

## Practice Problems on Production Function and Labor Market

1- What is a production function? What are some factors that can cause a nation's production function to shift over time? What do you have to know besides an economy's production function to know how much output the economy can produce?

A production function shows how much output can be produced with a given amount of capital and labor. The production function can shift due to supply shocks, which affect overall productivity. Examples include changes in energy supplies, technological breakthroughs, and management practices. Besides knowing the production function, you must also know the quantities of capital and labor the economy has.

2- The production function slopes upward, but its slope declines from left to right. Give an economic interpretation of each of these properties of the production function.

The upward slope of the production function means that any additional inputs of capital or labor produce more output. The fact that the slope declines as we move from left to right illustrates the idea of diminishing marginal productivity. For a fixed amount of capital, additional workers each add less additional output as the number of workers increases. For a fixed number of workers, additional capital adds less additional output as the amount of capital increases.

3- Explain why the profit-maximizing level of employment for a firm occurs when the marginal revenue product of labor equals the nominal wage. How can this profit-maximizing condition be expressed in real terms?

The marginal revenue product of labor represents the benefit to a firm of hiring an additional worker, while the nominal wage is the cost. Comparing the benefit to the cost, the firm will hire additional workers as long as the marginal revenue product of labor exceeds the nominal wage, since doing so increases profits. Profits will be at their highest when the marginal revenue product of labor just equals the nominal wage.

The same condition can be expressed in real terms by dividing through by the price of the good. The marginal revenue product of labor equals the marginal product of labor times the price of the good. The nominal wage equals the real wage times the price of the good. Dividing each of these through by the price of the good means that an equivalent profit-maximizing condition is the marginal product of labor equals the real wage.

4- What is the *MPN* curve? How is the *MPN* curve related to the production function? How is it related to labor demand?

The *MPN* curve shows the marginal product of labor at each level of employment. It is related to the production function because the marginal product of labor is equal to the slope of the production function (where output is plotted against employment). The *MPN* curve is related to labor demand, because firms hire workers up to the point at which the real wage equals the marginal product of labor. So the labor demand curve is identical to the *MPN* curve, except that the vertical axis is the real wage instead of the marginal product of labor.

5- Define the following: *labor force*, *unemployment rate*, *participation rate*, and *employment ratio*.

The labor force consists of all employed and unemployed workers. The unemployment rate is the fraction of the labor force that is unemployed. The participation rate is the fraction of the adult population that is in the labor force. The employment ratio is the fraction of adult population that is employed.

6- During 1980s and 1990s the average rate of unemployment in Europe was high. Some economists claimed that this rate was in part the result of “real-wage rigidity”, a situation in which unions kept real wages above their market-clearing levels.

a- Accepting for the sake of argument that real wages were too high in Europe in the 1980s and 1990s, show how this situation would lead to unemployment ( a situation where people who would like to work at the going wage cannot find jobs).

As shown in Fig. 3.17, when the real wage ( $w'$ ) is above its market-clearing level, labor supply ( $NS'$ ) exceeds labor demand ( $ND'$ ). The difference is the amount of unemployment ( $U$ ).

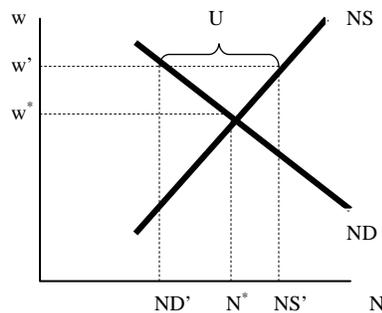


Figure 3.17

b- What is the effect of real-wage rigidity on the output actually supplied by firms, relative to the output they would supply if there were no real-wage rigidity?

Output is lower because of the real wage rigidity. With the real wage higher than the wage that clears the market at full employment, labor demand must be lower than it is at full employment, so employment and output are lower as well.

7- According to the growth accounting approach, what are the three sources of economic growth? From what basic economic relationship is the growth accounting approach derived?

The three sources of economic growth are capital growth, labor growth, and productivity growth. The growth accounting approach is derived from the production function.

8- For a particular economy, the following capital input  $K$  and labor input  $N$  were reported in four different years:

Year	$K$	$N$
1	200	1000
2	250	1000
3	250	1250
4	300	1200

The production function in this economy is:

$$Y = K^{0.3} N^{0.7}$$

where  $Y$  is total output.

Find total output, the capital-labor ratio, and output per worker in each year. Compare year 1 with year 3, and year 2 with year 4. Can this production function be written in per-worker form? If so, write algebraically the per-worker form of the production function.

Year	$K$	$N$	$Y$	$K/N$	$Y/N$
1	200	1000	617	0.20	0.617
2	250	1000	660	0.25	0.660
3	250	1250	771	0.20	0.617
4	300	1200	792	0.25	0.660

This production function can be written in per-worker form since  $Y/N = K^{0.3} N^{0.7} / N = K^{0.3} / N^{0.3} = (K/N)^{0.3}$ .

Note that  $K/N$  is the same in years 1 and 3, and so is  $Y/N$ . Also,  $K/N$  is the same in years 2 and 4, and so is  $Y/N$ .

9- What is a production function? In the context of the production function, what determines the growth rate of real GDP?

$$Y = A * K^{.3} * N^{.7}$$

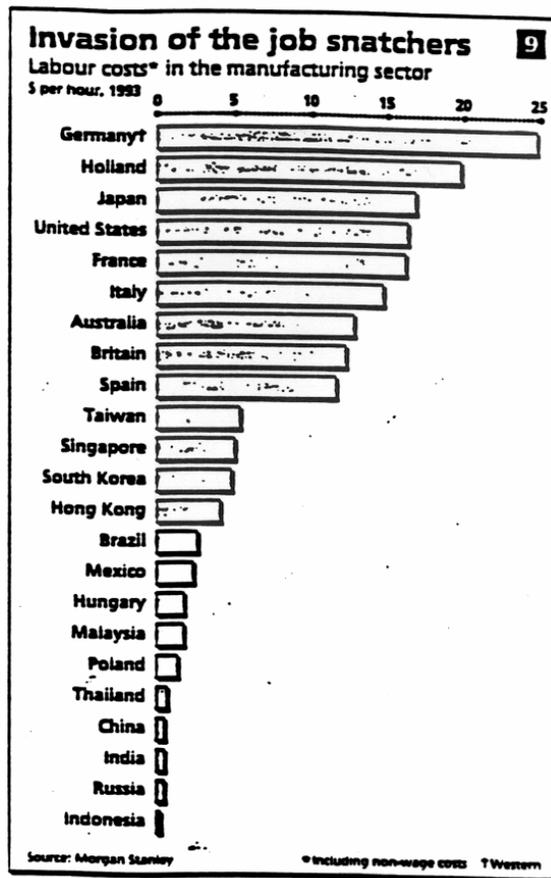
is the production function. It follows that

$$Y_{t+1}/Y_t = [K_{t+1}/K_t]^{.3} * [N_{t+1}/N_t]^{.7} * A_{t+1}/A_t$$

represents the growth rate of real GDP. Alternatively, real GDP growth is equal to growth in capital stock + growth in labor input + growth in productivity (i.e.,  $A$ ).

10- Wage differences

a. Below is a graph that shows the real wages across different countries. Why is the real wage different across different countries?



Real wages differ because the marginal productivity of labor is different across countries. More precisely, real wage =  $A \cdot 0.7 \cdot [K/L]^{0.3}$ , hence higher real wages reflect higher A.

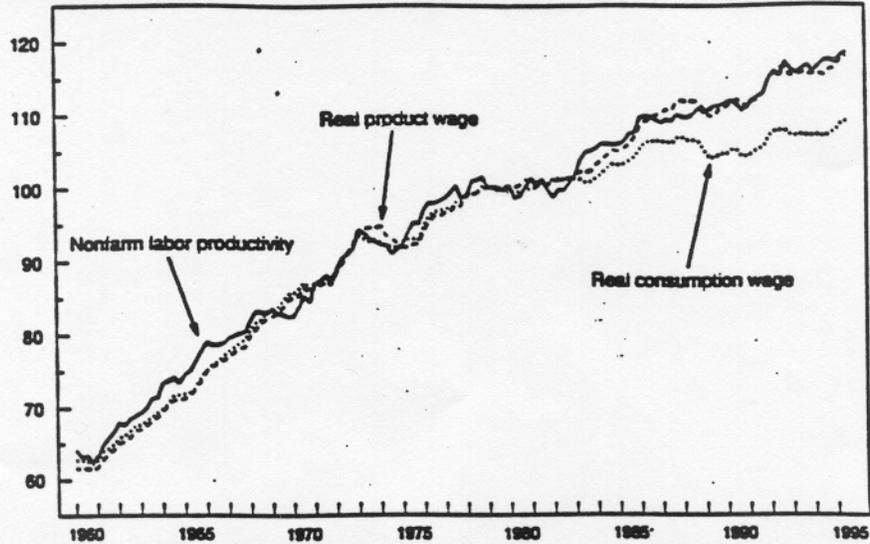
b. As an empirical fact, the income inequality has increased in the U.S., provide an explanation to this phenomenon for the U.S.

See discussion in text on skilled- based technical change.

c. On the next page is a graph that shows the real wages and productivity in the U.S.; why are productivity and real wages closely related?

**Chart 2-6 Measures of Real Compensation and Labor Productivity**  
 The real product wage has kept pace with productivity, whereas the real consumption wage has not.

Index, 1979=100



Note: Wages are compensation per hour in the nonfarm business sector divided by the consumption deflator for the real consumption wage and by the nonfarm business deflator for the real product wage.  
 Sources: Departments of Commerce and Labor, and Council of Economic Advisors.

As real wage = MPN, it is no surprise that labor productivity and real wages move closely in the data. Output per worker is the average product of labor (APN). Recall that  $MPN=0.7*APN$ , hence real wages and APN

12- The number of hours that a typical worker works in a week has fallen from an average of 50 to 40, however, the real wages per week has increased across the same time period. Why real wages per week have increased across time?

Real wages per hour equal the marginal product of labor. Hence, real wage per hour must satisfy

$$w = 0.7*A*(K/L)^{0.3}$$

real wages rise due to a rise in capital per unit of labor, or due to a rise in total factor productivity, i.e. A. Even though, the weekly supply of labor per worker has fallen by 20 percent, MPN per hour has risen more than 20 percent to raise weekly real wages of a typical worker.

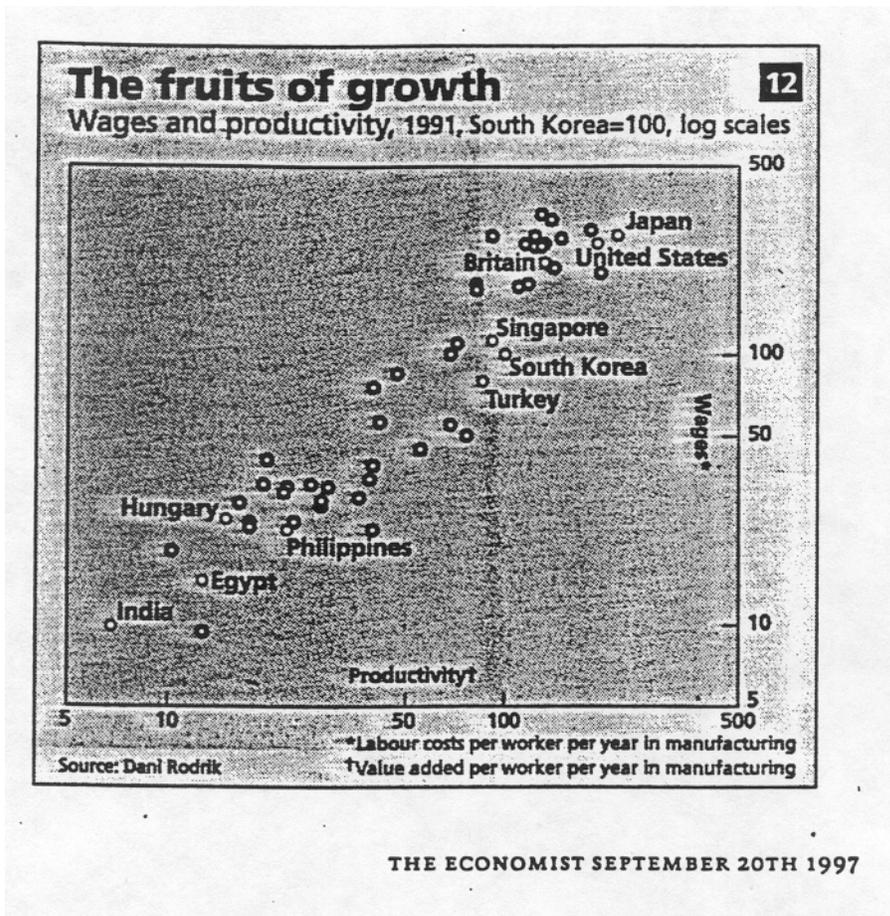
13- Below is a graph that shows the value-added per worker and real wages in various economies.

a. Why are real wages and value-added per worker positively related? Provide a precise economic argument.

Profit maximizing firms will only demand workers to the point where real wage = MPN, where MPN is the marginal product of labor. Hence in the economy real wages are determined by the marginal productivity of the worker (which is analogous to value-added per worker). Given this relationship between real wages and MPK it immediately follows that real wages will be higher in economies where value-added per worker is higher – this is exactly what the enclosed graph reveals. Developed economies have higher real wages as the average quality (education) of the workforce is higher.

b. On Average real wages are lower in the developing economies such as Kenya, Vietnam, and China (to name a few) relative to developed economies. Is this sufficient reason for firms in the developed economies to shift their capital and production activities to the developing economies? Using economic arguments justify your answer.

No, firms should not necessarily shift their capital to economies such as Kenya or Vietnam. This is so because the decision to shift capital is fundamentally based on evaluating the return to capital, MPK (ignoring risk). Capital is shifted in the direction where return to capital is highest -  $MPK = A(K/N)^{-0.7}$ . If A is higher in the developed economies, say due to a more educated work force, then the return to capital may be higher. Note this is consistent with the observation that real wages are higher in the developed economies. This argument is further strengthened by the fact that investments in the developing economies may be far more risky than in the developed economies.



14- Since early 1970's real wage rates for the lowest wage decile have fallen by about 1% per annum. The real wages for the topmost wage decile have risen by over 1% per annum. Provide an analysis regarding the changing pattern of real wages across time.

The real wages in the lowest decile have been decreasing and the real wages in the highest decile have been increasing. The economic interpretation of this phenomenon is that there have been skill-based improvements in technology, which increase the marginal productivity and hence real wages on the skilled workers. These technological changes also lower the marginal productivity of the unskilled workers and hence lower their real wages.

15- Consider the Cobb-Douglas production function:

$$Y_t = A_t K_t^b L_t^{1-b}$$

Where Y is output; K is capital; and L is labor hours worked, A is total factor productivity, and b is .3. Analyze how the marginal productivity of labor changes when:

- a. A increases by 10%
- b. K increases by 10%

- c. L increases by 10%
- d. b falls from .4 to .3

Answer

$$Y_t = A_t K_t^b L_t^{1-b}$$

The marginal productivity of labor (MPL) is:

$$(1-b) \cdot A_t (K_t / L_t)^b$$

where b is initially 0.3

- a. if A increases by 10% then so does the MPL
- b. If K increase by 10% then K/L is 10% higher and MPL is higher by a factor of  $(1.10)^{0.3} = 1.029$ . Thus MPL rises by 2.9%
- c. If L increase by 10% then K/L is 10% lower and MPL is lower by a factor of  $(1/1.10)^{0.3} = 0.972$ . Thus MPL falls by 2.8%
- d. If b were 0.4 and then fell to 0.3 (its current level) the new level of MPL relative to the old level is:

$$(1-0.3) \cdot (A_t (K_t / L_t)^{0.3}) / (1-0.4) \cdot (A_t (K_t / L_t)^{0.4}) = (0.7/0.6) \cdot (K_t / L_t)^{-0.1}$$

This is greater than one (and so MPL rises) if (K/L) is less than 4.67. If K/L is greater than 4.67 then the MPL falls as b declines from 0.4 to 0.3.

## Additional questions and definitions

### How is productivity defined?

Productivity is a measure of economic efficiency which shows how effectively economic inputs are converted into output.

### Why is productivity measurement important?

Advances in productivity, that is the ability to produce more with the same or less input, are a significant source of increased potential national income. The U.S. economy has been able to produce more goods and services over time, not by requiring a proportional increase of labor time, but by making production more efficient.

### What is the difference between labor productivity and TFP?

Labor productivity is the ratio of the output of goods and services to the labor hours devoted to the production of that output. TFP relates output to a combination of inputs used in the production of that output, such as labor and capital or labor, capital, energy and materials. Capital includes equipment, structures, inventories, and land.