Securing Distributed Machine Learning in High Dimensions

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• An attractive solution to large-scale problems
  ▶ Algorithms: [Boyd et al. 11], [Jordan, Lee and Yang 16], etc.
  ▶ Systems: [Map-Reduce, Dean and Ghemawat 08], etc.
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• The necessity of robustness:
Distributed Machine Learning: Robustness

• An attractive solution to large-scale problems
  ▶ Algorithms: [Boyd et al. 11], [Jordan, Lee and Yang 16], etc.
  ▶ Systems: [Map-Reduce, Dean and Ghemawat 08], etc.

• The necessity of robustness: Corrupted data
  ▶ Statistical noise: [Candes et al, JACM 11] [Loh and Wainwright, NIPS 11]
  ▶ Adversarial corruption: No structural assumptions [Chen, Caramanis and Mannor, ICML 13] [Diakonikolas et al., FOCS 16] [Charikar et al., STOC 17]
Our Goal

- **Implicit assumption of previous work:** Reliable learning system
  - Each computing device follows some designed specification

- **Our focus:** Unreliable learning system
  - Adversarial attacks: Some unknown subset of computing devices are compromised, and behave adversarially – such as sending out malicious messages
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- **Implicit assumption of previous work:** Reliable learning system
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- **Our focus:** Unreliable learning system
  - Adversarial attacks: Some unknown subset of computing devices are compromised, and behave adversarially – such as sending out malicious messages

**Goal:** Secure model training in unreliable learning system
Why consider unreliable learning system?
• Data is collected from providers and stored at clouds
Privacy Risk in Conventional Learning Paradigm

- Data is collected from providers and stored at clouds
- Serious privacy risks:
  - Facebook data scandal
  - PRISM: Facebook, Google, Yahoo!, Apple, Microsoft, Dropbox, etc.
New Learning Paradigm: Federated Learning

Key idea: Leave training data on mobile devices

- Learning with external workers (data providers)
- Proposed by Google researcher [McMahan 16]
- Tested by Gboard on Android and Google Keyboard
Security Risk in Federated Learning

- Less secured implementation environment
- External workers are prone to adversarial attack – reprogrammed by system hackers and behave maliciously

Cloud System

Leave training data on mobile devices
Goal: Secure model training in *unreliable* learning system
Challenges of Securing Unreliable Learning Systems

- Low local data volume versus high model complexity
  - Local estimator is statistically inaccurate
  - Hard to distinguish statistical errors from adversarial errors
  - Call for close interaction between the learner (cloud) and the workers

- Communication constraints: Data transmission suffers high latency and low throughput

Objectives

- Tolerate adversarial failures of the external workers
- Accurately learn highly complex models with low local data volume
- Use only a few communication rounds
Outline of the Remainder

1. Problem formulation

2. Algorithm 1: Geometric median of means

3. Algorithm 2 (Optimal Algorithm):
   Iterative rewriting + projecting + filtering

4. Summary and concluding remarks
Problem Formulation: Learning Model

- $N$ i.i.d. data points $X_i \sim \mu$

- Collectively kept by $m$ workers – each worker keeps $\frac{N}{m}$ data points

- The learner wants to pick a model in $\Theta \subseteq \mathbb{R}^d$

- loss function $f(x, \theta)$: loss induced by $x \in \mathcal{X}$ under the model choice $\theta \in \Theta$

Target: $\theta^* \in \arg \min_{\theta \in \Theta} F(\theta) \triangleq \mathbb{E}[f(X, \theta)]$

NOTE: the population risk $F(\theta)$ is unknown
Example: Linear Regression

- $N$ i.i.d. data points $X_i = (w_i, y_i)^{\text{i.i.d.}} \sim \mu$
  - $w_i$ can be the features of a house/apartment, and $y_i$ is its sold price
- $\Theta \subseteq \mathbb{R}^d$: the set of possible linear predictors
- Risk function $f(x, \theta) = \frac{1}{2} (y - \langle w, \theta \rangle)^2$

**Target:** $\theta^* \in \arg \min_{\theta \in \Theta} \mathbb{E} \left[ \frac{1}{2} (y - \langle w, \theta \rangle)^2 \right]$
Problem Formulation: Byzantine Fault Model

- In any iteration, up to $q$ out of $m$ workers are compromised and behave arbitrarily;
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- the set of faulty workers may be different across iterations;

![Diagram of parameter server and clients with some clients compromised.]
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Problem Formulation: Byzantine Fault Model

- In any iteration, up to $q$ out of $m$ workers are compromised and behave arbitrarily;
- the set of faulty workers may be different across iterations;
- faulty workers have complete knowledge of the system;
- faulty workers can collude
Algorithm: Byzantine Gradient Descent

The learner:

1. Broadcast the current model parameter estimator $\theta_{t-1}$;
2. Wait to receive all the gradients $g^{(j)}_t$ from all workers $j$;
3. Aggregate gradients to obtain $\hat{F}(\theta_{t-1})$;
4. Update: $\theta_t \leftarrow \theta_{t-1} - \eta_t \times \hat{F}(\theta_{t-1})$;

Non-faulty worker $j$:

1. Compute the sample gradient $g^{(j)}_t = \sum_{\text{local data}} X_i \nabla f(X_i, \theta_{t-1})$;
2. Send $g^{(j)}_t$ back to the learner;
Algorithm: Byzantine Gradient Descent

The learner:

1. Broadcast the current model parameter estimator $\theta_{t-1}$;
2. Wait to receive all the gradients $g_t^{(j)}$ from all workers $j$;
3. Aggregate gradients to obtain $\hat{F}(\theta_{t-1})$;
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Non-faulty worker $j$:

1. Compute the sample gradient $g_t^{(j)} = \sum_{\text{local data}} X_i \nabla f(X_i, \theta_{t-1})$;
2. Send $g_t^{(j)}$ back to the learner;

Averaging, i.e., taking $\hat{F}(\theta_{t-1}) = \frac{1}{m} \sum_{j=1}^{m} g_t^{(j)}$, is not robust to even a single Byzantine failure!
Algorithm: Byzantine Gradient Descent

The learner:

1. Broadcast the current model parameter estimator $\theta_{t-1}$;
2. Wait to receive all the gradients $g^{(j)}_t$ from all workers $j$;
3. **Robust gradient aggregate** to obtain $\hat{F} (\theta_{t-1})$;
4. Update: $\theta_t \leftarrow \theta_{t-1} - \eta_t \times \hat{F}(\theta_{t-1})$;

Non-faulty worker $j$:

1. Compute the sample gradient $g^{(j)}_t = \sum_{\text{local data}} X_i \nabla f(X_i, \theta_{t-1})$;
2. Send $g^{(j)}_t$ back to the learner;

Simple averaging, i.e., taking $\hat{F}(\theta_{t-1}) = \frac{1}{m} \sum_{j=1}^{m} g^{(j)}_t$, is not robust to even a single Byzantine failure!
### Generic Key Technical Challenges

**Target:** \( \theta^* \in \arg \min_{\theta \in \Theta} F(\theta) \triangleq \mathbb{E}[f(X, \theta)] \)

- **Suppose \( F(\theta) \) is known:** Perfect gradient descent –
  \[
  \theta_t = \theta_{t-1} - \eta \times \nabla F(\theta_{t-1})
  \]

- **But \( F(\theta) \) is unknown:** Approximate gradient descent –
  \[
  \theta'_t = \theta'_{t-1} - \eta_t \times \nabla \hat{F}(\theta'_{t-1}) = \theta'_{t-1} - \eta_t \times \nabla F(\theta'_{t-1}) + \epsilon(\theta'_{t-1}).
  \]
  - The elements in \( \{\epsilon(\theta'_{t-1})\}_{t=1}^{\infty} \) are dependent on each other;
  - Complicated interplay between the randomness and the arbitrary behaviors of Byzantine workers.
Target: $\theta^* \in \arg\min_{\theta \in \Theta} F(\theta) \triangleq \mathbb{E}[f(X, \theta)]$

- **Suppose $F(\theta)$ is known**: Perfect gradient descent –
  $$\theta_t = \theta_{t-1} - \eta \times \nabla F(\theta_{t-1})$$

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  - The elements in $\{\epsilon(\theta'_{t-1})\}_{t=1}^{\infty}$ are dependent on each other;
  - Complicated interplay between the randomness and the arbitrary behaviors of Byzantine workers.

**Our analysis plan**: show uniform convergence,
  
i.e., show $\epsilon(\theta) \approx 0$ uniformly for all $\theta \in \Theta$

Standard concentration results might not suffice
Algorithm I: Median of Means
Median of Means

Given $nk$ points $X_1, \ldots, X_{nk}$,

$$\hat{\phi}_{MM} \triangleq \text{median} \left\{ \frac{1}{n} \sum_{i=1}^{n} X_i, \ldots, \frac{1}{n} \sum_{i=(k-1)n+1}^{kn} X_i \right\}$$

Definition (Geometric median)

$$y^* \triangleq \text{med} \{ y_1, \ldots, y_m \} = \arg \min_{y \in \mathbb{R}^d} \sum_{i=1}^{m} \| y - y_i \|_2$$

Efficient computation of Geometric Median: Nearly linear time

[Cohen et al. STOC 2016]
Robustness of Geometric Median

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- **One-dimension case**: Geometric median = standard median
  If strictly more than \( \lfloor n/2 \rfloor \) points are in \([-r, r]\) for some \( r \in \mathbb{R} \), then median **ALSO** lies in \([-r, r]\)
Robustness of Geometric Median

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- **Multi-dimension case:**

**Lemma (Minsker et al. 2015)**

For any \( \alpha \in (0, 1/2) \) and given \( r \in \mathbb{R} \), if \( \sum_{i=1}^{n} \mathbf{1}_{\{\|y_i\|_2 \leq r\}} \geq (1 - \alpha) n \), then \( \| y^* \|_2 \leq C_\alpha r \), where \( C_\alpha = \frac{1 - \alpha}{\sqrt{1 - 2\alpha}} \).
Robustness of Geometric Median

**Definition (Geometric median)**

\[
y^* \triangleq \text{med}\{y_1, \cdots, y_m\} = \arg\min_{y \in \mathbb{R}^d} \sum_{i=1}^{m} \|y - y_i\|_2
\]

- **One-dimension case:** Geometric median = standard median
  If strictly more than \(\lfloor n/2 \rfloor\) points are in \([-r, r]\) for some \(r \in \mathbb{R}\), then median **also** lies in \([-r, r]\)

- **Multi-dimension case:**

**Lemma (Minsker et al. 2015)**

For any \(\alpha \in (0, 1/2)\) and given \(r \in \mathbb{R}\), if \(\sum_{i=1}^{n} 1\{\|y_i\|_2 \leq r\} \geq (1 - \alpha)n\), then \(\|y^*\|_2 \leq C_\alpha r\), where \(C_\alpha = \frac{1-\alpha}{\sqrt{1-2\alpha}}\).

Intuition: Majority voting in the noisy setting
Performance with Median of Means

(1) $q \geq 1$: the maximum $\#$ of Byzantine workers;
(2) $d$: model dimension, i.e., $\Theta \subseteq \mathbb{R}^d$

Theorem (Informal)

Suppose some mild technical assumptions hold, and $2(1 + \epsilon)q \leq k \leq m$. Assume $F(\theta)$ is $M$-strongly convex with $L$-Lipschitz gradient. Then whp

$$\|\theta_t - \theta^*\| \leq \rho^t \|\theta_0 - \theta^*\| + C\sqrt{\frac{dk}{N}}, \quad \forall t \geq 1,$$

where $\rho = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{M^2}{4L^2}} \in (0, 1)$.

- After $\log N$ rounds, $\sqrt{dq/N}$ becomes the dominant part
- When $q = 0$, we choose $k = 1$
- When $q$ is large, we choose $k = 2(1 + \epsilon)q$, resulting error of $O(\sqrt{dq/N})$
Drawbacks of Geometric Median in High Dimensions

$$y^* = \arg\min \sum_{i=1}^{m} \|y - y_i\| \iff \sum_{i=1}^{m} \frac{y_i - y^*}{\|y_i - y^*\|} = 0$$

- Good data $y_i \sim \mathcal{N}(\mu, I_d)$
- $\epsilon$ fraction is adversarially corrupted
- GM suffers from $\epsilon \sqrt{d}$ error
Drawbacks of Geometric Median in High Dimensions

\[ y^* = \arg \min_{y} \sum_{i=1}^{m} \|y - y_i\| \iff \sum_{i=1}^{m} \frac{y_i - y^*}{\|y_i - y^*\|} = 0 \]

- Good data \( y_i \overset{i.i.d.}{\sim} \mathcal{N}(\mu, \mathbf{I}_d) \)
- \( \epsilon \) fraction is adversarially corrupted
- GM suffers from \( \epsilon \sqrt{d} \) error
Drawbacks of Geometric Median in High Dimensions

\[ y^* = \arg \min \sum_{i=1}^{m} \| y - y_i \| \iff \sum_{i=1}^{m} \frac{y_i - y^*}{\| y_i - y^* \|} = 0 \]

- Good data \( y_i \sim \mathcal{N}(\mu, I_d) \)
- \( \epsilon \) fraction is adversarially corrupted
- GM suffers from \( \epsilon \sqrt{d} \) error
Algorithm II: Optimal Algorithm in High Dimension

[Su and Xu, 2018] improves the estimation error from $O\left(\sqrt{\frac{qd}{N}}\right)$ to $O\left(\sqrt{\frac{d}{N}} + \sqrt{\frac{q}{N}}\right)$ – matching the minimax error rate in the ideal failure-free setting as long as $q = O(d)$. 
If the center $\mu$ were known, from

$$uu^\top \in \arg\max \sum_i (y_i - \mu)^\top U (y_i - \mu)$$

s.t. $U \succeq 0$

$$\text{Tr}(U) \leq 1,$$

filter out outliers based on $\langle y_i - \mu, u \rangle^2$

However, $\mu$ is unknown!

Idea: represent $y_i$ through $\sum_j W_{ji} y_j$; $W_{ji}$ is constrained to be around $\frac{1}{(1-\epsilon)m}$
Define cost function

\[ \phi(W, U) = \sum_{i \in S} c_i \left( y_i - \sum_{j \in S} W_{ji} y_j \right)^\top U \left( y_i - \sum_{j \in A} W_{ji} y_j \right) \]

1. Compute saddle point

   (Center approxi.) \( W^* \in \arg \min_W \max_U \phi(W, U) \)
   
   (Extreme direction) \( U^* \in \arg \max_U \min_W \phi(W, U) \)

2. If \( \phi(W^*, U^*) \) is small enough, stop; otherwise, down-weight \( c_i \) proportional to \( \left( y_i - \sum_{j \in S} W_{ji}^* y_j \right)^\top U^* \left( y_i - \sum_{j \in S} W_{ji}^* y_j \right) \), throw away data points for which \( c_i \leq 1/2 \), and repeat.
Lemma (SCV ’18)

Define \( \mu_S = \frac{1}{m} \sum_{i=1}^{m} y_i \). Suppose that

\[
\left\| \frac{1}{m} \sum_i (y_i - \mu_S)(y_i - \mu_S)^\top \right\|_2 \leq \sigma^2.
\]

Then for \( \epsilon \leq \frac{1}{4} \), Iterative Filtering Algorithm outputs \( \hat{\mu} \) such that

\[
\| \hat{\mu} - \mu_S \| = O(\sigma \sqrt{\epsilon}).
\]

- Gradient vectors \( \{g_j(\theta_{t-1})\}_{j=1}^{m} \) are not i.i.d.
- Apply with \( y_j = \) gradient functions:

\[
g_j(\theta) = \frac{1}{|S_j|} \sum_{i \in S_j} \nabla f(X_i, \theta)
\]

- Need concentration of matrix \([g_1(\theta), \ldots, g_m(\theta)]\) uniformly over \( \theta \)
Uniform Concentration of Sample Covariance Matrix

- If gradient functions $g_j(\theta)$ is sub-Gaussian, use $\epsilon$-net
- However, in many cases such as linear regression, $g_j(\theta)$ is sub-exponential
- Existing tail bounds for matrices with sub-exponential columns are not tight

State-of-the-art: Standard concentration bounds [ALPTJ '10]:
$$\sqrt{md} + d$$

**Theorem (SX '18)**

Let $A$ be a $d \times m$ matrix whose columns $A_j$ are i.i.d. sub-exponential, zero-mean. Then with probability at least $1 - e^{-d}$,

$$\|A\|_2 \lesssim \sqrt{m} + d \log^3 d$$

**Remark:** Tight up to poly-log factors
Suppose some mild technical assumptions hold and \( N \gtrsim d^2 \). Let \( \nabla \hat{F}(\theta) \) be the aggregated gradient function by Iterative Filtering Algorithm. Then with probability at least \( 1 - 2e^{-\sqrt{d}} \),

\[
\|\nabla \hat{F}(\theta) - \nabla F(\theta)\| \lesssim \left( \sqrt{\frac{q}{N}} + \sqrt{\frac{d}{N}} \right) \|\theta - \theta^*\| + \left( \sqrt{\frac{q}{N}} + \sqrt{\frac{d}{N}} \right) \cdot N \gtrsim d^2
\]

- \( N \gtrsim d^2 \) is due to our sub-exponential assumption and is inevitable
- If assuming sub-Gaussian instead, only \( N \gtrsim d \) is needed
Main Convergence Result

**Theorem (SX ’18)**

Suppose *some mild technical assumptions* hold and $N \gtrsim d^2$. Assume $F(\theta)$ is $M$-strongly convex with $L$-Lipschitz gradient. Then whp,

$$\|\theta_t - \theta^*\| \lesssim \left(1 - \frac{M^2}{16L^2}\right)^t \|\theta_0 - \theta^*\| + \left(\sqrt{\frac{q}{N}} + \sqrt{\frac{d}{N}}\right).$$

- Improves over geometric median ($\sqrt{dq/N}$)
- If $q = O(d)$, error rate is optimal
- Tolerate up to $q/m = \Theta(1)$ fraction of Byzantine errors
- Exponential convergence $\rightarrow$ only logarithmic communication rounds
• Lili Su and Jiaming Xu, *Securing Distributed Machine Learning in High Dimensions*, arXiv:1804.10140, April 2018

• Yudong Chen, Lili Su, Jiaming Xu: *Distributed Statistical Machine Learning in Adversarial Settings: Byzantine Gradient Descent*
  - Conference version: SIGMETRICS 2018;