“Bricks-and-mortar” vs. “clicks-and-mortar”: An equilibrium analysis

Fernando Bernstein a,*, Jing-Sheng Song a, Xiaona Zheng b

a The Fuqua School of Business, Duke University, 1 Towerview Drive, Box 90120, Durham, NC 27708, United States
b Guanghua School of Management, Peking University, Beijing 100871, China

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Abstract

The Internet has provided traditional retailers a new means with which to serve customers. Consequently, many “bricks-and-mortar” retailers have transformed to “clicks-and-mortar” by incorporating Internet sales. Examples of companies making such a transition include Best Buy, Wal-Mart, Barnes & Noble, etc. Despite the increasing prevalence of this practice, several fundamental questions remain: (1) Does it pay off to go online? (2) Which is the equilibrium industry structure? (3) What is the implication of this business model for consumers? We study these issues in an oligopoly setting and show that clicks-and-mortar arises as the equilibrium channel structure. However, we find that this equilibrium does not necessarily imply higher profits for the firms: in some cases, rather, it emerges as a strategic necessity. Consumers are generally better off with clicks-and-mortar retailers. If firms align with pure e-tailers to reach the online market, we show that a prisoner’s dilemma-type equilibrium may arise.

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1. Introduction

The rapid development of information technologies has provided new means for retailers to reach the end market. As substantially more consumers have gained Internet access and found it both convenient and secure to shop online, e-commerce has become attractive to more firms. Various Internet-enabled business models have emerged. Among those, an important one deals with the integration of the Internet channel and the traditional retail channel, the so-called “clicks-and-mortar” business model. Indeed, retailers ranging from department stores to specialty stores, from low-end to high-end, have launched Internet sales sites beside their pre-existing retail channels. Examples include Bloomingdales, Best Buy, Barnes & Noble, Linens ’N Things, and WalMart, to name just a few of the most well-known.
In the case of traditional retailers, the Internet channel is often viewed as a logical extension of the store-front’s physical presence, a complement to existing customer relationships, business processes, and distribution systems (Zerega, 1999). Scott Silverman, director of Internet retailing at the National Retail Federation, says: ‘Branding is a tremendous advantage and cross-promoting it over the Internet and in physical stores will open up new selling opportunities.’ Melissa Bane, an analyst with the Yankee Group, agrees: ‘Some of the smarter ones are starting to realize that it is their game to lose. They already have the customers, and now they can use the Internet in their stores as a tool to expand their share of customers’ (Tedeschi, 1999).

These business practices and observations would seem to suggest that bricks-and-mortar (hereafter referred to as B) retailers are, or should necessarily be, transforming into clicks-and-mortar (hereafter referred to as C) retailers. However, if all retailers in the market adopt the C business model, the resulting competition may also decrease the value of a dual-channel structure. Therefore, the widespread shift to clicks-and-mortar supply chains may not bring a substantial benefit to retailers. Rather, it may favor only consumers. Motivated by these concerns, our research aims to address the following fundamental questions:

1. Are companies making the right decision in adopting the C business model?
2. What is the industry equilibrium business model?
3. Given that some retailers may not be capable of establishing and efficiently operating Internet sales, how does the industry equilibrium change when retailers need to align with pure Internet firms in order to go online?

To address these questions, we analyze the supply chain channel structure choice in an oligopoly setting. There are \( n \) existing B retailers, each selling one product. The products are close substitutes. When retailers have the capability of efficiently managing a C structure, their strategic decision concerns which supply chain model to adopt: B or C. The equilibrium of this channel structure game is derived by comparing the outcomes of three scenarios: B vs. B competition, C vs. B competition, and C vs. C competition. In each scenario, the retailers’ decision variables consist of the prices at the retail stores and the prices at the Internet stores (if applicable).

We differentiate the single traditional retail channel (B structure) from the dual clicks-and-mortar channel (C structure) in terms of the operational and transactional costs involved in managing each type of supply chain and in the way consumers assess the product value in each setting. Specifically, we model consumer demand by deriving a multinomial logit (MNL) model from consumer utility maximization. Consumers’ valuation for products depends on where the products are purchased (either at a retail store of a B retailer, at a retail store of a C retailer, or through a C retailer’s online channel).

We consider two variants of the MNL model that differ according to how consumers value the product. In the first setting, valuation for the product is high enough or, alternatively, consumers have very low price sensitivity for the product, to induce all consumers to make a purchase. In the second setting, there exists an outside option and some consumers may choose not to buy the product. Here, consumer valuation for the product is moderate and firms must compete with the outside alternative. Consumers select a variant by trading off the benefits of owning the product with the price they need to pay for it and the value obtained from resigning the purchase. Related to these two settings are the works of Rheec (1996) and Salop (1979). Rheec (1996) studies firms’ strategic quality decisions in a duopoly setting. Analyzing a scenario similar to our first setting, the author assumes that products’ consumption value is large enough for consumers to buy one of two products. The paper derives closed-form solutions under this assumption. Salop (1979) studies Chamberlinian monopolistic competition with outside goods. The author finds that the equilibrium behaves differently from that in the literature obtained without considering outside goods.

In each model setting, we characterize equilibrium prices, quantities and profits when each firm has a fixed channel structure (B or C). Based on that, we derive the equilibrium channel structure. We find that all firms adopt, in equilibrium, the C business model. In other words, C vs. C arises as the industry equilibrium structure. However, this industry equilibrium may not increase firms’ profitability compared to the base B vs. B setting. This actually turns out to be the case in the absence of an outside option. In those cases, adoption of the Internet becomes a strategic necessity, rather than a source of additional revenue for the retailer. In contrast, when there is an outside alternative, business profitability at the C vs. C industry equilibrium is strictly larger than in the B vs. B setting. These results have interesting implications. First, the underlying con-
sumer model has a non-trivial impact on the outcome of the game: without an outside option in the market, firms do not derive additional profits by adding online sales, while they do in settings where there is an outside alternative. In settings with an outside option, the addition of online sales may allow firms to capture a portion of the market that would have opted for the outside option had the Internet channels not been available. In contrast, when all customers make a purchase, the addition of online sales implies changes in the equilibrium prices that result in the same firms’ aggregate (retail and online) market shares as their retail market shares in the setting with traditional stores only.

We also show that the adoption of an Internet channel increases productivity by reducing transaction costs. Moreover, in this situation benefits are passed along to consumers. Consumer surplus increases for two reasons: First, a portion of reduced transaction costs are passed on to consumers in the form of lower prices, and second, the C structure typically increases customers’ valuation for the product, thus creating additional surplus. These findings are consistent with empirical observations by Hitt and Brynjolfsson (1996). These authors use firm-level data on IT (information technology) spending by 370 large firms and examine three measures of IT value: productivity, business profitability and consumer surplus. They suggest that IT has improved productivity and created value for consumers. They do not find evidence that these benefits have resulted in higher business profitability.

Traditional bricks-and-mortar firms may not necessarily have the capabilities to efficiently operate a C business structure. In those cases, retailers may need to align with existing Internet companies. For example, Toys-R-Us and Amazon formed an alliance in the year 2000 to sell toys online. In this context, we investigate the equilibrium industry structure that arises when one or more B retailers form an alliance with a pure Internet counterpart to establish a dual-channel market strategy. In particular, we study two duopoly settings. In the first setting, neither retailer is capable of efficiently operating its own Internet channel. Instead, they each may consider forming an alliance with an Internet retailer. In the second setting, one retailer can successfully operate a C supply chain, while the other retailer can sell online only by aligning with a pure e-tailer. In these settings, we find that a prisoner’s dilemma-type equilibrium may arise.

The focus of this paper is on existing traditional retailers and their decision on whether or not to sell over the Internet. That is, we consider entry decisions in terms of the retailers’ choice of expanding their operations by adding online sales. We consider both the cases when the retailer manages both its retail stores and its own Internet site (e.g., Wal-Mart), and cases when the traditional retailer aligns with a pure Internet retailer to sell online (e.g., Toys-R-Us and Amazon). In addition, we provide a brief discussion on how our results extend to settings with pure Internet firms in the market (see Footnote 7).¹

Finally, we briefly investigate the impact of Internet penetration on industry equilibrium. We find that at the industry’s equilibrium, as the additional market reached by use of the Internet increases, firms’ profits increase but their profits from the original market decrease. We also find that Internet prices are lower, mainly due to the expanded consumer reach of the Internet.

To the best of our knowledge, our work is among the first to study the industry equilibrium structure in response to the Internet as a new selling channel for traditional retailers, and to analytically investigate the value of the Internet as a selling channel for both retailers and consumers.

The related literature can be grouped into three streams. The first studies competition among single-channel retailers. Bernstein and McCardle (1997) analyze competing newsvendors and examine equilibrium inventory levels. Bernstein and Federgruen (2003) consider a multi-period model and study the inventory and pricing issues that arise when a supplier sells through multiple competing retailers. For other works on retailer competition, see Mahajan and van Ryzin (2001) and Bernstein and Federgruen (2005), and the references cited therein. Although our study also considers horizontal competition, the aforementioned studies focus on a fixed number of alternatives, while in our model the number of variants results from the retailer’s channel choice decisions.

The second related stream of research addresses price competition on the Internet. For example, Bakos (1997) explores the effect of buyer search costs on competition in markets with differentiated product offerings. The adoption of an electronic marketplace is not the decision of an individual seller and no dual-channel

¹ According to a special report sponsored by Triversity “The Nation’s Retail Power Players 2005,” by Schulz (2005), there is only one pure Internet retailer among the 100 largest retailers in the US – Amazon.com.
structure arises. Balasubramanian (1998) models horizontal competition between a direct marketer and conventional retailers. This work also examines the role of information in multi-channel markets and finds that high market coverage may depress profits. This paper does not consider competition among dual-channel players. Lal and Sarvary (1996) analyze competition between two retailers that each sell online and offline. They identify conditions under which the Internet might reduce price competition. They compare two settings: two traditional retailers vs. two dual-channel retailers. The retailers’ channel choice, however, is not explicitly modeled. Other works related to the Internet include Zettelmeyer (2000), who studies competition across channels in a context in which consumers are uncertain about their preferences, Dewan et al. (2003) that explores the effect of product customization on price competition in a duopoly setting, Balakrishnan et al. (2004) that analyzes how the ability of consumers to switch between retail and online channels affects prices, and Wu et al. (2004) that examines a seller’s incentive to provide information services online in the presence of free riding competitors. Although these studies consider price competition on the Internet, none of them explores the equilibrium industry structure that arises when firms choose between single- and dual-channel strategies.

Finally, the third stream of research explores the impacts of the Internet on supply chain performance and analyzes the choice of channel structure from a manufacturer’s perspective. See Bell et al. (2002), Cattani et al. (2003), Cattani et al. (forthcoming), Chiang et al. (2003), Kumar and Ruan (2004), Bernstein et al. (2004), Druehl and Porteus (2005), Tsay and Agrawal (2004b) and the surveys by Cattani et al. (2004) and Tsay (2003), Cattani et al. (forthcoming), Chiang et al. (2003), Kumar and Ruan (2004), Bernstein et al. (2004), Druehl and Porteus (2005), Tsay and Agrawal (2004a). These papers focus mainly on channel conflict and coordination in the presence of a manufacturer’s direct channel.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 analyzes the industry equilibrium structure without an outside alternative, while in Section 4 we explore settings with an outside option. We discuss extensions in Section 5, while Section 6 concludes the paper.

2. The model

We consider an n-firm oligopoly setting to study the supply chain channel structure game. Traditional retailer i, that sells a product to the end market through a retail store, may choose to open an Internet channel and move from a B to a C supply chain. In the B setting, retailer i chooses a unit retail price \( p_i \), and in the C setting, retailer i decides on two prices: the price at the traditional channel, \( p_i \), and the one at the Internet channel, \( p_{ei} \). (Our results remain valid if \( p_{ei} \) is constrained to equal \( p_i \)). We use \( \mathcal{I} = \{1, 2, \ldots, n\} \) to denote the set of retailers.

2.1. Consumer utility model

The means by which the product is distributed has an impact on consumer’s choice. A consumer derives different utilities when obtaining the product from a retailer’s physical store (alternatives i) than from a retailer’s Internet site (alternatives ei). Let the set of alternatives be denoted by \( \mathcal{A} \). We first assume that there is no outside option. Section 4 considers the setting with an outside alternative. Under the assumption of no outside option, the set of alternatives is \( \mathcal{A}^B = \{1, \ldots, n\} \) when all firms operate as bricks-and-mortar, while it is \( \mathcal{A}^C = \{1, e1, 2, e2, \ldots, n, en\} \) when all operate as clicks-and-mortar. When \( k \) firms, say firms \( 1, \ldots, k \), operate as clicks-and-mortar, while the remaining firms operate under the traditional structure, the set of alternatives is \( \mathcal{A}^{CB}(k) = \{1, e1, \ldots, k, ek, k + 1, \ldots, n\} \). (In the setting with an outside option, \( \mathcal{A} = \mathcal{A} \cup \{0\} \).)

A consumer associates a utility \( U_a \) for alternative \( a \in \mathcal{A} \). To model \( U_a \), we use the additive random utility model, which is composed of two parts, a deterministic representative component \( v_a - p_a \) and a random component \( \epsilon_a \) that accounts for the consumers’ idiosyncratic tastes about the options. Therefore, \( U_a = v_a - p_a + \epsilon_a \), \( a \in \mathcal{A} \), where \( v_a \) is the consumption value. The probability that variant \( a \) is chosen is given by \( q_a = P(U_a = \max_{b \in \mathcal{A} \cup \{0\}} U_b) \), \( a \in \mathcal{A} \), which depends on the random components \( \epsilon_a \). We assume that \( \epsilon_a \) are i.i.d. random variables with a Gumbel (or double exponential) distribution with mean zero and variance \( \frac{\pi^2}{6} \), where \( \sigma > 0 \) is a scale parameter representing the degree of heterogeneity among consumers. Then, the probability of choosing channel \( i \) is given by the multinomial logit (MNL) demand model

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2 This parameter represents the standard deviation of the taste distribution. See Anderson et al. (1992).
The own and cross-partial derivatives of the choice probabilities with respect to the prices are:

\[
\frac{\partial q_i}{\partial p_i} = -\frac{1}{\sigma} q_i (1 - q_i) < 0, \quad \frac{\partial q_i}{\partial p_j} = \frac{1}{\sigma} q_i q_j > 0.
\]

That is, increasing the price for a variant reduces its own demand but increases demands for other variants. Therefore, the MNL model captures the substitution effect among the alternatives.

2. If \( v_i - p_i = v_j - p_j \), all alternatives are equally probable, i.e., \( q_i = \frac{1}{|\mathcal{J}|}, \quad i \in \mathcal{J} \).

The MNL choice model allows us to study demand as a function of consumers’ valuations for the various channel alternatives. This model also captures consumers’ unobservable random preferences for different alternatives. In addition, the MNL model captures the idea of “bounded rationality,” because a consumer does not necessarily choose the alternative that yields the highest utility, but instead has a (positive) probability of choosing each of the possible alternatives. Finally, the MNL model lends itself to model changes in the number of alternatives (in this case, channels) available.

2.2. Internet features

We distinguish the Internet channel from the retail channel by comparing two features – the operating cost and consumer valuation. For products distributed through the physical store, retailer \( i \) incurs a unit cost \( c_r \), and for products sold at the Internet site, the unit cost is \( c_e \). We assume that \( c_{ei} < c_r \), reflecting differences in stocking costs, employee labor costs, facility maintenance costs, etc. In fact, empirical evidence shows that IT and the Internet reduces transaction costs and the cost of goods sold (Garicano and Kaplan, 2001; Zhu, 2004).

To model the impact of the Internet channel on consumer valuation, we introduce a parameter \( \beta \) and assume that a customer’s valuation for retailer \( i \)’s product changes from \( v_i \) to \( \beta v_i \) when a firm sells through both traditional and Internet channels. In Sections 3 and 4, we assume that \( \beta \geq 1 \), while in Section 5.1 we provide a discussion on settings where \( \beta < 1 \). The assumption \( \beta \geq 1 \) is reasonable in many settings because once a retailer changes its channel structure, a consumer’s perceived value for this retailer may improve. Indeed, consumers’ valuation may increase as the retailer provides them with an alternative channel in which to buy its product. Specifically, consumers’ perceived value for retailer \( i \)’s product may be positively influenced in several ways: (1) Consumers enjoy the convenience of buying the product either online or in the store; (2) consumers can obtain information from one channel but make the purchase in the other channel; (3) consumers are often allowed to return online products to a local store; (4) tailored emails may raise a consumer’s desire to purchase the product. Therefore, consumers’ value for the “brand” of retailer \( i \) may increase. In fact, many consumers report that a dual-channel structure increases their willingness to purchase. A 28-year-old Manhattan Web-advertising producer figures that she is spending 10–15% more at Gap ever since Gap launched its website and sends her tailored emails twice a month (Lee, 1999). Gaps’ chief executive, Jeanne Jackson, says, “this is about being clicks and mortar – letting customers access the Gap brands.” She believes that by aggressively marketing both the stores and the website and allowing each to leverage the strengths of the other, both channels will prosper (Lee, 1999). In this example, \( \beta \geq 1 \) makes sense. In some other settings, however, it may be reasonable to expect that \( \beta < 1 \). That is, for certain products, consumers’ valuation may be lower when firms sell through both channels than when they only operate through traditional retail stores. All the results in the paper continue to hold in these cases, provided that \( \beta \) is not too small. We provide a detailed discussion on the impact of other parameter ranges in Section 5.1. Note that for retailer \( j \) with a \( B \) structure, consumer valuation remains at \( v_j \). In Section 5.2.1, we explore a setting where the adoption of an Internet channel increases the overall market size.
Finally, while consumer valuation for a product obtained at the traditional store of a C retailer \( i \) is \( \beta v_i \), its valuation for a product obtained at this retailer’s Internet site is \( \beta \theta v_i \). The parameter \( \theta \) (which can take any positive value) measures the difference in valuation between the physical and Internet channels. Liang and Huang (1998) show that, overall, consumers prefer making their purchases at traditional retail stores rather than through the Internet. A survey by Kacen et al. (2002) shows that, for product categories such as books, shoes, toothpaste and DVD players, \( \theta \) varies from 0.769 to 0.904.

Throughout the paper, we assume symmetry of all retailers in the market. That is, \( c_i = c, c_{ei} = c_e \) and \( v_i = v \), for all retailers. This assumption ensures that our results are driven by purely competitive reasons and not by asymmetries among retailers.

3. Choice of channel structure: Without an outside option

Throughout this section, we assume that all consumers make a purchase. The absence of an outside alternative allows us to obtain closed-form expressions for equilibrium prices, quantities and profits in various competition scenarios. Later, we discuss additional implications of this assumption.

To study whether a retailer has incentive to open an Internet channel along with its existing traditional channel, we consider three scenarios: (I) All retailers operate a B business model (this is the base setting, referred to as B vs. B competition); (II) Some retailers adopt the C business model and the rest maintain the B business model (C vs. B competition); (III) All retailers adopt the C business model (C vs. C competition). Fig. 1 shows these scenarios with \( k \in \{1, 2, \ldots, n\} \).

We list notation for the equilibrium outcomes under different scenarios below. Subscripts are dropped when the results are identical for all retailers.

\[
\begin{align*}
& p_i^B \quad \text{equilibrium price for retailer } i \text{ under B vs. B competition;} \\
& p_i^{CB} \quad \text{equilibrium price at the traditional channel for retailer } i \text{ under C vs. B competition;} \\
& p_{ei}^{CB} \quad \text{equilibrium price at the Internet channel for a C retailer } i \text{ under C vs. B competition;} \\
& p_i^C \quad \text{equilibrium price at the traditional channel for retailer } i \text{ under C vs. C competition;} \\
& p_{ei}^C \quad \text{equilibrium price at the Internet channel for retailer } i \text{ under C vs. C competition (similar notation is used for quantities or market shares \( q \));} \\
& \Pi_i^B \quad \text{equilibrium profit for retailer } i \text{ under B vs. B competition;} \\
& \Pi_i^{CB} \quad \text{equilibrium profit for retailer } i \text{ under C vs. B competition;} \\
& \Pi_i^C \quad \text{equilibrium profit for retailer } i \text{ under C vs. C competition.}
\end{align*}
\]

3.1. Scenario I: B vs. B

In Scenario I, the set of alternatives for consumers is \( \mathcal{A}^B = \{1, 2, \ldots, n\} \). Demand in retailer \( i \)'s traditional store is given by (1) with the set \( \mathcal{A} = \mathcal{A}^B \). All retailers simultaneously set their prices \( p_i \). Each retailer \( i \)'s objec-

![Diagram](image-url)
tive is to maximize its profit \( P_i = (p_i - c)q_i \). The following result establishes the existence of a unique Nash equilibrium (the proof follows from Anderson et al., 1992, p. 222).

**Proposition 1.** Without an outside option, in Scenario I, the price-setting game has a unique Nash equilibrium with \( p^B = \frac{\sigma}{n-1} \sigma + c \), \( q^B = \frac{1}{\sigma} \) and \( \Pi^B = \frac{\sigma}{n-1} \).

We also compute the consumer surplus (denoted by \( s \)) as a basis for comparison with other scenarios. In this case,

\[
s^B = E[U] = v - p^B = v - \left( \frac{n}{n-1} \sigma + c \right).
\]

It follows from the proposition that as the number of firms increases, the equilibrium price, quantity and profit for each firm decreases. On the other hand, consumer surplus increases.

### 3.2. Scenario II: C vs. B

In Scenario II, we assume that \( k \) (1 \( \leq k \leq n \)) retailers (say, retailers 1,2,…,\( k \)) adopt a C business model. The remaining \( n - k \) retailers (retailers \( k + 1, k + 2, \ldots, n \)) maintain the B business model. The set of alternatives for consumers is \( \mathcal{A}^{CB}(k) = \mathcal{A}^{B} \cup \{e_1, e_2, \ldots, e_k\} \). The choice probabilities are as in (1), with \( v_i = \beta v \) and \( v_{ei} = \beta 0 v \) for \( i = 1,2,\ldots,k \), and \( v_j = v \) for \( j = k + 1, k + 2, \ldots, n \). For a C retailer, since \( \partial q_{ij} / \partial p_i < 0 \) and \( \partial q_{ei} / \partial p_i > 0 \), a decrease in \( p_i \) increases \( q_i \) but decreases \( q_{ei} \). Hence, in a sense, retailer \( i \)'s two channels are also "competing," but the retailer has control over the two prices.

For each C retailer, the problem is to choose \( p_i \) and \( p_{ei} \) to maximize its profit \( \Pi_i = (p_i - c)q_i + (p_{ei} - c_e)q_{ei} \). For each B retailer, the objective is to choose \( p_j \) to maximize \( \Pi_j = (p_j - c)q_j \), \( j = k + 1, k + 2, \ldots, n \). It will be helpful to define the following quantity

\[
\alpha = \exp\left(\frac{c - c_e - \beta(1-\theta)v}{\sigma}\right).
\]

For a C retailer,

\[
q_{ei} = \alpha \exp\left(\frac{(p_i - c) - (p_{ei} - c_e)}{\sigma}\right) q_i.
\]

The equilibrium is then characterized in the result below.

**Proposition 2.** Without an outside option, under Scenario II, the pricing game has a unique Nash equilibrium. Moreover, at equilibrium, any C-retailer’s profit margins in both channels are the same, i.e., \( p_i^{CB} - c = p_{ei}^{CB} - c_e \). In addition, each C-retailer chooses the same price \( p_i^{CB} \) (and \( p_{ei}^{CB} \)) and each B-retailer chooses the same price \( p_j^{CB} \).

Next, we compare the equilibrium solutions in Scenarios I and II. From (1), the definition of \( \alpha \) in (2), and Proposition 2, we have that

\[
q_i^{CB} = \frac{1}{k(1 + \alpha) + (n-k)} \exp\left(\frac{\beta^{B} - \beta^{B}}{\sigma}\right) \exp\left(\frac{(\beta - 1)\epsilon}{\sigma}\right), \quad i = 1,2,\ldots,k,
\]

\[
q_j^{CB} = \frac{1}{k(1 + \alpha) \exp\left(\frac{(\beta - 1)\epsilon}{\sigma}\right) + (n-k)} \exp\left(\frac{\beta^{B} - \beta^{B}}{\sigma}\right) + (n-k), \quad j = k + 1, k + 2, \ldots, n,
\]

since \( p_1^{CB} = \cdots = p_k^{CB} \) and \( p_{k+1}^{CB} = \cdots = p_n^{CB} \). This leads to the following result.

**Proposition 3.** Comparing the equilibria in Scenarios I and II, we have:

(i) \( p_j^{CB} < p^B < p_i^{CB} \),

(ii) \( q_i^{CB} < \frac{1}{(1+\alpha^2)} \) and \( q_j^{CB} < \frac{1}{\sigma} \),

(iii) \( \Pi_j^{CB} < \Pi^B = \frac{\sigma}{n-1} < \Pi_i^{CB} \)

for \( i = 1,2,\ldots,k \) and \( j = k + 1, k + 2, \ldots, n \).
The addition of an Internet channel allows a retailer to charge a higher store price and obtain a larger total market share, leading to a higher profit. Therefore, the \( k \) C-retailers benefit by moving from B to C, i.e., by establishing online sales. On the other hand, each remaining B retailer observes a decrease in market share and profit when some of its competitors adopt the C business model.

We now investigate how the equilibrium profits of all firms change as one additional retailer moves from a B structure to a C structure, leading to a market with \( \{1, \ldots , k, k + 1\} \) C-retailers and \( \{k + 2, \ldots , n\} \) B-retailers. (Note that this analysis makes sense when \( n \geq 3 \).) To facilitate the comparisons, we denote a C-retailer’s profit by \( \Pi_i^{CB}(l) \) and a B-retailer’s profit by \( \Pi_i^{CB}(l) \), when there are \( l \) C-retailers in the market.

First, we show how retailer \( k + 1 \)'s profit changes when it shifts its strategy from bricks-and-mortar to clicks-and-mortar. From Proposition 3, note that for any \( l \),

\[
\Pi_i^{CB}(l) > \frac{\sigma}{n-1}, \quad i = 1, \ldots , l, \quad \text{and} \quad \Pi_j^{CB}(l) < \frac{\sigma}{n-1}, \quad j = l + 1, \ldots , n.
\]

This implies that \( \Pi_i^{CB}(k + 1) > \Pi_i^{CB}(k) \). Note that the left-hand side of the inequality is retailer \( k + 1 \)'s profit when there are \( k + 1 \) C-retailers (including retailer \( k + 1 \)), while the right-hand side is retailer \( k + 1 \)'s profit when there are only \( k \) C-retailers (and retailer \( k + 1 \) operates under a B structure). Hence, the \( (k + 1) \)st retailer benefits by adopting a clicks-and-mortar structure. We next explore how \( \Pi_i^{CB}(l) \) and \( \Pi_j^{CB}(l) \) change when \( l \) increases from \( k \) to \( k + 1 \), for \( i < k + 1 \) and \( j > k + 1 \).

**Proposition 4.** For C-retailers \( i = 1, \ldots , k \), \( \Pi_i^{CB}(k + 1) < \Pi_i^{CB}(k) \). For B-retailers \( j = k + 2, \ldots , n \), \( \Pi_j^{CB}(k + 1) < \Pi_j^{CB}(k) \).

Proposition 4 shows that when the number of C-retailers increases from \( k \) to \( k + 1 \), the ‘old’ set of C- and B-retailers suffer, since they all experience intensified competition. Only retailer \( k + 1 \) benefits from the move.

### 3.3. Scenario III: C vs. C

In Scenario III, all retailers adopt a C business model. The set of alternatives is then \( \mathcal{A}^C = \{1, e_1, 2, e_2, \ldots , n, e_n\} \). Consumers’ valuation for a product obtained from a retailer’s physical store is \( \beta v \), and their valuation for a product obtained from a retailer’s website is \( \beta v \cdot \frac{\sigma}{n-1} \). The choice probabilities \( q_i \) are given by (1) with \( \mathcal{A} = \mathcal{A}^C \). The retailers simultaneously set their prices. In particular, each retailer \( i \) chooses \( p_i \) and \( p_{ei} \) to maximize its profit, \( \Pi_i = (p_i - c)q_i + (p_{ei} - c_e)q_{ei} \).

**Proposition 5.** Without an outside option, under Scenario III, the pricing game has a unique Nash equilibrium with \( p_i^C - c = p_{ei}^C - c_e \) for all \( i \). In particular, \( p_i^C = \frac{n}{n-1}\sigma + c \), \( p_{ei}^C = \frac{n}{n-1}\sigma + c_e \), and \( \Pi_i^C = \frac{n}{n-1}\sigma + c \) for \( i = 1, \ldots , n \).

Interestingly, in Scenario III, all retailers set store prices as in Scenario I (in which all of them sell only through physical retail stores) and make exactly the same profits. This occurs because \( p_i - c = p_{ei} - c_e \) implies that each retailer’s online market share and profit is proportional to its retail store’s market share and profit, respectively, with the proportionality constant being \( z \). When only a subset of the retailers adopt the C business model, a portion of the consumers that previously purchased at a retail store now buy online, so the remaining B retailers lose market share to the C retailers. When all firms adopt a dual-channel strategy, in equilibrium, they keep the same retail price and select a lower online price so that their aggregate (retail and online) market share is the same as their market share would be as pure traditional retailers. That is, because all consumers make a purchase, firms set their equilibrium prices to maintain the same aggregate market share as under the B vs. B scenario.

Given the equilibrium prices in Proposition 5, consumer surplus under the C vs. C scenario is

\[
s^C = E[U] = nq_i^C(\beta v - p_i^C) + nq_i^C\left(\beta v - p_i^C + c - c_e\right) = \frac{\beta v (1 + z\theta) + z(c - c_e)}{1 + z} - \left(\frac{n}{n-1}\sigma + c\right).
\]

Comparing the consumer surplus under Scenarios I and III, we have that

\[
s^C - s^B = \frac{(\beta(1 + z\theta) - (1 + z))v - z(c_e - c)}{1 + z}.
\]
Then, consumer surplus is higher in Scenario III when
\[ \beta > \frac{1 + \alpha}{1 + \alpha \theta} - \frac{\alpha (c - c_e)}{(1 + \alpha \theta)v}. \]

That is, if the increased valuation for purchases at a clicks-and-mortar retailer is sufficiently high (i.e., \( \beta \) sufficiently high), consumers are, on average, better off in a market with click-and-mortar retailers. Note that when \( \theta = 1 \), i.e., when valuation for purchases at the traditional retail store and the website are the same, consumer surplus in Scenario III is higher than in Scenario I for all \( \beta \geq 1 \). This is due to the lower cost to serve the Internet consumers, which is passed on to all consumers in the form of lower prices.

3.4. Equilibrium structure

Comparing the outcomes of Scenarios I, II, and III, we derive the following result.

**Theorem 1.** In an oligopoly setting, C vs. C is the equilibrium structure. In fact, adopting the C business model is a dominant strategy for any retailer.

We conclude that the resulting equilibrium consists of all clicks-and-mortar retailers. Moreover, if all customers purchase from one of the available channel alternatives, establishing an Internet channel becomes a strategic necessity rather than an additional source of revenue. That is, in equilibrium every firm launches an Internet channel, but none of them is better off. However, the value created by the Internet generally benefits consumers.

4. Choice of channel structure: With an outside option

Suppose now that customers have an outside option (labeled 0), which represents the default choice of not purchasing the product from any of the firms. The consumption value for this outside alternative is given by \( v_0 \). The set of possible alternatives is now \( \mathcal{A}^0 = \mathcal{A}^0 \cup \{0\} \), \( O = B, CB, C \). The choice probabilities are

\[ q_i^O = \frac{\exp \left( \frac{v - p_i}{\sigma} \right)}{\sum_{j \in \mathcal{A}^0} \exp \left( \frac{v - p_j}{\sigma} \right) + \exp \left( \frac{v_0}{\sigma} \right)}, \tag{3} \]

We next again analyze three scenarios corresponding to three different market compositions.

4.1. The equilibrium structure

4.1.1. Scenario I: B vs. B

In Scenario I, the set of alternatives is \( \mathcal{A}^B = \{0, 1, 2, \ldots, n\} \). Each retailer chooses \( p_i \) to maximize its profit, \( \Pi_i = (p_i - c) q_i \), where \( q_i \) is given by (3) with \( \mathcal{A}^O = \mathcal{A}^B \). The outcome of this game is characterized below (the proof follows from Anderson et al., 1992).

**Proposition 6.** With an outside option, under Scenario I, the pricing game has a unique Nash equilibrium. At equilibrium, \( p_1^B = \cdots = p_n^B = \bar{p}^B \), and

\[ \frac{\bar{p}^B - c}{\sigma} = \frac{1}{1 - \bar{q}^B}, \]

where \( \bar{q}^B_B \) is a retailer’s market share, given by \( \bar{q}^B = \frac{\exp \left( \frac{v - p^B}{\sigma} \right)}{n \exp \left( \frac{v - p^B}{\sigma} \right) + \exp \left( \frac{v}{\sigma} \right)} \). The equilibrium profit for each retailer is

\[ \Pi^B = \frac{\exp \left( \frac{v - p^B}{\sigma} \right)}{n \exp \left( \frac{v - p^B}{\sigma} \right) + \exp \left( \frac{v}{\sigma} \right)}. \]

In the setting with an outside option, it is not possible to derive closed-form expression for the equilibrium prices, quantities and profits. Nevertheless, the expressions obtained in **Proposition 6** allow us to conclude that \( \bar{q}^B < q^B, \bar{p}^B < p^B \) and \( \Pi^B < \Pi^B \). In other words, the existence of an outside option drives down the firms’ prices, market shares, and profits.
4.1.2. Scenario II: C vs. B

Suppose now that \( k (1 \leq k \leq n - 1) \) retailers (say, retailers 1, 2, \ldots, \( k \)) establish an Internet channel beside their traditional retail channels. The other \( n - k \) retailers (retailers \( k + 1, k + 2, \ldots, n \)) maintain the B business model. The set of alternatives for consumers is \( \mathcal{A}^{CB} = \{0, 1, e_1, 2, \ldots, e_k, k + 1, k + 2, \ldots, n\} \), and the choice probabilities are as in (3) with \( \mathcal{A}^{O} = \mathcal{A}^{CB} \), \( v_j = \beta v \) and \( v_{ei} = \beta \theta v \), for \( i = 1, 2, \ldots, k \), and \( v_i = v \), for \( i = k + 1, k + 2, \ldots, n \).

Following similar arguments as for the case of no outside alternative, we conclude the following:

**Proposition 7.** With an outside option, under Scenario II, the pricing game has a unique Nash equilibrium, \( \tilde{p}^{CB} \).

Moreover, at equilibrium, a C-retailer’s profit margins in both channels are the same, i.e., \( \tilde{p}^{CB}_i - c = \tilde{p}^{CB}_{ei} - c_e \). In addition, each C-retailer chooses the same retail price \( \tilde{p}^{CB}_i \) and the same Internet price \( \tilde{p}^{CB}_{ei} \), and each B-retailer chooses the same price \( \tilde{p}^{CB}_j \).

Recall the definition of \( a \) in (2). A C-retailer’s profit again reduces to \( \Pi_i = (p_i - c)(1 + x)q_i \), \( i = 1, 2, \ldots, k \).

Moreover, the equilibrium prices and quantities again satisfy Eqs. (6) and (7).

We now compare the equilibrium prices, quantities and profits under C vs. B, with and without an outside option. Because the pairs \( (\tilde{p}^{CB}_i, q^{CB}_i) \) and \( (\tilde{p}^{CB}_j, q^{CB}_j) \) satisfy (6) for \( 1 \leq l \leq k \) and (7) for \( k + 1 \leq l \leq n \), it follows that \( \tilde{p}^{CB}_i = \tilde{p}^{CB}_j \) and only if \( \tilde{q}^{CB}_i = \tilde{q}^{CB}_j \) for any \( i = 1, \ldots, n \). Thus, \( \tilde{p}^{CB}_i = \tilde{p}^{CB}_j \) for all \( i = 1, \ldots, n \) imply that \( \sum_{i=1}^{k} \tilde{q}^{CB}_i + \sum_{j=k+1}^{n} \tilde{q}^{CB}_j \geq 1 \), which cannot happen in the presence of an outside option. Suppose now that \( \tilde{p}^{CB}_i = \tilde{p}^{CB}_j \) for \( 1 \leq i \leq k \) (recall that all C retailers choose the same retail price, both with and without an outside option). This again implies that \( \tilde{q}^{CB}_i = \tilde{q}^{CB}_j \) for any \( 1 \leq i \leq k \). Comparing the expressions for \( \tilde{q}^{CB}_i \) and \( q^{CB}_i \), it follows that \( \tilde{p}^{CB}_i \geq \tilde{p}^{CB}_j \) for \( k + 1 \leq j \leq n \) needs to hold as well, again leading to a contradiction. Therefore, we conclude that \( \tilde{q}^{CB}_i > q^{CB}_i \), \( \tilde{p}^{CB}_i < \tilde{p}^{CB}_j \) for all \( i = 1, \ldots, n \), which in turn implies that \( \Pi_i^{CB} > \Pi_j^{CB} \) for all \( i = 1, \ldots, n \). Thus, the “competition” created by an outside option reduces the equilibrium prices, market shares, and profits, for all B and C retailers.

The following result establishes a comparison between Scenarios I and II.

**Proposition 8.** Comparing the equilibria in Scenarios I and II, we have:

(i) \( \tilde{p}^{CB}_i < \tilde{p}^{B}_i < \tilde{p}^{CB}_i \).

(ii) \( \Pi_i^{CB}(k + 1) > \Pi_i^{B}(k + 1) \), \( i = 1, 2, \ldots, k + 1 \),

\[ \Pi_j^{CB}(k + 1) < \Pi_j^{B}(k + 1) \], \( j = k + 2, k + 3, \ldots, n \),

which implies that \( \Pi_i^{CB}(k + 1) > \Pi_j^{CB}(k) \). In addition, following similar arguments as in Proposition 4, we have that for each C-retailer \( i \), \( \Pi_i^{CB}(k + 1) < \Pi_i^{CB}(k) \), and for each B-retailer \( j \), \( \Pi_j^{CB}(k + 1) < \Pi_j^{CB}(k) \).

4.1.3. Scenario III: C vs. C

In Scenario III, all retailers sell both through a traditional store and an Internet website. The set of alternatives is given by \( \mathcal{A}^C = \{0, 1, e_1, \ldots, e_n, en\} \). Consumers’ valuations are \( v_i = \beta v \) and \( v_{ei} = \beta \theta v \) for the traditional and Internet stores, respectively. All retailers simultaneously choose \( p_i \) and \( p_{ei} \) to maximize their profits \( \Pi_i = (p_i - c)q_i + (p_{ei} - c_e)q_{ei} \).

**Proposition 9.** With an outside option, under Scenario III, the pricing game has a unique Nash equilibrium. In particular, \( \tilde{p}^{C}_i - c = \tilde{p}^{C}_{ei} - c_e \) and \( \tilde{p}^{C}_i = \cdots = \tilde{p}^{C}_n = \tilde{p}^{C} \), which is characterized by

\[
\frac{\tilde{p}^{C} - c}{\sigma} = \frac{1}{1 - \tilde{q}^{C}},
\]
where \( q^C = \frac{(1+\alpha) \exp \left( \frac{b-v}{c_e} \right)}{n(1+\alpha) \exp \left( \frac{b-v}{c_e} \right) + \exp \left( \frac{b}{\theta} \right)} \) is a retailer’s market share. Each retailer’s profit is

\[
\Pi^C = \frac{\sigma q^C}{1-q^C}.
\]

Following similar arguments as for Scenario II, we again conclude that, when all firms adopt a C business model, the existence of an outside option reduces all retailers’ equilibrium prices, market shares, and profits.

Next, we compare the equilibrium prices and profits between Scenarios I and III.

**Proposition 10**

(i) \( p^C > p^B \),

(ii) \( q^C > q^B \), and

(iii) \( \Pi^C > \Pi^B \).

In contrast to the setting without an outside option, Proposition 10 suggests that in the presence of an outside alternative, the equilibrium profit achieved by any firm under C vs. C is strictly larger than that under B vs. B. Indeed, the addition of the online channels creates competition to the existing traditional channels and to the outside alternative. More precisely, the online channels steal some market share away from the traditional channels and from the outside alternative. In equilibrium, firms set prices in a way that the migration of customers from traditional stores to online sites does not reduce each firm’s aggregate customer base. However, in this setting, the newly introduced Internet channels offer prices that appeal to a portion of the customers that previously opted for the outside alternative. As a result, all firms’ market shares and profits increase. Indeed, note that part (ii) of Proposition 10 implies that the fraction of customers that select the outside alternative is larger under B vs. B than under C vs. C. Interestingly, it is easy to verify that \( \Pi^C > \Pi^B \) still holds when \( c_e = c \) and/or \( \beta = 1 \). In other words, even if the addition of an Internet channel does not lower the average production cost of a firm, and even if it does not increase consumers’ valuation, all firms are, at equilibrium, strictly better off by operating both in-store and online sales.

### 4.2. Equilibrium structure

Given the equilibrium outcomes under Scenarios I, II and III, we can establish the following result.

**Theorem 2.** The equilibrium supply chain structure consists of all firms choosing a clicks-and-mortar business model. In addition, the C business model is a dominant strategy for any retailer.

An important distinction between the results in this section and the previous one relates to the equilibrium profits of all firms under the equilibrium supply chain structure C vs. C. Without an outside alternative, establishing Internet sales becomes a strategic necessity. In settings with an outside alternative, however, the profit of each firm under the equilibrium clicks-and-mortar outcome is strictly higher than its profit under the B vs. B setting. In contrast to the setting studied in Section 3, the existence of a fraction of customers that opt for the outside alternative allows all firms to capture a portion of this market by establishing Internet sales.

We conclude this section by exploring the extent to which business profitability increases in the C vs. C equilibrium scenario. To that end, we define the difference between the profit under C vs. C and under B vs. B by \( \Delta \Pi = \Pi^C - \Pi^B \), and derive the following comparative statics.

**Proposition 11.** The difference in business profitability \( \Delta \Pi \) increases as (i) \( \beta \) increases, (ii) \( \theta \) increases, or (iii) \( c_e \) decreases.

The above result shows that the increase in business profitability at the equilibrium C vs. C structure is positively affected by increases in \( \beta \) and \( \theta \), and is negatively affected by increases in \( c_e \). An increase in \( \beta \) implies that the addition of Internet operations increases consumers’ valuation by a larger magnitude. In particular, more of the potential customers who would have chosen to forgo a purchase under the B vs. B scenario will actually make a purchase in a market composed of clicks-and-mortar retailers. A higher value of \( \theta \) means that the
difference in valuation between making a purchase at a store and online decreases, again increasing the fraction of customers that are willing to buy the product from one of the available channels. Finally, lower Internet operational costs translate into lower market prices which again induce more customers to buy the product.

5. Extensions

5.1. Other parameter ranges

We begin with a brief discussion on the impact of the assumption that $\beta \geq 1$ on the results in Sections 3 and 4. This assumption allows us to derive the inequalities shown in Propositions 2, 4 and 3, which imply that C vs. C is the resulting industry equilibrium structure. Closer inspection of the proofs of these results reveals that they continue to apply as long as the following condition holds:

$$z \equiv (1 + x) \exp \left( \frac{(\beta - 1)c}{\sigma} \right) > 1,$$

where $x$ is given by (2) (note that $\beta \geq 1$ implies $z > 1$). This condition holds if $\beta$ and/or $\theta$ are not too small, and/or if $c_e$ is sufficiently lower than $c$. All of these imply a dual-channel structure that operates relatively effectively and efficiently. Following the proofs of Propositions 2, 4 and 3, it can be verified that if $z < 1$, then all inequalities in these results are reversed, implying that B vs. B is the industry equilibrium structure. That is, $z$ can be thought of as a measure of the efficiency of a dual-channel structure. For example, a low value of $\beta$, representing a substantially decreased consumer valuation in the face of a dual structure (perhaps driven by the confusion created by uncertainty about shipping or return policies inherent to clicks-and-mortar retailers), would lead to a B vs. B equilibrium. Finally, if the parameters lead to $z = 1$, then profits are the same under any structure (B vs. B, C vs. B, or C vs. C) and multiple (identical) equilibria exist.

In what follows, we explore additional variations of the original model.

5.2. Alliance to sell online

In previous sections, we studied the choice of channel structure assuming that all firms had the capability to establish and operate their own Internet sales channels. However, not all have the ability to successfully implement e-commerce operations. For example, Toys-R-Us launched its Internet channel Toysrus.com in 1998 and soon found it “a corporate and public relations headache [...] things fell apart just as quickly as they came together” (Eisner et al., 2003). Due to its initial failure, Toys-R-Us formed an alliance with Amazon.com in 2000: “This deal solved the problem that many brick-and-mortar companies were having at the time. Toys-R-Us [...] had not figured out how to go from receiving an online order to getting products to the doorstep [...] the deal shows that Amazon no longer believed it could single-handedly be a global online shopping center. [...]” (Eisner et al., 2003). In this section, we explore the choice of channel structure in settings where some or all of the firms need to constitute an alliance with an existing e-tailer to establish an online presence. In such context, we again investigate the industry equilibrium structure.

We model an alliance following the Toys-R-Us/Amazon example. In that case, Amazon receives from Toys-R-Us regular fixed cash payments (which we do not incorporate, as they do not affect the price decisions) and a percentage payment for toys sold. We denote by $\xi$ that percentage, with $0 < \xi < 1$. In addition, we assume that the retailer makes the price decision for its store while the Internet firm decides on the online price. Although in reality the retailer may influence the price of its goods on the Internet, this assumption represents a reasonable approximation of how such alliances usually operate. Indeed, in the case of Toys-R-Us/Amazon, a consumer affairs website reports on how prices for toys may differ at the stores and on the web, and how the stores do not have control on prices charged over the Internet. For example, based on a customer’s experience at Toys-R-Us, “[a store manager explained that] Toys-R-Us on the web and the store were
two different entities” and continues saying that “a customer service manager [at the store] explained […] that they do not price match internet sales”. In addition, not all Toys-R-Us stores charge the same price. For example, stores in Manhattan charge from $2 to $10 more per item.\(^3\) However, Amazon charges a single price regardless of where the toy is shipped to (within the US).

It is worth noting that Zhang and Zhang (2003) model strategic alliances between firms in a similar way. As they describe, and consistent with our model, a strategic alliance differs from a merger in that the alliance partners remain separate business entities and retain their decision-making autonomy. In their locally autonomous setting, each firm in an alliance owns a share of the stock of its partner and thus makes its price/quantity decisions to maximize its own profit, plus a fraction of its partner’s profit (this is referred to as an equity alliance). The results and discussion in this section also apply in settings where the traditional retailer and the Internet firm form an equity alliance.

We investigate the equilibrium industry structure in the context of a duopoly. In this setting, a traditional retailer \(i\) in an alliance with an Internet retailer, selects a price \(p_i\) to maximize its profit

\[
\Pi_i = (p_i - c)q_i + \xi(p_e - c_e)q_e, 
\]

while its partner selects the online price \(p_e\) to maximize

\[
\Pi_e = (1 - \xi)(p_e - c_e)q_e. 
\]

Consumer valuation for the single/dual-channel and for store/online purchases preserve the same format as in previous sections. In other words, in a bricks-and-mortar/Internet alliance (which we denote by BI), consumers’ valuation for a product obtained from the B retailer is \(\beta v\) and for a product purchased online is \(\beta \theta v\). For simplicity, we assume that there is no outside option.

### 5.2.1. Scenario I: B vs. B

This setting is identical to Scenario I in Section 3.1 with \(n = 2\).

### 5.2.2. Scenario II: BI vs. B

In this setting, only one retailer, say retailer 1, establishes an alliance with an Internet firm. The three players’ objectives are to maximize the following profits:

\[
\Pi_1 = (p_1 - c)q_1 + \xi_1(p_e - c_e)q_e, \quad \Pi_1 = (1 - \xi_1)(p_e - c_e)q_e, \quad \Pi_2 = (p_2 - c)q_2, 
\]

with respect to \(p_1, p_e,\) and \(p_2\), respectively, where \(q_1\) is given by (1) with \(v_1 = \beta \nu, v_2 = \beta \theta v\) and \(v_2 = v\). Evaluating the second derivatives of these profit functions at any root of their first derivatives, leads to \(-q_i/\sigma, -(1 - \xi)q_e/\sigma\) and \(-q_e/\sigma\), respectively. The profit functions \(\Pi_i\) are therefore strictly quasi-concave, \(i = 1, 2\) (see Anderson et al., 1992). We conclude that there is a Nash equilibrium in the pricing game.

### 5.2.3. Scenario III: BI vs. BI

When both B retailers form an alliance with an Internet partner, the four firms select prices to maximize the following profit functions:

\[
\Pi_{11} = (p_1 - c)q_1 + \xi_1(p_e - c_e)q_e, \quad \Pi_{11} = (1 - \xi_1)(p_e - c_e)q_e, 
\]

\[
\Pi_{21} = (p_2 - c)q_2 + \xi_2(p_e - c_e)q_e, \quad \Pi_{21} = (1 - \xi_2)(p_e - c_e)q_e. 
\]

A similar argument as in Scenario II establishes the existence of a Nash equilibrium in the pricing game.

We conducted a set of numerical experiments to explore the outcome of the industry structure game in this setting. As discussed in Section 2, we assume that the firms are symmetric and \(\xi_1 = \xi_2\). Fig. 2 shows two possible industry structure equilibria. In this example, \(v = 20, \beta = 1.1, 0 = 0.9, \sigma = 2, c = 1, c_e = 0.7,\) and \(\xi = 0.3\) in Case 1 and \(\xi = 0.6\) in Case 2.

When \(\xi\) is small, the benefits from selling online do not offset the losses resulting from the competition created by the Internet channel. In this case, each firm’s incentive to form an alliance is weaker and, in equilibrium, both firms maintain the B model. For \(\xi\) sufficiently large, both retailers individually have an incentive to

\(^3\)http://www.consumeraffairs.com/toys/toys_r_us.htm.
sell online, resulting in an industry equilibrium where both firms form an alliance with an Internet partner. However, at the equilibrium, both retailers are worse off compared with a B vs. B setting. That is, a prisoner’s dilemma-type outcome arises.

Similar observations can be made in the presence of an outside option. For example, consider the following parameters: \( v = 20, v_0 = 10, \beta = 1.1, \theta = 0.9, \sigma = 2, c = 1, \) and \( c_e = 0.7. \) When \( \zeta = 0.2 \) profits are \((1.86, 1.86)\) for the scenario B vs. B, \((0.75, 1.81)\) for B vs. BI, and \((1.08, 1.08)\) for BI vs. BI. Therefore, B vs. B is the equilibrium. However, when \( \zeta = 0.6 \), profits are \((1.86, 1.86)\) for the scenario B vs. B, \((0.81, 2.18)\) for B vs. BI, and \((1.30, 1.30)\) for BI vs. BI. In this case, BI vs. BI is the equilibrium structure and profits for both firms are lower than under B vs. B.

A similar analysis follows in settings where one of the firms is capable of establishing and operating its own Internet outlet, while the other firm needs to form an alliance with an Internet partner in order to reach online customers. We next explore the possible equilibrium industry structure outcomes in this setting. We assume that retailer 1 can select between a pure bricks-and-mortar (B) structure and a clicks-and-mortar (C) structure, while retailer 2’s status quo is B and can only efficiently access the Internet market by aligning with a pure e-tailer (BI structure).

In this example, \( v = 20, \beta = 1.1, \theta = 0.9, \sigma = 2, c = 1, c_e = 0.7, \) and \( \zeta = 0.3 \) in case 1 and \( \zeta = 0.6 \) in case 2. In case 1, retailer 2’s incentive to form an alliance is weak – this firm only gets a 30% share of the profit, while it has to face competition from its own Internet partner. When that is the case, the resulting equilibrium structure is C vs. B. In case 2, when the gains from the alliance are higher for the retailer, both firms operate an online channel in equilibrium – retailer 1 by establishing its own Internet operations and retailer 2 by forming an alliance with an Internet partner. Again, in this case, a prisoner’s dilemma-type outcome may arise (see Fig. 3).

5.3. The internet reaches additional market

In contrast to physical retail stores, the Internet presents a diffused and ubiquitous network of points of access. Customers can shop anywhere and at any time. Establishing online sales allows retailers to reach a larger market than they would be able to by operating only physical stores. In this section, we briefly study the effects of the extended customer reach that Internet sales provide for a retailer. We incorporate this feature of the Internet by assuming, as before, that in a B vs. B setting total population is of size 1 and that any firm operating an Internet channel increases its total market size by \( \gamma \) (this parameter represents a population of customers that do not have access to or find it inconvenient to purchase from a physical store). In this setting, we allow for the existence of an outside alternative to purchases from any of the retailers, as in Section 4. To isolate the effect of an increased market reach, we focus on the case where \( \beta = \theta = 1. \)
Below, we introduce the relevant notation:

- $q_i$: proportion of consumers (out of total population of size 1) that purchase from retailer $i$’s physical store;
- $q_{ei}$: proportion of consumers (out of total population of size 1) that purchase from retailer $i$’s website;
- $q_{eiy}$: proportion of consumers (out of the extended market of size $\gamma$) that purchase from retailer $i$’s Internet channel.

Specifically, $q_i$ and $q_{ei}$ are given by (3) with $\mathcal{A}^B = \{1, 2\}$, $\mathcal{A}^{CB} = \{1, e1, 2\}$ (assuming retailer 1 is the C-retailer) and $\mathcal{A}^C = \{1, e1, 2, e2\}$. On the other hand, $q_{eiy}$ is given by (3) with $\mathcal{A}^B = \emptyset$, $\mathcal{A}^{CB} = \{e1\}$ and $\mathcal{A}^C = \{e1, e2\}$.

A bricks-and-mortar retailer’s profit is as in Section 4, while a C retailer’s profit function is

$$\Pi_i = (p_i - c) q_i + (p_{ei} - c_e) q_{ei} + \gamma (p_{ei} - c_e) q_{eiy}.$$ 

Existence of a Nash equilibrium follows from similar arguments as those in Section 5.2. We conducted an extensive numerical study based on a setting with two firms. Specifically, we consider the following parameters: $v = 15$, $v_0 = 10$, $c = 1$, $c_e = 0.7$, $\sigma = 2$, and $\beta = 1$. In one set of experiments, we varied $c_e$ from 0.5 to 1. In a second set, we varied $v_0$ from 5 to 15. In a final set, we varied $\sigma$ from 2 to 10. In all cases, $\gamma$ ranged from 0 to 1. To illustrate the findings, the following table exhibits the results for $v = 15$, $v_0 = 10$, $c = c_e = 1$, $\sigma = 2$, and some values of $\gamma$.

<table>
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<th>0</th>
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<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B vs. B</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>C vs. B</td>
<td>(1.7, 1.0)</td>
<td>(2.1, 1.0)</td>
<td>(2.5, 1.0)</td>
<td>(2.9, 1.0)</td>
<td>(3.3, 1.0)</td>
<td>(3.7, 1.0)</td>
</tr>
<tr>
<td>C vs. C</td>
<td>1.4</td>
<td>1.7</td>
<td>1.9</td>
<td>2.2</td>
<td>2.4</td>
<td>2.6</td>
</tr>
<tr>
<td>(P^C) original market</td>
<td>1.450</td>
<td>1.438</td>
<td>1.431</td>
<td>1.426</td>
<td>1.423</td>
<td>1.420</td>
</tr>
<tr>
<td>(p^C)</td>
<td>4.45</td>
<td>4.44</td>
<td>4.43</td>
<td>4.43</td>
<td>4.42</td>
<td>4.42</td>
</tr>
<tr>
<td>(p^e)</td>
<td>4.45</td>
<td>4.39</td>
<td>4.35</td>
<td>4.32</td>
<td>4.31</td>
<td>4.29</td>
</tr>
</tbody>
</table>

Based on the numerical study, we make the following observations:

1. C vs. C is the industry equilibrium.
2. Both online and store prices decrease with $\gamma$. Online prices decrease due to the increased total potential market, and store prices follow this decrease, albeit at a lower rate, since the two markets (traditional and online) compete for consumer demand.
3. As the additional Internet market reach increases, both firms’ profits increase at the C vs. C equilibrium structure. However, each firm’s profit from the original market (of size 1) decreases. That is, as the “new” market becomes larger, firms are willing to lose some profit in the “old” market to increase their total profit.

6. Concluding remarks

In the pre-Internet era, single channel was all that companies needed to deliver products or services to their customers. Today, responding to changes in the marketplace, companies have incorporated the Internet as a means to attract customers that find it convenient to shop from their homes or other places. “In five years’ time,” says Intel’s chairman Andy Grove, “all companies will be Internet companies, or they won’t be companies at all.”\footnote{Economist Survey: Business and the Internet, “the Net Imperative,” The Economist, June 26, 1999.} Despite this optimistic projection for e-commerce’s future, it is hard to predict its impact on the resulting industry market structure. This paper builds a model to explore this issue.

We consider two separate demand models, one in which consumers’ valuation for the product is high enough (or their price sensitivity for the product is low enough) so that all make a purchase, and another
one in which some customers may choose an outside alternative. For both demand models, we derive equilibrium prices, quantities and profits when each firm has a fixed channel structure (B or C). The two models lead to the same industry equilibrium outcome, but with one important distinction: in a setting without an outside option, the Internet does not bring additional value to firms. We suggest that, in such settings, the adoption of e-commerce operations by traditional retailers arises from strategic necessity. Even if firms do not benefit from the adoption of online sales, consumers do. The findings in this setting are similar to those observed in the banking industry with the introduction of ATMs (see Humphrey, 1994). Indeed, ATMs have helped reduce transaction costs by nearly 15% over the past years. At the same time, transaction volumes more than doubled and the benefits all went to consumers. Although ATMs do not add significant value to banks, they have become a strategic necessity in the banking sector. In contrast, in settings with an outside option, we show that all firms strictly benefit, in equilibrium, by establishing online operations.

We finally explore instances where some (or all) firms cannot efficiently operate their own Internet channel, but instead may align with pure e-tailers to reach the online market. In such settings, we show that a prisoner dilemma-type equilibrium may arise.7

The Internet is still in its early stages of development and the experiences companies have had thus far with it are limited. Therefore, in addition to many of the lessons learned from practice, the insights provided by our theoretical model contribute to the attempt to gain a clearer picture of this evolution.

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Appendix. Proofs

Proof of Proposition 2. Differentiating $\Pi_i$ with respect to $p_i$ and $p_{ei}$ respectively, we obtain

$$
\frac{\partial \Pi_i}{\partial p_i} = q_i - (p_i - c) \frac{1}{\sigma} q_i (1 - q_i) + (p_{ei} - c_e) \frac{1}{\sigma} q_i q_{ei},
$$

$$
\frac{\partial \Pi_i}{\partial p_{ei}} = q_{ei} - (p_{ei} - c_e) \frac{1}{\sigma} q_{ei} (1 - q_{ei}) + (p_i - c) \frac{1}{\sigma} q_i q_{ei}.
$$

Setting the above derivatives equal to zero and solving the resulting equations simultaneously, we verify that $p_i - c = p_{ei} - c_e$, reducing retailer $i$'s strategic variables to $p_i$, $i = 1, 2, \ldots, k$.

Since $p_i - c = p_{ei} - c_e$, we have that $q_{ei} = x q_i$ and $\frac{\partial \Pi_i}{\partial p_i} = -\frac{1}{\sigma} q_i (1 - (1 + x) q_i)$. Rewrite retailer $i$'s profit function as $\Pi_i = (p_i - c)(1 + x) q_i$. Then, the proof of existence and uniqueness of the Nash equilibrium is similar as in Proposition 1. Therefore, the equilibrium prices can be derived by solving

7 We have also investigated settings where, in addition to the existing traditional retailers, some pure Internet firms operate in the market. In these settings, we have been able to show that the clicks-and-mortar business structure continues to be the industry equilibrium, and that the traditional retailers strictly increase their market shares and profits by adding online operations. As in the setting with an outside option, the dual-channel firms absorb part of the consumers that previously purchased at the pure Internet firms. That is, pure Internet players may stand to lose as traditional bricks-and-mortar firms addition Internet sales to their retail operations. (Please contact the authors for details on the analysis.)
\[
\frac{p_i - c}{\sigma} = \frac{1}{1 - (1 + \alpha)q_i}, \quad i = 1, 2, \ldots, k, \tag{6}
\]
\[
p_{c} = p_i + c \alpha - c, \quad i = 1, 2, \ldots, k,
\]
\[
\frac{p_j - c}{\sigma} = \frac{1}{1 - q_j}, \quad j = k + 1, k + 2, \ldots, n. \tag{7}
\]

We verify that \(p^C_i\)'s are equal for \(i = 1, 2, \ldots, k\). For any two \(C\) retailers \(i\) and \(i'\), we have from (6)
\[
\frac{p_i - c}{p_{i'} - c} = \frac{1 - (1 + \alpha)q_i}{1 - (1 + \alpha)q_{i'}}. \tag{8}
\]

Suppose that \(p_i > p_{i'}\). Then, the left-hand side of (8) is greater than 1, while its right-hand side is less than 1 since \(q_i < q_{i'}\), a contradiction. Similarly, we can verify that \(p^C_j\)'s are equal for \(j = k + 1, k + 2, \ldots, n\). \(\square\)

**Proof of Proposition 3.** Because from Proposition 2, \(p^C_i = \ldots = p^C_k\) and \(p^C_{k+1} = \ldots = p^C_n\) and consequently \(q^C_i = \ldots = q^C_k\) and \(q^C_{k+1} = \ldots = q^C_n\), we show the results for \(i = 1\) and \(j = n\). Subtracting (7) from (6), we have
\[
\frac{p_1 - p_n}{\sigma} = \left[ 1 - \frac{1}{k + (n - k) \exp(\frac{(\alpha - \beta)\sigma}{\sigma}) / \exp((\alpha - \beta)\sigma)} \right]^{-1} - \left[ 1 - \frac{1}{k + (n - k) \exp(\frac{(\alpha - \beta)\sigma}{\sigma}) / \exp((\alpha - \beta)\sigma) + (n - k)} \right]^{-1}. \tag{9}
\]

Let \(x = \frac{\alpha - \beta}{\sigma}, z = (1 + \alpha) \exp(\frac{(\alpha - \beta)\sigma}{\sigma}) > 1, f_1(x) = x, \) and
\[
f_2(x, k) = \left[ 1 - \frac{1}{k + (n - k) \exp(x/z)} \right]^{-1} - \left[ 1 - \frac{1}{kz / \exp(x + (n - k))} \right]^{-1}.
\]

Note that since \(f_1(x)\) is an increasing function, \(f_2(x, k)\) is decreasing with \(f_2(0, k) > 0\) (since \(z > 1\)), and \(f_2(\ln z, k) = 0, f_1(x)\) and \(f_2(x, k)\) only cross once at a point \(0 < x^{CB} < \ln z\). Then \(\frac{\partial^{CB}}{\sigma} = x^{CB}\) and \(\frac{\partial^{CB}}{\sigma} > 1\), which implies that
\[
(1 + \alpha) \exp(\frac{(\alpha - \beta)\sigma}{\sigma}) / \exp\left(\frac{\partial^{CB}}{\sigma} \alpha\right) > 1. \tag{10}
\]

Thus,
\[
q^C_1 = \frac{1}{k + (n - k) \exp(\frac{\partial^{CB} - \partial^{B}}{\sigma}) / \exp\left(\frac{(\alpha - \beta)\sigma}{\sigma}\right)} > \frac{1}{1 + \alpha} \tag{11}
\]
\[
q^C_n = \frac{1}{k + (n - k) \exp\left(\frac{(\alpha - \beta)\sigma}{\sigma}\right) / \exp(\frac{\partial^{CB} - \partial^{B}}{\sigma}) + (n - k)} < \frac{1}{n}. \tag{12}
\]

Furthermore,
\[
P^C_1 = \frac{\sigma}{1 - (1 + \alpha)q^C_1} + c > \frac{n}{n - 1}\sigma + c,
\]
\[
P^C_n = \frac{\sigma}{1 - q^C_n} + c < \frac{n}{n - 1}\sigma + c,
\]
\[
\Pi^C_1 = (p^C_1 - c)(1 + \alpha)q^C_1 = \frac{\sigma}{1 - (1 + \alpha)q^C_1}(1 + \alpha)q^C_1 > \frac{\sigma}{n - 1},
\]
\[
\Pi^C_n = (p^C_n - c)q^C_n = \frac{\sigma}{1 - q^C_n}q^C_n < \frac{\sigma}{n - 1}. \tag{13} \]
Proof of Proposition 4. Consider the function \( f_3(x,k) \) defined in the proof of Proposition 3. It can be easily verified that \( f_3(x,k) > f_3(x,k+1) \) for all \( 0 \leq x < \ln z \) and \( f_3(\ln z,k) = f_2(\ln z,k+1) = 0 \). This implies that \( x^{CB}(k) > x^{CB}(k+1) \) (as in the proof of Proposition 3, \( x^{CB} \) is the unique root of \( f_2(x,l) - x \)).

Consider now \( q_j(x,l) = \frac{1}{l z/\exp(x^{CB}(1))} \), as in the proof of Proposition 3, for \( 0 < x < \ln z \) and \( j = k + 2, \ldots, n \). Then, it is easy to verify that \( q_j(x,l) > q_j(x,l) \). Since \( q_j(x,l) \) is increasing in \( x \), we then have that

\[
q_j^{CB}(k+1) = \frac{1}{(k+1)z/\exp(x^{CB}(k+1))} < \frac{1}{kz/\exp(x^{CB}(k))} = q_j^{CB}(k).
\]

Therefore, for \( j = k + 2, \ldots, n \),

\[
\Pi_i(k+1) = \frac{\sigma}{1 - q_j^{CB}(k+1)} q_j^{CB}(k+1) < \Pi_i(k).
\]

For \( i = 1, \ldots, k \), we have that

\[
f_2(x^{CB}(k),k) = \frac{1}{1 - (1 + x)q_i^{CB}(k)} - \frac{1}{1 - q_i^{CB}(k)} = x^{CB}(k)
\]

for any \( k + 2 \leq j \leq n \). Then,

\[
\frac{1}{1 - (1 + x)q_i^{CB}(k+1)} = x^{CB}(k+1) + \frac{1}{1 - q_i^{CB}(k+1)} < x^{CB}(k) + \frac{1}{1 - q_i^{CB}(k)} = \frac{1}{1 - (1 + x)q_i^{CB}(k)},
\]

which implies that

\[
\Pi_i(k+1) = \frac{\sigma}{1 - (1 + x)q_i^{CB}(k+1)} (1 + x)q_i^{CB}(k+1) < \Pi_i(k).
\]

Proof of Proposition 5. Fix any retailer \( i \). The proof for \( p_i^C - c = p_i^C - c_c \) is as in Proposition 2. Given the relationship between \( p_i \) and \( p_{ei} \), retailer \( i \)'s two decision variables reduce to one, namely \( p_i \). Substituting \( p_{ei} = p_i^C - c + c_c \), retailer \( i \)'s objective is to maximize \( \Pi_i = (p_i - c)(1 + \alpha)q_i \), where \( \alpha \) is defined in (2). Similar to the proof of Proposition 1, there is a unique Nash equilibrium given by \( p_i^C = \frac{\sigma}{n - 1} \sigma + c \) and \( p_{ei} = \frac{\alpha}{n - 1} \sigma + c_c \). In addition, \( \Pi_i^C = \frac{\sigma}{n - 1} \).}

Proof of Theorem 1. The structure with all B-retailers is not an equilibrium, since any one firm has an incentive to deviate to the C structure as \( \Pi_i^{CB}(1) > \frac{\sigma}{n - 1} = \Pi_i^B \). Similarly, in a market with \( k \) C-retailers and \( n - k \) B-retailers, any B-retailer has an incentive to deviate to the C structure. Finally, in a setting with only C-retailers, no retailer has incentive to deviate to the B structure.

Proof of Proposition 8. Rewriting the choice probabilities as

\[
q_i = \frac{\exp (\frac{p_i - p_{ei}}{\sigma})}{k(1 + x) \exp (\frac{p_i - p_{ei}}{\sigma}) + (n - k) \exp (\frac{p_{ei}}{\sigma}) + \exp (\frac{\alpha}{\sigma})},
\]

\[
q_j = \frac{\exp (\frac{p_j - p_{ej}}{\sigma})}{k(1 + x) \exp (\frac{p_j - p_{ej}}{\sigma}) + (n - k) \exp (\frac{p_{ej}}{\sigma}) + \exp (\frac{\alpha}{\sigma})},
\]

and subtracting (7) from (6), we can again verify (see the proof of Proposition 3) that

\[
0 < \frac{p_i^{CB} - p_j^{CB}}{\sigma} < \ln \left[ (1 + x) \exp \left( \frac{(\beta - 1)\sigma}{\sigma} \right) \right].
\]

Suppose that \( p_j^{CB} > p_{j}^{B} \). Then, from the second inequality in (11), we have that

\[
1 - q_j^{B} < \frac{1}{1 - q_j^{B}} = \frac{1}{1 - q_j^{B}} \left( 1 - \frac{1}{k(1 + x) \exp \left( \frac{(\beta - 1)\sigma}{\sigma} \right)} + \frac{1}{(n - k) + \exp (\frac{\alpha}{\sigma})} \right)^{-1}
\]

\[
> \frac{1}{1 - \frac{1}{n + \exp (\frac{\alpha}{\sigma})}} = \frac{1}{1 - q_j^{B}}
\]
leading to a contradiction, from (7) and (4). Therefore, \( \bar{p}^C < \bar{p}^B \) holds. Similarly, we can prove \( \bar{p}_i^C > \bar{p}_i^B \). Finally, it follows from (6) and (7) that \( \bar{p}^C > \bar{p}^B \) and \( \bar{p}_i^C > \bar{p}_i^B \). \( \square \)

**Proof of Proposition 10.** Define \( g_1(p) = \frac{p - c}{q} \).

\[
g_2(p) = \left( 1 - \frac{1}{n + \exp \left( \frac{\alpha \nu}{\sigma} \right) / (1 + x) \exp \left( \frac{\beta \nu}{\sigma} \right)} \right)^{-1} \quad \text{and} \quad g_3(p) = \left( 1 - \frac{1}{n + \exp \left( \frac{\alpha \nu}{\sigma} \right) / \exp \left( \frac{\beta \nu}{\sigma} \right)} \right)^{-1}.
\]

Note that \( \bar{p}^C \) is the unique intersection of \( g_1 \) and \( g_2 \), while \( \bar{p}^B \) is the unique intersection point of \( g_1 \) and \( g_3 \). It is easy to verify that \( g_2(p) > g_3(p) \) for all \( p \), implying that \( \bar{p}^C > \bar{p}^B \). Parts (ii) and (iii) then follow from (5). \( \square \)

**Proof of Proposition 11.** First note that

\[
A = \sigma \left( \frac{\hat{q}^C}{1 - \hat{q}^C} - \frac{\hat{q}^B}{1 - \hat{q}^B} \right) = \sigma \left( \frac{1}{1 - \hat{q}^C} - \frac{1}{1 - \hat{q}^B} \right) = \hat{p}^C - \hat{q}^B.
\]

We first examine how \( g_2(p) \) changes for a given \( p \) when \( \beta \) increases. Since

\[
\frac{\partial}{\partial \beta} \left[ (1 + x) \exp \left( \frac{\beta \nu}{\sigma} \right) \right] = \frac{\partial \nu}{\sigma} x \exp \left( \frac{\beta \nu}{\sigma} \right) + \frac{\nu}{\sigma} \exp \left( \frac{\beta \nu}{\sigma} \right) > 0,
\]

\( g_2(p) \) increases in \( \beta \). Therefore, when \( \beta \) increases, \( \Delta \Pi \) increases. Similarly, as \( \theta \) increases, \( \alpha \) (as defined by (2)) increases, and so does \( g_2(p) \). Therefore, \( \Delta \Pi \) also increases. Finally, as \( c_c \) decreases, \( \alpha \) increases and so does \( \Delta \Pi \). \( \square \)

**References**


