On the Benefits of Dynamic Bidding when Participation is Costly

David McAdams*

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Abstract

Consider a second-price auction with costly bidding in which bidders with i.i.d. private values have multiple opportunities to bid. If bids are observable, the resulting dynamic-bidding game generates greater expected total welfare than if bids were sealed, for any given reserve price. Making early bids observable allows high-value bidders to signal their strength and deter others from entering the auction. Nonetheless, as long as the seller can commit to a reserve price, expected revenue is higher when bids are observable than when they are sealed.

JEL Codes: D44. Keywords: dynamic bidding, multi-round auction, bidding cost, preemptive bid, optimal reserve price, communication cost

*Fuqua School of Business and Economics Department, Duke University, Email: david.mcadams@duke.edu, Post: Fuqua School of Business, 100 Fuqua Drive, Durham, NC 27708.
1 Introduction

Bids in sealed-bid auctions may arrive at different times but, since they are sealed, equilibrium play is the same as if bids were simultaneous. This paper considers the welfare and revenue implications of an alternative policy of publicly revealing all bids as they arrive, prior to an otherwise standard second-price auction with costly bidding in which bidders have i.i.d. private values. In particular, I consider a dynamic-bidding game that extends Samuelson (1985)’s costly-bidding model to a setting with multiple “bidding rounds” in which bids can be simultaneously submitted, with bids made in each round automatically revealed prior to the next round.

Bidders with higher values submit earlier bids in equilibrium, allowing them to deter lower-value bidders from competing. Such entry deterrence benefits higher-value bidders, by allowing them to obtain the object at a lower price, while also benefitting lower-value bidders as they avoid entering auction contests they would otherwise lose. Consequently, bidders’ interim expected surplus is higher under “dynamic bidding” when there are multiple bidding rounds than under “sealed bidding” when there is only one bidding round, for any given reserve price.

Equilibrium entry is not efficient under dynamic bidding, even when the reserve price is set to zero. The reason is that each bidder’s private benefit from deterring others’ entry, that he can win the object at the reserve price rather than the second-highest bidder value, differs from the social benefit of entry deterrence, that others do not incur the cost of bidding. This contrasts with the well-known result that equilibrium entry is efficient under sealed bidding with a zero reserve price; see e.g. Stegeman (1996).

Example 1 (Inefficient equilibrium entry). Two bidders have i.i.d. private values uniformly distributed on $[0, 1]$. The cost of bidding $c = \frac{1}{10}$ and there are two bidding rounds.
The efficient symmetric entry thresholds in this example are $\frac{5}{8}$ in the first round and $\frac{1}{4}$ in the second round, while the equilibrium entry thresholds are $\frac{3}{5}$ in the first round and $\frac{1}{5}$ in the second round. (See the online supplementary material for details.) Note that the object is more likely to be sold in equilibrium than is efficient ($\frac{1}{5} < \frac{1}{4}$) and bidders are more likely to enter early in equilibrium than is efficient ($\frac{2}{5} < \frac{5}{8}$).

Although equilibrium entry is inefficient under dynamic bidding, equilibrium expected total welfare is strictly higher under dynamic bidding than when bids are sealed, for any given reserve price (Theorem 1). For an intuition, note that allowing multiple bidding rounds has two sorts of effects on equilibrium play, each of which tends to increase expected total welfare. First, dynamic bidding facilitates welfare-enhancing entry deterrence, as bidders who would have entered but lost in a sealed-bid auction now avoid incurring the cost of bidding. Second, dynamic bidding reveals information about others’ values to those who choose not to enter early, encouraging some bidders who would have chosen not to participate in a sealed-bid auction to enter in a later round of the dynamic-bidding game.

What about expected revenue? For any given reserve price, the effect of dynamic bidding on seller expected revenue is ambiguous, as the revenue from new sales to lower-value bidders may or may not dominate the lost revenue from selling to higher-value bidders at lower prices. If the seller is able to commit to a reserve price however, expected revenue is higher under dynamic bidding than under sealed bidding. Intuitively, the reason is that since dynamic bidding makes the auction more attractive to bidders.

\footnote{The seller’s reserve price is set before the game begins and, in particular, does not depend on the realized timing of entry into the auction. The seller can obviously do even better if able to commit to a reserve price that changes over time or to more general dynamic mechanisms, the analysis of which is beyond the scope of this paper.}
at any given reserve price, the seller can raise the reserve without losing sales. Indeed, raising the reserve price allows the seller to extract all the welfare gains associated with better bidder coordination in the form of greater expected revenue.

The paper focuses on a setting in which bidding is costly and bids are publicly observable, but the analysis carries over to an alternative setting in which bidding is costless and unobservable but there are costs associated with participating in the auction and other bidders can observe when these costs are incurred. The notion that participation can be costly is well-accepted in the auction literature, but the analysis here also depends on the notion that (i) these costs can be incurred prior to the auction and (ii) the act of incurring these costs is observable to other bidders. For instance, the cost of traveling to an auction site would not fit within the framework studied here, since bidders cannot observe who has travelled to the auction site until they have already incurred the travel cost.

That said, there are several sorts of potentially substantial participation costs that must be incurred before an auction and which therefore have the potential, if observable by other bidders, to influence their decisions whether to participate.

Example: Establishing one’s qualifications. Bidders in auctions of high-value assets are often required to post bond and/or to receive third-party certification of their ability to pay. Similarly, bidders in complex procurement auctions must often first establish that they are capable of delivering the desired products or services.

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2 For instance, in a study of eBay coin auctions, Bajari and Hortacsu (2003) estimate that bidders faced participation cost of $3.20, a significant amount given that expected revenue in these auctions ranged from $40 to $50.

3 For example, cities that wish to host the Barack Obama Presidential Library must first fulfill a “request for qualifications,” submitting everything from aerial photographs of the proposed site to their strategic plan for engaging the surrounding community. See
Example: Securing necessary expertise. When competing to provide expert services, law firms, consulting firms, and others who coordinate such services may first need to secure a qualified expert. This may entail substantial cost even if the bid is unsuccessful, if the experts’ services are needed to prepare the bid.

Example: Building personal trust. In a corporate acquisition, top managers of each potential acquirer may need to meet at length with the target firm’s management to communicate their plans and build personal trust.⁴

Some of these “pre-participation costs” could potentially be kept secret. For instance, when a corporate acquisition target and potential suitor’s management teams meet, both sides could choose to keep quiet about it. Within the context of this paper’s model, however, the acquisition target benefits by committing to a transparent policy of revealing whenever such meetings take place (Theorem 2).

Other times, the seller may be unable to observe when bidders incur the costs of participating in the auction. In such cases, however, each bidder has an incentive to reveal to other bidders when it has incurred such costs, so as to deter others from participating in the auction. For instance, consider the example in which bidders need to secure an


⁴When Facebook CEO Mark Zuckerberg heard on Friday February 7th, 2014 that Google CEO Larry Page had scheduled a meeting with WhatsApp CEO Jan Koum for the following Tuesday, he scrambled Facebook’s management team over the weekend and invited Koum to his home on Monday night for several hours of preliminary discussions. Facebook ultimately faced no competing offer from Google but, according to Forbes magazine’s behind-the-scenes account of the deal (Olson (2014)), the fact that Google might have wanted WhatsApp induced Zuckerberg to accelerate his move to acquire the company. One reason why Google’s potential interest could have compelled Zuckerberg to make such an early move, consistent with this paper’s analysis, is that failure to move quickly would have signaled a lack of strong interest and encouraged Page to spend time and energy himself to compete for WhatsApp.
expert’s services in order to prepare a bid. In a “small world” with only a few qualified experts, each of whom is in regular contact with all of the bidders, each bidder would naturally be able to observe whenever anyone else secured an expert’s services, as that expert would then be unavailable. On the other hand, the seller might not be able to observe anything about which bidders have secured an expert until the auction itself.

Related literature. The paper fits into the costly-bidding literature following Samuelson (1985)\textsuperscript{5}, the novel feature here being that bidders have multiple opportunities to enter the auction. The only other paper I am aware of that allows for multiple bidding rounds in an auction with costly bidding is Li and Conitzer (2013), who characterize the optimal reserve-price path in a $K$-round second-price auction in which the seller can change the reserve price over time. The seller’s expected revenue is obviously higher under dynamic bidding with an optimal reserve-price path than under sealed bidding with an optimal reserve, since the seller could always replicate the sealed-bid environment by committing to an infinite reserve after the first round. This paper shows that, in fact, the seller’s expected revenue is higher under dynamic bidding even if the seller can only commit to an unchanging reserve\textsuperscript{6}.

Another closely related paper is Levin and Peck (2003) (hereafter “LP”), who consider a dynamic-entry game that can in some cases be interpreted as a second-price auction with zero reserve\textsuperscript{7}. The basic threshold structure of equilibrium entry is similar here and

\textsuperscript{5}Less closely related is the literature on auctions with sequential bidder arrivals, e.g. McAfee and McMillan (1987) and Bulow and Klemperer (2009), the key difference being that bidders here choose when to enter the auction.

\textsuperscript{6}Such limited commitment power arises naturally in some settings, such as the expert-services example mentioned earlier, in which the seller cannot directly observe when bidders incur the costs of participation but other bidders can.

\textsuperscript{7}In LP’s benchmark model, two firms with i.i.d. entry costs have multiple opportunities to enter a
in LP, but the papers take different (and complementary) analytical approaches. For instance, whereas LP use a contraction argument to establish equilibrium uniqueness, I provide a direct proof and a simple algorithmic method to compute the equilibrium.

The paper also touches more indirectly on several other literatures:

*Preemptive bidding and signaling:* A key feature of equilibrium bidding in this paper’s model is that bidders are more likely (ex ante) to enter in earlier bidding rounds, because earlier entry allows bidders to deter subsequent entry. As such, the paper is similar in spirit to the jump-bidding literature (e.g. Avery (1998) and Horner and Sahuguet (2007)) while also relating, more indirectly, to the broader literature on preemptive bidding (see e.g. Fishman (1988) and Hirschliefer and Png (1989)) and the diverse literature on bidder-to-bidder signaling in auctions.

*Market timing design:* A key result here is that, when bidding is costly, restricting bidding to occur at a single moment makes auctions less efficient. The paper therefore relates, at least in spirit, to others that consider when markets should be opened or closed to maximize the expected gains from trade. See e.g. Ockenfels and Roth (2006) on the benefits of not committing to a hard deadline in eBay-style auctions and Fuchs and Skrzypacz (2013) on the benefits of sometimes imposing a “lock-up period” (in which no trade is allowed) in a continuous-time lemons market.

*Disclosure in auctions:* This literature identifies optimal disclosure policies, as well as new market. Each firm enjoys monopoly revenue $R_m$ if it is the only one to enter or duopoly revenue $R_d$ if both enter. When $R_d = 0$, LP’s game can be interpreted as a second-price auction with zero reserve price, where bidders have known common value $v = R_m$ and i.i.d. entry costs.

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8 See e.g. Eso and Schummer (2004), where higher bribes signal higher values; McAdams and Schwarz (2007), where waiting until the deadline to bid signals a high value; and Daley, Schwarz, and Sonin (2012), where bidders can burn money and/or make costly investments.
optimal allocation rules, in multi-round sales mechanisms such as when a seller faces a sequence of potential buyers (e.g. Pancs (2013)) or when the winner of an auction may then resell the good (e.g. Calzolari and Pavan (2006), Zhang and Wang (2013)). This paper touches on the related issue of bid disclosure in a second-price auction, comparing two bid-disclosure regimes: never revealing any bids vs. always revealing all bids.

*Dynamic mechanism design:* This literature characterizes optimal mechanisms in a wide variety of dynamic environments with changing bidder preferences and/or random bidder arrivals. (See Bergemann and Said (2011) for an excellent survey.) This paper touches on this literature by analyzing a particular dynamic mechanism – the second-price auction with multiple bidding rounds – but differs by considering a static environment in which dynamics are driven by the presence of a participation cost.

2 Model

Potential bidders $i = 1, ..., N$ observe i.i.d. private values $v_i$ with continuous c.d.f. $F(\cdot)$, p.d.f. $f(\cdot)$, and full support on $[0, \bar{V}]$. The seller then holds a second-price auction with reserve price $r$, with the novel feature that there are $K > 1$ “bidding rounds.” In each bidding round, the bidders who have not yet submitted a bid simultaneously decide whether to submit a bid at cost $c > 0$, with all bids then becoming publicly observable at the end of the round. The analysis will focus on symmetric threshold equilibria.

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9The dynamic mechanism design literature has continued to grow quickly. A few recent notable contributions: Pavan, Segal, and Toikka (2014) considers a general setting with changing agent preferences in which all are available to participate when the principal fixes the mechanism; Hinmosaar (2011), Garrett (2013) and Gershkov, Moldavan, and Strack (2013) consider settings with fixed preferences in which agents arrive over time according to a random process; and Ely, Garrett, and Hinmosaar (2012) and Garrett (2014) consider environments with both changing preferences and random arrivals.
Definition 1 (Symmetric threshold equilibrium). A “symmetric threshold equilibrium (STE)” is a perfect Bayesian equilibrium in which each bidder \( i \) bids (and bids truthfully) in round \( k = 1, ..., K \) if and only if \( v_i \geq e_{K|K}^k \) and no one has bid previously, where \( e_{1|K}^1 > ... > e_{K|K}^K \) are entry thresholds.

Let \( v_{-i} = \max_{j \neq i} v_j \) and let \( G(\cdot) \) and \( g(\cdot) \) denote the c.d.f. and p.d.f. of \( v_{-i} \).

Benchmark case: Sealed bids. Samuelson (1985) characterized the unique STE when there is just one round of bids or, equivalently, when there are multiple rounds but bids are sealed. In this equilibrium, each bidder enters if and only if \( v_i \geq e_{1|1}^1 \), where the “simultaneous-entry threshold” \( e_{1|1}^1 \) is defined by the entry-indifference condition:

\[
(e_{1|1}^1 - r)G(e_{1|1}^1) = c. 
\] (1)

3 Dynamic bidding

Section 3.1 characterizes the unique symmetric threshold equilibrium (STE) when there are \( K \) bidding rounds, as well as the limit as \( K \to \infty \). Section 3.2 then explores the welfare and revenue effects of dynamic bidding.

3.1 Symmetric threshold equilibrium

Proposition 1. The \( K \)-round bidding game has a unique symmetric threshold equilibrium, with entry thresholds \( (e_{1|K}^1, ..., e_{K|K}^K) \) satisfying \( r + c < e_{K|K}^K < ... < e_{1|K}^1 < V \) and characterized by the following system of equations:

\[
(e_{K|K}^K - r)G(e_{K|K}^K) = cG(e_{K-1|K}^{K-1}) 
\] (2)

\[
\int_{e_{k+1|K}}^{e_{k|K}} (v_{-i} - r)dG(v_{-i}) = c(G(e_{k-1|K}^{k-1}) - G(e_{k|K}^k)) \text{ for } k = 1, ..., K - 1
\] (3)
where by convention $e^0|K = \nabla$.

Discussion: Any bidder whose value equals the round-$k$ threshold, $v_i = e^k|K$, must be indifferent between bidding or not in round $k$ when no one has bid prior to round $k$. In the last round, such indifference requires that a bidder with value $v_i = e^K|K$ earn zero expected profit. A bidder with value $v_i = e^K|K$ who enters in round $k$ will win when $\max_{j \neq i} < e^K|K$, with conditional probability $\frac{G(e^K|K)}{G(e^{K-1}|K)}$, and pay the reserve price. The zero-profit condition for this type is therefore $(e^K|K - r)\frac{G(e^K|K)}{G(e^{K-1}|K)} = c$, equivalent to equation (2) in which, for convenience, expected payoffs are expressed in ex ante terms.

In much the same way, equation (3) captures the equilibrium indifference condition for bidders having value $v_i = e^k|K$. In particular, the right-hand side of (3) is the (ex ante) expected loss to the threshold type $v_i = e^k|K$ from entering in round $k$ when others with higher values in $(e^k|K, e^{k-1}|K)$ also enter in round $k$, while the left-hand side of (3) is the expected benefit from entering in round $k$ associated with deterring others with lower values from entering in round $k + 1$.

Conditions (2,3) on the entry thresholds $(e^1|K, ..., e^K|K)$ are clearly necessary in any STE. The proof of Proposition 1 shows, in addition, that they are sufficient conditions with a unique solution, ensuring the existence of a unique STE (up to the behavior of a zero measure set of bidder types) for any given number of bidding rounds $K$.

**Proposition 2.** $e^k|K$ is increasing in $K$ but $e^k|\infty = \lim_{K \to \infty} e^k|K < \nabla$ for all $k = 1, 2, \ldots$. In particular, the limit-thresholds $(e^1|\infty, e^2|\infty, \ldots)$ are determined recursively by

\[
(e^1|\infty - r)G(e^1|\infty) - \int_{r+c}^{e^1|\infty} G(x)dx = c \tag{4}
\]

\[
(e^k|\infty - r)G(e^k|\infty) - \int_{r+c}^{e^k|\infty} G(x)dx = cG(e^{k-1}|\infty) \quad \text{for } k = 2, 3, \ldots \tag{5}
\]

where $\lim_{k \to \infty} e^k|\infty = r + c$. 

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Proof. The proof is in the Appendix.

Discussion: Proposition 2 characterizes bidder behavior in the limit as the number of bidding rounds goes to infinity. Note that a positive measure of bidders (anyone with value $v_i > e^{1|\infty}$) enters in the first round, no matter how many bidding rounds there may be. Even in the limit as the number of bidding rounds goes to infinity, then, there is a positive probability that two or more bidders will enter in the same round and compete.

Example. Suppose that there are two bidders with i.i.d. values uniformly distributed on $[0, 1]$, the reserve price is zero, and the bidding cost $c = \frac{1}{10}$. The simultaneous-entry threshold $e^{1|1}$ solves (1): $(e^{1|1})^2 - c = 0$, or $e^{1|1} = \sqrt{c} \approx .316$. By contrast, $e^{1|\infty}$ solves (4): $(e^{1|\infty})^2 - c = \int_c^{e^{1|\infty}} x dx$, or $e^{1|\infty} = \sqrt{2c - c^2} \approx .436$. Another interesting fact here is that $e^{2|\infty} = \sqrt{2e^{1|\infty}c - c^2} \approx .278 < e^{1|1}$. So, any bidder who waits until after the second round to bid in any $K$-round game would not bid at all if bids were sealed.

3.2 Welfare and revenue effects of dynamic bidding

This section investigates the welfare and revenue effects of having multiple bidding rounds ($K > 1$) compared to having sealed bids ($K = 1$). The main findings are that allowing dynamic bidding increases buyers’ interim expected surplus and expected total welfare under any fixed reserve price (Theorem 1) and increases the seller’s expected revenue under an optimal reserve price (Theorem 2).

Theorem 1. For any given reserve price, bidders’ interim expected surplus and expected total surplus are each strictly higher under dynamic bidding (for any $K < 1$) than under sealed bidding.

Proof. By a standard Envelope Theorem argument, each bidder’s interim expected equi-
librium payoff in the unique STE takes the integral form

$$
\Pi^K(v_i) = \int_{v_i}^{e^{K|K}} G(x) dx \text{ for all } v_i \geq e^{K|K},
$$

(6)

where $G(x)$ is the probability that each bidder wins given value $v_i = x$.10 (A bidder with value $v_i < e^{K|K}$ never enters and earns zero expected payoff.) The fact that bidder’s interim expected surplus is increasing in $K$ therefore follows immediately from the fact that $e^{K|K}$ is decreasing in $K$, which itself follows from inspection of (1,2).

The rest of the proof focuses on expected total surplus. Let $k^*$ be the last bidding round in the $K$-round game in which all entrants would have entered under sealed bidding, i.e., $e^{k^*|K} \geq e^{1|1} > e^{k^*+1|K}$. Let $v = (v_1, ..., v_N)$, $v^{(1)} = \max_i v_i$, and say that bidder $i$ is “type-E” if $v_i \geq e^{k^*|K}$, “type-S” if $v_i \in (e^{1|1}, e^{k^*|K})$, “type-L” if $v_i \in (e^{K|K}, e^{1|1})$, and “type-NO” if $v_i < e^{K|K}$.

**Case #1: the highest bidder is type-E.** The highest bidder enters and wins whether

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Figure 1: How dynamic bidding changes equilibrium outcomes, depending on the highest realized bidder value.
bidding is sealed or dynamic, but other bidders are less likely to enter under dynamic bidding. Conditional on the highest bidder being type-E, then, expected total welfare is strictly higher under dynamic bidding.

Case #2: the highest bidder is type-S. The highest bidder enters and wins whether bidding is sealed or dynamic, while other bidders are more likely to enter under dynamic bidding. In particular, since the highest bidder waits until round $k^* + 1$ to enter, type-L bidders with values $v_i \in (e^{k^*+1|K}, e^{1|1})$ enter and lose under dynamic bidding who would have stayed out under sealed bidding. Note that, in this case when the highest bidder is type-S, the decrease in (ex post) total welfare when one switches from sealed to dynamic bidding is equal to the decrease in (ex post) surplus of L-type bidders with values $v_i \in (e^{k^*+1|K}; e^{1|1})$.

Case #3: the highest bidder is type-L. Since the sealed-bid auction gives zero total welfare in this case, the change in (ex post) total welfare when one switches from sealed to dynamic bidding is equal to the auctioneer’s revenue plus the (ex post) surplus of the type-L bidders under dynamic bidding. Since the former is positive, the change in total surplus after the switch is greater than the surplus that the type-L bidders obtain under dynamic bidding.

Combining these observations, the overall effect of dynamic bidding on expected total welfare is greater than or equal to the expected surplus that type-L bidders obtain under dynamic bidding conditional on the combined event that the highest bidder is either type-S or type-L. This expected surplus is positive since this combined event is observed.

\footnote{To be precise, let $\hat{k} \leq k^*$ denote the round in which the highest bidder enters under dynamic bidding. Other type-E and type-S bidders with values $v_i \in (e^{1|1}, e^{\hat{k}|K})$ would have entered under sealed bidding but are deterred under dynamic bidding, while all type-L and type-NO bidders do not enter in either case, and type-E bidders with values $v_i \in (e^{\hat{k}|K}, v^{(1)})$ enter and lose in either case.}
when type-L bidders take their participation decision.

All together, then, dynamic bidding increases expected total welfare conditional on the highest bidder being type-H, and conditional on the highest bidder being either type-S or type-L, while obviously having no effect on welfare in the last case when all bidders are type-NO. We conclude that (unconditional) total expected welfare is higher under dynamic bidding (for any $K > 1$) than when bids are sealed.

**Theorem 2.** *If the seller can commit to a reserve price, expected revenue is strictly higher under dynamic bidding (for any $K < 1$) than under sealed bidding.*

**Proof.** Let $e^{1|1}(r)$, $e^{K|K}(r)$ denote the simultaneous-entry threshold in the 1-round game and the $K$-th round entry threshold for some $K > 1$, respectively, viewed as functions of the reserve price $r$. By inspection and comparison of (1) and (2), $e^{K|K}(r) < e^{1|1}(r)$ for all $r < V$. Moreover, it is straightforward to show that $e^{K|K}(r)$ is continuous and strictly increasing in $r$.

Let $r^S$ denote the optimal reserve price when bids are sealed. Define $\hat{r} > r^S$ so that $e^{K|K}(\hat{r}) = e^{1|1}(r^S)$, and note that the $K$-round game with reserve $\hat{r}$ has strictly less equilibrium entry than the 1-round game with reserve $r^S$. (Bidders with values in $(e^{K|K}(\hat{r}), e^{1|1}(\hat{r}))$ always enter when bids are sealed, but stay out when $\max_i v_i > e^{1|K}(\hat{r})$ in the $K$-round game.) On the other hand, the object’s allocation is identical in both cases – the object is sold to the highest-value bidder when $\max_i v_i > e^{1|1}(r^S)$ and otherwise not sold – as is interim expected surplus by (6) since $e^{1|1}(r^S) = e^{K|K}(\hat{r})$. Expected revenue must therefore be strictly higher in the $K$-round game with reserve $\hat{r}$, by an amount equal to the expected cost savings of having less equilibrium entry. This completes the proof since seller expected revenue in the $K$-round game is even higher under an optimal reserve.

$\square$
4 Concluding Remarks

A standard argument in favor of conducting a sealed-bid auction is that sealed bids can make it more difficult for bidder-cartels to monitor and enforce a collusive agreement (Marshall and Marx (2012)). This paper suggests a countervailing upside associated with making bidding activity observable during the auction, that such “dynamic bidding” leads bidders to economize on participation costs and, by doing so, allows the seller to generate greater expected revenue than when bids are sealed.

That said, more work needs to be done to characterize optimal disclosure rules in auctions with costly participation. An especially interesting direction for future work is in the study of auctions with costly communication. This paper’s model can be interpreted as one of costly communication, in which bidders must incur a fixed cost to communicate with the seller but the seller does not incur any cost to communicate with bidders. The optimal communication policy in this setting remains an open question. This paper shows that complete transparency is better for the seller than complete secrecy, but the optimal policy may be substantially more complex.\footnote{Because bidder congestion is greatest in early bidding rounds (see Corollary 1 to Proposition 1 in the appendix), the seller can decrease expected equilibrium entry costs – and thereby increase expected revenue – by encouraging bidders to enter later. One way to do so would be to reveal early bids with probability less than one, as such imperfect bid-disclosure decreases the entry-deterrence benefit of early bidding. For this reason, I suspect that the optimal policy involves stochastic disclosure.}
Appendix

Proof of Proposition 1

*Note on notation.* I will use shorthand \((e_0, e_1, ..., e_K)\) rather than \((e_0|K, e_1|K, ..., e_K|K)\) here for entry thresholds in the \(K\)-round bidding game. This should not create confusion, since the number of bidding rounds \(K\) is held fixed. (Recall that, by convention, \(e_0 = V\).)

*Part One: \([2,3]\) are sufficient for STE.* Consider first bidder \(i\)'s decision *what* to bid, assuming that he enters when specified by the entry thresholds \((e_1, ..., e_K)\). Bidding more than one's true value could be optimal, if such overbidding deters others from bidding later. However, since any round-\(k\) bid greater than \(e_k\) is sufficient to deter all future entry, and bidder \(i\)'s value \(v_i \geq e_k\) whenever he enters in round \(k\), truthful bidding is sufficient to deter all future entry when bidder \(i\) (and all other bidders) adopt symmetric \((e_1, ..., e_K)\)-threshold strategies. Moreover, as usual, truthful bidding ensures that bidder \(i\) only wins in round \(k\) when his value exceeds the price that he will pay to win.

Consider next bidder \(i\)'s decision *when* to enter the auction, assuming that all other bidders adopt \((e_1, ..., e_K)\)-threshold strategies where \((e_1, ..., e_K)\) satisfy \([2,3]\). Suppose first that \(v_i < e_K - 1\), so that round \(K\) can be reached with no prior bids (if bidder \(i\) does not bid prior to round \(K\)). By entering in round \(K\), bidder \(i\) wins at the reserve price when \(v_i < e_K\) and wins at price \(v_i\) when \(v_i \in (e_K, \min\{v_i, e_K - 1\})\), yielding expected payoff (expressed for convenience in ex ante terms)

\[
X^K(v_i) = (v_i - r)G(e_K) - cG(e_K - 1) \text{ if } v_i \leq e_K
\]

\[
= (v_i - r)G(e_K) + \int_{e_K}^{\min\{v_i, e_K - 1\}} (v_i - v) dG(v) - cG(e_K - 1) \text{ if } v_i \geq e_K
\]

Note that \(X^K(v_i)\) is strictly increasing in \(v_i\) and, by (2), \(X^K(e_K) = 0\). So, entering in round \(K\) is bidder \(i\)'s best response if and only \(v_i \geq e_K\).
Next, suppose that \( v_{-i} < e^{k-1} \), so that round \( k = 1, \ldots, K - 1 \) can be reached with no prior bids. Relative to waiting and entering in round \( k + 1 \), entering in round \( k \) has three effects, depending on others’ values.

**Case #1:** \( v_{-i} < e^{k+1} \). No one else enters in round \( k \) or would enter in round \( k + 1 \), so bidder \( i \) wins at the reserve price (for ex post payoff \( v_i - r - c \)) whether he enters in round \( k \) or waits to enter in round \( k + 1 \).

**Case #2:** \( v_{-i} \in (e^{k+1}, e^k) \). Bidder \( i \) is better off entering in round \( k \), since doing so deters others from entering in round \( k + 1 \). Such entry deterrence allows bidder \( i \) to win at the reserve price rather than at price \( v_{-i} \) when \( v_i \geq v_{-i} \) (for ex post gain \( v_{-i} - r \)), or to avoid losing the auction when \( v_i < v_{-i} \) (for ex post gain \( v_i - r \)). Overall, then, bidder \( i \)’s (ex ante) expected gain due to entering in round \( k \) rather than round \( k + 1 \) when \( v_{-i} \in (e^{k+1}, e^k) \) is

\[
Y^k(v_i) = \int_{e^{k+1}}^{e^k} (v_{-i} - r) dG(v_{-i}) \text{ if } v_i \geq e^k
\]

\[
= (v_i - r)(G(e^k) - G(\max\{v_i, e^{k+1}\})) + \int_{e^{k+1}}^{\max\{v_i, e^{k+1}\}} (v_{-i} - r) dG(v_{-i}) \text{ if } v_i \leq e^k
\]

**Case #3:** \( v_{-i} \in (e^k, e^{k-1}) \). Bidder \( i \) is at least weakly worse off entering in round \( k \), since there is an option value to waiting and observing what others’ round-\( K \) bids before deciding whether to enter the auction. In particular, waiting until round \( k + 1 \) allows bidder \( i \) to avoid incurring a loss of \( c - \max\{0, v_i - v_{-i}\} \) when \( v_{-i} > e^k \) and \( v_{-i} > v_i - c \). Overall, bidder \( i \)’s (ex ante) expected loss due entering in round \( k \) rather than round \( k + 1 \) is

\[\text{loss} = \int_{e^{k+1}}^{e^k} (v_{-i} - r) dG(v_{-i}) \text{ if } v_i \geq e^k
\]

\[
= (v_i - r)(G(e^k) - G(\max\{v_i, e^{k+1}\})) + \int_{e^{k+1}}^{\max\{v_i, e^{k+1}\}} (v_{-i} - r) dG(v_{-i}) \text{ if } v_i \leq e^k
\]

\[13\] If bidder \( i \) waits until round \( k + 1 \) and \( v_{-i} > e^k \) but \( v_{-i} < v_i - c \), bidder \( i \) will jump in and outbid the highest round-\( k \) bidder once round \( k + 1 \) is reached. So, in this case, he wins at price \( v_{-i} \) whether he bids in round \( k \) or waits until round \( k + 1 \).
$k + 1$ when $v_{-i} \in (e^k, e^{k-1})$ is

$$Z^k(v_i) = c(G(e^{k-1}) - G(e^k)) \text{ if } v_i \leq e^k$$

$$= \int_{\max\{e^k, v_i - c\}}^{\min\{e^{k-1}, v_i\}} (c - v_i + v_{-i})dG(v_{-i}) + c(G(e^{k-1}) - G(\min\{e^{k-1}, v_i\})) \text{ if } e^k \leq v_i \leq e^{k-1} + c$$

$$= 0 \text{ if } v_i \geq e^{k-1} + c$$

(To parse (9) in the most complex case when $v_i \in (e^k, e^{k-1}+c)$, note that (i) round-$k$ entry leads to a loss of $c$ when others also enter and bidder $i$ loses, i.e. when $v_{-i} \in (e^k, e^{k-1})$ and $v_{-i} > v_i$, and (ii) round-$k$ entry leads to a loss of $c - v_i + v_{-i}$ when others also enter and bidder $i$ wins at a price greater than $v_i - c$, i.e. when $v_{-i} \in (e^k, e^{k-1})$, $v_{-i} < v_i$, and $v_{-i} > v_i - c$.)

By (3), $Y^k(e^k) = Z^k(e^k)$. Note further by inspection of (8,9) that, for any $v'' > e^k > v'$, $Y^k(v'') = Y^k(e^k) > Y^k(v')$ and $Z^k(v'') < Z^k(e^K) = Z^k(v')$. So, $Y^k(v_i) \geq Z^k(v_i)$ for all $v_i \geq e^k$ while $Y^k(v_i) < Z^k(v_i)$ for all $v_i < e^k$. So, bidder $i$’s best response is to enter in round $k$ when $v_i \geq e^k$ but not enter in round $k$ when $v_i < e^k$.

So far, I have shown that bidder $i$’s best response if any round $k$ is reached with no prior bids is to enter and bid truthfully if and only if $v_i \geq e^k$. To complete the proof, observe that if bidder $i$ follows this rule in all subgames with no prior bids, bidder $i$’s best response is never to bid in subgames with prior bids. Why? Suppose that the first bids received were in round $k' < k$ and that bidder $i$ has not bid prior to round $k$. All bids submitted in round $k'$ are at least $e^{k'}$ but, since bidder $i$ did not bid in rounds 1, ..., $k'$, bidder $i$’s own value must be less than $e^{k'}$. Clearly, then bidder $i$ prefers not to bid. All together, then, bidder $i$’s best response when all others adopt $(e^1, ..., e^K)$-threshold strategies is to do so as well. This completes the proof that (2,3) are necessary and sufficient for existence of a STE with thresholds $(e^1, ..., e^K)$.

**Part Two: Uniqueness of STE.** To establish uniqueness, I need to show that (2,3) has
a unique solution. Note that (2,3) can be re-written as
\[ G(e^{K-1}) = G(e^K) \left( \frac{e^K - r}{c} \right) \] and
\[ G(e^{k-1}) - G(e^k) = (G(e^k) - G(e^{k+1})) \frac{E[v_{-i} - r_i | v_{-i} \in [e^{k+1}, e^k]]}{c} \]
for all \( k = 1, \ldots, K - 1 \). (Recall that \( e^0 = \overline{V} \), so this is a system of \( K \) equations with \( K \) unknowns.)

Suppose for a moment that \( e^K = r + c \). If so, (2) implies that \( e^{K-1} = r + c \) while (3) implies that \( e^{K-2} = \ldots = e^1 = e^0 = r + c \). This is a contradiction, clearly, since \( e^0 = \overline{V} > r + c \). Similarly, for \( e^K > r + c \), (2) determines \( e^{K-1} \) as a function of \( e^K \) (call it \( e^{K-1}(e^K) \)) while (3) inductively determines the other thresholds \( (e^{K-2}(e^K), \ldots, e^1(e^K), e^0(e^K)) \) of \( e^K \). (11) determines \( e^{K-2} \) as a function of \( (e^{K-1}(e^K), e^K) \). Since \( e^{K-1} \) is determined by \( e^K \), so is \( e^{K-2} \). Repeating this logic inductively determines \( e^{K-3}, \ldots, e^1, e^0 \) as functions of \( e^K \).

\( (e^0 = \overline{V}, e^{1*}, \ldots, e^{K*}) \) solves (2,3) if and only if (i) \( e^{1*} = e^1(e^{K*}), \ldots, e^{K-1*} = e^{K-1}(e^{K*}) \) and (ii) \( e^0(e^{K*}) = \overline{V} \) or, equivalently, \( G(e^0(e^{K*})) = 1 \). Next, note that (10) implies that \( G(e^{K-1}(e^K)) - G(e^K) > 0 \) is continuous and strictly increasing in \( e^K \), while (11) implies by induction that \( G(e^{k-1}(e^K)) - G(e^k(e^K)) \) is continuous and strictly increasing in \( e^K \), for all \( k = 1, \ldots, K - 1 \). In particular, \( G(e^0(e^K)) \) is continuous and strictly increasing in \( e^K \) if \( e^K > r + c \). So, there is a unique solution \( e^{K*} \) to \( G(e^0(e^{K*})) = 1 \) and hence a unique solution \( (e^{1*}, \ldots, e^{K*}) \) to (2,3).

\[ \square \]

**Corollary 1.** *In the unique STE, the ex ante probability that the first bid arrives in round \( k \) is decreasing in \( k \).*

*Proof.* The first bid arrives in round \( k \) when \( \max_i v_i \in [e^{k[K]}, e^{k-1}[K]) \), i.e. with probability \( F(e^{k-1[K])N} - F(e^{k[K])N} \). Since \( e^0[K] > e^1[K] > \ldots > e^{K[1]}[K], F(e^{k-1}[K])N - F(e^{k}[K])N \) is
decreasing in \(k\) if and only if \(G(e^{k-1|K}) - G(e^k|K) = F(e^{k-1|K})^N - F(e^k|K)^N\) is decreasing in \(k\). (Details for this step are straightforward and omitted.) Note that conditions (12) can be rewritten as

\[
\frac{e^{K-K} - r}{c} = \frac{G(e^{K-1|K})}{G(e^K|K)} \quad (12)
\]

\[
E[v_i - r|v_{-i} \in (e^{K+1|K}, e^{|K})] = \frac{G(e^{k-1|K}) - G(e^k|K)}{G(e^k|K) - G(e^{k+1|K})} \quad (13)
\]

for all \(k = 1, ..., K - 1\). Since \(e^{k+1|K} > r + c\), the left-hand side of (13) is greater than one, implying \(G(e^{k-1|K}) - G(e^k|K) > G(e^k|K) - G(e^{k+1|K})\) for all \(k = 1, ..., K - 1\), as desired.

\[\square\]

**Corollary 2.** \(e^k|K\) is decreasing in \(K\).

**Proof.** Suppose for the sake of contradiction that \(e^{K'|K'} \geq e^k|K\) for some \(K' > K\). This implies \(G(e^{K'|K'}) \geq G(e^k|K)\) and, by (12), \(\frac{G(e^{K'-1|K'})}{G(e^{K'|K'})} \geq \frac{G(e^{K-1|K})}{G(e^K|K)}\). So, \(e^{K'-1|K'} \geq e^{K-1|K}\) and \(G(e^{K'-1|K'}) - G(e^{K'|K'}) \geq G(e^{K-1|K}) - G(e^k|K)\). By (13), this then recursively implies both \(e^{K'-t|K'} \geq e^{K-t|K}\) and \(G(e^{K'-t|K'}) - G(e^{K'-t+1|K'}) \geq G(e^{K-t|K}) - G(e^{K-t+1|K})\) for all \(t = 2, ..., K\). Why? Consider \(t = 2\). Since \(e^{K'|K'} \geq e^k|K\) and \(e^{K'-1|K'} \geq e^{K-1|K}\), \(E[v_i - r|v_{-i} \in (e^{K'|K'}, e^{K'-1|K'})] \geq E[v_i - r|v_{-i} \in (e^k|K, e^{K-1|K})]\). Since \(G(e^{K'-1|K'}) - G(e^{K'|K'}) \geq G(e^{K-1|K}) - G(e^k|K)\), condition (13) for \(k = K - 1\) then implies that \(G(e^{K'-2|K'}) - G(e^{K'-1|K'}) \geq G(e^{K-2|K}) - G(e^{K-1|K})\), \(e^{K'-2|K'} \geq e^{K-2|K}\) then follows immediately from \(e^{K'-1|K'} \geq e^{K-1|K}\). Repeating this argument recursively, using condition (13) for \(k = K - 2, ..., 1\), establishes that \(e^{K'-K|K'} \geq e^0|K = V\). But this is a contradiction since, by Theorem 1, \(e^{K'-K|K'} < V\). \[\square\]
Proof of Proposition 2

Proof. Step One: Derive analogues to (4,5) for all finite $K$. By (6), each bidder’s interim expected equilibrium payoff $\Pi^K(v_i) = \int_{e^K|K} v_i e^K|K G(x) dx$ for all $v_i \geq e^K|K$. At the same time, for each threshold type $v_i = e^k|K$, we can also express $\Pi^K(e^k|K) = (e^k|K - r)G(e^k|K) - cG(e^{k-1}|K)$ since such a threshold type only wins if no one else enters in round $k$. In particular, for every $K$ and $k \leq K$, $e^k|K$ must solve

$$(e^k|K - r)G(e^k|K) - \int_{e^k|K} e^k|K G(x) dx = cG(e^{k-1}|K),$$

where $(e^{k-1}|K, e^K|K)$ are viewed in parameters that vary with $K$.

Step Two: $e^k|K$ is increasing in $K$ for all $k$. The fact that $e^k|K$ is increasing in $K$ emerges as a simple comparative static of the solution to (14). Consider first $k = 1$. Since $e^0|K = \mathcal{V}$ for all $K$, the right-hand side of (14) does not depend on $K$. On the other hand, since $e^K|K$ is decreasing in $K$ (Corollary 2 to Theorem 1), the left-hand side of (14) is decreasing in $K$. Since $\frac{d[(v_i - r)G(v_i)]}{dv_i} - \frac{d[\int_{e^k|K} G(x) dx]}{dv_i} = (v_i - r)g(v_i) > 0$, the solution $e^1|K$ of (14) must therefore be increasing in $K$. The rest of the proof is by induction on $k$. As long as $e^{k-1}|K$ is increasing in $K$, the right-hand side of (14) is increasing in $K$ while, as in the $k = 1$ case, the left-hand side of (14) is decreasing in $K$. So, the solution $e^k|K$ of (14) must be increasing in $K$, and the limit $e^{k|\infty}$ exists for all $k$.

Step Three: $\lim_{K \to \infty} e^K|K = r + c$. Let $\bar{v} = \lim_{K \to \infty} e^K|K$. Since $e^K|K > r + c$, $\bar{v} \geq r + c$. Suppose for the sake of contradiction that $\bar{v} > r + c$, and fix $\hat{v} \in (r + c, \bar{v})$. Since $e^K|K$ is decreasing in $K$, a bidder with value $\hat{v}$ must find it unprofitable to enter in every round $k$ of every $K$-round bidding game. Entering in round $k$ would allow a bidder with value $\hat{v}$ to win at the reserve price with (ex ante) probability $G(e^k|K)$ while incurring cost $c$ with probability $G(e^{k-1}|K)$. For this to be unprofitable, the conditional probability that someone else enters in round $k$ must be sufficiently high, namely, $\frac{G(e^{k-1}|K) - G(e^k|K)}{G(e^{k-1}|K)} \geq \frac{\hat{v} - r - c}{\hat{v} - r}$. 21
Now, consider any $K > \frac{\hat{v} - r}{G(r+c)(\hat{v} - r - c)}$ and define $\hat{k} \in \arg \min_k (G(e^{k-1|K}) - G(e^{k|K}))$.

Since $e^{1|K} > \ldots > e^{K|K}$, \[\min_k (G(e^{k-1|K}) - G(e^{k|K})) \leq \frac{1}{K} < \frac{G(r+c)(\hat{v} - r - c)}{\hat{v} - r}.\]

So, $\frac{\hat{v} - r - c}{\hat{v} - r} > \frac{G(e^{k-1|K}) - G(e^{k|K})}{G(r+c)}$ and entering in round $\hat{k}$ is profitable for a bidder with value $\hat{v}$, a contradiction.

To complete the proof, note that (4) and (5) follow from (14) for $k = 1$ and $k > 1$, respectively, by continuity in the limit as $e^{K|K} \to r + c$.

\[\square\]

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References


OLSON, P. (2014): “Inside the Facebook-WhatsApp megadeal: the courtship, the secret meetings, the $19 billion poker game,” *Forbes*.


