The Supermarket Game

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Life Experiences @Supermarket

Customers walk around to search for short queues

Waiting cost & Searching cost

Q1: How many queues will customers search?
Q2: Is searching more queues beneficial to others?

The Supermarket Game

Infinite buffer size

Poisson arrival
with rate N λ
λ<1

Sample L queues

FCFS

Join the
shortest
sampled
queue

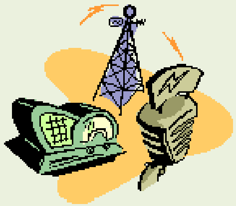
Exp.
service
rate one

Waiting cost per unit time: c
Cost of sampling a queue: c_s
Cost $= c \times E[W] + c_s \times$ (# of sampled queues)

Potential Applications

- Path selection
  - Traffic flows choose routes

- Dynamic Spectrum Access
  - Wireless users choose spectrum



- Cloud computing
  - Customized load balancer

Outline of Talk

I. Introduction (just completed)

II. Externalities for two queues (N=2)

III. Closing the loop: Nash equilibrium

IV. NE in the mean field model

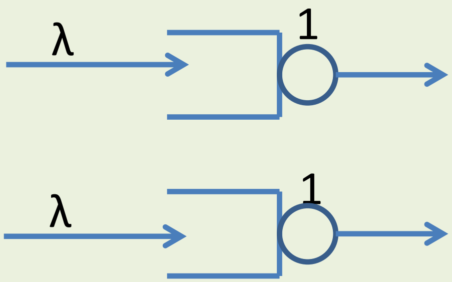
V. Externality in mean field model

VI. Summary/Comments

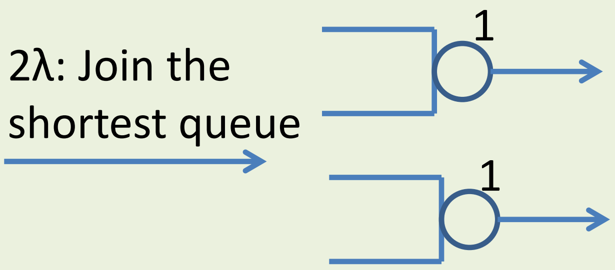
II. Externality for two queues

- Consider the effect on a reference customer, say Alice, of choices by others.

Sys1: all the others sample one queue



Sys2: all the others sample two queues

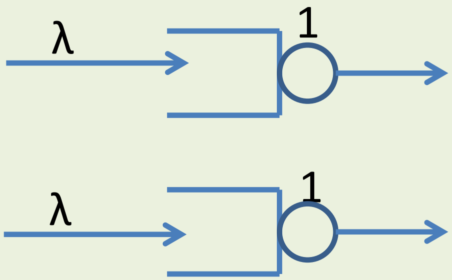


- If Alice samples one queue, which system gives her shorter mean waiting time? What if she samples two queues?

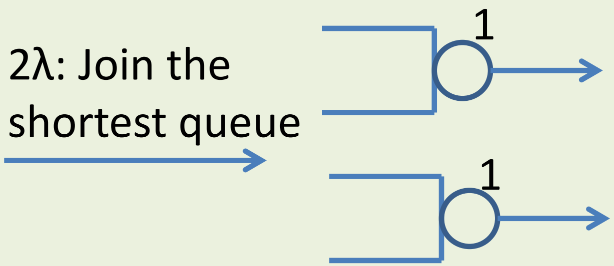
Externality on Alice is positive if she samples one queue

- The expected waiting time is proportional to the mean number of customers

Sys1: all the others sample one queue



Sys2: all the others sample two queues

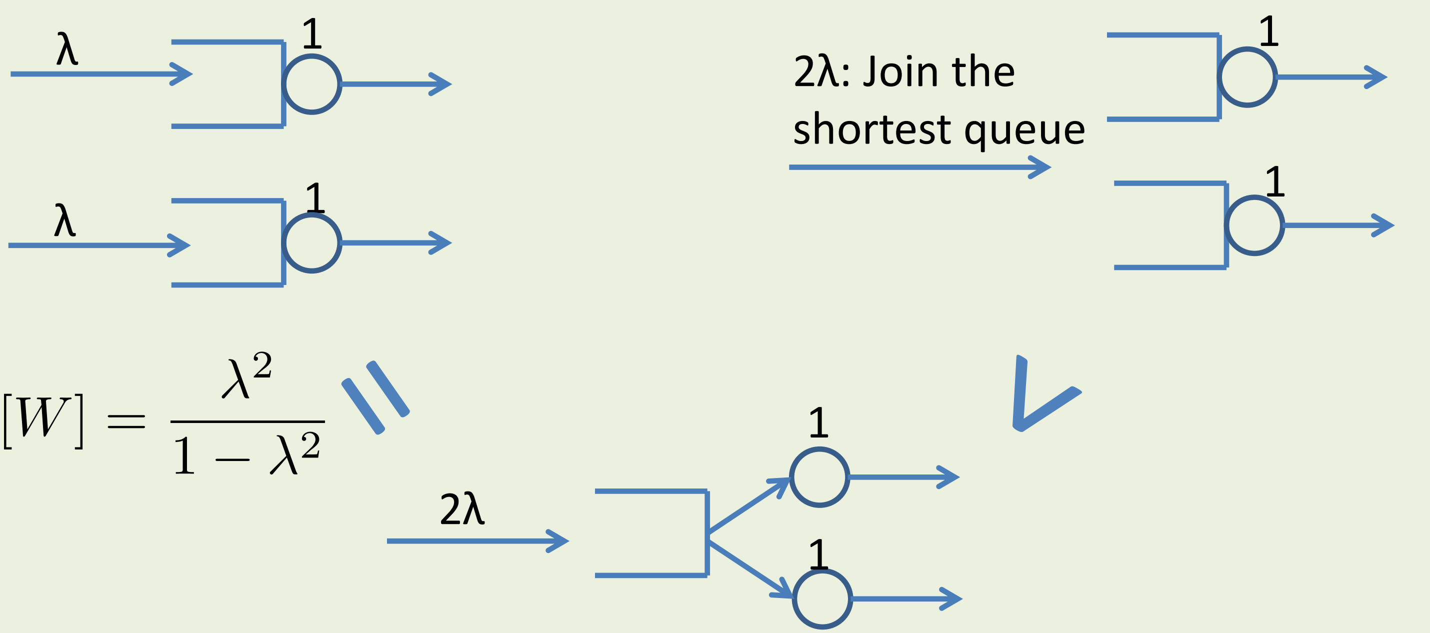


- A coupling argument (S. Turner) shows that Sys 2 has a smaller total number of customers

Externality on Alice is negative if she samples both queues

Sys1: all the others sample 1 queues

Sys2: all the others sample 2 queues



$$E[W] = \frac{\lambda^2}{1-\lambda^2}$$

Sys0:M/M/2

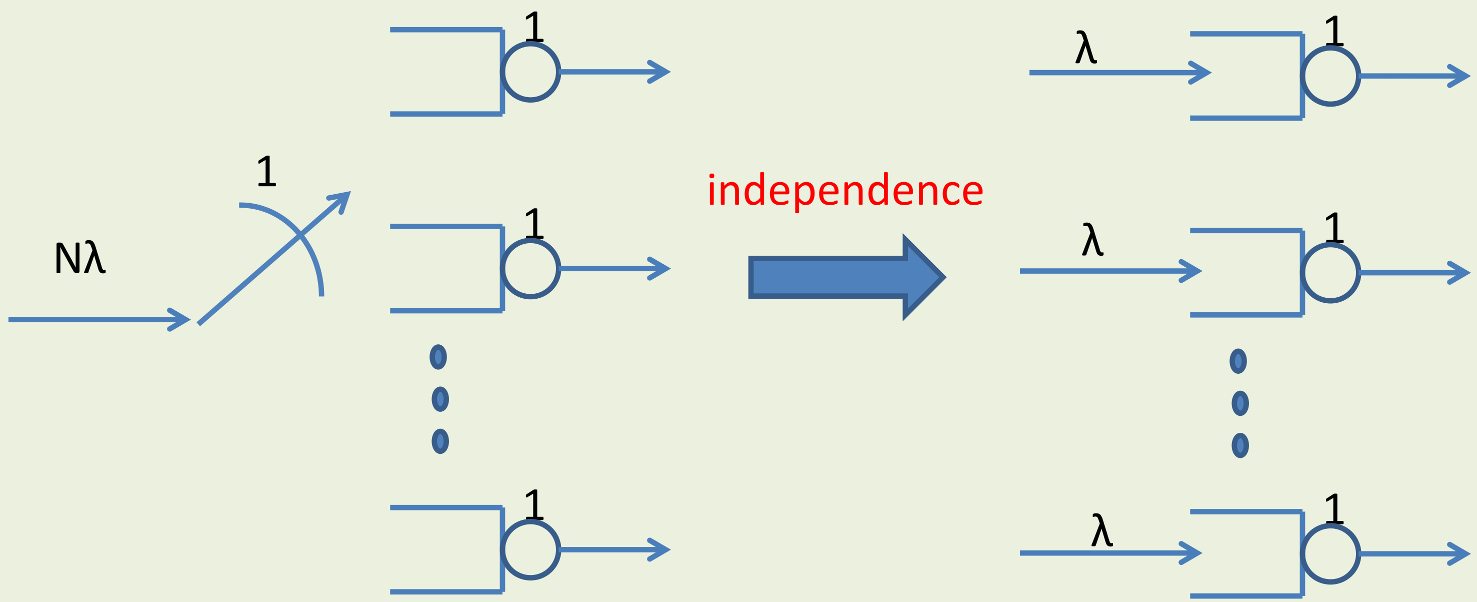
III. Closing the loop: Nash equilbrium (NE)

- Others’ decisions -> Alice’s best response

- Focus on (symmetric) Nash equilibrium
- Consider Alice in an example with two choices

Example of two choices

- All the others sample one queue



- Due to independence, the stationary queue length distribution has a simple form

Alice's Best Response

- If all the others sample one queue:
  - Alice samples one queue: $E[W] = E[Q]$
  - Alice samples two queues: $E[W] = E[\min(Q_1, Q_2)]$
  - Alice prefers to sample two queues if

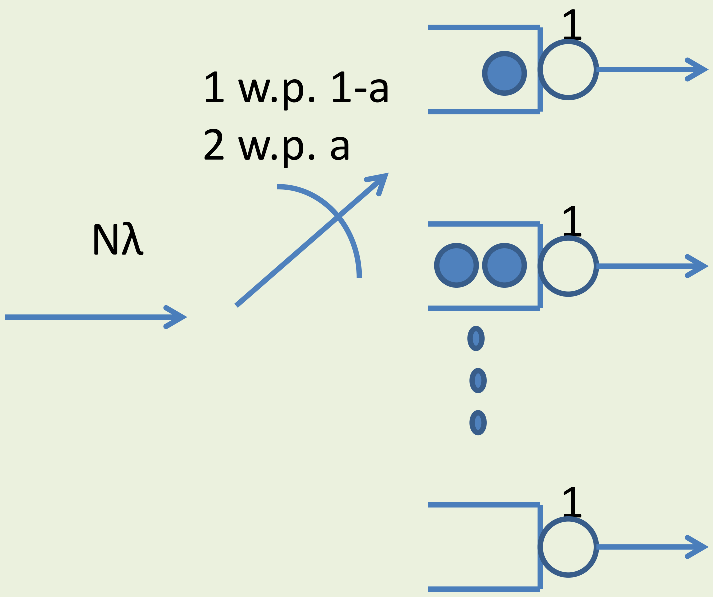
$$c(E[Q] - E[\min(Q_1, Q_2)]) > c_s$$

Reduction of waiting cost

Cost of sampling one more queue

Independence is missing if some customers sample more than one queue

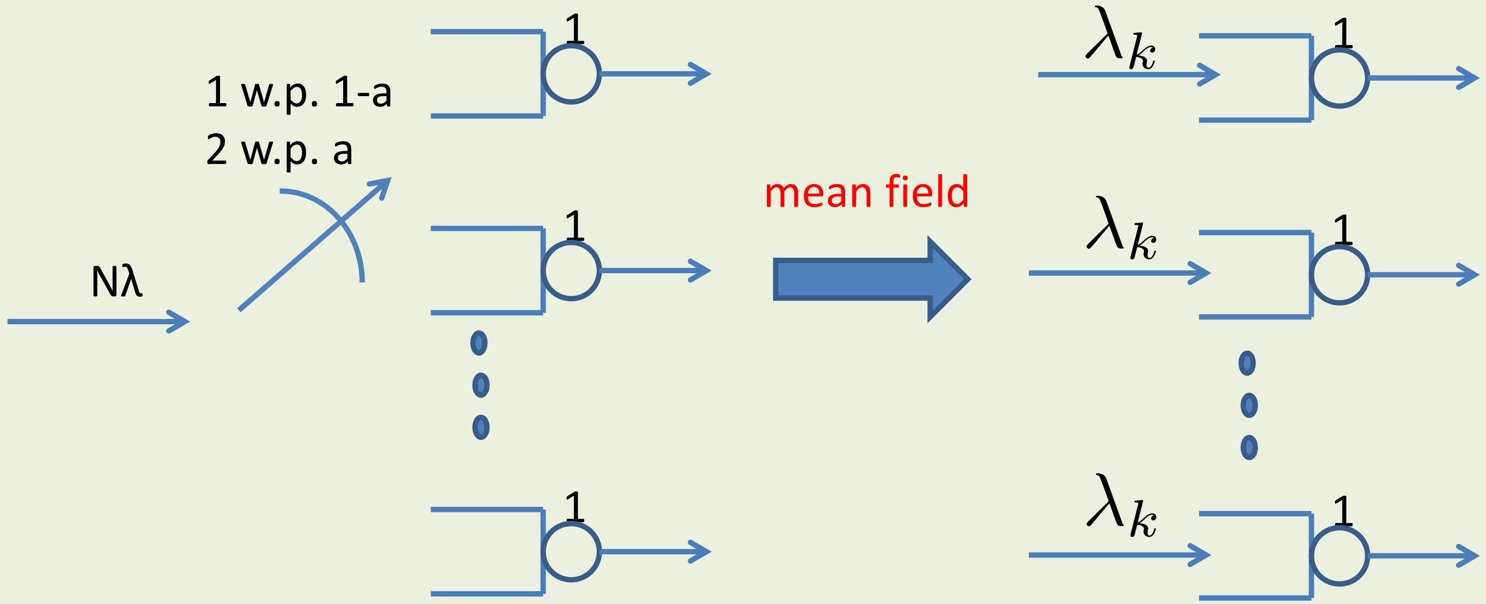
- All the others sample 1+a queues on average
  - Correlation among queues: if queue 1 and 2 sampled, then queue 1 receives arrival



$$P(Q \geq k) = ?$$

IV: Mean Field Limit under 1+a

- As $N \to \infty$, queues become independent
  - Intuition: queue 1 and queue 2 are sampled together with vanishing probability



Mean Field Equations

for random L, with distribution μ_{-i}

Let $r_t(k)$ be the tail of queue length distribution

$$\frac{dr_t(k)}{dt} = \sum_{l=1}^{L_{\max}} \lambda\mu_{-i}(l)\left(r_t^l(k-1) - r_t^l(k)\right) - \left(r_t(k) - r_t(k+1)\right),$$

Fraction of arrivals joining a queue with exactly k-1 customers

Fraction of queues with exactly k customers

Equilibrium distribution: $r_{\mu_{-i}}(1) = \lambda, \quad r_{\mu_{-i}}(k) = \lambda u_{\mu_{-i}}(r_{\mu_{-i}}(k-1)),$

where

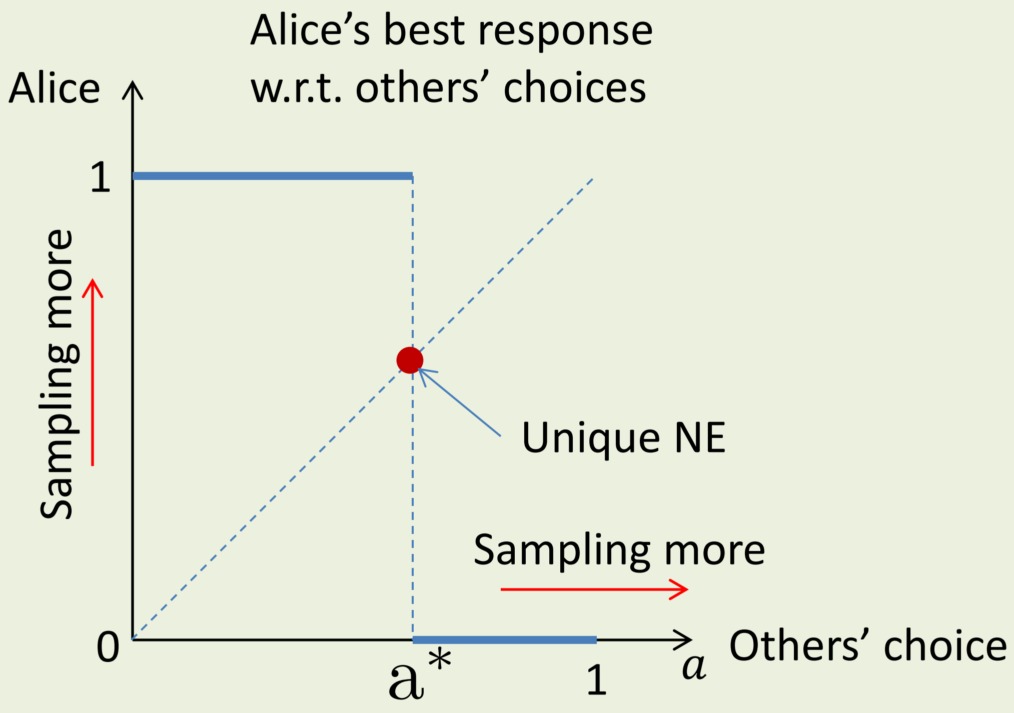
$$u_{\mu_{-i}}(x) := \mathbb{E}_{\mu_{-i}}[x^L]$$

Existence of NE in Mean Field

- If all the others sample $1 + a^*$ queues on average
  - Alice is indifferent from sampling one and two, then $1 + a^*$ is also optimal for Alice =>NE
- Existence of a^*?
  - Kakutani fixed point Theorem

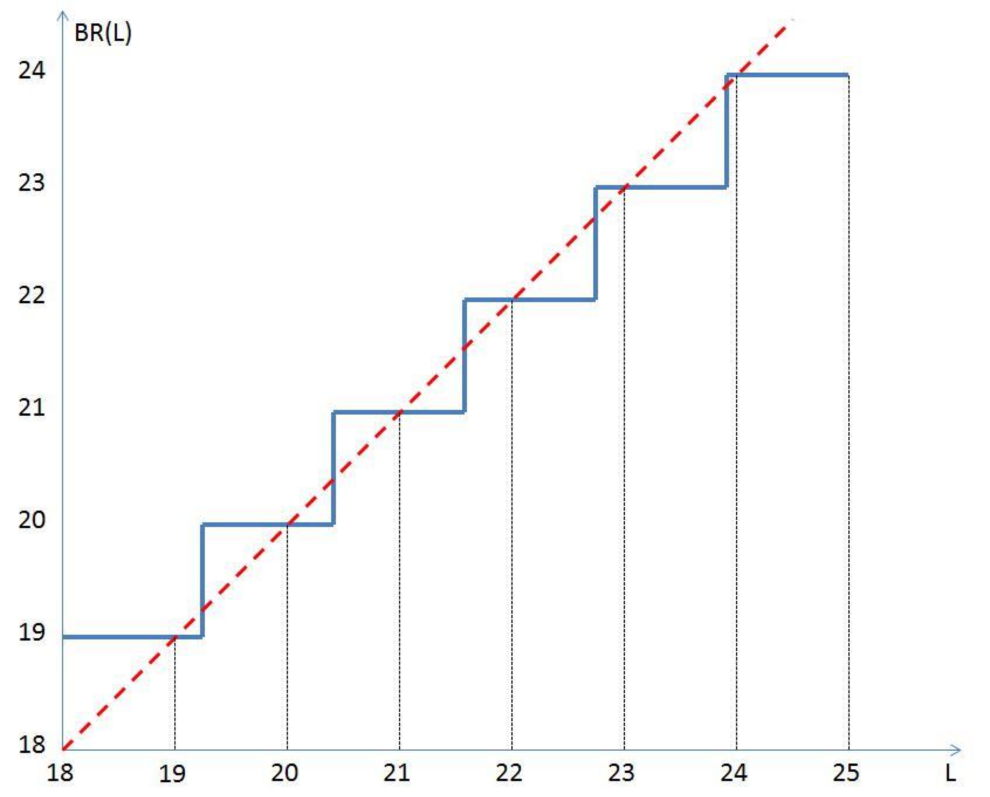
Uniqueness of NE in Mean Field

- Avoid the crowd
  - Valid for $\lambda^2 \leq \frac{1}{2}$
  - Not valid for λ near one



Multiple NE exist for some λ

For example, Alice's best response to L for $\lambda = 0.999$ and $\frac{c_s}{c} = 0.014$.



V: Externality is positive in Mean Field

- Sampling more
  - "occupy" short queues
  - balance total load
- Mean field: The second point dominates
  - load distribution is first-order stochastic decreasing

VI: Summary/Comments

- If Alice samples one queue or if N is very large, she is better off if others sample more queues.
- If Alice samples two queues and N=2, she is better off if others only sample one queue.
- Mean field model is much more tractable than finite N model.
- Often, but not always, have "avoid the crowd" behavior -- if others sample more, Alice samples less. NE unique in such cases.