Often we must make decisions in the face of uncertainties about what will happen in the future.

“An unbiased appreciation of uncertainty is a cornerstone of rationality – but it is not what people and organizations want.

“D. Kahneman, 2011.

Nor is thinking about uncertainties done well by most of us most of the time!
Examples of Probability Questions

• What is the probability that Duke will win the ACC Basketball Tournament in 2016?

• The DJIA closed at 17641 on January 12, 2015.
  – What will it close at on December 31, 2015?
  – Give a range such that you are 90% sure that the answer is between the low number and the high number. That is, give a number such that you think there is only 1 chance in 20 (5%) that the DJIA will close below that number. Do the same task for the upper number.

• Assume that the prevalence of breast cancer for women over the age of 40 is 1%. A widely used test, a mammography, gives a positive result in 10% of women without breast cancer, and in 80% of women with breast cancer.
  – What is the probability that a woman in this age class who tests positive actually has breast cancer?

• What is the probability that Sunday will be hotter than any other day during the next week?
  – What is the probability that next week the hottest day will be Sunday?
Some initial research questions about probabilistic reasoning - 1.

How good are people as assessors of probabilities?

Do people’s judgments about uncertainties correspond with real outcomes, e.g., the stock market returns? (Correspondence standard of rationality.)

Do people show good calibration in their judgments, e.g., how often do events for which I say there is an 80% probability actually occur?

Do people follow the basic laws of probability in assessing their beliefs about uncertain events? That is, if \((A \subseteq B)\) then is the assessed \(P(A) \geq P(B)\)? That is, are the judgments coherent?

*Note, no matter how good people are at assessing probabilities, they often do not feel comfortable doing so.*
Dealing with Uncertain Events is Essential and Often Uncomfortable

• “It’s science; there are uncertainties, and they get larger as you forecast further into the future. But the state can’t deal with uncertainties and wanted a specific number.”

• The quote above was from Stan Riggs who was a member of the NC Coastal Resources Commission Science Panel. Quote from a story in the News and Observer, June 21, 2012.
**Feelings about Uncertainty Scale: Measuring Individual Differences**

How well do each of the following statements describe you (your organization) in general. Please try to respond as truthfully as possible. That is, respond in accord with how you believe you actually are, not how you would like to be.

Please use the following scale to assess each statement in terms of how well it describes you (your organization).

<table>
<thead>
<tr>
<th>Completely Disagree</th>
<th>Neutral</th>
<th>Completely Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statement</th>
<th>Rating</th>
<th>Total Score =</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty makes life intolerable.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>My mind can’t be relaxed if I don’t know what will happen tomorrow.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncertainty makes me uneasy, anxious, and stressed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>One should always look ahead so as to avoid surprises.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A small unforeseen even can spoil everything, even with the best planning.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Being uncertain means that I am not first rate.</td>
<td></td>
<td></td>
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<tr>
<td>I always want to know what the future has in store for me.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Being uncertain means that I lack confidence.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can’t stand being undecided about my future.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The smallest doubt can stop me from acting.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Is being uncomfortable with uncertainty due to the nature of System 1 thinking which tends to operate with certainty?
Measures of Judgment Quality

• If we say there is a 70% chance of rain tomorrow, and it doesn’t rain, are we wrong in our judgment?

• How often do your 90% confidence intervals bracket (contain) the actual outcome? That is, how often are you surprised by an outcome that is outside your 90% confidence interval?
“Don’t blame me. It was a good forecast.”
The Concept of Good Calibration: A correspondence measure of rationality in Probability Judgments

(Example Democrat or Republican Wins the Presidential election in 2016)

• You say .60 probability that the Democrat wins
• .7
• .8
• .9
• 1.0

• 100 other events with $p = .60$, how many events should happen?
• 100 other events with $p = .70$, how many events should happen?

• Note, on any single event it is hard to determine if you were right or wrong.
More on The Concept of Good Calibration

<table>
<thead>
<tr>
<th>Forecast Probability of rain</th>
<th># of days</th>
<th>ideal # days with rain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>.1</td>
<td>1000</td>
<td>100</td>
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<td>.8</td>
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<tr>
<td>.9</td>
<td>500</td>
<td>450</td>
</tr>
<tr>
<td>1.0</td>
<td>250</td>
<td>250</td>
</tr>
</tbody>
</table>
Does good calibration ever exist?
An example of Good Calibration: Meteorologists
Physicians, after completing history and physical examination, estimated the probability that patients had pneumonia.
The typical Calibration Curve for events with few outcomes, (e.g., guilty or not), looks like the below, i.e., overconfident Eyewitness Confidence

Adapted from Granhag, P. (1997, J Applied Psych)
Recap

• Calibration provides one measure of good probability judgments. Calibration is a correspondence measure.

• People show good calibration sometimes, e.g., weather forecasters for whether it will rain tomorrow, however, people also show systematic patterns of poor calibration and poor coherence in probability judgments.

• Other measures include various coherence measures, e.g., the probability of living to age 85 or older should be 1 – the probability of dying by age 85 or younger. Is it?

• In addition, measures include match to normative models like Bayes’ Theorem.
A second set of key questions about probabilistic reasoning - 2.

• How do people actually process information when assessing probabilities?
  • Are people, for instance, a) approximately Bayesian (the normative model) in their reasoning processes when updating beliefs, or
  • b) do they think in very different ways? How sensitive are probability judgments to such task variables such as how you ask the probability question?
Thinking About Uncertainties: Bayesian Reasoning?

• The classic cab problem.

• Expert Judgment: The probability of breast cancer given a positive mammogram.

• Results and implications of various other tests of Bayesian updating.
The Cab Problem

• A cab was involved in a hit and run accident at night. Two cab companies, the Green and Blue, operate in the city. You are given the following data.

  – A) 85% of the cabs in the city are Green and 15% are Blue.

  – B) A witness identified the cab as Blue. The court tested the reliability of the witness and concluded that the witness correctly identified each of the two colors of cabs 80% of the time and failed 20% of the time.

  – C) What is the probability that the cab involved in the accident was Blue rather than Green?

• What is your estimate?
• What estimate does the average person provide?
The “Rational” Bayesian Model

Bayes’ Theorem

\[ P(H_i|D) = \frac{P(D|H_i) P(H_i)}{\sum P(D|H_i) P(H_i)} \]

or

\[ \frac{P(H_i|D)}{P(H_j|D)} = \frac{P(D|H_i)}{P(D|H_j)} \times \frac{P(H_i)}{P(H_j)} \text{ for all } j. \]

Posterior Odds  Likelihood Ratio  Prior Odds

The likelihood ratio is an index of data diagnosticity.
Note, the order in which data are processed makes no difference
to their impact on posterior opinion.

Example: P(H|D) = Cab was Blue given witness report.
= .80 * .15 / (.80 * .15) + (.20 * .85) = .41
Assume that the prevalence of breast cancer for women over the age of 40 is 1%. (This is sometimes referred to as the “Base-rate.”)

A widely used test, a mammography, gives a positive result in 10% of women without breast cancer, and in 80% of women with breast cancer.

What is the probability that a woman in this age class who test positive actually has breast cancer?

What was your answer?

What about (experts) doctors?
Bayes Theorem: A Medical Example - 2

The true answer is about .075.

P(cancer) = .01 & P(no cancer) = .9
P(pos test / cancer) = .80
P (pos test) = .107
(P (pos / no cancer) = .1

Eddy (1988) reports that when given this question, 95 out of 100 practicing physicians responded “About 75%.”
Another “Bias” due to System 1 thinking? The case of probability matching.
Probability Matching: An Classic Task

The subject must guess which of two random events will occur. A reward is given if the decision maker gets it right. The probability of the more likely event is specified by the experimenter but unknown to the subject, who learns through repeated trials.
Fig. 1. Distribution of choice index value across participants in the original and replication studies. An index value of 0 indicates probability matching; a value of 1 indicates maximizing. D.J. Koehler, G. James (2009)

Results are consistent with the interpretation of probability matching as a mistake rooted in a fast, intuitive response that is not reliably overridden by a more effortful reconsideration of whether it in fact produces the highest expected payoffs.

Notably, a substantial proportion of participants who engaged in probability matching on the choice task later acknowledged the superiority of maximizing when both strategies were explicitly described for comparison.

This suggests that one reason why people engage in probability matching is that it springs readily to mind as a strategy, while maximizing does not.
General Pattern of Results for Probability Matching Studies

• **Recap:** People generally probability match. If the probability of a red light is 70% (30% green), people will respond red approximately 70% of the time.

• Young children and many nonhuman animals are more likely to maximize and less likely to match than adult humans. Are pigeons smarter than people?

• People (adults) have a “bias” to think in terms of patterns over set of trials that can be overcome. Hardwired?

• Also, when memory load is increased, rate of maximization seems to increase. Does more thinking “hurt”?

• However, the “bias” to think in terms of patterns over set of trials can be overcome, somewhat.
Probability matching and the role of the Left hemisphere – Wolford et al. (2000)

- People are prone to search for and posit causal relationships among events. People look for patterns.

- The existence of an “interpreter” in the left hemisphere.

- Results show probability matching in the left hemisphere and movement toward maximizing in the right hemisphere.

- Is seeing patterns in noise a necessary consequence of how people are “hardwired”?
Are People Responding to the Right Variables?

Problem: Two bags - one bag with 70 red and 30 green chips a second bag with 30 red and 70 green chips. One bag is selected at random and a sample of 7 red and 3 green chips is drawn. What is the probability that the sample was drawn from the mostly red bag?

Bayes Calculation

\[(7/3)^7 \times (3/7)^3 \times 1 = 29.6 : 1\] or 97% probability

Actual responses = .70 or so.

K & T, 1972

The key is the difference, not the ratio.
Conclusion of Early Probability Studies

• “In his evaluation of evidence, man is apparently not a conservative Bayesian, he is not Bayesian at all.” Kahneman & Tversky.

• Note, there is still much debate about this proposition. Maybe people are just simple minded Bayesians.
  • Question: Does this include the use of an additive rule rather than a multiplicative rule for combining information? Is addition a default strategy?

• If people are not Bayesian in their reasoning, how are probabilistic judgments made?

• The “Heuristics and Biases” Paradigm.