Effect of Partial Cross Ownership on Supply Chain Performance

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Abstract

Partial cross ownership (PCO) in a dyad supply chain refers to a situation where each firm holds a portion of its partner’s shares. We study this topic in push and pull supply chains, and prove that neither the supply chain’s nor any member’s profit changes with the percentage of the leader’s shares the follower holds. However, while the profits of the chain and the leader increase with the percentage of the follower’s shares held by the leader, the follower’s profit does not necessarily increase or decrease. As a result, both partners can always achieve a win-win by setting a proper price for transferring the follower’s shares to the leader. Moreover, the equilibrium wholesale price may be greater than the retail price in the push chain, but less than the marginal production cost in the pull chain, contradicting the usual results found in the literature. We also derive a necessary and sufficient condition on the structure of PCO allowing a pull chain to perform better than a push one. This extends Cachon (2004)’s result that a pull chain always performs better than a push one (without PCO). Finally, PCO can coordinate a chain if and only if each one holds half of the other’s shares.

Keywords: Supply chain management; partial cross ownership; push; pull; win-win; leader

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1. Introduction

In 2009, Bosideng, the largest Chinese down wear manufacturer, acquired 1.76% of the shares of Dashang group, a local department store in northeast China. Also, Dashang group’s public notice showed that in the same year, Red Dragon, a leading Chinese shoe maker, purchased 2.2% of the shares of Dashang group. In these two cases of a push supply chain, a big supplier, the leader in a two echelon supply chain, holds its small downstream partner’s shares. Two firms holding each other’s shares is called partial cross ownership (PCO), with a special case where one firm holds the other’s shares unidirectionally.

PCO also exists in the pull supply chain between a big downstream firm and its small suppliers. For instance, as reported in The Machine That Changed the World (Womack, Daniel and Daniel 1991), Toyota held shares of its suppliers: 22 percent of Nippondenso, a maker of electrical components and engine computers; 14 percent of Toyoda Gosei, a maker of metal engine parts; and 19 percent of Koito, a maker of trim items, upholstery and plastics. In 2003, Mengniu, a leading Chinese milk producer, partially held shares of its supplier Modern Farming.¹

Other examples of PCO occur in the U.S. automobile industry (Alley, 1997), the Dutch Financial Sector (Dietzenbacher, Smid and Volkerink 2000), the Nordic power market (Amundsen and Bergman 2002) and the global steel industry (Gilo, Moshe and Spiegelthe 2004).

Although PCO is popular in practice, its impact on firms’ operations management is not studied enough. Instead, studies on PCO are mainly from the fields of law and economics, and focus on PCO between horizontal companies, especially rival competitors.

Thus, we want to explore from the viewpoint of operations management the following problems. How does the operational efficiency of a supply chain change under PCO? How does the chain efficiency change by allowing one firm to hold other’s

¹This report can be found from http://ns2.cbnweek.com/v/article?id=3262, accessed on May 20, 2016.
shares? Does PCO achieve a win-win for all partners in the chain? Or under what conditions can a win-win be achieved?

In this paper, we study the above problems in the frameworks of both a push and a pull supply chain consisting of one retailer and one supplier based on newsvendor setting. Our main findings are as follows. First, under PCO, not only the chain efficiency but also both players’ profits are independent of the percentage of the leader’s shares held by the follower. However, the profits of the chain and the leader increase with the percentage of the follower’s shares held by the leader, while the follower’s profit does not necessarily increase or decrease. This may help explain why Toyota, Bosideng and Red Dragon, as the leaders in their supply chains, hold part of their partners’ shares rather than let their own shares be partially held by their partners. Second, both members can always be better off and thus achieve a win-win via transferring the follower’s shares to the leader at a proper transferring price. Third, the equilibrium wholesale price may be greater than the retail price in the push chain and less than the marginal production cost in the pull chain. This contradicts the usual results in the literature on supply chain management. Fourth, we derive a necessary and sufficient condition on the structure of PCO allowing a pull supply chain to perform better than a push one. This extends Cachon (2004)’s result that the pull chain always performs better than the push one without PCO. Finally, PCO can coordinate the supply chain if and only if each player holds exactly 50% of the other’s shares.

The rest of the paper is organized as follows. Section 2 gives a brief literature review. In Section 3 we present how does the push supply chain efficiency change under PCO. In Section 4 we study the effects of PCO in the pull chain. In Section 5 we compare the push and pull supply chains under PCO. Section 6 is the concluding section. All the proofs are in the appendix.

2. Literature Review

One stream related to this paper is supply chain management. The supply chain under a wholesale price contract without PCO is well investigated in the literature.
Lariviere (1999) and Lariviere and Porteus (2001) provide a state-of-art analysis on push supply chain efficiency under a wholesale price contract. Cachon (2003) offers a review in both breadth and depth on both push and pull supply chains within the newsvendor setting. Cachon (2004) compares push, pull and advance-purchase discount contracts, and proves that there always exists some advance-purchase discount contract which Pareto dominates both the push and pull contracts. The main difference between a push supply chain and a pull one is who bears the inventory risk under demand uncertainty. In a push supply chain, all inventory risk is pushed onto the retailer, as with “a supplier selling to a newsvendor” (Lariviere and Porteus 2001), or “channel stuffing” (Cachon 2004), i.e., the supplier attempts to stuff the retailer with inventory. In a pull supply chain, the retailer pulls inventory from the supplier, replenishing as needed during the season. Thus, the supplier bears essentially all of the inventory risk. Similar allocation of inventory risk under the push and pull mode is studied in Beullens and Janssens (2011).

In general, the past researches prove that the simple wholesale price contract fails to coordinate the chain, due to the double marginalization and demand risk allocation. Consequently, many researchers investigate what incentive mechanisms can coordinate the supply chain with a random demand. Examples of such mechanisms include quantity discount, return policies, two part tariffs, buy back, sales rebate, quantity discount and revenue sharing contracts; see Cachon (2003) for an overview.

Our combination of PCO and the wholesale price contract is different from these coordination contracts. All these contracts are associated with an additional high administration cost in practice, such as keeping track of the realized revenue under the revenue sharing contract, checking and verifying the leftover inventory under the buy-back contract, incurring the fixed order cost twice under the advance-purchase discount contract. Instead, under our proposed PCO the transaction between two firms in a supply chain is governed by a single wholesale price, which is very easy to implement and free of additional administration overhand, and capable of bringing out a high chain efficiency.

It is worth mentioning that the weighted sum type of profit functions have been
used in other contexts of operations management too. For example, Bernstein, Song and Zheng (2008) use them to investigate the effect of horizontal alliance in joint ventures, while Chen, Hu and Song (2012) analyze the effect of altruism in a vertical chain.

In operations management, Kornbluth and Salkin (1994) use mathematical programming models to solve appropriate ownership structure in the United Kingdom, the Netherlands and the USA under different local laws and regulation. Levy (2011) leverages the graph theory to calculate the voting weight of each firm in a hierarchical network. None of them considers a bidirectional cross ownership structure like us, since both only consider one-directional top-down ownership structure. So far as we know, our work is the first to study the effect of PCO on efficiency and operation decisions in supply chains.

However, PCO is deeply studied in the fields of law and economics. In the field of law, for vertical PCO, Greenlee and Raskovich (2001) from the U.S. Department of Justice show that when a monopolist sells an input to an oligopoly, consumers’ and total surplus do not change if the downstream firms hold partial ownership of the upstream monopolist. We depart from their setting by modeling a price-taking follower rather than a price-setting oligopoly.

In the industrial economics field, Amundsen and Bergman (2002) investigate horizontal PCO between Norwegian-Swedish power companies, and find that horizontal PCO increases the electricity companies’ market power and thus the retail price of electricity. Similarly, Maleug (1992) finds that horizontal PCO can contribute to collusion between rivals, which decreases the public welfare. Florackis, Kanas and Kostakis (2015) empirically examine how does the managerial ownership affect the firm performance. Within this stream, our work is mostly related to Gütha, Niki- forakisc and Normann (2007) which is also on the vertical PCO. They consider a setting when a buyer buys one unit of product from a seller. Their experiment verifies that the mutual holding of a minority of shares between the buyer and the seller is sufficient to obtain efficient outcomes. There is only one unit good and no quantity decision considered in their model. Our paper instead focuses on the quantity
decision in a supply chain and the impact of PCO on profits of all the chain and players. However, in contrast to their result on the symmetric effect of PCO (i.e., the buyer holding shares of the seller has the same effect on efficiency as when the seller holds shares of the buyer), we prove the asymmetric effect of PCO (i.e., only the percentage of the follower’s share held by the leader increases the chain efficiency, while the percentage of the leader’s share held by the follower has no effect on the chain efficiency).

3. A Push Supply Chain

In this section we investigate a push supply chain under PCO, and compare the equilibrium with that without PCO given in the literature.

3.1. Model

The push supply chain studied here consists of one supplier and one retailer. The supplier first determines a wholesale price $w$ under which the supplier can produce as many as the retailer needs. Next, the retailer, facing an exogenous retail price $p$ and stochastic demand $\xi$, submits an order quantity $y$. Then, the supplier produces $y$ units at a constant marginal cost $c$ to fill the retailer’s order. To avoid the problem becoming trivial, we assume $p > c$. At last, the retailer receives the order quantity $y$ at the start of the selling season and bears the inventory risk of oversupply when realized demand is low. Hence, all inventory risk is pushed onto the retailer (Cachon 2004). Unmet demand is lost. To simplify our mathematical analysis and still capture the essential effects of PCO, we assume that any unsold inventory has no salvage value. However, a nonzero salvage value, whether positive or negative, does not change any of our conclusions.

Suppose the random demand $\xi$ has the distribution function (d.f.) $F(x)$ and the probability density function $f(x)$ with support $[a, b]$ or $[a, \infty)$ for some nonnegative real numbers $a$ and $b$. Let $\tilde{F}(x) = 1 - F(x)$ and $\tilde{F}^{-1}(\cdot)$ be the inverse function of $\tilde{F}(\cdot)$. Also, we assume that $F(\cdot)$ has an IGFR (increasing generalized failure rate), i.e., $g(y) := yf(y)/\tilde{F}(y)$ strictly increases with $y$, and that $f(x)$ is continuously differentiable for mathematical simplicity. Hence, $g'(y) \geq 0$ for $y \geq 0$ and $g(y) \geq g(0) = 0$. 
for $y > 0$. The IGFR captures most common distributions such as the normal, uniform, all the Gamma and Weibull and compound Poisson, and is often assumed in the supply chain management literature, e.g., Lariviere and Porteus (2001).

The demand d.f. $F(x)$, cost $c$ and retail price $p$ are common knowledge. For notational convenience, we denote $c^+ = \max\{c, 0\}$, $c \lor d = \max\{c, d\}$ and $c \land d = \min\{c, d\}$ for real numbers $c$ and $d$. Also, we denote by $S(y) = E(\xi \land y)$ the expected sales under order quantity $y$. Under order quantity $y$ and wholesale price $w$, the expected profits of the retailer, the supplier and the chain are, respectively,

$$
\pi_r(y, w) = -wy + pS(y),
$$

$$
\pi_s(y, w) = (w - c)y,
$$

$$
\pi_c(y) = -cy + pS(y).
$$

Obviously, $\pi_c(y)$ is concave with maximum point $y^L = \bar{F}^{-1}(c/p)$.

For the structure of PCO, suppose there are two firms, firm A and firm B. Firm A holds $1 - \beta$ percentage of the retailer’s shares and $\alpha$ percentage of the supplier’s shares, while firm B has $\beta$ percentage of the retailer’s shares and $1 - \alpha$ percentage of the supplier’s shares. Without loss of generality, we assume no one can get more than 50% of the other’s shares, i.e., $\alpha \leq 0.5$ and $\beta \leq 0.5$. When $\alpha > 0.5$ and $\beta \leq 0.5$ (or $\alpha \leq 0.5$ and $\beta > 0.5$), firm A (or B) holds the dominant shares in both the supplier and the retailer, and so is in charge of both in the two-dyad channel. This leads the supply chain to be a centralized one, and so the optimal order quantity is $y^L$. $\alpha \geq 0.5$ and $\beta \geq 0.5$ is similar to the case of $\alpha \leq 0.5$ and $\beta \leq 0.5$. In the standard financial terminology, firm A and firm B are both parent companies, whose profits are the main focus of our paper in addition to operational efficiency of the supply chain. For exposition, as firm A is in charge of the retailer, for simplicity we call firm A “the retailer” (she). Similarly, we call firm B “the supplier” (he).

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2We thank an anonymous referee for the suggestion on adding the compound Poisson distribution, which is not covered in Lariviere and Porteus (2001). Here, due to the easiness of analysis by using the first and second order derivatives, we assume that the demand distribution is continuous, as in the literature (e.g., Lariviere and Porteus 2001, Cachon 2003, Cachon 2004).
The Stackelberg game between the supplier and the retailer works as follows. In stage 1, the supplier offers a wholesale price \( w \). Then, in stage 2 the retailer faces the following problem given \( w \):

\[
\max_y \pi_{r,1}(y, w) = (1 - \beta)[pS(y) - wy] + \alpha(w - c)y \\
= (1 - \beta)pS(y) - (1 - \alpha - \beta)wy - \alpha cy. \tag{1}
\]

Let \( \tilde{y}_1(w) \) be the retailer’s best response as a function of \( w \) (the solution of the above optimization problem). Then, the supplier’s optimal wholesale price \( w^*_1 \) solves

\[
\max_w \pi_{s,1}(\tilde{y}_1(w), w) = (1 - \alpha)(w - c)\tilde{y}_1(w) + \beta[pS(\tilde{y}_1(w)) - w\tilde{y}_1(w)] \\
= \beta pS(\tilde{y}_1(w)) + (1 - \alpha - \beta)w\tilde{y}_1(w) - (1 - \alpha)c\tilde{y}_1(w). \tag{2}
\]

We first consider a special case of \( \alpha = \beta = 0.5 \). For this case, both players’ profit functions become

\[
\pi_{r,1}(y, w) = \alpha(w - c)y + (1 - \beta)[pS(y) - wy] = \frac{pS(y) - cy}{2} = \frac{\pi_c(y)}{2}, \\
\pi_{s,1}(y, w) = (1 - \alpha)(w - c)y + \beta[pS(y) - wy] = \frac{[pS(y) - wy] + (w - c)y}{2} = \frac{\pi_c(y)}{2}.
\]

In equilibrium, the order quantity will be \( y^I \), that is, the supply chain is coordinated with the chain profit being equally divided. This is still true for the pull chain. Hence, we consider \( \{(\alpha, \beta) | \alpha \leq 0.5, \beta \leq 0.5, \alpha + \beta < 1\} \) in the following. Note that for such \((\alpha, \beta)\), Proposition 1 below for the push chain says \( 0 \leq y^*_1 < y^I \) and Proposition 5 below for the pull chain states \( 0 \leq y^*_2 < y^I \), and so the supply chain can not be coordinated. Therefore, the chain under PCO can be coordinated if and only if \( \alpha = \beta = 0.5 \).

3.2. Main Results and Analysis

We need to solve the Stackelberg game by backward induction. First, we derive the retailer’s best response in stage 2. Given \( w \), the first-order condition for the optimal order quantity of the retailer is \( (1 - \beta)p\tilde{F}(\tilde{y}_1(w)) = (1 - \alpha - \beta)w + \alpha c \). This gives the retailer’s optimal order quantity. As the leader, the supplier can correctly
anticipate the retailer’s order quantity given any wholesale price $w$. Equivalently, the supplier faces the inverse demand curve

$$w_1(y) = \frac{(1 - \beta)p\bar{F}(y) - \alpha c}{1 - \alpha - \beta}. \quad (3)$$

Thus, the supplier’s problem of deciding a wholesale price $w$ is equivalent to setting an order quantity $y$ according to the following optimality problem:

$$\max_y \pi_{s,1}(y) := \pi_{s,1}(y, w_1(y)) = \beta p S(y) + (1 - \beta)py\bar{F}(y) - cy.$$  

At this time, the retailer’s profit becomes $r_{r,1}(y) := r_{r,1}(y, w_1(y)) = (1 - \beta)[pS(y) - y\bar{F}(y)]$. Denote by $y^*_1$ the optimal solution of $\max_y \pi_{s,1}(y)$, i.e., the optimal order quantity from the perspective of the supplier.

Interestingly, $y^*_1$ is independent of $\alpha$, and so neither the supplier’s profit nor the retailer’s profit in equilibrium depends upon $\alpha$. This holds for two reasons. First, as the first mover, the supplier can anticipate the retailer’s response, and then set the wholesale price accordingly to offset the effect caused by $\alpha$ percentage of his profit owned by the retailer. That is, according to the retailer’s response $w_1(y)$, the supplier just needs to set a wholesale price $w_1(y')$ if he wants the retailer to order $y'$. Thus, by substituting $w_1(y)$ into (1) and (2), $\alpha$ disappears in the two members’ profit functions $\pi_{s,1}(y)$ and $\pi_{r,1}(y)$. Moreover, another reason for the disappearance of $\alpha$ is the linear payment under the wholesale price contract: $wy$ is the payment of the supplier to the retailer under order quantity $y$. Note that the disappearance of $\alpha$ would not be true under a non-linear contract. Please see the following example.

**Example 1.** We consider a two-part tariff $L + wy$ in a push supply chain, where $L$ is the lump-sum payment term, $w$ is the wholesale price and $y$ is the order quantity. Under PCO, the retailer’s profit is $\pi_{r,1} = (1 - \beta)[pS(y) - L - wy] + \alpha[L + (w - c)y] = (1 - \beta)pS(y) - (1 - \alpha - \beta)(L + wy) - \alpha cy$ and the supplier’s profit is $\pi_{s,1} = (1 - \alpha)[L + (w - c)y] + \beta[pS(y) - L - wy] = \beta pS(y) + (1 - \alpha - \beta)(L + wy) - (1 - \alpha)cy$. Since $\pi_{r,1}$ is concave in $y$, the retailer’s response is $y = F^{-1}\left(\frac{(1 - \alpha - \beta)w + \alpha c}{(1 - \beta)p}\right)$ with its reverse function being $w_1(y) = \frac{(1 - \beta)p\bar{F}(y) - \alpha c}{1 - \alpha - \beta}$, which is the same as that under the wholesale price contract. However, with $w_1(y)$, the retailer’s profit becomes
\[ \pi_{r,1} = (1 - \beta)p[S(y) - y\bar{F}(y)] - (1 - \alpha - \beta)L, \text{ which depends on } \alpha \text{ through the term } -(1 - \alpha - \beta)L. \] Similarly, the supplier’s profit \( \pi_{s,1} \) depends also on \( \alpha \). Clearly, \(-(1 - \alpha - \beta)L\) is zero under the wholesale price contract (because \( L = 0 \)).

Now, \( y_1^* \) should satisfy the first order condition: \( \pi'_{s,1}(y) = p\bar{F}(y) - (1 - \beta)pyf(y) - c = 0 \), or equivalently,

\[ \bar{F}(y_1^*) - (1 - \beta)y_1^*f(y_1^*) = c/p. \quad (4) \]

From (4), we have the following proposition.

**Proposition 1.** (i) The equilibrium order quantity \( y_1^* \) is the unique solution satisfying (4) and the equilibrium wholesale price \( w_1^* := w_1(y_1^*) \).

(ii) \( \frac{\partial y_1^*}{\partial \alpha} \geq 0 \) but \( \frac{\partial y_1^*}{\partial \beta} = 0 \); \( y_1^* \leq y^* < y^I \).

(iii) \( \frac{\partial w_1^*}{\partial \alpha} \geq 0 \), \( w_1^* \geq c \).

Part (ii) of the proposition above says that the more the retailer’s shares the supplier holds, the greater the equilibrium ordering quantity \( y_1^* \) will be. The reason is that without PCO the equilibrium order quantity is low because of the double marginalization (Tirole 1988), under which each partner takes into account only its own marginal profit instead of the chain’s marginal profit. However, under PCO, partially holding the retailer’s shares, the supplier takes into consideration part of the retailer’s marginal profit in addition to his own marginal one. This increases the order quantity (from the supplier’s view) and at the same time increases the chain profit \( \pi_c(y_1^*) \). Part (iii) of the proposition says that \( w_1^* \) is increasing in \( \alpha \), however, \( \frac{\partial w_1^*}{\partial \beta} \) can be either positive or negative, as shown in Figure 1 below.

Note that when \( \beta = 0 \) and \( \alpha = 0 \), we have both partners’ profit functions without PCO. Denote by \( w_1^0 \) and \( y_1^0 \) the equilibrium wholesale price and order quantity. Due to equations (3) and (4), \( y_1^0 \) and \( w_1^0 \) uniquely satisfy

\[ \bar{F}(y_1^0) - y_1^0f(y_1^0) = c/p, \quad (5) \]

\[ w_1^0 = p\bar{F}(y_1^0), \quad (6) \]

and so the equilibrium expected profits of the retailer and supplier are, respectively,

\[ \pi_{r,1}^0 = -p\bar{F}(y_1^0)y_1^0 + pS(y_1^0), \quad (7) \]
\[ \pi_{s,1}^0 = [p\bar{F}(y_1^0) - c]y_1^0. \]  
(8)

This is consistent with those given by Lariviere and Porteus (2001).

3.2.1. Wholesale price \( w_1^* \)

From Proposition 1 (iii), \( w_1^* \geq c \). However, it is unclear whether \( w_1^* \leq p \). Interestingly, we find it is possible to have \( w_1^* > p \). As shown in Proposition 1 (iii), \( w_1^* \) increases with \( \alpha \), and so reasonably when \( \alpha \) is larger than a threshold, we can observe \( w_1^* > p \). For example, when the demand is uniformly distributed in the interval \([a, b]\) with \( a < b \), solving (4) we get

\[ y_1^* = \frac{(p - c)b + ca}{(2 - \beta)p}, \]  
(9)

\[ w_1^* = \frac{(1 - \beta)^2[(p - c)b + ca]}{(1 - \alpha - \beta)(2 - \beta)(b - a)} + c. \]  
(10)

Thus, \( w_1^* > p \) leads to \( \frac{(1-\beta)(a(c-p(2-\beta)+b(p-c))}{(2-\beta)(b-a)(p-c)} < \alpha < \frac{1}{2} \). In particular, when \( a = 0 \) and \( b = 1 \), i.e., the demand is uniformly distributed in the interval \([0, 1]\), \( w_1^* > p \) leads to \( \alpha \in (\frac{1-\beta}{2-\beta}, \frac{1}{2}) \). Since \( \frac{1-\beta}{2-\beta} \) decreases with \( \beta \), the length of the interval \((\frac{1-\beta}{2-\beta}, \frac{1}{2})\) increases with \( \beta \), and so \( w_1^* > p \) is more likely when \( \beta \) is large enough.

When the demand follows a truncated normal distribution in the interval \([0, 1]\) with \( \mu = 0.5 \) and \( \sigma = 0.24 \), together with \( c = 0.4 \) and \( p = 1 \), \( w_1^* \) is shown in Figure 1 for \( \alpha = 0.02 \) and \( \alpha = 0.4 \). Since the wholesale price \( w_1^* \) increases with \( \alpha \), the curve of \( w_1^* \) under \( \alpha = 0.02 \) is lower than that with \( \alpha = 0.4 \). We find from the figure that the wholesale price increases with \( \beta \) when \( \alpha = 0.4 \), but decreases with \( \beta \) when \( \alpha = 0.02 \). So, \( w_1^* > p \) when \( \alpha \) and \( \beta \) are large enough.

Remark 1. It is helpful to discuss whether the retailer’s profit is negative or not under \( w_1^* > p \). Under the uniform distribution in \([0, 1]\), the retailer’s profit \( \pi_{r,1} = (1 - \beta)(p - c)^2/(2(2 - \beta)^2) > 0 \). Under the truncated normal distribution, numerical results show that the retailer’s profit is also positive. The reason why the retailer earns a positive profit even under a negative profit margin \( p - w_1^* \) is that given the retailer’s profit function \((1 - \beta)[pS(y_1^*) - w_1^*y_1^*] + \alpha(w_1^* - c)y_1^* \), though \( pS(y_1^*) - w_1^*y_1^* \leq 0 \) under
$w_1^* > p$, the retailer is compensated from the second term $\alpha(w_1^* - c)y_1^* > \alpha(p - c)y_1^*$. Possibly there are cases that $\pi_{r,1} < 0$ given $w_1^* > p$, thus the retailer may choose to end the relationship with the supplier, and pursue her outside opportunity so that at least she can make ends meet.\footnote{For this issue, there are two remedies. One is a transfer payment to the retailer make her profit no smaller than that without PCO, which will be studied at the end of this section. The second is to set a restriction on $\beta$. As we show later, $\pi_{r,1}$ is independent of $\alpha$, but may decrease or increase with $\beta$. If $\pi_{r,1}$ increases with $\beta$, the retailer’s profit is always positive. However, if $\pi_{r,1}$ decreases with $\beta$ and is negative at $\beta = 1/2$, then an upper bound on $\beta$ should be set to make the retailer a nonnegative profit. We thank an anonymous referee for bringing this issue.}

3.2.2. Impacts of $\alpha$ and $\beta$

From equations (1) and (2), the profits of the retailer, the supplier and the chain are, respectively,

$$\pi_{r,1}(y_1^*) = (1 - \beta)p[S(y_1^*) - y_1^* F(y_1^*)], \quad (11)$$

$$\pi_{s,1}(y_1^*) = \beta pS(y_1^*) + (1 - \beta)p y_1^* F(y_1^*) - cy_1^*, \quad (12)$$

$$\pi_c(y_1^*) = pS(y_1^*) - cy_1^*. \quad (13)$$

Figure 1: The change of $w_1^*$ with $\beta$ in the push supply chain
We have the following proposition about the effects of PCO on profits, where IFR (increasing failure rate) means that the hazard rate \( h(y) := f(y) / \bar{F}(y) \) increases with \( y \).

The normal, uniform and Gamma and Weibull distributions with shape parameters greater than one are IFR (Lariviere and Porteus, 2001).

**Proposition 2.** (i) \( \frac{\partial \pi_{s,1}(y^*_1)}{\partial \alpha} = \frac{\partial \pi_{r,1}(y^*_1)}{\partial \alpha} = \frac{\partial \pi_{s,1}(y^*_1)}{\partial \alpha} = 0 \).

(ii) \( \frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} \geq 0 \) and \( \frac{\partial \pi_{s,1}(y^*_1)}{\partial \beta} \geq (\frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta})^+ \). Moreover, \( \frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} \leq 0 \) if \( \pi_{s,1}(y^*_1) \leq \frac{\pi_{r,1}(y^*_1)}{1-\beta} \) and \( F(x) \) is IFR.

Part (i) of Proposition 2 says that all the profits of the supplier and the retailer as well as the chain are independent of how much of the supplier’s shares are held by the retailer. This is due to the equilibrium order quantity being irrelevant to \( \alpha \). Whereas the more the retailer’s share held by the supplier, the greater the supplier’s profit and the chain profit will be. However, the change of \( \pi_{r,1}(y^*_1) \) with \( \beta \) is more complex, and is dependent of two opposite effects, as shown in the following expression:

\[
\frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} = \pi'_{r,1}(y^*_1) \frac{\partial y^*_1}{\partial \beta} - pS(y^*_1) + py^*_1 \bar{F}(y^*_1) = \pi'_{r,1}(y^*_1) \frac{\partial y^*_1}{\partial \beta} - p \int_0^{y^*_1} xf(x)dx.
\]

In the right hand side above, the first term is positive but the second negative. On one hand, the retailer will benefit from a larger chain profit; on the other hand, the retailer’s share of the chain profit becomes less. For example, under the truncated normal distribution in the interval \([0, 1]\) with \( \mu = 0.5, \sigma = 0.24 \) and \( p = 1 \), Figure 2(a) shows how does \( \pi_{r,1}(y^*_1) \) change with \( \beta \) under different values of \( c \). When \( c = 0.01, 0.02, 0.04, 0.08 \), the retailer’s profit is concave in \( \beta \). Also under the truncated normal distribution in the interval \([0, 1]\) with \( \mu = 0.5, c = 0.7 \) and \( p = 1 \), Figure 2(b) further shows that the retailer’s profit decreases with \( \beta \) when \( \sigma \) is very large (e.g., \( \sigma = 0.18 \)).

When the demand is uniformly distributed in \([a, b]\) with \( a < b \), from (9), \( \frac{\partial y^*_1}{\partial \beta} = \frac{y^*_1}{2-\beta} \), and from (11), \( \pi'_{r,1}(y^*_1) = \frac{(1-\beta)y^*_1}{b-a} \). Hence,

\[
\frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} = \pi'_{r,1}(y^*_1) \frac{\partial y^*_1}{\partial \beta} - p \int_0^{y^*_1} xf(x)dx = -\frac{p(y^*_1)^2}{b-a} \cdot \frac{\beta}{2(2-\beta)} \leq 0.
\]

The inequality above holds strictly only when \( \beta > 0 \). Therefore, under the uniform distribution we have \( \frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} \leq 0 \) while \( \frac{\partial \pi_{s,1}(y^*_1)}{\partial \beta} \geq 0 \). That is, the retailer is worse off, whereas the supplier is better off under PCO, i.e., \( \pi_{r,1}(y^*_1) < \pi_{r,1}^0 \) and \( \pi_{s,1}(y^*_1) > \pi_{s,1}^0 \).
3.2.3. Impacts of $c$ and $p$

Here, we examine the impacts of $c$ and $p$. First, we have the following result.

**Proposition 3.** (i) $\frac{\partial y_1^*}{\partial c} \leq 0$ and $\frac{\partial y_1^*}{\partial p} \geq 0$.

(ii) $\frac{\partial_{y_1}(y_1^*)}{\partial c} \leq 0$ and $\frac{\partial_{y_1}(y_1^*)}{\partial p} \geq 0$.

(iii) $\frac{\partial_{y_1}(y_1^*)}{\partial c} \leq 0$ and $\frac{\partial_{y_1}(y_1^*)}{\partial p} \geq 0$, while $\frac{\partial_{y_1}(y_1^*)}{\partial c} \geq 0$ and $\frac{\partial_{y_1}(y_1^*)}{\partial p} \geq 0$.

From the proposition, $p$ and $c$ have opposite effects on the quantity and profits: the optimal order quantity and all the profits of the retailer, the supplier and the chain decrease with production cost $c$ but increase with retail price $p$. However, whether PCO can lead to a higher chain efficiency $\frac{\pi_c(y_1^*)}{\pi_c(y^*)}$ with higher $p$ and lower $c$? This is a complex problem since the system optimal quantity $y'$ is also increasing in $p$ and decreasing in $c$. In the following, we study this problem for uniformly demand distributions and normal demand distributions, respectively.

First, suppose the demand is uniformly distributed in $[a, b]$ for some $a < b$. Then, the first best order quantity is $y' = b - \frac{(b-a)c}{p}$, and so from (9) we have $y_1^* = \frac{1}{2-\beta}$ and $\frac{\pi_c(y_1^*)}{\pi_c(y^*)} = \frac{(3-2\beta)}{(2-\beta)^2}$. Thus, the chain efficiency is independent of $c$ and $p$. Since the equilibrium quantity without PCO is $y_1^0 = \frac{(p-c)\beta+ca}{2p}$, the effect of PCO on the chain efficiency can be measured by $\frac{\pi_c(y_1^* y_1^0)}{\pi_c(y^*)} = \frac{\pi_c(y_1^*)}{\pi_c(y_1^0)} = \frac{4(3-2\beta)}{(2-\beta)^2} > 1$, which is also independent of $c$ and $p$. The reason for the above independence of $c$ and $p$ is that under
the uniform demand distribution the order quantities $y^I$, $y^0_1$ and $y^*_1$ are all linear in $c$ and $p$, and so the effects of $c$ and $p$ upon these ratios can be canceled out. Moreover, 

$$\frac{\pi_{s,1}(y^*_1)}{\pi_{s,1}(y^0_1)} = \frac{2}{2-\beta} \geq 1 \quad \text{and} \quad \frac{\pi_{r,1}(y^*_1)}{\pi_{r,1}(y^0_1)} = \frac{4(1-\beta)}{(2-\beta)^2} \leq 1,$$

which is consistent with the discussion below Proposition 2: when the demand follows a uniform distribution, PCO improves the supplier’s profit but reduces the retailer’s profit. Moreover, this is also explains why PCO increases the supplier’s share of the chain profit: the supplier’s share of the chain profit without PCO is 

$$s^0_{1} \left( y^0_1 \right) = \frac{2}{3}$$

while under PCO is 

$$s^0_{s,1} \left( y^*_1 \right) = \frac{2}{3-2\beta} \geq \frac{2}{3}.$$

However, when the demand follows a normal distribution, the situation becomes so complex that we cannot get a closed-form solution even for the first best quantity $y^I$. Numerical studies show that $\pi_{c}(y^*_1)/\pi_{c}(y^0_1)$ and $\pi_{s,1}(y^*_1)/\pi_{s,1}^0$ as well as $\pi_{r,1}(y^*_1)/\pi_{r,1}^0$ do not change with $c$ or $p$ monotonically.

### 3.2.4. Win-win

From our previous results, the supplier knows that holding shares of the retailer will increase his profit as well as the chain profit. However, this would not necessarily increase the retailer’s profit. Thus, the supplier should pay a price to the retailer for holding her shares. A question therefore arises: what is an adequate transferring price for the retailer’s $\beta$ shares? To simplify our exposition, we refer to the case in which both players’ expected profits are greater than those without PCO as a win-win, i.e., 

$$\pi_{r,1}(y^*_1) + T \geq \pi_{r,1}^0 \quad \text{and} \quad \pi_{s,1}(y^*_1) - T \geq \pi_{s,1}^0,$$

where $T$ is a payment transferred from the supplier to the retailer. Certainly, $T$ can be zero or even negative which is referred to as a win-win without transferring; otherwise, as a win-win with transferring.

Since $\frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} < 0$ may be true, as in the case of the uniform demand distribution, a payment

$$T \in \left[ \pi_{r,1}^0 - \pi_{r,1}(y^*_1), \pi_{c}(y^*_1) - \pi_{r,1}(y^*_1) - \pi_{s,1}(y^*_1) \right]$$

from the supplier to the retailer will make both partners better off, i.e., a win-win is achieved. Since $\pi_{c}(y^*_1) \geq \pi_{c}(y^0_1) = \pi_{s,1}^0 + \pi_{r,1}^0$, the domain of $T$ is nonempty. Therefore, we have the following proposition.

**Proposition 4.** (i) For a push supply chain there exists a transferring price $P(\beta) = T$
with $T \in [\pi_{r,1}^0 - \pi_{r,1}(y_1^*) - \pi_{r,1}(y_1^*) - \pi_{s,1}^0]$ under which a win-win is achieved. \(^4\)

(ii) Both the upper and lower bounds of $T$ are independent of $\alpha$.

From the proposition above we know that there are an upper bound and a lower bound for the price of transferring the retailer’s shares to the supplier. Under such a transferring price, the chain becomes more efficient and both the supplier and the retailer are better off, achieving a win-win.

If $\pi_{r,1}^0 - \pi_{r,1}(y_1^*) \leq 0$, such as discussed in the last subsection when $c$ is small or $p$ is large under normal demand distributions, the transferring price can be zero or even negative, and so the supplier may take the retailer’s $\beta$ shares for free; That is, win-win is achieved without transferring price. Otherwise, $\pi_{r,1}^0 - \pi_{r,1}(y_1^*)$ may be positive when $c$ is large or $p$ is small, even if $w_1^* > p$. \(^5\)In this case, $\pi_{r,1}^0 - \pi_{r,1}(y_1^*)$ is positive, and when moving to PCO, the retailer’s profit decreases, while the supplier earns more than $\pi_{s,1}^0$ because $\pi_{s,1}(y_1^*)$ increases with $\beta$. Therefore, to have win-win, the retailer should receive a positive payment, no less than $\pi_{r,1}^0 - \pi_{r,1}(y_1^*)$, from the supplier, if the retailer gives a portion of its shares to the supplier.

4. A Pull Supply Chain

4.1. Model

In the pull supply chain, everything is the same as in the push supply chain except that well before the selling season, the retailer first sets a wholesale price, then the supplier determines his production quantity $y$, produces and offers the product to the

\(^4\)In practice, the transaction between the supplier and the retailer lasts multiple periods, say, $n$ periods ($n = \infty$ means an infinite horizon). Then, using the discounted cash flow model in finance (Copeland, Koller and Murrin 2000) with a discounted rate $r$, the transferring price $P(\beta)$ is equal to the discounted value of a transfer payment $T_i$ in period $i$, i.e., $P(\beta) = \sum_{i=1}^{n} r^{i-1} T_i$. If $T_i = T$ for all $i$, then $P(\beta) = T^{\frac{1-r^n}{1-r}}$.

\(^5\)The retailer’s profit is independent of $\alpha$ from (i) of Proposition 2, while the wholesale price $w_1^*$ increases with $\alpha$ from (iii) of Proposition 1. According to (1), the retailer’s profit is $\pi_{r,1}(y_1^*) = (1-\beta)p S(y_1^*) - (1-\alpha-\beta)w_1^* y_1^* -acy_1^*$. When $w_1^* > p$, $\pi_{r,1}(y_1^*) < (1-\beta)p S(y_1^*) - (1-\alpha-\beta)py_1^* -acy_1^*$, and so $\pi_{r,1}^0 > \pi_{r,1}(y_1^*)$ if $\alpha < \frac{(1-\beta)p S(y_1^*) - (1-\alpha-\beta)py_1^* -acy_1^*}{(p-c)y_1^*}$.
retailer. That is, the retailer pulls inventory from the supplier, thereby leaving the supplier with all inventory risk. In other words, the retailer’s order is submitted during the selling season and, if the supplier has inventory available, goods are received immediately by the retailer with the retailer paying the supplier \( w \) per unit. Cachon (2004) refers to this case as “retailer buys from a newsvendor.” That is, the suppliers’ product is shipped to the retailer on consignment, or the supplier holds the inventory replenishing the retailer’s inventory frequently in small batches during the season.

Under a production quantity \( y \) and a wholesale price \( w \), the expected profits of the retailer and supplier as well as the chain are, respectively,

\[
\bar{\pi}_r(y, w) = (p - w)S(y), \quad (15)
\]
\[
\bar{\pi}_s(y, w) = wS(y) - cy, \quad (16)
\]
\[
\pi_c(y) = pS(y) - cy. \quad (17)
\]

Obviously, \( \pi_c(y) \) has the same expression as in the push supply chain, and is concave with the same maximum point \( y^* = F^{-1}(c/p) \).

The PCO structure is the same as that in Section 3. The Stackelberg game between the supplier and the retailer works as follows. In stage 1, the retailer offers a wholesale price \( w \). Then, in stage 2 the supplier faces the following problem for the given \( w \):

\[
\max_y \pi_{s,2}(y, w) = (1 - \alpha)(wS(y) - cy) + \beta(p - w)S(y) - (1 - \alpha)cy. \quad (18)
\]

Let \( \tilde{y}_2(w) \) be the supplier’s best response as a function of \( w \) (i.e., the solution of the above optimization problem). Then, the retailer’s optimal wholesale price \( w_r^* \) solves

\[
\max_w \pi_{r,2}(\tilde{y}_2(w), w) = (1 - \beta)(p - w)S(\tilde{y}_2(w)) + \alpha(wS(\tilde{y}_2(w)) - c\tilde{y}_2(w)) - (1 - \alpha)cy. \quad (19)
\]

Similar to that in the push chain, the pull chain can be coordinated if and only if \( \alpha = \beta = 0.5 \), and in the following we limit our study to the region of \( \{(\alpha, \beta) | \alpha \leq 0.5, \beta \leq 0.5, \alpha + \beta < 1\} \).
4.2. Main Results and Analysis

For the supplier’s best response in stage 2, given \( w \), the first order condition for
\[
\max_y \pi_{s,2}(y, w) \text{ is } \beta p \bar{F}(y) + (1 - \alpha - \beta) w \bar{F}(y) - (1 - \alpha) c = 0. \quad (20)
\]

There is a unique solution of \( y \) for the equation above given \( w \). Equivalently, in order to let the supplier set production quantity \( y \), the retailer just sets a wholesale price as follows:
\[
w_2(y) = \frac{(1 - \alpha)c - \beta p \bar{F}(y)}{(1 - \alpha - \beta)F(y)} = \frac{(1 - \alpha)c}{(1 - \alpha - \beta)F(y)} - \frac{\beta p}{1 - \alpha - \beta}, \quad (21)
\]

which increases with production quantity \( y \). Thus, the retailer’s problem of deciding a wholesale price is equivalent to setting a production quantity \( y \) according to \( w_2(y) \). Therefore, the retailer equivalently faces the following problem:
\[
\max_y \pi_{r,2}(y, w_2(y)) = pS(y) - \alpha cy - \frac{(1 - \alpha)cS(y)}{F(y)}. \quad (22)
\]

Interestingly, the retailer’s profit is independent of \( \beta \), since the payment \( wy \) is linear in \( y \). Denote by \( y^*_2 \) the optimal solution of \( \max_y \pi_{r,2}(y, w_2(y)) \). Then, \( y^*_2 \) is the optimal production quantity for the supplier from the retailer’s view, and should satisfy
\[
\pi'_{r,2}(y) = p \bar{F}(y) - \frac{(1 - \alpha)cf(y)S(y)}{F^2(y)} - c = 0, \quad \text{that is,}
\]
\[
p \bar{F}(y^*_2) - \frac{(1 - \alpha)cf(y^*_2)S(y^*_2)}{F^2(y^*_2)} = c. \quad (22)
\]

Then, we have the following proposition for the pull supply chain.

**Proposition 5.** (i) The equilibrium production quantity \( y^*_2 \) is the unique solution of equation (22) and the equilibrium wholesale price is \( w^*_2 := w_2(y^*_2) \).

(ii) \( \frac{\partial y^*_2}{\partial \alpha} \geq 0 \) but \( \frac{\partial y^*_2}{\partial \beta} = 0 \); \( y^0_2 \leq y^*_2 < y^f \).

(iii) \( \frac{\partial w^*_2}{\partial \beta} \leq 0 \) and \( w^*_2 \leq p \).

Proposition 5 for the pull supply chain is similar to that given in Proposition 1 for the push supply chain, but with the roles of \( \alpha \) and \( \beta \) changed. We then say that the results in Proposition 5 are dual to those in Proposition 1 for the push supply chain.
As implied by Proposition 5, the more the supplier’s shares the retailer holds, the higher the equilibrium production quantity $y_2^*$ will be. PCO mitigates the double marginalization since the retailer takes into consideration a part of the supplier’s profit in addition to her own. That helps to account for why Toyota takes part of its suppliers’ share to increase the chain efficiency, and none of her suppliers holds Toyota’s shares.\footnote{From Toyota’s annual report (which only discloses the name of the top 10 major shareholder), none of her suppliers is among the top 10 shareholder, and the 10th shareholder has about 0.44% of Toyota’s shares.}

Note that when $\beta = 0$ and $\alpha = 0$, (18) and (19) are both partners’ profit functions without PCO, for which we denote by $w_0^0$ and $y_0^0$ the equilibrium wholesale price and production quantity. Then, from equations (22) and (21), $y_0^0$ and $w_0^0$ uniquely satisfy

$$F(y_0^0) - \frac{cf(y_0^0)S(y_0^0)}{pF^2(y_0^0)} = \frac{c}{p},$$

and then the equilibrium profits of the retailer and the supplier are, respectively,

$$\pi_{r,2}^0 = (p - w_0^0)S(y_0^0),$$

$$\pi_{s,2}^0 = w_0^0S(y_0^0) - cy_0^0.$$

This is consistent with those given by Cachon (2004).

4.2.1. Wholesale price $w_2^*$

Proposition 5 (iii) says $w_2^* \leq p$. However, whether $w_2^* \geq c$ or not? Interestingly, $w_2^*$ may be smaller than $c$ when $\beta$ are large enough (since $\frac{\partial w_2^*}{\partial \beta} \leq 0$). Figure 3 (a) shows the change of $w_2^*$ with $\alpha$ under the uniformly distributed demand in $[0, 1]$ with $p = 1$ and $c = 0.2$, where the curve of $w_2^*$ under $\beta = 0.4$ is lower than that under $\beta = 0.01$. Moreover, $w_2^*$ is always lower than the retail price $p = 1$, and is negative when $\beta = 0.4$, which is equivalent to subsidizing the retailer’s acquisition of the product.\footnote{The reason why the supplier can earn a positive profit even under a negative profit margin $w_2^* - c$ is that he is compensated from $\beta$ percentage of shares of the retailer’s profit, i.e., $\beta(p - w_2^*)S(y_2^*)$. Possibly there are cases that $\pi_{s,2} < 0$ given $w_2^* < c$, then the supplier may choose to end the}
Figure 3: Changes of $w_2^*$ with $\alpha$

Under the truncated normal distribution in the interval $[0, 1]$ with $\mu = 0.5$ and $\beta = 0.2$ together with $c = 0.2$ and $p = 1$, Figure 3 (b) shows the change of $w_2^*$ with $\alpha$ under different values of $\sigma$. It shows that the lower the deviation of the random demand is, the lower the wholesale price will be. So, $w_2^* < c$ or even a negative $w_2^*$ is more likely when both $\alpha$ and $\beta$ are large enough or when the demand variance is small enough under a normal demand distribution.

4.2.2. Impacts of $\alpha$ and $\beta$

After solving the equilibrium production quantity and wholesale price, we go on to investigate impacts of $\alpha$ and $\beta$ on the profits in the following proposition. From equations (18) and (19) as well as (20), the profits of the retailer, the supplier and the chain in equilibrium are, respectively:

$$\pi_{s,2}(y_2^*) = \frac{(1 - \alpha)c \int_0^{y_2^*} xf(x)dx}{F(y_2^*)},$$

$$\pi_{r,2}(y_2^*) = pS(y_2^*) - \frac{cS(y_2^*)}{F(y_2^*)} + \frac{\alpha c \int_0^{y_2^*} xf(x)dx}{F(y_2^*)},$$

relationship with the retailer, and pursue his outside opportunity to earn at least a nonnegative profit. For this issue, two remedies similar to those in the push chain exist.
We have the following properties of the profits with $\alpha$ and $\beta$.

**Proposition 6.**

(i) $\frac{\partial \pi_r(y^*_2)}{\partial \beta} = 0$.

(ii) $\frac{\partial \pi_r(y^*_2)}{\partial \alpha} \geq 0$ and $\frac{\partial \pi_s(y^*_2)}{\partial \alpha} \geq \frac{(\frac{\partial \pi_r(y^*_2)}{\partial \alpha})^+}{\frac{1}{1-\alpha} (1+\alpha) \pi_s(y^*_2)}$.

Moreover, $\frac{\partial \pi_r(y^*_2)}{\partial \beta} \leq 0$ when $\pi_r(y^*_2) \leq \frac{\partial \pi_s(y^*_2)}{\partial \alpha}$.

Again, the results given in Proposition 6 for the pull supply chain are dual to those in Proposition 2 for the push supply chain. Especially, Proposition 6 says that all the profits are independent of $\beta$, but that the profits of the retailer and the chain increase with $\alpha$. However, the change of the supplier’s profit with $\alpha$ is complex. Figure 4 shows that the supplier’s profit decreases with $\alpha$ under the truncated normal demand distribution in the interval $[0, 1]$ with $\mu = 0.5$, $\beta = 0.2$ and $p = 1$, and $\sigma = 0.18$ in Figure 4 (a) and $c = 0.7$ in Figure 4 (b).

**4.2.3. Impacts of $c$ and $p$**

The following proposition characterizes the impacts of $c$ and $p$.

**Proposition 7.**

(i) $\frac{\partial y^*_2}{\partial c} \leq 0$ and $\frac{\partial y^*_2}{\partial p} \geq 0$. 

We have the following properties of the profits with $\alpha$ and $\beta$.

Figure 4: Changes of the supplier’s profit with $\alpha$ under a truncated normal demand.

\[
\pi_s(y^*_2) = pS(y^*_2) - cy^*_2. 
\] (29)
(ii) $\frac{\partial \pi_c(y_2^*)}{\partial c} \leq 0$ and $\frac{\partial \pi_c(y_2^*)}{\partial p} \geq 0$.

(iii) $\frac{\partial \pi_{s,2}(y_2^*)}{\partial c} \leq 0$ and $\frac{\partial \pi_{s,2}(y_2^*)}{\partial p} \leq 0$, while $\frac{\partial \pi_{r,2}(y_2^*)}{\partial c} \geq 0$ and $\frac{\partial \pi_{r,2}(y_2^*)}{\partial p} \geq 0$.

The proposition above is exactly the same as Proposition 3 for the push supply chain. Next, we investigate the impact of $c$ or $p$ when the demand follows a uniform distribution $[0, 1]$. For the impact of $c$ on the chain, we let $p = 1$. As shown in Figure 5, all $\pi_c(y_2^*)/\pi_c^0$ (in (a)), $\pi_{s,2}(y_2^*)/\pi_{s,2}^0$ (the dotted line in (b)) and $\pi_{r,2}(y_2^*)/\pi_{r,2}^0$ (the solid line in (b)) increase with $c$. Hence, when $c$ becomes larger, PCO leads to higher chain efficiency and greater profit for the both members.

![Figure 5: Impact of $c$ under uniform distributions for the pull chain](image)

For the impacts of $p$ on the chain, we let $c = 0.4$. Figure 6 (a) shows that $\pi_c(y_2^*)/\pi_c^0$ increases with $p$ (in (a)), but both $\pi_{s,2}(y_2^*)/\pi_{s,2}^0$ (the dotted line in (b)) and $\pi_{r,2}(y_2^*)/\pi_{r,2}^0$ (the solid line in (b)) decrease with $p$. Hence, when $p$ becomes larger, PCO increases the chain efficiency, but decreases the retailer’s profit with the supplier’s profit almost fixed. Thus, the impact of $p$ is rather different from that of $c$.

Moreover, from Figures 5 and 6, we find that both $\pi_c(y_2^*)/\pi_c(y_2^0)$ and $\pi_{r,2}(y_2^*)/\pi_{r,2}^0$ are always larger than 1, while $\pi_{s,2}(y_2^*)/\pi_{s,2}^0$ is always less than 1. So, we conclude that under a uniform demand distribution, in the pull chain PCO always benefits the chain and the retailer (the leader), but harms the supplier (the follower). However, the impact of $p$ on the chain is rather different from that of $c$. Note that in the push
chain, the improvement of PCO upon the chain profit is independent of c and p, due to that the order quantity is linear in c and p. However, the linearity does not hold in the pull chain. If the demand follows a normal distribution, like the push chain above, we find that the changes are complex, in fact, all of them are not monotone, given any value of α.

4.2.4. Win-win

Since \( \frac{\partial \pi_{s,2}(y_{2}^*)}{\partial \alpha} \geq 0 \) from Proposition 6 (ii), the retailer can increase her profit via holding shares of the supplier. However, this would not necessarily increase the supplier’s profit as in the case of the truncated normal demand distribution as shown in Figure 4. Thus, the retailer should pay a price to the supplier for transferring the shares of the supplier. So, a question arises: what is an adequate price for the supplier’s α shares? The answer is that any price

\[
T \in [\pi_{s,2}^{0} - \pi_{s,2}(y_{2}^*), \pi_c(y_{2}^*) - \pi_{s,2}(y_{2}^*) - \pi_{r,2}^{0}]
\]  

(30)

from the supplier to the retailer will make each partner better off, i.e., \( \pi_{s,2}(y_{2}^*) + T \geq \pi_{s,2}^{0} \) and \( \pi_{r,2}(y_{2}^*) - T \geq \pi_{r,2}^{0} \), which we call a win-win case. We therefore have the following proposition, with the proof similar to that of Proposition 4.

**Proposition 8.** (i) In the pull supply chain, there exists a transferring price for
\( \alpha \) percentage of the supplier’s shares \( P(\alpha) = T \) with \( T \in [\pi_{s,2}^0 - \pi_{s,2}(y_2^*) - \pi_{s,2}(y_2^*) - \pi_{r,2}^0] \) under which a win-win is achieved.

(ii) Both the upper and lower bounds of \( T \) are independent of \( \beta \).

If \( \pi_{s,2}^0 \leq \pi_{s,2}(y_2^*) \), then the transferring price may be zero or even negative. Thus, the retailer may take the supplier’s shares for free or even with a reward; then, both achieve a win-win without transferring.

5. Comparison of the Push and Pull Supply Chains under PCO

In summary, the more the follower’s shares the leader holds, the higher will be the equilibrium order/production quantity and so the chain efficiency and the leader’s profit. On the other hand, the equilibrium order/production quantity is independent of the leader’s shares held by the follower, and so is the chain efficiency. Both partners can achieve a win-win when the leader holds the follower’s share at an adequate transferring price, especially when the retail price is high or the production cost is low. Furthermore, we have the following theorem for the push/pull supply chains.

**Theorem 1.** \( y_1^* < y_2^* \) and \( \pi_c(y_1^*) < \pi_c(y_2^*) \) if and only if \( \alpha > 1 - \frac{(1-\beta)py^*_1F_2(y_1^*)}{cS(y_1^*)} \). Moreover, when \( \beta = 0, y_1^* < y_2^* \) and \( \pi_c(y_1^*) < \pi_c(y_2^*) \) for any \( \alpha \).

Note in the proof of Theorem 1 we show that when \( \beta = 0 \) the threshold \( 1 - \frac{(1-\beta)py^*_1F_2(y_1^*)}{cS(y_1^*)} \) \( \leq 0 \), and thus \( y_1^* \leq y_2^* \) is always true, which is consist with Cachon (2004) for \( \alpha = \beta = 0 \). Given \( \beta \), if and only if the percentage \( \alpha \) of the supplier’s share held by the retailer is larger than the threshold, the pull chain performs better than the push one. Hence, we extend the result obtained in Cachon (2004). This is shown in the northeast shadow area in Figure 7 (a) under a uniformly distributed demand in \([0, 1]\) with the same data as in Figure 3 (a), and in Figure 7 (b) under a truncated normal distribution in the interval \([0, 1]\) with \( \mu = 0.5, \sigma = 0.24, p = 1 \) and \( c = 0.9 \). Further from Figure 7 the threshold is increasing in \( \beta \) for both the uniform and normal demands. Moreover, through those figures we find that when \( \alpha = \beta \) (i.e., the leaders own the same share of the followers), the pull chain performs better than the push one.
6. Conclusions

In this paper, we investigate the effects of PCO in both push and pull supply chains. We show that PCO mitigates the double marginalization, and brings out a higher chain efficiency and a win-win for both partners. On the other hand, as we elaborate in the literature review section, compared with various coordination contracts, such as the revenue sharing, buy-back and advance-purchase discount contracts, our proposed combination of the wholesale price contract and PCO enjoys the simplicity of just setting the wholesale price, and thus incurring a lower administrative overhead. This helps to explain why PCO is so popular in reality. Moreover, we prove that only when the leader holds the follower’s shares, PCO can improve supply chain efficiency and benefit the leader and so win-win can always be achieved under an adequate transferring price; whereas under a normal demand distribution with small coefficient of variation (CV), PCO benefits the follower and thus win-win is achieved without any transferring payments. Hence, the managerial insight is that the leader should hold some of the follower’s shares. This also helps to explain why many firms such as Sharp and Samsung adopt vertical integration.

Furthermore, we find whether or not a pull chain performs better than a push
one depends on the structure of PCO. This extends Cachon (2004)’s result that the pull chain always performs better than the push one without PCO. We also find that the supply chain can be coordinated if and only if each one holds half of the other’s shares.

Our model is a first step in studying issues related to partial cross ownership in supply chains. Future research may include exploring a price-setting retailer setting. It is also interesting to check the effect of PCO in an asymmetric information setting.

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References


Section A: Proofs of all propositions

Proof of Proposition 1: (i) Similar to Lariviere and Porteus (2001), we can prove that $\pi_{s,1}(y)$ is unimodal in $y$ under the IGFR: Given $\pi_{s,1}(y) = \beta pS(y) + (1-\beta)pyF(y) - cy$, $\frac{d\pi_{s,1}(y)}{dy} = pF(y) - (1-\beta)pyf(y) - c = p\bar{F}(y)[1 - (1-\beta)\frac{f(y)}{F(y)}] - c$. Under the IGFR condition, $\frac{f(y)}{F(y)} > 1$ or $\frac{f(y)}{F(y)} = 0$. We first assume that $y^b$ exists. Clearly, for $y > y^b$, $\frac{d\pi_{s,1}(y)}{dy} < 0$, so the supplier would never choose $y > y^b$. For $y < y^b$, $\frac{d^2\pi_{s,1}(y)}{dy^2} = -pf(y)[1 - (1-\beta)\frac{f(y)}{F(y)}] - (1-\beta)p\bar{F}(y)(\frac{f(y)}{F(y)})' \leq 0$ under the IGFR condition. So, $\pi_{s,1}(y)$ is concave in $y \leq y^b$, and the equilibrium quantity $y^*_1$ is the unique solution satisfying (4) and $w^*_1 = w_1(y^*_1).$ If $y^b$ does not exist, the IGFR condition implies that $1 - (1-\beta)\frac{f(y)}{F(y)} > 0$ for all $y$, and so $\pi_{s,1}(y)$ is concave for all $y$, which implies that the optimal quantity is an interior solution satisfying (4).

(ii) First, from (4) it is obvious that $y^*_1$ is independent of $\alpha$, while

$$\frac{\partial y^*_1}{\partial \beta} = \frac{\bar{F}(y^*_1)g(y^*_1)}{f(y^*_1)[1 - (1-\beta)g(y^*_1)] + (1-\beta)\bar{F}(y^*_1)\frac{g(y^*_1)}{(1-\beta)F(y^*_1)}} > 0.$$ 

Also by comparing equation (4) with $\beta = 0$ and $\beta = 1$, $y^*_1 = y^0_1$ when $\beta = 0$. So, we have $y^*_1 > y^0_1$. From (4), $\bar{F}(y^*_1) > c/p$ and so $y^*_1 < y^f$. 

(iii) By differentiating $w^*_1 = w_1(y^*_1)$ in $\alpha$ we have $\frac{\partial w^*_1}{\partial \alpha} = \frac{(1-\beta)(p\bar{F}(y^*_1) - c)}{(1-\alpha-\beta)^2} \geq 0$. So, $w^*_1$ increases with $\alpha$. Moreover, from $y^*_1 < y^f$ shown in (i), $w^*_1 = \frac{(1-\beta)(p\bar{F}(y^*_1) - c)}{(1-\alpha-\beta)^2} \geq \frac{(1-\alpha-\beta)c}{1-\alpha-\beta} = c$. This completes the proof.

Proof of Proposition 2: (i) is obvious. So, we omit its proof.

(ii) We have

$$\frac{\partial \pi_{s,1}(y^*_1)}{\partial \beta} = \pi_{s,1}(y^*_1)\frac{\partial y^*_1}{\partial \beta} + pS(y^*_1) - py^*_1\bar{F}(y^*_1) = p\int_0^{y^*_1} x f(x) dx \geq 0,$$

$$\frac{\partial \pi_{c}(y^*_1)}{\partial \beta} = (p\bar{F}(y^*_1) - c)\frac{\partial y^*_1}{\partial \beta} \geq 0.$$

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8Here, an implicit condition is that $\frac{d\pi_{s,1}(y)}{dy} |_{y = a} \geq 0$, where $\tilde{a}$ is the minimum value in the support of the random demand. This together with $\frac{d\pi_{s,1}(y)}{dy} |_{y = b} = -c < 0$ and the concavity of $\pi_{s,1}(y)$ for $y \in [\tilde{a}, y^b]$ results in the interior solution characterized by (4). If $\frac{d\pi_{s,1}(y)}{dy} |_{y = a} < 0$, then the optimal solution is $y = \tilde{a}$, which leads to a case too trivial to be discussed.
\[ \frac{\partial \pi_c(y^*_1)}{\partial \beta} - \frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} = \frac{\partial \pi_{s,1}(y^*_1)}{\partial \beta} \geq 0. \]

Under the IFR, for any \( y \), \( (f(w))' = \frac{f'(w)}{F(w)} + \frac{f''(w)}{F(w)^2} \geq 0 \). Hence, \( \frac{uf'(y)}{f(y)} \geq \frac{-yf(y)}{F(y)} \). From (4), \( \frac{y'_i f(y^*_i)}{F(y^*_i)} \leq \frac{1}{1-\beta} \). Therefore, \( \frac{y'_i f(y^*_i)}{F(y^*_i)} \geq -\frac{y'_i f(y^*_i)}{F(y^*_i)} \geq -\frac{1}{1-\beta} \). From the proof of Proposition 1, \( \frac{\partial y^*_i}{\partial \beta} \leq \frac{y^*_i}{1-\beta} \). Therefore,

\[
\frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} = \pi_{r,1}(y^*_1) \frac{\partial y^*_1}{\partial \beta} - p \int_0^{y^*_i} x f(x)dx \\
\leq \frac{(p\bar{F}(y^*_1) - c)y^*_i}{1-\beta} - p \int_0^{y^*_i} x f(x)dx \\
= \pi_{s,1}(y^*_1) - \beta \pi_{r,1}(y^*_1)/(1-\beta) - \pi_{r,1}(y^*_1) \\
= \frac{\pi_{s,1}(y^*_1) - \pi_{r,1}(y^*_1)/(1-\beta)}{1-\beta}.
\]

The second equality is due to (11) and (12). Therefore, if \( \pi_{s,1}(y^*_1) \leq \frac{\pi_{r,1}(y^*_1)}{1-\beta} \) then \( \frac{\partial \pi_{r,1}(y^*_1)}{\partial \beta} \leq 0 \). This completes the proof.

**Proof of Proposition 3:** (i) since \( \frac{\partial^2 \pi_{s,1}(y)}{\partial y \partial c} = -1 < 0 \), \( \pi_{s,1} \) is submodular in \( y \) and \( c \), or supermodular in \( y \) and \( \bar{c} = -c \). According to Lemma 2.8.1 of Topkis (1998), \( y^*_1 = \text{argmax}_y \pi_{s,1}(y) \) increases with \( \bar{c} \), i.e., decreases with \( c \).

Meanwhile, because \( \frac{\partial^2 \pi_{s,1}(y^*_1)}{\partial y \partial p} = \bar{F}(y^*_1) - (1-\beta)y^*_if(y^*_1) = c/p > 0 \), \( \pi_{s,1} \) is supermodular in \( y \) and \( p \). Therefore, Lemma 2.8.1 of Topkis (1998) implies \( \frac{\partial y^*_1}{\partial p} \geq 0 \).

(ii) Because \( \pi_c(y^*_1) = pS(y^*_i) - cy^*_i \), \( \frac{\partial \pi_c(y^*_1)}{\partial c} = (p\bar{F}(y^*_1) - c) \frac{\partial y^*_1}{\partial c} = -y^*_i \). Since Proposition 1 (ii) says \( y^*_1 < y^*_i \), \( p\bar{F}(y^*_1) \geq c \). This together with \( \frac{\partial y^*_1}{\partial c} \leq 0 \) from (i) shows that \( \frac{\partial \pi_c(y^*_1)}{\partial c} \leq (p\bar{F}(y^*_1) - c) \frac{\partial y^*_1}{\partial c} \leq 0 \). Meanwhile, \( \frac{\partial \pi_c(y^*_1)}{\partial p} = (p\bar{F}(y^*_1) - c) \frac{\partial y^*_1}{\partial p} + S(y^*_1) \geq (p\bar{F}(y^*_1) - c) \frac{\partial y^*_1}{\partial p} \geq 0 \).

(iii) Differentiating (11) and (12) in \( c \) and \( p \), respectively, we get from (ii) of Proposition 1 that

\[
\frac{\partial \pi_{r,1}(y^*_1)}{\partial c} = (1-\beta)py^*_if(y^*_i) \frac{\partial y^*_1}{\partial c} \leq 0, \\
\frac{\partial \pi_{s,1}(y^*_1)}{\partial c} = y^*_i \frac{\partial y^*_1}{\partial c} - y^*_1 = -y^*_1 \leq 0, \\
\frac{\partial \pi_{r,1}(y^*_1)}{\partial p} = (1-\beta)p \int_0^{y^*_i} x f(x)dx + (1-\beta)py^*_if(y^*_i) \frac{\partial y^*_1}{\partial p} \geq 0, 
\]
\[
\frac{\partial \pi_{s,1}(y^*_1)}{\partial p} = \pi'_{s,1}(y^*_1) \frac{\partial y^*_1}{\partial p} + \beta S(y^*_1) + (1 - \beta)y^*_1 \bar{F}(y^*_1)
\]
\[
= \beta S(y^*_1) + (1 - \beta)y^*_1 \bar{F}(y^*_1) \geq 0.
\]

This completes the proof.

**Proof of Proposition 4:** (i) From (14) we have \(\pi_{r,1}(y^*_1) + T \geq \pi_{r,1}^0\) and \(\pi_{s,1}(y^*_1) - T = \pi_c(y^*_1) - \pi_{r,1}(y^*_1) - T \geq \pi_{s,1}^0\).

(ii) First, \(\frac{\partial \pi_{r,1}(y^*_1)}{\partial \alpha} = \frac{\partial \pi_{s,1}(y^*_1)}{\partial \alpha} = \frac{\partial \pi_c(y^*_1)}{\partial \alpha} = 0\) from Proposition 2. Thus, \(\pi_{r,1}^0 - \pi_{r,1}(y^*_1)\) and \(\pi_c(y^*_1) - \pi_{r,1}(y^*_1) - \pi_{s,1}^0\) are independent of \(\alpha\). This completes the proof.

**Proof of Proposition 5:** (i) Since \(\frac{d \pi_{r,2}(y)}{dy} = p \bar{F}(y) - \frac{(1 - \alpha)c f(y) S(y)}{F^2(y)} - c\) and \(\frac{d^2 \pi_{r,2}(y)}{d y^2} = -p f(y) - (1 - \alpha)f'\left(\frac{f(y) S(y)}{F^2(y)}\right)'\). From Lemma 1 in Cachon (2004), \(\frac{f(y) S(y)}{F^2(y)}\)' \(\geq 0\) under the IGFR. Thus, \(\pi_{r,2}(y)\) is concave in \(y\), and so \(y^*_2\) is the unique solution satisfying (22). Consequently, the equilibrium wholesale price is \(w^*_2 = w_2(y^*_2)\).

(ii) It is obvious that \(\frac{\partial y^*_2}{\partial \beta} = 0\) from (22). Note that because \(\frac{\partial^2 \pi_{r,2}(y)}{\partial y \partial \alpha} = \frac{c f(y) S(y)}{F^2(y)} \geq 0\), we conclude from properties of supermodular functions that \(\frac{\partial y^*_2}{\partial \alpha} \geq 0\). By comparing (23) with (22), \(y^*_2 = y_2^*\) when \(\alpha = 0\). Then, we have \(y^*_2 \geq y_2^*\). From (22), \(y^*_2 = y^*\) when \(\alpha = 1\). This together with \(\frac{\partial y^*_2}{\partial \alpha} \geq 0\) results in \(y^*_2 < y^*\).

(iii) Differentiating \(w^*_2 = w_2(y^*_2)\) in \(\beta\), due to \(y^*_2 < y^*\) from part (i), \(\frac{\partial w^*_2}{\partial \beta} = \frac{(1 - \alpha) p F(y^*_2)^2 - c}{(1 - \alpha - \beta)^2 F(y^*_2)^2} \leq 0\). So, \(w^*_2\) decreases with \(\beta\).

Next, since \(y^*_2 < y^*\) from part (i), \(w^*_2 = \frac{(1 - \alpha) c f(y^*_2) S(y^*_2)}{F^2(y^*_2)} \leq \frac{(1 - \alpha) p F(y^*_2) - \beta p F(y^*_2)}{F(y^*_2)^2 (1 - \alpha - \beta)} = p\).

This completes the proof.

**Proof of Proposition 6:** (i) Since \(y^*_2\) is irrelevant to \(\beta\), the result is apparent from (27), (28) and (29).

(ii) First, we have marginal profits as follows due to (22):

\[
\pi'_{s,2}(y^*_2) = \frac{(1 - \alpha) c f(y^*_2) S(y^*_2)}{F^2(y^*_2)} = p \bar{F}(y^*_2) - c,
\]
\[
\pi'_{r,2}(y^*_2) = p \bar{F}(y^*_2) - \frac{(1 - \alpha) c f(y^*_2) S(y^*_2)}{F^2(y^*_2)} - c = 0.
\]

From (22),
\[
\frac{\partial y^*_2}{\partial \alpha} = \frac{c f(y^*_2) S(y^*_2)}{-(\pi_{r,2}(y^*_2))^\alpha F^2(y^*_2)} \geq 0,
\]
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Proof of Proposition 7:

\[
\frac{\partial \pi_{r,2}(y^*_2)}{\partial \alpha} = \pi_{r,2}'(y^*_2) \frac{\partial y^*_2}{\partial \alpha} + \frac{c \int_0^{y^*_2} xf(x)dx}{F(y^*_2)} \geq \frac{c \int_0^{y^*_2} xf(x)dx}{F(y^*_2)} \geq 0,
\]

\[
\frac{\partial \pi_c(y^*_2)}{\partial \alpha} = (pF(y^*_2) - c) \frac{\partial y^*_2}{\partial \alpha} \geq 0,
\]

\[
\frac{\partial \pi_c(y^*_2)}{\partial \alpha} - \frac{\partial \pi_{r,2}(y^*_2)}{\partial \alpha} = \frac{\partial \pi_{s,2}(y^*_2)}{\partial \alpha} \geq 0.
\]

Moreover,

\[
\frac{\partial \pi_{s,2}(y^*_2)}{\partial \alpha} = \pi_{s,2}'(y^*_2) \frac{\partial y^*_2}{\partial \alpha} - \frac{c \int_0^{y^*_2} xf(x)dx}{F(y^*_2)}
\]

\[
= \pi_{s,2}'(y^*_2) \frac{cf(y^*_2)S(y^*_2)}{-(\pi_{r,2}(y^*_2))^2} - \frac{c \int_0^{y^*_2} xf(x)dx}{F(y^*_2)}
\]

\[
\leq \pi_{s,2}'(y^*_2) \frac{cS(y^*_2)}{pF(y^*_2)} - \frac{c \int_0^{y^*_2} xf(x)dx}{F(y^*_2)}
\]

\[
= \frac{c(pF(y^*_2) - c)S(y^*_2)}{pF(y^*_2)} - \frac{c \int_0^{y^*_2} xf(x)dx}{F(y^*_2)}
\]

\[
\leq \frac{(pF(y^*_2) - c)S(y^*_2)}{F(y^*_2)} - \frac{c \int_0^{y^*_2} xf(x)dx}{F(y^*_2)}
\]

\[
= \pi_{r,2}(y^*_2) - \frac{\alpha \pi_{s,2}(y^*_2)}{1 - \alpha} - \frac{\pi_{s,2}(y^*_2)}{1 - \alpha} = \pi_{r,2}(y^*_2) - \frac{(1 + \alpha) \pi_{s,2}(y^*_2)}{1 - \alpha},
\]

where the first inequality comes from differentiating \((.2)\) in \(y\) and applying \([f(y^*_2)S(y^*_2)]' \geq 0\) under the IGFR, the third equality from \((.1)\), the fourth equality from \((.2)\) in part (i) of Proposition 5, and the fifth equality from \((27)\) and \((28)\). Hence, \(\frac{\partial \pi_{s,2}(y^*_2)}{\partial \alpha} \leq 0\) when \(\pi_{r,2}(y^*_2) \leq \frac{(1+\alpha)\pi_{s,2}(y^*_2)}{1-\alpha}\). This completes the proof.

Proof of Proposition 7: (i) Since \(\frac{\partial^2 \pi_{r,2}(y)}{\partial y \partial c} = -1 - \frac{(1-\alpha)f(y)S(y)}{F^2(y)} < 0\) and \(\frac{\partial^2 \pi_{r,2}(y)}{\partial y \partial p} = \bar{F}(y) \geq 0\), \(\frac{\partial y^*_2}{\partial c} \leq 0\) and \(\frac{\partial y^*_2}{\partial p} \geq 0\) can be proved similar to the proof of Proposition 3 (i).

(ii) It can be proved similar to the proof of Proposition 3 (ii).

(iii) Differentiating \((27)\) and \((28)\) in \(c\) and \(p\), respectively, we get from Proposition 5 (ii) that

\[
\frac{\partial \pi_{r,2}(y^*_2)}{\partial c} = \pi_{r,2}'(y^*_2) \frac{\partial y^*_2}{\partial c} - \frac{S(y^*_2)}{F(y^*_2)} + \frac{\alpha \int_0^{y^*_2} xf(x)dx}{F(y^*_2)}
\]

\[
= \frac{S(y^*_2)}{F(y^*_2)} + \frac{\alpha \int_0^{y^*_2} xf(x)dx}{F(y^*_2)} \leq 0,
\]

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\[
\frac{\partial \pi_{s,2}(y_2^*)}{\partial c} = \pi_{s,2}'(y_2^*) \frac{\partial y_2^*}{\partial c} - \frac{\bar{F}(y_2^*) \int_0^{y_2^*} x f(x)dx}{S(y_2^*)} \leq \pi_{s,2}'(y_2^*) \frac{\partial y_2^*}{\partial c} \leq 0,
\]
\[
\frac{\partial \pi_{r,2}(y_2^*)}{\partial p} = \pi_{r,2}'(y_2^*) \frac{\partial y_2^*}{\partial p} + S(y_2^*) = S(y_2^*) \geq 0,
\]
\[
\frac{\partial \pi_{s,2}(y_2^*)}{\partial p} = \pi_{s,2}'(y_2^*) \frac{\partial y_2^*}{\partial p} = (p \bar{F}(y_2^*) - c) \frac{\partial y_2^*}{\partial p} \geq 0.
\]

This completes the proof.

**Proof of Theorem 1:** Due to (4),
\[
\pi_{r,2}'(y_1^*) = p \bar{F}(y_1^*) - c - \frac{(1 - \alpha) cf(y_1^*) S(y_1^*)}{F^2(y_1^*)} = (1 - \beta) py_1^* f(y_1^*) - \frac{(1 - \alpha) cf(y_1^*) S(y_1^*)}{F^2(y_1^*)} = \frac{f(y_1^*)}{F^2(y_1^*)} [(1 - \beta) py_1^* F^2(y_1^*) - (1 - \alpha) c S(y_1^*)].
\]

Thus, \(\pi_{r,2}'(y_1^*) > 0\) if and only if \(\alpha\) satisfies the given condition. Also we know that \(\pi_{r,2}(y)\) is concave in \(y\) and \(\pi_{r,2}'(y_2^*) = 0\). Therefore, if and only if \(\alpha\) satisfies the given condition, \(y_1^* < y_2^*\) and thus \(\pi_{c}(y_1^*) < \pi_{c}(y_2^*)\).

Note that as implied from Lemma 1 of Cachon (2004), \(\frac{f(y) S(y)}{F^2(y)}\) increases with \(y\). So, it is easy to obtain that \(p \bar{F}(y) - c - \frac{f(y) S(y)}{F^2(y)}\) decreases with \(y\). Also, under \(y_1^0 \leq y_2^0\) from Theorem 3 of Cachon (2004),
\[
p \bar{F}(y_1^0) - c - \frac{c f(y_1^0) S(y_1^0)}{F^2(y_1^0)} \geq p \bar{F}(y_2^0) - c - \frac{c f(y_2^0) S(y_2^0)}{F^2(y_2^0)} = 0,
\]
where the equality is from the definition of \(y_2^0\) in (23). Then,
\[
\frac{py_1^* F^2(y_1^0)}{c S(y_1^0)} \geq \frac{py_1^* f(y_1^0)}{p \bar{F}(y_1^0) - c} = 1,
\]
where the equality holds because under \(\beta = 0\), (4) and (7) imply that \(p \bar{F}(y_1^0) - c - py_1^0 f(y_1^0) = 0\). Therefore, when \(\beta = 0\)
\[
1 - \frac{(1 - \beta) py_1^* F^2(y_1^0)}{c S(y_1^0)} = 1 - \frac{py_1^* F^2(y_1^0)}{c S(y_1^0)} \geq 1 - \frac{py_1^* f(y_1^0)}{p \bar{F}(y_1^0) - c} = 0.
\]
This completes the proof.