Second sourcing and the experience curve: price competition in defense procurement

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We examine a dynamic model of price competition in defense procurement that incorporates the experience curve, asymmetric cost information, and the availability of a higher cost alternative system. We model acquisition as a two-stage process in which initial production is governed by a contract between the government and the developer. Competition is then introduced by an auction in which a second source bids against the developer for remaining production. We characterize the class of production contracts that are cost minimizing for the government and that induce the developer to reveal private cost information. When high costs are revealed, these contracts result in a credible cutoff of new system production in favor of the still higher cost alternative system.

1. Introduction

A popular solution to the problem of controlling defense costs in the United States has been to introduce competition into the procurement of defense systems. This solution is particularly attractive since most high-technology defense systems are developed and produced under conditions of bilateral monopoly (Beltramo, 1983). In this article we examine an important form of competition, known as second sourcing, in which the developer’s technology is transferred to a second contractor (or second source), who is then allowed to bid against the developer to take over production.

The basic argument in support of introducing reprocurement bidding competition into the acquisition process is that concern over losing the contract to a second source will cause the developer to bid more aggressively.1 There are, however, two important factors that favor the developer and limit the efficacy of second sourcing: an experience advantage and superior cost information.

If bidding occurs after the initial contractor has production experience, then this contractor will have lower costs of producing additional units. This, in turn, gives the incumbent

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1 Introducing competition will not, of course, reduce social costs. The objective is to transfer income from the contractor to the government.
an advantage over a second source. Since this experience advantage increases with the length of the initial production run, the ability of the second source to make an effective bid and, hence, the benefit of competitive bidding, decreases with the amount of initial production.

The benefits to competitive bidding are also negatively affected when the developer has cost information that is more accurate than the information possessed by either the government or the potential second source. In this situation a second source will bid higher on average than if it had perfect information to compensate for the "winner’s curse" phenomenon. Since the developer anticipates the second source's bidding strategy, the expected winning auction price will be higher under uncertainty than under certainty. Additional costs are likely to occur when the second source wins the contract, but finds costs to be higher than its bid. In such cases the government typically absorbs part of the contractor's loss to maintain quality and to avoid production delays.

Previous analyses of second sourcing (Daly and Schuttinga, 1982; Smith, 1983) have assumed that both contractors face the same costs and have the same information. In practice, however, insufficient attention to these effects partially explains why price competitions that are won by a second source often result in long-run problems. As Augustine (1983, p. 153) puts it, "the net impact of unknowledgeable bidders . . . is . . . [quoting Irving Bluestone] 'somewhat analogous to that of the cross-eyed discus thrower: he seldom comes out ahead, but he sure does keep the crowd alert.' " Thus, given the second source's experience and information handicaps, it is not immediately obvious that second sourcing benefits the government.

In this article we reexamine second sourcing in light of the experience curve and under the assumption that the gains to the revelation of cost information outweigh the gains to competition under asymmetric information. While adjusting procurement practice for the experience curve is straightforward in the absence of information considerations, the experience effect has important ramifications for designing incentives that will induce information revelation. In particular, we find that a policy that can credibly cut off procurement of the new system in the reprocurement stage in favor of a less efficient alternative system will reduce costs relative to a policy without such a cutoff. The second source is valuable to the government because it makes the cutoff credible.

\[ \square \] \textbf{Background on the procurement process.} For large defense systems the procurement process can be divided into four stages: initial design, development, initial production, and reprocurement. In the first stage the Department of Defense issues a request for proposal (RFP) that specifies the system it would like to have developed. Prospective contractors respond to the RFP with a proposed design and a price, after which the DoD chooses one, or more than one, of the designs for development. The choice criteria tend to weight design quality and perceived managerial and technical capability over price (Fox, 1974). In the competitive bidding for the development contract, contractors frequently submit bids below their expected cost, with the knowledge that price will be renegotiated over time to accommodate engineering change proposals that modify the original design. Since these negotiations occur under monopoly conditions, a "buy-in" strategy with respect to the original bid can be profitable because the contractor knows that the original bid will not be the final price that is received. Technically, it is not required that the development contractor be given the initial production contract. Because the DoD is usually under pressure to begin production as soon as possible and because the cost of transferring technology is quite high if initial production begins before the design is fully stabilized, however, the development contractor expects to win the initial production contract. A system is thought to be ready for reprocurement when a technology has achieved a stability of design such that further

\[ \text{\footnote{This assumption is necessary for a correct interpretation of our results. See the discussion of pooling in Section 2 for more on this point.}} \]
development work is minimal. Because design stability is approached incrementally, the logical time at which the technology could be transferred to a second source occurs when the costs of transferring technology begin to be dominated by the savings anticipated from second-source competition.³

☐ **Second sourcing and the benefits of competition.** We focus on initial production and reprocurement because these portions of the process are more tractable to economic analysis than are previous stages where complicating factors, such as military preferences for design or uncertainties about technological possibilities, are more salient. We model initial production and reprocurement as a sequential acquisition process in which the timing of the reprocurement stage is endogenous and the effects of the experience curve and asymmetric information are taken into account. In the initial-production stage the government offers a set of quantity-payment options to the developer. Competition is introduced in the reprocurement stage in the form of an auction between the developer and a second source. Alternatively, the government can choose to eschew competition and the auction by purchasing a previous generation system that, significantly, is known *ex ante* to be less efficient than the new system.

The nature of the reprocurement process is important because it will anchor the entire relationship between the government and the developer. The use of a competitive auction introduces the second source as a third party in an attempt to mitigate the adverse consequences associated with a situation of bilateral monopoly. Under an auction the developer is forced to bid against the second source, and the government must accept the lowest bid that results, subject to a reservation price. Thus, while the model we consider does not commit the government to reprocurring the new system, it does commit the government to using a sealed-bid auction as the method of reprocurement.⁴

A crucial aspect of competition through auctions is that it limits the ability of the government to dictate the terms of reprocurement. In particular, any information obtained about the developer’s cost during the initial-production stage cannot be used to force the developer to produce at cost in the reprocurement stage. The beneficial impact of this information must result from its use by the second source in its bidding and by the government in its decision to use the alternative system or to accept the price that is expected to result from the auction.

A second source is valuable to the government not only because it lowers auction prices, but also because the cost of developing the second source changes the government’s incentives to run a reprocurement auction in a way that reduces costs in equilibrium. Since transferring technology to a second source is costly, a second source is developed only if the government wants to run the auction. The decision to terminate new system procurement in favor of the alternative system precedes the decision to transfer technology and to run the auction. It is this aspect of second sourcing that makes procurement by way of the *ex ante* less efficient alternative credible.

We introduce the model in the next section. Section 3 studies the reprocurement stage, and we discuss several crucial aspects of the competitive auction reprocurement structure.

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³ Second sourcing has the obvious advantage of preventing the developer from taking full advantage of a monopoly position. The attractiveness of second-source bidding depends on the savings derived through the bidding process and on the costs of running the second-source competition. These costs—technology transfer, time lags, increased uncertainty, etc.—can exceed expected benefits. We do not deal explicitly with such costs in our model and thus do not conduct a full cost-benefit analysis; Section 3 indicates how technology transfer costs can be incorporated in the model.

⁴ The literature on bidding is extensive. Milgrom and Weber’s (1982) article is directly relevant, since they study the value of information in an auction where bidders are asymmetrically informed. The literature applying contracting theory (Ponsard, 1983) and bidding theory (Samuelson, 1983) to defense procurement is growing. Demski, Sappington, and Spiller (1987) also examine the role of an alternative supplier.
In Section 4 we characterize the set of initial-production contracts that result in the revelation of information by the developer. Also, we consider the role of commitment in relation to the work on multiperiod bilateral monopoly relationships governed by contracts (Laffont and Tirole, 1985; Baron and Besanko, 1985). The cost-minimizing contract for the government appears in Section 5. We discuss the policy implications of our results in Section 6.

2. The model

We study a sequential model for the acquisition of a newly developed defense system that is ready for production. The objective of the government is to minimize the expected cost of acquiring a given level of performance capability or "firepower." Let \( z > 0 \) denote this level with \( z \) measured in units of the new system. We assume that the government also has access to an older but less efficient system. The government may use this alternative technology and the new system either independently or in conjunction with each other to attain the needed firepower level.

We specify the production costs for both technologies below, and identify the asymmetries between the developer and the second source. Then we present and discuss the sequential acquisition model.

- **Production costs.** Production costs for the new system are characterized by an experience curve. We draw the basic structure governing this effect from Spence (1981). Suppose that the marginal cost of producing a flow of output is given by \( c(x) \), where \( x \) is total accumulated output. The function \( c(x) \) is the experience curve, and it satisfies \( c(x) > 0 \) and \( c'(x) < 0 \) for all \( x \geq 0 \). Thus, marginal cost declines with cumulated output. It follows that the total cost of producing a quantity \( x \) is

\[
\Gamma(x) = \int_0^x c(y) \, dy.
\]

The cost function \( \Gamma(x) \) is increasing, concave, and satisfies \( \Gamma(0) = 0 \).

An important consequence of the experience curve is that it provides a cost advantage to a firm with production experience. To see this suppose that the developer has produced a quantity \( x_1 \), while a potential second source has no previous production experience. The developer can produce an additional quantity \( x_2 \) at a cost of \( \Gamma(x_1 + x_2) - \Gamma(x_1) \). By concavity, this is less than \( \Gamma(x_2) \), the production cost of the second source. Thus, initial production by the developer will result in a cost advantage over the second source at later stages in the acquisition process.

The source of uncertainty in the model involves the position of the experience curve. This is described by the value of a parameter \( \theta \), which enters as a multiplicative factor in total cost. The value of \( \theta \) is the private information of the developer and can be viewed as specific knowledge of production costs acquired during a previous development stage.

All other parties are uninformed about the value of \( \theta \). Their initial beliefs are governed by a common prior probability distribution for the cost parameter. This distribution has a density \( f(\theta) \), which is positive and continuous on a closed interval \([\theta_\ell, \theta_u]\). \( F(\theta) \) denotes the cumulative distribution. The structure of the model, including the fact that the developer has private cost information, is common knowledge among all parties.

The old system is available at a known constant cost per unit. Let \( \delta \) denote the relative unit price of firepower from the old system in units of the new technology. Then, the linear cost function \( \delta x \) gives the cost of obtaining \( x \) units of firepower through the old system.

The following parameter conditions link the range of uncertainty in new system costs to the known cost of procurement through the old system. Define \( \bar{\theta} \) and \( \underline{\theta} \) by

\[
\bar{\theta} \Gamma(z) = \delta z \quad \text{and} \quad \underline{\theta} c(0) = \delta. \tag{1}
\]

The first condition states that the highest possible production cost for \( z \) units of the new system is the cost of the old system for \( z \). Thus, the old system must be less efficient than
the new one. The second condition states that at the lowest possible production cost it is just barely worthwhile to use the new technology for an arbitrarily small quantity of production.

The old system provides the government with a reservation value or benchmark, and thus enables identification of the relevant range of cost uncertainty for the new system. Values of $\theta$ beyond the high and low cost extremes add several uninteresting cases to the analysis. We note the extensions in Section 5.

□ The acquisition process. We now examine the sequential acquisition model and specify how the government may proceed in procuring firepower from the potential suppliers.

The developer and second source are profit-maximizing firms, and the government seeks to minimize the expected cost of acquiring $z$ units. All parties are risk neutral and future payments are not discounted. We abstract from issues involving risk and intertemporal efficiency to focus on the interaction between private information and the experience curve.\(^5\)

The government deals exclusively with the developer in the initial-production stage. The outcome of this stage is an exchange $(s, x)$, where $s$ is a payment to the developer for $x$ units of the new system. The exchange is governed by an initial procurement contract, defined as a pair of measurable real-valued functions on $[\tilde{\theta}, \bar{\theta}]$,

$$\{S(\tilde{\theta}), X(\tilde{\theta})\},$$

where

$\tilde{\theta}$ = a cost report from the developer;

$S(\tilde{\theta}) \in IR$ = the payment to the developer when $\tilde{\theta}$ is reported; and

$X(\tilde{\theta}) \in [0, z]$ = the quantity produced when $\tilde{\theta}$ is reported.

The government chooses the schedules $S$ and $X$. The developer responds with a report $\hat{\theta}$ for the cost parameter and thus determines the initial quantity and payment. Essentially, the developer is choosing from a menu of simple price-quantity pairs when a report is made.\(^6\)

An upper-case $S$ or $X$ will always refer to the entire schedule. A lower-case letter will refer to the value at a report. If the initial quantity $\hat{x} = X(\tilde{\theta})$ is less than $z$, then $z - \hat{x}$ units remain to be acquired. To distinguish the special case where all $z$ units are produced in the initial stage, we say that a contract involves reprocurement whenever $\hat{x} < z$.

In the reprocurement stage the government first decides whether to terminate new system procurement. If a cutoff is chosen, the old system is purchased at a cost of $\delta(z - \hat{x})$. If the government chooses to continue, technology is transferred to the second source at a fixed cost of $T$, and the reprocurement competition takes place. We model this competition as a sealed-bid, low-price auction with a reservation price equal to the cost of the old system. The low bidder wins the right to produce $z - \hat{x}$ in return for a payment equal to the low bid, provided that the low bid does not exceed the reservation price. In the event of a tie, the developer is chosen as the winner. Note that since the cutoff decision precedes technology transfer, the government can avoid the technology transfer cost if a reprocurement auction is not desired. Also, the old system is still available downstream as indicated by the use of the reservation price.

When $\theta$ is privately observed by the developer, the second-source bid must be based on information that is inferred from its priors, the initial contract offered by the government,

\(^5\) Note that an explicit consideration of real production time and discounting would require the introduction of capacity constraints or adjustment costs if the cost function is to be well defined.

\(^6\) Modelling the contract as a fee and quantity schedule that is contingent on the cost report simplifies analysis of the inference problem faced by uninformed parties. This contract form involves no loss in generality with respect to the separation of types. Alternatively, one may model the contract in a requirement form where the fee $s$ is a function (correspondence) of the quantity $x$. 

and the observed cost report of the developer. The same holds true for the government in the cutoff decision. Thus, the developer will strategically report his costs by taking into account the direct effect on initial-stage profits and the indirect influence on prospective auction profits. As the contract determines the direct effect, the government can influence the reporting incentive of the developer and, hence, can control the information content of the cost report.

In a separating equilibrium the developer reveals his type by the cost report he chooses. In this case the government and second source make optimal reprocurement-stage decisions based on the conjecture that the true value of the cost parameter was reported. This conjecture is correct when the contract provides an initial-stage profit incentive that is sufficient to induce revelation. That is, under the initial contract, the developer finds it optimal to report the true value of his costs if the reprocurement stage is governed by the above responses from the government and the second source. Such a separating contract is said to satisfy self-selection.

In a pooling equilibrium a range of developer types submits the same report. The government and the second source will then remain uncertain about production costs in the reprocurement stage. In an auction with asymmetric information and asymmetric valuations, the second source must bid conservatively, using a mixed strategy, to adjust for the “winner’s curse.” This will lead, with positive probability, to the second source’s winning the auction and incurring negative profits (assuming, of course, no bankruptcy constraints).  

If the government can rely on the second source to absorb losses, this bidding behavior can be used to mitigate the information and experience advantages of the developer. In practice, the government rarely allows a contractor to absorb large losses, since such losses will affect the rapidity of production, product quality, and long-run supply. A proper comparison of pooling and separation would therefore include the renegotiation aspect of large losses as a part of the acquisition model. We leave a consideration of pooling equilibria for future work.

In this article we investigate the potential for cost containment that arises when the government uses the initial contract to induce separation. In this case reprocurement-stage cost savings from better information must be balanced against increased costs from the profit incentive that the initial contract must provide. Separation occurs as a Nash equilibrium outcome in the “sequential acquisition” game that a given contract induces. We characterize the set of separating contracts and examine the properties of this incentive structure. From this set of contracts we then find the cost-minimizing choice for the government.

3. The reprocurement stage

- In this section we examine the auction equilibrium and the optimal cutoff decision for the government when the contract separates developer types. In Section 4 we use these results to characterize the set of separating contracts. As the reprocurement stage is also interesting in its own right, we discuss the general role an auction structure plays in determining the ability of the government to contain procurement costs.

☐ Auction equilibrium. The developer and the second source compete in a sealed-bid, low-price auction for the production of \( z - x \) units when the government chooses to use

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7 Sappington (1983) examines the role of limited liability in principal-agent relationships.
8 For example, in 1973 Grumman claimed losses of $1 million per plane on the first 86 F-14 Tomcats produced for the Navy and refused to produce any more. The Navy renegotiated the contract (Dorfer, 1983, p. 32).
9 Note that modelling renegotiation, given losses, is not necessary for analyzing separating contracts since all parties earn nonnegative profits.
the auction. In addition, the government reserves the right to reject all bids if the lowest bid exceeds $\delta(z - x)$, the price of the old system. Although both parties are fully informed about the value $\theta$ at this point, the experience curve puts the bidders in an asymmetric position with respect to production costs. Having produced $x$ initially, the developer faces costs of $\theta[\Gamma(z) - \Gamma(x)]$. The second source has no learning experience and can produce the residual quantity at a cost of $\theta \Gamma(z - x)$.

The unique perfect Nash equilibrium for this auction has the second source submitting a bid of $\theta \Gamma(z - x)$. When this bid is less than $\delta(z - x)$, the developer matches the second-source bid and wins the auction under the tie-breaking rule. If the second-source bid exceeds the price of the old system; the developer bids the government's reservation price of $\delta(z - x)$. Thus, the developer receives a payoff from the auction of

$$\Pi_2(\theta) = \min \{\theta \Gamma(z - x), \delta(z - x)\} - \theta[\Gamma(z) - \Gamma(x)],$$

which by the concavity of $\Gamma$ is always positive for $0 < x < z$.

The auction outcome is ex post efficient as the winner is also the lower-cost producer. The winning bid and, hence, the auction price the government pays for the residual $z - x$ are equal to the smaller of the second-source cost of production and the old-system price. As both bidders are aware that the developer has lower production costs, the developer is able to earn a rent from the experience advantage, and the role of the second-source is limited to setting the price that the developer receives.

- **Government cutoff decision.** The government decides to procure $z - x$ through an auction at the beginning of the reprocurement stage. If the government decides to use an auction, it pays the auction price plus the associated cost of transferring technology to the second source; otherwise, the old system is purchased for $\delta(z - x)$. Equilibrium requires that the government behave optimally at this point while taking $\theta$ and $x$ as given.

The government will terminate production of the new system whenever the auction price is expected to be $\delta(z - x)$, since the government avoids the cost of technology transfer by purchasing the old system and not running the auction. The auction is cost minimizing when

$$\delta(z - x) \geq \theta \Gamma(z - x) + T,$$

where $T$ denotes the cost of technology transfer. In this case the developer's bid is limited by the low production costs of the second source, and the auction price is held below the reservation price.

For any given $\theta$, the above inequality allows us to identify a critical quantity $x^c(\theta)$ such that the auction is cost minimizing if and only if $x \leq x^c(\theta)$ with the government indifferent when equality holds. Thus, for any contract involving first-period production above this critical value, the developer is cut off in the reprocurement stage. By an appropriate choice of the initial quantity schedule $X$, the government can create a credible cutoff threat for a set of $\theta$ types. Lemma 1 describes the cutoff decision. We set $T = 0$ since it simplifies the exposition while preserving the essential features of the cutoff.

**Lemma 1.** For any given $\theta$ there exists an $x^c(\theta)$ such that the auction is cost minimizing if and only if $x \leq x^c(\theta)$. The cutoff function $x^c(\theta) \rightarrow [0, z]$ is continuous and strictly decreasing with $x^c(\theta)$ given by the unique number less than $z$ satisfying

$$\delta(z - x) = \theta \Gamma(z - x^c).$$

All proofs are contained in the Appendix.

Figure 1 displays the cutoff function $x^c$. Note that $x^c(\theta) = z$ and that $x^c(\tilde{\theta}) = 0$. For a given $\theta$, low initial production $x$ implies no cutoff. A low initial $x$ results in a smaller experience advantage for the developer and, hence, a smaller cost differential between bidders in the auction. A relatively large value of $z - x$ allows the second-source production cost
to fall below the cost of the old system, and the auction is then optimal for the government. As $\theta$ rises, the relative efficiency of the new system is reduced, and the cutoff becomes increasingly stringent owing to the need for a larger value of $z - x$ to offset the old system cost.\footnote{The effect of a fixed cost of size $T$ is to reduce $x^*(\theta)$ for all $\theta$; this can be verified with Figure 1 by noting the vertical shift that occurs when $T$ is added to $\theta \Gamma(z - x)$. More generally, one expects the costs of technology transfer to decline with initial production. Again, the effect is a downward shift in the cutoff function.}

The threat of cancelling further production of the new system through a cutoff is potentially valuable in controlling acquisition costs. But such a threat is effective only if the government finds it optimal to terminate the new system. The cutoff function identifies initial quantities that constitute a credible threat.

\section*{Reprocurement costs} One can better appreciate the importance of the cutoff function under asymmetric information after considering the full-information case, where all parties know the value of $\theta$ from the outset. In such a situation the government is able to acquire $z$ units for a total payment of $\theta \Gamma(z)$, and the developer earns zero profits. To see this set the initial contract at $s = \theta [\Gamma(z) - \Gamma(z - x)]$, where $x < x^*(\theta)$. From Lemma 1 we know the government will choose to reprocure in this case. The resulting auction price is the second-source cost of producing $\theta \Gamma(z - x)$. Thus, total government payments are $s + \theta \Gamma(z - x) = \theta \Gamma(z)$, and the developer earns zero profits. Note that the initial fee is set to internalize the auction profits to the developer. The outcome is always socially efficient since the higher-cost old system is not used, and the social loss that arises if production of the new system is divided between the developer and second-source is avoided.

The situation is altered when $\theta$ is the private information of the developer. In this case the government’s cost is bounded from below by the highest $\theta$ for which the developer produces all $z$ units. Suppose for the moment that a developer with $\bar{\theta}$ ultimately produces all $z$ units. Then all lower-cost types are guaranteed access to a highly profitable strategy, namely, mimicking the behavior of type $\bar{\theta}$. As long as type $\bar{\theta}$ earns nonnegative profits, the government will pay at least $\bar{\theta} \Gamma(z)$. In contrast to the full-information case, the developer extracts all the surplus, and the government is forced to pay its reservation value as $\bar{\theta} \Gamma(z) = \delta z$, the cost of the old system.

In our model we limit the government’s ability to use acquired information to the cutoff decision by the way we structure the reprocurement stage. To see the importance of
this limitation, suppose that the government could specify a reservation price for the auction after the initial-production stage and that a separating equilibrium were achieved. The optimal choice for the government would then be to set the reservation price at the revealed cost of the developer. Such an action would not cut off the type \( \tilde{\theta} \). Consequently, the above argument implies that government cost is at least \( \tilde{\theta} T(z) \), since all types can ensure this total payment by mimicking the actions of type \( \tilde{\theta} \). Thus, to avoid paying the highest production cost, the government must use a cutoff and prevent the agent with private cost information from producing all \( z \) units.\footnote{Note that if the government could unilaterally restrict the developer to an arbitrarily small amount of initial production and then contract with the second source on an exclusive basis, it would be optimal to do so. This would effectively separate production activity from information acquisition. In practice, because initial production begins before the design is completely stable, technology-transfer considerations make some initial production by the developer inevitable. In turn, this experience advantage creates an \textit{ex post} incentive for the government to consider the developer for further production.}

The option to use a second source is valuable to the government because it makes the cutoff strategy credible. In this regard, both the bidding behavior of the second source and the technology-transfer cost associated with the reprocurement auction are essential. The importance of the second-source bid is clear from Lemma 1: for low initial quantities the developer is forced to match the second-source bid. This bid keeps the auction price below the reservation price of the old system and makes the auction attractive to the government. With larger initial production quantities, however, it is the price of the old system, rather than the second-source bid, that limits the auction price.

It is in the latter case that technology-transfer costs play a critical role. Since a cutoff avoids the cost of technology transfer to the second source, the government strictly prefers a cutoff, even though the developer’s bid equals the reservation price. As the technology transfer cost \( T \) goes to zero, we arrive at the cutoff rule in Lemma 1. This limiting form of the cutoff rule preserves the essential feature of the government’s choice: a cutoff is credible because a reprocurement auction involves additional costs.

Thus, even though the second source does not actually produce at equilibrium, its existence is essential. A costly second-source auction creates a credible threat to cut off the developer and to switch to the old system in the reprocurement stage. Since the developer will not inevitably produce the entire quantity, the option to second source strengthens the position of the government in the initial stage.

4. Contract design and self-selection

We now turn to the problem of designing a contract \( \{S, X\} \) that separates developer types and thus provides the information necessary for the reprocurement stage described above. In equilibrium the actual type is reported and the developer receives a payoff of

\[
\Pi(\theta) = \Pi_1(\theta) + \Pi_2(\theta) = \begin{cases} 
  s - \theta \Gamma(x), & x \geq x^*(\theta), \\
  s + \theta \Gamma(z - x) - \theta \Gamma(z), & x < x^*(\theta),
\end{cases}
\]

where \( s = S(\theta) \) and \( x = X(\theta) \).

A contract separates developer types if it provides no incentive for the developer to misreport its type. Suppose that a type \( \theta \) developer submits an initial report of \( \tilde{\theta} \). Initial profits \( \pi_1(\tilde{\theta}|\theta) \) are \( \delta - \theta \Gamma(x) \), where \( \delta = S(\tilde{\theta}) \) and \( x = X(\tilde{\theta}) \) are the fee and quantity specified by \( \{S, X\} \) for a report of \( \tilde{\theta} \). Let \( \pi_2(\tilde{\theta}|\theta) \) be the maximum reprocurement-stage profit that the developer can earn. Total profit \( \pi(\tilde{\theta}|\theta) \) is then \( \pi_1(\tilde{\theta}|\theta) + \pi_2(\tilde{\theta}|\theta) \).

A contract satisfies self-selection if reporting the actual value \( \theta \) is an optimal strategy for the developer. In this case the contract satisfies the continuum of constraints:
\[ \Pi(\theta) = \max_\tilde{\theta} \pi(\tilde{\theta}|\theta) \quad \text{for all } \theta. \] (5)

Note that \( \Pi(\theta) = \pi(\theta|\theta) \) by definition.

We assume that the government cannot force the developer to accept a contract that yields a negative total profit. This leads to a set of participation constraints:

\[ \Pi(\theta) \geq 0 \quad \text{for all } \theta. \] (6)

A necessary step in the characterization of contracts satisfying the self-selection and participation constraints is the calculation of \( \pi_2(\tilde{\theta}|\theta) \). A detailed treatment appears in the Appendix, so we only sketch the argument here.

In equilibrium the actual \( \theta \) is reported. Thus, beliefs of the government and second source regarding the value of the cost parameter are simply that the cost report \( \tilde{\theta} \) is the actual value with probability one. These beliefs govern the cutoff decision of the government and the auction bid of the second source. Then \( \pi_2(\tilde{\theta}|\theta) \) is calculated by finding the optimal bid of the developer in response to this second-source bid.

Three outcomes are possible if the type is misreported. First, if \( \tilde{x} \) is above \( x^c(\tilde{\theta}) \), then the government chooses to cut off production of the new system, and rep羅ercurement profits are zero. Otherwise, the auction ensues, and the second source submits the equilibrium bid of \( \tilde{\theta} \Gamma(z - \tilde{x}) \). The developer must consider whether it is profitable to win the auction. An overreport, \( \tilde{\theta} > \theta \), leads the second source to bid above his own actual production cost for \( z - \tilde{x} \). As the developer also enjoys an experience advantage, a positive profit is ensured by matching the second-source bid and winning the auction under the tie-breaking rule, a strategy we call “bid-to-win.” On the other hand, an underreport of \( \tilde{\theta} < \theta \) leads the second source to bid below his own actual production cost. Although the experience advantage is present, a low report may lead the second source to bid below the actual production cost of the developer. When this occurs, a “bid-to-lose” strategy is optimal, and the developer submits a bid above that of the second source. This strategy is motivated by an attempt to garner profits from the larger initial-production quantities associated with low-cost types, since bid-to-lose nets zero profit in the rep羅ercurement stage.

The full analysis of the bid-to-lose strategy is somewhat involved, since it depends on \( \theta \), \( \tilde{\theta} \), and \( \tilde{x} \) (see the Appendix). For the purposes of characterizing the self-selection and participation constraints, a simple condition on the \( X \) schedule is sufficient, namely, type \( \tilde{\theta} \) is not given an incentive to bid to lose. For each possible report \( \tilde{\theta} \), there is a critical quantity \( x^l(\tilde{\theta}) \) for which the equilibrium bid of the second source equals the developer’s actual production cost at \( \tilde{\theta} \). This is determined by

\[ \tilde{\theta} \Gamma(z - x^l) = \tilde{\theta}(\Gamma(z) - \Gamma(x^e)). \] (7)

If the production schedule \( X \) is chosen to be above the \( x^l \) function, then the second-source bid is above a type \( \tilde{\theta} \) developer’s actual production cost and hence, that of any lower type. Thus, bid-to-win is the optimal strategy for all types when \( X \) is so chosen. In addition, the Appendix shows that the \( x^l \) function is decreasing and strictly below the \( x^e \)-cutoff function.

Proposition 1 below provides the conditions on \( \{S, X\} \) that ensure self-selection and participation. In addition, it establishes that the total government cost under any contract that satisfies self-selection and participation can also be achieved by a contract that satisfies conditions (8)–(12) of Proposition 1 below. Figure 2 illustrates a typical production schedule.

The essential feature of a contract in a separating equilibrium is that there exists a critical type, denoted \( \theta^e \), such that any type above \( \theta^e \) is cut off and any type below \( \theta^e \) produces all \( z \) units over the two stages. Thus, cost reports are classified into two mutually exclusive sets in equilibrium: cutoff types where \( \theta > \theta^e \) and noncutoff types where \( \theta \leq \theta^e \). There are three kinds of conditions in Proposition 1: quantity schedule \( X \) restrictions, fee schedule \( S \) restrictions, and a condition on the profits earned by the critical type \( \theta^e \).
Proposition 1. In a separating equilibrium a contract \( \{S, X\} \) involving reprocurement satisfies (5) and (6) if conditions (8)–(12) are satisfied. Furthermore, if a contract satisfies (5) and (6), then the same level of government cost can be achieved for all \( \theta \) by a contract that satisfies (8)–(12).

The quantity schedule conditions are:

\[
x^l(\theta) \leq X(\theta) < x^c(\theta), \quad \text{for} \quad \theta \leq \theta^c; \tag{8}
\]

\[
x^c(\theta) \leq X(\theta) \quad \text{and} \quad X(\theta) \text{ nonincreasing, for} \quad \theta > \theta^c. \tag{9}
\]

The fee schedule conditions are:

\[
S(\theta) + \theta \Gamma(z - X(\theta)) = \pi^c + \theta^c \Gamma(z), \quad \theta \leq \theta^c, \tag{10}
\]

\[
S(\theta) = \tilde{\pi} + \theta \Gamma(X(\theta)) + \int_{\theta}^{\theta^c} \Gamma(X(t))dt, \quad \theta > \theta^c. \tag{11}
\]

The cutoff value is defined by \( \theta^c = \sup \{\theta | X(\theta) < x^c(\theta)\} \). The profits of \( \theta^c \) are

\[
\pi^c = \tilde{\pi} + \int_{\theta^c}^{\theta^c} \Gamma(X(t))dt, \tag{12}
\]

where \( \tilde{\pi} = \tilde{s} - \bar{\theta} \Gamma(\bar{x}) \geq 0 \) (set at zero under cost minimization).

Conditions (8) and (9) relate the production schedule \( X \) to the critical cutoff value \( \theta^c \). For \( \theta \leq \theta^c \) the \( X \) schedule lies between the \( x^l \) and \( x^c \) functions (see Figure 2). Thus, the government does not cut off these lower-cost types, and the bid-to-lose strategy is suboptimal for any type of developer. From (9) \( \theta > \theta^c \) leads to a cutoff; also, the \( X \) schedule is nonincreasing over this range (whenever it is constant, so is the fee).

The fee schedule \( S \) is given by (10) and (11). Over the noncutoff range, the initial fee \( S(\theta) \) and the auction price \( \theta \Gamma(z - X(\theta)) \) sum to a constant. Total developer revenues, which are equal to government payments in this range, are thus determined by the profit level \( \pi^c \) and production cost of the type \( \theta^c \). The fee schedule \( S \) is set so that no \( \theta \) type in the noncutoff range finds it profitable to report a \( \theta \neq \theta^c \) that also lies in the noncutoff range.
Over the cutoff range, the fee $S(\theta)$ is the sum of $\bar{\pi}$, the production cost for the initial quantity, and an incentive term that depends on the production level of all higher-cost types. This incentive term ensures that no $\theta$ type in the cutoff range finds it profitable to report a $\hat{\theta} \neq \theta$ that also lies in the cutoff range.

The profit level of $\theta^c$ is given in (12) and ensures that a type on one side of $\theta^c$ does not have an incentive to report a $\hat{\theta}$ that lies on the other side. As $X$ is above $\lambda^l$ over the noncutoff range, we know that a bid-to-win strategy is optimal for any type reporting a $\hat{\theta}$ in this range. Essentially, condition (12) forces the equilibrium payoff $\Pi(\theta)$ to be continuous at $\theta^c$, and thus links the fee schedule $S$ above and below $\theta^c$ to eliminate the incentive to misreport across ranges.

The value of $\theta^c$ is a crucial determinant of the ability of the government to contain procurement costs. Condition (11) formalizes the argument in Section 3 regarding the necessity of actually cutting off a range of developer types. Suppose, for instance, that the government ultimately procures $z$ units from the developer for all $\theta$ types (the $X$ schedule is such that a cutoff is never credible and $\theta^c = \bar{\theta}$). Then, (11) implies that total government payments to each $\theta$ are at least $\bar{\pi} + \bar{\theta} \Gamma(z)$, and the highest possible cost is always incurred.

The existence of a separating contract is intimately connected with the experience curve and the use of an auction for reprocurement. In the related problem of a buyer who is unable to commit to his future actions in a relationship with a supplier, Laffont and Tirole (1985) have shown that it is impossible for an initial contract to separate types when the buyer is free to use acquired information and the supplier cannot be forced to accept negative second-period profits. Baron and Besanko (1987) also discuss this result. Since the buyer's contract offer rather than an auction governs second-period purchases, it is always optimal for the buyer to use any revealed information and to offer a second-period contract that provides the supplier with second-period profits of zero. To ensure separation the initial contract must make it unprofitable for a firm to understate costs initially and then to reject the second-period contract offer (the bid-to-lose strategy is the counterpart in our model). An initial contract cannot simultaneously satisfy this incentive constraint and the more familiar constraints that are necessary to prevent high cost reports.

The acquisition model we consider does not commit the government to reprocurement (recall the cutoff), but it does commit the government to use a second-source auction if reprocurement is desired. This has the effect of limiting the ability of the government to use acquired information. If reprocurement is desired, then the auction outcome will determine profits. Since the developer acquires an experience advantage in return for revealing his private information and since low-cost developers are not cut off, the auction allows these types to earn a positive second-period profit (an indirect rent accruing to the private cost information). Owing to these profits, the bid-to-lose incentive constraint can be satisfied without also inducing low-cost types to overstate costs. Furthermore, the anonymity of the auction mechanism ensures a nonnegative profit for any developer type and any initial-stage outcome.

We take our assumption about the running of a second-source auction literally; that is, auctions of this sort are run in practice (Beltramo (1983) provides some case histories). One can, however, interpret the auction as a proxy for the polar outcome of bilateral monopoly bargaining in which the agent receives all of the surplus. The principal's ability to drive the agent to zero-second-period profits represents the other extreme.

5. The optimal contract

The objective of the government is to minimize the expected cost of acquiring $z$ units. For each report acquisition costs are the sum of the initial fee and the reprocurement cost. Over the cutoff range, the initial fee is given by condition (11), and the residual need is satisfied with the old system at a cost of $\delta[z - X(\theta)]$. Over the noncutoff range, total gov-
government payments are given by condition (10). Taking expectations over \( \theta \), we can write expected cost as

\[
\int_{\theta}^{\tilde{\theta}} [S(\theta) + \theta \Gamma(z - X(\theta))] f(\theta) d\theta + \int_{\tilde{\theta}}^{\infty} [S(\theta) + \delta(z - X(\theta))] f(\theta) d\theta.
\]

(13)

We can use conditions (10)–(12) to simplify this expression.

**Lemma 2.** If a contract \( \{S, X\} \) satisfies (5) and (6), then the expected cost of the government is equal to

\[
\theta^* \Gamma(z) F(\theta^*) + \int_{\theta}^{\tilde{\theta}} [\rho(\theta) \Gamma(X(\theta)) + \delta(z - X(\theta))] f(\theta) d\theta + \tilde{\pi},
\]

(14)

where \( \rho(\theta) = \theta + F(\theta)/f(\theta) \).

We can interpret expression (14) as a combination of direct cost effects and incentive terms. From the first term, government cost over the noncutoff regime is given by the production cost of \( z \) units for the highest noncutoff type, \( \theta^* \Gamma(z) \), weighted by the probability of the regime, \( F(\theta^*) \).

In the cutoff regime the term under the integral consists of the initial-stage cost and the cost of buying \( z - x \) units of the old system. Note that initial production is weighted by the \( \rho(\theta) \) function instead of \( \theta \). The weight \( \rho(\theta) \) includes an incentive term \( F(\theta)/f(\theta) \), the ratio of the cumulative distribution to the density function. In fact, when multiplied by \( \Gamma(X(\theta)) \), this ratio is the shadow price to the government of acquiring information on \( \theta \) from the developer. Thus, \( \rho(\theta) \) includes the added cost under private information of inducing the developer to self-select into the cutoff range. The profit level \( \tilde{\pi} \) enters as an additive term and is set to zero under cost minimization.

The government chooses a contract \( \{S, X\} \) to minimize the expected cost given by (14). The simplifying feature of (14) is that it expresses cost solely in terms of the cutoff value \( \theta^* \) and the production schedule \( X \). Thus, the cost-minimization problem is to choose a schedule \( X \) to minimize (14) subject to restriction (8) that \( X \) be between \( x^i \) and \( x^c \) for \( \theta \leq \theta^c \) and restriction (9) that \( X \) be above \( x^c \) for \( \theta > \theta^c \). Recall that \( \theta^* \) is determined endogenously by the \( X \) schedule and is crucial in fixing the price that the government pays to all noncutoff types.

**Proposition 2.** Assume that \( \rho(\theta) \) is strictly increasing on \([\theta, \tilde{\theta}]\). Then, in a separating equilibrium expected cost (14) is minimized subject to (8) and (9) by

\[
X^*(\theta) \in [x^i(\tilde{\theta}/\theta), x^c(\theta)), \quad \theta \leq \theta^*,
\]

(15)

\[
X^*(\theta) = x^c(\theta), \quad \theta > \theta^*.
\]

(16)

The optimal cutoff value \( \theta^* \) is interior and is determined by

\[
\rho(\theta^*)[\Gamma(z) - \Gamma(x^c(\theta^*))] = \delta(z - x^c(\theta^*)).
\]

(17)

The optimal value \( \theta^* \) is the boundary between the two regimes. All \( \theta > \theta^* \) are cut off and initially produce only the minimum amount \( x^c(\theta) \) needed to make a cutoff credible. Allowing \( \theta > \theta^* \) more production than \( x^c(\theta) \) only increases total procurement costs without changing the second-period incentive of the government to cut off \( \theta > \theta^* \). Under the cutoff rule the residual is procured from the producer of the old system, an outcome that is first-best inefficient. Thus, the experience curve gains from initial production are sacrificed to prevent low-cost types from announcing a high value for the cost parameter.

---

12 The \( \rho \) function occurs in many models of private information. Baron and Besanko (1987) provide a discussion and several further references. In our case the proof of Proposition 2 shows that monotonicity can be weakened somewhat without affecting the solution.
Over the lower interval the initial order is followed by reprocurement through the second-source auction. Since total cost over this regime is determined by the location of $\theta^*$, any $X$ schedule that lies between the $x^l$ and $x^c$ functions is sufficient. From condition (15) we see that reprocurement through an auction is credible, and no higher-cost producer can profitably pursue the bid-to-lose strategy in an attempt to garner initial profits.

The optimal cutoff $\theta^*$ is determined by (17). It is chosen to balance the cost gains over the noncutoff regime against the efficiency losses arising from the use of a cutoff. This is done by selecting $\theta^*$ to equate the residual production costs of the developer, adjusted for the added marginal cost of information, with the cost of procuring $z-x^{\theta}$ units of the old system.

The fee schedules associated with a production schedule in Proposition 2 are obtained directly from (10)-(12). Thus, we have

$$S^*(\theta) = \theta \Gamma(x^{\theta}(\theta)) + \int_\theta^{\hat{\theta}} \Gamma(x^{\theta}(t))dt, \quad \theta > \theta^*, \quad (18)$$

$$S^*(\theta) = \theta^* \Gamma(z) - \theta^* \Gamma(z - X^*(\theta)) + \int_{\theta^*}^{\hat{\theta}} \Gamma(x^{\theta}(t))dt, \quad \theta \leq \theta^*. \quad (19)$$

Over the cutoff regime, (18) shows that the fee decreases smoothly with the cost parameter. Properties of the fee over the noncutoff regime are determined once $X^*$ is specified; note that $S^*$ is decreasing if $X^*$ is decreasing.\textsuperscript{13}

Finally, note that we can easily extend the optimal contract to include the possibility of types outside the interval $[\theta, \hat{\theta}]$. For higher types set $X(\theta) = 0$, since these types can never produce below the cost of the old system. For lower types choose $X(\theta)$ to lie between $x^l(\theta)$ and $z$, since a cutoff is never credible in this range.\textsuperscript{14}

6. Policy implications and discussion

Previous analyses of second sourcing have assumed that there is no asymmetry in the contractors’ positions on the experience curve. Under this assumption the decision of when to begin reprocurement should depend only on tradeoffs among factors such as technology-transfer costs and the number of units over which competition will be applied; the effectiveness of second-source competition will not depend on the timing of reprocurement. When, more realistically, the developer’s production experience provides a cost advantage over the second source, the ability of a second source to provide effective competition is substantially reduced. The degree to which competition is weakened, however, is partially controlled by the government, since the developer’s cost advantage depends on the cumulative production experience that the government allows in the initial-production stage. Thus, other things being equal, the effectiveness of competition decreases with initial production, and the timing of reprocurement should be adjusted to reflect this factor.

The problem that results from differential experience disappears when the government

\textsuperscript{13} Although (18) appears to have a form that is analogous to a cost-plus incentive fee structure, the incentive fee in (18) is designed to induce information revelation. In a cost-plus incentive fee contract, the incentive fee is designed to induce the proper amount of effort on the part of the contractor. The fees given by (18) and (19) and the quantities from (15) and (16) can be used to obtain an equivalent representation of the contract in requirement form by eliminating $\theta$ and solving for $s$ as a function of $x$. See Riordan (1984) for further examples of the relationship between contingent and requirement forms.

\textsuperscript{14} If it is possible to contract for the entire production run in the initial stage, then a contract that specifies $X(\theta) = z$ and $S(\theta) = x^l + \theta \Gamma(z)$ for $\theta \leq \theta^*$ will achieve the same expected government cost for $T = 0$; see the proof of Proposition 2. If a contract that covered all future procurement needs could be written, it would economize on technology-transfer costs. At present, defense procurement typically proceeds on a year-to-year basis (Gansler, 1984).
chooses to skip the initial-production phase and to run the auction when both contractors face the same costs. But skipping the initial-production phase eliminates the government’s ability to elicit cost information from the developer and, thus, its ability to reduce the information asymmetry between the developer and the second source. As a result, the government is burdened with two additional costs. First, one-sided uncertainty about costs will raise the expected bid relative to bids made under perfect information. Second, without such knowledge the second source may bid so high that the developer wins the contract and secures large profits, or the second source may win the contract with a bid that is so low that he cannot break even. The danger in the latter outcome is that the government typically ends up absorbing a sizable fraction of the contractor’s loss, either directly through renegotiated contracts or indirectly through reduced quality and delays in production. Thus, the effective use of competition seems to require that the second source have reasonably accurate knowledge about the true costs of production.

An initial-production contract offers the government a natural opportunity for acquiring this private cost information. When the developer makes a choice from a menu of initial-production quantity and payment options, he does so on the basis of his private cost information. To induce information revelation, the government must not skip the initial-production phase (the most favorable timing if only the experience curve is considered); rather, the government should treat initial production as a strategic phase and use the contract menu to create incentives that lead the developer to reveal information. Since this method is based on the developer’s profit motive, it may be superior to alternative means of obtaining cost information.\(^{15}\)

The government can then use this information in the reprocurement phase when it decides either to transfer technology and to use the second-source auction or to terminate procurement of the new system and to procure the old system. Furthermore, without the burden of an information handicap, the second source can bid more competitively. Thus, an optimal procurement scheme trades off the benefits of information revelation with the costs of providing initial-production incentives.

The problem with eliciting information via contract choice and, hence, the cost of providing initial-production incentives is that the developer may find it profitable to misrepresent costs. Misrepresentation can take the form of low-cost types’ pretending to be high-cost, a strategy aimed at raising the second-source bid, or high-cost types’ pretending to be low-cost, so that they obtain a more profitable initial order and then bid to lose in the auction stage. As both of these incentives are influenced by the prospect of competing with an inexperienced and misinformed second source, the variation of downstream profits with cost reports is important for determining the developer’s incentive to reveal information.

A commitment to use a competitive reprocurement mechanism, such as an auction, limits the ability of the government to use revealed information. In particular, it is not possible to procure residual quantities of the new system at the “revealed” cost of the developer. While this feature of auction commitment is important for the incentive structure of the contract (especially with respect to making bidding-to-lose unprofitable), it is the structure of the reprocurement decision the government faces when using a second-source auction that allows the government to contain procurement costs.

The optimal policy of the government, implemented by the initial-production contract, is to use a cutoff rule. When reported costs are below a critical level, the auction is used. When reported costs are above this level, the government decides to cut off the new system.

\(^{15}\) For example, a program to transfer technology from the developer to the second source, while important from an engineering perspective, is not likely to provide the knowledge about experience-curve economies that is crucial to informed bidding. Production cost audits are another potential source of information, but such audits occur after the fact and may be unreliable, as there is little incentive to report cost data accurately, and, in the case of a large defense contractor, there are many ways to transfer costs among projects.
and no auction ensues. When the cutoff is used, residual needs are satisfied by the alternative system, an outcome that would be inefficient under full information. Although for high-cost types the government incurs higher total costs of acquisition by using a cutoff, low-cost types are denied access to the highly profitable strategy of pretending to be high-cost with the knowledge that ultimately all needed units of the system will be acquired from them at a price that reflects their cost report.

The effectiveness of this government strategy depends on the credibility of the threat to cut off procurement of the new system. Such a threat is not credible when the reprocurement decision is based solely on an auction involving the developer, the second source, and the alternative-system source, because the developer would win all such auctions. Thus, there would be no cutoff of developer production.

In settings involving complex technology, however, the desire to avoid the costs associated with transferring technology to the second source makes it natural for the government to make a cutoff decision on the new system before using a second-source auction. When the expected auction price is low (net of technology-transfer costs), it is optimal to transfer technology and to proceed with the auction. When the expected auction price is equal to the old-system price, no auction occurs, and the transfer costs are avoided by purchasing the old system directly. Thus, the second source is valuable to the government for two reasons. First, when the auction occurs, the second-source bid establishes the reprocurement price. Second, the inherent costs of transferring technology to the second source make it possible for the government to develop a credible cutoff threat.

The effectiveness of a second-source competition is ultimately dependent on the ability of the government to convince developers that an auction will be used for reprocurement. Defense acquisition policies are subject to change both by internal decisionmakers and by external bodies such as the Congress. On this issue Baron and Besanko (1987) point out that the courts may be more favorably inclined to consider disputes between an administrative agency and a contractor when these disputes involve methods and procedures as opposed to specific decisions regarding matters such as prices and quantities. On these grounds, an auction is appealing as it is a method instead of a particular contract specification. Thus, favorable prospects for legal remedies enhance the developer’s confidence in the government’s commitment to the auction mechanism.

Appendix

Proofs of Lemma 1, Proposition 1, Lemma 2, and Proposition 2 follow.

A straightforward argument based on the concavity of \( \Gamma \) establishes the existence of the cutoff function \( x^*(\theta) \).

**Proof of Lemma 1.** To begin, consider a given \( \theta \in (\theta, \tilde{\theta}) \). Then define a function \( A(y) = \theta \Gamma(y) - \delta y \) for \( y \in [0, z] \). Clearly, \( A(0) = 0 \) and \( A'(0) = \theta c(0) - \delta > 0 \) by (1), the parameter condition. Also, by (1) \( A(z) = \theta \Gamma(z) - \delta z < 0 \). Finally, note that \( A'(y) = \theta c'(y) < 0 \) for all \( y \), so that \( A \) is concave.

Then, by these properties the equation \( A(y) = 0 \) has a unique positive root. Denote this root by \( y(\theta) \) and then define \( x^*(\theta) = z - y(\theta) \). The remaining properties of \( x^*(\theta) \) are easily verified.

Now, \( x < x^*(\theta) \) if and only if \( z - x > z - x^*(\theta) \). This, in turn, is equivalent to \( A(z-x) < A(z-x^*(\theta)) \). By the definition of \( x^*(\theta) \), \( A(z-x^*(\theta)) = 0 \). Then, by the definition of \( A(\cdot) \), \( A(z-x) < 0 \) if and only if \( \theta \Gamma(z-x) < \delta (z-x) \). Thus, the auction is cost minimizing if and only if \( x < x^*(\theta) \). Q.E.D.

Before proving Proposition 1, we examine \( \pi_e(\tilde{\theta}|\theta) \), the maximum reprocurement-stage profit if \( \tilde{\theta} \) is reported and \( \theta \) is the type. To evaluate \( \pi_e(\tilde{\theta}|\theta) \), first consider a report \( \tilde{\theta} \) and a type \( \theta \). Let \( \tilde{x} = X(\tilde{\theta}) \) be the initial quantity for the report \( \tilde{\theta} \). If \( \tilde{x} > x^*(\theta) \), we have a credible cutoff and \( \pi_e(\tilde{\theta}|\theta) = 0 \). Thus, \( \tilde{x} < x^*(\theta) \), and the second source submits an auction bid of \( \tilde{\theta} \Gamma(z-\tilde{x}) \). A matching bid from the developer wins the auction, and any lower bid is dominated. This “bid-to-win” strategy yields a profit of

\[
\tilde{\theta} \Gamma(z-\tilde{x}) - \theta \Gamma(z) + \Gamma(\tilde{x}).
\]

Alternatively, the developer can “bid to lose” with a bid above the second-source bid.

In evaluating \( \pi_e(\tilde{\theta}|\theta) \) we must determine which of the bid-to-win and bid-to-lose strategies is optimal. Thus, we examine the sign of (A1) in relation to \( \theta \), \( \tilde{\theta} \), and \( \tilde{x} \). In the case of an overreport—\( \tilde{\theta} > \theta \)—\( \tilde{\theta} \Gamma(z-\tilde{x}) > \tilde{\theta} \Gamma(z) - \Gamma(\tilde{x}) \) by the concavity of \( \Gamma \). As \( \tilde{\theta} > \theta \), the latter expression is greater than \( \theta \Gamma(z) - \Gamma(\tilde{x}) \). Thus, (A1) is positive and bid-to-win is optimal. In the case of an underreport—\( \tilde{\theta} < \theta \)—if we take the ratio of \( \theta \) to \( \tilde{\theta} \) as given for the moment, we can define the concave function
\[ B(\bar{x}) = \Gamma(z - \bar{x}) - (\theta/\bar{\theta})[\Gamma(z) - \Gamma(\bar{x})]. \]

Bid-to-win is optimal if \( B(\bar{x}) > 0 \) and bid-to-lose is optimal if \( B(\bar{x}) < 0 \).

Consider solving for an interior \( L \) such that \( B(L) = 0 \). Then, if \( B(0) < 0 \) and \( B(z) = 0 \), if \( B'(z) < 0 \), there is a unique interior root \( L \) given the \( \theta \) to \( \bar{\theta} \) ratio. Clearly, \( B'(z) = -c(0) + (\theta/\bar{\theta})c(z) \). Then, by the concavity of \( \Gamma \), we have \( B'(z) < -c(0) + (\theta/\bar{\theta})\Gamma(z)/z \). Using (1), we then see that \( B'(z) < 0 \).

Thus, \( B(L) \) has a unique interior root \( L(\theta/\bar{\theta}) \). Differentiation shows that \( L(\theta/\bar{\theta}) \) is increasing in the ratio \( \theta/\bar{\theta} \). Bid-to-win is optimal if \( \bar{x} > L(\theta/\bar{\theta}) \) and bid-to-lose is optimal if the inequality is reversed.

Define
\[ x^*(\theta) = L(\theta/\bar{\theta}). \] (A2)

Thus, if the \( X \) schedule lies above the \( x^* \) function, bid-to-win is optimal for \( \bar{\theta} \) at any report \( \theta \). Furthermore, bid-to-win is optimal for all lower types. Finally, it is straightforward to show that \( x^* \) is strictly below \( x^c \).

**Proof of Proposition 1.** The proof is divided into two parts. In part (A) we show that if \( \{ S, X \} \) satisfies (8)–(12), then the self-selection constraints (5) and the participation constraints (6) are satisfied. In part (B) we demonstrate the cost-equivalence result.

(A) We begin by finding the equilibrium developer profits \( \Pi(\theta) \) that are implied by (8)–(12). For \( \theta > \theta^c \) condition (9) specifies a cutoff. Then
\[ \Pi(\theta) = \bar{x} + \int_{0}^{\bar{x}} \Gamma(X(t))dt \quad \text{for} \quad \theta > \theta^c \] (A3)
as follows from \( \Pi(\theta) = s - \theta \Gamma(X) \) and (11). For \( \theta \leq \theta^c \) condition (8) implies no cutoff. Then
\[ \Pi(\theta) = s + \theta \Gamma(z - x) - \theta \Gamma(z), \]
and by (10) and (12) we have
\[ \Pi(\theta) = \bar{x} + \int_{0}^{\bar{x}} \Gamma(X(t))dt + (\theta^c - \theta)\Gamma(z) \quad \text{for} \quad \theta \leq \theta^c. \] (A4)

From (A3) and (A4) constraint (6) clearly holds as \( \Pi(\theta) \geq \pi(\theta) \geq 0 \) for all \( \theta \).

Now consider constraint (5). We must verify that the equilibrium profit \( \Pi(\theta) \) given by (A3) and (A4) is at least as large as \( \pi(\theta/\bar{\theta}) \). For \( \bar{\theta} > \theta^c \) we have a cutoff, and \( \pi(\theta/\bar{\theta}) = \bar{x} - \bar{\theta} \Gamma(\bar{x}) \). Thus,
\[ \pi(\theta/\bar{\theta}) = \Pi(\bar{\theta}) + (\bar{\theta} - \theta)\Gamma(\bar{x}) \quad \text{for} \quad \bar{\theta} > \theta^c. \] (A5)

For \( \bar{\theta} \leq \theta^c \) condition (8) implies no cutoff, since \( \bar{x} < x^*(\bar{\theta}) \), and bid-to-win is the optimal strategy for any type \( \theta \), since \( x^*(\bar{\theta}) < \bar{x} \). Thus,
\[ \pi(\theta/\bar{\theta}) = \Pi(\bar{\theta}) + (\bar{\theta} - \theta)\Gamma(z) \quad \text{for} \quad \bar{\theta} \leq \theta^c \] (A6)
as total payment to the developer is \( \bar{x} + \bar{\theta} \Gamma(z - \bar{x}) - \theta \Gamma(z) \).

Using (A3)–(A6), it is easy to verify that \( \Pi(\theta) \geq \pi(\theta/\bar{\theta}) \) for all \( \bar{\theta} \) and \( \theta \).

(B) Let \( \{ S, X \} \) be a contract satisfying constraints (5) and (6). A contract satisfying conditions (8)–(12) is shown to achieve the same government cost for each \( \theta \).

For the schedule \( X \), define the critical value \( \theta^c \) by
\[ \theta^c = \sup \{ \theta | X(\theta) < x^*(\theta) \}. \]

Note that \( \theta^c \) is determined by \( X \) and, in general, may be anywhere in the interval \([\theta, \bar{\theta}]\). By definition all \( \theta \) above \( \theta^c \) are cut off. Without loss of generality, assume that \( \theta^c \) is not cut off. Otherwise, we could always find a \( \theta \) arbitrarily close to \( \theta^c \) that is not cut off, and the argument below would follow the same logic. Thus, \( \pi^* = \Pi(\theta^c) = S(\theta^c) + \theta^c \Gamma(z - X(\theta^c)) - \theta^c \Gamma(z) \).

To demonstrate the cost-equivalence result, we show that the self-selection constraints (5) and the participation constraints (6) imply that: (i) condition (10) holds for any \( \theta \leq \theta^c \); (ii) condition (11) holds for any \( \theta > \theta^c \); and (iii) the profit level \( \Pi(\theta^c) \) satisfies (12).

(i) For any \( \theta < \theta^c \), (10) holds, and total payment is \( \pi^* + \theta^c \Gamma(z) \). We show this by applying (5) to \( \theta \) and \( \theta^c \). Consider a specific \( \theta < \theta^c \). To simplify notation let \( x = X(\theta), s = S(\theta), \) and \( L = L(\theta/\theta^c), x^c = x^c(\theta) \). There are three cases for the value of \( x = X(\theta) \): (a) \( x \geq x^c \); (b) \( x^c > x \geq L \); and (c) \( L > x \). Of these, (a) and (c) result in a contradiction and (b) establishes condition (10). For brevity, we prove only (b).

Suppose \( x^c > x \geq L \). Then \( \theta \) is not cut off, and bid-to-win is optimal if type \( \theta^c \) reports \( \theta \). We have \( \Pi(\theta) = s + \theta \Gamma(z - x) - \theta \Gamma(z) \). Then, applying (5) to \( \theta \) and \( \theta^c \), we obtain the pair of inequalities
\[ s + \theta \Gamma(z - x) \geq \pi^* + \theta \Gamma(z) \quad \text{and} \quad \pi^* + \theta \Gamma(z) \geq s + \theta \Gamma(z - x). \]

Hence, government payments are \( s + \theta \Gamma(z - x) = \pi^* + \theta \Gamma(z) \).
(ii) For any \( \theta > \theta' \) (11) holds, and \( S(\theta) = \bar{x} + \theta \Gamma(X(\theta)) + \int_{\theta}^{\theta'} \Gamma(X(t))dt \). This is a standard result in the incentives literature. For instance, with a change of notation, the claim follows directly from Lemma 1 of Baron and Myerson (1982).

(iii) For \( \theta' \geq 0 \) holds, and \( \pi' = \bar{x} + \int_{0}^{\theta'} \Gamma(X(t))dt \). By (5) we have \( \pi' \geq \pi(\theta') \) for any \( \theta > \theta' \). Thus, \( \pi' \geq S(\theta) - \theta \Gamma(X(\theta)) \), and from (ii), \( \pi' \geq \pi' + (\theta - \theta') \Gamma(X(\theta)) + \int_{\theta}^{\theta'} \Gamma(X(t))dt \). Letting \( \theta \) approach \( \theta' \) yields \( \pi' \geq \bar{x} + \int_{\theta}^{\theta'} \Gamma(X(t))dt \). Going the other way, there are two cases. Again, for brevity, we prove only one.

Suppose \( X(\theta') \geq L(\theta'/\theta') \). Then bid-to-win is optimal if \( \theta \) reports \( \theta' \). By (5) \( \Pi(\theta) \geq \pi(\theta'/\theta) \). Thus,

\[ \bar{x} + \int_{\theta}^{\theta'} \Gamma(X(t))dt \geq \pi' - (\theta - \theta') \Gamma(z). \]

Letting \( \theta \) approach \( \theta' \) establishes the upper bound on \( \pi' \) for this case. \textit{Q.E.D.}

Lemma 2 is established through straightforward calculations. As Proposition 2 does not follow from standard optimization techniques, we provide a detailed proof.

\textbf{Proof of Lemma 2.} We begin with (13):

\[ \int_{\theta}^{\theta'} \left[ S(\theta) + \theta \Gamma(z - X(\theta)) \right] f(\theta)d\theta + \int_{\theta}^{\theta'} \left[ S(\theta) + \delta[z - X(\theta)] \right] f(\theta)d\theta. \]  

(13)

Use (10) to substitute for the first integrand and (11) to substitute for the fee \( S(\theta) \) in the second integrand. This yields

\[ [\pi' + \theta \Gamma(z)] f(\theta') + \int_{\theta}^{\theta'} \left[ \pi' + \theta \Gamma(X(\theta)) + \int_{\theta}^{\theta'} \Gamma(X(t))dt + \delta[z - X(\theta)] \right] f(\theta)d\theta. \]

To deal with the double integral, define \( H(\theta) = \int_{\theta}^{\theta'} \Gamma(X(t))dt \). Note that \( H'(\theta) = -\Gamma(X(\theta)) \) and \( H(\theta) = 0 \). Substituting (12) for \( \pi' \) above yields

\[ \bar{x} + \theta \Gamma(z) f(\theta') + \int_{\theta}^{\theta'} \left[ \theta \Gamma(X(\theta)) + \delta[z - X(\theta)] \right] f(\theta)d\theta + H(\theta) F(\theta') + \int_{\theta}^{\theta'} H(\theta) f(\theta)d\theta. \]

Using integration by parts, the last two terms add to \( \int_{\theta}^{\theta'} \Gamma(X(\theta)) f(\theta)d\theta \). Substituting with this in the above and then rearranging by using the definition of \( \rho(\theta) \) yield (14). \textit{Q.E.D.}

\textbf{Proof of Proposition 2.} Denote by (A7) the problem of choosing a schedule \( X(\cdot) \) to minimize (14) subject to (8) and (9):

\[ \min_{X} \int_{\theta}^{\theta'} V[X] = \theta \Gamma(z) F(\theta') + \int_{\theta}^{\theta'} \left[ \rho(\theta) \Gamma(X(\theta)) + \delta[z - X(\theta)] \right] f(\theta)d\theta \]  

(A7)

subject to

\[ x(\theta) \leq X(\theta) < x'(\theta) \quad \text{for} \quad \theta \leq \theta', \]

\[ x'(\theta) \leq X(\theta) \quad \text{and} \quad X(\theta) \text{ nonincreasing for} \quad \theta > \theta'. \]

Now define the function \( g(x, \theta) = \rho(\theta) \Gamma(x) + \delta(z - x) \). Consider the problem (A7a) of choosing a schedule \( Y(\cdot) \) to

\[ \min_{Y} \int_{\theta}^{\theta'} W[Y] = \int_{\theta}^{\theta'} g(Y(\theta), \theta) f(\theta)d\theta \]  

(A7a)

subject to

\[ x'(\theta) \leq Y(\theta) \quad \forall \theta. \]

First we show that (A7a) has a unique solution. Then we establish the relationship between the solutions of (A7) and (A7a).

\textbf{Lemma.} Let \( \theta' = \theta' \) as given by (17). The unique solution to (A7a) is given by \( Y^*(\theta) = z \) for \( \theta \leq \theta' \) and \( Y^*(\theta) = x'(\theta) \) for \( \theta > \theta' \).

\textbf{Proof.} Begin by observing that \( g(x, \theta) \) is strictly concave in \( x \) for each \( \theta \). Thus, for each \( \theta \) the solution to \( \min_{x} g(x, \theta) \) subject to \( x \in [x'(\theta), z] \) can be found by examining the end points of the constraint interval. To do this consider the function \( x'(\theta) \), which is implicitly defined by \( g(x^2, \theta) = g(z, \theta) \) and \( x^2 \neq z \) (see Figure A1). By the concavity of \( g \), \( x^2 \) exists for any \( \theta \) such that \( z \) does not provide a global maximum or minimum for \( g \) on the interval \([0, z] \). Also, \( x^2 > 0 \) in this case. Extend \( x^2 \) to all \( \theta \) by \( x^2(\theta) = 0 \) when \( z \) provides a global minimum and by \( x^2(\theta) = z \) when \( z \) provides a global maximum.

Thus defined, \( x'(\theta) \) is continuous in \( \theta \). For \( 0 < x^2 < z \) the function is also differentiable and

\[ dx^2/d\theta = \rho(\theta)[(\Gamma(z) - \Gamma(x^2))/((\rho(\theta)c(x^2) - \delta))]. \]
By the concavity of $\Gamma$ and the parameter condition (1), the denominator is positive. Then, as $\rho(\theta)$ is positive and increasing, $x^\theta$ is increasing. As can be seen in Figure A1, $x^\theta$ is constructed to represent the value of $g(z, \theta)$.

It is easily verified that $x^\theta(0) = 0$ and $x^\theta(\theta) > 0$. As the cutoff function $x^\theta$ continuously decreases from the value $z$ at $\theta$ to the value 0 at 0, the functions $x^\theta$ and $x^\theta$ have a unique crossing point. Thus, let $\theta^*$ denote this unique solution to $x^\theta(\theta) = x^\theta(\theta)$. Then $\theta^*$ is interior and (17) follows.

Now consider solving (A7a). For $\theta < \theta^*$ we have $x^\theta > x^\theta$. Hence, the choice $Y^\star(\theta) = z$ minimizes $g$ over $\{x^\theta(\theta), \theta\}$. When $\theta > \theta^*$, we have $x^\theta < x^\theta$ and the choice $Y^\star(\theta) = x^\theta(\theta)$ is the minimizer for $g$ over $\{x^\theta(\theta), \theta\}$. Q.E.D.

We use $Y^\star$, the unique minimization solution (A7a) to characterize solutions to the original program (A7). First, calculate the objective function of (A7a) at $Y^\star$:

$$W[Y^\star] = \int_0^\theta g(z, \theta)f(\theta)d\theta + \int_0^\theta g(x^\theta(\theta), \theta)f(\theta)d\theta$$

$$= \theta^* \Gamma(z)\rho(\theta^*) + \int_0^\theta g(x^\theta(\theta), \theta)f(\theta)d\theta,$$

which follows from $g(z, \theta) = \Gamma(z)\rho(\theta)$ and $\rho(\theta)f(\theta) = d[\theta]\Gamma(\theta)/d\theta$.

To solve (A7), define the class of schedules $X^\star(\theta)$ such that $x^\theta(\theta) \leq X^\star(\theta) < x^\theta(\theta)$ for $\theta \leq \theta^*$, and $X^\star(\theta) = x^\theta(\theta)$ for $\theta > \theta^*$. Any $X^\star$ is feasible for (A7), and $\theta^*$ is the cutoff value. Calculating, we have

$$V[X^\star] = W[Y^\star].$$  \hfill (A8)

Let $X$ be a feasible schedule for (A7) and let $\theta^\ast$ denote the cutoff value. Define an associated schedule by $Y(\theta) = z$ for $\theta \leq \theta^\ast$ and $Y(\theta) = X(\theta)$ for $\theta > \theta^\ast$. Then $Y$ is feasible for (A7), and direct substitution in the objective functions shows that $V[X] = W[Y]$. Since $Y^\star$ is a solution to (A7), we have $W[Y] \geq W[Y^\star]$. Then, by (A8), $V[X] \geq V[X^\star]$, and $Y^\star$ solves (A7).

Finally, consider uniqueness of the $X^\star$ solution class. The complete class of solutions to (A7) is obtained by extending the $X^\star$ class to $X^\star(\theta)$ such that $X^\star(\theta) = z$ for $\theta \in (\theta^\ast, \theta^\ast^\prime)$, and $X^\star^\prime = X^\star$ otherwise, where $\theta^\prime$ is any element of $[\theta^\ast, \theta^\ast]$. We omit the proof. Q.E.D.

References


