} \hat{\Pi}
\]

\[
\hat{V} = (1 - e^{-\phi} - \phi e^{-\phi}) \left( \frac{\theta - (1 - \beta)(1 - \rho) \beta \hat{\Pi} + \rho \beta \hat{V}}{1 - \beta(1 - \rho)} \right)
\]

\[
+ (e^{-\hat{\phi}} + \hat{\phi} e^{-\hat{\phi}}) \beta \hat{V}.
\]

Solving Eqs. (37') and (38') simultaneously, we obtain

\[
\hat{\Pi} = \frac{1 - \beta(1 - \rho)}{(1 - \beta) [e^{\hat{\phi}} - \beta(1 - \rho) \bar{\phi}]} \Theta
\]

\[
\hat{V} = \frac{(1 - \beta(1 - \rho))(e^{\hat{\phi}} - 1 - \bar{\phi})}{(1 - \beta) [e^{\hat{\phi}} - \beta(1 - \rho) \bar{\phi}]} \Theta.
\]

Equations (39) and (40) give closed-form solutions for the expected equilibrium payoffs accruing to candidates and vacancies under this payoff structure. These payoffs have the following intuitive properties. Both are increasing in the effective discount rate \( \beta(1 - \rho) \), since these payoffs are expressed as present values. Also, as in the static model, the tighter the market, the smaller is the payoff to each vacancy and the larger is the payoff to each candidate.\(^{12}\)

Having found \( \hat{\Pi} \) and \( \hat{V} \) it is then straightforward to calculate the values of the equilibrium contracts signed by workers, \( W(\hat{\Sigma}, \hat{m}_s) \). As indicated in Eq. (32), these contracts are a function of the number of employers that approach a candidate. We are now in a position to calculate the equilibrium degree of wage dispersion that this implies.

\(^{12}\) These comparative static results are derived in the Appendix.
**Wage dispersion in the large economy.** Let \( W(\bar{\xi}, 1) \) and \( W(\bar{\xi}, 2) \) denote the equilibrium expected values of contracts signed by workers who receive one offer, and more than one offer, respectively. The following proposition summarizes the equilibrium expected distribution of these contracts.

**Proposition 4.** The expected distribution of wage contracts in the large economy is given by

\[
\begin{align*}
\text{Fraction of employees} & \quad \text{Wage contract} \\
\frac{\theta e^{-\bar{\xi}}}{1 - e^{-\bar{\xi}}} & \quad W(\bar{\xi}, 1) = \frac{\beta(1 - \rho)\left[e^{\bar{\xi}} - 1 - \frac{\bar{\phi}}{\bar{\phi}}\right]}{e^{\bar{\xi}} - \beta(1 - \rho)\phi} \Theta \quad (41a) \\
\frac{1 - e^{-\bar{\xi}} - \theta e^{-\bar{\xi}}}{1 - e^{-\bar{\xi}}} & \quad W(\bar{\xi}, 2) = \frac{e^{\bar{\xi}} - \beta(1 - \rho)(1 + \frac{\bar{\phi}}{\bar{\phi}})}{e^{\bar{\xi}} - \beta(1 - \rho)\phi} \Theta. \quad (41b)
\end{align*}
\]

**Proof.** Substitution of Eqs. (39), (40), and (36') into Eq. (32) using (29) yields the wage contracts (41a) and (41b). The fractions of each type of employee are found by taking the fraction of each type of candidate, given in Subsection 2.5 above, and dividing by the fraction of candidates that become employed in any period \( (1 - e^{-\bar{\xi}}) \).

Given the results in Proposition 4, it is now possible to derive the different moments of the equilibrium wage distribution and to conduct comparative static exercises to see how these are affected by parameters in the model. Since both \( W(\bar{\xi}, 1) \) and \( W(\bar{\xi}, 2) \) are proportional to \( \Theta \), and \( \Theta \) is a function of some of the parameters in the model that we are interested in varying, comparative static arguments are more meaningful if we consider the effects on the wage shares \( W(\bar{\xi}, 1)/\Theta \) and \( W(\bar{\xi}, 2)/\Theta \). Similarly, we have chosen the coefficient of variation as our measure of dispersion, since this allows for more meaningful comparisons than standard deviation. The next proposition summarizes these results.

**Proposition 5.**

(i) The equilibrium mean wage share is an increasing function of labor market tightness \( \bar{\xi} \).

(ii) The effect of increases in the effective discount factor \( \beta(1 - \rho) \) on equilibrium mean wage shares is negative if \( \bar{\phi} < 1 \), zero if \( \bar{\phi} = 1 \), and positive if \( \bar{\phi} > 1 \).

(iii) Equilibrium wage dispersion is a decreasing function of the effective discount factor \( \beta(1 - \rho) \).

(iv) Equilibrium wage dispersion is a decreasing function of labor market tightness \( \bar{\xi} \).
Proof. See the Appendix.

The first part of this proposition is very straightforward and intuitive. As \( \overline{\phi} \) increases, with more employers per candidate, the bargaining power of candidates increases, increasing both \( W(\overline{\xi}, 1) \) and \( W(\overline{\xi}, 2) \). The second part is less obvious. This stems from the fact that \( W(\overline{\xi}, 1) \) is equal to the candidates’ outside option, while \( W(\overline{\xi}, 2) \) is a decreasing function of the employers’ (see Eq. (32)). As the effective discount factor \( \beta(1 - \rho) \) increases, both outside options become more valuable (since both outside options are increasing in the value functions, which are increasing in the effective discount factor). Hence, \( W(\overline{\xi}, 1) \) is an increasing function, and \( W(\overline{\xi}, 2) \) is a decreasing function, of \( \beta(1 - \rho) \). When \( \overline{\phi} < 1 \), employers have more bargaining power, and the negative effect of \( \beta(1 - \rho) \) on \( W(\overline{\xi}, 2) \) dominates the positive effect on \( W(\overline{\xi}, 1) \), so mean wages increase. When \( \overline{\phi} > 1 \), the relative strengths of these effects is reversed; and when \( \overline{\phi} = 1 \), these effects offset each other exactly.

The reasoning behind the third part of Proposition 5 is relatively simple. The wage premium enjoyed by workers who receive multiple offers exists solely due to the fact that, once employers have decided who to make offers to, it takes time for employers to make offers to other workers. This is seen by the fact that if \( \overline{\phi} = 0.95 \), then Eqs. (41a), (41b) imply that \( W(\overline{\xi}, 1) = W(\overline{\xi}, 2) \). Thus, employers are willing to bid workers’ wages above their reserve values once they know that other employers also have offers outstanding to these workers. Employers do this to try to avoid the delay inherent in not hiring in the current round and making an offer to another candidate in the next round of the market.

The fourth part of the proposition has the following reasoning, which is linked to the first part. When \( \overline{\phi} \) is low, then the mean wage share is low. With high returns, employers are eager to hire workers and are willing to pay a significant premium, in a bidding war, in order to employ a worker as soon as possible. In these cases, the “lucky” workers are paid significantly more than unlucky ones in relative terms, but both are still paid only a minor share of the surplus generated by the match.

To get a sense of the size of these effects, we constructed Table II. This table presents mean wage shares and coefficients of variation for different values of \( \beta(1 - \rho) \) and \( \overline{\phi} \). Values of \( \beta(1 - \rho) \) are listed across the top (varying from 0.95 to 0.999) and values of \( \overline{\phi} \) are given on the side (varying from 0.2 to 1.2). The cells present mean wages (expressed as percentages of total surplus) with coefficients of variation in brackets.

Examining Table II, it is clear that the effects of changing \( \overline{\phi} \) upon mean wage shares can be quite large. For example, examining the second column of Table II, when \( \beta(1 - \rho) = 0.99 \), and \( \overline{\phi} = 0.2 \) then the mean wage is only 2.1858% of the total surplus. With the same value of \( \beta(1 - \rho) \), but
TABLE II
Equilibrium Wage Dispersion

<table>
<thead>
<tr>
<th>$\bar{\phi}$ \ $\beta(1 - \rho)$</th>
<th>0.95</th>
<th>0.99</th>
<th>0.995</th>
<th>0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2.5435</td>
<td>2.1858</td>
<td>2.1407</td>
<td>2.1045</td>
</tr>
<tr>
<td>(68.785)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>13.648</td>
<td>13.089</td>
<td>13.018</td>
<td>12.961</td>
</tr>
<tr>
<td>(21.633)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>41.802</td>
<td>41.802</td>
<td>41.802</td>
<td>41.802</td>
</tr>
<tr>
<td>(9.0691)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>52.485</td>
<td>52.762</td>
<td>52.797</td>
<td>52.826</td>
</tr>
<tr>
<td>(7.2496)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Wage dispersion, however, is quite sensitive to changes in both $\beta(1 - \rho)$ and $\bar{\phi}$. For example, in Table II when $\bar{\phi} = 0.2$ (along the first row, with numbers in brackets), the standard deviation of the wage distribution is a huge 68.785% of its mean when $\beta(1 - \rho) = 0.95$, but is only 1.6788% when $\beta(1 - \rho) = 0.999$. Similarly, when $\bar{\phi} = 1.2$, the mean wage share varies only from 52.458% when $\beta(1 - \rho) = 0.95$ to 52.826% (when $\beta(1 - \rho) = 0.999$).

Overall, the implication that mean wage shares increase with labor market tightness is no surprise and is common to almost any labor market model. The effect of the discount rate on mean wage shares is not as straightforward; but this effect is small and it is difficult to imagine a data set rich enough to test this. However, the fact that wage dispersion significantly decreases as both the discount factor and labor market tightness increase is both novel and central to the theory. We believe that both of these implications are, in principle, testable.

4. CONCLUSIONS AND FURTHER RESEARCH

The overall goal of this project was to gain a deeper understanding of the role of different frictions in the labor market matching process. In this paper, we got a start by imposing an institutional environment which, we
believe, matches most labor markets quite closely. We then stripped away most frictions to focus on the basic problems of capacity constraints and coordination. We found that these problems, alone, can generate a matching function with reasonable properties, some unemployment, and some wage dispersion. Some testable implications of the model were drawn out. We also found that, despite the existence of an externality when vacancies enter, equilibrium entry in this environment is constrained-efficient when the market is large.

We hope that this very simple model can serve as something of a benchmark, when more frictions are introduced. Fortunately, the basic theoretical framework used here, competing auction theory, appears to be flexible enough to allow for a systematic study of the effects of each friction. Many of these frictions have already been analyzed in related settings, which could be extended to this environment. For example, a key assumption in this paper, which could be relaxed in a fairly straightforward way, is that of commitment in the dynamic model. When one relaxes this assumption, the possibility of endogenous separations arises—leading to a discussion of renegotiation-proof wage contracts. Work on commitment issues in the presence of repeat auctions (but not competing auctions) already exists (King and Welling, 1993) which can be drawn on. Heterogeneity can also be introduced into this model quite easily. Work along this line in wage-posting models (Montgomery, 1991) and preliminary work with competing auction models (Julien et al., 1999) have found that the heterogeneous quality of candidates raises the unemployment rate. Work on two-sided heterogeneity in wage posting models is given in Shi (1998). Informational asymmetries in competing auctions are also studied in Peters and Severinov (1997) and Julien (1997). Investment issues in auction-theoretic settings have also been considered (King, McAfee, and Welling, 1992, 1993). We believe that modeling the labor market in this way has the potential to shed new light on issues such as human capital choices, technology choices, and public policy.

APPENDIX

Proof of Proposition 1. Differentiating $V_i(\sigma_i, \sigma^*)$ with respect to $\sigma_i$ yields the first order conditions,

$$q_{i1} \left( 1 + \frac{\Phi}{p_i(\sigma_i, \sigma^*)(1 - p_i(\sigma_i, \sigma^*))} \frac{\partial p_i(\sigma_i, \sigma^*)}{\partial \sigma_i} \right) = 0 \quad \forall i,$$
where \( q_{i1} = M \phi_i(\sigma_i, \sigma^*)(1 - p_i(\sigma_i, \sigma^*))^{M-1} > 0 \) and \( \Phi = (M - 1)p_i(\sigma_i, \sigma^*)(\theta - \sigma_i - \gamma) + (1 - p_i(\sigma_i, \sigma^*)(\sigma_i - \alpha) > 0 \). Since

\[
\frac{\partial p_i(\sigma_i, \sigma^*)}{\partial \sigma_i} = -\frac{(N - 1)(\theta - \sigma_i - \gamma)^{1/(M-1)-1}(1 - p_i(\sigma_i, \sigma^*))}{(M - 1)\left[(\theta - \sigma^* - \gamma)^{1/(M-1)} + (N - 1)(\theta - \sigma_i - \gamma)^{1/(M-1)}\right]}
\]

then

\[
q_{i1}\left(1 - \frac{(\theta - \sigma_i - \gamma)^{1/(M-1)-1}\Phi}{(M - 1)p_i(\sigma_i, \sigma^*)\left[(\theta - \sigma^* - \gamma)^{1/(M-1)} + (N - 1)(\theta - \sigma_i - \gamma)^{1/(M-1)}\right]}\right) = 0 \quad \forall i.
\]

At this point, symmetry can be exploited. Setting \( \sigma_i = \sigma^* \) and \( p_i(\sigma^*, \sigma^*) = 1/N \) in the first order conditions yields Eq. (8),

\[
\sigma^* = \frac{(M - 1)(\theta - \gamma) + (N - 1)^2 \alpha}{(M - 1) + (N - 1)^2}.
\]

We now demonstrate that \( V_i(\sigma_i, \sigma^*) \) is strictly concave in \( \sigma_i \), over \( [\alpha, \theta - \gamma] \). Let \( \partial V_i(\sigma_i, \sigma^*)/\partial \sigma_i = q_{i1} Z \) where

\[
Z = \left(1 + \frac{\Phi}{p_i(\sigma_i, \sigma^*)(1 - p_i(\sigma_i, \sigma^*))} \frac{\partial p_i(\sigma_i, \sigma^*)}{\partial \sigma_i}\right).
\]

Then

\[
\frac{\partial^2 V_i(\sigma_i, \sigma^*)}{\partial \sigma_i^2} = \frac{\partial q_{i1}}{\partial \sigma_i} Z + q_{i1} \frac{\partial Z}{\partial \sigma_i}
\]

\[
= \frac{q_{i1}}{p_i(1 - p_i)} \frac{\partial p_i}{\partial \sigma_i} (1 - M \phi_i) \left(1 + \frac{\Phi}{p_i(1 - p_i)} \frac{\partial p_i}{\partial \sigma_i}\right)
\]

\[
+ q_{i1} \frac{(p_i(1 - p_i)(\partial \Phi/\partial \sigma_i) - \Phi(1 - 2p_i)(\partial p_i/\partial \sigma_i))}{(p_i(1 - p_i))^2}
\]

\[
\times \frac{\partial p_i}{\partial \sigma_i} + \frac{q_{i1}\Phi}{p_i(1 - p_i)} \frac{\partial^2 p_i}{\partial \sigma_i^2}.
\]
Since \( \frac{\partial \Phi}{\partial \sigma_i} = ((M - 1)(\theta - \gamma - \sigma_i) - (\sigma_i - \alpha)\frac{\partial p_i}{\partial \sigma_i} + (1 - M p_i) \),

then

\[
\frac{\partial^2 V_i(\sigma_i, \sigma^* \sigma_i)}{\partial \sigma_i^2} = \frac{2q_{i1}}{p_i(1 - p_i)} \frac{\partial p_i}{\partial \sigma_i} (1 - M p_i) \\
+ q_{i1} \left( (M - 1)(\theta - \gamma - \sigma_i) - (\sigma_i - \alpha) \right) \left( \frac{\partial p_i}{\partial \sigma_i} \right)^2 \\
- \frac{(M - 2)p_i q_{i1} \Phi}{p_i(1 - p_i)} \left( \frac{\partial p_i}{\partial \sigma_i} \right)^2 + \frac{q_{i1} \Phi}{p_i(1 - p_i)} \frac{\partial^2 p_i}{\partial \sigma_i^2}.
\]

Note

\[
\frac{\partial^2 p_i}{\partial \sigma_i^2} = \frac{M - 2}{\theta - \gamma - \sigma_i} \frac{\partial p_i}{\partial \sigma_i} - \frac{2}{1 - p_i} \left( \frac{\partial p_i}{\partial \sigma_i} \right)^2 < 0
\]

(that is, \( p_i \) is strictly concave in \( \sigma_i \)).

Now

\[
\frac{\partial^2 V_i(\sigma_i, \sigma^* \sigma_i)}{\partial \sigma_i^2} = \frac{q_{i1}}{p_i(1 - p_i)} \frac{\partial p_i}{\partial \sigma_i} \left( 2(1 + p_i) + M p_i (M - 5) \\
+ \frac{(M - 2)(1 - p_i)(\sigma_i - \alpha)}{\theta - \gamma - \sigma_i} \right) \\
+ \frac{q_{i1}}{p_i(1 - p_i)} \left( (M - 1)(\theta - \gamma - \sigma_i) \\
- (\sigma_i - \alpha) - \frac{M \Phi}{p_i(1 - p_i)} \right) \left( \frac{\partial p_i}{\partial \sigma_i} \right)^2.
\]

Examining this expression, it is easily shown that the last term is negative by substituting in the expression for \( \Phi \) and expanding. The first term is clearly negative if \( M \geq 5 \), in which case the entire expression is negative. Moreover, it can easily be shown that the entire expression is negative for the cases \( M = 1, 2, 3, 4 \). In these cases, the term \( M p_i (M - 5) \) is negative but not large enough in absolute value to dominate the whole expression.

Hence, \( \frac{\partial^2 V_i(\sigma_i, \sigma^* \sigma_i)}{\partial \sigma_i^2} < 0 \) and the unique global maximum over \( [\sigma, \theta - \gamma] \) occurs at \( \sigma^* \).
Proof of Proposition 3. Differentiating $V_i(\Sigma, \Sigma^*)$ with respect to $\Sigma_i$ (bearing in mind that $\partial V^*/\partial \Sigma_i = \partial \Pi^*/\partial \Sigma_i = 0 \forall i$), and collecting terms, yields the first order conditions

\[
(M - 1) p_i \frac{\partial p_i}{\partial \Sigma_i} (\Theta - \hat{\beta} \Pi^* - \Sigma^*)
\]

\[
= (1 - p_i) \left( (\hat{\beta} V^* - \Sigma^*) \frac{\partial p_i}{\partial \Sigma_i} - p_i \right) \quad \forall i.
\]

Now, from Eq. (34), using the fact that $\Lambda - \beta \Pi^* = \Theta - \hat{\beta} \Pi^*$, we have

\[
\frac{\partial p_i}{\partial \Sigma_i} = \frac{-(N - 1)^2 \left( \Theta - \hat{\beta} \Pi^* - \Sigma_i \right)^{1/(M-1)}}{(M - 1)(\Theta - \hat{\beta} \Pi^* - \Sigma_i) \left( 1 + (N - 1) \left( \frac{\Theta - \hat{\beta} \Pi^* - \Sigma_i}{\Theta - \hat{\beta} \Pi^* - \Sigma^*} \right)^{1/(M-1)} \right)}.
\]

At this point, symmetry can be exploited. Setting $\Sigma_i = \Sigma^*$ in this expression, and in Eq. (34), one obtains

\[
\frac{\partial p_i}{\partial \Sigma_i} = \frac{-(N - 1)^2}{(M - 1)(\Theta - \hat{\beta} \Pi^* - \Sigma^*) N} \quad \text{and} \quad p_i = \frac{1}{N}.
\]

Substituting these two expressions into the above first order conditions and solving for $\Sigma^*$ yields Eq. (36),

\[
\Sigma^* = \frac{(M - 1)(\Theta - \hat{\beta} \Pi^*) + (N - 1)^2 \hat{\beta} V^*}{(M - 1) + (N - 1)^2}.
\]

Now, using an argument completely analogous to the one given in the proof of Proposition 1, it is straightforward to show that $V_i(\Sigma, \Sigma^*)$ is strictly concave in $\Sigma_i$ over $[\hat{\beta} V^*, \Theta - \hat{\beta} \Pi^*]$. This implies that the unique global maximum over the relevant domain occurs at $\Sigma^*$.
Properties of the equilibrium payoff functions, Eqs. (39) and (40).

\[
\frac{\partial \check{\Pi}}{\partial \beta(1 - \rho)} = \frac{\check{\phi} \theta}{(1 - \beta) \left[ e^\check{\phi} - \beta (1 - \rho) \check{\phi} \right]^2} > 0
\]

\[
\frac{\partial \check{V}}{\partial \beta(1 - \rho)} = \frac{\left( e^\check{\phi} - 1 - \check{\phi} \right) \check{\phi} \theta}{(1 - \beta) \left[ e^\check{\phi} - \beta (1 - \rho) \check{\phi} \right]^2} > 0
\]

\[
\frac{\partial \check{\Pi}}{\partial \check{\phi}} = -\frac{\left[ e^\check{\phi} - \beta (1 - \rho) \right] \theta}{(1 - \beta) \left[ e^\check{\phi} - \beta (1 - \rho) \check{\phi} \right]^2} < 0
\]

\[
\frac{\partial \check{V}}{\partial \check{\phi}} = \frac{\left[ \check{\phi} e^\check{\phi} [1 - \beta (1 - \rho)] + \beta (1 - \rho) (e^\check{\phi} - 1) \right] \theta}{(1 - \beta) \left[ e^\check{\phi} - \beta (1 - \rho) \check{\phi} \right]^2} > 0.
\]

**Proof of Proposition 5.** From Proposition 4, the equilibrium mean wage share is

\[
\mu = \frac{\check{\phi} e^{-\check{\phi}}}{1 - e^{-\check{\phi}}} W(\check{\xi}, 1) + \frac{1 - e^{-\check{\phi}} - \check{\phi} e^{-\check{\phi}}}{1 - e^{-\check{\phi}}} W(\check{\xi}, 2) \Theta
\]

which simplifies to

\[
\mu = \frac{\left[ e^\check{\phi} \beta (1 - \rho) \right] \left[ e^\check{\phi} - 1 - \check{\phi} \right]}{\left[ e^\check{\phi} - 1 \right] \left[ e^\check{\phi} - \beta (1 - \rho) \check{\phi} \right]}
\]

which yields

\[
\frac{\partial \mu}{\partial \beta(1 - \rho)} = \left. \frac{\check{\phi} \left[ e^\check{\phi} - 1 - \check{\phi} \right] (\check{\phi} - 1)}{\left[ e^\check{\phi} - 1 \right] \left[ e^\check{\phi} - \beta (1 - \rho) \check{\phi} \right]^2} \right| < 0 \quad \text{if } \check{\phi} < 1
\]

\[
\frac{\partial \mu}{\partial \check{\phi}} = \left. \frac{\left( e^\check{\phi} - 1 \right) \left[ \beta (1 - \rho) (e^\check{\phi} - 1) - (1 - \beta (1 - \rho)) e^\check{\phi} \right] + (1 - \beta (1 - \rho) \check{\phi} e^{\check{\phi}}}{\left[ e^\check{\phi} - 1 \right]^2 \left[ e^\check{\phi} - \beta (1 - \rho) \check{\phi} \right]} \right| > 0 \quad \text{if } \check{\phi} > 1
\]
Since the variance is
\[
\operatorname{var} = \frac{\Phi e^{-\beta}}{1 - e^{-\beta}} \left( \frac{W(\hat{\Sigma}, 1)}{\Theta} \right)^2 + \frac{1 - e^{-\beta} - \Phi e^{-\beta}}{1 - e^{-\beta}} \left( \frac{W(\hat{\Sigma}, 2)}{\Theta} \right)^2 - \mu^2
\]
using Proposition 4, we have
\[
\operatorname{var} = \frac{(1 - \beta(1 - \rho))^2(e^\beta - 1 - \Phi)e^2\beta}{(e^\beta - 1)^2 \left( e^\beta - \Phi(1 - \beta(1 - \rho)) \right)^2}.
\]
Now, using \(\mu\) from above,
\[
\frac{\operatorname{var}}{\mu^2} = \frac{(1 - \beta(1 - \rho))^2\Phi}{(\beta(1 - \rho))^2(e^\beta - 1 - \Phi)}.
\]
Since the coefficient of variation \(\nu = \sqrt{\operatorname{var}} / \mu\), then
\[
\frac{\partial \nu}{\partial \Phi} = \left( \frac{1 - \beta(1 - \rho)}{\beta(1 - \rho)(e^\beta - 1 - \Phi)} \right)^2 \left( e^\beta(1 - \Phi) - 1 \right) \mu \frac{1}{2 \operatorname{var}} < 0
\]
and
\[
\frac{\partial \nu}{\partial \beta(1 - \rho)} = - \frac{(1 - \beta(1 - \rho))\Phi \mu}{(\beta(1 - \rho))^3(e^\beta - 1 - \Phi) \operatorname{var}} < 0.
\]

REFERENCES


