The Timing of Moves in 2x2 Games

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Abstract

This paper provides a novel taxonomy of all strictly ordered 2x2 games, characterizing how each player ranks moving first, moving last, and moving simultaneously in every such game.

1. Introduction

How does each player rank moving first, moving last, and moving simultaneously in 2x2 games? Determining such rankings is trivial in any given game, but the mechanical process of computing and comparing the Nash equilibria (NE) and subgame-perfect equilibria (SPE) of a given game (under different timings) sheds little light on the connections between different games. The contribution of this paper is to provide simple necessary and sufficient conditions on payoffs for players to have each feasible combination of preferences over the timing of moves.²

The main finding is that, for the purpose of characterizing how each player ranks moving first, moving last, and moving simultaneously, there are only eight types of strictly ordered 2x2 games in which players care about the timing of moves: three having exactly one pure-strategy Nash equilibrium (PSNE), called “Nash Only”, “Lazy Husband”, and “Sir Robin’s Minstrel”; two having two PSNE, called

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²Several papers following Rapoport and Guyer (1966) provide an exhaustive examination of all strictly ordered 2x2 games. This paper’s contribution is its novel taxonomy of all such games.
“Assurance” and “Chicken” (of which Chicken has three variants); and four having no PSNE, called “Hide & Seek”, “Gossip”, “Chores”, and “Hold-Up”. Furthermore, to identify a game’s type, it suffices to answer at most four simple questions about each player’s payoffs in the game, as illustrated in the “Question Tree” of Figure 1. (Each type of game will be defined and discussed later.)

![Figure 1: The Question Tree.](image)

Different move timings are “ranked” by comparing the set of NE under simultaneous moves with the unique SPE given each sort of sequential moves. When there is a unique NE, this is straightforward. When there are three NE, a player ranks simultaneous moves above (or below) some given sequential moves if all NE are
weakly better (or weakly worse) for that player than the unique SPE given the sequential moves in question. Otherwise, if the SPE is better than some NE but worse than another NE, simultaneous moves and the sequential moves in question “cannot be unambiguously ranked”.

Players’ preferences over move timing in these games are as follows.

1. **Nash Only.** Neither player cares about the timing of moves.

2. **Assurance.** Each player prefers sequential moves over simultaneous moves, but neither player cares who moves first.

3. **Chicken.** Each player’s best timing is to move first. In addition:
   a. **Chicken(2).** Each player’s worst timing is to move last.
   b. **Chicken(0).** Each player cannot unambiguously rank moving last vs. moving simultaneously.
   c. **Chicken(1).** One player’s worst timing is to move last, while the other cannot unambiguously rank moving last vs. moving simultaneously.

4. **Lazy Husband.** Each player’s best timing is for Husband to move first. Each player is indifferent between simultaneous moves or sequential moves with Wife moving first.

5. **Sir Robin’s Minstrel.** Sir Robin’s best timing is to move first, but this is the worst timing for the Minstrel. Each player is indifferent between simultaneous moves or sequential moves with Minstrel moving first.

6. **Hide & Seek.** Each player’s best timing is to move last and worst timing is to move first.
7. **Chores.** Each player’s best timing is to move last. Child’s worst timing is to move first, while Parent’s worst timing is simultaneous.

8. **Gossip.** Each player’s best timing is for Rowena to move first. Each player’s worst timing is for Colin to move first.

9. **Hold-Up.** Each player’s best timing is for Supplier to move first. Supplier’s worst timing is to move last, while Buyer’s worst timing is simultaneous.

The rest of the paper is organized as follows. Section 2 provides more detail on the Question Tree. The bulk of the paper then analyzes in more depth those games with one PSNE (Section 3), two PSNE (Section 4), and no PSNE (Section 5). Section 6 offers some concluding remarks, including an overview of the prior literature classifying strictly ordered 2x2 games.

### 2. The Question Tree

This section provides a series of simple questions (“The Question Tree”) about the payoffs in a given 2x2 game, to determine its type. See Figure 1 above.

**Question #1:** How many pure-strategy Nash equilibria (PSNE) are there?

- **If two,** proceed to Question #2-2.
- **If one,** proceed to Question #2-1.
- **If zero,** proceed to Question #2-0.

**Games with two PSNE**

**Question #2-2:** Do Row and Column have the same best outcome?

- **If yes,** the game is **Assurance**.
- **If no,** proceed to Question #3-2-N.

**Question #3-2-N:** How many players’ best outcome is the other’s second-worst?
• If two, the game is Chicken(2).
• If one, the game is Chicken(1).
• If zero, the game is Chicken(0).

Games with one PSNE.

Question #2-1: How many players have a dominant strategy? (When there is exactly one PSNE, at least one player must have a dominant strategy.)

• If two, the game is Nash Only.
• If one, proceed to Question #3-1-1.

Question #3-1-1: Let Row be the player with the dominant strategy. (Otherwise, proceed in the obvious way by flipping players’ identities in the questions to follow.)

In the Nash equilibrium, does Row get one of its two best outcomes?

• If yes, the game is Nash Only.
• If no, proceed to Question #4-1-1-N.

Question #4-1-1-N: In the Nash equilibrium, does Column get its best outcome?

• If yes, the game is Sir Robin’s Minstrel.
• If no, the game is Lazy Husband.

Games with no PSNE.

Question #2-0: (i) Are Row’s two best outcomes in different columns? (ii) Are Column’s two best outcomes in different rows? (If yes to (i) and/or (ii), I will say that Row and/or Column “controls its own destiny”.

• If yes to (i) and (ii), the game is Hide & Seek.

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3 Suppose that Row’s two best outcomes are in the same column. Column’s choice determines whether Row gets to enjoy one of its two best outcomes, or whether Row must settle for one of its two worst. In that sense, Row does not control its own destiny.
• If yes to (i) but no to (ii), or vice versa, proceed to Question #3-0-1.

• If no to (i) and (ii), the game is Hold-Up.

Question #3-0-1: Do Row and Column have the same second-worst outcome?

• If yes, the game is Gossip.

• If no, the game is Chores.

3. Games with one pure-strategy Nash equilibrium

This section considers strictly ordered 2x2 games possessing exactly one pure-strategy Nash equilibrium (“PSNE”). Such games fall into three categories: “Nash Only”, “Lazy Husband”, and “Sir Robin’s Minstrel”.

3.1. Nash Only

Definition. A strictly ordered 2x2 game is “Nash Only” whenever

1. There is a unique PSNE.

2. The SPE outcome coincides with the Nash outcome, whoever moves first.

Clearly, in any Nash Only game, neither player cares about the timing of moves.

What games are Nash Only? In any strictly ordered 2x2 game with a unique PSNE, at least one player has a dominant strategy. Further, if a player with a dominant strategy moves last, the resulting SPE outcome will coincide with the Nash outcome. So, the game is Nash Only whenever both players have a dominant strategy.

Suppose now that just one player (say Row) has a dominant strategy. See Figure 2 in which, without loss, (Up,Left) is the Nash equilibrium but, if Row were to play Down, Column would respond by playing Right. Row’s decision as first-mover
amounts to a choice between the Nash outcome (Up,Left) for payoff “A” and the opposite-to-Nash outcome (Down,Right) for payoff “B”. If A > B and Row moves first, then Row will play Up, the SPE outcome will be (Up,Left), and the game is Nash Only. Otherwise, if B > A, the SPE outcome will be (Down,Right) and the game is not Nash Only.

\begin{figure}[h]
\centering
\begin{tabular}{cc}
\hline
 & Left & Right \\
\hline
Up & A, W & C, X \\
Down & D, Y & B, Z \\
\hline
\end{tabular}
\caption{A>D, C>B, W>X, Z>Y}
\end{figure}

So, the games illustrated in Figure 2 are Nash Only if and only if A > B or, equivalently, if and only if A = 3 or A = 4 (where outcomes are ordinally ranked with payoff “1” for the worst outcome and “4” for the best outcome).\(^4\)

We conclude that all games illustrated in Figure 2 are Nash Only except for those shown in Figure 3. These games will be split into two categories, depending on whether W > Z (Sir Robin’s Minstrel) or W < Z (Lazy Husband).

\begin{figure}[h]
\centering
\begin{tabular}{cc}
\hline
 & Left & Right \\
\hline
Up & 2, W & 4, X \\
\hline
\end{tabular}
\caption{}
\end{figure}

\(^4\) To prove this equivalence, suppose first that A = 3 or A = 4. B>A requires that B = 4 and A = 3. However, since C > B, it must be that B < 4, a contradiction. Next, suppose A = 2. Now it must be that D = 1, C = 4, and B = 3, in which case B > A.
3.2. Lazy Husband

The story. A young couple (Row = Husband and Column = Wife) each work long days but don’t have enough money to afford a housekeeper. If either of them chooses to clean up, then both will be able to enjoy a clean house. Unfortunately, Husband is lazy, with a dominant strategy not to clean. More precisely, for Husband, the best outcome is to relax while Wife cleans, the worst outcome is to clean while Wife refuses to do so, and he would prefer to share the cleaning with Wife than to live in a dirty house. On the other hand, Wife is willing to clean the house, but only if Husband shares the load. More precisely, for Wife, the best outcome is to share the cleaning and, if Husband does not clean, then she also prefers not to.

This story does not pin down every detail about Wife’s payoffs. In particular, how does she feel if Husband ends up doing all of the cleaning by himself: sorry for him (“1”); pleased that he now understands what it feels like to clean all alone ("3"); or
something in between (“2”)? Each is possible in Lazy Husband.

**Definition.** A strictly ordered 2x2 game is “Lazy Husband” whenever

1. One player (Husband) has a dominant strategy.
2. Both Husband and Wife prefer the Nash outcome over the opposite-to-Nash outcome.

**Preferred timing of moves.** If players move simultaneously or Wife moves first, Nash outcome (Relax, Refuse) is the unique equilibrium outcome. If Husband moves first, the SPE outcome is the opposite-to-Nash outcome (Clean, Clean), making both players better off. To summarize:

1. Both players’ best timing is for Husband to move first; and
2. Both are indifferent between simultaneous moves and Wife moving first.

### 3.3. Sir Robin’s Minstrel

**The story.** The name “Sir Robin’s Minstrel” comes from a scene in the 1974 movie “Monty Python and the Holy Grail”. If you have not seen this classic British comedy, here is what you need to know for our present purposes. Sir Robin (Row) has embarked on a quest, accompanied by his Minstrel (Column). Unfortunately, after Sir Robin runs away from a fight, the Minstrel torments him with songs about his cowardice. Sir Robin would like nothing more than to be rid of his Minstrel, but the Minstrel follows Sir Robin everywhere. Ironically, the Minstrel provides a motivation for Sir Robin to seek danger, if doing so also puts the Minstrel in danger!

Namely, both Sir Robin and the Minstrel decide whether to take a safe or dangerous route to their destination. Sir Robin has a dominant strategy to take the
safe route, but prefers the outcome in which both he and the Minstrel are exposed to danger over that in which both are safe. By contrast, the Minstrel does not have a dominant strategy, preferring to accompany Sir Robin wherever he goes and most preferring to accompany him on the safe route.

<table>
<thead>
<tr>
<th></th>
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<th>Dangerous</th>
</tr>
</thead>
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<td>Safe</td>
<td>2, 4</td>
<td>4, 1 or 2 or 3</td>
</tr>
<tr>
<td>Dangerous</td>
<td>1, 1 or 2</td>
<td>3, 2 or 3</td>
</tr>
</tbody>
</table>

*Figure 5: Sir Robin’s Minstrel.*

This story does not pin down every detail about the Minstrel’s payoffs. In particular, how would he feel if taking the dangerous route alone: scared witless (“1”); pleased by his own bravery (“3”); or something in between (“2”)? Each is possible in Sir Robin’s Minstrel.

**Definition.** A strictly ordered 2x2 game is “Sir Robin’s Minstrel” whenever

1. One player (Sir Robin) has a dominant strategy.
2. Sir Robin prefers the opposite-to-Nash outcome over the Nash outcome.
3. The Nash outcome is Minstrel’s best outcome.

**Preferred timing of moves.** If players move simultaneously or the Minstrel moves first, Nash outcome (Safe, Safe) is the unique equilibrium outcome. If Sir Robin moves first, the SPE outcome is the opposite-to-Nash outcome (Dangerous, Dangerous), making Sir Robin better off but the Minstrel worse off. To summarize:

1. Sir Robin’s best timing is to move first;
2. Minstrel’s worst timing is to move last; and

3. Both are indifferent between simultaneous moves and Minstrel moving first.
4. Games with two pure-strategy Nash equilibria

This section considers strictly ordered 2x2 games possessing two pure-strategy Nash equilibria (“PSNE”). Such games fall into two well-known categories: Assurance and Chicken.

4.1. Assurance

The story. Two friends each decide where to go lunch, at the Burger Joint or the Salad Bar. Each would prefer to go the Burger Joint if the other will be there, or to the Salad Bar if the other will be there. However, since both friends are trying to lose weight, they each view having lunch together at the Salad Bar as the best outcome. See Figure 6.

<table>
<thead>
<tr>
<th></th>
<th>Salad</th>
<th>Burger</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salad</td>
<td>4, 4</td>
<td>1 or 2, 1 or 2 or 3</td>
</tr>
<tr>
<td>Burger</td>
<td>1 or 2 or 3, 1 or 2</td>
<td>2 or 3, 2 or 3</td>
</tr>
</tbody>
</table>

Figure 6: Assurance.

This story does not pin down every detail about either friend’s payoffs. In particular, how does Row feel if at the Burger Joint alone: ashamed that only he is eating fatty foods (“1”); thrilled that only he has a tasty lunch (“3”); or something in between (“2”)? All three are possible in Assurance, for each player.
“Stag Hunt”, a game first described by Jean-Jacques Rousseau, is a famous example of Assurance. Two hunters must decide whether to hunt stag or hare. Catching a stag is best, but stag can only be caught when both hunt stag. Thus, each hunter prefers to hunt stag if the other hunts stag, but prefers to hunt hare if the other hunts hare. This is an Assurance game, ironically, when we re-label “Stag” = “Salad” and “Hare” = “Burger”.

**Definition.** A strictly ordered 2x2 game is “Assurance” whenever

1. There are two PSNE (plus one mixed-strategy Nash equilibrium).
2. Both players have the same best outcome.

**Preferred timing of moves.** If Row moves first or Column moves first, both play Salad in the unique SPE, so that each player gets his best outcome. If players move simultaneously, matters are more complex. There are three Nash equilibria: (i) pure-strategy equilibrium (Salad,Salad), (ii) pure-strategy equilibrium (Burger,Burger), and (iii) a mixed-strategy equilibrium in which each player randomizes. Equilibrium (i) induces the same outcome as sequential moves (payoff “4” for both players), while equilibria (ii) and (iii) are obviously worse for both. Thus, both players rank simultaneous moves below sequential moves, whoever moves first. To summarize:

1. Both players’ worst timing is to move simultaneously; and
2. Both are indifferent between moving first and moving last.

### 4.2. Chicken Game

**The story.** In 1950s America, two teenage boys race cars toward each other to see who will “chicken out” and swerve to avoid a fatal crash. If the other boy is going to
swerve, each boy would rather go straight to look tough. However, if the other is
going straight, each boy would rather swerve to avoid a crash. See Figure 7.

<table>
<thead>
<tr>
<th></th>
<th>Swerve</th>
<th>Straight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swerve</td>
<td>1 or 2 or 3, 1 or 2 or 3</td>
<td>2 or 3, 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Col first</td>
</tr>
<tr>
<td>Straight</td>
<td>4, 2 or 3</td>
<td>1 or 2, 1 or 2</td>
</tr>
<tr>
<td></td>
<td>Row first</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7: Chicken.

This story does not pin down every detail about either boy’s payoffs. In
particular, how does Row feel if both boys swerve: glad that at least Column is
equally shamed (“3”); disappointed in Column for not showing more guts (“1”); or
something in between (“2”)? All are possible in Chicken, for each player.

**Definition.** A strictly ordered 2x2 game is “Chicken” whenever

1. There are two PSNE (plus one mixed-strategy Nash equilibrium).
2. Both players do not have the same best outcome.

**Preferred timing of moves.** If Row player moves first, Row will go straight and
Column will swerve in the unique SPE, so that Row gets his best payoff (“4”).
Similarly, Column gets his best payoff when moving first. When players move
simultaneously, matters are more complex. There are three Nash equilibria: (i) pure-
strategy equilibrium (Straight,Swerve), (ii) pure-strategy equilibrium
(Swerve,Straight), and (iii) a mixed-strategy equilibrium in which players randomize.

While moving first is clearly each player’s best timing, it is less clear how to rank
moving last vs. moving simultaneously, or even whether there is an unambiguous
ranking. To make headway on this question, we first need to dig deeper into the properties of the mixed-strategy equilibrium (“MSE”). Most important for our purposes is a result that each player’s payoff in the MSE is “between 2 and 3”.

Indeed, this result holds in all 2x2 games that possess a MSE, not just Chicken.

**Mixed-Strategy Equilibrium Claim:** Consider any strictly ordered 2x2 game having a mixed-strategy Nash equilibrium (MSE). Each player’s expected payoff in this equilibrium is better than his second-worst outcome and worse than his second-best outcome, i.e. payoff is “between 2 and 3”.

**Proof:** Consider Row player. (A similar argument establishes the result for Column.) Without loss, suppose that (Up,Left) is Row’s best outcome. First, note that (Up,Right) cannot be Row’s second-best outcome, since then Up would be Row’s dominant strategy and the game’s only Nash equilibrium would be one in pure strategies. Thus, Row’s payoff in (Up,Right) must be “1” or “2”. These two cases are illustrated in Figures 8A and 8B.

<table>
<thead>
<tr>
<th></th>
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<th>Right</th>
<th></th>
<th>Left</th>
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<tbody>
<tr>
<td>Up</td>
<td>4 , ??</td>
<td>1 , ??</td>
<td>Up</td>
<td>4 , ??</td>
<td>2 , ??</td>
</tr>
<tr>
<td>Down</td>
<td>2 or 3 , ??</td>
<td>2 or 3 , ??</td>
<td>Down</td>
<td>1 or 3 , ??</td>
<td>1 or 3 , ??</td>
</tr>
</tbody>
</table>

Figure 8A: Case #1
Figure 8B: Case #2

Case #1: (Up,Right) is Row’s worst outcome. In any MSE, Row must be indifferent between playing Up and Down. Since Row must be indifferent between playing Up
or Down in any MSE, Row’s expected payoff in any MSE must therefore be “between 2 and 3”, as desired.

Case #2: (Up, Right) is Row’s second-worst outcome. When playing Down, Row’s realized payoff is sometimes “1” and sometimes “3”. Thus, Row’s expected payoff when playing Down is less than “3”. On the other hand, when playing Up, Row’s realized payoff is sometimes “2” and sometimes “4”. Thus, Row’s expected payoff when playing Down is greater than “2”. So, again, Row’s expected payoff in any MSE must be “between 2 and 3”, as desired. QED

Now, let us return to the problem of comparing moving simultaneously vs. moving last in Chicken. Consider Row. If Row moves last, the unique equilibrium outcome is (Swerve, Straight), yielding Row a payoff of “2” or “3”.

Case #1: (Swerve, Straight) is Row’s second-worst outcome (payoff “2”). Moving last yields payoff “2”, at least weakly worse than in all three equilibria given simultaneous moves: (i) PSNE (Straight, Swerve) yields a better payoff (“4”); (ii) PSNE (Swerve, Straight) yields the same payoff (“2”); and (iii) the mixed-strategy equilibrium yields a better expected payoff (‘between 2 and 3”). Thus, moving last is Row’s worst timing of moves.

Case #2: (Swerve, Straight) is Row’s second-best outcome (payoff “3”). Row’s preferred PSNE (Straight, Swerve) yields a better payoff (“4”), while the mixed-strategy equilibrium yields a worse expected payoff (“between 2 and 3”). Thus, Row cannot unambiguously rank moving last vs. moving simultaneously. To summarize:

1. Both players’ best timing is to move first;
2. For any player whose second-worst outcome is the other’s best outcome, the worst timing is moving last; and

3. For any player whose second-best outcome is the other’s best outcome, moving last and moving simultaneously cannot be unambiguously ranked.

5. Games with no pure-strategy Nash equilibrium

This section considers the last and most interesting case of strictly ordered 2x2 games with no pure-strategy Nash equilibrium (PSNE). Such games fall into four categories: “Hide & Seek”, “Gossip”, “Chores”, and “Hold-Up”.

5.1. Hide & Seek

The story. Two children play hide and seek; one is Hider (Column) and the other is Seeker (Row). There are two places to hide, “Inside” and “Outside”. The Hider’s two best outcomes are those in which the players choose different locations, while the Seeker’s two best outcomes are those in which the players choose the same location.

<table>
<thead>
<tr>
<th></th>
<th>Inside</th>
<th>Outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside</td>
<td>3 or 4, 1 or 2</td>
<td>1 or 2, 3 or 4</td>
</tr>
<tr>
<td>Outside</td>
<td>1 or 2, 3 or 4</td>
<td>3 or 4, 1 or 2</td>
</tr>
</tbody>
</table>

Figure 9: Hide & Seek

This story does not pin down how players compare the two outcomes in which Hider is found and those in which Hider is not found. For example, if it is a pleasant outside, each player may (all else equal) prefer to be outside, so that both are better
off in (Out,Out) than (In,In) (Figure 10A). By contrast, in the famous “Zero-Sum Game” (Figure 10B), players’ interests are completely opposed, in that any change that makes one player better off always makes the other player worse off.

<table>
<thead>
<tr>
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<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>3,1</td>
<td>1,4</td>
</tr>
<tr>
<td>Out</td>
<td>2,3</td>
<td>4,2</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th></th>
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<th>Out</th>
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<tr>
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<td>3,2</td>
<td>2,3</td>
</tr>
<tr>
<td>Out</td>
<td>1,4</td>
<td>4,1</td>
</tr>
</tbody>
</table>

**Figure 10A: Pleasant day**

**Figure 10B: Zero-Sum**

**Definition.** A strictly ordered 2x2 game is “Hide & Seek” whenever

1. There is no PSNE.
2. Row’s two best outcomes are in different columns.
3. Column’s two best outcomes are in different rows.

The conditions of this definition are satisfied if and only if payoffs are as in Figure 9, after one allows for re-labeling of players’ identities and strategies. (The proof of this fact is relatively straightforward and omitted to save space.)

**Preferred timing of moves.** If Seeker moves first, Hider will respond by going wherever Seeker does not, ensuring that Seeker gets one of his two worst outcomes. As first-mover, Seeker will therefore choose whatever strategy leads to his second-worst outcome (payoff “2”) when Hider best-responds, while Hider gets one of his two best outcomes (payoff “3” or “4”). By the same reasoning, if Hider moves first, Hider’s SPE payoff will be “2” while Seeker will get “3” or “4”. Finally, if players move simultaneously, each player’s expected payoff in the unique MSE will be
“between 2 and 3” – better for each player than when he moves first, but worse than when he moves last. To summarize:

1. Each player’s best timing is to move last.
2. Each player’s worst timing is to move first.

### 5.2. Chores

**The story.** A Parent (Row) decides which of two tasks to leave to a Child (Column), while the Child decides whether to work or shirk. One task is enjoyable for the Child but of little value to the Parent. The other task is more important, but the Child finds it boring and would choose to shirk if certain to be assigned that task. Overall, the Child’s worst outcome is shirking on the enjoyable task and second-worst outcome is working on the important task. The Parent wants the Child to work and Parent’s best outcome is when the Child works on the important task, while Parent’s worst outcome is when the Child shirks on the important task.

<table>
<thead>
<tr>
<th></th>
<th>Work</th>
<th>Shirk</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Enjoyable</strong></td>
<td>3, 3 or 4</td>
<td>2, 1</td>
</tr>
<tr>
<td><strong>Important</strong></td>
<td>4, 2</td>
<td>1, 3 or 4</td>
</tr>
</tbody>
</table>

*Row first*  

*Col first*  

**Figure 11: Chores**

This story does not pin down every detail of the Child’s payoffs. In particular, the Child’s best outcome could be to work on the enjoyable outcome or to shirk on the important task.

**Definition.** A strictly ordered 2x2 game is “Chores” whenever
1. There is no PSNE.

2. One player controls its own destiny. (This player is the Child.)

3. Both players do not share the same second-worst outcome.

The conditions of this definition are satisfied if and only if payoffs are as in Figure 11, after one allows for re-labeling of players’ identities and strategies.

**Chores Claim:** A strictly ordered 2x2 game is “Chores” if and only if its players and strategies can be labeled so that payoffs correspond to those in Figure 11.

**Proof:** Without loss, suppose that Row’s two best outcomes are in the same column, Column’s two best outcomes are in different rows, and Column’s best outcome is (Up,Left). Row’s two best outcomes are either in the Left column or the Right column. First, suppose they are in the Left column. Under the restrictions imposed so far, payoffs must be as in Figure 12.

```
<table>
<thead>
<tr>
<th></th>
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<td>1 or 2, 1 or 2</td>
</tr>
<tr>
<td>Down</td>
<td>3 or 4, 1 or 2 or 3</td>
<td>1 or 2, 1 or 2 or 3</td>
</tr>
</tbody>
</table>
```

Figure 12

(a) (Up,Left) cannot be Row’s best outcome, since then (Up,Left) would be a PSNE. So, Row’s payoff must be “3” in (Up,Left) and “4” in (Down,Left). (b) Neither player can have a dominant strategy, since otherwise the game must have a PSNE. So, Row must prefer (Up,Right) over (Down,Right) and Column must prefer
(Down,Right) over (Down,Left). In particular, Row’s payoff must be “2” in (Up,Right) and “1” in (Down,Right) while Column’s payoff must be “3” in (Down,Right). (c) Since players do not share the same second-worst outcome, Column’s payoff must be “1” in (Up,Right) and “2” in (Down,Right). All together, payoffs must be those in Figure 13A, corresponding to one of the two possibilities in Figure 11 when we re-label Up = Enjoyable and Left = Work.

\[
\begin{array}{c|cc}
  & \text{Left} & \text{Right} \\
\hline
\text{Up} & 3,4 & 2,1 \\
\text{Down} & 4,2 & 1,3 \\
\end{array}
\quad
\begin{array}{c|cc}
  & \text{Left} & \text{Right} \\
\hline
\text{Up} & 1,4 & 4,2 \\
\text{Down} & 2,1 & 3,3 \\
\end{array}
\]

Figure 13A  Figure 13B

By similar logic, payoffs must be those in Figure 13B when Row’s two best outcomes are in the Right column. These payoffs correspond to the other possibility in Figure 11, once strategies are re-labeled Up = Important and Left = Shirk. QED

**Preferred timing of moves.** If Parent moves first, the SPE outcome is (Enjoyable,Work), yielding Parent his second-best outcome (“3”) and Child one of his two best outcomes (“3” or “4”). If Child moves first, the SPE outcome is (Important,Work), yielding Parent his best outcome (“4”) and Child his second-worst outcome (“2”). Finally, if players move simultaneously, each player’s expected payoff in the unique MSE will be “between 2 and 3”. To summarize:
1. Each player’s best timing is to move last.
2. The Parent’s worst timing is to move simultaneously.
3. The Child’s worst timing is to move first.

5.3. Gossip

The story. Colin (Col) has a secret. His friend Rowena (Row) knows that he has a secret and asks about it. Colin must decide whether to tell Rowena the truth or a lie. Rowena must decide whether to spread Colin’s story as gossip or keep quiet. Colin’s worst outcome is for the truth to become gossip, while his second-worst outcome is to lie and then have Rowena keep quiet (since then he will have been a bad friend). Rowena’s best outcome is to spread true gossip, her worst outcome is to spread false gossip and, if she is keeping quiet, Rowena would prefer Colin’s story to be true.

<table>
<thead>
<tr>
<th></th>
<th>Truth</th>
<th>Lie</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiet</td>
<td>3, 3 or 4</td>
<td>2, 2</td>
</tr>
<tr>
<td>Row first</td>
<td></td>
<td>Col first</td>
</tr>
<tr>
<td>Gossip</td>
<td>4, 1</td>
<td>1, 3 or 4</td>
</tr>
</tbody>
</table>

Figure 14: Gossip

This story does not pin down every detail about Colin’s payoffs. In particular, Colin’s best outcome could be to tell Rowena his secret in confidence or it could be for Rowena to spread false gossip. (In the latter case, Colin might be especially pleased that he chose not to trust Rowena.)

Definition. A strictly ordered 2x2 game is “Gossip” whenever
1. There is no PSNE.

2. Exactly one of the following statements is true:
   a. Row’s two best outcomes are in different columns.
   b. Column’s two best outcomes are in different rows.

3. Both players share the same second-worst outcome.

The conditions of this definition are satisfied if and only if payoffs are as in Figure 14, after one allows for re-labeling of players’ identities and strategies. (The proof is very similar to that of the Chores Claim in Section 5.2 and omitted to save space.)

**Preferred timing of moves.** If Rowena moves first, the SPE outcome is (Quiet,Truth), yielding Rowena her second-best outcome (“3”) and Colin one of his two best outcomes (“3” or “4”). If Colin moves first, the SPE outcome is (Quiet,Lie), yielding Rowena and Colin their mutually second-worst outcome (“2” for each).

Finally, if players move simultaneously, each player’s expected payoff in the unique MSE will be “between 2 and 3” – better for each player than when Colin moves first, but worse than when Rowena moves first. To summarize:

1. Each player’s best timing is for Rowena to move first.
2. Each player’s worst timing is for Colin to move first

### 5.4. Hold-Up

**The story.** A Supplier (Column) would like to sell a customized product to a Buyer (Row), but the Buyer may either buy now or wait (and maybe buy later). For the product to be valuable to Buyer now, Supplier needs to “invest” in product quality. More specifically, Supplier decides whether to invest or not, and Buyer decides whether to buy now or wait. For Supplier, the best outcome is to make a sale now
without investing, while the worst outcome is not making a sale and not investing (future sale may be lost). Further, Supplier is willing to invest if doing so will induce Buyer to buy now. For Buyer, the best outcome is to buy now if Supplier invests, while the worst outcome is to buy now when Supplier hasn’t invested. Further, when Buyer chooses to wait, it would be better for the Buyer if Seller invests (Buyer may want to purchase in the future).

<table>
<thead>
<tr>
<th></th>
<th>Invest</th>
<th>Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy Now</td>
<td>4,3</td>
<td>1,4</td>
</tr>
<tr>
<td></td>
<td><em>Col first</em></td>
<td></td>
</tr>
<tr>
<td>Wait</td>
<td>3,2</td>
<td>2,1</td>
</tr>
<tr>
<td></td>
<td><em>Row first</em></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 15: Hold-Up**

**Definition.** A strictly ordered 2x2 game is “Hold-Up” whenever

1. There is no PSNE.
2. Row’s two best outcomes are in the same column.
3. Column’s two best outcomes are in the same row.

The conditions of this definition are satisfied if and only if payoffs are as in Figure 15, after one allows for re-labeling of players’ identities and strategies.

**Hold-Up Claim:** A strictly ordered 2x2 game is “Hold-Up” if and only if its players and strategies can be labeled so that payoffs correspond to those in Figure 15.

**Proof:** Without loss, suppose that Row’s two best outcomes are in column Left and Column’s two best outcomes are in row Up. The outcome (Up,Left) must therefore be one of each player’s two best outcomes. However, (Up,Left) cannot be both players’
best outcome (“4,4” payoffs), since then it would be a PSNE. Furthermore, (Up,Left) cannot be both players’ second-best outcome (“3,3” payoffs). To see why, note that Row’s best outcome would have to be (Down,Left) while Column’s best outcome have to be (Up,Right). But then at least one of (Down,Left), (Up,Right), or (Down,Right) must be a PSNE, as illustrated in Figure 16.5

So, it must be that (Up,Left) is one player (say Row)’s best outcome and the other’s second-best outcome, with payoffs as in Figure 16B. Since Row prefers (Down,Right) over (Up,Right), it must be that X = 1 and Z = 2. Similarly, D = 1 and C = 2. So, payoffs are as in Figure 15, as desired. QED

Preferred timing of moves. If Supplier moves first, the SPE outcome is (BuyNow,Invest), Buyer’s best outcome (“4”) and Supplier’s second-best outcome (“3”). If Buyer moves first, the SPE outcome is (Wait,Invest), Buyer’s second-best outcome (“3”) and Supplier’s second-worst outcome (“2”). Finally, if players move simultaneously, each player’s expected payoff in the unique MSE is “between 2 and 3” by the Mixed Strategy Equilibrium Claim. To summarize:

5 If (Down,Left) and (Up,Right) are not PSNE, it must be that Column prefers (Down,Right) over (Down,Left) and Row prefers (Down,Right) over (Up,Right). But then (Down,Right) is a PSNE.
1. Both players’ best timing is for the Supplier to move first.

2. The Buyer’s worst timing is to move simultaneously.

3. The Supplier’s worst timing is for Buyer to move first.

6. Concluding remarks

“Successful business strategy is about actively shaping the game you play, not just playing the game you find.” – Raymond W. Smith.  

This paper provides a novel taxonomy of all strictly ordered 2x2 games that characterizes each player’s preferences over the timing of moves. There are (only!) eight types of games in which players care about the timing of moves, and I have provided a simple set of questions about payoffs (“The Question Tree”) that allows one to identify any given game’s type.

The results developed here shed light on the extent to which players have common or conflicting interests in the “meta-game” to shape the timing of moves. In four types of games, both players have the same favorite timing of moves:

- Lazy Husband – “Husband moves first” is best for both;
- Assurance – “Either player moves first” is best for both;
- Gossip – “Rowena moves first” is best for both; and
- Hold-Up – “Supplier moves first” is best for both.

In one type of game, players disagree about the best timing of moves but agree on how to rank at least some timing options:

- Chores – “Parent moves first” is better than “Simultaneous moves” for both.

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6 Smith was chairman of Bell Atlantic during the 1990s. This quotation is from “Business As War Game: A Report From the Battlefront”, Fortune, September 30, 1996.
Finally, in Chicken(2),\textsuperscript{7} Sir Robin’s Minstrel, and Hide & Seek, players’ preferences over the timing of moves are in complete conflict, in the sense that any timing change that benefits one player hurts the other player.

**Related literature.** This is not the first paper to categorize all strictly ordered 2x2 games in a useful way. To the best of my knowledge, Rapoport and Guyer (1966) was first, proposing 24 categories based on “stability” properties of these games.\textsuperscript{8} Barany, Lee, and Shubik (1992) proposed a different set of 24 categories, based on the shape of the convex hull of payoffs. More recently, Robinson and Goforth (2005) describe the “topology” of strictly ordered 2x2 games and highlight relationships among games that are nearby in this topology.\textsuperscript{9}

Most closely related to this paper is Walliser (1988), who identified a taxonomy of all strictly ordered 2x2 games that characterizes how each player ranks *moving first* vs. *moving last*. Since moving simultaneously is never either player’s strictly best timing of moves in any 2x2 game, one can interpret Walliser’s taxonomy as characterizing each player’s best timing of moves. This paper goes beyond Walliser (1988) by providing a new taxonomy that characterizes each player’s best *and worst* timing of moves.

**References**


\textsuperscript{7} Chicken (0) and Chicken(1) are less clear-cut. For instance, suppose players in Chicken(0) expect the mixed-strategy equilibrium to be played if moves are simultaneous. Players will then agree that simultaneous moves is worse than sequential moves, no matter who moves first.


\textsuperscript{9} Other papers consider all 2x2 games, including those in which players may be indifferent between some outcomes. See e.g. Kilgour and Fraser (1998).


