Standard Auctions with Financially Constrained Bidders.

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Outline

1. Motivation
2. Model
3. Results
4. Implications
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1. Motivation
Motivation

▶ So far we’ve seen that buyers have private info over their willingness-to-pay, given by $f(v_i)$.
▶ But it seems plausible that buyers may also be heterogeneous over their ability-to-pay. Examples:

  ▶ Liquidity constraints may be binding if capital markets are imperfect, e.g. spectrum auctions (Klemperer et al), mineral leases, etc.
  ▶ Divisional budget constraints imposed by HQ due to moral hazard issues, e.g. Timber auctions (Athey, Levin, Seira).

⇒ Marginal cost of expenditure is increasing.
Motivating Questions

What does an increasing marginal cost of expenditure imply for:

- Revenue equivalence across auction mechanisms?
- Auction efficiency?
- Bidding strategies?

What other endogenous responses might we see from sellers and/or buyers?
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Motivation

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Motivation

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2. Model
Model Setup

- One indivisible item for sale, with $v_0 = 0$.
- $N$ risk neutral bidders, with two-dimensional private info of value and budget drawn iid from $f(v, w)$, with support on $[\underline{w}, \overline{w}] \times [\underline{v}, \overline{v}]$. NB: $v_i, w_i$ may be correlated, but independent across individuals.
- Bidder incurs cost $C(x, w)$ if he pays $x$ with budget $w$.
- Winner’s utility given by, $u_i = v_i - C(x_i, w_i)$. 
Assumptions on $C(x, w)$,

- MC of spending is positive and increasing in expenditure - formally: $C(x, w)$ is strictly increasing and (weakly) convex in $x$ for $x \leq \delta(w)$ where $\delta(w)$ is some upper bound.

- Buyers are approximately homogeneous in the cost function - formally: for any $w$ and $x < \delta(w)$, $\lim_{w' \to w} C(x, w') = C(x, w)$

- Larger budget sets $\implies$ less costly to raise a bid by a given amount.

- $C(x, \bar{w}) = x$ for $x \leq \bar{v}$, and $C(x, w) = x$ for $x \leq v$ $\implies$ unconstrained at the top and bottom.
Auction Properties

They are going to consider a family of auction rules which satisfy,

1. Highest bidder receives the object.
2. There exists a symmetric pure-strategy BNE in which $b = B(w, v)$, with $B)$., .) continuous - allows us to characterize an equilibrium using isobid curves.
3. $B(w, v)$ is increasing in $(w, v)$, and strictly increasing if both $w, v$ increase.
4. For any equilibrium bid, there exists an unconstrained type with budget $\bar{w}$ that makes the same bid - guarantees unconstrained type on each isobid curve.
Single Crossing

Define $\Omega_M(w, v)$ as the set of types that do not participate in auction $M$ or bid below that of a type $(w, v)$ bidder. Then,

Definition: Single Crossing
Auction $A$ satisfies the single-crossing property with respect to auction $B$ if, for any type $(w, v)$ that’s in both auctions,

$$\Omega_A^-(w, v) \subseteq \Omega_B^-(w, v), \quad \text{and} \quad \Omega_A^+(w, v) \supseteq \Omega_B^+(w, v)$$

where the superscript relates to whether $w > w'$ or $w < w'$, respectively.

Implication: isobid curves in space $(w, v)$ in auction $A$ cut those in $B$ from below, and at most once.
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Model

Single Crossing

Figure 1
Note,

- If bidder wins \( B \) with \((w, v)\) then \( \Omega_B(w, v) \) is the shaded area.

- By SCP the shaded area sits below the isobid curve for \( A \) \( \Rightarrow \) these bidders also lose in \( A \).
Implications

Key Theorem 1
Let $\hat{v}_A$ be the lowest valuation for a participating bidder. If $\hat{v}_A \leq \hat{v}_B$ and auction $A$ satisfies the SCP, then $A$ yields higher social surplus than $B$. Intuition: see previous figure.

Corollary
Flatter isobid curves $\implies$ financial constraints are weaker $\implies$ increased bidding competition, so that if $A$ satisfies the SCP wrt $B$ then $A$ yields (weakly) higher expected revenue (strict is the SCP is strict).
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Auction Comparisons

Consider absolute limits on expenditure so that it’s a dominated strategy to bid above one’s budget,

\[ C(x, w) = \begin{cases} x, & \text{for } x \leq w \\ \infty, & \text{for } x > w \end{cases} \]

and where \( v \leq w \leq \bar{v} \leq \bar{w} \).

SPA

- Participants are the set with \( \min \{w_i, v_i\} \geq r_s \)
- Vickrey’s results still hold - see PS.
Comparison of FPA and SPA

FPA

- Participants are the set with $\min\{w_i, v_i\} \geq r_s$
- Equilibrium bids are of the form $B_f(w, v) = \{w, b_f(v)\}$ for some continuous and increasing $b()$.
- Under some regularity conditions this bid is the same as in the unconstrained case, i.e. $b(v) = E[\max\{Y_1, r_s\}|Y_1 < v]$ where $Y_1$ is the highest of the other $N - 1$ values. Except...
- Valuations and budgets are drawn from a joint distribution $F(v, w)$. They illustrate that this leads to shading of bids relative to the unconstrained case, i.e. $b(v)$ is below $v$ by more than when $w$ is unconstrained.
Key Proposition 1: RET doesn’t hold.

For a given \( r_s \in [v, \bar{v}) \) a FPA yields higher social surplus, strictly higher expected social surplus and strictly higher expected revenue.
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Why? For a given reserve we have the same set of participants. In the FPA the bidder has \( b(v) < v \), so that the constraint is more likely binding in the SPA see figure 3 in the paper.
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Corollaries

- In SPA good is less likely to be allocated to the bidder who values it more \( \implies \) social surplus is lower in SPA.
- Increased likelihood of being budget constrained reduces seller’s expected revenue \( \implies \) revenue lower in SPA.
- Might account for why open outcry auction are used less frequently in public auctions (concern for social welfare).
Introducing Capital Markets

Buyers now participate iff $v \geq C(r, w)$
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Key Proposition 2: RET still doesn’t hold.

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Intuition: similar to the risk aversion result. There bidders are more aggressive in FPA and invariant to risk premia in SPA. Here bidders are less aggressive in both FPA and SPA, but more so in the SPA because the winning payment is now random.
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Note here that the social surplus comparison is no longer possible, generally.
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2. How should competition policy respond to mergers of bidders? Bigger firms may have easier access to capital markets → allow the merger, yet fewer bidders leads to lower revenue and increased possibility of collusion → disallow the merger.
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3. What of entry fees instead of a reserve? Allows for the seller’s value to be spread across all bidders, hence alleviating the financial constraint → Che and Gale (1996) show that entry fees with financial constraints increases revenue.