Little Patents and Big Secrets: Managing Intellectual Property *

James J. Anton† and Dennis A. Yao‡

August 5, 2000

We examine how an innovator should manage its intellectual property when confronted with limited intellectual property rights and possible imitation. Exploitation of an innovation commonly requires some disclosure of enabling knowledge to selected firms or to the public (e.g. to obtain a patent or induce complementary investment). When property rights offer only limited protection, the value of the disclosure is offset by the created threat of imitation. Our model incorporates three features critical to understanding this decision: innovation creates asymmetric information, innovation often has only limited legal protection, and disclosure facilitates imitation by transferring enabling knowledge. Imitation depends in part on inferences the imitator makes about the innovator’s advance. We find an equilibrium in which small inventions are fully disclosed, medium inventions are protected by both legal property rights and secrecy, and large inventions are protected primarily through secrecy when property rights are weak. Our discussion is framed in terms of a firm’s decision of what to patent, what to disclose, and what to keep secret, but the model is adaptable to other intellectual property settings.

*The authors thank Tracy Lewis, Marvin Lieberman, Rob Merges, Scott Stern, and seminar participants at Berkeley, Florida, FTC, NBER, Penn, and UCLA for helpful comments and the Fuqua Business Associates Fund for research support.

†Associate Professor, Fuqua School of Business, Duke University, Durham, NC 27708-0120 and Visiting Scholar, Economics Department, University of North Carolina, Chapel Hill; james.anton@duke.edu

‡Associate Professor, Wharton School, University of Pennsylvania, Philadelphia, PA 19104-6372 and Visiting Scholar, Haas School, University of California, Berkeley; yao@wharton.upenn.edu
1. Introduction

We examine how an innovator should manage its intellectual property (IP) when confronted with limited intellectual property rights and possible imitation. Exploitation of an innovation commonly requires some disclosure of enabling knowledge to selected firms or to the public (e.g. to obtain a patent, obtain an alliance partner, or to induce investment in complementary assets). When property rights offer only limited protection, however, the value of the disclosure is offset by the created threat of imitation.

Our analysis can be applied to IP settings involving patents, copyrights, contractual property rights, trade secrets and confidentiality agreements, as well as to the polar case of no property rights. We focus on the decision of a firm concerning how much of an innovation should be disclosed (with and without legal protection) and how much should be kept secret. A major business concern is that disclosure through patenting or voluntary disclosure will provide competitors with usable information. In a survey of U.S. firms in twelve industries Mansfield 1986 [14] found that a substantial fraction of patentable inventions were not patented. This finding plus Cohen, Nelson, and Walsh’s 2000 [4] finding from a general survey of U.S. manufacturing firms that secrecy was viewed as more important than patenting for appropriability indicate the importance of understanding the secrecy-patent decision. Along these lines we explore, among other issues, why firms (holding legal specificities constant but allowing changes in the strength of property rights) are likely to employ secrecy more heavily as the significance of the invention increases.

Three features of the economic environment of innovation are critical for under-

---

1 Jolly 1997 [11], for example, argues that an important aspect of a business strategy for mobilizing interest in a technology is “formulating a communication strategy that balances interest creation with secrecy..” (p.83)

2 There are, of course, numerous anecdotes supporting the selection of secrecy over patenting. Jackson 1998 [10], p.41, for example, discusses Intel’s decision to keep a portion of a patentable “reflow” manufacturing process secret because they were concerned that a full patent would provide competitors with too much information and also provides an example of a manufacturing trade secret (“walking out”) that allowed Intel to maintain a complete monopoly over EPROMs for nearly two years because of Intel’s superior yield from its wafers. Milgrim 1974 [16] gives the example that a French company kept a valuable process for producing cellophane secret. Du Pont spent millions of dollars in an unsuccessful attempt to duplicate this process. Many firms including Kodak involve their business managers in the decision of what parts of the underlying technology should be patented and what parts kept secret. (“How Kodak, Fearing Theft of Trade Secrets, Mounted Its Own Sting” Wall Street Journal 11/25/96 p.1).
standing the management of IP. First, incomplete information about the extent of innovation is often fundamental. Second, it is common for inventions to have available only limited intellectual property protection. Third, enabling knowledge revealed through disclosures makes imitation feasible.

Asymmetric information gains force when property rights are limited. If an innovation were fully protected, full disclosure would facilitate appropriation of the benefits associated with signaling the extent of innovation and would entail no risk of imitation or other unauthorized use. With limited protection, however, disclosure risks imitation and incomplete information remains a primary concern.

Imitation depends on each of these factors. Disclosure determines what imitation is possible and limited property right protection frequently makes unauthorized imitation economically attractive despite the possibility of legal damages. Asymmetric information also figures prominently in the imitation decision. For instance, if an innovation were known a priori to be minor, a competitor might prefer to remain with existing technology rather than imitate and risk legal damages. If an innovation were known to be major, it could render the status quo technology noncompetitive and trigger imitation or perhaps exit. Under incomplete information an imitation decision must necessarily be based on an inference about the extent of innovation and an assessment of the downstream competitive position relative to the innovator. The extent of disclosure and protection chosen by the innovator provide an important clue. Thus, signaling concerns are important for managing IP vis-a-vis competitors. For example, Ford Motor Company’s disclosure of substantial amounts of unprotected technical knowledge about its revolutionary moving assembly line system may have been motivated in part for its value as a signal to competitors of its dominant low-cost manufacturing position.

Inferences are also pivotal when the innovator discloses enabling knowledge to obtain sales (e.g. publications by management consulting firms),

---

3. There are a number of benefits to making signaling disclosures beyond that of obtaining patents. Such disclosures can induce weaker competitive responses or discourage rival innovation (e.g. some biotechnology firms avoid or abandon projects for which others are perceived to be ahead). The disclosures are also valuable for inducing third-party complementary investment or securing financing.

4. Mansfield, Schwartz, and Wagner 1981[15] found, for example, imitation within four years of 60% of the patented successful innovations in their sample of 48 innovations.

5. See, e.g., Nevis and Hill 1954 [18] for a discussion of the Ford disclosures. Publication is a common form of disclosure. Based on extensive interviews with European and Japanese R&D managers, Hicks 1995 [7] argues that publication is used by firms in part to gain “a reputation for possessing useful tacit knowledge” and “signals the area ...and the quality of that work.” (p.420)
secure third-party complementary investments or financing (e.g., business plans revealed to venture capitalists, academic research proposals) or in strategic alliance negotiations (e.g., licensing, contract disputes over rights to IP) because the decisions of these third parties depend ultimately on an assessment of the relative advantages of the innovator over its competition. Our basic model provides a foundation for understanding such interactions.

Thus, the amount of the innovator’s disclosure is critical to the imitation decision. In economic terms, one can view the innovator’s disclosure choice as a decision of how to “manage” market competition between direct competitors when imitation is a central concern. The relative positions of the competitors are managed by moderating the amount of disclosed knowledge, and competition occurs in the shadow of an ongoing legal dispute over IP rights (e.g., Polaroid and Kodak in instant photography).

The strength of intellectual property rights is critical to our story and merits further comment. If patents, copyrights, and trade secret law could fully protect all economically important inventions, circumvention and possible infringement would be of secondary importance to the management of intellectual property. But clearly this is not the case. With respect to patents, for example, infringement suits are common and surveys of firms on the question of appropriability of IP suggest that firms in a majority of industries do not see patents as providing strong appropriability (Levin, Klevorick, Nelson, and Winter 1987 [13] and Cohen, Nelson, and Walsh 2000 [4]). Some reasons for these results include the ease with which some patents can be circumvented, the possibility that a patent will be invalidated if challenged, and the sometimes modest damages awarded in successful infringement suits. Especially for process innovations, lack of appropriability may also simply result from the difficulty of detecting infringement.

---

6 A common form of voluntary disclosure is a conference presentation or other public disclosure made subsequent to a patent filing but before the patent is issued and becomes public. (e.g., “Fina filed the patent application...and shortly afterword Dr. Ewen gave a speech about it [a plastic] at a symposium organized by Exxon [Fina’s competitor].” “Battle Over Patents Pits Two Oil Concerns Against One Scientist,” Wall Street Journal 3/1/96, p.1; “[O]n December 14 he mailed in his patent application. The next day, a joint article by him and Dr. Srinivasan appeared in the American Journal of Ophthalmology.” “Patent Challenges Face Leader in Laser Surgery for Nearsightedness,” Wall Street Journal, 5/26/99).

7 Other prominent examples include Intel and AMD over microprocessor technology, Kimberly-Clark and Procter & Gamble in disposable diaper technology, Visix and Nidek in vision-correcting laser surgery, and Fonar, Johnson & Johnson and GE in MRI technology.

8 An example of the difficulty of detection is provided in Northern Petrochemicals Co. v. Tomlinson, 484 F.2d 1057 (7th Cir.) 1973 in which a trade secret theft was discovered only

4
We model these trade-offs in a duopoly competition setting where an innovating firm (the “innovator”) has private information regarding an invention, giving it (possibly) lower costs of production than the other firm (the “follower”). The degree of innovation is a continuously valued variable, ranging from no advance over the status-quo to major innovations that, under complete information, could induce the follower to exit. The innovator makes two choices: whether to patent and how much knowledge to disclose. Having observed the protection and disclosure choices of the innovator, and using these as a basis for assessing the inventor’s total knowledge, the follower decides whether to use the inventor’s disclosed knowledge and risk infringement or to stay with the old technology. Thus, our analysis conforms best to a process innovation setting. The subsequent market competition is modeled as Cournot. The Cournot structure, while special, captures the basic tension between the cost of enabling knowledge transfer and the benefits of signaling toughness to a competitor, which is a central concern in a variety of relevant competitive settings. Market competition takes place under the shadow of infringement, and legal damages, if any, are assessed after market competition. We assume that damages take the form of payments linear in the imitating firm’s sales.9

Preview of Results Our analysis leads to four main results. For small innovations we find an exclusion effect under which small innovations are patented and fully disclosed and no imitation occurs. The second result involves a licensing effect where larger innovations are protected both through patents and secrecy and after the victim acquired the perpetrator. See also Milgrim 1974 [16].

Although the advent of the Federal Circuit (and its changes in policy) has resulted in increased protection for IP in the United States in recent years, effective legal property right protection in a wide variety of settings and industries should arguably still be characterized as limited. One problem is that the patent office issues patents somewhat leniently, relying, in part, on subsequent litigation to make the ultimate validity determination on economically meaningful patents. The weaknesses in the protection regime are multiplied when one considers IP protection internationally. Copyright protection is limited by its inherent narrowness while proving a trade secret violation is quite difficult.

9It is common for litigation involving infringement to last for many years and sometimes to be resolved after the effective economic life of the invention has ended. A common remedy involves assessing a royalty on an infringer’s past sales that represents a “reasonable royalty” (sometimes augmented with a punitive component) and an injunction of the use into the future. By statute, the reasonable royalty constitutes the floor with respect to the damage award. (35 U.S.C. § 284) In recent years, the US courts have been more willing to grant preliminary injunctions but such injunctions are difficult to obtain because of the burden of proof imposed on the patentee. See, e.g., Rhodes 1997 [20]. The economic implications of disclosure under a preliminary injunction are analyzed in Anton and Yao 1999 [2].
imitation occurs, leading to an implicit licensing relationship between competitors. Third, we find a waiver effect where, for sufficiently weak property rights, very large innovations are not protected via patent but instead through a strong reliance on secrecy. Finally, we have a no-exit result under which innovations that would be drastic (force exit) if the follower knew the innovator’s cost (i.e. under complete information) become incremental when the follower does not know the innovator’s cost (i.e. under incomplete information). The common thread in these results is that weak property rights imply disclosure incentives that are relatively stronger for smaller innovations and, as a result, larger innovations are protected more through secrecy as a response to the problem of imitation by a competitor.

Property rights provide protection to the extent that they discourage imitation and create an expectation of a damage payment for the use of protected knowledge. For a small innovation relative to the status quo technology, even a relatively weak patent will discourage imitation because the gain to using the new knowledge is insufficient to justify a possible infringement payment. This creates a strong incentive to patent and disclose fully since the threat of imitation can be discounted. As a result, we find that for a range of small innovations, which expands as property rights become stronger, that weak patents are economically equivalent to strong patents and, in equilibrium, the patent will fully disclose the innovator’s enabling knowledge because of the downstream competitive benefit to appearing to have lower costs.

When the disparity between costs associated with old and new technology is greater, a sufficiently large (protected) disclosure will trigger imitation. A switch to an imitation regime means that expected damages no longer perform the “exclusionary” role. Instead, the competitors find themselves in what amounts to a licensing relationship governed by the property rights regime. In this interpretation of imitation and infringement, the innovator chooses the technology transfer (via the disclosure) and the license fee is set exogenously by the court (via ex post expected per unit damages). Imitation can then be viewed as an exercise of the implicit option to license.

Because the exclusion effect leads high-cost types in the nonimitation range to disclose fully, greater absolute-sized disclosures are required in equilibrium to achieve separation by innovators with more substantial advances. When imitation occurs, the innovator has an incentive for partial disclosure to preserve an advantage over the rival. The disclosure-secrecy-signaling trade-off consists of the cost of the rival’s use of the disclosed knowledge and the benefits of expected damage payments (the licensing effect) and an anticipated weaker competitive
response when low costs are signaled. In equilibrium, the enabling knowledge behind a large innovation is not completely disclosed and, as the innovation becomes larger, the gap between disclosed and actual knowledge increases: partial disclosure and imitation go hand-in-hand.

Weak patent protection exacerbates the imitation problem. As the size of the innovation increases, the relative benefit from expected damages decreases because the larger gap between disclosed and actual knowledge implies an increasingly inefficient production allocation for the innovator and the imitator. Together with the substantial amount of disclosure required to separate in equilibrium, this raises the question of whether a different signal, if it exists, may be preferable and this leads to our third result.

The signal we identify is for the innovator to give up the benefits of the patent, say by eschewing the patent or through licensing the patent rights for a nominal lump sum fee. Such an action substantially reduces the amount of disclosure (knowledge transfer) that is needed to separate from higher types and causes the imitator to infer that the innovator has low costs. Because the knowledge disclosure is effectively unprotected, though, only big-innovation low-cost firms are willing to take this action. Thus, weak property rights and incomplete information regarding the innovation lead to an incentive cost for large innovations under which the value of protection is outweighed by a combination of secrecy and unprotected, but smaller, disclosures.

One prominent example that can be interpreted through the lens of disclosure to signal low costs involves the actions of the Ford Motor Company in 1913-1915 during their implementation of the arguably revolutionary moving assembly line process for mass production of automobiles. During this period Ford allowed a number of journalists to write extensively about its (unpatented) processes. Hounshell 1984 [9] in a historical study of American manufacturing technologies remarks that Ford “...educated the American technical community in the ways of mass production.” (p. 261) One reason Ford may have done this was to signal its competitors that it had extremely low costs and that a head-to-head competition with Ford would have been foolish.10

10One series of articles in Engineering Magazine resulted in a 440 page book on Ford methods of manufacturing which was published in 1915. Ford also disclosed much about its pre-moving assembly line mass production system. While the systems were described in detail, this was clearly only a partial disclosure of the knowledge it took to make the system work and adapt the system to various applications. Further, the efficiency of the system depended in part on economies of scale that Ford could exploit given its dominant sales position in the industry. Ford may have also signaled its low costs with aggressive pricing, its $5/day wage (double the
The first three results of the model (exclusion, licensing, and waiver effects) when taken together lead to an interesting implication for the traditional distinction between incremental and drastic innovations. When the range of potential innovation is large relative to the status quo technology, a setting with complete information will lead the follower to exit the market when the innovator has a sufficiently large success. Now consider a large success under incomplete information and weak property rights. Disclosure incentives lead to imitation which may make it feasible for the follower to remain in the market: here disclosure and protection choices signal a major innovation but simultaneously reduce the cost differential between the innovator and the follower. The distinction between drastic and non-drastic is endogenous in our analysis and, ceteris paribus, we find that, in equilibrium, the follower cannot be forced from the market given weak property rights. The difference in conclusions under complete versus incomplete information highlights the value to studying IP transactions and decisions in an incomplete information setting.

Our model implements protection via an ex post expected transfer of some damage payment back to the innovator. This feature is not specific to patenting, but is a general feature of property rights. Thus, the model allows an exploration of a wide variety of IP settings, including negotiations held under a confidentiality agreement between an innovator and a supplier that is threatening entry\textsuperscript{11}, departure of key employees who had access to trade secrets, and the sale of an invention to a buyer that is skeptical of the invention’s value but is unwilling to talk unless the seller waives confidentiality rights (e.g. in the toy industry and with venture capitalists). Further, the approach is helpful for developing intuition relevant to more complex IP settings such as (potentially) licensing an invention to a firm that does not know the value of the IP. This value depends on the resulting relative competitive position offered by the license, which, in turn, depends on how much is licensed and how much remains exclusive to the innovator. We discuss various applications in the conclusion.

**Literature Review** The decision of whether to patent has been explored directly in Horstmann, MacDonald, and Slivinski 1985 [8] and is an integral part of the analysis in the models of Scotchmer and Green 1990 [21] and Gallini 1992 [5]. The patent decision has two critical components that have not been fully

\textsuperscript{11}See, e.g., “Hardball Beans Citrix, but it Recovers,” USA Today June 11, 1997 for a story about negotiations between software-maker Citrix and Microsoft.
explored in this previous work. First, what is patented (and/or publicly disclosed) is a decision on the amount of enabling knowledge to transmit to one’s competitor and, second, the amount that is disclosed is a signal of the total knowledge the innovator possesses. Both of these aspects are important for understanding the imitation choice of the noninnovators.

Horstmann, MacDonald, and Slivinski (HMS) model an information signaling problem but not the strategic choice in the amount of knowledge transfer. The innovator chooses whether to patent and the follower chooses either to stay out, imitate (without infringing), or directly duplicate (only possible if no patent). The follower’s choice directly affects the innovator’s payoff. The innovator has private information about the competitor’s payoff for each action and, while this information has no direct effect on the innovator’s payoff, the patent choice can provide a signal that influences the competitor. The optimal innovator strategy involves mixing between patenting and not patenting; the follower stays out of the market when the innovator patents and imitates when the innovator does not. In contrast, we find infringement-risking imitation (closest to HMS’s duplication) in the face of a patent. This contrast underscores some basic differences between our work and that of HMS. Imitation in our model is more attractive for a follower as the patent decision and disclosure transmits enabling knowledge as well as information about the innovator’s costs. Second, HMS do not examine cases where safe imitation or staying in the market will always be profitable, cases that one would frequently encounter when innovation provides small cost reductions from current technology. Finally, no duplication is permitted against a patent. These are critical differences and we interpret HMS to be most appropriate for cases where property rights are strong.

Scotchmer and Green and Gallini focus on the impact of patent policy on the incentives to innovate. The decision to patent or suppress the innovation is important to their models, but is handled as a binary choice in which patents disclose fully and there are no incomplete information problems regarding whether innovation has occurred. These properties make our concern with the interaction between enabling knowledge transmission and information (cost) signaling essentially moot as the knowledge transmission and information...

---

12 The signaling and disclosure elements in our paper explore issues related to those examined in Bhattacharya and Ritter 1983 [3], Milgrom and Roberts 1986 [17], Okuno-Fujiwara, Postlewaite, and Suzumura 1990 [19], and Anton and Yao 1994 [1].

13 HMS employ a leader-follower equilibrium concept which involves differences in observability and commitment relative to the equilibrium concept we use.
We begin with a discussion of the model. Sections 3 through 5 provide the core analysis leading to our main propositions which characterize the patenting, disclosure, and infringement-risking imitation decisions under incomplete information. We end with a discussion of the results in the wider context of managing intellectual property, suggest some testable implications of our model, and indicate how the model informs our understanding of licensing, the incentives to innovate, and disclosure to induce demand.

2. The Model

We examine the choices to protect intellectual property and disclose information and the resulting market interactions in a model with two firms: an innovating firm (innovator) and a competing firm (imitator or follower). Each firm is risk-neutral and seeks to maximize expected profits. The model has three stages. First, in the protection and disclosure stage, the innovator realizes an R&D outcome and decides whether to protect via patent and how much of the invention ("enabling knowledge") to disclose. Next, after observing these choices, the follower decides whether to use the disclosed knowledge or to stay with the prior status-quo technology. For simplicity we assume that the follower has not engaged in R&D. Finally, there is a competition stage in which market outcomes are determined and, following the market outcomes, a third party (court) determines if the follower is liable for use of knowledge disclosed by the innovator. We specify each stage in turn and then define equilibrium.

2.1. Protection and Disclosure Stage:

The innovator, \( i \), privately observes the realized outcome of \( i \)'s prior R&D investment. This outcome involves the discovery of a process innovation which entails an associated marginal cost of producing (fixed costs are set to zero). The innovation is fully summarized by this marginal cost, \( c \), and we assume that \( c \) is drawn from a c.d.f. \( F \) with support \([0, \bar{c}]\). The upper bound, \( \bar{c} \), is the cost of the prior status-quo technology (an R&D failure is an atom in \( F \) at \( \bar{c} \)). We set the lower

---

14 Our result that small innovations (in terms of variable cost reduction) are fully disclosed and not imitated in equilibrium is similar in spirit to a result in Gallini 1992 [5] in which a short patent life in conjunction with a fixed cost of imitation implies no imitation.
bound at zero as we seek to examine a wide range of potential cost innovations (results with a positive lower bound are a minor extension).

The innovator chooses whether to protect its innovation with a patent. We use \{P, S\} to denote the choice of patent, \(P\), and secrecy (no patent), \(S\). Firm \(i\) also chooses how much enabling knowledge \(s\) to disclose. When \(i\) chooses to patent, \(P\), the disclosure \(s\) can be interpreted as the amount of enabling knowledge that is patented.\(^{15}\) An alternative interpretation is that \(s\) is a disclosure outside the patent and that the underlying patent can potentially block the free use of this disclosed knowledge. Of course, we may also have disclosure when \(i\) chooses secrecy, \(S\), although the disclosure is then not protected.

A disclosure transfers technological information that makes it feasible for firm \(j\) to produce at cost \(s\). We require that

\[ s \geq c \]  

so that the innovator cannot disclose more knowledge than is actually possessed. The innovator can disclose less. We refer to \(s > c\) as partial disclosure; \(j\) does not directly observe how much enabling knowledge remains confidential since \(c\) is private information of \(i\).

Our model does not allow licensing a competitor as a strategic option, but this is not unrealistic for licensing involving direct competitors in a concentrated industry where antitrust considerations would severely circumscribe licensing possibilities. We discuss licensing in the concluding section.

### 2.2. Infringement-Risking Imitation Stage

The follower observes whether the innovator patented or not and the disclosed knowledge. Given \((s, S)\), there is no patent in place and the follower is free to use disclosed knowledge without penalty and produce at cost \(s\). Given \((s, P)\), the follower decides whether to use the disclosed knowledge of \(s\) or the old technology. These choices are denoted by \(I\), infringement-risking imitation, and \(N\), no imitation. As we specify below, a choice to imitate is actually a choice to risk a legal finding of infringement. We assume that the innovator can observe whether the follower has chosen to imitate or not prior to the subsequent competition stage.\(^{16}\)

---

\(^{15}\)It is feasible to patent any invention \((c < \bar{c})\) and the direct cost of obtaining a patent is assumed to be zero. Because the imitator can always use the prior technology without legal risk, there is no economic force to a patent with \(s = \bar{c}\).

\(^{16}\)There are many settings in which the infringement-risking imitation choice is known either before or shortly after competition commences, e.g., Polaroid-Kodak and Intel-AMD. In other
2.3. Competition Stage

This stage consists of a duopoly market competition which we model as quantity setting (Cournot) with linear market demand

\[ p(Q) = \alpha - \beta Q \]  

(2.2)

where \( Q \equiv q_i + q_j \) is the sum of outputs. Market competition takes place under one of three possible regimes, depending on prior moves. First, at \((s, S)\) where no patent is in place and \(s\) was disclosed, the market reduces to pure Cournot competition between \(i\) at cost \(c\) and \(j\) at cost \(s\) (in which \(j\) may remain uncertain about \(i\)'s cost). Second, at \((s, P, N)\), the market reduces to pure Cournot between \(i\) at \(c\) and \(j\) at \(\bar{c}\) (again, \(j\) may remain uncertain about \(i\)'s cost). Third, and most importantly, at \((s, P, I)\) the market competition occurs under the shadow of infringement. Firms first choose quantities and this determines price, revenues, and production costs (with \(i\) at \(c\) and \(j\) at \(s\)). Then, infringement (court outcome) is determined as follows. With probability \(\gamma\), \(j\) is found to have infringed and is forced to pay damages to \(i\). Infringement damages are assessed at a royalty rate of \(\tau\) on the realized market price for each unit \(j\) produced. Thus, \(j\) is required to pay \(\tau pq_j\) to \(i\) with probability \(\gamma\) and nothing with \(1-\gamma\). As the expected penalty is what matters, we define \(g \equiv \gamma \tau\) where \(0 < g < 1\). To avoid nuisance division by zero, we treat full property rights \((g = 1)\) and no property rights \((g = 0)\) as limiting cases of the analysis.

We assume that \(\bar{c} < \alpha\) so that both firms would be active in the absence of innovation. Cases for \(\alpha, \bar{c}\) and \(g\) are introduced below.

2.4. Equilibrium

Strategic options are as follows. A protection and disclosure strategy for the innovator is a map from \([0, \bar{c}]\) into \(\{P, S\} \times [0, \bar{c}]\). By feasibility, disclosures must satisfy 2.1. We use \(\varphi_p(c)\) to denote the disclosure of an innovator with cost draw \(c\) who decides to patent and \(\varphi_S(c)\) for the disclosure when no patent is chosen.

An imitation strategy for the follower is a choice from \(\{I, N\}\) based on the observed protection and disclosure choice of the innovator, which is of the form \((s, P)\) or \((s, S)\). Quantity strategies for \(i\) and \(j\) at the competition stage are
choices for output based on the observed prior disclosure, protection and imitation history and, for $i$, the privately observed cost $c$. Thus, for example, if $i$ is of type $c$ and $(s, P, I)$ is the history, then quantity choices are denoted by $q_j(s, P, I)$ and $q_i(c, s, P, I)$. Finally, a belief of $j$ regarding $i$’s cost conditional on an observed protection and disclosure choice by $i$ is a c.d.f. on $[0, \bar{c}]$ and Bayes Law implies that a belief must put zero probability on all cost types greater than the observed disclosure.

A perfect Bayesian equilibrium (PBE) is a protection and disclosure strategy for $i$, an imitation strategy for $j$, and quantity strategies for $i$ and for $j$ as well as beliefs for $j$ such that, given these strategies and beliefs, i) quantity choices are optimal for each history at the competition stage, ii) the imitation strategy is optimal for $j$ at the second stage, and iii) the disclosure and protection strategy is optimal for $i$ at the first stage. A PBE is separating if, in equilibrium, each observed disclosure and protection choice is made by a unique cost type of $i$. We focus on equilibria of this type.

3. Market Competition

Competition between the innovating firm $i$ and the follower $j$ depends on their relative cost positions and on the property right positions they have chosen. At the point when $i$ and $j$ choose quantities, the history of the game consists of a disclosure, $s$, and property right choice $\{P, S\}$ by $i$ and, given $P$ by $i$, an infringement-risking imitation choice, $\{I, N\}$, by $j$. In a separating equilibrium, $j$ infers $i$’s cost $c$ from the observed disclosure and property right choices.

For any given $c$ and $s$, one of three cases for competition arises: i) if $i$ chose $S$, then we have (pure) Cournot competition between $i$ with cost $c$ and $j$ with cost $s$; (ii) if $i$ chose $P$ and $j$ chose $N$, we have Cournot competition between $i$ at cost $c$ and $j$ at cost $\bar{c}$; (iii) if $i$ chose $P$ and $j$ chose $I$, we have Cournot competition under the imitation regime with $i$ at $c$ and $j$ at $s$.

We focus on case (iii) since a full understanding of the consequences of infringement-risking imitation by $j$ is essential for the rest of the analysis. Summary results are provided for cases (i) and (ii) which are standard Cournot situations.

3.1. Patent and Infringement-Risking Imitation

Consider the competition stage given that $i$ chose to patent and disclose $s$ and that $j$ chose to imitate. Thus, the observed history is $(s, P, I)$. In equilibrium,
j infers that the innovator $i$ has cost $c = \varphi_{p}^{-1}(s)$.

Imitation allows $j$ to produce at cost $s$, an improvement over $\bar{c}$, but it also exposes $j$ to the risk of paying infringement damages. Given imitation, if $i$ produces $q_i$ and $j$ produces $q_j$, the resulting market price of $p = \alpha - \beta(q_i + q_j)$ leads to payoffs of $\pi_j = (p - s)q_j - gpq_j$ and $\pi_i = (p - c)q_i + gpq_j$ where $g$ is the expected infringement damages rate.\footnote{In this model $g$ is not a function of the amount of the patented enabling knowledge, so imitation always involves the maximum use possible. When key disclosures are made outside of the patent, $g$ is not likely to be a function of $s$.} The best response for $j$ to $q_i$ is given by

$$q_j^{BR} = \frac{1}{2\beta} \left[ \alpha - \frac{1}{1-g}s - \beta q_i \right],$$

when the interior term is positive (and zero if not). In the strategic response to a given $q_i$, the damages payment leads $j$ to be more timid and produce as if it had a higher marginal cost of $\frac{1}{1-g}s$ rather than $s$. Thus, as property rights weaken, $j$ will produce more aggressively. The best response for $i$ is given by

$$q_i^{BR} = \frac{1}{2\beta} [\alpha - c - \beta(1+g)q_j],$$

when the interior term is positive (and zero if not). Thus, infringement damages lead the innovator to be less aggressive in its response to a given $q_j$. The prospect of an infringement payment provides $i$ with an incentive to keep prices higher than otherwise as the damages payment is a function of $j$’s revenue.

The resulting competition stage outcome is given by:

**Lemma 1.** Consider an equilibrium and suppose $(s, P, I)$ is observed at the competition stage. Let $c = \varphi_{p}^{-1}(s)$. Then the unique outcome is given by

$$q_j(s, P, I) = \frac{1}{\beta(3-g)} \left[ \alpha - \frac{2}{1-g}s + c \right]$$

$$q_i(c, s, P, I) = \frac{1}{\beta(3-g)} \left[ \alpha(1-g) - 2c + \frac{1+g}{1-g}s \right]$$

if $s < \frac{1-g}{2}(\alpha + c)$ and by monopoly for $i$ at an output of $\frac{(\alpha-c)}{2g}$ if $s \geq \frac{1-g}{2}(\alpha + c)$.

Note two effects. The first relates to changes in cost differentials between the innovator and follower. If $i$ were to disclose more information (at a given
inference of \( c \)), then an imitating follower has lower costs as \( s \) is lower. Firm \( j \) produces more while \( i \) produces less. A higher cost \( c \) for \( i \), in equilibrium, also leads \( j \) to produce more and \( i \) to produce less (at a given disclosure of \( s \)).

Second, weaker property rights (smaller \( g \)) lead \( j \) to increase output while \( i \) reduces output. When the follower imitates, it is as if \( i \) and \( j \) have entered a de facto licensing agreement with the royalty rate \( g \) set exogenously through the legal system of intellectual property rights. Due to the infringement payment, \( i \) has a weaker incentive to increase quantity as this reduces the market price and, with it, the (expected) infringement payment from \( j \) to \( i \).

In equilibrium, the payoffs at the competition stage under imitation are given by

\[
\pi_j(s, P, I) = \frac{1 - g}{\beta(3 - g)^2} \left[ \alpha - \frac{2}{1 - g} s + c \right]^2
\]

(3.1)

\[
\pi_i(c, s, P, I) = \frac{1}{\beta(3 - g)^2} \left[ \alpha - (2 - g)c + s \right]^2 + \frac{g}{\beta(3 - g)} \left[ \alpha + c - \frac{2}{1 - g} s \right] c
\]

(3.2)

We see that \( i \)'s profit increases when \( i \) is (correctly) inferred to have lower costs and that \( i \)'s profit falls as more information is disclosed. These incentive properties form the basis for \( i \) to signal low costs through disclosure.

### 3.2. Patent and No Infringement-Risking Imitation

In this case, \( j \) operates at cost \( \bar{c} \) since \( j \) did not imitate. Also, in equilibrium, the disclosure \( s \) leads \( j \) to infer that \( i \) has cost \( c = \varphi_P^{-1}(s) \). Thus, at a history of \((s, P, N)\), equilibrium competition is analogous to a full information Cournot setting with \( i \) at cost \( c \) and \( j \) at cost \( \bar{c} \).

**Lemma 2.** Consider an equilibrium and suppose \((s, P, N)\) is observed at the competition stage. Let \( c = \varphi_P^{-1}(s) \). Then the unique outcome is given by \( q_j(s, P, N) = \frac{1}{\alpha} (\alpha - 2\bar{c} + c) \) and \( q_i(c, s, P, N) = \frac{1}{\alpha} (\alpha - 2c + \bar{c}) \) if \( c > \max\{2\bar{c} - \alpha, 0\} \) and by monopoly for \( i \) with output of \((\alpha - c)/2\beta \) if \( c \leq \max\{2\bar{c} - \alpha, 0\} \).

The payoffs are given by \( \pi_i(c, s, P, N) = \beta q_i(c, s, P, N)^2 \) and \( \pi_j(s, P, N) = \beta q_j(s, P, N)^2 \).
3.3. No Patent

In this case, as i did not patent, j produces using the disclosed information of s and faces no damages payment. In equilibrium, j infers that i has cost \( c = \varphi_{S}^{-1}(s) \). Thus, at a history of \((s, S)\), equilibrium competition is analogous to full information Cournot between i at cost c and j at cost s.

**Lemma 3.** Consider an equilibrium and suppose \((s, S)\) is observed at the competition stage. Let \( c = \varphi_{S}^{-1}(s) \). Then the unique outcome is given by \( q_{j}(c, s, S) = \frac{1}{3\beta}(\alpha - 2s + c) \) and \( q_{i}(c, s, S) = \frac{1}{3\beta}(\alpha - 2c + s) \) if \( c > \max\{2s - \alpha, 0\} \) and by monopoly for i with output of \((\alpha - c)/2\beta\) if \( c \leq \max\{2s - \alpha, 0\} \).

The payoffs are given by \( \pi_{i}(c, s, S) = \beta q_{i}(c, s, S)^{2} \) and \( \pi_{j}(s, S) = \beta q_{j}(s, S)^{2} \).

The key aspect of market competition captured in our model is that there are profits benefits to (a) having a differential cost advantage over one’s competitor and (b) making this fact known prior to some downstream competitive interaction. That is, it pays to be (relatively) strong or to appear to be relatively strong at the start of the market competition phase. Many competitive settings other than a straightforward market competition (e.g., a choice of capacity followed by market competition or a second-phase R&D competition in which lagging firms reposition the direction of or reduce their innovation efforts) have this strategic substitutes feature and, to a first approximation, should create similar incentives for disclosure and imitation to those captured here.

4. Infringement-Risking Imitation

Because it is always technically feasible for j to access disclosed knowledge, we must consider when j will find it profitable to imitate rather than stay with the noninfringing old technology. Suppose that i chose to patent, \( \mathcal{P} \), and disclose s and that, based on these actions, j infers that i has cost \( c = \varphi_{P}^{-1}(s) \). By not imitating, \( \mathcal{N} \), j faces a cost disadvantage of \( \bar{c} \) versus c. By imitating, \( \mathcal{I} \), j reduces the cost disadvantage to s versus c but risks the infringement payment. To decide which is better, the anticipated payoffs from the competition stage at an observed history of \((s, \mathcal{P}, \mathcal{N})\) and at \((s, \mathcal{P}, \mathcal{I})\) must be compared.

The results from Section 3 are used to compare the payoffs for j under \( \mathcal{N} \) and \( \mathcal{I} \). The choice depends on the relative cost positions of i and j and the strength
of the property right. The essential economic features of the imitation decision are illustrated in Figure 4.1.\footnote{For the purposes of Figure 4.1, we have assumed that $0 < 2\bar{c} - \alpha < s^* \equiv \frac{1-\bar{c}^2}{1+g^2}\bar{c} < \bar{c}$. While useful for the graph, these conditions are much stronger than necessary; Lemma 4 provides much weaker necessary and sufficient conditions on $\alpha$, $\bar{c}$, and $g$ for the imitation choice.}

First, consider the vertical line at $2\bar{c} - \alpha$. Under $N$, $j$ is active in the competition stage and earns a positive profit, $\pi^N_j > 0$, provided that $c$ is to the right of the vertical line at $2\bar{c} - \alpha$. Since $j$ operates at cost $\bar{c}$ under $N$, the disclosure $s$ only matters indirectly via the inference $c = \varphi^{-1}(s)$. To the left, where $c < 2\bar{c} - \alpha$, $j$ is inactive under $N$ and earns zero as, with $i$ at $c$ and $j$ at $\bar{c}$, the cost advantage is sufficiently large to force $j$ from the market.

Next, consider the upward sloping line, given by $s = \frac{1-\bar{c}^2}{2}(\alpha + c)$, that crosses the $45^\circ$ line at $s^* \equiv \frac{1-\bar{c}^2}{1+g^2}\alpha$. Under $I$, $j$ is active and earns a positive profit, $\pi^I_j > 0$, provided that the observed disclosure $s$ and inferred $c$ lie below this line. Above the line, $j$ is inactive under $I$ and earns zero. In this case, the cost gap between $s$ and $c$ is large enough, relative to the damages payment implied by $g$, that $j$ is forced from the market.

The $I$ versus $N$ choice has substance in the lower right region (where $c > 2\bar{c} - \alpha$ and $s < \frac{1-\bar{c}^2}{2}(\alpha + c)$) as this is the only case for which $j$ is active under both $I$
and \( N \). We compare the payoffs under \( I \) and \( N \). In the lower right region of Figure 4.1, \( j \) is active at each of \((s, P, I)\) and \((s, P, N)\) against an inferred cost of \( c = \varphi_p^{-1}(s) \) for the innovator, and we have

\[
\pi_j^I \geq \pi_j^N \iff \frac{1-g}{\beta(3-g)^2} \left[ \alpha - \frac{2}{1-g} s + c \right]^2 \geq \frac{1}{9\beta} [\alpha - 2\bar{c} + c]^2
\]

\[
\iff \left[ \frac{1-g}{2} - \frac{(3-g)\sqrt{1-g}}{6} \right] (\alpha + c) + \left[ \frac{(3-g)\sqrt{1-g}}{3} \right] \bar{c} \geq s \quad (4.1)
\]

Equality in 4.1 defines a linear relationship between \( s \) and \( c \), denoted by \( s = e(c) \), along which \( j \) is indifferent between \( N \) and \( I \). This equal payoff (EP) line has a negative slope and always passes through the point \( c = 2\bar{c} - \alpha \), \( s = (1-g)\bar{c} \). In Figure 4.1 the EP line begins at point D, which is at the edge of the monopoly region for \( i \), and falls as \( c \) rises, hitting the 45° line at the point labelled \( c^* \). Above the EP line, \( j \) will choose \( N \) and, below EP, \( j \) will choose \( I \). Thus, as (4.1) and Figure 4.1 suggest, the follower will imitate when the cost disadvantage for \( j \) at \( s \) with \( i \) at \( c \) is small enough, given the expected damages payment implied by \( g \) under imitation, relative to the larger cost disadvantage for \( j \) at \( \bar{c} \) with \( i \) at \( c \), under \( N \).

The final step is to characterize the infringement-risking imitation decision formally with respect to the underlying parameters of the model. Two properties are necessary for \( j \) to have a non-trivial imitation decision. First, \( j \) must be active in the competition stage under \( N \) and under \( I \) for a nonempty set of \( c \) and \( s \) values; otherwise, whenever one choice has a positive payoff, the other will have a zero payoff. Second, given that \( j \) is active under \( N \) and \( I \) at a set of \( c \) and \( s \) values, one of the choices must not strictly dominate the other across the set. When either of these features is not present, we will necessarily have a trivial imitation choice for \( j \).

In terms of the graph, we see that \( 0 < c^* < \bar{c} \) is the necessary property with respect to a non-trivial imitation choice. If \( c^* > \bar{c} \), then \( N \) is never chosen by \( j \) and, if \( c^* < 0 \), then \( I \) is never chosen. Solving for the intersection of EP with the 45° line, we find

\[
c^*(g, \alpha, \bar{c}) = \frac{2\bar{c} + \alpha[(1-g)h(g) - 1]}{1 + (1+g)h(g)} \quad (4.2)
\]

where \( h(g) \equiv \frac{3}{(3-g)\sqrt{1-g}} \). For reference, define the set

\[
B \equiv \left\{ (c, s) \mid \max\{0, 2\bar{c} - \alpha\} < c \leq \min\{\bar{c}, s^*\} \text{ and } \frac{1}{2}(\alpha + c) \leq s \right\}.
\]
\( B \) corresponds (in general) to the lower right region in Figure 4.1 where \( j \) is active under \( I \) and \( N \). Analysis of the EP line and the \( N \) versus \( I \) choice yields

**Lemma 4.** Suppose that \( c < \frac{\alpha}{1+g} \). Consider the infringement-risking imitation decision for \( j \) given that \( i \) patented and disclosed \( s \). Then

i) the set \( B \) is non-empty and for any \((c, s) \in B\) the follower \( j \) is active in the competition stage at \((s, P, N)\) and at \((s, P, I)\);

ii) for \((c, s) \in B\), a choice of \( N \) is optimal for \( j \) when \( s \geq e(c) \) and \( I \) is optimal when \( s \leq e(c) \);

iii) \( c^* < c \);

iv) \( c^* > 0 \) and \( c^* \) is strictly decreasing in \( g \) iff \( c > \alpha k(g) \equiv \frac{\alpha}{2} [1 - (1 - g)h(g)] \).

The condition \( c < \frac{\alpha}{1+g} \) necessarily holds as property rights become weak. This implies \( B \neq \emptyset \), the first necessary feature. When \( c > \frac{\alpha}{1+g} \), the active regions for \( j \) in Figure 4.1 do not intersect, as \( 2c - \alpha > s^* \), and the choice of \( I \) versus \( N \) is trivial at any disclosure \( s \) and inferred cost \( c \). The second essential feature is captured by \( 0 < c^* < c \). This ensures that the EP line crosses the 45° line at a possible innovation cost draw and, hence, that \( j \) will be induced to choose \( I \) and \( N \) in response to the observed disclosure and inferred cost type.

An important implication of Lemma 4 is that the innovator will not, in equilibrium, be able to force the follower to exit the market, no matter how large the innovation (when \( c < \frac{\alpha}{1+g} \)). Refer to Figure 4.1 and note that \( i \) can induce \( j \) to exit the market (when \( c < 2c - \alpha \)) only if a relatively small fraction of the innovation is disclosed. But if the equilibrium has \( j \) exiting, then the partial disclosure necessarily implies that innovators with a much smaller innovation will be able to mimic the disclosure, ensuring exit and a monopoly payoff for themselves. This cannot be an equilibrium outcome. Referring again to Figure 4.1, we see that the size of the disclosure required to induce \( j \) to exit also necessarily makes it feasible and desirable for types above \( c^* \) to mimic such a disclosure. However, \( j \) will not exit against types above \( c^* \). Formally, we have
Corollary 1. Suppose $\bar{c} < \frac{\alpha}{1+g}$. Then, in equilibrium, the follower is active in the competition stage (does not exit) for all patent and disclosure choices of the innovator.

This result is a consequence of incomplete information regarding innovation. If partial disclosure could induce exit, then higher-cost types would have a strong incentive to mimic such a disclosure. In contrast, under complete information, if $j$ had cost $\bar{c}$ and knew that $i$ had cost $c$, firm $j$ would exit whenever $c < 2\bar{c} - \alpha$. Thus, we find that there are no drastic innovations in equilibrium for $\bar{c} < \frac{\alpha}{1+g}$. While other factors not considered here might moderate this result, the analysis does highlight how incomplete information impacts the market structure effect of major innovations.

In summary, the imitation decision depends critically on the follower’s assessment of the cost position of the innovator. This underscores the focus of this paper on the signaling aspect of the competitive interaction: an optimal imitation decision when there is the option of producing with noninfringing technology necessarily turns on how the follower assesses the advantage of the innovator.

5. Equilibrium Protection and Disclosure

With the results for the competition and infringement-risking imitation stages in hand, we can examine the incentives of the innovator in the protection and disclosure stage. The equilibrium involves three distinct regions: i) high-cost types who have a relatively small innovation and choose to patent and disclose fully, ii) medium-cost types with a more significant innovation who, while still choosing to patent, disclose partially and thus rely in part on secrecy, and iii) low-cost types with large innovations who eschew a patent entirely, disclose partially, and rely more extensively on secrecy. We develop the economic analysis of each region and then state our main result, Proposition 1, which establishes existence of this three-region equilibrium. Finally, we consider uniqueness issues in Proposition 2.

5.1. Existence of a Three-Region Equilibrium

Small Innovation Region: Patent and Full Disclosure Suppose $c \geq c^*$ so that the innovation is relatively small. Suppose further that $i$ patents and, in equilibrium, discloses $s = \varphi_p(c)$. Then, we know from the imitation analysis that $j$ will not imitate since, with $s \geq c \geq c^*$, the cost reduction benefit of $s$
Figure 5.1: Equilibrium Protection and Disclosure

over $\bar{c}$ is insufficiently attractive to justify infringement-risking imitation. As a result, $i$ would earn a payoff of $\frac{1}{99} (\alpha - 2c + \bar{c})^2$. The important point is that this payoff does not depend directly on $s$; the disclosure only impacts the inference of $j$ regarding $i$’s innovation since $j$’s cost remains at $\bar{c}$ absent imitation.

The innovator, therefore, has a strong incentive to disclose fully in the small innovation region. If some type $c$ were to disclose partially, say, at $s = \varphi_P(c) > c$, then all higher-cost types $\hat{c}$ between $c$ and $s$ would find it feasible and profitable to disclose $s$ in order to be perceived as the lower cost type of $c$. Significantly, for types above $c^*$, even a very weak property right (small $g$) is sufficiently strong to deter imitation when the innovation is relatively small. Thus, we find that $i$ will fully disclose its innovation under a patent and that $\varphi_P(c) = c$ for $c \geq c^*$. In Figure 5.1, $\varphi_P$ coincides with the 45° line above $c^*$.

**Medium Innovation Region: Patent and Partial Disclosure** The innovation is more significant when $c < c^*$. We know from above that the follower will find infringement-risking imitation to be attractive in this range provided the disclosure $s$ is not too far above the inferred cost type of the innovator ($s$ and $c$ lie below the EP line). Recall that types above $c^*$ disclose fully. Thus, for a disclosure to signal cost below $c^*$, it must be that $s < c^*$. Then, in equilibrium, $j$
will choose to imitate.

The payoff for the innovator, from patenting and disclosing \( s = \varphi_P(c) \) is then given by equation (3.2). Consider the incentive of a different innovator type, say \( \hat{c} \), who finds it feasible to disclose such an \( s < c^* \). This would lead \( j \) to infer that the innovator is type \( c = \varphi_{P}^{-1}(s) \) and, hence, to imitate and produce \( q_j(s, P, I) \). By choosing a best response to this quantity, type \( \hat{c} \) can obtain the deviation payoff of

\[
\frac{1}{\beta(3 - g)^2} \left[ \alpha - \frac{3 - g}{2} \hat{c} - \frac{1 - g}{2} c + s \right]^2 + \frac{g}{\beta(3 - g)} \left[ \alpha - \frac{2}{1 - g} s + c \right] \hat{c}.
\]

In equilibrium, since \( j \) imitates and operates at cost \( s \), the innovator no longer has an incentive to disclose fully. Instead, as \( s \) varies, we find a trade-off between signalling low costs (a larger innovation) and transferring enabling knowledge to an imitating follower. A simple incentive compatibility argument based on (5.1) then establishes that as \( c \) rises \( \varphi_P(c) \) must rise at the rate of \( \frac{1 - g}{2} \). As this rate is below one, we find partial disclosure of innovations when \( c < c^* \). See \( \varphi_P \) in Figure 5.1 in the medium innovation region.

### Large Innovation Region: Partial Disclosure Without Patent Protection

The final question to consider before presenting the formal equilibrium is why an innovator should patent rather than rely exclusively on secrecy. For small innovations \( (c \geq c^*) \) and for medium-sized innovations \( (c < c^* \) but not too much smaller), we find that the patent incentive is dominant. For large innovations, however, the economic trade-off to signalling via partial disclosure in a patent becomes less attractive: high-cost types disclose fully, which pushes disclosure down to \( c^* \), and then medium-cost types disclose partially, which forces still more information knowledge disclosure by the innovator. The innovator, however, has the option not to patent. In equilibrium, the choice to give up property rights signals a large innovation and permits less disclosure of valuable enabling knowledge. See \( \varphi_S \) and the jump from \( \varphi_P \) at \( c_L \) in Figure 5.1.

We can assess the economic strength of this incentive in the following way. Suppose the innovator has achieved total success and that \( c = 0 \). Then, rather than patenting and disclosing \( \varphi_P(0) \), as implied by extending \( \varphi_P \), suppose the innovator does not patent and discloses \( c^* \). Provided that the follower beliefs regarding \( i \) are not too unfavorable, the innovator necessarily finds it more profitable to give up a patent when \( g \) is sufficiently small.\(^{19}\) It can be more profitable

\(^{19}\)If \( c = 0 \) deviates to \((\hat{s}, S)\), then \( j \) is free to operate at cost \( \hat{s} \). If \( \hat{c} \), where \( \hat{c} \leq \hat{s} \), is \( j \)'s belief,
not to patent large innovations than to patent them. This result contrasts with the value even weak patents have for protecting small innovations.

Let \( c_L \) denote the upper boundary of the large innovation range. The formal parameter condition on \( \overline{\alpha}, \alpha \) and \( g \) that ensures \( c_L > 0 \) is provided next.

**Lemma 5.** Suppose \( \alpha k(g) < \overline{c} < \frac{\alpha}{1+g} \). Then, there exists a unique \( c_L \) that satisfies

\[
\pi_i(c_L, c^*, \mathcal{S}) = \pi_i(c_L, \frac{1+g}{2}c^* + \frac{1-g}{2}c_L, \mathcal{P}, \mathcal{I})
\]

provided that \( g < 1/3 \) and \( \overline{c} > \alpha m(g) \) where \( m(g) \equiv k(g) + \frac{2[1+(1+g)h(g)]}{3-5g} \). Further, we have \( c^* > c_L > \max\{0, 2c^* - \alpha\} \). Otherwise, when \( g > 1/3 \) or when \( g < 1/3 \) and \( \overline{c} < \alpha m(g) \), we have \( \pi_i(c, c^*, \mathcal{S}) < \pi_i(c, \frac{1+g}{2}c^* + \frac{1-g}{2}c, \mathcal{P}, \mathcal{I}) \) for all \( c \in [0, c^*] \).

The type \( c_L \) is indifferent in equilibrium between patenting at a high level of disclosure and secrecy with a lower level of disclosure. Regarding the parameter conditions, a small value for \( g \) is important. In particular, as \( g \to 0 \) both \( m(g) \) and \( k(g) \) approach zero and, consequently, the jump type exists for any value \( \overline{c} \) of the older technology cost. When the existence conditions in Lemma 5 do not hold, we effectively have \( c_L \) at zero and a special case of our analysis provides the corresponding equilibrium.

For reference, define the parameter set

\[
A \equiv \left\{ (g, \overline{c}, \alpha) \mid g \in (0, 1/3) \text{ and } \alpha m(g) < \overline{c} < \frac{\alpha}{1+g} \right\}.
\]

We now show the equilibrium exists.

**Proposition 1.** Assume that \( (g, \overline{\alpha}, \alpha) \in A \). Then an equilibrium exists and is given by the following strategies:

i) the innovator patents and discloses according to
a) in the small innovation range, \( c \geq c^* \), i patents and fully discloses with 
\( \varphi_P(c) = c \); 
b) in the medium innovation range, \( c^* > c > c_L \), i patents and partially discloses with 
\[
\varphi_P(c) = \frac{1 + g}{2} c^* + \frac{1 - g}{2} c
\]
c) in the large innovation range, \( c_L \geq c \), i eschews a patent and partially discloses with 
\[
\varphi_S(c) = \left( c^* - \frac{c_L}{2} \right) + \frac{1}{2} c;
\]

ii) the follower imitates under the risk of infringement according to (a) \( j \) chooses not to imitate, \( N \), when \( (s, P) \) is observed and \( s > c^* \) and (b) \( j \) chooses to imitate, \( I \), when \( (s, P) \) is observed and \( s \leq c^* \).

iii) at each observed history on the equilibrium path, \( (s, P, N) \) for \( s \geq c^* \), 
\( (s, P, I) \) for \( s < c^* \), and \( (s, S) \) for \( s \leq c^* \), the innovator and follower produce in the competition stage according to the implied Cournot outputs.\(^{20}\)

The proof of Proposition 1 involves verifying that each of \( i \) and \( j \) finds it optimal to follow the specified strategies and this entails a set of profit comparisons for deviations from the equilibrium strategies. With the formal result established, we now discuss three additional important properties: the impact of imitation on market structure, the switch to a no patent strategy for large innovations when patent protection is weak, and the importance of the strength of property rights.

5.2. Discussion

Boundary Between Small and Medium Innovations: An Implicit Licensing Interpretation Consider what happens to market competition between the innovator and the follower at \( c^* \), the innovation level that just triggers infringement-risking imitation. Once \( c \) falls below \( c^* \), the follower strictly prefers to imitate because the disclosure \( s = \varphi_P(c) \) allows \( j \) to operate at a small cost

\(^{20}\)The equilibrium also requires that we specify out-of-equilibrium beliefs and actions. See the Appendix.
disadvantage relative to \( i \) at \( c \) while remaining at \( \sigma \) now implies a large cost disadvantage. At \( c^* \), the follower is exactly indifferent between \( N \) and \( I \).

An intriguing feature of the equilibrium is that the innovator is not indifferent with respect to \( j \)'s imitation choice because there is a discrete jump upwards in profit for \( i \) at \( c^* \) when \( j \) imitates. The innovator benefits from imitation by \( j \) because imitation leads to a qualitative change in the competitive relationship.

Fix \( c = c^* \) for purposes of discussion and examine the competition stage as \( j \) switches from \( N \) to \( I \). Imitating and producing at cost \( c^* \) leads \( j \) to increase output while \( i \) is led to reduce output relative to no imitation (\( q_I^j > q_N^j \) and \( q_I^i < q_N^i \)). The net effect is that aggregate output falls, the market price rises, and total profits for \( i \) and \( j \) rise. Thus, imitation has the beneficial joint effect of creating a market relationship in which \( i \) is effectively “licensing” \( j \). The “royalty rate” is determined implicitly by the strength of patent protection via \( g \), the infringement penalty. The extent of technology transfer is determined by the disclosure of the innovator.

**Boundary Between Medium and Large Innovations: Declining Value of the Implicit License** Once the innovation size reaches \( c_L \) (which occurs if \( g \) is small), the innovator chooses not to patent and this functions, in equilibrium, as a strong signal that \( i \) has low costs. The associated benefit is the discrete jump to \( c^* \) in disclosure: although the follower can operate at cost \( c^* \) without risking infringement, the cost disadvantage jumps to \( c^* - c_L \) from \( \varphi_P(c_L) - c_L \) and, so, \( j \) becomes a weaker competitor. In essence, a large innovation leads \( i \) to give up the benefits from the “licensing relationship” under imitation.

The innovator sacrifices patent protection and the associated “licensing” revenues because the value of licensing falls as \( c \) falls. Recall that the innovator has two sources of profit under imitation: “licensing” revenues of \( gp_I^j q_I^j \), and operating profits of \( (p_I^j - c)q_I^j \). In turn, the price and the quantities depend on the cost differential between \( i \) and \( j \) (\( s = \varphi_P(c) \) and \( c \)) measured relative to the strength of property rights, \( g \). Consider, how the innovator’s profit sources change as \( c \) falls from \( c^* \).

At \( c = c^* \), where imitation commences, we have \( \varphi_P(c^*) = c^* \) and there is no cost differential in absolute terms. As \( c \) falls, \( i \) is led to disclose more knowledge but, significantly, a cost differential opens up since the gap \( \varphi_P(c) - c \) increases as \( c \) falls. It is easy to verify that \( i \) increases output as \( c \) falls. The follower, however, does not change quantity: the rate at which equilibrium disclosure \( \varphi_P(c) \) falls exactly balances the effect of lower costs for \( j \) against that of facing a lower
cost opponent. In sum, the effect of lower cost is to increase industry output and reduce price with the cost advantage leading \( i \) to assume the more dominant role.

These effects shift the innovator’s profit source from licensing revenue towards operating profit. With \( p^T \) falling and \( q^T_j \) steady, licensing revenue of \( gp^T q^T_j \) falls. The price-cost margin for \( i \), \( p^T - c \), rises as \( c \) falls because of the widening cost differential relative to \( j \) and, consequently operating profit, \( (p^T - c)q^T_i \), is driven up by each of the margin and volume changes.

A large innovation leads \( i \) to rely more strongly on the cost advantage against \( j \) and less on expected infringement revenues. When \( g \) is sufficiently small and \( \bar{c} \) is sufficiently large, the cost advantage effect necessarily becomes dominant. Then, the innovator switches strategy at \( c = c_L \) and abandons patent protection to open up a wider cost advantage by disclosing less information. This is why eschewing the patent “signals” a large innovation. Licensing revenue is high for smaller innovations and only a large innovation (large cost advantage) makes it profitable to give up the patent.

Finally, we need to explain why the jump is to \( c^* \) and not higher. Low-cost innovators prefer the jump to be as large as possible since this would mean less “free” disclosure. What limits the size of the jump is the necessity of maintaining a consistent signal of low costs. A jump above \( c^* \) would make it feasible for innovator types in the no-imitate region \( (c > c^*) \) to mimic this signal. Moreover, these types would find it profitable to deviate if the jump point were above \( c^* \). Thus, the jump is capped at \( c^* \).

**Impact of Property Rights** Consider how the strength of property rights, as measured by \( g \), impacts the equilibrium. We focus on the two limiting cases in which property rights vanish, \( g \to 0 \), or become perfect, \( g \to 1 \).

Weaker property rights lead the innovator to rely more heavily on secrecy (disclose less). As \( g \) falls, imitation becomes more attractive for the follower. \( c^* \) rises and the full-disclosure, patent region shrinks. In the medium innovation region, we find less disclosure as \( \varphi_P \) shifts upwards. At the type \( c_L \), the jump to secrecy involves less disclosure as \( c^* \) is larger. In the limit, as \( g \to 0 \), there is no economic distinction between \( P \) and \( S \). Both \( c^* \) and \( c_L \) converge to \( \bar{c} \) and disclosure under \( P \) and \( S \) approach a common limit of \( \frac{1}{2}(\bar{c} + c) \).

The impact of stronger property rights depends on the status quo technology. If \( 2\bar{c} - \alpha > 0 \), then \( \bar{c} \) is relatively large and a drastic innovation is possible under complete information. As \( g \) rises, we eventually reach the case of \( \bar{c} > \frac{\alpha}{1+g} \). At this point, types above \( 2\bar{c} - \alpha \) disclose fully. Further, there is no overlap in the
versus \( I \) choice of the follower, and upon observing \( \varphi_P(2\bar{\tau} - \alpha) = 2\bar{\tau} - \alpha \), the follower exits the market. Thus, types below \( 2\bar{\tau} - \alpha \) achieve the monopoly outcome. Once \( c > \alpha + 1 + g \) holds, property rights are effectively perfect for the innovator although equilibrium requires that the disclosure must still be partial (\( s \) above the \( \frac{1+g}{2}(\alpha + c) \) line) in order to deter an imitation choice.

When \( 2\bar{\tau} - \alpha < 0 \), the ex ante range for process innovation is smaller. Now, as \( g \) rises, we eventually cross into the case of \( c < \alpha k(g) \) where \( c^* \) has been pushed to zero. This means that the size of \( g \) has rendered \( I \) unprofitable relative to \( N \) and the innovator chooses to patent and disclose fully. Due to the small value of \( \bar{\tau} \), however, the follower cannot be forced from the market.\(^{21}\)

5.3. Uniqueness of the Separating PBE

In Proposition 2 we characterize the set of equilibria (separating PBE). For reference, define \( \sigma = \frac{1}{2(3-g)}(2\alpha + 3(1+g)c^*) \); for \((g, \bar{\tau}, \alpha) \in A\), we have \( \sigma < c^* \). A generalization of Lemma 5 (see Lemma A5 in the Appendix) shows that for each disclosure \( \sigma \in [\sigma, c^*] \), there is a unique type \( c_\sigma \) at which \( \pi_L(c_\sigma, \sigma, S) = \pi_i(c_\sigma, \frac{1+g}{2} \sigma + \frac{1-g}{2} c_\sigma, P, I) \). We then have

**Proposition 2.** Assume \((g, \bar{\tau}, \alpha) \in A\). For each disclosure \( \sigma \in [\sigma, c^*] \) and associated crossing type \( c_\sigma \), the strategies in Proposition 1 constitute an equilibrium when we replace \( c_L \) with \( c_\sigma \) and set \( \varphi_S(c) = (\sigma - \frac{c_\sigma}{2}) - \frac{\beta}{2} \). Further, every equilibrium (separating PBE) is of this form. Finally, for all types, the equilibrium of Proposition 1 is maximal with respect to the payoff of the innovator.

Thus, equilibrium choices and outcomes for all types above \( c_L \) are unique.\(^{22}\) The switch from a patent to secrecy strategy can occur, in equilibrium, at any type below \( c_L \). All innovator types in \([0, c_L]\), however, strictly prefer the equilibrium in which secrecy is used to the maximum extent possible.

\(^{21}\)As specified in Lemma 5 there is also an intermediate region where \( \frac{\sigma}{1+g} > \bar{\tau} > \alpha k(g) \) and \( c_L \) has been pushed to zero.

\(^{22}\)Consider the possibility of all types pooling at no disclosure (\( s = \bar{c} \)) so that the distinction between \( P \) and \( S \) is moot. Letting \( \mu \) denote the mean of the prior \( F \) on innovation draws, the payoff to type \( c \) is given by \( \frac{1}{1+g}(\alpha - \frac{3}{2} c - \frac{1}{2} \mu + \bar{c})^2 \). Suppose type \( c \) deviates to \((s, P)\) and that \( j \) forms the (most) pessimistic belief of \( \hat{\mu} = s \). For a choice of \( s > c^* \), \( j \) optimally chooses \( N \) and the deviation payoff to \( i \) is given by \( \frac{1}{1+g}(\alpha - \frac{3}{2} c - \frac{1}{2} s + \bar{c})^2 \). Then the deviation is profitable if \( \mu > s \). Thus, pooling at \( \bar{c} \) is not an equilibrium if \( \mu > c^* \).
6. Discussion: Managing Intellectual Property

Our model focuses on three features of innovation and the patent system: innovation implies incomplete information, property rights often provide only limited protection from imitation, and disclosures make imitation feasible. In settings where imitation is a real possibility, the interplay between property rights, disclosure and the imitation decision is key to managing IP.

In terms of patents and managing IP, we interpret disclosure in our model in two ways. First, the patent itself may be the primary vehicle for disclosure of the enabling knowledge. A second interpretation is that disclosure is separate from the patent, though the patent is the means of overall protection against unauthorized use of the disclosed knowledge. We discuss the implications of disclosure in this section and note some empirical implications of our theory and some modeling extensions.

6.1. Information Transmission and Signaling

In the last decade, some firms have begun to proactively manage their IP. The older lawyer-driven conventional wisdom that emphasized the value of patenting all that is worth patenting is being supplanted by a more strategic business-driven logic that balances the advantages and disadvantages of protection and disclosure. The strategic logic implies as well that the choice of what to patent and what to disclose serves an important signaling role. This aspect of the management choice is especially salient when patent protection is not strong. Weaker patent protection makes infringement-risking imitation an economically attractive option for the follower. The question of how much to patent then turns on the economic

---

23 One might also consider the IP management as a portfolio of technologies problem (which of a set of related technologies to patent) instead of as a problem involving a single technology.

24 See, e.g., Grindley and Teece 1997 [6] for a discussion of these practices in some semiconductor and electronics firms.

trade-off between the cost of imitation enablement and the benefit of signaling a large innovation.

Small innovations are fully disclosed and protected because imitation through the use of that disclosed knowledge is not attractive to a competitor. Medium and large innovations, on the other hand, invite imitation; hence a portion of the innovation is kept secret so that the innovator can maintain its competitive edge.

Somewhat more surprising is the result that with weak property rights an innovator will choose to give its competitor a free road to the disclosed technology by making unprotected disclosures (e.g., Ford Motor Company’s disclosures of its moving assembly line processes) or, alternatively, by patenting but licensing the enabling knowledge for a nominal payment. When property rights are weak, the amount of disclosure under a patent needed to signal low costs is increasingly unattractive for large-innovation firms and this makes the patent increasingly less valuable relative to reducing disclosure. By sacrificing expected infringement damages the innovator can signal with much less disclosure. This sacrifice of expected damages is the core signal. Eschewing the patent and then making unprotected disclosures is one implementation of the signal, though this implementation has practical deficiencies because the choice to not patent is a nonaction and even if noticed might be attributed to a problem with the legal patentability of the invention (e.g. too close to prior art). The patent plus nominal lump-sum license, on the other hand, has the same expected damages feature, but is better tailored to creating the desired and ineluctable signal. Actions consistent with such signals include public disclosures of enabling knowledge in conferences and papers and licenses of technology for (surprisingly) low fees.26

Because the management of IP has traditionally been handled as a legal rather than business matter, our model can be viewed as partly positive and partly normative. The cost-oriented model does, however, offer some predictions relating innovation size, market structure, imitation, and disclosure. To the extent that firms have been treating IP in a business-sophisticated way, the model predicts that small process innovations will not be imitated, in contrast to medium and large process innovations which will be imitated, where imitation will be associated with an infringement lawsuit or perhaps licensing. The size of the innovation may be estimated by examining the change between pre and post innovation market share. Large cost differentials–larger innovations–will lead to a larger relative

26 This approach also avoids in part the problem associated with having some other firm inventing and then patenting the technology that the innovator kept secret.
market share for the innovator. In industries where property rights are generally considered weak, we further predict very large cost differentials will not be associated with infringement suits, though some (low royalty) licensing may occur. In such cases, for example, our model suggests that an exogenous increase in the strength of property rights might lead a firm that had previously relied on nominal license fees and (more) secrecy to try to collect substantial license fees (through the threat of enforcement of the patent) and to increase the fee revenue by disclosing additional enabling knowledge which in turn encourages greater output by the competition. The change in the strength of patent protection after the advent of the Federal Circuit provides a natural experiment with which to look for such changes.\textsuperscript{27}

\subsection*{6.2. Investment Incentives}

A potentially interesting extension of our model would be to add consideration of R&D incentives. Investment in R&D will reflect the level of appropriability expected for the R&D outputs and this, in turn, depends on the strength of the property rights and the endogenous actions of the players post invention. When a firm faces pathways offering ex ante different profiles with respect to the probability and extent of the resulting innovation, the profit outcomes from the anticipated downstream protection, disclosure, and imitation decisions are critical for assessing each path’s attractiveness. Along these lines, we speculate that where property rights are weaker (small \(g\) in our model), there may exist a bias towards investment in smaller over larger innovations because small innovations have effectively better IP protection.\textsuperscript{28}

\subsection*{6.3. Licensing}

Licensing is a strategic option that is not directly considered in our analysis. We omit this option because abstracting away from licensing allows a more precise investigation of the information transmission and signaling issues associated with IP

\footnotesize

\textsuperscript{27}There is some evidence that supports a change in the policy of some firms from a very low licensing rate to a much higher rate in the years subsequent to the new patent regime. There are, however, a plethora of possible explanations for these changes, of which our theory is only one.

\textsuperscript{28}Other important elements relating to the management or administration of IP such as the role of cumulative innovation and blocking (Scotchmer and Green 1990 [21]), reverse engineering, etc. would also be valuable to bring to our framework. See also Katz and Shapiro 1987 [12] for an analysis of how imitation influences the incentive to innovate.

30
protection and exploitation and licensing is a legally questionable option between direct competitors.

Under the assumptions adopted in our model, antitrust laws present an impediment to the use of licensing as the innovator and follower compete in the market even when the follower is using the old technology.\textsuperscript{29} Even where licensing might be (legally) permissible, the form of licensing is likely to be constrained in “close call” situations. For example, per unit licensing fees which appear to have the greatest potential for interfering with competition tend to be frowned upon by antitrust law. If detection probabilities are low, however, some firms may not be deterred from licensing by the legal risk, so augmenting our analysis with some licensing possibilities, especially those not involving per unit licensing fees would be a useful extension.

The analysis in our “implicit licensing” case, however, does provide a preliminary result—the fall back or threat points for each party to the license negotiation—helpful for the analysis of fully strategic licensing in the joint presence of incomplete information and limited property rights.

Licensing is a more complex variant of the problem we analyze. The basic inference and imitation issues remain critical: the potential licensee needs to know the value of the license before it can strike a deal and as the value is difficult to establish absent disclosure, some amount of partial, but enabling, disclosure will typically be necessary.

6.4. Signaling to Third Parties and Induced Demand

Innovating firms often disclose enabling knowledge to third parties to induce market demand (e.g., indirectly through the development of complementary products or facilitating financial backing, or directly to obtain buyers). Such disclosures are particularly important when the innovation makes a relatively large break from the past, say, as a completely new product or in the form of a process innovation that substantially decreases costs and makes new applications for the original product economically feasible. These disclosures induce additional market demand, but often at the expense of transferring enabling knowledge (either

\textsuperscript{29} “[A]ntitrust concerns may arise when a licensing arrangement harms competition among entities that would have been actual or likely potential competitors in a relevant market in the absence of the license.” Antitrust Guidelines for the Licensing of Intellectual Property, U.S. Department of Justice and U.S. Federal Trade Commission, 1995.
directly from the public disclosures or indirectly through leaks) to competitors.\(^{30}\)

The interest of noncompetitors will turn on their assessment of the relative advantage of the new technology over the old. That assessment requires noncompetitors to make inferences based on disclosed knowledge, just as the competitors do in our basic model.\(^{31}\) Further, even though the targets of the signal may be noncompetitors, the signal is likely to reach competitors as well. Thus, information flow strategies directed to noncompetitors should not be divorced from issues affecting direct competitors such as we have analyzed in this paper.

References


---

\(^{30}\) Management consultants, for example, frequently publish their general frameworks in hopes of attracting business. Revelation of confidential information to encourage sales is common in manufacturing as well. For example, Tippens, a steel plant construction company, alleged that confidential knowledge it disclosed to a potential buyer was transferred to competing company that was hired to do the job. (see, e.g., "Tippens Sues Lukens over Steckel-mill Technology," *Iron Age New Steel* 12 (3) March 1996, pp. 10-14.).

\(^{31}\) Induced demand could be included in our model by allowing the demand function to increase with disclosure. This would capture the public good aspect of the signal for the follower firm.


7. Appendix

A. Proofs of Lemmas 1, 2, and 3 and Related Results

Lemmas 1, 2, and 3 are special cases of a more general Cournot game with one-sided incomplete information. We establish existence and uniqueness of a Bayesian equilibrium for the more general game in Lemma A1 and apply the result.

Suppose payoffs satisfy \( \pi_j = (p - c_j)q_j - gpq_j \) and \( \pi_i = (p - c_i)q_i + gpq_i \) and each firm chooses quantity.

Lemma A1 Let \( t \in [0, c] \) and \( g \in [0, 1] \). Suppose firm \( j \) has cost \( c_j \in [t, c] \).

Suppose firm \( i \)'s cost type is private information and takes values in \([0, t]\) with c.d.f. \( G \) and let \( \mu \equiv \int_0^t c_i dG(c_i) \) be the mean cost type of \( i \). Strategies are non-negative quantity choices of \( q_j \) for \( j \) and \( q_i(c_i) \) for each \( c_i \in [0, t] \). Then a unique Bayesian equilibrium exists and is given by

\[
q_j^* = \frac{1}{\beta(3 - g)} \left[ \alpha - \frac{2}{1 - g} c_j + \mu \right]
\]
and

\[
q_i^*(c_i) = \frac{1}{\beta(3 - g)} \left[ \alpha(1 - g) - \frac{3 - g}{2} c_i - \frac{1 + g}{2} \mu + \frac{1 + g}{1 - g} c_j \right]
\]

when \((1 - g)(\alpha + \mu) > 2c_j\) and by monopoly for \( i \) with \( q_j^* = 0 \) and \( q_i^*(c_i) = \frac{(\alpha - c_i)}{2\beta} \) when \((1 - g)(\alpha + \mu) \leq 2c_j\).

Proof: Consider the best-response for \( i \) and \( j \). For a given \( q_j \), type \( c_i \) of \( i \) maximizes profit at \( q_i^{BR}(c_i, q_j) = (\alpha - c_i - \beta(1 + g)q_j)/2\beta \) when the numerator is positive, and at \( q_i^{BR}(c_i, q_j) = 0 \) if not. Given \( q_i : [0, t] \to [0, \infty) \), let \( \mu^i \equiv \int_0^t q_i(c_i) dG(c_i) \) be the mean output of \( i \). Then, \( j \) maximizes expected profit at \( q_j^{BR}(q_i) = (\alpha - \frac{1}{1 - g} c_j - \beta \mu^i)/2\beta \) when the numerator is positive, and at \( q_j^{BR}(q_i) = 0 \) if not.

It is then straightforward to verify that \( q_j^* \) and \( q_i^*(c_i) \) as specified in Lemma A1 satisfy the best-response conditions and constitute a Bayesian equilibrium.

Consider uniqueness. Suppose \( q_j \) and \( q_j : [0, t] \to [0, \infty) \) are a Bayesian equilibrium. To begin, we show \( q_i(c_i) > 0 \) for all \( c_i \in [0, t] \). First, suppose \( \alpha(1 - g) \leq c_j \). Then \( q_j^{BR}(q_i) = 0 \) as \( \alpha - \frac{1}{1 - g} c_j - \beta \mu^i \leq \alpha - \frac{1}{1 - g} c_j \leq 0 \) for any
\( \mu^i \geq 0 \). Hence, \( q_j = 0 \) in equilibrium and we must have \( q_i(c_i) = q_i^{BR}(c_i, 0) = \frac{\alpha - c_i}{2\beta} > 0 \). Now suppose \( \alpha(1 - g) > c_j \). If \( \mu^i = 0 \) in equilibrium, then \( q_j = (\alpha - \frac{1}{1 - g} c_j) / 2\beta \). We then have \( q_i^{BR}(c_i, q_j) > 0 \iff \alpha - c_i - \beta(1 + g)q_j > 0 \iff \frac{1}{2} \left[ \alpha(1 - g) + \frac{1 + g}{1 - g} c_j \right] > c_i \). With \( \alpha(1 - g) > c_j \), this last inequality holds strictly for all \( c_i \leq t \). Hence, \( q_i(c_i) > 0 \) must hold and, in fact, \( q_i(c_i) = \frac{1}{2\beta} \left[ \frac{\alpha}{2}(1 - g) - c_i + \frac{1 + g}{1 - g} c_j \right] > \frac{1}{2\beta} \left( \frac{c_i}{1 - g} - c_i \right) \). Hence, \( \mu^i > \frac{1}{2\beta} \left( \frac{c_i}{1 - g} - \mu \right) \geq 0 \), a contradiction. Thus, \( q_i(c_i) > 0 \) for all \( c_i \) in any equilibrium.

Now, suppose that \( q_j > 0 \) in equilibrium. Then \( q_j \) and \( q_i(c_i) \) satisfy the best-response conditions at equality. Solving simultaneously directly implies that \( q_j \) and \( q_i(c_i) \) must assume the values in Lemma A1 for \( j \) at positive output. Then, a positive output best response for \( j \) implies that \( (1 - g)(\alpha + \mu) > 2c_j \). Now, suppose that \( (1 - g)(\alpha + \mu) > 2c_j \). If \( q_j = 0 \), then we have \( q_i(c_i) = \frac{\alpha - c_i}{2\beta} \) in equilibrium. With \( \mu^i = \frac{(\alpha - \mu)}{2\beta} \), however, the best response for \( j \) has positive output as \( \left[ \alpha - \frac{1}{1 - g} c_j - \beta \mu^i \right] > 0 \). Hence, we must have \( q_j > 0 \) when \( (1 - g)(\alpha + \mu) > 2c_j \).

Next, suppose \( q_j = 0 \) in equilibrium. Then, \( q_i(c_i) = \frac{(\alpha - c_i)}{2\beta} \) must hold and \( \mu^i = \frac{(\alpha - \mu)}{2\beta} \). A best response of zero for \( j \) then implies that \( (1 - g)(\alpha + \mu) \leq 2c_j \). Going the other way, suppose \( (1 - g)(\alpha + \mu) \leq 2c_j \). If \( q_j > 0 \), then the values in Lemma A1 for \( q_j \) and \( q_i(c_i) \) must apply for \( j \) at positive output. A positive best response for \( j \), however, then implies \( (1 - g)(\alpha + \mu) > 2c_j \), a contradiction. Hence, \( q_j = 0 \) when \( (1 - g)(\alpha + \mu) \leq 2c_j \).

Thus, the equilibrium in Lemma A1 is unique when \( (1 - g)(\alpha + \mu) > 2c_j \) and when \( (1 - g)(\alpha + \mu) \leq 2c_j \). \( \square \)

Lemmas 1, 2, and 3 are special cases of Lemma A1. For Lemma 1 set \( t = c_j = s \) and let \( G \) be degenerate with an atom at \( c_i = \varphi_P^{-1}(s) \), so that \( \mu = c_i \) holds. For Lemma 2 set \( t = s, c_j = \bar{c} \), let \( G \) be degenerate at \( c_i = \varphi_P^{-1}(s) \), and set \( g \equiv 0 \) as the infringement damage does not apply when \( j \) chooses \( N \). Finally, for Lemma 3 set \( t = c_j = s \), let \( G \) be degenerate at \( c_i = \varphi^{-1}(s) \), and set \( g \equiv 0 \).

Let \( \pi_i^*(c_i) \) be the payoff to type \( c_i \) for the game in Lemma A1. We record the following result for future reference.

**Lemma A2** \( \pi_i^*(c_i) \) is strictly decreasing in the mean belief \( \mu \) whenever firm \( j \) is active and it is constant in \( \mu \) if \( j \) is inactive.
Proof: Begin with the case where $0 < \frac{2}{1-g}c_j - \alpha < \bar{c}$. By Lemma A1, $j$ is inactive for $\mu \leq \frac{2}{1-g}c_j - \alpha$, and $i$ earns a monopoly profit which is independent of $\mu$. Firm $j$ is active when $\mu > \frac{2}{1-g}c_j - \alpha$. Then, substituting $q^*_j$ and $q^*_i(c_i)$ into the definition of $\pi_i$ yields

$$
\pi^*_i(c_i) = \frac{1}{\beta(3-g)^2} \left[ \alpha - \frac{3-g}{2}c_i - \frac{1-g}{2}\mu + c_j \right]^2 + \frac{gc_i}{\beta(3-g)} \left[ \alpha - \frac{2}{1-g}c_j + \mu \right].
$$

Differentiating with respect to $\mu$, $\frac{\partial}{\partial \mu} \pi^*_i < 0 \Leftrightarrow (3-g)c_i < 2s^*+2 \left( \frac{1-g}{1+g} \right) \left[ c_j - \frac{1-g}{2}\mu \right]$, where $s^* \equiv \left( \frac{1-g}{1+g} \right) \alpha$. The right-hand side is strictly increasing in $c_j$ and strictly decreasing in $\mu$. As $\mu \leq t \leq c_j$, the right-hand side is bounded below by the value when we set $c_j = \mu = t$. Further, since $c_i \leq t$, it is sufficient to show that $(3-g)t < 2s^* + (1-g)t$. This reduces to $t < s^*$ which is valid by $j$ active and $c_j \geq t \geq \mu$.

Finally, for the case of $\frac{2}{1-g}c_j \geq \alpha$, firm $j$ is active for all $\mu \in [0,t]$. For $\frac{2}{1-g}c_j \geq \alpha + \bar{c}$, firm $j$ is never active for any $\mu \in [0,t]$. The result for these cases follows trivially. □

B. Proof of Lemma 4 and Corollary 1 and Related Results

Proof of Lemma 4: (i) We show that $j$ is active under $(s, P, I)$ and $(s, P, N)$ for a disclosure $s$ and inferred type $c = \varphi^{-1}_P(s)$ iff $(c, s) \in B \neq \emptyset$. Note that $\bar{c} < \frac{\alpha}{1+g}$ implies $(1-g)\bar{c} > 2\bar{c} - \alpha$; in Figure 4.1, the point D thus lies above the 45° line. Otherwise, as when $\bar{c} \geq \frac{\alpha}{1+g}$, the point D lies below the 45° line.

Assume $\bar{c} < \frac{\alpha}{1+g}$. This implies $s^* > 2\bar{c} - \alpha$. All feasible disclosures satisfy $\bar{c} \geq s \geq c \geq 0$. Now, by Lemma 2, $j$ is active under $N$ at any $(c, s)$ where $\max\{0, 2\bar{c} - \alpha\} < c \leq \bar{c}$ and $c \leq s$, which is a non-empty set. Also, by Lemma 1, $j$ is active under $I$ at any $(c, s)$ where $0 \leq c < \min\{\bar{c}, s^*\}$ and $c \leq s < \frac{1}{2}(\alpha + c)$, which is a non-empty set. Note that $\max\{0, 2\bar{c} - \alpha\} < \min\{\bar{c}, s^*\}$ and, as a result, the intersection of these two sets, defined as $B$, is non-empty.

(ii) This follows directly from (4.1) in the text.

(iii) From (4.2), we have $c^* < \bar{c} \iff \bar{c} [1 - (1+g)h(g)] < \alpha [1 - (1+g)h(g)]$, after simplifying terms. Since $h(g) > 1$ and, therefore, positive, we have $[1 - (1+g)h(g)] < [1 - (1+g)h(g)]$. Then, with $\bar{c} < c^*$, we are done and $c^* < \bar{c}$.

(iv) The denominator in (4.2) is always positive and, hence, $c^* > 0$ iff $\bar{c} > \frac{\alpha}{g} [1 - (1-g)h(g)] \equiv \alpha k(g)$. The function $k(g)$ is easily seen to be strictly increasing, strictly convex, and rising in value from zero as $g \downarrow 0$ to $1/2$ as $g \uparrow 1$. Thus, $\alpha k(g) < \frac{\alpha}{1+g}$ for all $g < 1$. 

37
To show that $c^*$ is strictly decreasing in $g$, note that the numerator in (4.2) is positive when $\bar{c} > \alpha k(g)$. Thus, it is sufficient to show that the numerator in (4.2) is strictly decreasing while the denominator is strictly increasing. Differentiation shows that $h(g)$ and, hence, $(1 + g)h(g)$ are strictly increasing, while $(1 - g)h(g)$ is strictly decreasing. Hence, $c^*$ is strictly decreasing in $g$.

When $\bar{c} < \alpha k(g)$, we have $c^* < 0$. As $\lim_{g \to 1} c^*(g, \bar{c}, \alpha) = 0$, $c^*$ cannot be decreasing for all of this part of the $(g, \bar{c}, \alpha)$ parameter space.\[\blacksquare\]

Table 7.1 summarizes the optimal choice in $\{I, N\}$ for $j$. We employ Lemma A1 to cover off-equilibrium cases. Thus, suppose $j$ has observed a choice of $(t, P)$ by $i$, where $t \in [0, \bar{c}]$, and holds (mean) belief $\mu \in [0, t]$.

In Figure 4.1, consider a horizontal line at vertical height $t$ with $\mu$ ranging from 0 to $t$. We can divide $[0, t]$ into at most three sub intervals, say $K_M, K_I$, and $K_N$. As $\mu$ ranges over $[0, t]$, $j$ chooses to be inactive (a monopoly for $i$) when $\mu \in K_M$, $j$ chooses $I$ when $\mu \in K_I$, and $j$ chooses $N$ when $\mu \in K_N$. Note that when $t$ lies on the boundary between cases, one of these intervals typically collapses to a single point; we omit these details from the table. Finally, recall that $t = e(\mu)$ is the EP line and $e^{-1}$ denotes the inverse; the domain for the EP line is $\mu \in [2\bar{c} - \alpha, c^*]$ when $2\bar{c} - \alpha > 0$ and $[0, c^*]$ when $2\bar{c} - \alpha \leq 0$.

Table 7.1: Follower $I$ Versus $N$ Choice

<table>
<thead>
<tr>
<th>Disclosure $t$</th>
<th>$K_M$</th>
<th>$K_I$</th>
<th>$K_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case: $2\bar{c} - \alpha &gt; 0$</td>
<td>$[0, 2\bar{c} - \alpha]$</td>
<td>$\phi$</td>
<td>$(2\bar{c} - \alpha, t]$</td>
</tr>
<tr>
<td>$t &gt; (1 - g)\bar{c}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha k(g) &gt; t &gt; M$</td>
<td>$[0, \frac{2}{1 - g}t - \alpha]$</td>
<td>$(\frac{2}{1 - g}t - \alpha, e^{-1}(t)]$</td>
<td>$[e^{-1}(t), t]$</td>
</tr>
<tr>
<td>Case: $M &gt; t &gt; c^*$</td>
<td>$\phi$</td>
<td>$[0, e^{-1}(t)]$</td>
<td>$e^{-1}(t), t]$</td>
</tr>
<tr>
<td>Case: $M &gt; t &gt; \frac{1 - g}{2} \alpha$</td>
<td>$[0, \frac{2}{1 - g}t - \alpha]$</td>
<td>$(\frac{2}{1 - g}t - \alpha, t]$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>$m &gt; t$</td>
<td>$\phi$</td>
<td>$[0, t]$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Case: $2\bar{c} - \alpha &lt; 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t \geq e^{-1}(0)$</td>
<td>$\phi$</td>
<td>$\phi$</td>
<td>$[0, t]$</td>
</tr>
<tr>
<td>$e^{-1}(0) &gt; t &gt; c^*$</td>
<td>$\phi$</td>
<td>$[0, e^{-1}(t)]$</td>
<td>$e^{-1}(t), t]$</td>
</tr>
<tr>
<td>$c^* &gt; t$</td>
<td>$\phi$</td>
<td>$[0, t]$</td>
<td>$\phi$</td>
</tr>
</tbody>
</table>

$M \equiv \max \left\{ \frac{1 - g}{2\alpha}, c^* \right\}, m \equiv \min \left\{ \frac{1 - g}{2\alpha}, c^* \right\}$
With the optimal choice of \( j \) between \( \mathcal{I} \) and \( \mathcal{N} \) (and inactivity) characterized, we can establish the following result for the payoff to \( i \). Whenever a disclosure \( t \) (with \( P \)) and a mean belief \( \mu \) lie on the EP line, \( j \) is indifferent between \( \mathcal{I} \) and \( \mathcal{N} \). Firm \( i \), however, strictly prefers that \( j \) chooses \( \mathcal{I} \) in this case. To show this, define \( \pi^I_i(c) \) to be the payoff to type \( c \) for game in Lemma A1 when we set \( c_j = t \) (i.e., \( j \) chose \( \mathcal{I} \)). Define \( \pi^N_i(i) \) to be the corresponding payoff for the game when we set \( c_j = \bar{c} \) and \( g \equiv 0 \) (i.e., \( j \) chose \( \mathcal{N} \)). We have

Lemma A4 Suppose \( a(k(g) < \bar{c} < \alpha/(1+g) \), so that \( c^* \in (0, \bar{c}) \). For the Bayesian game of Lemma A1, suppose \((\mu, t)\) lies on the EP line. Then \( \pi^I_i(c) > \pi^N_i(c) \) for all \( c \in [0, t] \).

Proof: We have \( \mu \in [\max\{0, 2\bar{c} - \alpha\}, c^*\] and \( t = \alpha(\mu) \). Let a superscript \( \mathcal{I} \) denote a variable in the Bayesian game of Lemma A1 with \( c_j = t \) and \( g \in (0, 1) \); let a superscript \( \mathcal{N} \) refer to the game with \( c_j = \bar{c} \) and \( g \equiv 0 \). We suppress the arguments of variables when no confusion arises. Lemma A1 implies \( q^\mathcal{I}_i = \frac{1}{\beta}(\alpha - \frac{2}{3}c - \frac{2}{3}+ \bar{c}) \), \( q^\mathcal{N}_i = \frac{1}{\beta}(\alpha - 2\bar{c} + \mu) \) and, with \( Q^\mathcal{N} \equiv q^\mathcal{I}_i + q^\mathcal{N}_i \), we find \( p^\mathcal{I} > q^\mathcal{I}_i + q^\mathcal{N}_i \), \( p^\mathcal{I} > p^\mathcal{N} \). We claim \( p^\mathcal{I} > p^\mathcal{N} \). Comparing expressions, \( p^\mathcal{I} > p^\mathcal{N} \Leftrightarrow (3-g)\bar{c} < 3t + g(\alpha + \mu) \). Since \( t = \alpha(\mu) \), we can substitute for \( t \) in terms of \( \mu \) using (4.1). This yields \( p^\mathcal{I} > p^\mathcal{N} \Leftrightarrow 2\bar{c} - \alpha < \mu \), which is valid on the EP line. Thus, \( p^\mathcal{I} > p^\mathcal{N} \).

Next, we have as an accounting identity that \( \pi^I_i > \pi^N_i \Leftrightarrow (t-c)q^\mathcal{I}_j - (\bar{c} - c)q^\mathcal{N}_j < (p^\mathcal{I} - c)Q^\mathcal{I} - (p^\mathcal{N} - c)Q^\mathcal{N} \), as follows from the definition of profits and the fact that \( \pi^I_j = \pi^N_j \) on the EP line. We first show \( (p^\mathcal{I} - c)Q^\mathcal{I} > (p^\mathcal{N} - c)Q^\mathcal{N} \). The monopoly profit function defined by \( (p(x) - c)x \) has unique maximum at \( x = \frac{1}{\beta}(\alpha - c) \) and it is strictly decreasing at larger \( x \) values. Since \( Q^\mathcal{I} < Q^\mathcal{N} \), as follows from \( p^\mathcal{I} > p^\mathcal{N} \), we need only show \( Q^\mathcal{I} > \frac{1}{\beta}(\alpha - c) \) to establish the result. Comparing, we have \( Q^\mathcal{I} > \frac{1}{\beta}(\alpha - c) \Leftrightarrow t < \frac{1-2g(\alpha + \mu)}{2} \), which is valid on the EP line (as \( q^\mathcal{I}_j > 0 \)).

Next, we show \( (t-c)q^\mathcal{I}_j - (\bar{c} - c)q^\mathcal{N}_j < 0 \). Note this is a decreasing function of \( c \) provided that \( q^\mathcal{N}_j > q^\mathcal{I}_j \) (note that \( j \)'s output depends on \( \mu \), but not \( c \)). Comparing
and simplifying via $t = e(\mu)$ from (4.1), we find that $q_j^N > q_j^T \iff 2c - \alpha < \mu$, which is valid. Thus, we are done if we can show the function is negative at $c = 0$. Thus, we must show $tq_j^T < cq_j^N$. Note that $t < (1 - g)c$ holds on the EP line. Thus, it is sufficient to show $(1 - g)q_j^T < q_j^N$. Comparing and simplifying, we see this holds $\iff (3 - g)c < 3t + g(\alpha + \mu)$ and, as above, this is valid on the EP line.

Combining the above results, we have $(i - c)q_j^T - (c - c)q_j^N < 0 < (p^T - c)Q^T - (p^N - c)Q^N$. By the accounting identity, this implies $\pi_i^T > \pi_i^N$. ■

**Proof of Corollary 1:** We must show that $q_j > 0$ holds for any possible equilibrium path preceding the competition stage. Recall that each path takes one of three forms, namely, $(s, \mathcal{P}, I), (s, \mathcal{P}, \mathcal{N})$ or $(s, \mathcal{S})$.

Case 1: $2c - \alpha \leq 0$. For any type $c \in [0, \check{c}]$, we apply Lemmas 1, 2, and 3 and it is easy to verify that $j$ is active for each path.

Case 2: $2c - \alpha > 0$. For a type $c \in (2c - \alpha, \check{c}]$, the argument from Case 1 applies. Now consider $c \in [0, 2 \check{c} - \alpha)$. We first show that if $j$ is inactive in equilibrium, then type $c$ necessarily discloses $s \geq \frac{1 - g}{2}(\alpha + c)$. There are three possibilities.

First, if $i$ chose $(s, \mathcal{S})$, then Lemma 3 implies $j$ is inactive $\iff s \geq (\alpha + c)/2$. Next, if $i$ chose $(s, \mathcal{P})$ and $j$ chose $\mathcal{N}$, then Lemma 2 implies $j$ is inactive, since $c \leq 2c - \alpha$ holds. Finally, if $i$ chose $(s, \mathcal{P})$ and $j$ chose $I$, then Lemma 1 implies $j$ is inactive $\iff s \geq \frac{1 - g}{2}(\alpha + c)$. Combining, we see that $s \geq \frac{1 - g}{2}(\alpha + c)$ is necessary for $j$ to be inactive in the competition stage; otherwise, $j$ could always produce profitably whether $i$ chose $(s, \mathcal{S})$ or $(s, \mathcal{P})$.

Suppose that, in equilibrium, $j$ is inactive following the disclosure and patent choice of some type $c_0 \in [0, 2c - \alpha]$. Then, from above, type $c_0$ must disclose $s_0 \geq \frac{1 - g}{2}(\alpha + c_0)$ and, as $j$ is inactive, type $c_0$ earns the monopoly payoff (of type $c_0$). Let $c_1 \equiv \frac{1 - g}{2} \alpha$ and note that $c_1 \leq s_0$. Then the disclosure $s_0$ is feasible for any type $c \in [0, c_1]$. Further, since $j$ is inactive at the equilibrium disclosure and patent choice of type $c_0$, a deviation by type $c$ must yield the monopoly payoff (for $c$). Hence each type $c \in [0, c_1]$ must earn the monopoly payoff in equilibrium and $j$ must be inactive following the disclosure and patent choice of type $c$. Otherwise, if $j$ were active, then by Lemma A2, type $c$ would necessarily earn a payoff below the monopoly level.

Consider type $c_1$. Since $j$ must be inactive, we know from the above analysis that $c_1$ discloses $s_1 \geq \frac{1 - g}{2}(\alpha + c_1) \equiv c_2$. Then all types in $[c_1, c_2]$ must earn a monopoly payoff and $j$ must be inactive, in equilibrium, as the disclosure $s_1$ is feasible. Repeating this argument, we construct a sequence via $c_n = \frac{1 - g}{2}(\alpha + c_{n-1})$. By induction, $c_n = \rho \alpha \sum_{k=1}^{n-1} \rho^k$, where $\rho \equiv \frac{1 - g}{2} \in (0, 1)$. Hence, $c_n \to \frac{\alpha p}{1 - \rho} = s^*$. 40
However, $2\bar{c} - \alpha < s^*$ by $\bar{c} < \frac{\alpha}{1+g}$. Then, $j$ must be active for sufficiently large $n$ since, eventually, we have $c_n > 2\bar{c} - \alpha$. Thus, there can be no type $c_0 \leq 2\bar{c} - \alpha$ for which $j$ is inactive in equilibrium.

C. Proof of Lemma 5

We generalize Lemma 5 (to Lemma A5). Consider the payoff $\pi_i(c, \sigma, S)$ for type $c \in [0, \sigma]$ and a disclosure $\sigma \in [0, c^*]$; Consider $\pi_i(c, r(c), \mathcal{P}, \mathcal{I})$ for $c \in [0, c^*]$ and the disclosure $r(c) \equiv \frac{1+g}{2}c^* + \frac{1-g}{2}c$. Lemma A5 establishes when there exists a type-disclosure pair $(c_\sigma, \sigma)$ such that these payoff functions cross at $c_\sigma$.

For these disclosures, the payoff functions are as follows. At $(r(c), \mathcal{P})$ by type $c$, we have $r(c) < \frac{1-g}{2}(\alpha + c) \iff c^* < s^*$, which is valid, and Lemma 1 implies $j$ is active. Further, Lemma 4 and Table 7.1 imply that $\mathcal{I}$ is optimal for $j$. Then, substitution with $s = r(c)$ in (3.2) of the text implies

$$v(c) \equiv \pi_i(c, s, \mathcal{P}, \mathcal{I}) = \frac{1}{\beta(3-g)^2} \left[ \alpha - \frac{3-g}{2}c + \frac{1+g}{2}c^* \right]^2 + \frac{gc}{\beta(3-g)} \left[ \alpha - \frac{1+g}{2}c^* \right],$$

for any $c \in [0, c^*]$. Next, at $(\sigma, S)$ by a type $c$, where $c \in [0, \sigma]$ and $\sigma \in [0, c^*]$, we apply Lemma 3 to find $j$ is active $\iff c > 2\sigma - \alpha$. Substituting into the payoff for $i$, we have

$$w(c, \sigma) \equiv \pi_i(c, \sigma, S) = \begin{cases} \frac{1}{\beta(3-g)^2} \left( \alpha - 2c + \sigma \right)^2 & \text{for } c > M \equiv \max\{2\sigma - \alpha, 0\}, \\ \frac{1}{\beta(3-g)^2} \left( \alpha - c \right)^2 & \text{for } c \leq 2\sigma - \alpha, \end{cases}$$

where the lower branch irrelevant if $2\sigma - \alpha \leq 0$; $w$ is defined for types $c \in [0, \sigma]$ and $\sigma \in [0, c^*]$. Note that the interval $[M, \sigma]$ is non-empty as $M < \sigma$ always holds; the cases of $2\sigma - \alpha \geq 0$ both arise (as $2c^* - \alpha \geq 0$ occurs across $(g, \bar{c}, \alpha)$ values) and must be dealt with.

Define $\Delta(c, \sigma) = w(c, \sigma) - v(c)$ for $c \in [0, \sigma], \sigma \in [0, c^*]$. It is easy to verify the following properties. First, $\Delta$ is strictly convex for $c \in [M, \sigma]$. Next, if $2\sigma - \alpha > 0$, then i) $\Delta$ is linear in $c$ over $(0, 2\sigma - \alpha)$, ii) $\Delta$ is continuous at $c = 2\sigma - \alpha$, iii) there is a kink in $\Delta$ at $c = 2\sigma - \alpha$ and the partial derivative of $w$ satisfies $0 > w^-_c > w^+_c$ at $(2\sigma - \alpha, \sigma)$.

We now show $\Delta(\sigma, \sigma) < 0$. Since $c = \sigma > 2\sigma - \alpha$, we apply the upper branch of $w$. Note that the second term in $v(\sigma)$ is always positive as $\frac{gc}{\beta(3-g)} \left( \alpha - \frac{1+g}{2}c^* \right) > 0 \iff \frac{1-g}{1+g} \alpha \equiv s^* > c^*$, which holds. Hence, it is sufficient for $\Delta(\sigma, \sigma) < 0$ to show $w(\sigma, \sigma)$ is less than the first term in $v(\sigma)$. Comparing, this reduces to $0 < g\alpha + \frac{3}{2}(1+g)c^* - \frac{1}{2}(3-g)\sigma$. This expression is linear decreasing in $\sigma$, so we
need only show it is positive at $\sigma = c^*$ and this reduces to $0 < g(\alpha + 2c^*)$, which is valid. Hence, $\Delta(\sigma, \sigma) < 0$ for $\sigma \in [0, c^*]$.

Now consider the value of $\Delta(0, \sigma)$. First, suppose $2\sigma - \alpha \leq 0$. Then, upon simplifying, we have $\Delta(0, \sigma) > 0 \iff \sigma > \sigma$ where $\sigma = \frac{1}{2(3-g)} [2g\alpha + 3(1 + g)c^*]$. We also note, for later use, $\sigma \geq c^* \iff 2g\alpha \geq (3 - 5g)c^*$.

Now suppose $2\sigma - \alpha > 0$. We claim $\Delta(c, \sigma) > 0$ holds for $c \in [0, 2\sigma - \alpha]$. We employ a monopoly versus duopoly payoff argument. By Lemma 3, $j$ is inactive at $(\sigma, S)$ for $c \leq 2\sigma - \alpha$ and $i$ earns the monopoly payoff of $w(c, \sigma) = \frac{1}{4g}(\alpha - c)^2 = \text{Max} (p - c)q$. From above, $j$ chooses $I$ and is active at $(r(c), P)$ by $i$ and $j$ earns $\pi^I_j = \pi_j(r(c), P, I)$ from Lemma 1 and (3.1). Letting superscript $I$ denote values at $(r(c), P, I)$ for $j$ when $i$ is type $c$ and similarly for $i$, we have $\pi^I_i < \pi^I_i + \pi^I_j = (p^T - c)q^I_i + gp^T q^I_j + (p^T - c)q^I_j - gp^T q^I_i = (p^T - c)q^I_j + (p^T - c)q^I_i < (p^T - c)(q^I_j + q^I_i)$, where the last step follows from $r(c) > c$ and we used the accounting definition of profits. Combining, $\Delta(c, \sigma) > 0 \iff$ monopoly profit exceeds $\pi^I_i$. This holds if $q^I_i + q^I_j$ exceeds the monopoly output of $\frac{1}{4g}(\alpha - c)$. Substituting for quantities from Lemma 1, this reduces to $s^* > c^*$, which is valid. Hence, $\Delta(c, \sigma) > 0$ for $c \in [0, 2\sigma - \alpha]$ when $2\sigma - \alpha > 0$.

We now sort out the parameter cases for $(g, c, \alpha)$ that lead to $2c^* - \alpha \geq 0$.

**Lemma 6.** If $g < 1/3$ and $\bar{c} > \alpha r(g) \equiv \alpha \left[3 - (1 - 3g)h(g)\right]$, then $2c^* - \alpha > 0$. If $g < 1/3$ and $\bar{c} < \alpha r(g)$ or if $g > 1/3$, then $2c^* - \alpha < 0$.

**Proof:** From Lemma 4 and (4.2), we see that $c^*(g, \alpha, \bar{c})$ rises in value from 0 at $\bar{c} = \alpha k(g)$ to $s^*$ at $\bar{c} = \frac{\alpha}{1 + g}$. When $g > 1/3$, we have $s^* < \frac{\alpha}{2}$ and, hence, $2c^* - \alpha < 0$. When $g < 1/3$, we have $s^* > \frac{\alpha}{2}$. Thus, $c^*$ crosses $\frac{\alpha}{2}$ as $\bar{c}$ varies. Simplifying with (4.2) then shows that $c^* \geq \frac{\alpha}{2}$ as $\bar{c} \geq \alpha r(g)$. $\square$

One further preliminary result will be useful.

**Lemma 7.** If $g < 1/3$ and $\bar{c} > \alpha m(g)$, then we have $(3 - 5g)c^* > 2g\alpha$. If $g < 1/3$ and $\bar{c} < \alpha m(g)$ or if $g > 1/3$, then we have $(3 - 5g)c^* < 2g\alpha$.

**Proof:** At $\bar{c} = \alpha k(g)$, we have $c^* = 0$. At $\bar{c} = \frac{\alpha}{1 + g}$, we have $c^* = s^*$. Note that $(3 - 5g)s^* \geq 2g\alpha \iff 3 - 10g + 3g^2 \geq 0$. This is a convex quadratic with roots at 1/3 and 3. If $g > 1/3$, it is negative and, by $c^* < s^*$, we have $(3 - 5g)c^* < (3 - 5g)s^* < 2g\alpha$. If $g < 1/3$, the quadratic is positive and, with $(3 - 5g)s^* > 2g\alpha$, we see that $(3 - 5g)c^*$ crosses $2g\alpha$ as $\bar{c}$ varies. From (4.2), we have $(3 - 5g)c^* \geq 2g\alpha \iff \bar{c} \geq \alpha m(g)$. $\square$
Direct calculations show that \( m(g) \) rises from \( m(0) = 0 \) to \( m(1/3) = 3/4 \) as \( g \) varies and that \( r(g) \) rises from \( r(0) = 1/2 \) to \( r(1/3) = 3/4 \) as \( g \) varies. Further, \( m(g) < r(g) \) for \( g < 1/3 \). See Figure 7.1 for reference.

We now have Lemma A5.

**Lemma A5** Assume \( \alpha k(g) < \bar{c} < \frac{\alpha}{1+g} \). Suppose \((g, \bar{c}, \alpha) \) satisfy \( g < 1/3 \) and \( \bar{c} > \alpha m(g) \). Then, for each \( \sigma \in [\sigma, c^*] \) there exists a unique type \( c_\sigma \) such that \( \Delta(c_\sigma, \sigma) = 0 \). Further, we have i) \( \Delta(c, \sigma) \geq 0 \) as \( c \lesssim c_\sigma \), ii) \( c_\sigma \) increases with \( \sigma \), and iii) \( c_\sigma \) is between \( \max\{2\sigma - \alpha, 0\} \) and \( \sigma \), with \( c_\sigma = 0 \) for \( \sigma = \sigma \).

Suppose, instead, that \( g \geq 1/3 \) or that \( g < 1/3 \) and \( \bar{c} < \alpha m(g) \). Then \( \Delta(c, \sigma) < 0 \) for all \( c \in [0, \sigma] \) and any \( \sigma \in [0, c^*] \).

**Proof:** We apply the above results on \( \Delta \) to \((g, \bar{c}, \alpha) \) in each of regions I, II and III in Figure 7.1. Take \((g, \bar{c}, \alpha) \) \in Region III. Then \( 2c^* - \alpha < 0 \) by Lemma 6 and, hence, \( 2\sigma - \alpha < 0 \) for any \( \sigma \in [0, c^*] \). We know \( \Delta(\sigma, \sigma) < 0 \) and that \( \Delta \) is strictly convex in \( c \in [0, \sigma] \). We are done if \( \Delta(0, \sigma) < 0 \). From above, \( \Delta(0, \sigma) < 0 \Leftrightarrow \sigma < \sigma \). But \( \sigma > c^* \Leftrightarrow (3 - 5g)c^* < 2g\alpha \) and this holds in III by Lemma 7. Hence, \( \sigma \leq c^* < \sigma \) and \( \Delta(0, \sigma) < 0 \).

Consider \((g, \bar{c}, \alpha) \) \in Region II. Again, by Lemma 6, we have \( 2\sigma - \alpha \leq 2c^* - \alpha < 0 \) for \( \sigma \in [0, c^*] \). Also, \( \Delta(\sigma, \sigma) < 0 \) and \( \Delta \) is strictly convex in \( c \in [0, \sigma] \). From Lemma 7, we have \( \sigma < c^* \) as \( 2g\alpha < (3 - 5g)c^* \) in II. Now, note that \( \sigma < \bar{c} \) implies
\[ \Delta(0, \sigma) < 0 \] from above, so strict convexity implies \( \Delta(c, \sigma) < 0 \) for all \( c \in [0, \sigma] \) in this case. Take \( \sigma > c \) so that \( \Delta(0, \sigma) > 0 \), from above. By continuity, \( \Delta \) crosses zero at some \( c_\sigma \) between 0 and \( \sigma \), and convexity implies \( c_\sigma \) is unique. Hence, \( \Delta(c, \sigma) \geq 0 \) as \( c \leq c_\sigma \). Since the partials satisfy \( \Delta(c(\sigma, \sigma)) < 0 < \Delta(c_\sigma, \sigma) \), we see that \( c_\sigma \) increases with \( \sigma \).

Let \( (g, c, \alpha) \in \text{Region I} \). Then \( 2c^* - \alpha > 0 \) by Lemma 6 and \( \sigma < c^* \) by Lemma 7. Further, with \( g < 1/3 \) we have \( \sigma < \alpha/2 < c^* \). First, suppose, \( \sigma < \alpha/2 \). Then, the analysis follows the same logic as that for Region II. Next, suppose \( \sigma > \alpha/2 \). Then, as \( 2\sigma - \alpha > 0 \), we know from above that \( \Delta(c, \sigma) > 0 \) for \( c \in [0, 2\sigma - \alpha] \). Since \( \Delta(c, \sigma) < 0 \), continuity and strict convexity of \( \Delta \) for \( c \in [2\sigma - \alpha, \sigma] \) imply a unique \( c_\sigma \) where \( \Delta \) crosses zero. Further, \( \Delta(c, \sigma) \geq 0 \) as \( c \leq c_\sigma \). As above, we find \( c_\sigma \) increases with \( \sigma \).

Lemma 5 in the text follows directly as a special case of Lemma A5 where we take \( \sigma = c^* \) and let \( c_L \) denote the crossing value \( c_\sigma \).

D. Proof of Proposition 1

The main task is to verify that equilibrium payoff to \( i \) for each \( c \in [0, \bar{c}] \) at the candidate disclosure and patenting choice exceeds the payoff for any feasible deviation. We also verify that \( j \) is choosing optimally from \( \{I, N\} \) and that quantities are optimal. Out-of-equilibrium supporting beliefs are specified at the end of the proof. Note that \( (g, c, \alpha) \in A \) implies that the three ranges are well defined as \( 0 < c_L < c^* < \bar{c} \) holds.

Consider the equilibrium payoff for type \( c \in [0, \bar{c}] \) of \( i \), denoted by \( U(c) \). For \( c \geq c^* \), we have full disclosure with \( \varphi_P(c) = c \). Then, firm \( j \) optimally chooses \( N \) since \( (c, \varphi_P(c)) \) lies above the EP line for \( c \geq c^* \) (see Table 7.1). Further from Lemma 2, \( j \) is active in the competition stage at \( (\varphi_P(c), P, N) \) since \( c^* > 2\bar{c} - \alpha \). We then calculate \( U(c) = \frac{1}{2\beta} (\alpha - 2c + \bar{c})^2 \).

For \( c^* > c > c_L \), we have partial disclosure at \( \varphi_P(c) \). Since \( (c, \varphi_P(c)) \) lies below the EP line, firm \( j \) optimally chooses \( I \) (see Table 7.1). Further, it is easy to show \( \varphi_P(c) < \frac{1-g}{2}(\alpha + c) \) and, hence, \( j \) is active at \( (\varphi_P(c), P, I) \) by Lemma 1. Then, from (3.2), we calculate

\[
U(c) = \pi_i(c, \varphi_P(c), P, I) = \frac{1}{\beta(3-g)^2} \left( \alpha - \frac{3-g}{2}c + \frac{1+g}{2}c^* \right)^2 + \frac{g}{\beta(3-g)} \left( \alpha - \frac{1+g}{1-g}c^* \right) c.
\]

Next, for \( c_L \geq c \), we have partial disclosure at \( \varphi_S(c) \). By Lemma 3, \( j \) is active at \( (\varphi_S(c), S) \) provided \( c > \max\{2\varphi_S(c) - \alpha, 0\} \); since \( c > 2\varphi_S(c) - \alpha \iff c_L > 2c^* - \alpha \), we see from Lemma 5 that \( j \) is active. We then calculate from Lemma 3, \( U(c) = \pi_i(c, \varphi_S(c), S) = \frac{1}{2\beta} \left( \alpha - \frac{3}{2}c + c^* - \frac{\alpha L}{2} \right)^2. \)

44
For later reference, we note the following properties of $U(c)$. First, it is strictly decreasing and convex in each of the regions. This is obvious for $c \leq c_L$ and for $c > c^*$. For $c_L < c \leq c^*$, differentiation shows that $U'(c) < 0 \Leftrightarrow g \left[ \alpha - \frac{1 + g}{1 - g} c^* \right] < [\alpha - (1 - g)c^*]$. To see that this last inequality is valid, note that we have $(1 + g)c^* > c^* > (1 - g)^2 c^*$ and this implies $\alpha - (1 - g)c^* > \alpha - \frac{1 + g}{1 - g} c^*$. Since $g < 1$, we have $U'(c) < 0$ for $c_L < c < c^*$.

Next, consider $c = c_L$ and $c = c^*$. By construction of $c_L$ (see Lemma 5), $U$ is continuous at $c = c_L$. At $c = c^*$, however, there is a downward jump in $U$. This is because $j$ is indifferent between $I$ and $N$ since $(c^*, \varphi_P(c^*))$ lies on the EP line. The equilibrium specifies a choice of $I$ by $j$ in this case and, by Lemma A4, we have $U(c^*) > \lim_{c \downarrow c^*} U(c)$. Because of $j$’s indifference, we could also specify a choice of $N$ by $j$; this would require that type $c_L$ choose $(\varphi_P(c_L), P)$ rather than $(c^*, S)$. Either specification works equally well and has no impact on the structure of the equilibrium.

We will need the following result to compare the equilibrium payoff, $U(c)$, for $c \leq c^*$, to deviation payoffs.

**Lemma 8.** Assume $(g, \bar{c}, \alpha) \in A$. Let $c_0$ and $s_0$ satisfy $0 < c_0 \leq c_L$ and $\frac{1 + g}{2} c^* + \frac{1}{2} (c_0 - s_0) \leq s_0 < \frac{1}{2} (\alpha + c_0)$. Suppose $\sigma_0$ satisfies $\pi_i(c_0, \sigma_0, S) = \pi_i(c_0, s_0, \varphi_P(c_0), I)$. Then $\pi_i(c, \sigma_0 - \frac{1}{2} (c_0 - c), S) \geq \pi_i(c, s_0 - \frac{1}{2} (c_0 - c), \varphi_P(c_0), I)$ as $c \leq c_0$, for any $c \in [0, c_0']$, where $c_0' > c_0$ is defined by $e(c_0') = s_0 - \frac{1}{2} (c_0 - c_0')$.

**Proof:** Consider the line $s = s_0 - \frac{1}{2} (c_0 - c)$. Note that $c_0'$ is defined by where this line crosses the EP line. From Lemma 4 and Table 7.1, it is easy to verify that a choice of $(s, \varphi_P)$ by $c \in [0, c_0']$, where $s = s_0 - \frac{1}{2} (c_0 - c)$ implies that $j$ chooses $I$ and is active. Consequently, $j$ is active at $(\sigma_0, S)$ by $c_0$ since $i$ earns less than the monopoly payoff at $(s_0, \varphi_P)$; in turn, $j$ is found to be active at $(\sigma, S)$ choice by $c$, for $\sigma = \sigma_0 - \frac{1}{2} (c_0 - c)$.

Define $W(c) \equiv \pi_i(c, \sigma_0 - \frac{1}{2} (c_0 - c), S)$ and $V(c) = \pi_i(c, s_0 - \frac{1}{2} (c_0 - c), \varphi_P(c_0), I)$. Let $\delta(c) \equiv W(c) - V(c)$. Then $\delta(c_0) = 0$, by construction, and differentiation show $\delta(c)$ is linear. We are done if we show $\delta(0) > 0$. Evaluating the profit functions at $c = 0$, we have $\delta(0) > 0 \Leftrightarrow (3 - g) \sigma_0 > g \alpha + 3s_0 + gc_0$. The next step is to show this inequality is valid.

Substituting $s = s_0 - \frac{1}{2} (c_0 - c)$ into $\pi_i(c, s, \varphi_P, I)$ from (3.2) in the text, we have

$$V(c) = \frac{1}{\beta(3 - g)^2} \left[ \alpha - \frac{3 - g}{2} c + s_0 - \frac{1 - g}{2} c_0 \right]^2 + \frac{gc}{\beta(3 - g)} \left[ \alpha - \frac{2}{1 - g} s_0 + c_0 \right].$$

45
\[ V_1(c) + V_2(c). \] Clearly, \( V_2(c_0) > 0 \) holds as \( \frac{1 - \sigma}{2}(\alpha + c_0) > s_0. \) Therefore, \( W(c_0) > V_1(c_0). \) Simplifying the profit expressions then reveals that this is equivalent to the inequality for \( W(0) > V(0). \)

For Proposition 1 we will apply Lemma 8 with \( c_0 = c_L, s_0 = \varphi_P(c_L), \) and \( \sigma_0 = c^*. \) We now consider deviations for \( i \) types in each of the three ranges.

Case 1: \( \hat{c} \geq c > c^*. \) Note that all types below \( c \) either disclose fully or disclose no more than \( c^* \). Thus, the only feasible deviation for \( c \) is to patent and disclose \( \hat{c} = \varphi_P(\hat{c}) \) for some \( \hat{c} > c. \) Upon inferring type \( \hat{c}, \) the same argument as above implies that \( j \) chooses \( N \) and produces (actively) at \( \hat{q}_j = q_j(\hat{c}, \mathcal{P}, N) \) as given by Lemma 2. The best response for \( i \) of type \( c \) is positive and follows

\[ q_i^{BR} = \frac{1}{g_{ij}}(\alpha - c - \beta \hat{q}_j) \]

and results in a payoff of \( \beta (q_i^{BR})^2. \) Simplifying then yields the deviation payoff of \( u(\hat{c}, c) = \frac{1}{g_{ij}} (\alpha - \frac{3}{2}c - \frac{1}{2} \hat{c} + \hat{c})^2. \) Since this is strictly decreasing in \( \hat{c}, \) we have \( U(c) > u(\hat{c}, c) \) and the equilibrium choice \( \varphi_P(c) = c \) is optimal for \( c. \)

Case 2: \( c^* \geq c > c_L. \) There are three kinds of feasible deviations: i) to the small innovation region at \( \hat{c} \) where \( \hat{c} > c^*, \) ii) within the medium innovation region to a \( \hat{c} \) where \( c < \varphi_P(\hat{c}) \leq c^*, \) and iii) to the large innovation region at \( \hat{c} \) where \( c \leq \varphi_S(\hat{c}) \leq c^*. \) We take each of these sub-cases in turn.

i) \( \hat{c} > c^*: \) By the same argument as in case 1, we find \( u(\hat{c}, c) = \frac{1}{g_{ij}} (\alpha - \frac{3}{2}c - \frac{1}{2} \hat{c} + \hat{c})^2. \) This is decreasing in \( \hat{c}. \) Thus, to rule out a deviation to \( \hat{c} > c^*, \) it is sufficient to show \( U(c) \geq \lim_{\hat{c} \downarrow c^*} u(\hat{c}, c) \). We apply Lemma A4. Noting that \((c^*, c^*)\) lies on the EP line, set \( t = \mu = c^* \) in Lemma A4. Then, we see that \( U(c) = \pi_i^T(c) \) and \( \lim_{\hat{c} \downarrow c^*} u(\hat{c}, c) = \pi_i^N(c) \) as constructed in Lemma A4. As Lemma A4 asserts \( \pi_i^T(c) > \pi_i^N(c), \) we are done.

ii) \( c < \varphi_P(\hat{c}) \leq c^*. \) Upon observing \( \varphi_P(\hat{c}), \) firm \( j \) infers type \( \hat{c} \) and, therefore, chooses \( \mathcal{I} \) and the (positive) quantity \( \hat{q}_j = q_j(\varphi_P(\hat{c}), \mathcal{P}, \mathcal{I}) \) as in Lemma 1. The best response for \( i \) of type \( c \) is positive and follows \( q_i^{BR} = \frac{1}{g_{ij}} [\alpha - c - \beta (1 + g) \hat{q}_j]. \) Calculating the payoff to \( i \) (as in (5.1) in the text) reveals that \( U(c) = u(\hat{c}, c); \) as we see later, \( \varphi_P(c) \) necessarily involves weak incentive compatibility in \((c_L, c^*)\).

iii) \( c \leq \varphi_S(\hat{c}) \leq c^*. \) Upon observing \( \varphi_S(\hat{c}), \) firm \( j \) infers type \( \hat{c} \) and chooses the (positive) quantity \( \hat{q}_j = q_j(\varphi_S(\hat{c}), \mathcal{S}), \) as in Lemma 3. Calculating the best response for \( i \) of type \( c \) and the resulting payoff yields \( u(\hat{c}, c) = \frac{1}{g_{ij}} (\alpha - \frac{3}{2}c + c^* - \frac{c\hat{c}}{2})^2; \) thus, \( c \) is indifferent across the set of feasible \( \hat{c} \) deviations into the large innovation region. Since \( c > c_L, \) we apply Lemma 8 directly and \( U(c) > u(\hat{c}, c) \) holds.

Case 3: \( c_L \geq c. \) The three kinds of feasible deviations are i) \( \hat{c} > c^*, \) ii)
\(c^* \geq \hat{c} > c_L\), and iii) \(c^* \geq \varphi_S(\hat{c}) \geq c\). We take each in turn.

i) \(\hat{c} > c^*\). As above, we find that \(u(\hat{c}, c) = \frac{1}{2\beta} (\alpha - \frac{3}{2} c - \frac{1}{2} \hat{c} + \hat{c})^2\) is decreasing in \(\hat{c}\), so it is sufficient to show \(U(c) \geq \lim_{\hat{c} \downarrow c^*} u(\hat{c}, c)\). From case 2, we know \(U(c') > \lim_{\hat{c} \downarrow c^*} u(\hat{c}, c')\) for \(c' \in (c_L, c^*)\). Since \(U\) is continuous at \(c_L\), we have \(U(c_L) > \lim_{\hat{c} \downarrow c^*} u(\hat{c}, c_L)\) and this implies \(3c^* \geq c_L + 2\hat{c}\). Comparing \(U(c)\) and \(\lim_{\hat{c} \downarrow c^*} u(\hat{c}, c)\), we see this last inequality implies \(U(c) \geq \lim_{\hat{c} \downarrow c^*} u(\hat{c}, c)\) holds.

ii) \(c^* \geq \hat{c} > c_L\). We calculate \(u(\hat{c}, c)\) exactly as in case 2 (ii). For \(c < c_L\), we apply Lemma 8 directly and \(U(c) > u(\hat{c}, c)\) holds. At \(c = c_L\), we have equality and the type \(c_L\) is indifferent.

iii) \(c^* \geq \varphi_S(\hat{c}) \geq c\). Calculate \(u(\hat{c}, c)\) exactly as in case 2 (iii). This yields \(U(c) = u(\hat{c}, c)\); as we see later, \(\varphi_S(c)\) necessarily involves weak incentive compatibility in \([0, c_L]\).

Finally, we must specify supporting beliefs for out-of-equilibrium \((s,P)\) and \((s,S)\) choices. The simple linear extension of \(\varphi_P(c)\) to \([0, c_L]\) and the mean belief \(\mu = \varphi_P^{-1}(s)\) for \(s \in [\varphi_P(0), \varphi_P(c^*)]\), along with \(\mu = c\) for \(s < \varphi_P(0)\) is sufficient to support the equilibrium. A linear extension of \(\varphi_s\) to \((c_L, \hat{c}]\), however, will induce deviations (by types near \(c^*)\). It suffices to take beliefs at \((s,S)\) for \(s > c^*\) to be \(\mu = \hat{c}\) or \(\mu = s\). Intermediate inferences also work. 

E. Proof of Proposition 2

To prove that the conditions are sufficient for a PBE, we simply apply the proof of Proposition 1. The only change is that we replace \(c_L\) and \(\varphi_S(c_L) = c^*\) with \(c_\sigma\) and \(\varphi_S(c_\sigma) = \sigma\).

To establish payoff dominance, simply compare the equilibrium with \(c_L\) and \(\varphi_S(c_L) = c^*\) from Proposition 1 to an equilibrium with \(c_\sigma\) and \(\sigma\) for \(c_\sigma < \sigma < c^*\). Since \(c_\sigma < c_L\), all types above \(c_L\) earn the same payoff. Since \(\pi_i(c, c^* - \frac{1}{2}(c_L - c), S) > \pi_i(c, \frac{1}{2}c_\sigma + \frac{1}{2}c_\sigma, P, I)\) for \(c \leq c_L\), by Lemma 8, all types \(c \in [c_\sigma, c_L]\) strictly prefer the equilibrium from Proposition 1.

This leaves types \(c \in [0, c_\sigma]\). We must show \(\pi_i(c, c^* - \frac{1}{2}(c_L - c), S) > \pi_i(c, \sigma - \frac{1}{2}(c_\sigma - c), S)\). This reduces to \(c^* - \frac{1}{2}c_L > \sigma - \frac{1}{2}c_\sigma\). From Lemma 5, we know \(c_\sigma\) and \(\sigma\) satisfy \(\pi_i(c_\sigma, \sigma, S) = \pi_i(c_\sigma, \frac{1}{2}c^* + \frac{1}{2}c_\sigma, P, I)\). Combining with the inequality from Lemma 8 in the previous paragraph evaluated at \(c = c_\sigma\), we have \(\pi_i(c_\sigma, c^* - \frac{1}{2}(c_L - c), S) > \pi_i(c_\sigma, \sigma, S)\) and this implies \(c^* - \frac{1}{2}c_L > \sigma - \frac{1}{2}c_\sigma\).

We now turn to necessary conditions.

For reference, \((s,P)\) or \((s,S)\) denotes a disclosure and patent choice by \(i\). We reserve \(\varphi_P(c)\) and \(\varphi_S(c)\) for candidate equilibrium choices. From Corollary 1, \(j\) is active at any equilibrium choice by \(i\). Finally, let \(r(c) = \frac{1}{2}c^* + \frac{1}{2}c\) for all \(c\).

The first step is to derive continuation payoffs in a PBE for any feasible patent
and disclosure choice by $i$. Suppose $j$ observes $(t, S)$ by $i$ and holds mean belief $\mu \in [0, t]$. We apply Lemma A1 with $c_j = t$ and $g \equiv 0$ to find the optimal quantity for $j$; further, the quantity choice for $i$ in Lemma A1 for each type $c$ is a best-response to this choice by $j$. The continuation payoff for type $c$ of $i$ from $(t, S)$ is

$$u_i(c, t, S; \mu) = \begin{cases} \frac{1}{18} (\alpha - \frac{3}{2} c - \frac{1}{2} \mu + t)^2, & \text{for } t < \frac{\alpha + \mu}{2} \\ \frac{1}{48} (\alpha - c)^2, & \text{for } t \geq \frac{\alpha + \mu}{2} \end{cases}$$

If $(t, S)$ is observed in equilibrium, then $\mu = \varphi_S^{-1}(t)$. Otherwise, $(t, S)$ is off the equilibrium path and the belief is only required to support the equilibrium and satisfy Bayes’ rule ($\mu \leq t$); by Lemma A1, the mean is the only payoff relevant property of $j$’s belief.

Now, suppose $j$ observes $(t, P)$ and has mean belief $\mu \in [0, t]$. PBE requires that $j$ makes an optimal choice between $I$ and $N$ at $(t, P)$ for belief $\mu$. For each possible $t$ and $\mu$, we apply Table 7.1 to find the optimal $j$ choice of $I$ or $N$, and then apply Lemma A1 as above to find the quantities. The continuation payoff for type $c$ of $i$ is

$$u_i(c, t, P; \mu) = \begin{cases} \frac{1}{18} \left[\alpha - \frac{3}{2} c - \frac{1-g}{2} \mu + t\right]^2 + \frac{gc}{18} \left[\alpha - \frac{2}{1-g} t + \mu\right], & \mu \in K_I \text{ at } t \\ \frac{1}{9} \left[\alpha - \frac{3}{2} c - \frac{1}{2} \mu + \bar{c}\right]^2, & \mu \in K_N \text{ at } t \\ \frac{1}{48} (\alpha - c)^2, & \mu \in K_M \text{ at } t \end{cases}$$

As before, if $(t, P)$ occurs in equilibrium, then $\mu = \varphi_P^{-1}(t)$ must hold. Note that $u_i$ is single-valued except in the case where $\mu$ and $t$ are on the EP line, $t = c(\mu)$ and $\mu \in K_I \cap K_N$. In this case, each of $N$ and $I$ is optimal for $j$ and the continuation payoff for $i$ can assume two possible values.

The following result for $u_i$ is useful

**Lemma 9.** The continuation payoff $u_i$ is non-increasing in $\mu$ and strictly decreasing in $\mu$ whenever $j$ is active in the production stage.

**Proof:** This is obvious for $u_i(c, t, S; \mu)$. For $u_i(c, t, P; \mu)$, the result follows directly from Lemma A2 when $\mu$ is in the interior of the $K_M$, $K_I$ or $K_N$ intervals (see Table 7.1). Across the various boundary cases, we find that $u_i$ is continuous and, hence, the result is established, except for the boundary between $K_I$ and

48
\[ K_N. \] Recall that this occurs when \( \mu \in \{ \max\{0, 2\bar{c} - \alpha\}, c^* \} \) and \( t = e(\mu) \). From Lemma A4, we see that \( \pi_i^T(c) > \pi_i^N(c) \) are the two continuation payoff values for \( u_i \). Hence, the jump in \( u_i \) is downward since we cross from \( K_T \) to \( K_N \) as \( \mu \) rises.

We now examine the necessary structure of an equilibrium (separating PBE). Clearly, under separation, a disclosure of \( \bar{c} \) leads \( j \) to infer \( i \) is type \( \bar{c} \) since, with no innovation, \( \bar{c} \) is the only feasible disclosure for type \( \bar{c} \). It is convenient to adopt the convention that \( i \) chooses \( \mathcal{P} \) in this case. Hence, \( \varphi_\mathcal{P}(\bar{c}) = \bar{c} \).

Now consider types between \( c^* \) and \( \bar{c} \). First, we show that only \( \mathcal{P} \) is used in equilibrium for \( c \in (c^*, \bar{c}) \). Suppose, instead, that such a \( c \) type chooses \((s, S)\) in equilibrium for some \( s \in [c, \bar{c}] \). From above, separation implies \( s < \bar{c} \). In equilibrium, \( j \) infers type \( c \) for \( i \) upon observing \((s, S)\) and, by Lemma 3, \( i \) must earn the payoff of \( \pi_i(c, s, S) = \frac{1}{y_0} (\alpha - 2c + s)^2 \).

Consider a deviation by type \( c \) to \((t, \mathcal{P})\) for some \( t \geq c \). By Lemma 9, the lowest possible deviation payoff is \( u_i(c, t, \mathcal{P}; t) \), where \( j \) holds the pessimistic mean belief of \( \mu = t \). Note that \( c^* < c \leq t \) implies \( j \) will choose \( N \) in this case. Comparing, we have \( u_i(c, t, \mathcal{P}; t) > \pi_i(c, s, S) \Leftrightarrow (\alpha - \frac{3}{2} c - \frac{1}{2} t + \bar{c}) > (\alpha - 2c + s) \Leftrightarrow \frac{1}{2}(c - t) + \bar{c} > s \). Under a full-disclosure deviation of \( t = c \), this reduces to \( \bar{c} > s \) and the deviation is strictly profitable. Therefore, all types \( c \in (c^*, \bar{c}) \) necessarily use \( \mathcal{P} \) in equilibrium.

It is straightforward to show that each \( c \in (c^*, \bar{c}) \) chooses full disclosure under a patent in equilibrium. Suppose, instead, that some type \( c \in (c^*, \bar{c}) \) chooses \((s, \mathcal{P})\) with \( s > c \). This implies that \((s, \mathcal{P})\) is a feasible deviation for any type \( \hat{c} \in (c, s) \). Let \((\hat{s}, \mathcal{P})\) be the equilibrium choice of such a \( \hat{c} \) type. Note that firm \( j \) necessarily chooses \( N \) at each of \((s, \mathcal{P})\) and \((\hat{s}, \mathcal{P})\) since each of \((c, s)\) and \((\hat{c}, \hat{s})\) lies above the EP line. Then, \( \hat{c} \) prefers the deviation \((s, \mathcal{P})\) to \((\hat{s}, \mathcal{P}) \Leftrightarrow u_0(\hat{c}, s, \mathcal{P}; c) > \pi_i(\hat{c}, s, S, \mathcal{P}, N) \Leftrightarrow (\alpha - \frac{3}{2} \hat{c} - \frac{1}{2} c + \bar{c}) > (\alpha - 2\hat{c} + \bar{c}) \Leftrightarrow \hat{c} > c \), which is valid. Therefore, in equilibrium, we must have \( \varphi_\mathcal{P}(c) = c \) for \( c \in (c^*, \bar{c}) \).

Now consider types \( c \leq c^* \). We claim that if \((s, S)\) is chosen by any \( c \leq c^* \) in equilibrium, then \( s \leq c^* \). Suppose, instead, that some \( c \leq c^* \) chooses \((s, S)\) with \( s > c^* \). Since \( c \leq c^* \), it is feasible for type \( c \) to deviate to \((\hat{c}, \mathcal{P})\) for any \( \hat{c} \) \( > c^* \). In equilibrium, type \( c \) must prefer \((s, S)\) to \((\hat{c}, \mathcal{P})\) and this holds \( \Leftrightarrow \pi_i(c, s, S) \geq u_i(c, \hat{c}, \mathcal{P}; c) \Leftrightarrow (\alpha - 2c + s) \geq (\alpha - \frac{3}{2}c - \frac{1}{2}\hat{c} + \bar{c}) \Leftrightarrow s \geq \hat{c} - \frac{1}{2}(\hat{c} - c) \). As this must hold for any \( \hat{c} \) \( > c^* \), we then have \( s \geq \hat{c} - \frac{1}{2}(c^* - c) \).

Consider the type \( \hat{c} = \bar{c} - \frac{1}{2}(c^* - c) \). Clearly, we have \( c^* < \hat{c} \leq \bar{c} \) since \( c \leq c^* \). Further, a deviation by type \( \hat{c} \) to \((s, S)\) is feasible since \( s \geq \hat{c} \). Then, type \( \hat{c} \) strictly prefers \((s, S)\) to \((\hat{c}, \mathcal{P}) \Leftrightarrow u_i(\hat{c}, s, S; c) > \pi_i(\hat{c}, \hat{c}, \mathcal{P}, N) \Leftrightarrow (\alpha - \frac{3}{2}\hat{c} - \frac{1}{2}c + s) > (\alpha - 2\hat{c} + \bar{c}) \Leftrightarrow s \geq \hat{c} - \frac{1}{2}(\hat{c} - c) \). This last inequality necessarily holds since \( \hat{c} > c^* \). Thus, we must have \( s \leq c^* \) for any equilibrium choice of \((s, S)\).
The above argument implies that type $c^*$ chooses $(c^*, \mathcal{P})$ in equilibrium. This is because the only possible equilibrium choice of the form $(s, \mathcal{S})$ for type $c^*$ has $s = c^*$. The payoff $\pi_i(c^*, c^*, \mathcal{S})$, however, is strictly dominated by $u_i(c^*, \hat{c}, \mathcal{P}, \hat{c})$ for $\hat{c} > c^*$ and $\hat{c}$ sufficiently close to $c^*$, which type $c^*$ can obtain by choosing $(\hat{c}, \mathcal{P})$. Further, we have $\pi_i(c^*, c^*, \mathcal{P}, \mathcal{N}) > u_i(c^*, \hat{c}, \mathcal{P}, \hat{c})$ for any $\hat{c} > c^*$. Thus, separation implies type $c^*$ must choose $(c^*, \mathcal{P})$ in equilibrium and this holds whether $j$ (who is indifferent) chooses $\mathcal{I}$ or $\mathcal{N}$ upon observing $(c^*, \mathcal{P})$ and inferring type $c^*$ for $i$.

Now consider types $c \in (c_L, c^*)$. We claim all such types use $\mathcal{P}$ in equilibrium. Suppose, instead, some $c \in (c_L, c^*)$ chooses $(s, \mathcal{S})$. From above, $s \leq c^*$ holds and the payoff to type $c$ is $\pi_i(c, s, \mathcal{S})$. Consider a deviation by type $c$ to $(t, \mathcal{P})$ for some $t \in (c, c^*)$. By Lemma 9, the deviation payoff is at least $u_i(c, t, \mathcal{P}, t)$, where $j$ holds the pessimistic belief of $\mu = t$. Since $\mu = t < c^*$, we see from Table 7.1 that $\mu = t \in K_T = [0, t]$ in this case and, hence, $j$ necessarily chooses $\mathcal{I}$.

This deviation is strictly profitable for a choice of $t$ sufficiently close to $c^*$. First, $\pi_i(c, s, \mathcal{S}) \leq \pi_i(c, c^*, \mathcal{S})$, by Lemma A2. Next, (from the proof of Lemma 5) $0 > \Delta(c) \equiv \pi_i(c, c^*, \mathcal{S}) - \pi_i(c, r(c), \mathcal{P}, \mathcal{I})$ for $c \in (c_L, c^*)$. Now, $u_i(c, t, \mathcal{P}, t)$ converges to $\pi_i(c, r(c), \mathcal{P}, \mathcal{I})$ as $t$ increases to $c^*$. Combining, we then have $\pi_i(c, s, \mathcal{S}) < u_i(c, t, \mathcal{P}, t)$ for $t < c^*$ and $t$ sufficiently close to $c^*$. Thus, the deviation is strictly profitable and no type $c \in (c_L, c^*)$ uses $\mathcal{S}$ in equilibrium.

We now show all types $c \in (c_L, c^*)$ choose partial disclosure and $\mathcal{P}$. From above, we know any such $c$ chooses $\mathcal{P}$ and separation implies $s < c^*$ for an equilibrium choice $(s, \mathcal{P})$ by $c$. We also see $j$ chooses $\mathcal{I}$ at $(s, \mathcal{P})$, by Table 7.1. Hence, the equilibrium payoff for $c$ is $\pi_i(c, s, \mathcal{P}, \mathcal{I})$.

Consider a deviation by type $c$ to $(\hat{s}, \mathcal{P})$, where $(\hat{s}, \mathcal{P})$ is the equilibrium choice of a type $\hat{c}$ for $c < \hat{c} < c^*$. In equilibrium, type $c$ must prefer $(s, \mathcal{P})$ to $(\hat{s}, \mathcal{P})$ and this requires $\pi_i(c, s, \mathcal{P}, \mathcal{I}) \geq u_i(c, \hat{s}, \mathcal{P}, \hat{c})$. Define the function $n(c, x) = \frac{1}{3(3-g)^2} \alpha - \frac{3-2g}{2} c + x \right)^2 + \frac{8c}{3(3-g)} \left[ \alpha - \frac{2}{1-g} x \right]$. From the proof of Lemma A2, we see $n(c, x)$ is strictly increasing in $x$. Further, we have $\pi_i(c, s, \mathcal{P}, \mathcal{I}) = n(c, s - \frac{1-g}{2} c)$ and $u_i(c, s, \mathcal{P}, \hat{c}) = n(c, s - \frac{1-g}{2} \hat{c})$. Hence, we must have $s - \frac{1-g}{2} c \geq \hat{s} - \frac{1-g}{2} \hat{c}$ in equilibrium. As this holds for $\hat{c}$ arbitrarily close to $c^*$, we must have $s - \frac{1-g}{2} c \geq c^* - \frac{1-g}{2} c^* = \frac{1-g}{2} c^*$ since $\hat{c} \leq \hat{s} < c^*$. Then, $s \geq \frac{1+g}{2} c^* + \frac{1-g}{2} c \equiv r(c) > c$, as $c < c^*$, and we have partial disclosure.

Now, partial disclosure implies that a deviation to $(s, \mathcal{P})$ is feasible for types $\hat{c}$ such that $c < \hat{c} \leq s$. Since type $\hat{c}$ must prefer $(\hat{s}, \mathcal{P})$ to $(s, \mathcal{P})$, the same argument as above implies $\hat{s} - \frac{1-g}{2} \hat{c} \geq s - \frac{1-g}{2} c$. Hence, $\hat{s} = s + \frac{1-g}{2} (\hat{c} - c)$.

We can now show $\varphi_P(c) = r(c)$ for $c \in (c_L, c^*)$. Suppose some $c_0 \in (c_L, c^*)$
chooses \((s_0, \mathcal{P})\) in equilibrium with \(s_0 > r(c_0)\). Define \(c_1 = s_0\) and, noting that \(s_0\) is feasible for \(c_1\), the above incentive compatibility argument implies that \(\varphi_{\mathcal{P}}(c_1) = s_1 = s_0 + \frac{1-g}{2}(c_1 - c_0)\). Define \(c_2 = s_1\) and continue on to construct a sequence \((c_n)\) where \(c_n = c_{n-1} + \frac{1-g}{2}(c_{n-1} - c_{n-2})\) for \(n \geq 2\). Then \((c_n)\) is easily found to converge to \([2s_0 - (1-g)c_0]/(1+g); \) since \(s_0 > r(c_0)\), this limit exceeds \(c^*\). But then \(\varphi(c_n) > c_n > c^*\) holds for large \(n\), which is not possible. Hence, \(\varphi_{\mathcal{P}}(c) = r(c)\) for \(c \in (c_L, c^*)\).

Now consider \(c \in [0, c_L]\). Define \(\Sigma\) to be the set of types who choose \(\mathcal{S}\) in equilibrium. If \(\Sigma = \emptyset\), then all types choose \(\mathcal{P}\) and the arguments above imply that \(\varphi_{\mathcal{P}}(c) = r(c)\). Thus, suppose \(\Sigma \neq \emptyset\). We find that an equilibrium choice \((s, \mathcal{S})\) by \(c \in \Sigma\) must satisfy i) \(s \leq c^*\); ii) \(c < \hat{c} - \frac{1}{2}(c^* - c) < s\); iii) if \(\hat{c} \in \Sigma\) and \(\hat{c} < c\), then \(s > \hat{s} \geq s - \frac{1}{2}(\hat{c} - c)\), with equality if \(c \leq \hat{s}\). Claim i) is from above. Claim ii) follows from considering a deviation by \(c\) to the equilibrium choice of types above \(c^*\). Claim iii) follows from a deviation by \(c\) to \((s, \mathcal{S})\) and, when feasible, vice versa.

Define \(c_\sigma \equiv \sup \Sigma\) and \(\sigma \equiv \sup \{s | s = \varphi_{\mathcal{S}}(c) \text{ for some } c \in \Sigma\}\). From above, \(c_\sigma \leq c_L\) and \(\sigma \leq c^*\) must hold. Also, by property iii) of \(\Sigma\), for any sequence of types in \(\Sigma\) converging to \(c_\sigma\), we must have \(\varphi_{\mathcal{S}}(c)\) converging monotonically to \(\sigma\). From our earlier arguments, all \(c > c_\sigma\) choose \((r(c), \mathcal{P})\) in equilibrium.

We claim that \(c_\sigma\) and \(\sigma\) must satisfy \(\pi_i(c_\sigma, \sigma, \mathcal{S}) = \pi_i(c_\sigma, r(c_\sigma), \mathcal{P}, \mathcal{I})\). To see this, take \(c \in \Sigma\) such that \(c_\sigma - \epsilon < c \leq c_\sigma\) for \(\epsilon > 0\). By property (ii) of \(\Sigma\), we have \(\varphi_{\mathcal{S}}(c) > c\). For \(\epsilon\) small enough, \((\varphi_{\mathcal{S}}(c), \mathcal{S})\) is feasible for type \(\hat{c}\) where \(c_\sigma < \hat{c} < c_\sigma + \epsilon\). With \(c < c_\sigma\), it is clearly feasible for \(c\) to deviate to \((\varphi_{\mathcal{S}}(\hat{c}), \mathcal{P})\). The claim then follows from the incentive compatibility conditions as \(\epsilon \to 0\). From Lemma A5, we see that this profit equality condition requires \(\sigma \leq c_\sigma\).

The next step is to show that all \(c < c_\sigma\) choose \(\mathcal{S}\) and disclose \(\varphi_{\mathcal{S}}(c) = \sigma - \frac{1}{2}(c_\sigma - c)\) in equilibrium. Suppose some \(c < c_\sigma\) chooses \((s, \mathcal{P})\) in equilibrium. Since \(j\) must be active, the payoff to \(c\) is \(\pi_i(c, s, \mathcal{P}, \mathcal{I})\). By construction of \(c_\sigma = \sup \Sigma\), we can find \(c \in \Sigma\) such that \(c < c_\sigma\). Then \((\hat{s}, \mathcal{S})\), where \(\hat{s} = \varphi_{\mathcal{S}}(\hat{c})\), is feasible for \(c\) since \(c < c_\sigma\) and incentive compatibility implies \(\pi_i(c, s, \mathcal{P}, \mathcal{I}) \geq u_i(c, \hat{s}, \mathcal{S}; \hat{c})\). Letting \(\hat{c}\) approach \(c_\sigma\) (or taking \(\hat{c} = c_\sigma\) if \(c_\sigma \in \Sigma\)), we know that \(u_i(c, \hat{s}, \mathcal{S}; \hat{c})\) converges to \(u_i(c, \sigma, \mathcal{S}; c_\sigma)\). Thus, we must have \(\pi_i(c, s, \mathcal{P}, \mathcal{I}) \geq u_i(c, \sigma, \mathcal{S}; c_\sigma)\).

It is also feasible for \(c\) to deviate to \((r(\hat{c}), \mathcal{P})\), the equilibrium choice of \(\hat{c}\) for \(c_\sigma < \hat{c} < c^*\), and incentive compatibility implies \(s > r(c)\). We can now rule out the possibility of such a \(c\) type in a neighborhood of \(c_\sigma\): suppose \(r^{-1}(c_\sigma) < c < c_\sigma\) so that \((s, \mathcal{P})\) is feasible for \(c > c_\sigma\) sufficiently close to \(c_\sigma\). Then incentive compatibility implies \(s = r(c)\). With \(s = r(c)\), however, we can apply Lemma 8.
(set \(c_0 = c_\sigma, s_0 = r(c_\sigma)\) and \(\sigma_0 = \sigma\)) to see that \(u_i(c, \sigma, S; c_\sigma) = \pi_i(c, \sigma - \frac{1}{2} (c_\sigma - c), S) > \pi_i(c, r(c_\sigma) - \frac{1}{2} (c_\sigma - c), P, I) = \pi_i(c, r(c), P, I)\), which violates incentive compatibility. Thus, we must have \(c \in \Sigma\) for all \(c \in (r^{-1}(c_\sigma), c_\sigma]\).

Then, we show \(\varphi_S(c) = c - \frac{1}{2}(c_\sigma - c)\) for all \(c\) in this lower neighborhood of \(c_\sigma\) by an analogous argument to the one we employed to show \(\varphi_P = r\) for types between \(c_L\) and \(c^*\). Note that if \(c_\sigma < r(0)\), then we have shown \(c \in \Sigma\) and \(\varphi_S(c) = \sigma - \frac{1}{2}(c_\sigma - c)\) for all \(c\) below \(c_\sigma\). If, instead, \(c_\sigma \geq r(0)\), we can employ a variation of our argument to rule out \(P\) for all types below \(c_\sigma\). Define \(\eta = \sup\{c \mid c < c_\sigma\ \text{and} \ c \notin \Sigma\}\). We find that \(\eta\) and \(s_\eta\) must satisfy \(\pi_i(\eta, s_\eta, P, I) = \pi_i(\eta, \sigma - \frac{1}{2}(c_\sigma - \eta), S)\). But Lemma 8 implies that incentive compatibility will be violated and some type at or below \(\eta\) will strictly prefer a deviation to \((\varphi_S(c), S)\) for \(\eta < c < c_\sigma\). Thus, all types below \(c_\sigma\) choose \(S\) and disclose \(\varphi_S(c) = \sigma - \frac{1}{2}(c_\sigma - c)\).

Finally, note that type \(c_\sigma\) can choose \((\sigma, S)\) or \((\varphi_P(c_\sigma), P)\) in equilibrium when \(\sigma < c^*\). For \(\sigma = c^*\), however, the equilibrium connection with \(j\) indifference between \(I\) and \(N\) at \((c^*, P)\) by type \(c^*\) of \(i\) comes into play. If \(j\) chooses \(I\), then equilibrium requires that type \(c_L\) choose \((c^*, S)\); if \(j\) chooses \(N\), then \(c_L\) must choose \((\varphi_P(c_L), P)\). Except for these minor open set considerations, the equilibrium structure is not impacted.