Decentralized Pricing and Capacity Decisions in a Multi-Tier System with Modular Assembly

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A On Line Appendix: Proofs

Proof of Proposition 1. Consider any Nash equilibrium $Q^*$. For any two players $h_1$ and $h_2$, $Q^*_{h_1} \leq Q_{h_1}^{min}(Q^*) \leq Q^*_{h_2} \leq Q_{h_2}^{min}(Q^*) \leq Q^*_{h_1}$, so that $Q^*_{h_1} = Q^*_{h_2}$. Also, $Q^*_{h} \leq Q^*_d$ for all $h$, so that $Q^*_{h} \leq Q^*_d$. Wang and Gerchak (2003) show that any $Q$ such that $Q_{ij} = Q_i = Q_0 \leq Q^*_d(\alpha, \gamma)$ is a Nash equilibrium and that $Q^*_d$ is uniquely Pareto-optimal. ■

Proof of Lemma 1. The proof follows a similar logic to the proofs of Lemma 1 in Wang and Gerchak (2003) and Lemma 23 in Tomlin (2000). ■

Proof of Proposition 2. Given any capacity choices by the other subassemblers, if subassembler $i$ chose $Q_i > \min \{\min_{x \neq i} Q_x, Q_I^L(\alpha)\}$, then she could increase her expected profit by reducing her capacity to $Q_i = \min \{\min_{x \neq i} Q_x, Q_I^L(\alpha)\}$, thus increasing $\gamma_i(\alpha)_0$ (see (8)). Also, for $Q_i < \min \{\min_{x \neq i} Q_x, Q_I^L(\alpha_0)\}$ subassembler $i$’s expected profit is the same as in its isolated pricing problem, so if $Q_i^S(\alpha_i) < \min \{\min_{x \neq i} Q_x, Q_I^L(\alpha_0)\}$ it is optimal to choose $Q_i = Q_i^S(\alpha_i)$. As a result, subassembler $i$’s best-response capacity in this game is $\min \{\min_{x \neq i} Q_x, Q_I^L(\alpha_0), Q_i^S(\alpha_i)\}$. It is then easy to verify that any $\{Q_1, \cdots, Q_n\}$ such that $Q_1 = \cdots = Q_n \leq Q^S(\alpha)$ is a Nash equilibrium for the subassemblers’ pricing game. Similar to the proof of Proposition 1, consider now $Q^*$ a Nash equilibrium. For any two players $h_1$ and $h_2$, $Q^*_{h_1} = \min \{\min_{x \neq h_1} Q_x^*, Q_I^L(\alpha_0), Q^S_{h_1}(\alpha_{h_1})\} \leq Q^*_{h_2} = \min \{\min_{x \neq h_2} Q_x^*, Q_I^L(\alpha_0), Q^S_{h_2}(\alpha_{h_2})\} \leq Q^*_{h_1}$, so that $Q^*_{h_1} = Q^*_{h_2} \leq Q^S(\alpha)$. ■
Since each subassembler $i$’s expected profit is increasing on $Q_i \leq Q^{S*}(\alpha)$, the equilibrium with $Q_i = Q^{S*}(\alpha)$ for $i = 1, \ldots, n$ is uniquely Pareto-optimal. ■

**Proof of Lemma 2.** Observe from (12) that $\frac{\partial^2 \pi^S_i(Q)}{\partial Q^2} = T'(Q_i) > 0$. Also, from (12) and the fact that $l(\cdot)$ is increasing, we have that

$$\frac{\partial^2 \pi^S_i(Q)}{\partial Q^2} = -(\alpha_i - C^i)f(Q_i) - \sum_{j=1}^{m_i} k_{ij} l'(Q_i) \leq -(\alpha_i - C^i)f(Q_i). \quad (22)$$

Since $Q^S_i(\alpha_i)$ is the solution of (12), we can compute $\frac{\partial Q^S_i}{\partial \alpha_i} = -\frac{\partial^2 \pi^S_i / \partial Q / \partial \alpha_i}{\partial^2 \pi^S_i / \partial Q^2}$ by the Implicit Function Theorem, and conclude, from (22), that

$$0 \leq \frac{\partial Q^S_i}{\partial \alpha_i} \leq \frac{\alpha_i - C^i}{F(Q^S_i)}. \quad (\alpha_i - C^i)$$

We thus conclude that $Q^S_i(\alpha_i)$ is increasing in $\alpha_i$, and this increase can only be achieved by subassembler $i$ passing on to her suppliers some of the price increase she receives from the assembler – i.e., $\alpha_i \gamma^S_{ij}(\alpha_i)$ is increasing in $\alpha_i$. It is also possible to identify an upper bound on how fast $\alpha_i \gamma^S_{ij}(\alpha_i) = c_{ij} + k_{ij}/F(Q^S_i(\alpha_i))$ increases in $\alpha_i$, for $j = 1, \ldots, m_i$, that is

$$0 \leq \frac{\partial (\alpha_i \gamma^S_{ij})}{\partial \alpha_i} = \frac{k_{ij} f(Q^S_i(\alpha_i))}{F(Q^S_i)} \leq \frac{k_{ij}}{F(Q^S_i(\alpha_i))}. \quad (\alpha_i - C^i)$$

This in turn implies that

$$\frac{\partial (\alpha_i \gamma^S_{ij})}{\partial \alpha_i} = 1 - \sum_{j=1}^{m_i} \frac{\partial (\alpha_i \gamma^S_{ij})}{\partial \alpha_i} \geq 1 - \sum_{j=1}^{m_i} \frac{k_{ij}}{F(Q^S_i(\alpha_i))} = \frac{\alpha_i - C^i}{F(Q^S_i(\alpha_i))}.$$  

The last expression can be shown to be positive as follows: (3), (4) and (5) imply that for $j = 1, \ldots, m_i$,

$$\frac{c_{ij} + k_{ij}}{\alpha_i} \leq \gamma^S_{ij} \leq \frac{k_{ij}}{K^i} + \frac{c_{ij}}{\alpha_i} - \frac{k_{ij} C^i}{K^i}. \quad (23)$$

Taking the second inequality in (23), summing both sides over $j = 1, \ldots, m_i$ and noting that $\alpha_i - \alpha_i \gamma^S_{ij0} = \alpha_i \sum_{j=1}^{m_i} \gamma^S_{ij}$, yields $\alpha_i \gamma^S_{ij} \geq c_{ij0} + c_i + \frac{k_{ij} + k_i}{K^i} (\alpha_i - C^i)$. We then conclude that

$$\frac{\partial (\alpha_i \gamma^S_{ij})}{\partial \alpha_i} \geq \frac{\alpha_i \gamma^S_{ij0} - c_{ij0} - c_i}{\alpha_i - C^i} \geq \frac{k_{ij} + k_i}{K^i} > 0. \quad (\alpha_i - C^i)$$

**Proof of Proposition 3.** (a) Suppose the assembler sets $\alpha$ such that $Q^S_j(\alpha_j) > Q^S_0(\alpha_0) \geq Q^S_i(\alpha_i)$ for any two subassemblers $i$ and $j$. By Lemma 2, $Q^S_j(\alpha_j)$ is increasing in $\alpha_j$. Also, $Q^S_0(\alpha_0)$ is increasing in $\alpha_0$, i.e., it is decreasing in $\alpha_j$. Thus, by slightly decreasing $\alpha_j$ to $\hat{\alpha}_j$ so
that $Q^S_j(\hat{\alpha}_j) > Q^I_0(\hat{\alpha}_0) > Q^S_i(\alpha_i)$, the system capacity does not change and $\hat{\alpha}_0 > \alpha_0$, where $\hat{\alpha}_0$ reflects the change in $\alpha_j$. This implies an increase in the assembler’s profit (13).

Suppose now that the assembler sets $\alpha$ such that $Q^S_j(\alpha_j) > Q^I_0(\alpha_0)$ for all $j = 1, \ldots, m_i$. Then, $Q^{S*}(\alpha) = Q^I_0(\alpha_0)$ so that $\pi_0(\alpha) = \min_{\alpha} \{-k_0Q_0 + (\alpha_0 - c_0)E[\min\{Q_0, D\}])\}$, which is increasing in $\alpha_0$. Therefore, by slightly decreasing $\alpha_j$, the assembler’s profit increases.

(b) Suppose that the assembler selects $\alpha$ so that $Q^S_i(\alpha_i) < Q^S_j(\alpha_j) \leq Q^I_0(\alpha_0)$ for any two subassemblers $i$ and $j$. Then, a slight decrease in $\alpha_j$ to $\hat{\alpha}_j$ implies $Q^S_i(\alpha_i) \leq Q^S_j(\hat{\alpha}_j) < Q^I_0(\hat{\alpha})$, so that $Q^{S*}(\alpha) = Q^{S*}(\hat{\alpha})$ and $\hat{\alpha}_0 > \alpha_0$, where $\hat{\alpha}$ and $\hat{\alpha}_0$ reflect the change of $\alpha_j$ to $\hat{\alpha}_j$. This implies an increase in the assembler’s profit (13). It is then optimal for the assembler to choose $\alpha$ so that $Q^{S*}(\alpha) = Q^S_1(\alpha_1) = \cdots = Q^S_n(\alpha_n)$.

Proof of Proposition 4. Let $\alpha$ be a vector of assembler’s prices for the general system satisfying Proposition 3 and let $Q^*_d(\alpha, \gamma^{S*}(\alpha))$ be the resulting equilibrium system capacity. Then, Proposition 3 implies that for each $i = 1, \ldots, n$, $Q^*_d(\alpha, \gamma^{S*}(\alpha))$ satisfies (12), i.e.,

$$-(k_{i0} + k_i) + (\alpha_i - C^d)\overline{F}(Q^*_d(\alpha, \gamma^{S*}(\alpha))) - \sum_{j=1}^{m_i} k_{ij}l(Q^*_d(\alpha, \gamma^{S*}(\alpha))) = 0.$$

Summing these expressions over $i$ yields

$$-K_{SA} + \left(\sum_{i=1}^{n} \alpha_i - C_S - C_{SA}\right)\overline{F}(Q^*_d(\alpha, \gamma^{S*}(\alpha))) - K_Sl(Q^*_d(\alpha, \gamma^{S*}(\alpha))) = 0. \quad (24)$$

Note that (24) is just the first-order optimality condition for the series system if the assembler chooses a price $\bar{\alpha} = \sum_{i=1}^{n} \alpha_i$, so that choice of price yields an equilibrium capacity in the series system of $\bar{Q}^*_d(\bar{\alpha}, \gamma^{S*}(\bar{\alpha})) = Q^*_d(\alpha, \gamma^{S*}(\alpha))$. This choice allows the assembler to achieve the same system capacity and the same profit margin as in the general system, so he will earn the same expected profit.

Conversely, consider now $\bar{\alpha}$, an assembler’s price in the series system, satisfying Proposition 3 (part (a)). The equilibrium capacity for the series system is then given by $\bar{Q}^*_d(\bar{\alpha}, \gamma^{S*}(\bar{\alpha}))$, and by Proposition 3(a), $\bar{Q}^*_d(\bar{\alpha}, \gamma^{S*}(\bar{\alpha})) \leq Q^I_0(\bar{\alpha}_0)$, where $\bar{\alpha}_0 = 1 - \alpha$. The capacity $\bar{Q}^*_d(\bar{\alpha}, \gamma^{S*}(\bar{\alpha}))$ satisfies

$$-K_{SA} + (\bar{\alpha} - C_S - C_{SA})\overline{F}(\bar{Q}^*_d(\bar{\alpha}, \gamma^{S*}(\bar{\alpha}))) - K_Sl(\bar{Q}^*_d(\bar{\alpha}, \gamma^{S*}(\bar{\alpha}))) = 0. \quad (25)$$

Following (14), define now

$$\alpha_i = C^i + \frac{k_{i0} + k_i + l(\bar{Q}_d^{\gamma^{S*}(\bar{\alpha}))}\sum_{j=1}^{m_i} k_{ij}}{\overline{F}(\bar{Q}^*_d(\bar{\alpha}, \gamma^{S*}(\bar{\alpha})))}.$$
Then, by its definition, $Q_i^S(\alpha_i) = \tilde{Q}_i^*(\tilde{\alpha}, \gamma^{S*}(\tilde{\alpha}))$, for $i = 1, ..., n$, and
\[
\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} C^i + \frac{1}{F(Q_i^*(\tilde{\alpha}, \gamma^{S*}(\tilde{\alpha})) (K_{SA} l(Q_i^*(\tilde{\alpha}, \gamma^{S*}(\tilde{\alpha}))) + K_S) = \tilde{\alpha},
\]
since $\tilde{Q}_i^*(\tilde{\alpha}, \gamma^{S*}(\tilde{\alpha}))$ satisfies (25) and $\sum_{i=1}^{n} C^i = C_S + C_{SA}$. We then have that $\alpha_0 = 1 - \sum_{i=1}^{n} \alpha_i = 1 - \tilde{\alpha} = \tilde{\alpha}_0$, so that the assembler’s profit margin is the same as in the series system, and $Q_i^S(\alpha_i) \leq Q_i^d(\tilde{\alpha}_0) = Q_i^d(\alpha_0)$ for $i = 1, ..., n$. This implies that the vector of prices $\alpha$ yields, in the general system, an equilibrium capacity of $Q_d^*(\alpha, \gamma^{S*}(\alpha)) = Q_i^S(\alpha, \gamma^{S*}(\alpha))$.

We conclude that the same assembler profit can be achieved in the general system with the vector of prices $\alpha$. □

**Proof of Proposition 5.** Without loss of generality (by Proposition 4) we restrict attention to a series system with a single supplier and a single subassembler. For any initial cost configuration let $\alpha_0^*$ be the optimal assembler’s price and let $Q_d^*$ be the resulting equilibrium system capacity so that $Q_d^*$ satisfies (12), i.e.,
\[
-(k_{10} + k_1) + (\alpha_1^* - (c_{10} + c_{11} + c_1)) F(Q_d^*) - k_{11} l(Q_d^*) = 0. \tag{26}
\]

It is immediately clear from (26) that a shift in unit component production cost between $c_{10}$ and $c_{11}$ will have no impact on the equilibrium system capacity, and as a result will have no impact on the assembler’s optimal profit.

Now consider a unit assembly capacity cost shift $\varepsilon > 0$ from supplier to subassembler, i.e., a shift from $k_{11}$ and $k_{10}$ to $k_{11} - \varepsilon$ and $k_{10} + \varepsilon$. Define $\alpha_1 = \alpha_1^* + \frac{\varepsilon}{F(Q_d^*)} (1 - l(Q_d^*)) \leq \alpha_1^*$, since $l(Q_d^*) \geq 1$. If the assembler responds with the new price $\alpha_1$, then $Q_d^*$ is the solution to the first order condition
\[
-(k_{10} + k_1 + \varepsilon) + (\alpha_1 - (c_{10} + c_{11} + c_1)) F(Q) - (k_{11} - \varepsilon) l(Q) = 0.
\]

Substituting this same system capacity, along with the new price, into the assembler’s profit function, we have that
\[
\pi_0(\alpha_1 | \varepsilon) = -k_0 Q_d^* + (\alpha_0^* - \frac{\varepsilon}{F(Q_d^*)} (1 - l(Q_d^*)) - c_0) \int_0^{Q_d^*} F(x) dx
\geq -k_0 Q_d^* + (\alpha_0^* - c_0) \int_0^{Q_d^*} F(x) dx = \pi_0(\alpha_1 | 0).
\]

Since this is just one feasible price response, the assembler’s optimal profit will be at least as high after the cost shift. □
Proof of Proposition 6. Without loss of generality (by Proposition 4) we restrict attention to a series system with a single supplier and a single subassembler. For any initial cost configuration let \( \alpha_1^* \) be the optimal assembler’s price and let \( Q_d^* \) be the resulting equilibrium system capacity so that \( Q_d^* \) satisfies (12), i.e.,

\[
-(k_{10} + k_1) + (\alpha_1^* - (c_{10} + c_{11} + c_1))F(Q_d^*) - k_{11}l(Q_d^*) = 0.
\]

(27)

First consider a unit assembly cost shift \( \varepsilon \) (positive or negative) from subassembler to assembler, i.e., a shift from \( c_1 \) and \( c_0 \) to \( c_1 - \varepsilon \) and \( c_0 + \varepsilon \). If the assembler responds with a new price \( \alpha_1 = \alpha_1^* - \varepsilon \), we have \( \alpha_1 - (c_{10} + c_{11} + c_1 - \varepsilon) = \alpha_1^* - (c_{10} + c_{11} + c_1) \), so (27) defines the equilibrium system capacity for this modified system as well. Since the system capacity is the same and the assembler faces the same margin \( \alpha_0 - (c_0 + \varepsilon) = \alpha_0^* + \varepsilon - (c_0 + \varepsilon) = \alpha_0^* - c_0 \), the assembler achieves the same profit as in the original system. Since this is just one feasible price response, the assembler’s optimal profit will be at least as high after the cost shift. A similar argument shows that given the optimal payment from the assembler in the case where a unit assembly cost \( \varepsilon \) is shifted, one can construct a corresponding payment in the original system that leads to the same profit for the assembler. This proves the result for \( c \).

Now consider a unit assembly capacity cost shift \( \varepsilon > 0 \) from subassembler to assembler, i.e., a shift from \( k_1 \) and \( k_0 \) to \( k_1 - \varepsilon \) and \( k_0 + \varepsilon \). If the assembler responds with a new price \( \alpha_1 = \alpha_1^* - \frac{\varepsilon}{F(Q_d^*)} \), then (27) defines the equilibrium system capacity for this modified system as well. Substituting this same system capacity, along with the new cost and price, into the assembler’s profit function yields

\[
\pi_0(\alpha_1|\varepsilon) = -(k_0 + \varepsilon)Q_d^* + \left( \alpha_0^* + \frac{\varepsilon}{F(Q_d^*)} - c_0 \right) \int_0^{Q_d^*} F(x)dx
\]

\[
\geq -k_0Q_d^* + (\alpha_0^* - c_0) \int_0^{Q_d^*} F(x)dx - \varepsilon Q_d^* + \frac{\varepsilon}{F(Q_d^*)} Q_d^* F(Q_d^*)
\]

\[
= -k_0Q_d^* + (\alpha_0^* - c_0) \int_0^{Q_d^*} F(x)dx,
\]

which is the assembler’s optimal profit before the cost and price shift. Since this is just one feasible price response, the assembler’s optimal profit will be at least as high after the cost shift. Interpreting this in the reverse direction establishes the result for \( k \). ■

Proof of Theorem 2. Following Proposition 4 and Remark 1, we will consider traditional and modular assembly systems in their pure-series form. We will compare the assembler’s expected profit under a traditional system to his profit under a modular system in which he
outsources unit assembly costs $c_\Delta$ and assembly capacity costs $k_\Delta$. That is, the traditional system has one supplier with costs $c_0 + c_1$ and $k_0 + k_1$ (or, equivalently, two suppliers with costs $c_0$ and $k_0$, and $c_1$ and $k_1$, respectively) and the assembler has costs $c_0$ and $k_0$, while the modular system has a single supplier with costs $c_{10}$ and $k_{10}$, a subassembler with costs $c_{10} + c_\Delta$ and $k_{10} + k_\Delta$, and an assembler with costs $c_0 - c_\Delta$ and $k_0 - k_\Delta$.

The superscript $m$ denotes the modular assembly system, and the superscript $t$ denotes the traditional system. Also, define $C = c_0 + c_{10} + c_{11}$ and $K = k_0 + k_{10} + k_{11}$.

Take $\alpha^{*m}_{1}$ to be the optimal assembler payment in the modular assembly system and $Q^{*m}$ the system’s equilibrium capacity. Observe from (15) that

$$\pi_0^m(\alpha^{max}_{1}) = -(k_0 - k_\Delta)Q^m_0(\alpha_{1}^{max}) + (\alpha_{0}(\alpha_{1}^{max}) - (c_0 - c_\Delta))E[\min\{Q^m_0(\alpha_{0}(\alpha_{1}^{max})), D]\] = \max_Q \{- (k_0 - k_\Delta)Q + (\alpha_{0}(\alpha_{1}^{max}) - (c_0 - c_\Delta))E[\min\{Q, D]\]} > 0 = \pi_0(\alpha^{min}_{1})$$

This implies that $\alpha^{*m}_{1} > \alpha^{min}_{1} = c_0 + c_{10} + c_{11}$ and $k_0 + k_{10} + k_{11}$, and the optimal expected profit for the assembler is $\pi_0^m(\alpha^{*m}_{1}) = -(k_0 - k_\Delta)Q^{*m} + (1 - \alpha^{*m}_{1} - (c_0 - c_\Delta))E[\min\{Q^{*m}, D]\}$.

In general, for any $\alpha_1$ chosen by the assembler, we have from (12) that

$$\frac{\partial \pi_1^m(Q|\alpha_1)}{\partial Q} = -(k_0 + k_\Delta) + (\alpha_1 - c_0 - c_{10} - c_\Delta)\frac{\partial Q}{\partial Q} - k_{11}l(Q) = 0.$$  

From the concavity of $\pi_1^m(Q|\alpha_1)$, which follows from (11), we have that (28) has a unique solution for any value of $\alpha_1$. This allows us to represent

$$\alpha_1(Q) = c_0 + c_{10} + c_{11} + \frac{k_0 + k_\Delta + k_{11}l(Q)}{F(Q)},$$

so that $Q^{*m}$ maximizes $\pi_0^m(Q) = -(k_0 - k_\Delta)Q + \left(1 - C - \frac{k_0 + k_\Delta + k_{11}l(Q)}{F(Q)}\right)E[\min\{Q, D\}]$. In particular,

$$\frac{\partial \pi_0^m(Q^{*m})}{\partial Q} = -(k_0 - k_\Delta) + (1 - C)F(Q^{*m}) - (k_0 + k_\Delta + k_{11})l(Q^{*m}) + k_{11}l(Q^{*m}) (1 - l(Q^{*m})) - \frac{k_{11}l(Q^{*m})}{F(Q^{*m})} \int_0^{Q^{*m}} F(x) dx = 0.$$  

For the traditional system, we know that given a payment $\alpha_1$ by the assembler, the supplier will set a capacity level given by

$$Q'(\alpha_1) = \frac{k_{10} + k_{11}}{\alpha_1 - c_{10} - c_{11}},$$

which leads to assembler’s expected profit given by $\pi_0^t(\alpha_1) = -k_0 Q'(\alpha_1) + (1 - \alpha_1 - c_0)E[\min\{Q'(\alpha_1), D\}]$.  

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Replacing \( Q'(\alpha_1^*) \) in the left-hand side of (28), we get that
\[
\frac{\partial \pi^m(Q'(\alpha_1^*))(\alpha_1)}{\partial Q} = k_{11} \left( 1 - l(Q'(\alpha_1^*)) \right) - k_\Delta - c_\Delta \overline{F}(Q'(\alpha_1^*)) < 0,
\]
so that \( Q^{\ast m} < Q'(\alpha_1^*) \). Since \( Q'(\alpha_1) \) is increasing in \( \alpha_1 \), let \( \tilde{\alpha}_1 < \alpha_1^* \) be such that \( Q^{\ast m} = Q'(\tilde{\alpha}_1) \). More specifically, from (29) and (31) we have that
\[
\alpha_1^* = \tilde{\alpha}_1 + c_\Delta + \frac{k_\Delta + k_{11} \left( l(Q'(\tilde{\alpha}_1)) - 1 \right)}{\overline{F}(Q'(\tilde{\alpha}_1))} > \tilde{\alpha}_1 + c_\Delta + \frac{k_\Delta}{\overline{F}(Q'(\tilde{\alpha}_1))}.
\]
Thus,
\[
\pi^m_0(\alpha_1^*) = -(k_0 - k_\Delta)Q'(\tilde{\alpha}_1) + (1 - \alpha_1^* - (c_0 - c_\Delta))E[\min\{Q'(\tilde{\alpha}_1), D\}] < -(k_0 - k_\Delta)Q'(\tilde{\alpha}_1) + \left( 1 - \tilde{\alpha}_1 - c_0 - \frac{k_\Delta}{\overline{F}(Q'(\tilde{\alpha}_1))} \right)E[\min\{Q'(\tilde{\alpha}_1), D\}] = -(k_0 - k_\Delta)Q'(\tilde{\alpha}_1) - k_\Delta \frac{E[\min\{Q'(\tilde{\alpha}_1), D\}]}{\overline{F}(Q'(\tilde{\alpha}_1))} + (1 - \tilde{\alpha}_1 - c_0)E[\min\{Q'(\tilde{\alpha}_1), D\}] \leq -k_0Q'(\tilde{\alpha}_1) + (1 - \tilde{\alpha}_1 - c_0)E[\min\{Q'(\tilde{\alpha}_1), D\}] = \pi^*_0(\tilde{\alpha}_1),
\]
where the first inequality follows from (32) and the second one from the fact that \( E[\min\{Q, D\}] > Q\overline{F}(Q) \). The amount \( \tilde{\alpha}_1 \) is one feasible payment for the traditional system, so that the assembler’s expected profit in the traditional system is higher than his profit in the modular system.

From the expression in (31), we can write
\[
\pi^t_0(Q) = -k_0Q + \left( 1 - C - \frac{k_{10} + k_{11}}{\overline{F}(Q)} \right)E[\min\{Q, D\}],
\]
which is again concave in \( Q \) by (11). Its unique maximum \( Q^{\ast t} \) solves
\[
\frac{\partial \pi^t_0}{\partial Q} = -k_0 + (1 - C)\overline{F}(Q) - (k_{10} + k_{11})l(Q) = 0.
\]
Since \( l(Q) > 1 \) and \( l'(Q) > 0 \), we have from (30) and the first equality of (33) that
\[
\frac{\partial \pi^t_0(Q^{\ast m})}{\partial Q} = k_\Delta \left( l(Q^{\ast m}) - 1 \right) - k_{11}l(Q^{\ast m}) \left( 1 - l(Q^{\ast m}) \right) + \frac{k_{11}l'(Q^{\ast m})}{\overline{F}(Q^{\ast m})} \int_0^{Q^{\ast m}} \overline{F}(x)dx > 0,
\]
implying that \( Q^{\ast m} < Q^{\ast t} \).

Finally, let \( Q^C = \overline{F}^{-1}\left( \frac{k_{11}}{1 - l(Q^C)} \right) \) be the centralized optimal capacity for the system, i.e., the maximum of the concave profit function \( \pi^c(Q) = -KQ + (1 - C)E[\min\{Q, D\}] \). Note that \( \frac{\partial \pi^t_0(Q^C)}{\partial Q} = (k_{10} + k_{11})(1 - l(Q^C)) < 0 \), which implies that \( Q^{\ast m} < Q^{\ast t} < Q^C \). From the concavity of \( \pi^C \), we conclude that \( \pi^C(Q^{\ast m}) < \pi^C(Q^{\ast t}) < \pi^C(Q^C) \).