Quality, Upgrades, and Equilibrium in a Dynamic Monopoly Model

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November 5, 2010
What do we know about dynamic durable goods monopoly?

Most work is on a good of a single quality - lit on Coase Conjecture

Most goods do not fit single quality good framework.

Examples

- ‘Upgrade’ Goods
  - Software:
    - Operating Systems (Microsoft), Applications (Scientific Word, Adobe)
    - Commercial Airplane Manuf (Boeing, Airbus)
    - Defense Systems: Planes, Ships
    - Cellular Networks

- ‘Independent’ Goods
  - Computer, Television, Car
This paper studies upgrade goods

- Important market features
  - Infinite horizon environment
  - Firm - new quality increments to sell in the future
  - Firm - can offer any bundles of quality increments
  - Buyer - private information
  - Consumers need previous quality increments for next increment to be valuable (upgrade)
Key questions:

- What determines the equilibrium division of surplus?
- Will the market be efficient?

Answers hinge on buyer credible threat

- Rests on the ability of seller to tempt a buyer to buy (jump ahead of market) when others do not
Preview of Main Result:

- Market with homogeneous buyers- cleanest environment to understand pricing
- For efficient equilibrium: Any division of surplus between the one period flow value of a good and its PDV is an equilibrium for ANY discount factor
- For high enough discount factors, inefficient equilibrium exist, and buyers always get positive surplus and seller more than flow value
- Key: Growth in surplus + buyer implicit coordination leads to possible loss in market power. It gives buyers credible threat.
- Examples: Microsoft ME and Vista
Policy Implications:

- US-Microsoft anti-trust case of the late 1990s - Did Microsoft have monopoly power?
- Schmalensee (Microsoft)- fear of entry- limit pricing argument
- Fisher and Rubinfeld (Government) - network effects of consumers having the same operating systems
- Other economists - buy Microsoft’s application programs
- We offer a different interpretation
Outline for rest of talk:

- Model
- Benchmarks
- Efficient Equilibria
- Inefficient Equilibria
- Discussion
- Conclusion
Model

Infinite horizon $t = 1, 2, \ldots, \infty$

Monopolist:

- A new quality increment in each period
- No commitment to future pricing decisions
- Production costs are 0
- Can offer any feasible set of qualities in a period
- Maximizes discounted ($\delta$) profit
Buyers:

- Measure one of identical consumers in $[0, 1]$
- $v$ flow value of a unit of quality
- For a quality increment to be valuable in a period, buyer must possess all previous increments (upgrade structure)
- Common discount factor $\delta$
- Maximize expected discounted utilities:
  - value from quality - payments
Information and Timing

- All cost and valuations are known
- Any price or bundle is available to any consumer (no conditioning on individual behavior)
- All players know aggregate quality shares

Timing

- Each period - firm offers bundles and prices for the bundles
- Buyers then decide which, if any, bundles to purchase

Interpretation

- Value flow $\nu$ - marginal utility and quality increment
- Discount factor $\delta$ - time preference and innovation frequency
Example of Path

Period 1 - Seller offers Unit 1 for \( p_1 \) purchased

\[ \Rightarrow \text{flows of } p_1 \text{ for seller and } v - p_1 \text{ for each buyer} \]

Period 2 - Units \{1, 2\} feasible, seller makes no offer

\[ \Rightarrow \text{flows of 0 for seller and } v \text{ for each buyer} \]

Units \{1, 2, 3\} feasible, seller offers bundle \{2, 3\} for \( p_3 \), buyers purchase

\[ \Rightarrow \text{flows of } p_3 \text{ for seller and } 3v - p_3 \text{ for each buyer} \]

Continue on to later periods

Seller payoff of \( p_1 + \delta^2 p_3 + \ldots \)

Buyer payoff of \( (v - p_1) + \delta v + \delta^2 (3v - p_3) + \ldots \)
Efficiency and Surplus

For an efficient equilibrium, buyers acquire each unit when first available.

**Efficiency:** Buyers acquire new unit of quality in each period.

**PDV of flows on efficient path:**

\[\nu + \delta^2 \nu + \delta^4 \nu + \ldots\]

\[= \nu (1 + \delta + \delta^2 + \ldots) \quad \text{unit 1}\]

\[+ \delta \nu (1 + \delta + \delta^2 + \ldots) \quad \text{unit 2}\]

\[+ \ldots = \frac{\nu}{1-\delta} (1 + \delta + \delta^2 + \ldots) = \frac{\nu}{(1-\delta)^2} = S_1\]
Markov Perfect Equilibria

- Stationary
- Simple cyclical structure
- Flexible enough to generate entire subgame perfect payoff range for both efficient and inefficient eq.
- State \((t, q)\), \(t\) is maximal feasible quality, \(q\) highest quality held at start of period
- Players condition strategies on \((t - q)\) "quality gap"
- Implications: Past prices and path of qualities do not matter to players’ strategies
Benchmarks

Efficient Allocation and Buyer Extraction

1. Finite Horizon $T > 1$.
   - Does not depend on number of buyers, stationarity, upgrade/independent units

2. Infinite Horizon, Single Buyer, Quality Growth
   - No buyer coordination issue

3. Infinite Horizon, Continuum of Buyers, No Growth
   - Special case of FLT 85
Final Period $T$: state of the form $(T, q_{T-1})$
Upgrade from $q_{T-1}$ to $T$ at extraction price

Period $T-1$: state of the form $(T-1, q_{T-2})$
Upgrade from $q_{T-2}$ to $T-1$ at extraction price
Buyers expect no future surplus increment
Path to final period state $(T, T-1)$

- Work backwards to period 1
- Efficient path and surplus extraction
No delay in equilibrium - "speed up" argument

Example - No sale in period 1, then sell 2 units in period 2 for $p$ and cycle

$$\pi_1 = \delta p + \delta^2 \pi_1$$

$$u_1 = \delta (2v - p) + \delta^2 \left[ \frac{2v}{1 - \delta} + u_1 \right]$$

Seller can offer unit 1 in period 1 for $\hat{p}$, buyer accepts and seller increases payoff if

$$v - \hat{p} + \frac{\delta v}{1 - \delta} + \delta u_1 > u_1 \text{ and } \hat{p} + \delta \pi_1 > \pi_1 \iff$$

$$\frac{v}{(1 - \delta)^2} > u_1 + \pi_1$$
⇒ Efficient path - sell current unit
⇒ Continuation outcome in any \((t,0)\) is sale of \(t\) units
⇒ Buyer extracted at price 
\[ p_1 = \frac{\nu}{1-\delta} \]

If infinite horizon, continuum, no growth, special case of FLT '85

Benchmark message - Efficiency and Extraction if either finite horizon, finite buyers, and no growth
Basic Results: Flow Dominance

? - If seller offers $t$ units at price $p < vt$ in state $(t,0)$

All buyers must accept- current surplus, future options

$\Rightarrow$ Lower bounds on seller payoff

$$\pi_1 \geq vt + \delta \pi_1 = vt + \frac{\delta v}{1 - \delta}$$

$$\pi_t \geq vt + \delta \pi_1 = vt + \frac{\delta v}{1 - \delta}$$

Flow dominance

$\Rightarrow 0 \leq u_1 \leq \delta S_1$

extraction $\nearrow$  \hspace{1cm} \searrow$ static one period monopoly
Basic Results: Cycles

*t*-cycle equilibrium- a sale occurs every *t* periods, and *t* units are sold in each sale period

**Proposition**

_Every equilibrium is a *t*—cycle equilibrium._

Why? pure ‘speed-up’, but must have *τ* < *t*

No implication of a sale in every period

Argument breaks down when *τ* = *t* > 1

- Feasibility
- Not necessarily optimal for an individual buyer to accept the offer
Speed-Up Intuition:

Suppose $t$ is date of first sale but only $\tau < t$ units

In $t-1$, seller offers $\tau$ for price $\hat{p} = \nu \tau + \delta p - \epsilon$

Individual buyer accepts even if others reject:

$$(\nu \tau - \hat{p}) + \delta \nu \tau + \delta^2 u(t+1, \tau) > 0 + \delta (\nu \tau - p) + \delta^2 u(t+1, \tau)$$

Seller offer successfully speeds up path

$$\hat{p} + \delta \pi_{t-\tau} = \hat{p} + \delta^2 \pi_{t-\tau+1} > 0 + \delta p + \delta^2 \pi_{t-\tau+1}$$
Efficient Equilibria

New quality units sold immediately at price $p_1$

equilibrium path of $(1, 0) \rightarrow (2, 1) \rightarrow (3, 2) \rightarrow \ldots$

Need to specify continuation payoffs

propose ‘cash-in’ support off-equilibrium-path

in $(\tau, 0)$ have sale of $\tau$ units at price $p_\tau$

It must be optimal for the seller to offer $\tau$ at $p_\tau$

versus delay or partial cash-in

Buyer strategies follow simple cut-off rule:

accept $\sigma$ units in state $(\tau, 0)$ iff $p \leq p(\sigma, \tau)$

Must hold for all $\tau \geq 2$ and cut-off rules $p(\sigma, \tau)$ for all $\sigma \leq \tau$ (and $\tau = 1$)
Example 1: Efficient Equ. (constant utility support)

- Equ. path - sell new unit at price $p_1 \Rightarrow$ payoffs

$$\pi_1 = p_1 \left(1 + \delta + \delta^2 + \ldots\right) = \frac{p_1}{1 - \delta}$$

$$u_1 = (v - p_1) + \delta (2v - p_1) + \delta^2 (3v - p_1) + \ldots$$

$$= \frac{1}{1 - \delta} \left(\frac{v}{1 - \delta} - p_1\right)$$

- Support - prices rise by $\frac{v}{1 - \delta} \Rightarrow u \equiv u_1 = u_2 = \ldots$
  If delay, then seller is residual claimant of growth

- Delay incentive $\Rightarrow u(1 - \delta) < v$
  $v$ is loss from delay (surplus) and $(1 - \delta) u$ is gain
? Why buyers refuse $\hat{p} = p_1 + \epsilon$, if all others reject $\Rightarrow \delta u_2$

If individual buyer accepts: $\nu - \hat{p}$ today plus option

$$\max \left\{ \frac{\nu}{1 - \delta}, u \right\} = \frac{\nu}{1 - \delta}$$

So must have

$$\delta u > \left[ \frac{\nu}{1 - \delta} - p_1 \right] = (1 - \delta)u \Leftrightarrow \delta > \frac{1}{2}$$

Interpret - coordinate on share of first unit surplus

$\Rightarrow$ ? why not coord on Units 2, 3, ...
Summary: Example 1

- Constant utility support $\Rightarrow$ seller residual claimant
- Delay incentive limits buyer payoff
  - Coord on rejecting high prices for positive payoff
  - Extraction is special case where $u = 0$
  - Positive buyer payoffs in equilibrium
- Potential for coord on future surplus
Efficient Equ - Analysis

- Buyer Strategies (symmetric): cut-offs $p(\sigma, \tau)$

If seller offers $\sigma$ units upgrade for $p$ in state $(\tau, 0)$

When all other buyers accept, individual payoff

$$v\sigma - p(\sigma, \tau) + \delta \left[ \frac{v\sigma}{1 - \delta} + u_{\tau+1-\sigma} \right]$$

(accept) 0 (reject)

Thus, equ. $\Rightarrow$ upper bound

$$\frac{v\sigma}{1 - \delta} + \delta u_{\tau+1-\sigma} \geq p(\sigma, \tau)$$

Fall behind path $\Rightarrow$ zero (inessential)
When all other buyers reject, individual buyer payoff

\[ \delta u_{\tau+1} \text{ (reject) } v\sigma - p + \delta \max \left\{ \frac{v\sigma}{1 - \delta}, u_{\tau+1} \right\} \text{ (accept)}. \]

Define ‘Net option value’

\[ g(\sigma, u) \equiv v\sigma + \delta \max \left\{ \frac{v\sigma}{1 - \delta}, u \right\} - \delta u \]

Then, cut-off rules require

\[ g(\sigma, u_{\tau+1}) \leq p(\sigma, \tau) \leq \frac{v\sigma}{1 - \delta} + \delta u_{\tau+1 - \sigma} \]

- "Price Wedge" Always exist
- Buyer Implicit Coordination

\[ u_{\tau+1} > \frac{v\sigma}{1 - \delta} \Rightarrow g = v\sigma \]

pushes net option value down to flow surplus
Given buyer responses, seller must find it optimal to offer $\tau$ units at price $p_\tau$ in state $(\tau, 0)$.

Seller deviations:
- delay via $\sigma = 0$,
- partial cash-ins via $1 \leq \sigma \leq \tau - 1$
- offer $\tau$ upgrade at different price from $p_\tau$.

Seller optimality requires

$$
\pi_\tau \geq p(\sigma, \tau) + \delta \pi_{\tau + 1 - \sigma}
$$

for $\sigma = 0, 1, \ldots, \tau$
Support Condition - combine buyer and seller

- Recall $S_\tau$ is total available surplus.

$$S_\tau = \frac{v\tau}{1 - \delta} + \delta S_1 \quad S_1 = \frac{v}{(1 - \delta)^2}$$

- Cash-in support (efficient path) has

$$S_\tau = \pi_\tau + u_\tau$$

- Support conditions that need to be satisfied

$$S_\tau - \delta S_{\tau+1-\sigma} \geq u_\tau - \delta u_{\tau+1-\sigma} + g(\sigma, u_{\tau+1})$$

for all $\sigma \leq \tau$ and all $\tau \geq 1$
Claim:

- We can support maximal range of equ. payoffs

\[ u_1 \in [0, \delta S_1] \]

(recall flow dominance bound for seller).

- Introduce \( T \)-Stage Support

\[ u_\tau = v\tau + \delta u_{\tau+1} \quad \text{for} \quad (u_1, \ldots, u_T) \]

\[ u_\tau = u_T \quad \text{for larger} \quad \tau \]

- Keeps seller indifferent delay versus cash-in

? Why - flow surplus to buyers

- Must truncate eventually: if not, support \( \sigma = \tau \) is

\[ S_\tau - \delta S_1 \geq u_\tau - \delta u_1 + g(\tau, u_{\tau+1}) \Rightarrow \]

\[ \delta u_1 \geq v\tau \]

at large \( \tau \) and this will fail.

? Why - flow dominance offer
Key Properties for $T$-Stage Support

- At stage $T$, seller strictly prefers to make cash-in offer
  
  \[ u_T < \frac{\nu T}{1 - \delta} \]

- At stages $\tau < T$, buyers willing to pay no more than $\nu \tau$ (flow value) if others reject
  
  \[ u_\tau > \frac{\nu \tau}{1 - \delta} \]

- Need to verify support conditions
- Need to find length $T$ relative to $u_1$ and $\delta$
Choosing Length $T$

- Pick utility level between 0 and $\delta S_1$.
- If $u_1 \leq (1 - \delta)S_1$, then $u_\tau = u_1$ for $\tau > 1$.  \[[T = 1]\]
- If $u_1 \in [(1 - \delta)S_1, \delta S_1]$, set $u_2$ via $u_1 = \nu + \delta u_2$.
- If $\delta S_1 \leq (1 - \delta^2)S_1$, then $u_\tau = u_2$ for $\tau > 2$. \[[T = 2]\]
- If not, set $u_3$ via $u_2 = 2\nu + \delta u_3$.
- Keep following logic until reach $T$ where

$$
(1 - \delta^{T-1})S_1 \leq \delta S_1 \leq (1 - \delta^T)S_1
$$
Verifying equ. support conditions:

- If support holds at $T$ it holds at all $\tau > T$
  utility is in constant range; now finite #
- ‘Cash-in’ incentive sufficient for $\tau \leq T$
- Then choose $T$ to satisfy ‘Cash-in’

**Proposition**

*Every buyer payoff $u_1 \in [0, \delta S_1]$ can be supported in an efficient equilibrium if $\delta \in [1/2, 1]$.***
Corollary

If \( \delta \in [1/2, 1] \), then \( \pi_1 \in [S_1(1 - \delta), S_1] \)

- Interpret - as if seller only has static monopoly power
  - each unit sold for price of \( v \)
  - no ability to capture future value

Corollary

The buyer share of the surplus, \( \frac{u_1}{S_1} \), is between 0 and \( \delta \).

Discussion: payoffs relative to total surplus.

- Case: \( \delta < 1/2 \)

Can support any buyer payoff from 0 to \( \delta S_1 \)

? Why special - when

\[ 1 > 2\delta \]

\( \Rightarrow 1 \) unit now dominates 2 units tomorrow.
Delay and inefficient equilibria

- Every equilibrium is a $t$-cycle equilibrium
- No sales in periods 1 through $t - 1$, then sell $t$ units at $p_t$ in period $t$
- Approach conditions- Minimum $\delta$—preventing early cash-in
- No delay equ if $\delta < 1/2$
- Buyers must receive positive utility⇒
  if observe delay and bundling, then buyers are not extracted
- Sellers must get more than flow payoff
- Thus, payoff bounds are compressed relative to efficient eq.
Bundling in Practice

- Sometimes the good is just added onto existing version (upgrade)- adding existing programs to a machine
- Other times, the good is completely replaced - new software version
- There is not necessarily a technological reason - Microsoft anti-trust case and the browser, PDF for Word or Sci Word

- Generation version with price contingency same as an upgrade version - same set of equilibria
- Results robust to network, lack of compatibility, and adoption costs harder to get consumer to jump ahead
- Unbreakable versions - hurts market power Fishman & Rob
- Independent Goods
Future Research

- Price Discrimination
- Innovation
  - Rate of Innovation
  - Scope of IP