The Ten Games: A Novel Categorization of All 2x2 Games

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Abstract

This paper provides a pedagogically useful categorization of all 2x2 games as one of ten types: (i) Slam Dunk, (ii) Immovable Object, (iii) Prisoners' Dilemma, (iv) Happy Marriage, (v) Food Fight, (vi) Master and Beloved Servant, (vii) Master and Annoying Servant, (viii) Assurance, (ix) Chicken, and (x) Hide and Seek. For each type of game, I characterize the commitment strategies (e.g. “moving first” or “making a promise”) that allow each player to enjoy his/her best achievable payoff.

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1. Introduction

All students of economics learn at least a little game theory. They learn at least a few important concepts, such as dominant strategy and Nash equilibrium, and how to apply those concepts in at least a few important examples of games, such as the Prisoners' Dilemma and the Battle of the Sexes. Should a student find herself playing one of these well-understood games, she will know how to apply what she has learned and enjoy a strategic advantage. But what if that same student finds herself in a game that was not covered in class? Even if she understands how to analyze a game fully and correctly, such an analysis may take more time than is available. (Time constraints can be especially pressing in job interviews, when students are often asked to provide their quick assessments of a strategic situation.)

This paper provides a welcome short-cut to identify and analyze any 2x2 game, i.e. any game played once by two players who each have two strategic options. First, given any 2x2 game, I provide a sequence of five questions (“The Question Tree”) to identify what type of game it is:² (i) Slam Dunk, (ii) Immovable Object, (iii) Happy Marriage, (iv) Food Fight, (v) Prisoners' Dilemma, (vi) Master and Beloved Servant, (vii) Master and Annoying Servant, (viii) Assurance, (ix) Chicken, or (x) Hide and Seek. For each of these “Ten Games”, the paper then characterizes each player’s “optimal commitment strategy”, e.g. move first, move last, make a promise, make a threat, etc.

All together, my goal is to empower students who face a 2x2 game to quickly

² Many famous games are special cases, e.g. Stag Hunt is an Assurance Game, while Battle of the Sexes is a Chicken Game.
identify what sort of game they are playing and what sort of strategy is likely to provide them with the greatest strategic advantage.

The rest of the paper is organized as follows. Section 2 has preliminaries, including the definitions for several concepts that will be used later. Section 3 provides the Question Tree that students can use to identify any given game. Section 4 then analyzes each of the Ten Games, showing what sort of “commitment strategy” will allow each player to enjoy his/her best achievable outcome. Section 5 concludes with several examples.

2. Preliminaries

2.1 When are two games “the same”?

A 2x2 game is one in which two players each have just two options. Although most real-world games are much more complicated – having many players or many (often highly complex) strategic options per player – thinking about simplified 2x2 versions of a game can be useful as a starting-point for brainstorming and strategy development. Unfortunately, there are so many 2x2 games that it might seem hopeless to develop a ready-made understanding of even this simple class of games. In any 2x2 game, each player has four payoffs, corresponding to the four possible outcomes of the game. Thus, one needs eight numbers – which could be anything – to describe payoffs fully.

In some cases, games with different payoffs are obviously “the same”. For example, consider the two payoff matrices in Figure 1. The players are called “Row” and “Col”, each box corresponds to a possible outcome of the game, and payoffs according to each outcome are expressed as “Row’s payoff, Col’s payoff”.


Suppose that the games in Figure 1 differ only in that payoffs in Figure 1A are expressed in pennies while those in Figure 1B are expressed in dollars. In this case, it is natural to say that these are in fact the “same game”. In order to make progress on a classification of all 2x2 games, I will lump together even more sorts of games as “the same”. In particular, I will focus on how players rank the possible outcomes of the game, using either the numbers 1-4 (with 4 being best and 1 being worst) or with happy- and unhappy-faces (with ☺☺ being best and ☻☻ being worst). For example, both of the games of Figure 1 can be described by the same ordinal payoffs:

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<tr>
<th></th>
<th>High</th>
<th>Low</th>
</tr>
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<tbody>
<tr>
<td>High</td>
<td>300 , 300</td>
<td>100 , 400</td>
</tr>
<tr>
<td>Low</td>
<td>400 , 100</td>
<td>200 , 200</td>
</tr>
</tbody>
</table>

Figure 1A

<table>
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<tr>
<th></th>
<th>High</th>
<th>Low</th>
</tr>
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<tbody>
<tr>
<td>High</td>
<td>3 , 3</td>
<td>1 , 4</td>
</tr>
<tr>
<td>Low</td>
<td>4 , 1</td>
<td>2 , 2</td>
</tr>
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</table>

Figure 1B

Figure 2: Ordinal version of games in Figures 1A and 1B

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3 The term “ordinal” reflects the fact that only the order of the outcomes (from best to worst) is captured.
Restricting attention to ordinal payoffs does entail some loss of generality, and might lead one to develop poor strategic advice in some situations. For example, since ordinal payoffs ignore magnitudes, they fail capture the risks that players take on by adopting different strategies. Obviously, your own and others’ attitudes toward risk are important considerations when developing strategy, but the ordinal analysis here will be silent on inherently cardinal issues such as risk.

On the other hand, a great advantage of ordinal analysis is that it strips away distracting details, allowing one to focus more effectively on the heart of the strategic issues at hand. For example, suppose in a price-setting game that each firm always prefers to set its price below that of its opponent and to avoid being undercut itself but, all else equal, prefers for both firms’ prices to be higher. Such preferences lead to ordinal payoffs as in Figure 2, regardless of why each player ranks the outcomes in this way:

- **Best outcome** (笑脸 = 4): I set Low price while you set High price
- **Second-best outcome** (รอย = 3): We both set High price
- **Second-worst outcome** (รอย = 2): We both set Low price
- **Worst outcome** (รอยรอย = 1): I set High price and you set Low price

Unfortunately, there remain 576 (!!) 2x2 games, even when we restrict attention to ordinal payoffs. The pedagogical contribution of this paper is to show that many of these games can be usefully lumped together, when viewed from the perspective of

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4 There are 4! = 24 ways that each player can rank the four outcomes. 576 = (24)². If one allows for indifference – that a player may prefer two outcomes equally – the number of games rises to (81)³=6561!! Some of these games can be shown to be equivalent, by symmetry considerations. For example, there are actually “only” 144 distinct games with strict preferences.
**how one ought to play the game to maximize one’s own strategic advantage.** Indeed, I will classify all 2x2 games as belonging to one of ten families:

1. Slam Dunk
2. Immovable Object
3. Happy Marriage
4. Prisoners’ Dilemma
5. Food Fight
6. Master & Beloved Servant
7. Master & Annoying Servant
8. Assurance
9. Chicken
10. Hide & Seek

### 2.2. Definitions

In this section, I will define and discuss four important concepts: (i) superdominant strategy; (ii) dominant strategy; (iii) pure-strategy Nash equilibrium; and (iv) commitment strategy. In addition, I will discuss four notable types of commitment strategies: moving first, moving last, promise, and threat.

**Definition: Superdominant strategy.** Strategy X is “superdominant” over strategy Y for a player if the *worst possible outcome* when he plays X yields a better outcome for that player than the *best possible outcome* when he plays Y.

In Figure 3A, each player has a **superdominant** strategy to play High. Consider Row. His worst possible outcome after playing High is (High, High) for payoff “3”, while
his best possible outcome after playing Low is (Low, Low) for “2”. By contrast, High is not superdominant in Figure 3B, since Row prefers (Low, Low) over (High, High).

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<thead>
<tr>
<th></th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>3 , 3</td>
<td>4 , 1</td>
</tr>
<tr>
<td>Low</td>
<td>1 , 4</td>
<td>2 , 2</td>
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</table>

Figure 3A: High superdominant

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<th>High</th>
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<tbody>
<tr>
<td>High</td>
<td>2 , 2</td>
<td>4 , 1</td>
</tr>
<tr>
<td>Low</td>
<td>1 , 4</td>
<td>3 , 3</td>
</tr>
</tbody>
</table>

Figure 3B: High not superdominant

Discussion: Whenever a player has a superdominant strategy, it is impossible to dissuade him from playing that strategy. As we shall see, this limits your ability to shape a game to your advantage when the other player has a superdominant strategy.

**Definition: Dominant strategy.** Strategy X is “dominant” over strategy Y for a player if that player prefers to play X rather than Y, regardless of the other players’ strategy – *holding the other player’s strategy fixed.*

For example, consider the two games illustrated in Figure 4. In Figure 4A, each player has a dominant strategy to play High. Consider Row. He prefers to play High if Col plays High since “2” > “1”, and he also prefers to play High if Col plays Low since “4” > “3”. By contrast, High is not a dominant strategy in Figure 4B, since each player prefers to play Low if the other player has chosen Low.
Discussion: I often hear students who have not yet acquired a deep understanding of game theory saying something like the following: “You should always play a dominant strategy when you have one, since you always get a higher payoff from doing so.” This sort of statement is wrong, in a subtle but important way. A dominant strategy yields a higher payoff *when we hold fixed the other player’s strategy*, but can yield a lower payoff if the other player *reacts* to your move. For example, in Figure 4A, Row can improve his payoff from “2” to “3” by *promising* to play Low if Col plays Low.

**Definition: Pure-strategy Nash equilibrium.** A profile of strategies (X,Y) – where Row plays X and Col plays Y – is a “pure-strategy Nash equilibrium” if each player’s strategy is a best response to the other player’s strategy.

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5 Some readers may feel that the statement is *ethically* wrong, as well as factually incorrect, if one “should” seek to maximize total welfare rather than one’s own selfish desires. Game theory provides a useful set of tools for exploring such ethical issues as “payoffs” can and do incorporate such social concerns. Viewed this way, the ethical issue here is more about the payoffs that one should have, rather than about whether one should seek to maximize one’s own payoffs.

6 Nash equilibria may also exist in which one or both player adopts a random (or “mixed”) strategy. The analysis in this paper restricts attention to non-random strategies.
For example, (High, High) is the unique pure-strategy Nash equilibrium in the games of Figure 3A and Figure 3B/4A. By contrast, both (High, High) and (Low, Low) are pure-strategy Nash equilibria in the game of Figure 4B.

**Discussion**: The notion of Nash equilibrium is often used as a “solution concept”, i.e. as a way of predicting what will or should happen when a game is played. For our purposes, it is more helpful to view the set of Nash equilibria of a game as a property of the payoffs that is useful primarily for determining what sort of game is being played. As we shall see, players can sometimes gain a strategic advantage by committing not to play their Nash equilibrium strategy.

**Definition: Commitment strategy.** A commitment strategy for (say) the Row player is a credible announcement of what Row will do, depending on Col’s chosen move. By assumption, Col chooses its move after observing Row’s commitment strategy.

Several types of commitment strategies are worth special note.

**Definition: Moving first.** Suppose Row announces: “No matter what Col does, I will play X”. I will call this commitment strategy “moving first”. (Since Row always plays X, it is as if Row moved first and played X.)

**Definition: Moving last.** Suppose Row announces: “No matter what Col does, I will play my best response to Col’s chosen move”. I will call this commitment strategy “moving last”. (Since Row always plays a best response, it is as if Row moves last.)

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7 This paper characterizes optimal commitment strategies in each game, assuming that only one player is able to commit. If both players can make credible commitments, the analysis becomes much richer and beyond our present scope.

8 I use notation “X” to denote a generic strategy, i.e. X = High or X = Low.
**Definition: Promise.** Suppose Row announces: “If Col chooses X, I will respond by NOT playing my best response.” Any commitment strategy that entails such a commitment is called a “promise” if Col’s best response is to choose X.

**Definition: Threat.** Suppose Row announces: “If Col chooses X, I will respond by NOT playing my best response.” Any commitment strategy that entails such a commitment is called a “threat” if Col’s best response is NOT to choose X.

The following example may be helpful to clarify some of these definitions.

**Example: Team Project.** Two students have a team project. Suppose that (i) if neither works on the project, both will fail, (ii) if just one works, both get the grade “A”, and (iii) if both work, both get the grade “A+”. Each is willing to work to avoid failure, but only one student (Row) is willing to work to increase their grade from “A” to “A+”. Namely, Row ranks the four possible outcomes (Work, Work) > (Not, Work) > (Work, Not) > (Not, Not), while Col ranks them as (Work, Not) > (Work, Work) > (Not, Work) > (Not, Not). See Figure 5.

<table>
<thead>
<tr>
<th></th>
<th>Work</th>
<th>Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work</td>
<td>☺️ , ☺️</td>
<td>☹️ , ☺️</td>
</tr>
<tr>
<td>Not</td>
<td>☺️ , ☹️</td>
<td>☹️ , ☹️</td>
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</tbody>
</table>

Figure 5: Team Project Game
Strategic advice for Row? Col gets its best outcome in the Nash equilibrium, so only Row needs help here. Can Row achieve its best outcome (High, High)? Not by moving first: if Row plays High, then Col will respond by playing Low. Nor by moving last: anticipating that Row will play High, Col will play Low as the first-mover. However, there is hope. Row can achieve its best outcome (Work, Work) by committing to the following threat:

“If you work, then so will I. But if you don’t work, then neither will I.”

If Row is committed to follow through on this, Col’s best response is to Work, since (Work, Work) is better than (Not, Not). The part of Row’s commitment that “If you don’t work, then neither will I” is a threat, since Row would rather work if Col does not, to avoid failure. (On the other hand, the part of Row’s commitment saying “If you work, then so will I” is technically not a promise, since Row is just stating his true preference to work if Col works.)
3. The Question Tree

To determine which of the Ten Games is being played, it suffices to answer the following five simple questions, which I refer to as the Question Tree.

➢ QUESTION #1. How many players have a superdominant strategy?
   o Both  ➔ Slam Dunk.
   o One  ➔ The Immovable Object.
   o None  ➔ Proceed to Question 2.

➢ QUESTION #2. How many pure-strategy Nash equilibria are there?
   o Two  ➔ Proceed to Question #3
   o One  ➔ Proceed to Questions #4 & 5
   o Zero  ➔ Hide & Seek.

➢ QUESTION #2. Is Row’s best outcome also Col’s best outcome?
   o Yes  ➔ Assurance.
   o No  ➔ Chicken.

➢ QUESTIONS #4 & #5. How many players have a dominant strategy? Of these, how many get their best outcome in the Nash equilibrium (i.e. “love Nash”)?
   o Two & Both love Nash  ➔ Happy Marriage.
   o Two & One loves Nash  ➔ Food Fight.
   o Two & None love Nash  ➔ Prisoners’ Dilemma.
   o One & This player loves Nash  ➔ Master & Beloved Servant.
   o One & This player does not love Nash  ➔ Master & Annoying Servant.
   o Zero  ➔ START OVER. You answered Question #1 or #3 incorrectly!!
4. Gaining a Strategic Advantage in the Ten Games

A player who can make observable and irreversible strategic commitments can gain a strategic advantage in games, without the need for deception. This section characterizes each player’s optimal commitment strategy in each of the Ten Games.

4.1 Slam Dunk

Defining features of Slam Dunk

1) Each player has a superdominant strategy.

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<th>High</th>
<th>Low</th>
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<tbody>
<tr>
<td>High</td>
<td>3 or 4 , 3 or 4</td>
<td>3 or 4 , 1 or 2</td>
</tr>
<tr>
<td>Low</td>
<td>1 or 2 , 3 or 4</td>
<td>1 or 2 , 1 or 2</td>
</tr>
</tbody>
</table>

Figure 6: Slam Dunk games

Why that name? In American-English idiom, a decision is a “slam dunk” if its superiority over any alternative is so clear and convincing that no further discussion is necessary. For example, a Duke student might be overheard in the hallway saying to a friend: “Taking Professor McAdams’ game-theory course is a slam dunk! But what else will I take with my remaining credits?” In the Slam Dunk Game, playing High is a slam dunk for each player, since it is superdominant over Low.

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9 Effective deception can sometimes allow a player to achieve an even better payoff than what is possible with credible commitment. However, a strategic opponent will expect and counter deceptive tactics.
**Optimal commitment strategy:** Since each player has a superdominant strategy, there is no way for either player to dissuade the other from playing High. Thus, commitment strategies are not helpful. The best that any player can do is just to play High itself and enjoy the Nash equilibrium payoff associated with (High, High).

### 4.2. The Immovable Object

**Defining features:**

1) Only one player has a superdominant strategy.

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<tbody>
<tr>
<td>High</td>
<td>3 or 4 , A</td>
<td>3 or 4 , B</td>
</tr>
<tr>
<td>Low</td>
<td>1 or 2 , C</td>
<td>1 or 2 , D</td>
</tr>
</tbody>
</table>

Figure 7: The Immovable Object (A > B)\(^{10}\)

**Why that name?** A player with a superdominant strategy cannot be dissuaded from playing that strategy. Consequently, such a player is akin to an “immovable object”.

**Optimal commitment strategy for Col:** Since Row has a superdominant strategy, Col cannot dissuade Row from playing High. Thus, Col’s best achievable outcome is (High, High) for payoff A. Depending on C and D, this could be Col’s best, second-best, or third-best outcome.

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\(^{10}\) Players and strategies can always be re-labeled in any Immovable Object so that (i) High is Row’s superdominant strategy and (ii) High is Col’s best response when Row plays High. (The arrow in Figure 8 means that Col prefers to play High when Row plays High, i.e. A > B.)
**Optimal commitment strategy for Row:** If Row prefers outcome (High, High) over (High, Low), then all is well for Row. Namely, Row achieves his best possible outcome in the (unique) Nash equilibrium. This outcome is easily achievable, for example by (i) moving first and playing High or (ii) moving last. (If Row moves last, Col will choose High as the first-mover.) So, the only interesting case is when Row prefers (High, Low) over (High, High):

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<th>Low</th>
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<tbody>
<tr>
<td>High</td>
<td>3 , A</td>
<td>4 , B</td>
</tr>
<tr>
<td>Low</td>
<td>1 or 2 , C</td>
<td>1 or 2 , D</td>
</tr>
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</table>

Figure 8: The Immovable Object when (High, Low) is Row’s best outcome

In this scenario, Row would like to persuade Col to play Low, while being sure to remain playing High itself. The only possible way to do so is for Row to commit to a threat:

“If you play High, I will play Low even though my best response is to play High. If you play Low, I will play my best response of High.”

*If B > C*, such a threat will induce Col to play Low, so that Row achieves its best outcome for payoff “4”. On the other hand, *if B < C*, such a threat is ineffective – Col will still choose to play High. In this case, there is no way for Row to achieve its best possible outcome, and Row must settle for the Nash equilibrium payoff of “3”.
4.3. Happy Marriage

**Defining features:**

1. Each player has a dominant but not superdominant strategy.
2. The Nash equilibrium is both players’ best outcome.

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<th>High</th>
<th>Low</th>
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<tbody>
<tr>
<td>High</td>
<td>4, 4</td>
<td>2, 3</td>
</tr>
<tr>
<td>Low</td>
<td>3, 2</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

Figure 9: Happy Marriage

*Why that name?* This game is named after the following strategic vignette. Husband and Wife each decide whether to exert “high effort” or “low effort” in their marriage. Husband and Wife love one another so much that they each want to work hard for their partner. However, if only one person were to work, each spouse would prefer for the *other* person to be the one who works.

*Optimal commitment strategy:* In any Happy Marriage, the players can each easily achieve their best outcome (High, High), since this is the (unique) Nash equilibrium.

4.4. Food Fight

**Defining features:**

1. Each player has a dominant but not superdominant strategy.

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11 Any game satisfying the defining features of Happy Marriage must have ordinal payoffs exactly as in Figure 9, after strategies are appropriately re-labeled. The proofs of this and similar results in the paper are straightforward, and omitted to save space.
2. The Nash equilibrium is only one player’s best outcome. (This player is “Baby”.)

<table>
<thead>
<tr>
<th></th>
<th>High = dessert</th>
<th>Low = no dessert</th>
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</thead>
<tbody>
<tr>
<td>High = no veggies</td>
<td>4, 2</td>
<td>2, 1</td>
</tr>
<tr>
<td>Low = veggies</td>
<td>3, 4</td>
<td>1, 3</td>
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</table>

Figure 10: Food Fight

*Why that name?* This game is named after the following strategic vignette. Dad and Baby sit down to dinner. Dad (Col) has served Baby some vegetables. Baby (Row) decides whether to eat the veggies while Dad decides whether to also serve dessert. Dad would like most of all for Baby to eat veggies but, secondarily, he would like for Baby to be happy. Baby would like most of all to have dessert but, secondarily, would like to avoid eating veggies.

Baby has a dominant strategy not to eat veggies: if Dad is serving dessert, Baby gets his best outcome by eating only dessert; and if Dad is not serving dessert, then Baby would rather go hungry than eat those veggies. However, not eating veggies fails to be a superdominant strategy since Baby is willing to eat desserts *if doing so will cause Dad to serve dessert.*

Similarly, Dad has a dominant strategy to serve dessert: if Baby has eaten vegetables, Dad gets his best outcome by also serving dessert; and if Baby is not eating veggies, Dad prefers to serve dessert to avoid the disaster outcome of an angry, hungry Baby at bedtime. Even so, serving dessert is not a superdominant strategy for
Dad since he is willing to give up serving dessert if doing so will cause Baby to eat his veggies (… not likely, Dad …).

**Optimal commitment strategies:** Baby’s best outcome (High, High) is the unique Nash equilibrium of the game. So, Dad is the only player who needs strategic advice here. Fortunately, Dad can achieve his best outcome (Low, High) by committing to a threat:

“If you play High, I will go against my best response and play Low. If you play Low, I will play my best response of High.”

Or, translating into plain English recognizable to any parent of a young child:

“No dessert unless you eat your veggies!!”

### 4.5. Prisoners’ Dilemma

The Prisoners’ Dilemma is the most famous of all 2x2 games.

**Defining features:**

1. Each player has a dominant but not superdominant strategy.
2. The Nash equilibrium is neither player’s best outcome.

![Figure 11: Prisoners’ Dilemma](image.png)
Why that name? This game is named after the following, famous vignette. Two criminals are caught after a serious crime (e.g. murder) but the police only have enough evidence to convict each of them for a lesser offense (e.g. auto theft). The police put the criminals in separate cells and make the following offer to each:

“If you confess to the murder, I guarantee that you will get at most 20 years in jail and I will let you off with 5 years if you confess and the other guy does not. However, if your partner confesses and you don’t, I will lock you up and throw away the key. If no one confesses, you will still serve 10 years for auto theft.”

As long as each criminal wants to minimize his time in prison, the four possible outcomes can be ranked as in Figure 11.

Optimal commitment strategies: Since each player always has a choice – Confess or Don’t – the other player cannot be induced to accept its worst outcome, via any commitment strategy. Since each player’s best outcome in the Prisoners’ Dilemma is also the other player’s worst outcome, this means that each player’s best outcome cannot be achieved. However, each player can achieve its second-best outcome (Don’t, Don’t) by committing to the following promise:

“If you confess, then so will I. But if you don’t, then neither will I.”

Col’s best response to this promise is Don’t Confess, since Col prefers (Don’t, Don’t) over (Confess, Confess), allowing both players to “escape” the Prisoners’ Dilemma.

Unfortunately, implementing such a promise may be impossible for the criminals, since the police have placed them in separate cells. Each player must
decide what to do without knowing the other’s choice, making this a “simultaneous move” game in which promises and threats are not possible.

4.6. Master & Beloved Servant

*Defining features of Master & Beloved Servant*:

1. One player (“Master”) has a dominant but not superdominant strategy.
2. The Nash equilibrium is Master’s best outcome.

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<tr>
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<th>Safe</th>
<th>Dangerous</th>
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<tbody>
<tr>
<td>Safe</td>
<td>4 , A</td>
<td>2 , B</td>
</tr>
<tr>
<td>Dangerous</td>
<td>3 , C</td>
<td>1 , D</td>
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Figure 12: Master & Beloved Servant [A > B and D > C]

*Why that name?* This game is named after the following vignette. A medieval knight (Row) must decide whether to take the safe or the dangerous route to the castle, as must his loyal Servant (Col). All else equal, the Master prefers to take the safer route, while the Servant prefers to accompany his Master regardless of the route. Further, the Master loves his Servant and wants to keep his Servant safe, namely: (i) Master’s best outcome is when both he and Servant take the safe route; (ii) Master prefers the outcome in which he alone takes the dangerous route over the outcome in which Servant alone does so; but (iii) Master prefers the safe route if Servant is already committed to take the dangerous route. (Without (iii), the game would be either Assurance or Chicken.)
Optimal commitment strategy for Row [i.e. Master]: Master achieves his best outcome in the unique Nash equilibrium. So, only the Servant needs strategic advice here.

Optimal commitment strategy for Col [i.e. Servant]: Servant’s best outcome is either (Safe, Safe) or (Dangerous, Dangerous). If (Safe, Safe) is the best outcome, Servant has no problem to solve. So, for the rest of this discussion, I shall add an addition “defining feature” to the set of games being considered:

3. Servant’s best outcome is (Dangerous, Dangerous), as in Figure 13.

In terms of the “story”, this corresponds to a Servant who, for reasons of his own, really enjoys travelling along the dangerous route with his master. (Perhaps he thinks his master could use a little peril.)

<table>
<thead>
<tr>
<th></th>
<th>Safe</th>
<th>Dangerous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe</td>
<td>4, A</td>
<td>2, B</td>
</tr>
<tr>
<td>Dangerous</td>
<td>3, C</td>
<td>1, 4</td>
</tr>
</tbody>
</table>

Figure 13

Unfortunately, since (Dangerous, Dangerous) is the Master’s worst outcome, there is no commitment strategy that can achieve that outcome. (See discussion of a related point in our previous analysis of the Prisoners’ Dilemma.)

Can the Servant achieve his second-best outcome? Yes, but different approaches are needed depending on which outcome is second-best for the servant.
(Note that \( B < 3 \) since \( A > B \), i.e. (Safe, Dangerous) cannot be second-best for the Servant.)

Suppose that \( A = 3 \), so that (Safe, Safe) is second-best. This is the Nash equilibrium, and so is easily achieved. (For example, one simple approach is to move first and play Safe.)

Suppose that \( C = 3 \), so that (Dangerous, Safe) is second-best. Since the Master prefers (Dangerous, Safe) over (Safe, Dangerous), Servant can achieve this outcome by committing to the following combination of a threat and promise:

“I will stay Safe if you take the Dangerous route. But, if you stay Safe, I will go on the Dangerous route myself!”

4.7. Master & Annoying Servant

Defining features of Master & Annoying Servant:

1. One player (“Master”) has a dominant but not superdominant strategy.

2. The Nash equilibrium is not Master’s best outcome.

<table>
<thead>
<tr>
<th></th>
<th>Safe</th>
<th>Dangerous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe</td>
<td>2 , A</td>
<td>4 , B</td>
</tr>
<tr>
<td>Dangerous</td>
<td>1 , C</td>
<td>3 , D</td>
</tr>
</tbody>
</table>

Figure 14: Master & Annoying Servant

Why that name? This game is named after the same “medieval knight” vignette as the Master & Beloved Servant. The key difference is that (Safe, Safe) is not the Master’s
favorite outcome. Indeed, the Master would prefer to be rid of his Servant!\footnote{For an example, watch the movie “Monty Python and the Holy Grail.”} Namely, (i) the master’s best outcome is when he takes the safe route and his servant takes the dangerous route and (ii) the master is willing to risk his own life to endanger the servant, preferring the outcome in which both travel on the dangerous route over the outcome in which both take the safe route. (Without condition (ii), Safe would be a superdominant strategy for the Master, making this an Immovable Object.)

**Optimal commitment strategy for Row [i.e. Master]:** The Master can easily achieve his second-best outcome (Dangerous, Dangerous) by moving first and choosing Dangerous. (The Servant will respond by also choosing Dangerous.) On the other hand, the Master cannot achieve its best outcome (Safe, Dangerous) by moving first, since the Servant will respond by also choosing Safe. If $C < B$ – meaning that the Servant prefers to substitute himself on the dangerous route – the Master can achieve (Safe, Dangerous) by committing to the following threat:

“I will stay Safe if you take the Dangerous route. But, if you stay Safe, I will go on the Dangerous route myself!”

This commitment strategy is identical to the Master’s optimal commitment strategy in the Master & Beloved Servant Game (see Section 3.6), but the game-theoretic interpretation is different. In particular, while Master’s announcement continues to include a threat – Master says he will choose Dangerous if Servant chooses Safe, even though Master would prefer not to do so – the Master’s statement that “I will stay Safe if you take the Dangerous route” is no longer a “promise” since it does not
require commitment power. (The Master now prefers Safe if the Servant chooses Dangerous.)

If \( C > B \), however, there is no commitment strategy that will induce the Master’s best outcome (Safe, Dangerous). The reason is that both of the possible outcomes when Servant chooses Safe – either (Safe, Safe) or (Dangerous, Safe) – are then better for the Servant than Master’s preferred (Safe, Dangerous).

*Optimal commitment strategy for Col [i.e. Servant]:* See the discussion in the Master & Beloved Servant, which carries over to the Master & Annoying Servant without modification.

### 4.8. Assurance

*Defining features of Assurance:*

1. There are two pure-strategy Nash equilibria.
2. Both players have the same best outcome.

![Figure 15: Assurance](image)

*Why that name?* Since each player has the same best outcome, the players’ interests are well-aligned. In particular, both players would like to “assure” one another to each play High.
Optimal commitment strategy: Each player achieves its best outcome (High, High) by moving first or by moving last.\textsuperscript{13}

4.9. Chicken

Defining features of Chicken:

1. There are two pure-strategy Nash equilibria.
2. Both players do NOT have the same best outcome.

![Figure 16: Chicken](image)

Where does the name come from? This game is named after a famous vignette, that Dixit and Skeath (2004) describe as follows: “Two teenagers [in 1950s America] take their cars to opposite ends of Main Street, Middle-of-Nowhere, USA, at midnight and start to drive toward each other. The one who swerves to prevent a collision is the “chicken”, and the one who keeps going straight is the winner. If both maintain a straight course, there is a collision in which both cars are damaged and both players are injured.” The teenager’s payoffs in this vignette are a special case of those in Figure 16, when strategies are re-labeled as in Figure 17.

\textsuperscript{13} A player who moves first will choose High, anticipating that the second-mover will also choose High. Thus, it does not matter to either player who moves first, as long as someone does so.
Optimal commitment strategy: Each player achieves its best outcome by moving first.

As first-mover, Row chooses High, anticipating that Col will choose Low.

### 4.10. Hide & Seek

**Defining features of Hide & Seek:**

1. There is no pure-strategy Nash equilibrium,

<table>
<thead>
<tr>
<th></th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>4, A</td>
<td>X, B</td>
</tr>
<tr>
<td>Low</td>
<td>Y, C</td>
<td>Z, D</td>
</tr>
</tbody>
</table>

Figure 18: General payoffs in Hide & Seek (Y > Z, B > A, C > D)

Note that, without loss of generality, we may re-label strategies as “High” and “Low” so that Row finds (High, High) to be its best outcome. Otherwise, payoffs can be of many forms, as long as Y > Z, B > A, and C > D so that players’ incentives are aligned with the arrows depicted in Figure 18.
Why that name? This famous game is known by many names, such as Hunter & Hunted by Dixit and Skeath (2004). Suppose that one player is trying to hide from another. The Hider (Col) always prefers to choose a different location than the Seeker (Row) – thereby avoiding detection – while the Seeker always prefers to choose the same location. Given this hide-and-seek dynamic, there can be no pure-strategy Nash equilibrium since at least one player always prefers to change its strategy given what the other has done.

**Optimal commitment strategy [for Row]:** Hide & Seek is a rich game, with payoffs taking a variety of forms. In particular, the optimal commitment strategy in Hide & Seek depends on some further details about player payoffs.

**Case #1: A > D.** If Col prefers (High, High) over (Low, Low), Row can achieve its best outcome of (High, High) by committing to the following threat:

> “I will play High if you play High. But I will play Low if you play Low.”

If A < D, there is unfortunately no commitment strategy for Row that will induce Row’s favorite outcome (High, High). The reason is that both of the possible outcomes when Col chooses Low – either (High, Low) or (Low, Low) – are then better for Col than (High, High). Fortunately, Row can achieve its second-best outcome, although the right approach to do so depends on which outcome is second-best. (Since Z > X, X < 3 and (High,Low) cannot be Row’s second-best outcome.)

**Case #2: A < D and Y = 3.** Suppose first that (Low, High) is Row’s second-best outcome, i.e. Y = 3. This outcome can be achieved by moving first and playing Low, since Col will respond by playing High.
Case #3: A < D and Z = 3. Suppose next that (Low, Low) is Row’s second-best outcome, i.e. Z = 3. This outcome can be achieved by moving last. To see why, note that Col anticipates that Row will play High if Col plays High, or Low if Col plays Low. However, since A < D by presumption in the case considered here, Col prefers (Low, Low) over (High, High) and will play Low as first-mover.

Other comments: The most famous sub-class of Hide & Seek games are those referred to as “Zero-Sum Games”, in which players are opponents. That is, one player’s gain is always the other’s loss, as in Figure 19. Note that every zero-sum game falls within “Case #3” above. So, each player’s optimal commitment strategy in any zero-sum game is to move last. This is quite intuitive, since moving last allows you to observe and take advantage of your opponent’s move.

\[
\begin{array}{c|cc}
& \text{High} & \text{Low} \\
\hline
\text{High} & 4,1 & 2,3 \\
\text{Low} & 1,4 & 3,2 \\
\end{array}
\]

Figure 19: Zero-Sum Games

5. Examples

This section provides several vignettes that illustrate how to apply the Question Tree of Section 3, and the commitment-strategy analysis of Section 4.
5.1. John Nash and the Blonde Beauty\textsuperscript{14}

John Nash and a friend are drinking beers at the local bar when a beautiful blonde enters the room. Each wants to maximize the probability that she becomes his girlfriend, but knows that this is only possible if he goes to talk with her. Unfortunately, if both talk to her at the same time, the odds that either of them will succeed decreases dramatically ($2\% + 2\% < 10\%)$.

<table>
<thead>
<tr>
<th></th>
<th>Talk</th>
<th>Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talk</td>
<td>2% , 2%</td>
<td>10% , 0%</td>
</tr>
<tr>
<td>Not</td>
<td>0% , 10%</td>
<td>0% , 0%</td>
</tr>
</tbody>
</table>

Figure 20: Nash and the Blonde

\textit{Discussion}: Talking here is a superdominant strategy since the worst outcome when one talks with the blonde is 2\% chance of success, while not talking with her guarantees no chance at all. Thus, within the context of the game as currently described, there is no way for either player to achieve his best outcome of being the only one to talk with the blonde. \textbf{This game is a Slam Dunk}.

In fact, of course, the game is likely to be richer than this 2x2 simplification. Since the players have a long-term relationship, they may be able to credibly commit to strategies that hurt themselves today but improve future payoffs by maintaining their relationship. For example, the player might agree to alternate each night who

\textsuperscript{14} This example is inspired by a scene from “A Beautiful Mind”, the movie about game theorist John Nash.
gets to talk alone with the prettiest girl in the bar, making such an arrangement credible by threatening never to cooperate again should it be breached.

Even if the players are never likely to interact again, the one-shot game they play may include additional strategies and/or additional players that change the game significantly. For example, one guy might try to “bribe” the other not to talk with the blonde. Or, the blonde herself might try to change the game. For example, the blonde might not like to be crowded and, to avoid this, she might commit (say) to reject any pair of men that approach her at the same time. Such a commitment would change the payoffs in the game, since each guy will fail and be humiliated if both go to talk with her. Assuming that such failure is worse for each guy than not approaching her at all, the resulting game is that illustrated in Figure 7. This new game is no longer a Slam Dunk, but rather the Chicken Game.

<table>
<thead>
<tr>
<th></th>
<th>Talk</th>
<th>Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talk</td>
<td>&lt;0%</td>
<td>10%</td>
</tr>
<tr>
<td>Not</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Figure 21: … when the Blonde dislikes a crowd

This example illustrates the broader point that the best game-theoretic analysis does not end with a simplified 2x2 version of the game in question. Such a simplification is useful as a starting-point for strategy development, but often abstracts from important features of the environment (such as strategies or players that have not been
accounted for) and/or oversimplifies crucial elements of the “game within the game” at hand.

5.2. Britney Spears and the Paparazzi

Tabloid favorite Britney Spears (Row) decides one night whether to party at LA’s hottest dance club. Paparazzi (Col) decide whether to stalk the club, hoping to take a photograph of her. Britney wants most to party, but would also prefer not to be bothered by the paparazzi. The paparazzi want to be at the club to take a photograph if Britney shows up, but would prefer to stay home otherwise.\textsuperscript{15}

\begin{figure}[h]
\centering
\begin{tabular}{|c|c|}
\hline
 & Stalk & Not \\
\hline
Party & 3, 4 & 4, 2 \\
\hline
Home & 1, 1 & 1, 2 \\
\hline
\end{tabular}
\caption{Britney Spears and the Paparazzi}
\end{figure}

Britney has a superdominant strategy to party, while the Paparazzi do not have a dominant strategy. \textbf{This game is an Immovable Object.}

In the unique Nash equilibrium of this game, the paparazzi get their best outcome: Britney parties and they stalk. So, only Britney needs strategic advice here. Fortunately, Britney can achieve her best outcome by committing to a \textit{threat}:

\textsuperscript{15} In the payoffs of Figure 11, the paparazzi are assumed to be indifferent between the outcomes (Party, Not) and (Home,Not), since they do not get a photograph in either case. This is not important for the analysis of this example. In particular, the advice to Britney Spears does not change if paparazzi prefer (Home, Not) over (Party, Not) because they feel regret at
“I will only come to the club if you are not stalking it.”

Or, translated in game-theory jargon:

“Even though partying is always my best response, I commit to stay home if you stalk the club. On the other hand, if you don’t stalk it, I will play my best response and party.”

If feasible and credible, such a threat dissuades paparazzi from stalking the club. Of course, both feasibility and credibility are challenging here for Britney. First of all, such a threat requires the paparazzi to “move first”, which in turn requires that (i) Britney can detect their move and that (ii) the paparazzi’s move is irreversible. But how can Britney detect whether paparazzi are stalking the club? And what if stalkers show up only after she has arrived?

Second, Britney has to be able to commit credibly to stay home should the paparazzi decide to stalk. Given her current reputation as a party animal, the paparazzi might simply not believe her threat. If so, Britney will either be forced to abandon her dreams of a paparazzi-free night out, or endure some lonely nights at home to establish her credibility.

5.3. Team Incentives

Two partners each decide whether to work on a team project or to work on an individual outside project. Each player’s outside project is safe and worth $10,000, whereas the team project is risky but brings in $100,000 if successful, which is contractually split equally between the partners (regardless of who worked on the team project). If neither player works on the team project, it never succeeds. If one
player does, it succeeds 50% of the time. And if both players work on the team project, it succeeds 100% of the time.

Assuming that the partners seek to maximize their *expected* total profit, payoffs in this game are illustrated as follows. (For example, Row’s payoff in (Not, Team) is $35,000 = $10,000 + 50% \times $100,000 / 2.)

![Payoff Matrix]

Both partners have a dominant – but not superdominant – strategy to work on the team project, and each views (Team, Team) as the best possible outcome. Thus, this game is a Happy Marriage.

### 5.4. North Korea’s Nukes

North Korea’s new leader Kim Jong-un decides whether to keep or dismantle North Korea’s nuclear weapons. Western nations (“West”) decide whether or not to provide economic aid.
Kim has a dominant strategy not to dismantle, while the West has a dominant strategy to provide aid. In the Nash equilibrium (No Dismantle, Aid), however, only Kim gets his best outcome. Thus, this game is a Food Fight.

Discussion. The payoffs represented in Figure 24 are based on several assumptions, some of which may be inaccurate when it comes to the actual game being played with North Korea. These assumptions include:

- **The West prefers to provide aid**, regardless of whether Kim has nuclear weapons. That is, the West prefers (Dismantle, Aid) over (Dismantle, No Aid) and also prefers (No Dismantle, Aid) over (No Dismantle, No Aid). This could be reasonable if the West prefers avoiding a humanitarian crisis in North Korea.

- **Kim prefers not to dismantle.** Whether or not he receives economic aid today, Kim may view continued possession of nuclear weapons as a valuable asset that can strengthen his internal political position, deter foreign invasion, be sold for cash, or be used to extract economic aid from the West in the future.\(^{16}\)

\(^{16}\) The fact that Western leaders change every few years makes it difficult for the West to commit irreversibly to withhold economic aid until Kim dismantles his weapons systems.
Kim is willing to dismantle in exchange for aid. That is, Kim prefers (Dismantle, Aid) over (No Dismantle, No Aid). This assumption could be reasonable if Kim views his reign as being near its end, so that immediate aid is worth more than the prospect of future aid that can be secured by keeping nuclear weapons. Without this assumption, No Dismantle would be a superdominant strategy, making this an Immovable Object Game with Kim Jong-un as the immovable object.

The West’s optimal commitment is a threat:

“No aid unless you dismantle!!”

Of course, the trouble is making such a commitment credible, since the outcome (No Dismantle, No Aid) could be the very worst for the West – sending North Korea into a crisis in which Kim might be more likely to sell or launch his nuclear weapons. Also, since the decision to stop providing aid is reversible, Kim will have a strong incentive to test the West’s resolve, making the situation even more perilous than otherwise.

A more credible approach could involve “carrots” instead of “sticks”. Although the West cannot commit to provide no aid, they could credibly provide a baseline level of aid regardless of Kim’s actions plus additional inducements to Kim should he dismantle or take steps toward dismantling. The West can easily commit to withhold such inducements should Kim not dismantle (or attempt to reacquire
weapons later) since doing so will not create any “collateral damage” in North Korea itself.  

### 5.5. Study or Party?

Two students are enrolled in a class that, unfortunately, has zero educational value. Grading in the class depends solely on a final exam and is curved: the professor is constrained to give one student the grade “A” and one student the grade “B” based on their relative performance on the exam. The night before the exam, each student decides whether to study or to party with other students. If only one studies, that student will be sure to get the “A” while the other gets the “B”. However, if both study an equal amount, each student has a 50% chance of getting the “A”. Both students care most about their grade and, secondarily, would like to party.

![Figure 25: Students’ Dilemma](image)

In this game, each player has a dominant strategy to study since, regardless of what the other student does, studying always increases one’s chance of getting the “A” by

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17 Such an approach appears to have been adopted with Libya’s strongman leader Muammar al-Gaddafi, who was able – at least for a while – to amass much greater personal riches after his renunciation of nuclear weapons led to greater acceptance by the international community.
50%. However, the Nash equilibrium (Study, Study) is neither player’s best outcome. Thus, this game is a Prisoners’ Dilemma.

The students appear doomed to miss the party while studying in vain. Fortunately, to escape this Prisoners’ Dilemma and each enjoy the party, it suffices for either player to make a promise:

“If you attend the party, so will I. But you stay home to study, so will I.”

Such a promise can easily be made credible, ironically, because it is so quickly reversible by either player. Indeed, attending the party is a sort of dynamic game in which either student can leave at any time. As long as both players are at the party, neither has any incentive to sneak off to study because he knows that the other student will do the same shortly, leaving the original player worse off since he will miss the party and still only get the “A” with probability 50%.

Once at the party, the students will find themselves in a similar Prisoners’ Dilemma, regarding the extent to which they enjoy themselves. Each has a dominant strategy not to “enjoy themselves too much”, as they will have a sharper mind for the exam the next morning. Perhaps such dilemmas could be resolved in much the same way, with a promise to enjoy the party together.

5.6. Entry

A Monopolist faces potential competition from an Entrant in an undifferentiated product market, e.g. memory disks. Monopolist currently operates one factory and must decide whether or not to build one more. Entrant must similarly decide whether to build a factory. A factory costs either firm $1.5 billion to build. Total profits
depend on how many factories have been built and are divided between the two firms in proportion to how many factories they operate. In particular, total gross profits – in units of $billions, including production costs but not factory construction costs – take the form \( \Pi(Q) = Q(4 - Q) \).\(^{18}\) Thus, firm’s net profit, in $billions, are as follows. (I will leave it to the interested reader to verify these payoffs. Recall that Monopolist already has one factory and each extra factory costs $1.5 billion.)

![Figure 26. Entry Game (payoffs in $millions)](image)

Neither player has a superdominant strategy, and only the Monopolist has a dominant strategy – not to build another factory. Further, the Monopolist does not get its best outcome in the Nash equilibrium. Thus, this game is **Master & Annoying Servant**.

**Optimal commitment strategies:** The Entrant gets its best outcome (building a factory while Monopolist does not build) in the Nash equilibrium, so only the Monopolist needs help here. Fortunately, the Monopolist can achieve its best outcome (no one builds a factory) by committing to a threat:

“If you build a factory, I will build a factory as well even though doing so will hurt us both. But, if you don’t build a factory, neither will I.”

\(^{18}\) Such gross profits would arise given inverse demand of the form \( P(Q) = 4-Q \) and zero production costs.
5.7. Counter-insurgency Warfare

An Insurgency (Row) seeks to inflict damage on an Occupier (Col), while the Occupier would like to destroy the Insurgency. Both players must decide whether to locate their forces in the Hills or in the Valley. The Insurgency can inflict the most damage in the Valley but, if the Occupier is also in the Valley, the Occupier will be able to force the Insurgency into open combat where it is most vulnerable. By contrast, the Insurgency is safer in the Hills, although it is also less capable of inflicting damage.

<table>
<thead>
<tr>
<th></th>
<th>Hills</th>
<th>Valley</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hills</td>
<td>2,3</td>
<td>3,2</td>
</tr>
<tr>
<td>Valley</td>
<td>4,1</td>
<td>1,4</td>
</tr>
</tbody>
</table>

Figure 27

As I have described it, this insurgency conflict is actually a zero-sum game. That is, any outcome that is better for the Occupier is worse for the Insurgent, and vice versa. And, in any zero-sum game, the optimal commitment strategy is simple: move last.

For each player, “moving last” means gathering intelligence on the other’s location and moving nimbly to take advantage of that knowledge. As Che Guevara (1961) wrote: “The fundamental characteristic of a guerrilla band is mobility.”

Classic counter-insurgency seeks to change the scope of such a game, by seeking to “win hearts” and control territory rather than just chasing the insurgent. This approach can constrain the Insurgent’s freedom of movement, transforming the
Occuper’s natural first-mover disadvantage (since it is naturally slower) into a first-mover advantage.

References