Partial identification and testable restrictions in multi-unit auctions

D. McAdams (2008)

Presented by
Pamela Medina

Duke University

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Motivation

- Multi-unit auctions → Multiple identical objects are bought, sold or traded.
- Result in single object auction: distribution from bidders' values is point identified from the distribution of bids given IPV.

**Question** → How can we interpret the data in multi-unit auctions given IPV?

1. Distribution of bidders' values is *not necessarily* point-identified from the distribution of bids under the assumption of equilibrium bidding, and asymmetric IPV. We can get bounds if bidders have non-increasing marginal values (NIMV) in both discriminatory and uniform-price auctions.

2. Testable implications of the hypothesis that bidder i’s strategy is a best response to the distribution of other bids, assuming bidders have IPV and bidder i has NIMV.
Guerre et al. (2000) used FOC of optimal bidding to identify the distribution of bidder values from the distribution of equilibrium bids.

Not the same in multi-unit auctions. Why?

- Equilibrium bid in an auction with S identical units is an S-dimensional demand schedule and a “value” is an S-dimensional marginal value schedule.
- Many different marginal value schedules that can rationalize a given bid as a best response.

What to do? → Use partial identification: find bounds.
The Model

- S identical indivisible objects ("units") are sold to N risk-neutral bidders.
- Each bidder i’s marginal value schedule (or "value") takes the form $v_i = (v_{i,1}, \ldots, v_{i,S})$.
- IPV. Distribution of values is common knowledge among the bidders, but unknown to the econometrician.
- Auction rules:
  - Permissible bids. Each bidder submits a bid $b_i = (b_{i,1}, \ldots, b_{i,S})$ such that $b_{i,q} \geq b_{i,q'}$ for all $q < q'$.
  - Allocation rule: Highest S unit-bids win.
  - Payment rule: Bidder i pays the sum of his unit-bids on what he wins.
- Winning probability: $w_{i,q}(x) = \Pr(x > \tilde{s}_{i,q})$ where $\tilde{s}_{i,q} = b_{-i}^{S-q+1}$ which is defined as the maximum $(S-q+1)$th bid made by of the others.
- Assume winning probability is continuously differentiable at $b_{i,q}$ for all q.
Bidder’s i interim payoff:

\[ \Pi_i(b_i, v_i; \tilde{s}_i) = \sum_{q=1}^{S} \Pi_{i,q}(b_{i,q}, v_{i,q}; \tilde{s}_{i,q}) \]

where \( \Pi(b_{i,q}, v_{i,q}; \tilde{s}_{i,q}) = w_{i,q}(x)(v_{i,q} - b_{i,q}) \).

Here you assume q things: (i) bidder i has some probability of winning all S units for all q and (ii) bidder i faces no “gaps” in the distribution of competing bids. However, you lose generality.

Gaps: A bidder i faces a ”gap” at price p for quantity q if \( w'_{i,q}(p) = 0 \) and \( w_{i,q}(p) > 0 \).

Usually, gaps are not best responde. BUT they might be for discriminatory auctions of more than 2 units.
The Model

- The distribution of values $\tilde{v}$ rationalizes a given distribution of bids $\tilde{b}$ if, when values are distributed as $\tilde{v}$, there exists a Bayesian Nash equilibrium profile of (possibly mixed) strategies $\sigma$ such that $\sigma(\tilde{v})$ is distributed as $\tilde{b}$.

- Characterizing the set of best responses in this problem might be a challenging task. Therefore, consider a larger set of values that satisfy FOC associated with bid $b$ (necessary conditions): $V_{i}^{FOC}(b_i)$.

- Let $v_{i,X}^{*}(b_i)$ be the indifference level of marginal values where bidder $i$ is indifferent between raising or lowering his unit-bids on all quantities in $X$. For every bid, $(v_{i,1}(b_i), \ldots, v_{i,S}(b_i)) \in V_{i}^{FOC}(b_i)$.

- In the discriminatory auction, this is the only marginal value schedule than can rationalize bidder $i$’s bid schedule if that bid schedule is strictly decreasing in quantity. If same for several units, then, we can have several marginal value schedules.
The Model

- NIMV: \( v_{i,q} \geq v_{i,q'} \) for all \( q < q' \).

- Step of quantities: \( Q(b) = \{ q \in \{1, ..., S\} : b_{i,q} = b \} \).

- Theorem 2: Bounds
  - \( v_i \in V_i^{FOC}(b_i) \cap V_i^{NIMV} \) implies \( v_{i,q} \in [v_{i,q}(b_i), \overline{v}_{i,q}(b_i)] \).
  - As long as \( V_i^{FOC}(b_i) \cap V_i^{NIMV} \neq \emptyset \), there exists \( v_i', v_i'' \in V_i^{FOC}(b_i) \cap V_i^{NIMV} \) such that \( v_{i,q} = \overline{v}_{i,q}(b_i) \) and \( v_{i,q} = \overline{v}_{i,q}(b_i) \).
The Model

![Diagram showing the model with price on the y-axis and quantity on the x-axis. The diagram includes points labeled with mathematical expressions: \( \min Q(b_{i,q}) \), \( q \), and \( q' \), and a horizontal line at a constant price level.](image-url)
Outline

1 Introduction

2 The Model

3 Testable Restrictions
Testable Restrictions

- Suppose that an econometrician does not know the true distribution of bids but observes a sample of $M$ discriminatory auctions, in which the same bidders have IPV drawn from the same distribution and play the same equilibrium strategies in each auction.

- Consider the discretized version of the hazard rates

$$
\Delta + \frac{w_{i,q_1}(b)}{w_{i,q_1}(b+\Delta) - w_{i,q_1}(b)} \geq \frac{w_{i,q_2}(b - \Delta)}{w_{i,q_2}(b) - w_{i,q_2}(b - \Delta)}.
$$

- Evaluating DISCRIM–$(q_1, q_2)$ given a sample of $M$ identical auctions is equivalent to a standard problem of testing a moment inequality involving multinomial probabilities given $M$ iid draws from the relevant multinomial distribution.