This assignment is about using dynamic programming in an environment with heterogeneous agents and no macroeconomic uncertainty.

1. Aiyagari (1994)

We will verify some of the results in Aiyagari (1994), Table 2. You should obtain your results by solving for a piecewise linear approximation of the true (stationary) equilibrium decision rule of for the model.

A typical household solves the following problem

$$\max_{\mathbf{c}_t} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\mu}}{1-\mu}, \quad \mu > 0$$

subject to

$$c_t + a_{t+1} = w_l t + (1 + r_t)a_t$$

$$\log l_{t+1} = \rho \log l_t + \sigma(1 - \rho^2)^{1/2} \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1)$$

$$a_{t+1} \geq 0$$

The variable $c_t$ is consumption, $l_t$ is labor endowment, and $a_t$ is asset holdings (claims to productive capital) in period $t$. The labor endowment shock is distributed independently across a continuum of households. In addition, households are liquidity constrained; they cannot have negative asset holdings.

Output is produced using a technology given by $k^{\theta n} (1 - \theta)$, where $k$ is the stock of capital and $n$ is labor. The stock of capital is assumed to depreciate at the rate $\delta$ each period. Given that we are interested in studying a stationary equilibrium, the stock of capital, labor, wage rate and interest rate will be constant over time. This will be our trick to solve this model without getting into too much trouble (no macroeconomic uncertainty).

Let $\beta = .96$, $\mu = 3$, $\theta = .36$, $\delta = .08$, $\rho = .6$, and $\sigma = .4$.

1) Define a stationary recursive competitive equilibrium for this economy (see chapter 2 of the book by Cooley cited on the Syllabus).

2) Calibrate a two-stationary Markov chain for the $\log l_t$.

3) Compute a piecewise linear approximation of the household's optimal decision rule, $a' = g(a, l)$. Graph this decision rule (you will need two curves, one for $s=e$ and one for $s=u$).

4) By simulating for 100,000 periods and discarding the first 10,000 observations, compute the invariant asset distribution for this economy and graph it. Use the Matlab command `hist` with 100 equal sized bins.

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1 This problem was prepared by Prof. Gary Hansen.
5) Compute the interest rate and aggregate savings rate for this economy. How do your results compare with what is given in Table 2 of Aiyagari’s paper? How do your results compare with the interest rate and savings rate under complete markets? Explain carefully.