Seniority and Efficiency

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Abstract
The role of seniority in efficient compensation and employment arrangements between a firm and its labor pool is examined. Seniority is treated as an endogenous factor in a dynamic model with uncertainty. Nonseparability over time in preferences and production emerges as a pivotal aspect of seniority provisions in efficient allocations. The analysis focuses on two dynamic factors that have been emphasized in the literature: nonseparable worker preferences in leisure and a learning-by-doing process of human capital accumulation.

I. Introduction
The role of seniority in compensation and employment arrangements between a firm and its labor pool is examined in this paper. Seniority provisions and effects, whereby the compensation and employment of junior and senior workers exhibit a systematic relationship with some measure of tenure or experience, are treated as endogenous elements that are determined by underlying dynamic factors in productivity and preferences. We focus on the effects of human capital accumulation and intertemporally nonseparable preferences and, for each case, we assess the efficiency basis for the use of seniority and identify how seniority provisions can be used to achieve efficient allocations.

To frame the basic issues, suppose a firm must reduce total labor input due to a negative shock. This reduction has to be distributed across the available labor pool in some manner. If wages rise with seniority, for instance, then short-run labor costs can be reduced by concentrating work

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reductions on senior employees. The utilization of junior employees, however, necessarily entails an increase in their seniority level (as measured by total past work effort) and, when wages rise with seniority, this will contribute to higher labor costs in future periods. Thus, the utilization decision involves a tradeoff between current and future labor costs.

An intertemporal tradeoff also arises when the potential for human capital accumulation exists, as in the case of learning-by-doing increases in worker productivity. While senior workers have a greater marginal product, concentrating work reductions on junior employees is an implicit reduction in human capital investment. Again, an intertemporal tradeoff has to be evaluated.

To model these tradeoffs and analyze the implications for seniority effects, we employ an overlapping-generations model as it allows for a rolling horizon in which the employer faces a repeated choice with respect to junior and senior workers. The efficiency properties of compensation and employment are jointly derived in terms of the underlying preferences and technology. Thus, it is efficiency rather than bargaining power that constitutes the driving force behind seniority effects in our model. An excellent example of the insights provided by an efficiency analysis is Carmichael (1983b). He demonstrates that ex-post productive efficiency in a model with on-the-job training leads to a seniority pattern where layoffs fall on senior before junior workers.

We examine seniority with respect to two kinds of "economic glue" that can bind firms and workers: (i) gains from income smoothing that arise from the life-cycle preferences of workers and (ii) human capital accumulation when learning-by-doing leads to productivity increases; see Hall (1980). In the specification of preferences under (i), labor effort in the previous period reduces the current utility of a worker. This effect figures prominently in the recent literature on labor market fluctuations including Johnson and Pencavel (1984) and Hotz, Kydland and Sedlacek (1988). As our focus on (i) and (ii) suggests, we argue that seniority provisions are a feature of efficient allocations when there are nonseparabilities in preferences and production; for preferences and production that are separable over time, such as when the effects in (i) and (ii) are allowed to vanish, we find that efficiency entails complete work sharing and a general absence of any seniority patterns.

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1 Seniority and experience coincide in our model; see Abraham and Farber (1987) and Altonji and Shakatko (1987) for an empirical analysis of this distinction.

2 See also Oswald (1981). In a firm-union relationship, seniority issues can arise because some workers may have important bargaining powers; see e.g. Grossman (1983), Weiss (1985) and Oswald (1986). On the incentive role of seniority, see Lazear (1979), Harris and Holmstrom (1982), Carmichael (1983a), Ioannides and Pissarides (1983) and Ito and Kahn (1986).
Efficient allocations under (i) and (ii) can be implemented with long-term contracts between firms and individual workers. Seniority patterns then emerge by examining how the contractual provisions for compensation and employment vary in response to shocks. Under (i) or (ii), past shocks influence current payoffs through previous employment and compensation choices and, as a consequence, efficient decisions must take this historical dependence into account. In many cases, a simple rule based on a worker's seniority (measured by accumulated work effort) can be used to achieve efficient allocations.

Efficiency under nonseparability involves a set of seniority premiums that compensate an employee for previous work effort. In an efficient allocation, senior employees earn more than their junior counterparts and employment reductions follow a strict seniority pattern whereby junior workers precede senior ones whenever hour reductions or layoffs are called for. This pattern is efficient because it minimizes the creation of future seniority premiums.

When learning-by-doing effects are present, the efficiency aspects of seniority reveal several striking properties. In an interesting benchmark case, where learning-by-doing effects augment the labor input of senior workers in a Harrod-neutral fashion, junior workers are completely insulated from current shocks as they work a fixed number of hours and receive a fixed compensation payment. While senior workers are unambiguously more productive and work more hours on average, all fluctuations in current shocks are absorbed by adjusting the hours and compensation of senior employees. Intuitively, this occurs because the relevant margin for senior workers hinges on their current marginal product while that for junior workers is based on an expected value, namely, the return from the implicit learning-by-doing investment. For the benchmark case, as well as more general ones, the familiar seniority patterns (that arise with the preference formulation) are absent.

The paper is organized as follows. The model is introduced in Section II and a basic result on compensation schemes is developed in Section III. Efficiency under nonseparable preferences is examined in Sections IV and V. The results under learning-by-doing are developed in Section VI. Equilibrium contracting is analyzed in Section VII and Section VIII concludes.

II. The Model

Consider an infinite-horizon, discrete-time model of labor exchange and production. In each period $t$, the production of a perishable consumption good is governed by $\theta_t f(L_t)$, where $\theta_t$ is a transitory shock and $L_t$ is total

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labor input. The function \( f(\cdot) \) is smooth, increasing and strictly concave with \( f'(0) = \infty \) and \( f'(\infty) = 0 \).

The gains-to-trade between a firm and its employees fluctuate as the multiplicative shock shifts the marginal product of labor schedule. The shock sequence \( \{\theta_t\}_{t=0}^{\infty} \) is i.i.d. with a common c.d.f. \( P(\cdot) \) and a finite mean. The support of \( P(\cdot) \) is an interval \( \Theta = (\underline{\theta}, \overline{\theta}) \) where \( \underline{\theta} \) is positive and the case of \( \overline{\theta} = \infty \) is allowed. Under this specification, the prediction of future shocks is trivial, and we focus, instead, on the dynamics generated by asymmetries in production and preferences across different vintage workers.

Total labor input, \( L_t \), is drawn from a workforce that includes junior and senior employees. Each worker lives for two periods, consuming and working in each. Workers are identical except for their arrival times so that a firm faces a repeated choice with respect to junior and senior employees. With one worker in each generation (an extensive margin is introduced in Section VII), total labor input is given by \( L_t = n_t^1 + n_t^2 - g(n_t^1) \), where \( n_t^i \geq 0 \), denotes the labor effort (hours) of a vintage \( i \) worker in stage \( i = 1, 2 \) of life. The function \( g(\cdot) \) describes a learning-by-doing effect of past effort on current productivity. Assume that \( g \) is smooth, increasing and strictly concave with \( g(0) = 1 \) and \( g(n) \leq g \) for all \( n \geq 0 \).

The utility function of workers allows for intertemporal nonseparability with respect to labor effort and it is specified as

\[
U[c_1 - r(n_1)] + \gamma U[c_2 - r(n_2) - s(n_1)],
\]

where \( c_i \) and \( n_i, i = 1, 2 \), are consumption and labor effort in each period, and \( \gamma \in (0, 1) \) is a discount factor. Assume that the single-period utility payoff \( U(\cdot) \) is smooth, increasing and strictly concave and that the functions \( r(\cdot) \) and \( s(\cdot) \) are smooth, increasing and weakly convex. \( U \) maps \( [-B, \infty) \), where \( B \geq 0 \), into a finite interval \( [\underline{U}, \overline{U}] \). Note that the value of \( B \) relates to the set of feasible consumption and labor effort values; \( B \) can reflect an upper bound on labor effort or an underlying income endowment.

The argument of \( U \) can be viewed as a composite good in which labor effort is converted into a consumption equivalent.\(^3\) The utility function is not separable since early-life work has a negative impact on utility in the next period. Specifications such as (1) are familiar from the literature on dynamic labor supply, including Johnson & Pencavel (1984), Hotz,

\(^3\)This means the familiar static income effect in labor supply is zero; dynamic labor supply under (1) is discussed in Section V. The risk-sharing and employment properties of static efficiency problems without income effects are well known; see Hart and Holmstrom (1985) and Topel and Welch (1987). In our case, this feature of (1) allows us to apply dynamic programming techniques; the analysis is substantially more complicated when income effects are introduced.

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Kydland and Sedlacek (1988), and Eckstein and Wolpin (1989) as well as aggregate fluctuations, such as Kydland and Prescott (1982) and Kennan (1988).

The transitory shocks, \( \{ \theta_i \} \), are a potential source of uncertainty in consumption and labor effort for workers. Any compensation and employment pattern with respect to the two shocks over each worker’s lifetime can be specified with a contract \( \delta = \{ w_1(\theta), n_1(\theta), w_2(\theta'), n_2(\theta') \} \), where \( w_i \) refers to total compensation and \( n_i \) refers to hours for \( i = 1, 2 \). For a worker of generation \( t \) and a contract \( \delta^t \), expected utility is (suppressing \( \theta_t \) and \( \theta_{t+1} \))

\[
V(\delta^t) = \mathbb{E}_t \{ U[w_1 - r(n_1^t)] + \gamma U[w_2^t - r(n_2^t) - s(n_1^t)] \},
\]

where the expectation \( \mathbb{E}_t \) is over \( \theta_t \) and \( \theta_{t+1} \) by the i.i.d. specification.

When the contracts \( \delta^{t-1} \) and \( \delta^t \) apply to workers \( t - 1 \) and \( t \), respectively, the profit of the firm in period \( t \) is (suppressing \( \theta_t \) and \( \theta_{t+1} \))

\[
\pi_t = \theta_t f[n_1^t + n_2^{t-1} g(n_1^{t-1})] - w_1^t - w_2^{t-1}.
\]

Firms are risk neutral with a discount rate \( \beta \in (0, 1) \).

An allocation for this economy specifies compensation and employment for each vintage worker, \( t \), for each of periods \( t \) and \( t + 1 \), where each choice in each period \( t \) may be contingent on past shocks, \( \theta = (\theta_t, \theta_{t-1}, \ldots, \theta_0) \). The welfare of a vintage \( t \) worker is evaluated as expected utility over \( \theta_t \) and \( \theta_{t+1} \), as in (2), \( \theta^{t-1} \), and the welfare of the firm is evaluated as the unconditional expected value of discounted profits. Any allocation can be achieved with a sequence of contracts, \( \{ \delta^t \}_{t=0}^\infty \), where each \( \delta^t \) specifies explicit contingencies for \( \theta_t \) and \( \theta_{t+1} \), and where the contract for period \( t \), \( \delta^t \), can vary with previous shocks, \( \theta^{t-1} \).

Pareto efficient allocations are found by solving for the sequence of contracts that maximizes the expected discounted value of profits subject to an expected utility constraint for each vintage worker:

\[
\max_{\{ \delta^t \}_{t=0}^\infty} \mathbb{E}_0 \left\{ \sum_{t=0}^\infty \beta^t \pi_t \right\} \text{ s.t. } V(\delta^t) \geq \Lambda, t \geq 0.
\]

Thus, efficient allocations are characterized in terms of contracts between the firm and workers, where the firm can choose \( \delta^t \) based on past shocks, \( \theta^{t-1} \).

\( ^4 \) Variations in \( \Lambda \), the expected utility level, trace out the efficient frontier; the only effect, however, of variations in \( \Lambda \), including time and state contingencies, is to generate lump-sum transfers between the firm and workers. In addition, initial conditions with respect to the worker of generation \( -1 \) turn out to be unimportant. Suppose, for concreteness, that this worker receives a one period contract where the utility level \( \Lambda \) and labor input \( n_1^{t-1} \) are given.
The rationale for approaching efficiency in terms of (M) is as follows. Seniority provisions always involve an element of historical dependence since current compensation and employment are linked to past events. In solving (M), we show how seniority, as measured by previous labor effort, can function as a state variable and proxy for the payoff-relevant aspects of previous shocks, thus eliminating the need for explicit contingencies on past shocks. The result is that efficient allocations can be achieved with contractual policies for junior and senior workers based on seniority measures. Thus, seniority emerges as an endogenous feature of efficient contracts.

III. Compensation Rules

We now examine cost-minimizing contracts between a firm and a single worker, and identify the gains-to-trade that exist with respect to compensation. We then employ the resulting compensation scheme to reduce the efficiency problem, (M), to a dynamic programming problem.

By definition, a cost-minimizing contract minimizes the expected present value of labor costs, \( E[\bar{w}_1 + \beta \bar{w}_2] \), for a given pattern of employment contingencies, \( n_1 \) and \( n_2 \), and a given expected utility level, \( \Lambda \). The cost-minimizing compensation scheme is given by (see Appendix 1 for the derivation)

\[
W_1(\theta; \Lambda) = Z_1(\Lambda) + r(n_1(\theta))
\]

\[
W_2(\theta', \theta; \Lambda) = Z_2(\Lambda) + r(n_2(\theta', \theta)) + s(n_1(\theta)).
\]

The payments \( Z_1 \) and \( Z_2 \) establish a base level of compensation in each period. They are determined by the utility constraint, \( \Lambda = U(Z_1) + \gamma U(Z_2) \), and \( \beta/\gamma = U'(Z_2)/U'(Z_1) \). These payments constitute a fixed cost for the firm as they are determined independently of the \( n_1 \) and \( n_2 \) policies. When \( \gamma = \beta \) we have \( Z_1 = Z_2 \), and when \( \Lambda > (1 + \gamma)U(0) \) the base levels are positive. The present value of the compensation base is \( c(\Lambda) = Z_1(\Lambda) + \beta Z_2(\Lambda) \), which is increasing and convex with \( c((1 + \gamma)U(0)) = 0 \) and \( c((1 + \gamma)\bar{U}) = \infty \). While \( c(\Lambda) \) is a fixed cost when a worker is under contract, it is a variable cost with respect to the number of workers hired in each period (the extensive margin).

The compensation scheme in (6-7) has three important features. First, the familiar insurance system between a risk-neutral employer and a risk-averse worker is clearly present. Since the single period utility function displays no income effect in leisure, keeping marginal utility constant reduces to keeping utility constant over shock realizations in each period. With compensation given by (6-7), the worker is indifferent to variations.
in hours and the firm is free to set \( n_1 \) and \( n_2 \) as it desires. The employment terms in (6-7) thus emerge as a variable cost for the firm.

The compensation scheme exhibits a distinct seniority effect. Any work effort in early life will shift up the \( W_e \) schedule by the amount \( s(n_1) \). While this seniority effect becomes a fixed cost in the later period, it is a variable cost for the \( n_1 \) choice in the initial period.

In addition to these seniority and insurance features, there is a simple Fisherian life-cycle interpretation that applies to the time pattern of compensation. To see this, consider the hypothetical savings decision of a worker with wealth \( c(A) \) and access to a capital market at the interest rate \( r = \beta^{-1} - 1 \). Clearly, the optimal savings decision involves equating the MRS over time with the interest factor while satisfying the wealth constraint.

Hall (1980) has pointed out that, when contracts function as binding bilateral commitments, the expected present value of compensation payments and not their time pattern is of direct concern to the firm. In the model at hand, the firm can structure compensation payments to provide a worker with three services: (i) seniority premiums, (ii) insurance and (iii) a life-cycle pattern of consumption. These are the efficiency gains-to-trade between the two parties with respect to compensation. Because it reduces the cost of attracting workers, the firm has an incentive to offer a cost-minimizing compensation scheme and internalize these services through the contract whenever a worker is unable to obtain the services elsewhere.

With regard to employment variations, the compensation scheme in (6-7) ensures that the firm faces a marginal cost of labor that respects the worker's underlying MRS (within and across periods). In particular, a marginal increase in \( n_1 \) will not only raise current labor costs but will also entail a seniority premium in the subsequent period. As the marginal cost of a senior worker is given by \( r(n_2) \), the seniority premium creates an asymmetry between junior and senior workers in any given period.

We can employ the compensation scheme, (6-7), to simplify the efficiency problem, (M), by substituting for the compensation payments in (M) to obtain

\[
\max_{n_1, n_2^{-1}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left[ \theta_t f(n_1 + n_2^{-1} g(n_1^{-1})) - r(n_1) - \beta s(n_1) - r(n_2^{-1}) \right] \right].
\]

(S')

Solving (S') for employment choices and then using (6-7) to find compensation payments yields a solution to (M). The two problems are equivalent because cost-minimizing compensation schemes are a dominant strategy for the firm, as they implement any desired employment policy at the minimum possible labor cost (a formal proof is available from the author). (S') is a dynamic program and the values for \( n_1 \) and \( n_2^{-1} \) are
no longer restricted by the utility constraint; these choices can vary freely over \( \theta \), and, if desired, \((\theta_{i-1}, \ldots, \theta_0)\).

IV. Seniority and Preferences

Next we examine the implications of nonseparability in preferences for seniority effects. This is done by solving the program \((M')\) with \(g(\cdot) = 1\) (no learning-by-doing). Note that the hours of junior and senior workers, \(n'_1\) and \(n'_2 - 1\), appear exclusively in the current-period profit flow of \((M')\). Thus, to find the efficient choices for junior and senior hours, we can solve the static pointwise program in the current shock \(\theta\) (dropping time superscripts)

\[
\max_{n_1, n_2} \theta f(n_1 + n_2) - r(n_1) - \beta s(n_1) - r(n_2),
\]

where \(n_1\) and \(n_2\) are viewed as scalars. As \(\theta\) varies the solutions for \(n_1\) and \(n_2\) trace out the contingencies in the efficient contract.

The first-order conditions for this standard program are

\[
r'(n_1) + \beta s'(n_1) = \theta f'(n_1 + n_2) = r'(n_2)
\]  

\((8)\)

\(n_1\) and \(n_2\) are positive if \(r'(0) = s'(0) = 0\). As (8) indicates, the marginal cost of hours for a junior worker exceeds the marginal cost for a senior worker by the amount of the discounted marginal seniority premium, \(\beta s'(n_1)\). Because of this, we have \(n_1(\theta) < n_2(\theta)\) for all \(\theta\) since each marginal cost is equated with marginal product. Thus, variations in \(\theta\) reveal a seniority pattern in the efficient employment policy as senior employees always work more than junior employees.  

Compensation policies then follow directly from the employment policies. With \(n_1(\theta_i)\) and \(n_2(\theta_i)\) from (8), substitution into (6–7) yields

\[
W_1(\theta_i; \Lambda) = Z_1(\Lambda) + r(n_1(\theta_i))
\]

\((9)\)

\[
W_2(\theta_i, \theta_{i-1}; \Lambda) = Z_2(\Lambda) + r(n_2(\theta_i)) + s(n_1(\theta_{i-1})).
\]

\((10)\)

5 The base levels for compensation, \(Z_1\) and \(Z_2\), act as a fixed cost and have no influence on determination of work effort. Consequently, the total cost to the firm of \(\beta(1 - \beta)^{-1} c(\Lambda)\) has been excluded from \((M')\) since it amounts to a lump-sum transfer which is spread out over the sequence of workers.

6 For an example, suppose that \(n_i [0, 1]\) for each worker, and that \(r(n) = Rn\) and \(s(n) = \alpha Rn\) for parameters \(R > 0\) and \(\alpha > 0\). Then \(n_1(\theta)\) and \(n_2(\theta)\) are as follows. For \(\theta > (1 + \alpha \beta)R / f'(2)\), we have \(n_1 = n_2 = 1\). As \(\theta\) falls, the senior worker remains at 1 but junior hours decline to zero according to \(\theta' f'(1 + n_1) = (1 + \alpha \beta)R\). Subsequent decline in \(\theta\) leaves the senior worker at 1 and the junior worker at 0, reflecting the discrete gap of \(\alpha \beta R\) between the marginal cost of junior and senior workers. For very low \(\theta\), the hours of the senior worker are reduced.

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Compensation varies with the shocks through current and past employment levels. For a junior worker there is no dynamic aspect to current compensation. For a senior worker the employment level from last period determines the seniority premium and raises the base for current compensation. Seniority, as measured by previous work effort, is sufficient to describe the historical dependence of compensation.

The seniority premium for early-life work is the basic force behind the firm’s adoption of a seniority rule for scheduling hours. With separability \( s(\cdot) = 0 \), there is no seniority premium and complete work sharing prevails \( n_1 = n_2 \). With nonseparability, the seniority rule for employment emerges as an efficient policy as it minimizes the creation of seniority premiums.

This system of seniority premiums provides an example of the type of increasing wage schedule examined by Lucas (1970). Suppose each worker lives \( T \) periods and the current-period utility function of a worker of age \( K \) takes the form \( U(c_t - \Sigma_{t=1}^{K} r_i(n_{t+1-i})) \), where \( r_i(\cdot) \) describes the effect of \( n_{t+1-i} \) on current utility. When the total labor pool contains \( N_{t-i} \) workers of vintage \( t-i \), then the cost to the firm of labor input \( L_t = \Sigma_{t=1}^{T} N_{t+1-j}n_{t-j}^{-i}/j \) becomes

\[
C(L_t) = \sum_{j=1}^{T} N_{t+1-j} \sum_{i=1}^{T+1-j} \beta^{i-1} r_i(n_{t-j}^{-i}).
\]

Cost minimization implies that \( C(L_t) \) is convex and increasing. For instance, when each \( r_i \) is linear, \( C(L_t) \) is piecewise-linear and convex in \( L_t \). While labor cost in terms of current compensation is a straight line (as in a spot market), labor cost after accounting for the implicit seniority premiums exhibits an increasing “wage” as labor input \( L_t \) rises.

As a general feature, the cost-minimizing compensation scheme allows efficient allocations to be implemented with a two-step procedure. First, by internalizing the consumption-leisure tradeoffs, the compensation scheme captures the gains from income smoothing (seniority premiums, insurance and life-cycle payments) while also ensuring that the firm faces a marginal cost of utilizing junior and senior workers that takes into account the effects of current work on future utility. Then, when the firm equates marginal product with these cost margins, \textit{ex-post} production efficiency is guaranteed.

V. Capital Markets and Seniority Premiums

The seniority premium is part of the internal capital market created by an efficient contract. In addition to its role in employment determination, the seniority premium also reflects an implicit savings and borrowing
transaction. This dual efficiency role can be made explicit by examining trade with a spot market for labor and a capital market for borrowing and lending.

In a spot market for labor, trade occurs after \( \theta \) is realized and each worker receives the same wage rate, \( W \). For the firm, workers are perfect substitutes in production and, with a common wage, the firm faces no cost differential across junior and senior workers. Letting \( n_i \) denote the labor supply choice of a worker of age \( i \) in a spot market, the market-clearing wage in the spot market satisfies \( \theta f'(n_1 + n_2) = W \).

The labor supply of a senior worker is given implicitly by the first-order condition \( W = r'(n_2), \) and \( n_2(W) \) is determined by \( W \), the current wage. A junior worker faces a dynamic labor supply decision under uncertainty. To introduce borrowing and lending, suppose that a worker has access to the familiar type of capital market where it is possible to borrow/lend at a given interest rate of \( \rho \) and no contingencies on future events enter the loan agreement; cf. e.g. Hall (1978). Let \( b \) denote borrowing while young (saving if \( b < 0 \)) so that \( (1 + \rho)b \) is to be repaid when old. A junior worker then solves

\[
\max_{n_1, b} U(\hat{w}n_1 + b - r(n_1)) + \gamma E[U(\hat{w}n_2(\hat{w}) - (1 + \rho)b - r(n_2(\hat{w})) - s(n_1))],
\]

where \( W \) is the current wage and the expectation is taken with respect to the distribution of \( \hat{w} \), the random wage for next period.

The first-order conditions for \( n_1 \) and \( b \) are

\[
W = r'(n_1) + (1 + \rho)^{-1} s'(n_1) \tag{11}
\]

\[
(1 + \rho)^{-1} = \frac{\gamma E[MU_2]}{MU_1}, \tag{12}
\]

where \( MU_i \) denotes current and future marginal utility. From (11), \( n_1 \) depends on the current wage and interest rate. Borrowing is then determined with (12) by equating the market discount factor with the expected MRS for consumption between periods. As indicated by the last term in (11), a junior worker evaluates the spot wage as if it contains a seniority premium and then uses the capital market to transfer this implicit premium into the next period.

In an efficient contract, the marginal cost of a senior worker is \( r'(n_2) \) and the marginal cost of a junior worker is \( r'(n_1) + \beta s'(n_1) \). Efficient employment levels are given by (8), where \( n_1 \) and \( n_2 \) are set to equalize marginal product with both cost margins. Significantly, if \( \beta = (1 + \rho)^{-1} \) then the market-clearing condition for the spot market determines employment according to the efficient pattern specified in (8). While the lack of insurance opportunities renders the overall allocation *ex-ante* inefficient, a

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spot market in labor with access to a capital market leads to \textit{ex-post} efficiency in employment determination.

Without the capital market, employment outcomes in a spot market will be inefficient (given $\theta$). In a spot market, the firm does not internalize any insurance or capital market services and the burden of intertemporal adjustment falls on the worker. Setting $b = 0$ in the $n_t$ decision, the analogue of (11) has a weight of $\gamma E\{MU_2\}/MU_1$ on the $s(n_t)$ disutility term, reflecting the lack of saving/borrowing options, and this leads to an \textit{ex-post} inefficient level of employment.

Similarly, contracts without seniority premiums would lead to employment inefficiencies as the firm would then face an incorrect marginal cost for junior workers. Just as access to capital markets improves the labor supply decision, the use of seniority premiums improves the utilization decision. Thus, the efficiency gains associated with seniority premiums reflect the value of capital market transactions that are part of the internal capital market created by the contract.

\section{Learning-By-Doing and Seniority}

When learning-by-doing effects are present, senior workers are more productive than junior ones. To focus on the consequences of learning-by-doing, set $s(\cdot) = 0$ so that preferences are time separable. We begin with the static features of the formulation and then proceed to the dynamic analysis.

With a current input of $(n_1, n_2)$ in labor effort from the junior and senior worker, output is $\theta f(n_1 + n_2g)$ where $g \geq 1$ summarizes the effect of past work on the effective labor input of the senior worker. Note that a senior worker is unambiguously more productive than his junior counterpart since $\theta f'g$ is always greater than $\theta f'$. Therefore, in a spot market where wage equals marginal product, the senior worker earns a higher wage. In addition, the comparative statics of labor demand in a spot market reveal that $(\partial n_1/\partial \theta) = (\partial n_2/\partial \theta) g$ and, since $g \geq 1$, labor demand for the junior worker is more sensitive to shock movements.

In contrast to these static seniority patterns, the investment dynamics of learning-by-doing in (M') lead to very different conclusions. Recall that

$$\max_{(n_t, n_t^{-1})_t} E \left\{ \sum_{\tau=0}^{\infty} \beta^\tau [\theta \tau f(n_1^\tau + n_2^\tau g(n_1^{\tau-1})) - r(n_1^\tau) - r(n_2^\tau)] \right\}. \quad (M')$$

Profits in $t$ depend on the current shock $\theta_n$, the experience level of the senior worker $n_1^{t-1}$ (the learning-by-doing effect), and the employment choices.
To formulate the value function for \(\langle M'\rangle\), let \((e, \theta)\) denote the experience level and current shock and then define \(v(e, \theta)\) as the value of optimal employment choices in state \((e, \theta)\). The value function satisfies

\[
v(e, \theta) = \max_{n_1, n_2} \theta f[n_1 + n_2 g(e)] - r(n_1) - r(n_2) + \beta E\{v(n_1, \theta')\},
\]

where \(n_1\) and \(n_2\) are chosen in response to \((e, \theta)\). Because the term \(n_2 g(e)\) makes the problem non-concave, establishing the existence and properties of interior solutions requires a detailed argument (see Appendix 2).

Seniority effects are determined by how the optimal choices for \(n_1\) and \(n_2\) vary with \(e\) and \(\theta\). We employ the first-order conditions for (13) to analyze \(n_1\) and \(n_2\) comparative statics (Appendix 2 contains the formal analysis):

\[
\theta f'[n_1 + n_2 g(e)] + \beta u'(n_1) = r'(n_1)
\]

\[
\theta f'[n_1 + n_2 g(e)] g(e) = r'(n_2),
\]

where \(u(\cdot) = E\{v(\cdot, \theta')\}\). For the senior worker, \(n_2\) is determined by equating marginal cost with current marginal product, which is augmented by \(g(e)\). For the junior worker, marginal cost is equated with current marginal product plus the expected return on the implicit investment in human capital.

To sort out the effects of dynamics from uncertainty, let \(\theta = 1\) and consider a steady state where \(n_1 = e\). In the special case of \(r(n) = \alpha n\) and \(g(e) = 1 + \alpha e\), we see that \(\beta n_2 = n_1\). Intuitively, the senior employee works more because the return is immediate whereas the return to learning-by-doing investment with the junior worker is always one period away. We also find \(n_2 > n_1\) for the general case (where learning-by-doing enters as \(g(n_2, e)\)).

Reintroducing uncertainty, the case of \(r(n) = \alpha n\) leads to a striking seniority pattern. Substituting (15) into (14) and rearranging results in

\[
\beta u'(n_1) = \left(1 - \frac{1}{g(e)}\right) R.
\]

Thus, the choice of \(n_1\) is independent of \(\theta\) and the hours of the junior worker do no vary with the shock. In contrast, the labor effort of the senior worker absorbs all of the variation in \(\theta\) as (15) reveals that \(n_2\) is adjusted in order to keep his marginal product of \(\theta f' g\) equal to his marginal cost of \(R\). Because of the multiplicative learning-by-doing effect, setting \(n_2\) according to (15) exactly offsets the influence of \(\theta\) on the marginal product of the junior worker. Thus the \(n_1\) choice reduces to a pure investment decision as, by (16), the discounted, expected return on the (implicit) learning-by-
doing investment is equated with an experience-adjusted measure of marginal cost.

In this case, efficiency entails a reversal of the typical seniority pattern as junior employees provide a fixed amount of labor input and, by the compensation scheme, receive a fixed payment in return. The complete insulation of junior workers generalizes the result of Carmichael (1983b) on efficient layoff patterns. Utilizing a discrete work/layoff context, he demonstrates that *ex-post* productive efficiency in the presence of learning-by-doing requires that senior workers be laid off ahead of junior ones. In general, (14–15) reveal that insulation is a useful benchmark. While the investment margin of \( \beta u'(n_t) \) is insensitive to current shocks (absent serial correlation), the cost margin varies with the current shock whenever the functional-form assumptions underlying (16) are relaxed.\(^7\) The hours of both workers are then increasing in \( \theta \) and no clear seniority pattern emerges.

### VII. Equilibrium Contracting

We now examine equilibrium contracting for the case of nonseparable preferences. In each market period, firms decide on the number of junior workers to hire, \( N_t \), and the contract offer, \( \theta_t \), subject to an expected utility level, \( \Lambda_{n} \), given by the market.\(^8\) Next, the shock, \( \theta_{n} \) is realized and the contracts are executed. The process then repeats in \( t + 1 \). Each generation consists of a continuum \([0, \bar{N}]\) of identical workers, and there is a finite number, \( J \), of identical firms. Let \( \mu = \bar{N}/J \). We begin with the contract choice and then proceed to the hiring choice.

It is optimal for firms to offer contracts with the cost-minimizing compensation scheme for \( \Lambda_{r} \). The hours decision is then unconstrained. Let \( L_t = N_t n'_1 + N_t n'_2^{-1} \) denote total labor input. Then, by (6–7), profits are

\[
\pi_t = \theta_t f(L_t) - c(\Lambda_t) N_t - [r(n'_1) + \beta s(n'_2)] N_t - r(n'_2^{-1}) N_t^{-1},
\]

and costs are the sum of hiring costs and utilization costs. Because \( n'_1 \) and \( n'_2^{-1} \) appear exclusively in the period \( t \) profit flow, for any given \( N_t \) and \( N_t^{-1} \), hours are set to satisfy the necessary and sufficient first-order

\[^{7}\text{These are multiplicative learning-by-doing and a constant reservation wage. For a payoff of } f(n_1 + g(n_2, e), \theta) - r(n_1) - r(n_2), \text{ the problem is concave and, via comparative statics, these restrictions turn out to be necessary for insulation.}\]

\[^{8}\text{It is a dominant strategy to offer the same contract to all new hires (by Jensen’s Inequality and convex indifference curves of workers). Thus work sharing always prevails within each generation. Also, as long as the market expected utility level is at least } (1 + \gamma) U(0), \text{ all workers will accept contracts.}\]
conditions of $r'(n_i^{t-1}) = \theta_i f'(L_i) = r'(n_i^t) + \beta s'(n_i^t)$. These conditions yield solutions for junior and senior hours, $n_i^t = n_i^t(\theta_i; N_i, N_{i-1})$ and $n_i^{t-1} = n_i^{t-1}(\theta_i; N_i, N_{i-1})$. We find that $n_i^* > n_i^t$ and that the seniority patterns in compensation and employment discussed in Section IV always apply.

The hiring choice, $N_i$, precedes the current shock, $\theta_i$. As a result, hiring is influenced by the expected value over $\theta_i$ of profits from utilizing workers. We can incorporate the above results on utilization by defining

$$\Pi(N_i, N_{i-1}) =$$

$$\int_c \{ \theta_i f(n_i^* N_i + n_i^* N_{i-1}) - [r(n_i^*) + \beta s(n_i^*)] N_i - r(n_i^*) N_{i-1} \} dP(\theta_i),$$

where $n_i^*$ and $n_i^*$ are evaluated at $(\theta_i; N_i, N_{i-1})$. Then, profits in $t$ are given by $\Pi(N_i, N_{i-1}) - c(\Lambda_i) N_i$ and $(N_{i-1}, \Lambda_i)$ emerge as state variables in the $N_i$ choice. A one-time cost of $c(\Lambda_i)$ for each new hire is associated with $\Lambda_i$. This utility level, however, does not influence utilization (the absence of income effects) and, consequently, $\Pi$ is independent of $\Lambda_i$. Past hiring, $N_{i-1}$, impacts directly on utilization and is the only direct link to previous periods.\(^9\)

We find an equilibrium in which all workers enter contracts $(N_i = \mu)$ and the utility level is constant over time $(\Lambda_i = \Lambda)$. Intuitively, the supply of new workers willing to enter contracts is inelastic while the demand for new workers depends on past hiring at firms. If the utility level adjusts to clear this market, then the market is essentially in the same position each time the hiring decision is made — each firm already has $\mu$ workers under contract, there are $\mu$ new workers per firm seeking to enter contracts, and expected profitability (over $\theta_i$) is stationary.

To formalize this argument, define a value function by

$$v(N_{i-1}, \Lambda) = \max_{N_i} \{ \Pi(N_i, N_{i-1}) - c(\Lambda) N_i + \beta v(N_i, \Lambda) \}.$$

From the comparative statics of $n_i^*$ and $n_i^*$, $\Pi$ is concave in $(N_i, N_{i-1})$. Then, $v(N, \Lambda)$ is concave and differentiable in $N$; cf. Benveniste and Scheinkman (1979). In Appendix 3 we show that there exists $\Lambda^*$ such that a firm with $N_{i-1} = \mu$ workers under contract will choose to hire $N_i = \mu$ new workers.

\(^9\)Recall that compensation for senior workers depends on the shock from the previous period via the seniority premium. This is accounted for in the $\Pi$ expression by the term $\beta s(n_i^*)$, reflecting the property that, with respect to the hours decision, seniority premiums act as a variable cost for junior workers and a fixed cost for senior workers. Thus, $\theta_{i-1}$ determines the realized seniority premium of generation $t-1$, but it has no impact on period $t$ decisions.

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In equilibrium, each firm hires $\mu$ new workers, maintaining a workforce of $2\mu$, and contracts specify a cost-minimizing compensation scheme for $\Lambda^*$. Together with compensation for current work, $r(n^*_1)$ and $r(n^*_2)$, the seniority premium for early work, $s(n^*_1)$, then leads the firm to set hours efficiently over the workforce in response to the subsequent $\theta$ shock. Equilibrium contracting thus results in an efficient allocation for firms and workers.

VIII. Conclusion

Seniority provisions emerge endogenously as features of contracts in a first-best allocation when there are dynamic links in preferences or technology. Nonseparable preferences provide an example in which senior workers earn more than their junior counterparts and layoff/hour reductions are concentrated on junior workers. In contrast, learning-by-doing provides an example where junior workers are insulated from transitory shocks.

Seniority provisions thus emerge as implications of hypotheses regarding the structure of preferences and technology (and efficient behavior) rather than as a consequence of bargaining power. This provides a benchmark for assessing seniority and experience effects in models where information and enforcement problems are critical in that the efficiency and incentive basis for these effects can be distinguished. In addition, the joint determination of seniority effects in compensation and employment, including the associated restrictions, may prove to be useful in empirical analysis.

References


Appendix 1. Cost-Minimizing Compensation Rules

Let $\delta$ be a contract with given employment policies $n_1(\theta)$ and $n_2(\theta', \theta)$. We must show that the compensation rules in (6–7) minimize $E[w_1(\theta) + \beta w_2(\theta', \theta)]$ given $n_1(\theta)$ and $n_2(\theta', \theta)$ and the expected utility constraint $V(\delta) \geq \Lambda$, where the expectation is over $\theta$ and $\theta'$. 

To begin, the pointwise utility of the worker in each period, is given by $\lambda_1(\theta) = U[\omega_1(\theta) - r(n_1(\theta))]$ and $\lambda_2(\theta', \theta) = U[\omega_2(\theta', \theta) - r(n_2(\theta', \theta)) + s(n_1(\theta))]$. Inverting $U(\cdot)$ and rearranging then yield

$$w_1(\theta) = U^{-1}[\lambda_1(\theta)] + r(n_1(\theta))$$

$$w_2(\theta', \theta) = U^{-1}[\lambda_2(\theta', \theta)] + r(n_2(\theta', \theta)) + s(n_1(\theta)).$$

Given $n_1$ and $n_2$, Jensen’s Inequality implies that $Ew_1$ and $Ew_2$ are minimized by keeping the utility levels $\lambda_1$ and $\lambda_2$ constant over $\theta$ and $\theta'$. Let $z_i = U^{-1}[\lambda_i]$ denote the consumption value corresponding to a constant $\lambda_i$, $i = 1, 2$. Then

$$E[w_1(\theta) + \beta w_2(\theta', \theta)] = z_1 + \beta z_2 + E[r(n_1(\theta))] + \beta s(n_1(\theta)) + r(n_2(\theta', \theta))].$$

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With pointwise utilities constant, the expected utility constraint reduces to 
\[ \Lambda = V(\delta) = U(z_1) + \gamma U(z_2). \]
The optimal choices, \( Z_1(\Lambda) \) and \( Z_2(\Lambda) \), are then uniquely 
determined by the utility constraint and \( \beta U'(Z_1) = \gamma U'(Z_2) \), and the compensation 
rules (6–7) follow directly.

**Appendix 2. The Value Function and Interior Solutions**

We can apply standard dynamic programming arguments, e.g. Harris (1987), to the 
mapping
\[
(Tv)(e) = \sup_{(n_1, n_2) \in \mathcal{R}_+^2} \left[ \theta f(n_1 + n_2 g(e)) - r(n_1) - r(n_2) + \beta v(n_1) \right] \text{d}P(\theta)
\]
defined on bounded \( v : [0, \infty) \to \mathcal{R}. \) The mapping \( T \) has a unique fixed point \( u \), which is 
bounded, continuous and increasing. The value function \( v(e, \theta) \) is
\[
v(e, \theta) = \max_{(n_1, n_2) \in \mathcal{R}_+^2} \{ \theta f(n_1 + n_2 g(e)) - r(n_1) - r(n_2) + \beta u(n_1) \}.
\]

Standard results cease to be useful at this point because the learning-by-doing effect 
can make the value function nonconcave. In the analysis that follows we focus on 
\( r(n) = Rn \) as insulation of junior workers is the result of interest (the extension to the 
general \( r(n) \) case is straightforward).

To begin, note that \( n_2 \) appears exclusively in the current-period objective 
so that an optimal \( n_2 \) choice always satisfies the Kuhn–Tucker condition of 
\[ \theta f'(n_1 + n_2 g(e)) g(e) \leq R, \] with equality if \( n_2 > 0. \) Thus, \( n_2 > 0 \) iff we have 
\[ \theta f'(n_1) g(e) > R. \] Now define two functions by \( d(e) = \{1 - |g(e)|^{-1} \} R \) and 
\[ \pi(e, \theta) = \max_{n \geq 0} \{ \theta f(n) - |g(e)|^{-1} Rn \}. \]
The optimal choice of \( n \) for \( \pi(e, \theta) \) is \( F(R[\theta g(e)]^{-1}) \) where \( F = [f']^{-1}. \) The current-
period payoff in (P) is then
\[
\theta f(n_1 + n_2 g(e)) - n_1 R - n_2 R = \begin{cases} 
\pi(e, \theta) - d(e) n_1 & n_1 \leq F \left( \frac{R}{\theta g(e)} \right) \\
\theta f(n_1) - Rn_1 & > 
\end{cases}
\]
The r.h.s. above is concave and differentiable in \( n_1. \) Note that 
\[ \pi(e, \theta) - d(e) n \geq \theta f(n) - Rn \] for all \( n \geq 0. \) This means that if an optimal \( n \) choice for the problem
\[
\max_{n \geq 0} [\pi(e, \theta) - d(e) n + \beta u(n)]
\]
lies in the interval \( [0, F(R[\theta g(e)])] \), then this \( n \) is an optimal choice for \( n_1 \) in the 
original (P) program. An optimal \( n_2 \) choice then follows from the Kuhn–Tucker 
condition.

Since \( \pi(e, \theta) \) does not vary with \( n \), the problem reduces to \( \max \beta u(n) - d(e) n. \) The 
optimal \( n \) choice depends on \( e \) but not on \( \theta. \) As \( u(\cdot) \) is bounded above, say by \( \tilde{u}, \) and
$d(e)$ is linear in $n$, we have $\beta u(n) - d(e) n \leq \beta \bar{u} - d(e) n$ for all $n \geq 0$. Define

$$\bar{n}(e) = \frac{\bar{u} - u(0)}{d(e)},$$

a convex decreasing function of $e$. If $n > \bar{n}(e)$, then $\beta u(n) - d(e) n \leq \beta \bar{u} - d(e) n < \beta \bar{u} - d(e) \bar{n}(e) = \beta u(0)$, and $n$ cannot be optimal since a choice of 0 yields a bigger payoff. Therefore, the interval $[0, \bar{n}(e)]$ contains all optimal choices for $n \geq 0$.

Let $e > 0$. Then there exists a critical $\theta$-value, denoted by $\phi(e)$, such that $\bar{n}(e) < F(R \mid \theta g(e))$ whenever $\theta > \phi(e)$. To see this, note that $\bar{n}(e)$ is continuous and decreasing with $\bar{n}(0) = \infty$. Also, $F(R \mid \theta g(e))$ is continuous and increasing in $e$. Since

$$\lim_{\theta \to \infty} F(R \mid \theta g(e)) = \infty,$$

$\phi(e)$ exists and is decreasing and continuous in $e$. Therefore, for any $(e, \theta)$ pair with $\theta > \phi(e)$, we have $\bar{n}(e) < F(R \mid \theta g(e))$ and all optimal $n$ choices for (P) are contained in $[0, \bar{n}(e)]$.

It remains to show that $n_1$ and $n_2$ are both positive for some $(e, \theta)$ pairs. Suppose, to the contrary, that $\forall (e, \theta)$ it is never the case that optimal choices satisfy $n_1(e, \theta) > 0$ and $n_2(e, \theta) > 0$.

To obtain the contradiction, take $(e, \theta)$ such that $\theta > \phi(e)$. From above, we know that any optimal $n_1$ choice satisfies $n_1(e, \theta) \leq \bar{n}(e) < F(R \mid \theta g(e))$ and, therefore, $n_2(e, \theta) > 0$ is implied by the Kuhn–Tucker condition. Hence, we have $n_1(e, \theta) = 0$ since, by assumption, one of the choices is always zero, and

$$0 \in \arg\max_{n \geq 0} [\beta u(n) - d(e) n].$$

Now consider a shock $\theta' < \phi(e)$. Clearly, the optimal policy has $n_1(e, \theta') = 0$ and $n_2(e, \theta') > 0$. Thus for any $e > 0$ we have $v(e, \theta) = \pi(e, \theta) + \beta u(0)$, and $u(e) = \int_0^\theta \pi(e, \theta) \ dP(\theta) + \beta u(0)$ then follows by integration. Note that $u(\cdot)$ is strictly increasing. Consider the value of $\beta u(n) - d(e) n$ once again. Let $n = \epsilon > 0$ be arbitrarily small. Since $d(e) \to 0$ as $\epsilon \to 0$, we can find a $\delta > 0$ such that $e < \delta$ implies that $d(e) < \beta [u(e) - u(0)] / \epsilon$, or equivalently, $\beta u(e) - d(e) e > \beta u(0)$. This contradicts the optimality of $n_1(e, \theta) = 0$.

**Appendix 3. Equilibrium Contracting**

The envelope theorem applies and it is straightforward to calculate the partial derivatives $\Pi_1$ and $n_1$. Define $\Lambda^*$ by

$$c(\Lambda^*) = \Pi_1(\mu, \mu) + \beta \int_0^\theta [r'(n_2^*) n_2^* - r(n_2^*)] \ dP(\theta),$$

where $n_2^* = n_2^*(\theta, \mu, \mu)$. Then, at $(N_{i-1}, \Lambda) = (\mu, \Lambda^*)$, the choice $N_i = \mu$ is optimal.

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