Information Capital, Firm Dynamics and Macroeconomic Performance*

José Wynne†

12/21/02

Abstract

This paper studies the role of information capital accumulation in firm dynamics and the macroeconomic performance of economies that deal with repeated adverse selection in credit markets. With asymmetric information, it takes time for good firms to build up financial reputation and net worth. Over time the best firms pay lower interest rates and grow at a speed positively associated with macroeconomic conditions. An unexpected temporary shock partially destroys the firms’ net worth, slowing down the accumulation of information capital and deteriorating aggregate output performance. This channel is shown to be responsible for generating both strong endogenous persistence and amplification at the aggregate level. Aggregate and firm dynamics are consistent with stylized facts.

*I gratefully acknowledge helpful comments and suggestions from Michele Boldrin, John Coleman, Gary Hansen, Hugo Hopenhayn, Tim Kehoe, Pablo Lopez Murphy, Lee Ohanian, Joseph Ostroy, Victor Rios-Rull, Jean-Laurent Rosenthal, Carolyn Sissoko, Aaron Tornell, Federico Weinschelbaum, Kirk White and participants in seminars at UCLA, University of Texas, Austin, University of Chicago, University of Illinois, Urbana-Champaign, Duke University, 2002 SED Conference and the Dynamic Macroeconomic Workshop at Vigo, Spain. This paper is an improved version of Chapter 1 of my PhD thesis at UCLA, chaired by David Levine, to whom I am indebted for efficient advice and support.

†Duke University, Fuqua School of Business.
1 Introduction

In the last decade we have witnessed episodes of successful and unsuccessful speculative attacks on domestic currencies, especially in emerging economies. Interestingly, in some of these episodes we observed that these economies entered long recessions even when the attacks were unsuccessful and confidence was recovered swiftly.\(^1\) The aftermath of these episodes was generally characterized by financial distress, particularly for small firms (most of which are dependent on banks for financing investments).

In this work I attempt to rationalize how an unexpected and uncorrelated shock to interest rates is capable of generating a long lasting recession through an endogenous transmission mechanism. At the same time, I try to explain why small firms experience financial distress even after the cost of capital for the economy returns to normal levels.

The most important feature of the environment studied here is the existence of an asymmetric information problem between entrepreneurs and banks about the entrepreneur’s managerial talent. These talents determine the firms’ chances of failing versus staying in business from one period to the next.\(^2\) While each entrepreneur knows his own talent, banks are unable to observe it. Thus adverse selection within each cohort arises as a consequence of the informational asymmetry in credit markets. This is inefficient since highly productive entrepreneurs pay the same cost of external finance as entrepreneurs with lower productivity. Moreover, the adverse selection problem is repeated in nature because entrepreneurs keep the same talent over time and it takes time to build financial reputation.

This environment resembles some of the features in Diamond (1989) where firms face repeated adverse selection and reputation is acquired over time. This economy differs from the one in Diamond (1989) in dimensions that are essential for these credit market imperfections to explain both typical firm dynamics and aggregate output responses to shocks. First, firms are allowed to build up net worth, which plays an important role in solving the asymmetric information problem with banks. When entrepreneurs self-finance a higher fraction of the

\(^1\) See the case of Argentina 1995 below.

\(^2\) Firms might also exit due to macroeconomic reasons, although these chances are independent of entrepreneurs’ talents.
cost of production, they signal lenders that they are a better type and have access to lower interest rates, as casual observation suggests. Eventually, when the best entrepreneurs accumulate enough wealth the adverse selection problem within a cohort is resolved—although it takes time for this to happen. This implies that the reputation “effect” where worse types mimic the best type is only temporary.\(^3\) In the meantime, the cost of capital for the best firms falls.

Second, firms use a decreasing return to scale technology over their lifetime and their size is endogenously determined together with their financial reputation. Because of this, adverse selection produces under-investment instead of over-investment in the economy—this is a crucial element in reconciling the fact that credit market imperfections worsen in the downturn of the business cycle. When an unexpected aggregate shocks hit the economy, it destroys some of the firms’ net worth and slows down the information revelation process. Consequently the amount of information capital in the economy shrinks, the adverse selection problem worsens, and aggregate investment and output fall below steady state levels, displaying a recession. With over-investment, output would expand when financial imperfections are exacerbated after a shock.

This feature of the environment is also responsible for generating firm dynamics that are consistent with stylized facts both in steady state and in the downturn of the business cycle.\(^4\) The best firms survive with higher probability, accumulate net worth, and hence have access to lower interest rates. Firms with high enough probability of surviving would be willing to initially reinvest all their cash flows to gradually separate from lower types. As lower types separate, the cost of capital decreases for the best firms. Because technology exhibits decreasing returns, firms grow as they age. Also, smaller firms have higher productivity and optimally tend to finance externally a higher fraction of their cost of production. Higher leverage makes them more vulnerable to unexpected shocks, resulting in smaller firms facing

---

\(^3\)In Diamond (1989) this reputation effect can be permanent.\(^4\)See Jovanovic (1982) for a story where firm dynamics are also driven by learning about the firms’ productivity. In that environment there is no financial imperfection because everybody learns the firms’ type at the same time (no asymmetry of information).
more severe financial problems in the downturn of the business cycles.5

Third, a contract where banks screen the entrepreneurs’ talents from the start are not optimal even though banks live forever and have full commitment technology. This is a more subtle point, although an important one. In such contracts banks would collect all the firms’ cash flows for a number of periods and then rebate them to the firm’s owner contingent on their survival. That contract could screen the types and make the adverse selection problem disappear because only those with low probability of failure would be willing to take it. But this contract is not optimal in the environment developed in this paper. I show that the optimal contract is the one that provides incentives for entrepreneurs to close without defaulting whenever it is possible, and that this is achieved by a contract where banks break even in every period.

The model is capable of producing a long-lasting endogenous transmission mechanism after a one period shock to interest rates. This happens for two reasons. First, the speed at which information is revealed is decreased when firms are surprised by a bad shock that reduces their net worth. Slow recovery of the firms’ net worth leads to a slow information revelation process since banks use this variable as a screening device in financial contracts. In the meantime productive firms pay higher interest rates compared to steady state levels (while bad firms pay lower rates). Thus, aggregate economic performance deteriorates because of this inefficiency. I call this the “net worth” effect. Second, when macroeconomic conditions deteriorate, more firms might exit the industry compared to normal times. This increase in the exit rate destroys not only present but also future output since the production levels of exiting firms can only be resumed once younger generations pass through the costly screening process of building reputation over time.

The model economy generates strong output correlation after a one period shock to the interest rate but it fails to fully replicate the sizable economic downturns experienced in these economies when reasonable parameter values are adopted. Nonetheless output responses are

---

5See Evans (1987) on the relationship between firm growth, age and size in the US. Also see Gertler and Gilchrist (1994) and Fazzari, Hubbard and Peterson (1988) for facts on aggregate shocks, liquidity constraints and investment dynamics.
greatly amplified when there are other distortions in the economy that look like a total factor productivity (TFP) drop. Importantly, the TFP process needed to generate the desired output response is shown to be sterile on its own. It is the feedback from this process to the informational asymmetry that matters. A lower TFP shrinks the firms’ profits. In turn, lower profits slow down the accumulation of net worth and with that the information revelation process. Slowing down the information revelation process exacerbates the net worth effect and amplifies the aggregate response.

To understand the role of firm dynamics in this story I present simulations for firm performance in steady state and in the aftermath of the shock. Time series and cross sectional firm data produced by simulations confirm that the information revelation process is slowed down in the business cycle. This is reflected in temporarily higher lending rates, lower net worth and hence lower input-output scales of firms along the recession compared to steady state levels.

1.1 Related Literature on the Credit Channel

In the last fifteen years there has been an increasing mass of literature emphasizing the importance of asymmetric information problems in financial relationships to the credit cycle. Most of the theoretical literature focuses on the idea that it is costly for lenders to verify the output produced by ex-ante identical borrowers. Williamson (1987), Bernanke and Gertler (1991), Gertler (1992), Fuerst (1995) and its comment by Gertler (1995) and Cooley and Nam (1998) are part of this literature. I depart from this literature in that in my story firm heterogeneity and dynamics are essential for the behavior of aggregate output.

The adverse selection problem modeled in this work is similar in spirit to Bernanke and Gertler (1990). There firms differ ex-ante in their success probability –which is private information– and net worth helps banks to imperfectly screen firms’ types. In my work I exploit this idea, but it differs in two important aspects. First, I allow the scale of production to be endogenously chosen while they have a fixed scale. Second, I study the adverse selection problem in a dynamic setting while theirs is a static one.

Cooley and Quadrini (1998), Albuquerque and Hopenhayn (2002) and Clementi and
Hopenhayn (2002) develop models with exogenous and endogenous financial imperfections to explain some stylized facts for US firms. Some of these facts are also matched by firm dynamics in this work, although I have reduced the heterogeneity (and limited the realism) to handle firm dynamics not only in steady state but also along the downturn of the business cycle. This allows me to study the interactions between firm dynamics and aggregate output performance.

Heterogeneous firms have been incorporated into dynamic models in the literature on credit channels before. Bernanke, Gertler and Gilchrist (1998) present a model where heterogeneity is due to ex-post realizations of output. My work differs from theirs in three respects. First, they build the financial accelerator on top of a general equilibrium model with exogenously driven fluctuations. Second, reputation plays no role in their story while in my work firms that have already gained a good reputation will continue funding their production at low interest rates in the aftermath of the shock. Finally, firm heterogeneity plays no role in the business cycle (the ratio of debt to net worth and the lending rates are the same for all firms in the same sector).

Cooley and Quadrini (1999) study the differential effects of a monetary policy on small and large firms under one period financial contracts. They impose an AR(1) stochastic processes to obtain output persistence. Financial frictions do not add much to persistence because only a small number of firms is financially constrained. In a another paper with Marimon (2000) they show in a fully optimal dynamic and stochastic environment that limited contract enforceability amplifies the response of output. But again persistence is exogenous: vintages of capital arrive with a certain productivity that they keep over time. As they state: “In this economy a business cycle expansion is driven by the arrival of more productive technologies…”

Aghion, Bachetta and Banerjee (1999) study a dynamic open economy model with tradable and non-tradable goods where the non-tradable good is an input of production in the

---

6 For a study on entry and exit in a stationary environment with no financial frictions see Hopenhayn (1992).
7 The total factor productivity shocks are highly autocorrelated (0.95).
8 In Kiyotaki and Moore (1997) the level of leverage is exogenously assumed.
tradable sector like here, but there are differences in several important dimensions. Financial frictions are built into the tradable sector. Firms dynamics are driven by exogenous borrowing constraints and saving rates. And the non-tradable output is fixed along the cycle.

1.2 An example: Argentina 1995

The Mexican crisis that took place in December 1994 spilled over to the Argentine economy in the first quarter of 1995. The average deposit rate in Argentina sharply increased in that quarter, and it returned to original levels right away. Yet, this short-period shock had long-lasting and profound effects on this economy. The economy entered into a recession that lasted almost three years, as the series on the deviations from trend of the Industrial Production Index below shows. This fact seems to suggest that there are strong aggregate endogenous transmission mechanisms at work, a feature that the standard literature on real business cycles is not able to explain.

![Av. Deposit Rate and Industrial Production](image)

Figure 1:

Aside from these macroeconomic facts, policy makers in Argentina have repeatedly shown

---

concern regarding the inability of small firms to recover in the aftermath of the crisis mainly because of their limited credit access, which motivates the credit channel as a natural mechanism for transmission.\footnote{See Ramos (1998) for evidence on a credit crunch for this episode.}

Although the case of Argentina in 1995 is a neat example of an economy that enters a long recession after a temporary shock to the interest rate, one should expect the same mechanisms studied here to be present in cases where shocks exhibit more autocorrelation.

\section{The model}

There are two types of agents in the economy, workers and entrepreneurs, and three sectors, the tradable, the non-tradable goods sectors and the financial sector. Workers and entrepreneurs consume tradables, which are produced using capital and non-tradable goods. The non-tradable good is produced using capital and labor.\footnote{The non-tradeable good is also non-storable.}

There is a mass $\mu$ of infinitely lived workers. They are infinitely endowed with labor at every period of life and they consume only tradable goods. They maximize the following utility function:

\begin{equation}
U^W_t = E_t \sum_{j=t}^{\infty} \left( \frac{1}{r} \right)^{j-t} \left( \frac{c^W_j - l^W_j}{1 - \sigma} \right)^{1-\sigma} \omega, \sigma > 0
\end{equation}

where $c_t$ and $l_t$ represent the consumption of tradable goods and labor supplied at time $t$. Superscript $W$ stands for workers. Labor can be supplied at the market wage rate $w_t$. The discount parameter is set equal to $1/r$, where $r$ is one plus the international interest rate faced by this economy in good times. Uncertainty in this economy comes from the international interest rate, with $r_t$ being i.i.d., $r_t \in \{r, \tau\}$ where $\tau > r$ and probability $P(r_t = r) = \rho$ being high. Note that this process is uncorrelated because we want to study how our economy reacts to a non-correlated shock to the interest rate.

At each period of life workers decide how much of their wealth to allocate in consumption and savings, carried via riskless assets or bonds. I assume that all asset holdings between
period $t$ and $t+1$ are represented by a non-contingent portfolio $\Gamma_t$ expressed in consumption goods. Hence, the workers’ intratemporal budget constraint at every period $t$ is given by,

$$c^w_t + \Gamma_t \leq w_t l_t + r_{t-1} \Gamma_{t-1} \quad \forall t \geq 0$$

(2)

where $r_t$ is the international interest rate between period $t$ and $t+1$.

Entrepreneurs are also infinitely lived agents and consume tradable goods. A unit mass of them is born at every period and they maximize their utility given by

$$U^E_t = E_t \sum_{j=t}^{\infty} \gamma^{j-t} [c^E_j - \gamma g(m_t, q_t)]$$

(3)

where superscript $E$ stands for entrepreneur, $\gamma$ is the discount factor and the function $g(m_t, q_t)$ is the cost of exerting managerial effort ($m_t \in [0,1]$) to make the firm survive one period, which depends on the firms cash flow $q_t$ as I specify and explain in detail below. Entrepreneurs are endowed with some labor time in their first period of life and with the talent to produce and sell non-tradable goods in all remaining periods, as long as they remain in business. Additionally let $\gamma < 1/r$, which implies that entrepreneurs prefer to consume rather than save at the rate $r$. However, they will end up contributing most of the savings in the economy because they have access to very profitable investment opportunities.

In each period entrepreneurs face a probability of being driven out of the market that depends on their own type and on aggregate conditions (see Figure 2). At every date and after inputs have been allocated to the firm, entrepreneurs can be driven out of the market with probability $1 - p$, where the parameter $p$ constitutes each entrepreneur’s type or characteristic. When this is the case, they obtain no output and the firm disappears. On the other hand, with probability $p$ two things can happen to the entrepreneur. If he is lucky and with probability $\xi_t$ he encounters no problems (good times keep rolling for the firm), in which case it succeeds to produce and survives one more period. But with probability $(1 - \xi_t)$ trouble arises and the situation requires the entrepreneur’s managerial effort $m_t$ to complete its production plan for the period. After the entrepreneur chooses his effort level $m_t$, the firm completes its plan with probability $m_t$, and fails to do that (getting no
output) with probability $1 - m_t$. Thus, this variable is both the managerial effort exerted by the entrepreneur and the probability that the manager successfully fulfill his plan in times of distress. In either of these two cases and after “trouble” arrives the firm exits and the entrepreneur is out of business.

Output is unobservable but verifiable at a cost. I assume it is costly for third parties to distinguish successful from distressful situations—the first three states at each period. For simplicity I also assume it is costless to verify when firms go out of business for own reasons—the last state at each period. Verification costs are some function of output, say $\lambda P_{t+1} y_{t+1}^N$, but verification will never happen in equilibrium.

Only entrepreneurs know their own type, and there is a continuum of them. When born, these types are drawn from a density function $f(p)$ with support $p \in [0, 1]$ and $f(p) > 0 \forall p$, which is public knowledge. The asymmetry of information about their characteristics is the core of this paper because it drives the main results. Effort is assumed to be non-observable (or non-verifiable) which implies that it is driven by incentives since it cannot be contracted. The non-observability of effort creates a moral hazard problem that is essentially

---

13 This is without loss of generality
Probability $\xi_t$ is the same for all entrepreneurs and it is correlated with aggregate shocks (to the interest rate) in the following way: $\xi_t = \xi$ in good times and $\xi_t = \overline{\xi}$ in bad times, with $\overline{\xi} \leq \xi$. This allows us to experiment with different exit rates to study the impact of information capital loss in the economy.

Overall, firms’ exit rates depend on the lack of own managerial talent –which drives the probability $1 - p$– and aggregate conditions –which determine the probability of times of distress $p(1 - \xi_t)$. Again, in the latter case firms only fulfill their production plan with probability $m_t$, which is chosen by the manager at this stage of the (sub)game.

**Assumption 1:** $(1 - \xi)\lambda$ is big enough.

This assumption is important to make the incentive issues in the financial contract relevant from the ex-ante perspective. I discuss this point in detail below.

Furthermore let $\bar{p}_n(p^*) \equiv E[p/p > p^*, f(p)p^{n-1}]$ be the average value of $p$ conditional on $p > p^*$ and for a density function $f(p)p^{n-1}$.

**Assumption 2:** For a given $p^* \in [0, 1]$, $\bar{p}_n(p^*) - \bar{p}_{n+1}(p^*)$ is not too big $\forall n$.

The assumption is satisfied by most density functions. It is not essential for the model but it simplifies computations. It simply says that the average type on the same support $[p^*, 1]$ for two consecutive generations should not be too far apart.

Entrepreneurs produce homogenous goods and are subject to idiosyncratic shocks. They have the managerial talent to produce and sell non-tradable goods. Production of non-tradable goods at time $t + 1$ requires capital $(k^N)$ –which is a tradable good– and labor $(l)$ to be input at $t$ after the realization of the current interest rate, and managerial effort if needed. The function adopted is

$$y^N_{t+1} = \theta_{t+1}(k^N_t)^\alpha(l_t)^\beta \begin{cases} 0, & \text{if } \alpha + \beta < 1 \\ \theta_{t+1}, & \text{if } \alpha + \beta \geq 1 \end{cases} \text{ i.i.d. } \theta_{t+1} = \begin{cases} 0, & \text{with prob } p(\xi_t + (1 - \xi_t)m_t) \\ \overline{\theta}, & \text{otherwise} \end{cases}$$

where $y^N$ stands for non-tradable output. The random variable $\theta_{t+1}$ can take two values high, $\overline{\theta}$, or 0, and it is realized once inputs have been chosen. When entrepreneurs go out of business $\theta_{t+1} = 0$ for all successive periods. The decreasing returns technology can be
interpreted as management being a fixed indivisible factor of production where one manager is required for each firm to operate.

To gain in simplicity the cost function for effort is chosen so that the marginal cost of effort is linear and increasing in the firm’s cash flows $q_t$, defined as current sales minus per-period cost of the loan. In particular it is convenient to use the following simple functional form.

**Assumption 3:** $g(m, q) = \frac{1}{\chi} m^\chi q$, with $\chi$ greater than, but close to 1.

Under Assumption 3 it is easier to characterize sufficient conditions for which one period contracts are optimal in this environment.

Because this paper focuses on the aggregate consequences of macroeconomic shocks, I assume that this uncertainty is not diversifiable for entrepreneurs. Although the non-diversifiability could have been generated by introducing some small fixed cost of contracting in terms of aggregate variables I choose to clearly state this as an assumption.14

**Assumption 4:** Entrepreneurs’ financial contracts cannot be written in terms of the aggregate shocks.

Capital in the non-tradable sector ($k_N$) depreciates at a rate $\delta^N$; it is a tradable good; it is rented at a price $r_k$ per unit of time and the rental price of capital is settled before the realization of the interest rate. This last assumption allows isolation of the effect of the interest rate shock on the credit channel since the rental price of capital in this sector will not change.

The tradable sector is composed of a mass of firms with constant return to scale technology of the form

$$y_{t+1}^T = A_t F(k_t^T, y_t^N)$$  

This sector produces tradables at time $t+1$ by inputting capital ($k^T$) and non-tradable goods ($y^N$) at time $t$. Technology exhibits the usual assumptions on marginal products and concavity. Notice that the total factor productivity $A_t$ depends on time. In the simulation

14 A very small fixed cost would suffice to support the result as the probability of getting a bad macroeconomic shock goes to zero (as in this paper).
exercises I let the total factor productivity depend on past aggregate non-tradable output to explore some interesting interactions between financial frictions and other sources of imperfections in the economy. Capital in this sector depreciates at the rate $\delta^T$ and can be produced by transforming tradable goods in the same period.

Finally, the model is completed with the financial sector. There is a mass of infinitesimal banks. They borrow from depositors and international investors, lend to firms and do verifications whenever necessary. Moreover, they are owned by risk-neutral-foreign agents with unlimited liability.

**Assumption 5:** Banks are able to fully commit to financial contracts (one side commitment).

Lastly, but not least importantly

**Assumption 6:** Contracts are private information for the parties.

Third parties of the contract observe neither the terms nor the outcome. Thus banks observe the entrepreneur’s credit history provided that the relationship lasts. This is consistent with evidence for the US and likely in less developed financial markets.\(^{15}\)

In the rest of the paper I analyze the limiting case of this economy when $\rho \rightarrow 1$, that is, when the probability of getting to a high interest rate state goes to zero for all periods. Then we simulate the dynamics of the economy for a one period realization of a bad interest rate shock ($r_t = \bar{r}$). The computational advantages of this case are enormous since it is in practice a non-stochastic economy. After a large enough sequence of good realizations of the interest rate, prices will be constant. Then all state variables will be computed for the steady state (note that the state space is big due to the heterogeneity of the model). After the shock the dynamics of the economy generate a deterministic sequence of prices and state variables returning to their steady state values. The non-stochastic experiment allows us to simulate the model and gain insights into the transmission mechanisms underlying business cycle downturns in economies with informational imperfections in credit markets, which is the focus of this paper.

The roadmap for the next sections is as follows. First I make a conjecture about the

\(^{15}\)See Petersen and Rajan (1994).
nature of the financial contracts that arise in equilibrium. I present the agents’ problems under those contracts, define the equilibrium for this case, and show that equilibrium exists under this conjecture. Finally I show that the conjectured contracts are equilibrium contracts under the assumptions of the model. Thus,

Conjecture 1  Equilibrium financial contracts are such that banks are expected to break even each period.

Financial contracts are neither front-loaded nor back-loaded.

2.1 The worker’s problem

Workers solve the following problem

$$\max_{\{c^w_t, l_t, \Gamma_t\}} U^W_t = E_t \sum_{j=t}^{\infty} \left( \frac{1}{r} \right)^{j-t} \frac{(c^W_j - l^\omega_j)^{1-\sigma}}{1-\sigma}$$

subject to

$$c^w_t + \Gamma_t \leq w_t l_t + r_{t-1} \Gamma_{t-1} \quad \forall t \geq 0$$

$$\Gamma_0 \text{ and } \{w_t, r_{t-1}\}^{\infty}_{t=0} \text{ given.}$$

$$\lim_{t \to \infty} \frac{\Gamma_t}{\Pi_{\tau=0}^{t} r_{\tau}} \geq 0$$

Equation (8) rules out Ponzi schemes. The first order conditions for this problem in the limiting case where the probability of the shock goes to zero are:

$$c^W_t - l^\omega_t = \left( \frac{r}{r_{t}} \right)^{1/\sigma} \left( c^W_{t+1} - l^\omega_{t+1} \right) \quad \forall t > 0$$

$$l_t = \left( \frac{w_t}{\omega} \right)^{\frac{\omega}{1-\sigma}} \quad \forall t > 0$$

$$\sum_{t=0}^{\infty} [c^w_t + \Gamma_{t+1} - w_t l_t - r_{t-1} \Gamma_t] \leq 0$$
and the transversality conditions for assets.

Equation (9) is the law of motion for consumption and Equation (10) is the worker’s labor supply. See that labor supply has no income effect. Thus allocations in the supply side of the economy can be computed independently of the consumption path, simplifying the work a great deal. See that with these preferences, workers try to smooth \((c_{t+1}^w - l_{t+1}^w)\) but not consumption.

### 2.2 The tradable sector’s problem

There is no uncertainty for this sector because firms buy inputs at known spot prices and have a deterministic production process. They maximize profits and the first order conditions of this sector’s problem are

\[
A_t F_y(k_t^T, y_t^N) = P_t^N r_t \tag{12}
\]

\[
A_t F_k(k_t^T, y_t^N) = (r_t - 1 + \delta^T) \tag{13}
\]

Implying that the value of the marginal product of inputs should equal their marginal cost. Taking into account that \(A_t\) is a state variable of the model we get

**Proposition 2** For each possible \(A_t\), there exists only one equilibrium non-tradable good price corresponding to each interest rate, \(P_N^N(r, A_t)\) and \(P_N^N(\bar{r}, A_t)\) with \(P_N^N(r, A_t) \geq P_N^N(\bar{r}, A_t)\). Moreover \(\frac{\partial P_N^N(.,A_t)}{\partial A_t} > 0\).

**Proof.** See Appendix B.

The fall in the intermediate non-tradable output price surprises firms expecting good conditions (high prices). Furthermore it triggers a net worth (or wealth) effect in the non-tradable or bank-dependent sector that impacts on allocations as I show below.

---

16 Given our assumptions workers don’t have a stationary steady state consumption process in the stochastic version of this environment. Although there are ways of dealing with the assumptions to get stationarity, I choose not to because of two reasons: 1) consumption is not the focus of the paper, and 2) in practice we deal with a non-stochastic economy that is hit by a once and for all high interest rate shock.
2.3 The entrepreneurs’ problem

Entrepreneurs differ across talents, ages, and particular history since useful information is revealed over time. For this reason it is convenient to use the following notation. Let subscript $\tau$ be the period of the entrepreneur’s birth and let $nt$ denote an entrepreneur of age $n = t - \tau$ at time $t$. Superscript $p$ denotes the type but I only use it when is strictly necessary.

In the first period of life the entrepreneur supplies his labor endowment and saves all his income to get ready for next period. In all successive periods, he allocates wealth $W_{nt}$ between consumption $c_{nt}$ and investment in the firm $e_{nt}$ (equity). Capital $k_{nt}^N$, labor $l_{nt}^N$, and effort level $m_{nt}$ are also chosen, given input prices, expected output prices and the bank’s participation constraint under our conjecture.

By letting $\rho \rightarrow 1$, financial contracts and decision variables at time $t$ for an entrepreneur of age $t - \tau$ will arise from the following problem

$$\max_{\{c_{nt}^E, e_{nt}, m_{nt}, k_{nt}^N, l_{nt}^N, \bar{y}_{t+1}, i_{nt}, M_{nt}, \bar{p}_{nt}\}} U_{nt}^p = E_t^p \sum_{j=t}^{\infty} \gamma^{j-t} \left[ c_{nt}(j), e_{nt}(j), m_{nt}(j) - \frac{\gamma}{\chi} m_{nt}(j), q_{nt}(j) \right]$$

subject to

$$c_{nt}^E + e_{nt} \leq W_{nt}$$

$$\bar{y}_{t+1} = \bar{q} (l_{nt}^{\alpha} i_{nt}^{\beta})$$

$$q_{nt} = \max \left\{ P_{t+1} \bar{y}_{t+1} - i_{nt} M_{nt}, 0 \right\}$$

$$W_{n+1t+1} = \begin{cases} q_{nt} \text{ w/Pr } p[\xi + (1 - \xi)m_{nt}] \\ 0 \text{ otherwise} \end{cases}$$

$$r_k k_{nt}^N + w_{nt} l_{nt}^N \leq c_{nt} + M_{nt}$$

$$\bar{p}_{nt} (1 - \xi) (1 - \bar{m}_{nt}) \lambda E_t P_{t+1} \bar{y}_{t+1} + r_t M_{nt} = \bar{p}_{nt} [\xi + (1 - \xi) m_{nt}] i_{nt} M_{nt}$$

$$\bar{p}_{nt} = E_t [p | p \in \mathcal{Y}(e_{nt}, i_{nt}, M_{nt}, \bar{p}_{n-1t-1}), f_{nt}(p)]$$

$$\{r_k, w_t, P_{t+1} (r_{t+1}, A_t+1)\}, \text{ processes for } r_t \text{ and } A_t \text{ given}$$

$$\forall n \text{ and } t$$

\(^{17}\)He doesn’t save at the interest rate $r$ because $\gamma \leq 1/r$. 

16
The problem maximizes the expected utility of a type \( p \) entrepreneur subject to a set of constraints that I now explain. Equation (15) is the budget constraint. Consumption plus investment in the firm \( (e_{nt}) \) cannot exceed the entrepreneur’s wealth, a state variable. Equation (16) defines output when \( \theta_{t+1} = \bar{\theta} \). Equation (17) is the firm’s cash flow when \( \theta = \bar{\theta} \): the max between sales minus the cost of borrowing for the period and zero, given that firms have limited liability. This borrowing cost is just \( i_{nt}M_{nt} \), where \( i_{nt} \) is one plus the lending rate of the financial contract and \( M_{nt} \) is the amount borrowed. Equation (18) describes the law of motion for wealth. If the firm is successful, which occurs with probability \( p[\xi + (1-\xi)m_{nt}] \), next period wealth is just the firm’s cash flow of the current period. Otherwise it is zero (and the firm goes out of business). Equation (19) is the firm’s resource constraint: total cost of inputs must be financed with equity and liability. Note that the rental price of capital is constant because capital is priced before the realization of the aggregate shock. Equation (20) is the bank’s participation constraint under Conjecture 1: banks’ expected profits are zero each period. The RHS is the bank’s expected revenue, while the LHS is its expected cost (note the verification cost). See that the bank’s expected revenues and costs are computed using: 1) bank’s expectation about the average type taking the contract \( \bar{p}_{nt} \), because banks do not observe the true type \( p \), and 2) the bank’s estimate of the entrepreneur’s effort level in case of distress, \( \bar{m}_{nt} \). Under rational expectations \( \bar{m}_{nt} = m_{nt} \) but since effort is not observable—or verifiable—for the bank it cannot be contracted; hence \( \bar{m}_{nt} \) is not part of the entrepreneurs’ decision variables. This is the moral hazard problem. Finally Equation (21) is the bank’s expectation about the entrepreneur’s type \( p \) given the bank’s information set \( \Upsilon(.) \). This information set depends on all relevant information that is verifiable: terms of the contract \( \{e_{nt}, i_{nt}, M_{nt}\} \), credit history \( \bar{p}_{n-1t-1} \) and mass of survivors in the cohort \( f_{nt}(p) \).\(^{18}\) Credit history is summarized by the bank’s belief about the quality of this firm in the previous period \( \bar{p}_{n-1t-1} \), as long as the entrepreneur is a client (Assumption 6). History only matters after a shock as I will explain later.

\(^{18}\)The mass is computed recursively in the following way: \( f_{1t}(p) = f(p) \) and \( f_{nt}(p) = f_{n-1t-1}(p)\xi_{t-1}p \) for all \( n > 1 \).
In this problem effort $m_{nt}$ affects the objective function but not the feasible set; $\bar{m}_{nt}$ but not $m_{nt}$ enters in the constraints. The objective function is\(^{19}\)

$$U_{nt}^p = c_{nt}^E + \gamma p \left\{ \xi U_{n+1}^p + (1 - \xi) \left( m_{nt} W_{nt}' - \frac{1}{\chi} m_{nt} q_{nt} \right) \right\}$$

and the first order condition for effort is

$$W_{nt}' = m_{nt}^{\chi^{-1}} q_{nt} \quad (22)$$

But $W_{nt}' = q_{nt}$, implying that $m_{nt} = 1 \forall nt$. Entrepreneurs exert full effort whenever the firm is in distress. I will show later that only one period contracts can fully resolve the moral hazard problem. For now we postpone this discussion because we are solving the model under Conjecture 1.

Under Assumption 3, $\chi$ is greater than but close to one. For convenience let $\chi \rightarrow 1$. Now the entrepreneur’s objective function simplifies to $c_{nt}^E + \gamma p \xi U_{n+1}^p$.

Given $c_{nt}$, optimality requires that inputs are chosen to maximize the firm’s expected cash flow $q_{nt}$ such that the resource constraint for firms binds (Equation (19)) and the bank participates (Equation (20)). See that the banks’ rational belief about the entrepreneur’s probability of repayment is computed for some $\bar{p}_{nt}$ and for $\bar{m} = 1$. This implies that monitoring never occurs. From the bank’s participation constraint

$$i_{nt} = \frac{r_t}{\bar{p}_{nt}} \quad (23)$$

As the bank’s perception about the entrepreneur’s type increases, the interest rate banks charge him decreases.

Let $E_t^NP_{t+1}^N$ be the expected next period price of the non-tradable good. Because $\rho \rightarrow 1$ we know that $E_tP_{t+1}^N = P_{t+1}^N$ all the time except when the shock occurs. Keeping that in mind, maximizing the firm’s cash flow implies that the optimal input choices are

$$k_{nt}^N = \left[ \frac{\bar{p}_{nt}E_t^NP_{t+1}^N \theta \alpha^{1-\beta} \beta^\delta}{w_t^\beta r_t^\gamma k_t^{1-\beta}} \right]^{1/(1-\beta)} \quad (24)$$

---

\(^{19}\) $W_{nt}' = W_{n+1}t+1$. The proof of Proposition 11 shows why payoffs in the $(1-p)$-probability state are zero in the optimal contract.
\( l_{nt} = \left[ \bar{p}_{nt} E_t P_{t+1}^N \bar{\theta} \alpha^\alpha \beta^{1-\alpha} \right]^{-1/a-\beta} \) 

(25)

Inputs depend negatively on their prices and positively on the expected final output price and the productivity parameter \( \bar{\theta} \). More meaningfully, both inputs depend positively on the average quality of the pool since the lending rate depends on it. The higher the bank’s belief about the entrepreneur’s type the lower the interest rate on loans and the greater the demand for both inputs. It is interesting to notice that inputs are not determined by the entrepreneur’s true type \( p \). Total output is determined only by the bank’s perception about the firms average quality \( \bar{P} \). This occurs because banks are the marginal suppliers of funds in the economy since they have a lower opportunity cost of funds.

From this result I state the first proposition

**Proposition 3**: (Modigliani and Miller’s Neutrality Theorem). Under complete information, the optimal amount of labor and capital hired to produce non-tradable goods is independent of the firms’ wealth.

**Proof.** See that under perfect information \( \bar{p}_{nt} = p \). ■

The basic intuition behind this theorem is simple: when entrepreneurs and banks have the same information regarding the success probability of the firm, agency problems in financial contracts disappear. Then, we know there is a financial contract such that the efficient scale of production is implemented. In a world with full information –or even a world with imperfect but symmetric information like in Jovanovic (1982)— macroeconomic shocks affecting entrepreneurs’ wealth are not capable of generating dynamics either at the firm or at the aggregate level. Under perfect information firms do not grow over time since they start up right away at the efficient level of production (different for each type). Under asymmetric information firms exhibit interesting dynamics as the banks’ perception about their quality improves over time.

In a non-stochastic environment like this one \( E_t P_{t+1}^N = P_{t+1}^N \) and the path for wealth is expected to be determined by
\[ E_t W'_{nt} = (1 - \alpha - \beta) \left( \frac{\tilde{p}_n^{\alpha+\beta} E_t P_{t+1}^{N} \tilde{\theta} \alpha^\alpha \beta^\beta}{w_t^{\alpha+\beta} r_t^{\alpha} r_k^{\beta}} \right)^{\frac{1}{1-\alpha-\beta}} + \frac{r}{\tilde{p}_n} e_{nt} \] (26)

For all periods in which there is no shock \( E_t W'_{nt} = W'_{nt} \). But because the financial contract is determined before sales realize, when the shock occurs the price of non-tradable goods will be lower than expected and so will the firm’s cash flow. I call this the net worth effect.

Before proceeding, see that the entrepreneur can only affect the bank’s belief through \( e_{nt} \), because all other variables in Equation (21) are state variables or the contract itself.

From the mechanism design standpoint we can re-think our problem as one solving for an announcement of the entrepreneur’s type \( \hat{p} \) and the financial contract \( \{i_{nt}(\hat{p}), M_{nt}(\hat{p}), e_{nt}(\hat{p})\} \). With the help of the revelation principle we can concentrate on truth telling contracts. The optimal contract under Conjecture 1 is found following two steps: 1) solving the entrepreneur’s optimal announcement \( \hat{p} \) given the financial contract \( \{i_{nt}(\hat{p}), M_{nt}(\hat{p}), e_{nt}(\hat{p})\} \) satisfying Equations (19)-(20) and \( \tilde{p}_n = \hat{p} \), and 2) solving for the financial contract given that it provides the entrepreneur with incentives for truth telling. Thus step 1) is

\[ U^p_{nt}(W_{nt}, \hat{p}_{n-1t}) = \max_{\{c^E_{nt}, \tilde{p}\}} \left\{ c^E_{nt} + \gamma \xi U^p_{n+1t+1}(W'_{nt}, \tilde{p}_n) \right\} \] (27)

subject to

\[ c^E_{nt} + e_{nt}(\hat{p}) \leq W_{nt} \] (28)

\[ W'_{nt} = (1 - \alpha - \beta) \left( \frac{\tilde{p}_n^{\alpha+\beta} E_t P_{t+1}^{N} \tilde{\theta} \alpha^\alpha \beta^\beta}{w_t^{\alpha+\beta} r_t^{\alpha} r_k^{\beta}} \right)^{\frac{1}{1-\alpha-\beta}} + \frac{r}{\tilde{p}_n} e_{nt}(\hat{p}) \] (29)

\( \{r_k, w_1, P_{t+1}^{N}\} \) and \( e_{nt}(\hat{p}_n) \) given \( \forall n \) and \( t \)

Solving this problem involves solving for the optimal announcement given the financial contracts (see that \( \{e_{nt}, i_{nt}, M_{nt}\} \) depend on the announcement \( \hat{p}_n \)).

Computing the first order condition with respect to the announcement gives

\[ \frac{\partial e_{nt}}{\partial \tilde{p}} \leq \gamma \xi \frac{\partial U^p_{n+1t+1}}{\partial W^t_{nt}} \left( \frac{\partial W'_{nt}}{\partial \tilde{p}} + \frac{\partial W'_{nt}}{\partial e_{nt}} \frac{\partial e_{nt}}{\partial \tilde{p}} \right) \quad \forall n, t \]

if \( < 0 \) for some \( nt \), then \( e_{nt} = W_{nt} \) and \( c^E_{nt} = 0 \) (30)
If $W_{nt}$ is big enough an interior solution at $t$ exists for this problem. Moreover, an interior solution is expected to exist in successive periods as long as $E_{nt} > e_{n+1t+1}(p)$. Applying the Envelope Theorem, $\frac{\partial U_{nt}^{p+1}}{\partial W_{nt}} = 1$. After rearranging terms in this FOC and imposing truth telling ($\hat{p} = p$), we obtain

$$(\alpha + \beta)\gamma\xi \left( \frac{E_t P_{t+1}^N}{u_t^{\alpha} r_t^{\alpha + \beta} r_k^{\beta}} \right)^{\frac{1}{1 - \alpha - \beta}} \hat{p}^{\frac{\alpha + \beta}{1 - \alpha - \beta}} = \frac{r_t \gamma \xi}{\hat{p}} \hat{e}_{nt}(\hat{p}) + (1 - r_t \gamma \xi) \frac{\partial \hat{e}_{nt}(\hat{p})}{\partial \hat{p}} \quad (31)$$

$$\forall n \text{ and } t.$$

Also note that the optimality condition under truth-telling gives a differential equation for net worth as a function of the announcement. Luckily this differential equation has a closed form solution.

**Proposition 4** A truth telling contract is given by

$$e_{nt}(\hat{p}) = \frac{(1 - \alpha - \beta)\gamma\xi r_t (\alpha + \beta)}{[1 - \gamma \xi r_t (\alpha + \beta)]} \left( \frac{E_t P_{t+1}^N}{u_t^{\alpha} r_t^{\alpha + \beta} r_k^{\beta}} \right)^{\frac{1}{1 - \alpha - \beta}} \hat{p}^{\frac{1}{1 - \alpha - \beta}}$$

$$i_{nt}(\hat{p}) = \frac{r_t}{\hat{p}}$$

$$M_{nt}(\hat{p}) = r_k k_{nt}^N(\hat{p}) + w_t l_{nt}(\hat{p}) - e_{nt}(\hat{p})$$

$$\forall \ n \text{ and } t.$$

**Proof.** See Appendix.

Interestingly, the amount financed internally under a truth telling contract increases with $\hat{p}$, a parameter that also represents the size of the project, and with $\gamma$, indicating that banks will lend proportionally more when entrepreneurs are more impatient. As a thought experiment see that by letting $\gamma \xi r = 1$, the net worth required becomes

$$TC_{nt}(p) = r_k k_{nt}^N(p) + w_{nt}(p) = (\alpha + \beta) \left( \frac{E_t P_{t+1}^N}{u_t^{\alpha} r_t^{\alpha + \beta} r_k^{\beta}} \right)^{\frac{1}{1 - \alpha - \beta}} \hat{p}^{\frac{1}{1 - \alpha - \beta}} \quad (32)$$
which is the total cost of production for a firm with characteristic $p$. Thus when $\gamma \xi r = 1$, $M_{nt} = 0$ for all announcements: the owner will only have incentives to truthfully reveal his type when there is no borrowing! When $\gamma \xi r < 1$ there is borrowing under truth telling.

These contracts are only truth telling if the firm takes the same contract in the future. In equilibrium this occurs because banks can fully commit to offer the same contract even though the true type is revealed to the lending bank (Assumption 5).

We conclude that when entrepreneurs are wealthy enough the allocation of credit in the economy is efficient. In other words this economy will produce as much as an economy with no informational imperfections. Unfortunately, initially entrepreneurs are not that wealthy and the interior condition that we just found does not exist for all types. In particular, higher types won’t be able to signal their success probability even though they will be investing all their wealth. For what follows it is useful to define the two types of contracts that will arise in equilibrium

**Definition 5** A separating financial contract $\{i_{nt}^{Sep}(p), M_{nt}^{Sep}(p), e_{nt}(p)\}$ is a simple debt contract in which only one type of entrepreneur participates.

**Definition 6** A pooling financial contract $\{i_{nt}^{Pool}(p), M_{nt}^{Pool}, e_{nt}\}$ is a simple debt contract in which more than one type of entrepreneur participates.\(^{20}\)

See that we use separating contracts in the intertemporal truth telling sequence of contracts computed for the case where entrepreneurs are wealthy enough. Figure 3 plots $e_{nt}(\hat{p}_{nt})$ when $W_{nt} \geq \hat{e}$ and there is an interior solution for all types.

When entrepreneurs don’t have enough wealth, types above $p^A$ cannot be screened. In such cases pooling will arise. Below I show that there will be only one such contract in equilibrium. In this case allocations come from solving the following problem

\[
U_{nt}^p(W_{nt}, \bar{p}_{n-1t-1}) = \max_{\{i_{nt}^E, e_{nt}, \bar{p}_{nt}, \bar{p}_n\}} \{c_{nt}^E + \gamma p \xi U_{n+1t+1}^p(W_{nt}^f, \bar{p}_{nt})\}
\]  

\(^{20}\)Note that the pooling contract does not depend on types.
subject to

\[ c_{nt}^E + e_{nt} \leq W_{nt} \quad (34) \]

\[ W_{nt}' = q_{nt}(\bar{p}_{nt}, e_{nt}) \quad (35) \]

\[ \bar{\pi}_{nt} \equiv E [p | p > p_{nt}^* > p_{n-1t-1}^*, f_n(p)] \quad (36) \]

\[ \gamma p_{nt}^* \bar{\pi}_{nt} q_{nt}(\bar{p}_{nt}, e_{nt}) \leq \gamma p_{nt}^* \bar{\pi}_{nt} q_{nt}(p_{nt}^*, e_{nt}(p^*)) + [e_{nt} - e_{nt}(p_{nt}^*)] \quad (37) \]

\{r_k, w_t, P_{t+1}^N\} given \forall n and t

where

\[ q_{nt}(\bar{p}, e) = (1 - \alpha - \beta) \left( \frac{\bar{p}^{\alpha+\beta} E_{t+1} P_{t+1}^N \bar{\theta} \alpha^\alpha \beta^\beta}{w_t^{\alpha+\beta} r_t^{\alpha+\beta} r_k^{\alpha+\beta}} \right)^{\frac{1}{1-\alpha-\beta}} + \frac{r}{p} \quad (38) \]

The banks’ beliefs are computed as follows

\[ \bar{p}_{nt} = E_t(p/p > p_{nt}^* > p_{n-1t-1}^*, f_n(p)) = \frac{\int_{p_{nt}^*}^{1} f(p) \ p^n \ dp}{\int_{p_{nt}^*}^{1} f(p) \ p^{n-1} \ dp} \quad \text{with} \quad p_{nt}^* \in [p_{n-1(t-1)}^*, 1] \quad (39) \]

where \( p_{nt}^* \) is the lowest quality type taking a pooling contract at time \( t \) in a cohort of age \( n \).
See that from $\bar{p}_{nt}$ we can always infer $p_{nt}^*$ for any $nt$. Thus the lowest type $p_{nt}^* \geq p_{(n-1)(t-1)}^*$ because a firm that took a pooling contract at $t-1$ is considered by its bank to have a type of at least $p_{n-1t-1}^*$. Thus the lowest type $p_{nt}^* \geq p_{(n-1)(t-1)}^*$ because otherwise it would not have taken it.

Equation (37) is the incentive compatibility constraint for low types to truthfully announce their own type. Here we work under the following conjecture

**Conjecture 7** A type that is indifferent between taking a pooling and a separating contract at time $t$ expects to prefer a separating contract next period.

We will see below that this conjecture holds under Assumption 2. The incentive compatibility constraint compares the utility of an entrepreneur that is indifferent between taking a separating contract for the first time now or next period. In either case this type expects to invest in the firm an amount $e_{n+1t+1} = e_{n+1t+1}(p_{nt}^*)$ next period, given our conjecture. From then on, consumption streams are the same regardless of whether he chooses to separate at $t$ or at $t+1$.

Thus, if type $p_{nt}^*$ takes a separating contract his expected consumption stream for $t$ and $t+1$ is

$$c_t + \gamma p_{nt}^* \xi c_{t+1} = [W_{nt} - e_{nt}(p_{nt}^*)] + \gamma p_{nt}^* \xi [q_{nt}(p_{nt}^*, e_{nt}(p_{nt}^*)) - e_{n+1t+1}(p_{nt}^*)]$$

Since $W_{nt} > e_{nt}(p_{nt}^*)$ from Expression (30). In this case $c_t > 0$ while $c_{t+1}$ is given by the difference between the firm’s cash flow after taking a separating contract at $t$, that is $q_{nt}(p_{nt}^*, e_{nt}(p_{nt}^*))$, and what he will choose to re-invest in the firm at $t+1$, $e_{n+1t+1}(p_{nt}^*)$.

On the other hand if type $p_{nt}^*$ takes a pooling contract, he invests all his wealth in the firm and his expected consumption stream is just

$$c_t + \gamma p_{nt}^* \xi c_{t+1} = \gamma p_{nt}^* \xi [q_{nt}(\bar{p}_{nt}(p_{nt}^*), e_{nt}) - e_{n+1t+1}(p_{nt}^*)]$$

given that the $c_t = 0$.

---

21 The product $\xi_{t-1}\xi_{t-2}...\xi_{t-n}$ in the numerator and denominator cancels.

22 Again, this Assumption is not necessary for an equilibrium to exist but it facilitates solving the model.

23 If the conjecture didn’t hold, we would just need to compare longer consumption streams.
Simple algebra demonstrates that \( q_{nt}(\bar{p}_{nt}(p^{*}_{nt}), e_{nt}) > q_{nt}(p^{*}_{nt}, e_{nt}(p^{*}_{nt})) \).\(^{24}\)

The incentive compatibility constraint illustrates the trade-off faced by type \( p^{*}_{nt} \): free ride the pool by risking all the wealth in the project, versus separate, risk less and consume a fraction of the wealth today. Types below \( p^{*}_{nt} \) know they will succeed with a very small probability and prefer to separate right away.

Figure 3 helps to understand this trade off. Types above \( p^{*} \) will take a pooling contract even though types between \( p^{*} \) and \( p^{A} \) have enough wealth to take a separating contract and distinguish from lower types. Why? Take type \( p^{A} \). He would have to invest the same both under pooling and separating contracts (\( W = e(p^{A}) \)) while the lending rate is lower under pooling. These incentives drive the adverse selection problem in the economy.

After some algebraic manipulation type \( p^{*}_{nt} \)'s incentive compatibility constraint is

\[
\left( E_t \frac{P^{n}_{t+1}}{w_{t}^{\alpha} r_{t}^{\beta} r_{k}^{\alpha}} \right)^{1-\alpha-\beta} \left[ \frac{\alpha+\beta}{\bar{p}_{nt}^{\alpha+\beta}} - \frac{(1 - \alpha - \beta)}{(1 - r_{t}^{\gamma} \xi (\alpha + \beta))^{\alpha+\beta}} p^{\alpha+\beta}_{nt} \right] \leq e_{nt} \frac{(\bar{P}_{nt} - r_{t}^{\gamma} \xi p^{\gamma}_{nt})}{(1 - \alpha - \beta) r_{t}^{\gamma} \xi p^{\gamma}_{nt} \bar{P}_{nt}}
\]

for all \( nt \).

Having completed the description of the problem under Conjecture 7, I now proceed to characterize the solution. First see that

**Proposition 8** The average quality firm in a pooling contract, \( \bar{p}_{nt} \), is an increasing function of both \( p^{*}_{nt} \) and \( n \).

**Proof.** See Appendix and Figure 4.

Furthermore, from the incentive compatibility constraint and from the fact that \( p^{*}_{nt} \geq p^{*}_{n-1t-1} \) I state the following result

**Proposition 9** The lowest and average type participating in a pooling financial contract, \( p^{*}_{nt} \) and \( \bar{p}_{nt} \) are non-decreasing functions of the entrepreneurs net worth \( e_{nt} \).

\(^{24}\)See that \( q_{nt}(p, e) \) is increasing in both arguments. In particular

\[
\frac{\partial q_{nt}(p, e)}{\partial p} = \frac{r_{t}}{p^{2}} [TC_{nt}(p) - e] > 0
\]

because there is always some borrowing.
Proof. See appendix.

This happens because of the trade off emphasize before: only high enough types would be willing to risk more of their own wealth in the firm. In particular for a level of net worth equal to \( e_{nt} = e_{nt}(1) \) (or equal to \( \bar{e} \) in Figure 3) the incentive constraint binds for a type \( p_{nt}^* = 1 \). That is no type will be willing to participate in the pooling contract other than type 1 itself! On the other hand if \( W_{nt} = e_{nt} = 0 \) all types would be willing to participate simply because the lending rate in the pooling contract would be lower (than infinite) and they would have nothing to loose.

**Proposition 10** Conjecture 7 holds.

**Proof.** See Appendix.

Because all entrepreneurs start with the same wealth we see

**Proposition 11** Given a sequence of prices, equilibrium contracts under Conjecture 1 are such that all entrepreneurs that belong to the same cohort of age \( n \) with types \( p < p_{nt}^* \) will take separating contracts and all those with types \( p \geq p_{nt}^* \) will take the same pooling contract and hence have the same wealth.

**Proof.** See Appendix.

An example of the mass of firms for each cohort of age 1,2 and 3 are plotted in Figure 4 for \( f(p) = 6p(1 - p) \). For a newborn cohort , all types to the right of \( p_1^* \) will take the same pooling contract and those to the left will take separating ones.

We now can get an idea of how the firm dynamics work. Entrepreneurs that shared the pool before choose between sharing a pooling contract again or taking a separating one. We know that \( p_{n+1t+1}^* \geq p_{nt}^* \). If there is no shock next period’s wealth is greater than today’s \( (W_{nt} > e_{nt}) \) and generally more types separate as \( p_{n+1t+1}^* > p_{nt}^* \) as in Figure 4. Repeating this analysis we conclude that eventually \( W_{nt} > e_{nt}(1) = \bar{e} \) and \( p_{n+1t+1}^* = 1 \), or the threshold splitting separating and pooling contracts reaches the upper bound of the distribution of types. At this point the asymmetry of information in the cohort is fully resolved.

In particular
Proposition 12 Given constant prices of inputs and output, the dynamics of a type $p$ firm are such that

1) firms pay decreasing lending rates and increase the production scale as long as $p_{nt} \leq p$,

2) firms pay a higher interest rate and scale down production when taking the first separating contract

3) firms pay the same lending rate and maintain the same production scale from then on conditional on surviving.

Proof. See Appendix.

Interestingly firm dynamics occurs not due to technological improvements, since technology doesn’t change over the firm’s lifetime, but due to financial reasons. Because of the asymmetry of information, it takes time for good firms to build up financial reputation and convince banks about their own quality. As they acquire reputation, interest rates on loans decrease and firms grow. These results are broadly consistent with empirical findings in Evans (1987).
3 Aggregation

At each moment in time there is a mass $\mu$ of workers. The mass of entrepreneurs productively active at $t$ is the sum of those that are one, two and so on periods old. In the absence of an aggregate bad shock history ($\xi_t = \xi \ \forall t$) the total mass of entrepreneurs is

$$ M^E = \int_0^1 [f(p) + \xi pf(p) + (\xi p)^2 f(p) + ...] dp = \int_0^1 \frac{f(p)}{1 - \xi p} dp \ \forall t \tag{41} $$

The total mass when there was one bad shock at $j$ is just

$$ M^E_t(j) = \int_0^1 f(p)\left[\frac{1 - (\xi p)^{t-j}}{1 - \xi p}\right] dp + \bar{\xi}^{t-j-1}p^{t-j}M^E $$

where $M^E_t(j)$ is increasing in $t - j$ and bounded above by $M^E$ as $\lim_{t-j \to \infty} M^E_t(j) = M^E$. That is, total mass recovers over time after a bad shock.

Let $\eta_{nt}$ be the mass of firms of age $n$ taking a pooling contract at time $t$. Thus,

$$ \eta_{nt} = \int_0^1 \tilde{\xi}^{\zeta_t(n)}\xi^{n-\zeta_t(n)-1}p^{n-1} f(p) dp $$

where $\zeta_t(n)$ is the number of aggregate shocks in the history of an $n$ periods old firm at $t$. The mass of firms of each cohort taking separating contracts is computed similarly.

Because this is a small open economy model, equilibrium requires that both labor and non-tradable good markets clear.

4 Equilibrium

We now define equilibrium under Conjecture 1 to then show that the conjecture holds. Let $T^*_t = \left[ p^*_1, p^*_2, ... \right]$ be the (infinite dimensional) vector of thresholds for all cohorts and let $\Omega(\mu, \{T^*_t\}_{t=0}^\infty, f(p))$ be the economy described above.

**Definition 13** A competitive equilibrium for economy $\Omega(\mu, \{T^*_t\}_{t=0}^\infty, f(p))$ is a collection of initial state variables $\{T^*_{t-1}, [y^N_{n-1t}(p)]_{p \in [0,1]}, W_{nt}(p)_{p \in [0,1]}, \zeta_t(n)_{n=1}^\infty, \Gamma_{t-1}\}_{t=0}$, a collection of inputs, financial contracts and output for the entrepreneurs taking a pooling contract,
\( \left\{ \left( k_{nt}, l_{nt}, i_{nt}^{Pool}, M_{nt}^{Pool}, e_{nt}, y_{nt+1}^N \right)_{n=1}^{\infty} \right\}_{t=0}^{\infty} \) a collection of inputs, financial contracts and output as function of their announcements for all entrepreneurs taking separating contracts, 
\[ \left\{ \left[ k_{nt}^{\hat{p}}, l_{nt}^{\hat{p}}, y_{nt+1}^{\hat{p}}, M_{nt}^{\hat{p}}, e_{nt}^{\hat{p}}, y_{nt+1}^N (\hat{p}) \right]_{n=1}^{\infty} \right\}_{t=0}^{\infty}, \] inputs and output for the tradable sector, 
\( \left\{ Y_t^N, K_t^T, Y_t^T \right\}_{t=0}^{\infty}, \) all entrepreneurs’ consumption and effort allocations
\( \left\{ \left( c_{nt}^w, m_{nt} \right)_{n=1}^{\infty} \right\}_{t=0}^{\infty}, \) workers’ consumption allocation, labor supplied and portfolio choices 
\( \left\{ c_{it}^w, l_t, \Gamma_t \right\}_{t=0}^{\infty} \) and prices \( \left\{ r_t, w_t, P_t^N \right\}_{t=0}^{\infty} \) such that,

- \( \left\{ \left( k_{nt}, l_{nt}, i_{nt}^{Pool}, M_{nt}^{Pool}, e_{nt}, y_{nt+1}^N \right)_{n=1}^{\infty} \right\}_{t=0}^{\infty} \) is the solution to all entrepreneurs’ problems of age \( n \) at time \( t \) with parameter \( p \geq p_{nt}^* \) and net worth \( W_{nt}(p) \).
- \( \left\{ \left[ k_{nt}^{\hat{p}}, l_{nt}^{\hat{p}}, i_{nt}^{\hat{p}}, M_{nt}^{\hat{p}}, e_{nt}^{\hat{p}}, y_{nt+1}^N (\hat{p}) \right]_{n=1}^{\infty} \right\}_{t=0}^{\infty} \) is the solution to all entrepreneurs’ problems for all owners of firms of age \( n \) at time \( t \) with parameter \( p = \hat{p} < p_{nt}^* \) and wealth \( W_{nt}(p) \).
- \( \left\{ Y_t^N, K_t^T, Y_t^T \right\}_{t=0}^{\infty} \) is the solution to the tradable sector’s problem,
- \( \left\{ \left( c_{nt}^{Ep}, m_{nt} \right)_{n=1}^{\infty} \right\}_{t=0}^{\infty} \) are the consumption and effort allocations of entrepreneurs of type \( p \) and age \( n \) at every period \( t \).
- \( \left\{ c_{it}^w, l_t, \Gamma_t \right\}_{t=0}^{\infty} \) is the solution to the workers’ problem. Finally,
- Labor and non-tradable goods market clear\textsuperscript{25}

\[
\sum_{n=1}^{\infty} \xi^{x_t(n)} \xi^{n-x_t(n)-1} \left[ \int_{0}^{P_{nt}^*} l_{nt}(p)p^{n-1}f(p)dp + l_{nt} \int_{P_{nt}^*}^{1} p^{n-1}f(p)dp \right] = b + \mu l_t \quad \forall t \geq 0.
\]

\[
\sum_{n=1}^{\infty} \xi^{x_t(n)} \xi^{n-x_t(n)-1} \left[ \int_{0}^{P_{nt}^*} y_{nt-1t}^N(p) p^n f(p)dp + y_{nt-1t}^N \int_{P_{nt}^*}^{1} p^n f(p)dp \right] = Y_t^N \quad \forall t \geq 0.
\]

I have shown that the financial contracts proposed are equilibrium contracts under Conjecture 1 in Proposition 11. Now I finally show
\textsuperscript{25}Remember that a type \( p \) entrepreneur produces \( p y_{nt+1}^N \) on average

---

25 Remember that a type \( p \) entrepreneur produces \( p y_{nt+1}^N \) on average
Proposition 14  Equilibrium exists for economy $\Omega(\mu, \{T^*_t\}_{t=0}^\infty, f(p))$ under Conjecture 1.

Proof. See Appendix.

Existence of equilibrium follows by showing that aggregate excess demand functions for labor and non-tradable goods in this economy are well behaved. All previous propositions hinge on Conjecture 1 being true. Finally

Proposition 15  Conjecture 1 holds.

Because of Assumption 5, if banks could offer multi-period financial contracts they would be back-loaded. In such contracts, banks retain part of the entrepreneur’s earnings to then rebate them to surviving firms at some date in the future. Indeed, this could allow banks to screen types within cohorts because higher types value such contracts more than lower types.

Let $x_{nt}$ be the earnings retained by the bank in period $t$ under such a multi-period contract. In this case, the first order condition for effort at $t$ would be

$$W'_{nt} - x_{nt} = m_{nt}^{\chi^{-1}}q_{nt}$$  \hspace{1cm} (42)

Given our assumption about $\chi$ being bigger but very closed to one, we get that $m_{nt} \rightarrow 0$. As long as $x_{nt} > 0$, only a tiny managerial effort level will be exerted when the firm is in difficulty. Even if a multi-period contract allowed banks to screen the best type Equation (20) shows that the cost of capital in such case is $^{26}$

$$i_{nt} = \frac{r_t}{\xi} + \frac{(1 - \xi)\lambda E_t P^N_{t+1} \tilde{y}^N_{t+1}}{\xi M_{nt}}$$

which can be very big depending on $(1 - \xi)\lambda$. That is where Assumption 1 kicks in. The trade off is the following. Multi-period contracts increase $p^*_n$ since lower types are being screened, and hence reduce the cost of capital for firms in the pool. But they also reduce effort $m_{nt}$, increasing the cost of capital. For this reason, as long as $\xi < 1$ there will be a $\lambda$ such that Conjecture 1 holds. This concludes the proof of Proposition 15.

$^{26}$In this case $m_{nt} = 0$ and $\tilde{p}_{nt} = 1$. 
Other than generating dynamics at the firm level, the model also sheds light on the role of information capital in aggregate performance. To see this, let \( T_\Omega \) be the (infinite dimensional) vector of thresholds for cohorts 1, 2, ..., in economy \( \Omega \), and \( T_\Lambda \) be the vector of thresholds for all cohorts in an otherwise equal economy \( \Lambda \). Also let economy \( \Omega \) be information-capital intensive compared to economy \( \Lambda \) if and only if \( T_\Omega \geq T_\Lambda \) with strict inequality for at least one element of the vectors.

**Proposition 16** Information-capital intensive economies produce more output, have more labor employment and exhibit higher wages than otherwise equal economies.

**Proof.** See Appendix.

Economies where entrepreneurs start up with more wealth will be information-capital intensive and hence outperform economies with less wealthy entrepreneurs. Thus economies with more severe informational asymmetries will under-invest, as common wisdom suggests. Proposition 16 contradicts the main result in De Meza and Web (1990) claiming that “in a pooling equilibrium there is always too much investment”.

## 5 Quantitative exercises

The model economy is simulated to conduct quantitative comparisons of output performance between different exercises under “reasonable” parameter values. First I investigate Bernanke and Gertler’s net worth effect in this environment with repeated adverse selection and firm heterogeneity after an interest rate shock. To isolate the net worth effect the simulation is done under the assumptions that \( A_t = A \) at all times and \( \bar{\xi} = \xi \). Thus, exit rates and TFP are kept constant along the business cycle. In the second simulation exercise I investigate what are the macroeconomic consequences of additionally having the exit rate be correlated with the aggregate shock \((\bar{\xi} < \xi)\) under the assumption that \( A_t = A \). In the third simulation I study output performance in this economy when some other form of market failure that looks like a TFP process is present. I model this productivity process as an externality in
the following way

\[ A_t(Y_{t-1}^N, \bar{Y}^N) = A \left[ 1 - \nu \left( \frac{Y_N^N - Y_{t-1}^N}{\bar{Y}^N} \right) \right] \]

where \( \nu \geq 0 \) and \( \bar{Y}^N \) is the steady state level of aggregate non-tradable output. The TFP does not change on impact because aggregate non-tradable output does not change; it only decreases in successive periods. In steady state \( A_t = A \). This externality can be interpreted in a number of ways, such as coordination costs, adjustment costs, etc. The parameter \( \nu \) determines the relative importance of the externality and can potentially drive the persistence in the model. I show below that some interesting interactions arise between the informational asymmetry and these market failures that look like a TFP process. Moreover I show that these interactions are meaningful even for values of \( \nu \) that are incapable of generating, per se, relevant output dynamics.

The simulations also let us examine in detail the firm dynamics both in the macroeconomic steady state and after a shock for all kinds of firms.

### 5.1 Parameters

For simulation purposes the interest rate levels are \( \{r_l, r_h\} = \{1.0147, 1.035\} \).

The tradable goods production function adopted for the simulation is a standard CES

\[ Y_{t+1}^T = A_t \left[ \phi (K_t^T)^{-\phi} + (1 - \phi) (Y_t^N)^{-\phi} \right]^{-\frac{1}{\phi}} \tag{43} \]

where \( A_t \) follows the process specified above. The parameter values \( A, \varphi \) and \( \phi \) were chosen to: 1) roughly match the share of labor in total output to 50 percent, and 2) produce a fall in the intermediate good’s price of 10% as a response to the change in interest rates. These parameters are \( A \approx 1.349, \varphi = 7.0678 \) and \( \phi = 0.2039 \). A low elasticity of substitution between inputs in this sector is necessary to produce a fall in non-tradeable good prices of approximate 10% and, at the same time, a fall in the capital stock of no more than 4% on impact. This high complementarity can be relaxed at the cost of increasing the volatility of investment in the tradable sector. The parameter \( \nu \) is left as a free parameter for reasons that I explain below.
The sum of capital and labor contributions to non-tradable output \((\alpha + \beta)\) were set close to one according to microeconomic evidence at the plant level for the US and emerging economies.\(^{27}\) Moreover I chose \(\beta > 1 - \beta > \alpha\) to capture the idea that small firms in this sector are labor intensive. In my simulations \(\alpha = .35\) and \(\beta = .61\), but results are robust to changes in these values as long as they are not extreme. Productivity \(\bar{\theta} = 2.75\) so that labor employed by the biggest firms is approximately 300 times bigger than labor employed by a newborn firm.

Parameters for the worker’s utility function are \(\omega = 3\) and \(\sigma = 3\). This implies a labor supply elasticity of \(\frac{1}{\omega - 1} = .5\), which is conservative for the purpose of our model and consistent with previous studies.\(^{28}\) Output performance is sensitive to this parameter but the main message is not as I explain below. The intertemporal elasticity of substitution was chosen according to evidence for small emerging economies, but it is only relevant for the dynamics of consumption as the FOC of the worker’s problem illustrates. For simulation purposes I assume that \(\Gamma_0\) is some negative number such that the long run consumption level after the shock is the same as before the shock.

Estimates of US small business turnover for 2001 reveal that 9.5% (2.3% per quarter) of firms closed without defaulting. Roberts and Tybout show that turnover is bigger in the type of economies we focus on here. This fact makes Conjecture 1 an important and endogenous feature of emerging economies. To be conservative, I let the probability that firms exit without defaulting be 2% per quarter in steady state and 4% on impact. Thus \(\xi = .98\) and \(\bar{\xi} = .96\). Also according to the model I let \(\gamma = \frac{1}{.01 + r} < \frac{1}{r}\). The entrepreneur discount rate was chosen so that the firm’s debt-to-equity ratio is approximately .8 once they are completely screened.\(^{29}\)

\(^{27}\)See Davis, Haltiwanger and Schuh (1996) for evidence and discussions on these issues for the US. See Roberts and Tybout (1996) for evidence in developing countries.

\(^{28}\)See Rebelo and Vegh (1995).

\(^{29}\)From Proposition 4 we have that

\[
\frac{M_{nt}(p)}{e_{nt}(p)} = \frac{TC_{nt}(p)}{e_{nt}(p)} - 1 = \frac{1 - \gamma r_t}{(1 - \alpha - \beta) \gamma r_t}.
\]
The density function for entrepreneurs’ types utilized in this numeric example is \( f_1(p) = 6p(1 - p) \) as in Figure 4.

Finally, depreciation rates for capital in the tradable and non-tradable sectors were fixed at 6%. Results in the model have shown to be robust to different depreciation rates, although lower depreciation rates require higher complementarity between capital and non-tradable inputs in the tradable production function to generate a 10% drop in non-tradable prices on impact, given that investment is irreversible.

5.2 Simulations

I now compare the performance of final output in the three economies described above. Note that in steady state all economies generate exactly the same values for all variables, including aggregate tradable output, because exit rates are the same and \( A_t = A \).

Final output performance for these economies is presented in the following chart. Model 1 refers to case where only the net worth channel is at work. The net worth effect in this dynamic environment is capable of producing very long-lasting and endogenous persistence, but quantitatively the fall in output is not enough to replicate recessions in small emerging economies. Model 2 refers to the case where the exit rate increases on impact. Simulations show how the loss of information capital affects output performance since firms that had acquired financial reputation are being replaced by newborn firms with no reputation. Again we see that the second economy generates strong endogenous output autocorrelation after an uncorrelated aggregate shock, but it cannot fully account for output drops of more than 1%. This result is sensitive to labor supply elasticity. Other simulations show that with a labor supply elasticity of 3 this model economy produces persistent output declines of 2%, but the output drop on impact is much bigger than 4% as it is in the graph shown below.

For the model economy with externalities we need to specify \( \nu \). The approach I followed here is to find the lowest value for this parameter that is able to generate a smoothed output response after a shock. The chart shows the output performance of the economy with \( \nu = .3 \), implying that a one percent drop in total non-tradable output at \( t \) decreases total factor productivity by 0.3% in the period that follows. As shown, this economy can
now replicate a typical output response.

Does this imply that the financial imperfections can account for only 1% of the drop in aggregate output while TFP-look-like externalities explain the rest? To answer this question I have simulated the output dynamics of an economy with externalities only (labeled Ext. in the chart). This is done by switching off the net worth effect and by letting $\xi = \bar{\xi}$. The simulation demonstrates that the same externalities alone produce very weak and short-lasting output dynamics. Therefore the answer to our question is no.

![Model Comparison](image)

Indeed there are powerful interactions between financial imperfections and externalities in this model, and they operate in the following way. After the shock the non-tradeable output continues below $\bar{Y}^N$ because: 1) entrepreneurs lose net worth that enables the very best entrepreneurs in each cohort to distinguish from lower types, making the informational asymmetry more persistent, and 2) the total mass of firms is lower than in steady state ($M_t^E < M^E$). For this reason $A_{t+1} < A$. From Proposition 2 we know that $P_{t+1}^N < P^N$ which affects the profitability of firms. Consequently the accumulation of net worth for growing firms is affected as shown by Equation (26). Moreover Proposition 9 shows that lower wealth generally implies lower future thresholds $p_{nt}^*$, implying more adverse selection within each cohort. According to Proposition 16 when the adverse selection worsens a lesser amount of non-tradeable output will be produced by each cohort, hence keeping $A_{t+2}$ below its steady state value $A$. Thus the cycle continues.
In other words, a bad macroeconomic environment (in the sense of $A_{t+1} < A$) slows down the accumulation of wealth and with it the information revelation process in the economy, aggravating the recession.

Next I present the main macroeconomic variables simulated under the case with externalities.
The simulation, as in the previous cases, was done by assuming that the interest rate increases at period 0, and it returns to normal levels right away. Wages in the non-tradable sector are pro-cyclical. Employment decreases as a response to lower wages. Capital in both sectors decreases on impact in response to a higher interest rate. After the shock capital remains low because non-tradable output is below steady state levels and the two are highly complementary by assumption. Investment in both sectors drops sharply on impact and then increases so that capital reverts to steady state levels. Aggregate consumption is mainly the workers’ consumption but entrepreneurs’ consumption is very sensitive to the net worth effect.\textsuperscript{30} Non-tradable output falls on impact and it recovers slowly as well because of the reasons explained above. Consequently, tradable output drops and remains low for many quarters. Finally the mass of firms drops 2\% by assumption and it recovers quickly.\textsuperscript{31}

The simulations show that the shock has effects on aggregate output that propagate through time despite the fact that the cost of capital changes only at $t=0$. Wages, tradable and non-tradable output, investment and consumption decline and it takes a while for the economy to return to its full potential output and consumption levels. The informational asymmetry mainly via the net worth effect drives the persistence in the economy, while externalities and the feedback to the net worth effect drive the amplification. Both account for a significant and persistent recession in response to a one time spike in interest rates.

To have a better understanding of the forces at work I present the microeconomic data generated by this simulation.

### 5.3 Information capital at the firm level

In this subsection I present firm data simulated for the model with externalities, both in and out of the steady state.\textsuperscript{32} The graphs below show the evolution of the main microeconomic variables for a firm of type $p = 1$ over its lifetime, assuming that the economy is in steady state.

\begin{itemize}
  \item \textsuperscript{30} Aggregate consumption dynamics are driven by assumptions and do not affect allocations in production in any way.
  \item \textsuperscript{31} The trade balance is sharply improved on impact mostly due to the drop in investment.
  \item \textsuperscript{32} Again, microeconomic data corresponding to steady state levels are the same for the three models.
\end{itemize}
state. Note that in this case cross-section and time-series microeconomic variables coincide.

Net worth and amounts loaned are positively correlated, evidence that the banks utilize
the firms’ wealth as a screening device. Remember that in a world with symmetric informa-
tion these variables would not be correlated (see Proposition 3). Also these variables increase
with age until these entrepreneurs have built enough wealth to screen all lower types. From
then on, this type keeps a constant scale of production as long as he stays in business. In-
puts and output also increase with age of the firm as we see in the plot for labor demanded.
Leverage, expressed as the ratio of loans to net worth, is monotonically decreasing with the
firm’s age until this type separates. At that point leverage is constant at around .8 as Foot-
note (26) explains. Finally, the interest rates paid on loans by these firms decreases with
age as the bank’s perception of this firms’ quality improves. Younger firms pay higher rates
because they face more severe adverse selection problems. These results coincide with the
qualitative results in Proposition 12 and empirical finds on relationships among firm growth,
firm size and firm age by Evans (1987).

The simulation shows that only after 33 quarters is the best firm able to fully develop.
This means that asymmetric information problems within members of each cohort are fully
resolved after a long time.
In the next graphs I present micro simulated data when the economy is hit by the interest rate shock. Now cross-sectional and time-series generated data differ because a firm’s performance will depend on the age of the firm at the moment of the shock. In the next set of graphs I plot time-series data for the highest quality firms that belong to a four-period-old cohort when the shock occurs. For comparison, I present the ratio of the actual time-series of the variable considered versus the time-series of that variable if no shock had occurred.

All ratios are equal to one in the first 4 quarters because this cohort suffers the shock at the age of 4 quarters. After that, the ratio of actual net worth to that in steady state conditions is lower than one. Moreover the ratio increases over time, showing that firms that are hit by the shock will only recover after 35 periods. In the aftermath of the shock the ratio for labor demand will also be lower because of the net worth effect. Indeed the ratio for labor demanded by these firms falls by more than 20% even after the shock. Furthermore the actual leverage is temporarily higher than the steady state level of leverage in all these

33 This cohort is hit particularly hard on impact because of its high leverage (see previous graphs).
34 Keep in mind that a firm with \( p = 1 \) and age \( n + 1 \) in a pooling contract at time \( t + 1 \) is perceived by its bank to be of type bigger than \( p^*_t \).
periods. Finally, the ratio of actual interest rates paid by firms during recessions to those rates paid in the steady state are higher throughout the recession.\textsuperscript{35} Older firms exhibit similar patterns, although changes in those variables are less significant because leverage is lower for these firms when the shock strikes, and losses are not as large.

These simulations show how firm dynamics explain the behavior of aggregate output performance.

\section{Concluding remarks}

This paper shows that adverse selection problems in financial markets have important implications for both firm and aggregate dynamics. The financial imperfection arises because entrepreneurs have more information than banks about the probability of being driven out of the market every period. I show that the asymmetry of information can be resolved when entrepreneurs are wealthy enough. But when they start with a limited amount of wealth, as is usually the case, the informational asymmetry generates an adverse selection problem because banks cannot perfectly infer the quality of all firms. Then different types end up taking the same financial contract, which is inefficient since productivity differs across types.

The adverse selection problem between firms in each cohort mitigates as firms grow older. Successful firms build up net worth and by re-investing it all, they are able to signal to the bank that they are better types. Because only those firms with higher success probability are willing to follow such an aggressive path of investment, banks perceive them as better types. Eventually the adverse selection problem disappears as the best entrepreneurs achieve a high enough level of wealth. Nonetheless it takes a long time for this to happen. How long depends on macroeconomic conditions because they determine the pace at which firms build up net worth. Firm dynamics in this economy are shown to be consistent with results in previous empirical work (both under stationarity and along the recession).

\textsuperscript{35}There is no discontinuity in the last graph. For a type $p = 1$, lending rates decrease over the firm’s lifetime both in steady state and after the shock. The graph only shows that the ratio of actual and steady state rates does not necessarily decrease with age.
The model economy predicts very significant and long-lasting output responses to negative and uncorrelated aggregate large shocks. The demand for goods produced by small firms shrinks and they lose net worth on impact. The screening process is slowed down in the downturn of the business cycle and cohorts produce less. If the shock implies a higher exit rate aggregate performance deteriorates even further because firms that have developed a good financial reputation disappear and it takes time before new firms develop their own. These two effects generate strong endogenous aggregate persistence. I also show that in the presence of other distortions that look like a TFP process the net worth effect intensifies as firm profitability worsens. This feedback amplifies the output response to shocks.

This paper has policy implications. See that the shock reduces welfare to both workers and entrepreneurs. Consumers are worse off because their labor income dropped and they faced incomplete markets. Entrepreneurs’ welfare decreases because wealth is lost on impact and the stream of profits along the recession falls. In this environment sterilizing the capital outflow, for example, may improve welfare when it can be done at a relatively low cost. This would avoid the massive net worth losses of small firms –by keeping their demand strong in the short run. The model economy predicts that a policy that shuts down the net worth effect would quickly recover, like the case with externalities alone simulated in the graph on Model Comparison. A crude measure of the benefits of sterilizing is given by the area between output responses in this case and that in Model 3, while costs would depend on the instruments available for the government to smooth out short-term capital outflow. This kind of policy has been implemented before. Argentina received assistance from the International Monetary Fund in the aftermath of the crisis although, as has been argued by Calvo (1996), it was insufficient and inadequate. Nonetheless, whether this kind of policy is effective in mitigating massive net worth losses or even optimal is a matter of further research.

36 Contingent credit lines to emerging economies were also common after the Mexican crisis.
Appendix

Proof. of Proposition 2. The result follows from the zero profit condition for this sector, which is \( \pi_t^T(A_t, P_t^N, r_t) = 0 \) and from the fact that the function increases with the TFP and decreases with input prices.

Proof. of Proposition 4. Equation (37) is a differential equation that fits into the following general form of linear differential equations

\[
 w(\hat{p}) = u(\hat{p}) e(\hat{p}) + e'(\hat{p})
\]

and its closed form solution is given by

\[
 e(\hat{p}) = \exp(- \int u d \hat{p}) \left( A + \int w \exp(\int u d \hat{p}) d \hat{p} \right)
\]

Finally, by noting that an entrepreneur with characteristic \( p = 0 \) never invests \( (e(0) = 0) \), the proof is completed.

Proof. of Proposition 8. Dropping subscripts and taking partial derivatives to \( \bar{p}_n(p^*) \equiv E[p/p > p^*, f(p)p^{n-1}] \), give us

\[
 \frac{\partial \bar{p}}{\partial p^*} = \frac{f(p^*)p^m}{f(x)x^{n-1}dx} \left( \frac{\bar{p}}{p^*} - 1 \right) > 0 \ \forall \ p^* \in [0, 1]
\]

Also,

\[
 \frac{\partial \bar{p}}{\partial n} = \frac{n}{(n-1)} \int_{p^*_n}^{1} f(p) p^{n-1} dp > 0 \ \forall p^* \in [0, 1].
\]

Proof. of Proposition 9. From Proposition 8 the proof consists in showing that \( p^*_n \) is a non-decreasing function of the entrepreneur’s net worth \( e_{nt} \). There are two cases. On one hand, if the participation constraint is not binding because \( p^*_n = p^*_{n-1t-1} \) local changes in the entrepreneur’s net worth do not change \( p^*_n \). On the other hand, when the participation constraint is binding, then \( p^*_n \) will change with \( e_{nt} \). Dropping subscripts and rearranging the participation constraint, we obtain

\[
 e = C \frac{p^* \bar{p} \left[ \bar{p}^{\alpha + \beta} - \frac{(1-\alpha-\beta)}{(1-r\gamma(\alpha+\beta))} p^* \left( \bar{p}^{\alpha + \beta} \right) \right]}{\left( \bar{p} - r\gamma \bar{p} \right)}
\]

42
where $C$ is a constant that depends on parameters and prices. Call \[1\] the expression between brackets. Differentiating the participation constraint with respect to $p^*$, and simplifying gives

$$\frac{\partial e}{\partial p^*} = \frac{\bar{p}}{(\bar{p} - r\gamma \xi p^*)^2} \left[ \bar{p} [1] - (\bar{p} - r\gamma \xi p^*) \frac{\alpha + \beta}{(1 - r\gamma \xi (\alpha + \beta))} p^* \frac{\alpha + \beta}{(1 - \alpha - \beta)} \right] + \frac{\partial \bar{p}}{\partial p^*} \frac{p^*}{(\bar{p} - r\gamma \xi p^*)^2} \left[ -r\gamma \xi p^* [1] + (\bar{p} - r\gamma \xi p^*) \frac{\alpha + \beta}{(1 - \alpha - \beta)} \bar{p}^* \frac{\alpha + \beta}{(1 - \alpha - \beta)} \right]$$

Where $r\gamma \xi \leq 1$ by assumption. Now, let \[2\] and \[3\] be the first and second expressions between brackets in this derivative. The proof follows by showing that these two expressions are positive for all possible values of $p^*$. Since $\frac{\partial e}{\partial p^*}$ is always positive, then $\frac{\partial e}{\partial p^*} > 0$ for all values of $p^*$.

Rearranging terms, \[2\] becomes

$$\frac{1}{\bar{p}^* (1 - \alpha - \beta)} \left[ (1 - r\gamma \xi (\alpha + \beta)) - x \frac{\alpha + \beta}{(1 - \alpha - \beta)} \right]$$

$$\frac{1}{(1 - r\gamma \xi (\alpha + \beta))} F(x)$$

where $x = \frac{p^*}{\bar{p}} \in [0, 1)$. It is easy to show that $F(0) > 0$, $F(1) = 0$, and $F'(x) < 0 \forall x$. This implies that \[2\] > 0. Similarly,

$$\frac{1}{\bar{p}^* (1 - \alpha - \beta)} \left[ r\gamma \xi (1 - \alpha - \beta)^2 \frac{1}{(1 - r\gamma \xi (\alpha + \beta))} - r\gamma \xi x + (\alpha + \beta) \right]$$

$$\frac{1}{(1 - \alpha - \beta)} G(x)$$

where $x$ is defined as before. Now, $G(0) = \alpha + \beta > 0$, $G(1) = (\alpha + \beta)(1 - r\gamma \xi)^2 > 0$, and $G'(x) < 0 \forall x$.

**Proof. of Proposition 10.** The incentive compatibility constraint for $nt$ under the conjecture is

$$e_{nt} = C_t \frac{p^*_{nt} \bar{p}_{nt}}{(\bar{p}_{nt} - r\gamma \xi p^*_{nt})} \equiv C_t H_t(p^*_{nt}, \bar{p}_{nt}, (p^*_{nt}))$$
with
\[ C_t = (1 - \alpha - \beta) r_t \gamma \xi \left( \frac{E_t \Pi^N_{t+1}}{w_t^2 \alpha \beta} \right)^{1/(1-\alpha-\beta)} \]
and where the average type depends on \( n \) and \( p_{nt}^* \). Conjecture 7 holds as long as
\[ q_{nt}(\bar{p}_{nt}(p_{nt}^*), e_{nt}) \geq C_{t+1} H_{t+1}(p_{nt}^*, \bar{p}_{n+1t+1}(p_{nt}^*)) \tag{44} \]
meaning that \( p^* \) expects to prefer to separate at \( t + 1 \). We know that the LHS of this expression is expected to increase since
\[ q_{nt}(\bar{p}_{nt}(p_{nt}^*), e_{nt}) = \text{profits}_n + \frac{\gamma}{\bar{p}_{nt}} e_{nt} >> e_{nt}, \]
thus relaxing the constraint. Also \( C_{t+1} \) is never much bigger than \( C_t \) (in fact \( C_t \) is constant in steady state and decreases after the shock).

But the RHS is increasing in \( n \) since
\[ \frac{\partial \text{RHS}}{\partial \bar{p}_{nt}} \frac{\partial \bar{p}_{nt}}{dn} = \frac{p^*}{(\bar{p} - r \gamma \xi p^*)^2} G(\frac{p^*}{\bar{p}}) \geq 0 \]
where \( G(.) \) was defined in the previous Proof and shown to be positive for all possible values of \( p^*/\bar{p} \). Nonetheless, because \( \bar{p}_{n+1t+1}(p^*) - \bar{p}_{nt}(p^*) \) is not too big (from Assumption 2), \( RHS_{t+1} \) is not much bigger than \( RHS_t \) and thus Conjecture 7 holds.

**Proof. of Proposition 11.** The proof follows by showing: 1) Separating contracts are equilibrium contracts, and 2) there is only one pooling contract per cohort.

To show 1) I use the notion of Reacting Equilibrium analyzed by Enger and Fernandez (1987). They show that pooling with a subset of lower types might dominate separating depending on the distribution of types, but they also show that no bank would offer such a contract because it would trigger a corresponding reaction by other banks that would skim the cream and produce losses to the first contract. The same logic applies to contracts offering positive payoffs to entrepreneurs in the state that occurs with probability \( 1 - p \). This contract would separate the types only in a monopolistic environment. In our environment, a contract with no payoff in such a state would skim the cream and produce losses to the previous contract. Hence such a contract would not arise in equilibrium.

For 2) first see that all entrepreneurs in a newborn cohort have the same wealth. If there are two pooling contracts in this cohort they must have different \( e_{nt} \), as otherwise all entrepreneurs would choose the one with lower lending rates. But the one with lower net
worth is such that $e < W$ which means there is room for sequential financial offers by banks to attract the best types. This again would produce losses for the bank offering pooling contracts where $e < W$ which implies that there is only one pooling contract in equilibrium with $e_{nt} = W_{nt}$. For successive cohorts the argument is the same (note that surviving firms in a pooling contract obtain the same cash flow).

**Proof. of Proposition 12.** First see that $p^*_{nt} \geq p^*_{n-1t-1}$ since banks remember the entrepreneur’s history. Also $\bar{p}_{nt}$ is increasing in $n$. Thus $\bar{p}_{nt} < \bar{p}_{n+1t+1}$ which implies that as long as the entrepreneur takes a pooling contract ($p^*_{nt} < p$) he gets cheaper lending rates over time and hence the scale increases. 3) Follows from Proposition 4, and 2) from the fact that $\bar{p}_{nt} > p^* \geq p$. Note that there is a discontinuity in the firm’s demand for inputs.

**Proof. of Proposition 14.** Given $A_t$, from Proposition 4 we obtain $P_t^N(r_t, A_t)$. Because of the CRS assumption in the tradable sector and capital being tradable, demand for non-tradable $Y_t^N$ always adjust to meet the supply at $P_t^N(r_t, A_t)$. Then the proof consists in showing that there exists a $w_t$ such that excess demand for labor is zero.\(^{37}\) It is easy to verify that the excess labor demand is continuous, negative for $w \rightarrow 0$ and positive for $w \rightarrow \infty$.

**Proof. of Proposition 16.** Let the total amount of output produced and labor demanded by cohort $n$ with threshold $p^*$ be

$$Y_n(p^*) = \int_0^{p^*} y^N(x)f_n(x)dx + y^N[\bar{p}_n(p^*)] \int_{p^*}^{1} f_n(x)dx$$

$$L_n(p^*) = \int_0^{p^*} l^N(x)f_n(x)dx + l^N[\bar{p}_n(p^*)] \int_{p^*}^{1} f_n(x)dx$$

where

$$y^N(p) = \frac{\theta^{\alpha^\gamma} \beta^{\alpha^\gamma}}{u^{\beta^\gamma} r^{\alpha^\beta} r_k^{\alpha^\beta}} \frac{1}{p^{1-\alpha-\beta}} = C_y p^{\frac{1}{1-\alpha-\beta}}$$

is the average output produced by a type $p$ and $l^N(p) = C_l p^{\frac{1}{1-\alpha-\beta}}$ is given by Equation (25).

\(^{37}\)See that because labor supply has no income effects and under Conjecture 1, the computation of equilibrium is done sequentially period after period.
First I prove that $Y_n(p^*)$ and $L_n(p^*)$ are increasing in $p^*$ for each cohort $n$. Thus

$$\frac{\partial Y_n(p^*)}{\partial p^*} = \frac{\partial y^N[p_n(p^*)]}{\partial \bar{p}} \frac{\partial \bar{p}}{\partial p^*} \int_{p^*}^1 f_n(x)dx - \{y^N[p_n(p^*)] - y^N(p^*)\} f_n(x)$$

From Proposition 8 we know that $\frac{\partial \bar{p}}{\partial p^*} \int_{p^*}^1 f_n(x)dx = f_n(p^*)[\bar{p}^N - p^*]$. With this and the expression for average output we get that

$$\frac{\partial Y_n(p^*)}{\partial p^*} = C \bar{p}^{\frac{1}{1-\alpha-\beta}} f_n(p^*) \left[\alpha + \beta - x + (1 - \alpha - \beta)x^{1-\alpha-\beta}\right]$$

$$= C \bar{p}^{\frac{1}{1-\alpha-\beta}} f_n(p^*) H(x) > 0$$

where $x = \frac{p^*}{\bar{p}} \in [0, 1]$ as before and $H(0) > 0$, $H(1) = 0$, and $H'(x) < 0 \forall x$, showing that $H(x) > 0 \forall x$. Similarly $\frac{\partial L_n(p^*)}{\partial p^*} > 0$. Thus aggregate labor demand (by all cohorts) is greater in the information-capital intensive economy. This proves the proposition because labor supply functions are the same in both economies. ■
References


