Homework #2
Econ 305. Prof Jose Wynne

These homework is about solving dynamic programming with the help of the computer.

1. Brock-Mirman Model (1972)

We solved this problem in class. Remember that the problem is

$$\max E \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

subject to $c_t + k_{t+1} \leq \theta_t A k_t^\alpha$, $1 > \alpha > 0$, $\ln(\theta_t)$ is i.i.d. $N(0, \sigma^2)$ and $k_0$ given. Let $\alpha = .6$, $\beta = .99$, and $A = 1$.

1) Discretize the space of $\theta_t$ in two states $\{\theta, \theta^c\}$. Calibrate these grid to match the first and second moments of the distribution for $\theta_t$. What do these two values depend on? Calibrate the transition matrix to match the TFP autocorrelation.

2) Provide the policy functions and the value functions for both $k_{t+1}$ and $c_t$ as functions of the state variables $(k_t, \theta_t)$ assuming $\sigma^2 = 1$.

Let $\sigma^2 = 1$.

3) Use Matlab to plot the value functions and the policy function for $k_{t+1}$ as functions of the state variables, $k_t$ at both $\theta$ and $\theta^c$ states. Using algebra, find the ergodic set for $\sigma^2 = .5$ and $\sigma^2 = 1$.

4) Simulate the process for $\theta_t$ for 100 shocks. Suppose $k_o = \frac{k_{max} + k_{min}}{2}$. Use the process for theta to generate a series for capital for 100 periods.

5) Repeat for 3000 periods. Discretize the ergodic set in 100 equally spaced intervals. Plot the histograms corresponding to the unconditional distribution of the capital stock for $\sigma^2 = 0$ and $\sigma^2 = 1$. For $\sigma^2 = 1$, plot the histograms corresponding to the conditional distributions for capital (conditioned on $\theta$ and $\theta^c$).

2. Practical Dynamic Programming

Let the problem be

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to $c_t + k_{t+1} \leq \theta_t A k_t^\alpha + (1 - \delta)k_t$, $1 > \alpha > 0$, $1 > \delta > 0$, $\ln(\theta_{t+1}) = \rho \ln(\theta_t) + \varepsilon_{t+1}$ with $\varepsilon_{t+1}$ being i.i.d. $N(0, \sigma^2)$ and $k_0$ given. Let $\alpha = .6$, $\delta = .1$, $\rho = .95$, $\beta = .99$, and $A = 1$. Assume $u(c_t) = c_t^{1-\phi}$. Let $\phi = 2$.

1) Discretize the space for capital in 150 equally spaced intervals. Discretize the space
of $\theta_t$ in two states $\{\breve{\theta}, \bar{\theta}\}$. Calibrate the grid to match the first and second moments of the distribution for $\theta_t$. Calibrate the transition probability matrix to match the TFP autocorrelation (Hint: the stochastic process for theta is symmetric).

2) Find the policy function for $k_{t+1}$ and the value functions as functions of the state variables $(k_t, \theta_t)$ assuming $\sigma^2 = .1$.

3) Find the ergodic set for $\sigma^2 = .1$.

4) Simulate the process for $\theta_t$ for 100 shocks. Suppose $k_o$ is right the average of the two limits of the ergodic set. Use the process for theta to generate a series for capital for 100 periods.

5) Repeat for 3000 periods. Plot the histograms corresponding to the unconditional distribution of the capital stock for $\sigma^2 = 0$ and $\sigma^2 = .1$. What does it mean to have a $\sigma = 0$? For $\sigma^2 = .1$, plot the histograms corresponding to the conditional distributions for capital (conditioned on $\breve{\theta}$ and $\bar{\theta}$).

6) Compute the sequence of consumption for these innovations. Plot the unconditional distribution of consumption using a histogram. Compute the expected utility and the certainty equivalent consumption for this economy. What is the lost in terms of consumption goods (Lucas)?

7) What economy is that with $\rho = 0$, $\phi = 1$ and $\delta = 1$?

3. **Howard’s Improvement Algorithm**

Use the Howard improvement algorithm to solve the above problem. Let $\sigma^2 = .1$. Given the convergence criteria you used before,

1) Compute the number of iterations needed for convergence without the Howard’s improvement algorithm.

2) How many iterations would you need if you implement this algorithm?

3) Is this algorithm more useful for many state variable problems? For many control variable problems? Explain.