Homework #6

Prof. Jose Wynne

This assignment is about applying the log-linearization method to approximate rational expectation models.

1. The Optimal Growth Model

Simulate the optimal growth model with labor and adjustment costs by solving

\[
\max \sum_{t=0}^{\infty} \beta^t \left[ c_t^\alpha (1 - h_t)^{1-\alpha} \right]^{1-\mu} / (1 - \mu)
\]

subject to

\[
c_t + i_t = s_t A_t^{1-\theta} k_t^{1-\theta} - \frac{\phi}{2} [k_{t+1} - (1 + \lambda)\bar{k}]^2
\]

\[
k_{t+1} = (1 - \delta)k_t + i_t
\]

\[
A_t = (1 + \lambda)A_{t-1}
\]

\[
s_t = \rho s_{t-1} v_t, \quad v_t \sim i.i.d \text{ LogN}(1, \sigma^2_v)
\]

\[
k_0, s_0, A_0 \text{ given}
\]

The variable \( c_t \) is consumption, \( h_t \) is labor supply, and \( k_t \) is the capital stock at \( t \) as usual. Let \( \bar{k} \) be the capital stock in steady state. This problem solves the allocations of a competitive economy where agents don’t internalize the full cost of adjustment.\(^1\) For your simulations let \( \beta = .98, \alpha = .5, \mu = .5, A_0 = 1, \lambda = .025, \theta = .36, \phi = .003 \delta = .025, \rho = .95 \) and \( \sigma_v = .00712 \). See that \( \beta(1 + \lambda)\alpha(1-\mu) < 1. \)\(^2\)

1) Find the steady state values of the de-trended variables.
2) Find the first order conditions of the de-trended dynamic problem.
3) Log-linearize the system of first order conditions around the non-stochastic steady state.
4) Generate impulse responses for de-trended capital, investment, consumption, employment and output variables for a one standard deviation shock. Simulate 5 periods before and 25 after the innovation.
5) Simulate this economy during 50 periods starting at the de-trended steady state by generating the a vector of shocks for each period. Show all the variables like before after using the H-P filtering approach. Compute the business cycle statistics like you did in

1 How would this problem change with adjustment costs of the following form?

\[
\frac{\phi}{2} [k_{t+1} - (1 + \lambda)k_t]^2
\]

2 What is the importance of this condition?
HW#5.

6) Sensitivity analysis. What are the consequences of increasing $\phi$ in the economy? And $\mu$?

7) What would you say are the drawbacks of this method?

Report your computational strategy clearly. Hand in your homework and codes as requested by your TA.

2. **Letting the computer do the Log-linearization**

Program an algorithm that does the log-linearization for you. That is, program a matlab function that returns the matrices of the log-linear system of equations for the Optimal Growth Model. Repeat exercises 1) to 5) of Problem 1 for the same parameter values. Is your matlab function giving you about the same results?