A Theory of Auction and Competitive Bidding
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Outline

- Introduction
- Overview of the received auction theory
- General model and its variations
- Conclusion
1. Introduction

2. Overview of the received auction theory

3. General model and its variations

4. Conclusion
Research Questions

- What accounts for the popularity of such common auction forms as the English auction, the Dutch auction, the first-price sealed-bid auction, and the second-price sealed-bid auction?
- What determines which form should be used in any particular circumstance?
- What is the relationship between auction theory and traditional competitive theory?
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The Independent Private Values Model

- a single indivisible object is to be sold to only one bidder
- risk-neutral bidders, private values assumption
- values are independently drawn from some continuous distribution
- competitively bidding, noncooperative game
Main Conclusions

1. Dutch auction and the first-price auction are strategically equivalent.
2. The second-price sealed-bid auction and the English auction are equivalent.
3. The outcome of the English and second-price auctions is Pareto optimal.
4. All four auction forms lead to identical expected revenues for the seller.
5. The expected revenue generated for the seller is precisely the expected value of the object to the second-highest evaluator.
6. The four standard auction forms with suitably chosen reserve prices or entry fees are optimal auctions.
7. If the sellers or the buyers are risk averse, the seller will strictly prefer the Dutch or first-price auction to the English or second-price auction.
Mineral Rights Model

- Risk-neutral bidder make independent estimates of the common value where the estimates are drawn from a single underlying distribution parameterized by $V$.

- Winner’s Curse: even if all bidders make unbiased estimates, the winner will find that he had overestimated (on average) the value of rights he has won at auction.

- A bidder’s expected profits in a mineral rights auction depend more on the privacy of his information than on its accuracy as information about $V$. 
Limitations: A Painting Example

- Assumptions needed for independent private values model:
  1. each bidder knows his value for the painting
  2. the values are statistically independent

- Assumption 1 rules out the possibilities: (i) resold; (ii) “prestige” value; (iii) authenticity doubt

- Assumption 2 rules out the possibility that several bidders may have relevant information concerning the painting’s authenticity, or that a buyer, thinking that the painting is particularly fine, may conclude that other bidders also are likely to value it highly

- Little guidance is forthcoming: the four most common auction forms lead to the same expected price

- The mineral rights theory allows for statistical dependence among bidders’ value estimates, but offers no role for differences in individual tastes.
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value estimates or signals:  \( X = (X_1, \ldots, X_n) \)

bidder \( i \)'s actual value:  \( V_i = u_i(S, X) \)

\( S \) might be observed by the seller, can capture things like appraisals obtained by the seller

\( V_i \) may not observed by bidder \( i \) at the time of the auction

joint probability density of the random elements:  \( f(s, x) \)
Assumptions

Assumption 1

There is a function $u$ on $\mathbb{R}^{m+n}$, such that for all $i$, $u_i(S, X) = u(S, X_i, \{X_j\}_{j \neq i})$. Consequently, all of the bidders’ valuations depend on in the same manner, and each bidder’s valuation is a symmetric function of the other bidders’ signals.

Assumption 2

There is a function $u$ is nonnegative, and is continuous and nondecreasing in its variables.

Assumption 3

For each $i$, $E[V_i] < \infty$.

Assumption 4

$f$ is symmetric in its last $n$ arguments.
Assumptions

Assumption 5

The variables $S_1, \ldots, S_m, X_1, \ldots X_n$ are affiliated.

Definition

Let $z$ and $z'$ be points in $\mathbb{R}^{m+n}$. Let $z \vee z'$ denote the component-wise maximum of $z$ and $z'$, and let $z \wedge z'$ denote the component-wise minimum. We say that the variables of the model are affiliated if, for all $z$ and $z'$, $f(z \vee z')f(z \wedge z') \geq f(z)f(z')$.

Remark

This condition means that large values for some of the variables make the other variables more likely to be large than small. A high value of one bidder’s estimate makes high values of the others’ estimates more likely.
The Dutch and first-price auctions are strategically equivalent in the general model.

The English auction generally leads to larger expected prices, when bidders are uncertain about their value estimates.

The second-price auction generates a higher average price than does the first-price auction, when the bidders’ value estimates are statistically dependent.

Honesty is the best policy for the first-price, second-price, and English auctions.
Introducing Reserve Prices and Entry Fees

- A bidder will participate in the auction if and only if his expected profit from bidding (given the reserve prices) exceeds the entry fee. (known as screening level)

- For each type of auction we study, any particular screening level can be achieved by a continuum of different combinations of reserve prices and entry fees.

- So long as regularity is preserved and the screening level is held fixed, it pays to raise entry fees and reduce reserve prices.
Revenue-maximizing fee is positive in the first-price auction with reserve prices and entry fees.

With constant absolute risk aversion, the English auction generates higher average prices than the second-price auction.

The best information-reporting policy: reveal fully the bidder’s information.
Most analyses of competitive bidding situations are based on the assumption that each auction can be treated in isolation.

Little is understood about simultaneous auctions, or about the effects of the resale market in drilling rights on the equilibria in the auction games.

The noncooperative game formulation of auctions seems to be reasonable.

Few bidding literature concerns auctions for shares of a divisible object.

Much remains to be done in the theory of auctions.