Abstract

We examine the asset pricing implications of innovation and R&D in a stochastic model of endogenous growth. In equilibrium, R&D endogenously drives a small, persistent component in productivity growth, consistent with the empirical evidence. These productivity dynamics induce uncertainty about the long-term growth prospects in the economy, reflected in long-term cycles and growth waves in quantities and asset market valuations. With recursive preferences, households are very averse to such movements in growth rates and command high risk premia in asset markets that helps the model quantitatively rationalize a variety stylized facts in asset pricing. The resolution of these puzzles is inherently linked to the strong propagation mechanism exhibited by the model, which is absent in standard macroeconomic models. Extending the model to account for stochastic volatility and innovation shocks strengthens our findings. We find strong empirical support for innovation-driven low-frequency movements in aggregate growth rates and asset market valuations in the data.

Keywords: Endogenous growth, asset pricing, low-frequency cycles, R&D, innovation, business cycle propagation, recursive preferences.

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1 Introduction

Innovation and the development of new technologies have long been identified as important determinants of economic growth. At the aggregate level, such development of new technologies is reflected in the sustained growth of capital and labor productivity. Innovation, as measured by R&D expenditures, is characterized by being quite volatile, fairly persistent, and procyclical. Given that R&D is a driving force of growth, such movements should then be reflected in growth rate dynamics, and hence in asset market valuations. Indeed, US post-war data exhibit significant fluctuations in aggregate growth rates and asset market valuations at higher and lower frequencies. Put differently, US data display substantial long-term cycles or growth waves induced by the development of new technologies.

In this paper, we quantitatively examine the asset pricing implications of innovation. Specifically, we ask how the dynamics of innovation are themselves reflected in the movements of the aggregate economy and in asset prices. That is, we focus on the link between innovation, long-term growth prospects and asset prices. Our setup has two distinguishing features. First, we use a stochastic endogenous growth model in which in contrast to standard macroeconomic models, firms actively engage in R&D in order to generate sustained growth. Second, we assume that households have recursive preferences, so that they care about long-term growth prospects.

Our results suggest that accounting for the endogeneity of innovation, and therefore long-run growth, goes a long way towards establishing a macroeconomic framework that quantitatively captures the joint dynamics of quantities and asset prices in a general equilibrium setting. More specifically, the model generates a strong propagation mechanism for shocks which is absent in standard macroeconomic frameworks. In particular, this propagation mechanism induces significant movements in aggregate quantities and asset market valuations at lower frequencies, a prediction for which we find strong support in the data. With recursive preferences agents are very averse to such low-frequency dynamics in both consumption and cash flows; hence, they command high risk premia in asset markets. These innovation-driven dynamics allow the model to be consistent with a variety of stylized facts about asset markets, such as a high equity premium and a low and smooth risk free rate. Our results imply that there is a tight connection between macroeconomic risk, growth and risk premia in asset markets. Put differently, we view stochastic models of endogenous growth as useful tools for both quantitative macroeconomic modeling and general equilibrium asset pricing.

We first show that in this model innovation and R&D endogenously drive a small, but persistent component in the growth rate of aggregate productivity. More specifically, productivity will contain a high-frequency component driven by exogenous shocks, as well as an endogenous component that is linked to R&D activity in the economy. Crucially, this endogenous component operates at lower frequencies than the exogenous component; hence, productivity endogenously exhibits high- and low-frequency movements.
Interestingly, these dynamics arise in a model that is driven by a single exogenous shock to the level of technology. While this shock induces fluctuations at business cycle frequency comparable to standard macroeconomic settings, the innovation process in the model translates this disturbance into an additional, slow-moving component in productivity growth. Recently, in the finance literature, such dynamics have been referred to as long-run productivity risk (Croce (2008)). Hence, the shock generates a term structure of productivity growth. Naturally, these productivity dynamics induce uncertainty about the economy’s long-term growth prospects that will be reflected in the dynamics of aggregate quantities. In other words, the innovation process induces persistent growth waves and hence provides a strong propagation mechanism for shocks. In contrast to standard macro models, our setting endogenously generates fluctuations at various frequencies, including significant movements at a low frequencies. Such persistent movements are inherently linked to innovative activity. Indeed, as predicted by the model, we empirically show that R&D intensity has significant forecasting ability for aggregate growth rates including productivity, consumption, output and cash flows.

Such consumption and cash flow dynamics have important implications for risk premia in asset markets given our assumption of agents’ preference specification. In line with much of the recent asset pricing literature we assume that households have Epstein-Zin recursive preferences. This implies that not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth. Under the standard assumption that these preferences exhibit a preference for early resolution of uncertainty, households are strongly averse to the persistent innovations to expected growth rates implied by the propagation mechanism of the model. Hence, the model’s propagation mechanism translates into long-run risks in asset markets. Quantitatively, this is reflected in a substantial equity premium and a low and stable risk-free rate. Moreover, persistence in cash flow dynamics will also be reflected in asset valuations. In this respect, the model predicts productivity-driven low-frequency cycles in stock market values and price-dividend ratios, which is consistent with the empirical evidence.

At the heart of these dynamics of the model is the innovation process, which arises endogenously. In the model, the consumption good is produced with standard inputs such as labor or capital, and additionally, a bundle of intermediate goods. These intermediate goods are most readily thought of as patents or blueprints for machines that facilitate the production of the consumption good by making the production process more efficient. Consumption good producers purchase intermediate goods from intermediate good firms, which possess monopoly rights on their respective products. In accordance with the real business cycle literature we assume that production of the consumption good is subject to a random exogenous disturbance (a level technology shock), which in contrast, we assume to be stationary. Thus, long-run growth in aggregate output must result from growth in the number of intermediate goods (stock of R&D). In order to create new patents, firms need to engage in R&D. The reward for creating a new patent is the stream of monopoly
profits associated with supplying the intermediate good to the representative firm each period. Given that both demand for new patents and monopoly profits rise after a positive disturbance, incentives to engage in R&D rise as well. This raises the number of intermediate goods the representative firm utilizes, and given the production technology, leads to a sustained rise in the growth rate of aggregate output, which spills over into a sustained rise in the growth rate of dividends and output. Given a realistically persistent stationary specification of the level technology shock, this mechanism leads to long swings in the growth rate of productivity. Crucially, we show that that the resulting model resembles a version of the real business cycle model in which TFP has an endogenous component driven by R&D activity.

To this basic benchmark model, we provide two extensions. First, following the literature on long-run risks in asset markets, we provide a quantitative analysis of stochastic volatility in a setting with endogenous growth. While, as in consumption-based models, this allows the model to rationalize the predictability of stock returns, it also, in stark contrast to a consumption-based environment, reduces the equity premium. The intuition is simple: In a setting with endogenous growth, consumption falls after a volatility shock, yet expected consumption growth increases. With recursive preferences, both these movements are priced, with opposite signs. Quantitatively, the net effect is a decrease of the equity premium. Second, we introduce a second source of uncertainty, namely a shock to the productivity in the R&D sector. This provides a natural interpretation of a ‘news shock’ in our setting, a shock to expected productivity orthogonal to current productivity. This is in contrast to the benchmark model, where shocks to productivity and expected productivity are perfectly correlated.

Our paper is related to a number of different strands of literature in asset pricing, economic growth and macroeconomics. The economic mechanisms driving the asset pricing implications are closely related to Bansal and Yaron (2004). In a consumption-based model, Bansal and Yaron specify both consumption and dividend growth to contain a small, persistent component, which leads to long and persistent swings in the dynamics of these quantities. This specification along with the assumption of Epstein-Zin recursive utility with a preference for early resolution of uncertainty, allows them to generate high equity premia as compensation for these ‘long-run risks’. The ensuing literature on long-run risk quantitatively explains a wide range of patterns in asset markets, such as those in equity, government, corporate bond, foreign exchange and derivatives markets. While somewhat hard to detect in the data, we show that the growth rate dynamics that Bansal and Yaron specify directly follow naturally from agents’ equilibrium R&D decisions in our endogenous growth model. In other words, the equilibrium growth rate dynamics that a broad class of stochastic endogenous growth models generate are precisely what Bansal and Yaron refer to as long-run.

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risk. In short, equilibrium growth is risky.

Our paper is also very closely related to a number of recent papers seeking to understand how long-run risks arise endogenously in production economies (Croce (2008), Kaltenbrunner and Lochstoer (2008), Campanale, Castro and Clementi (2009), Ai (2008), Kuehn (2008), Fernandez-Villaverde, Kojien, Rubio-Ramirez, van Binsbergen (2010)). These papers typically work in versions of the standard real business cycle model, where growth is given exogenously. One conclusion from calibrated versions of these important contributions is that while long-run risks do arise endogenously in such settings, they are typically not quantitatively sufficient to rationalize key asset market statistics. This is in contrast to our specification, where the incentives to engage in R&D deliver quantitatively significant long-run risks in both consumption and dividend growth, suggesting that stochastic endogenous growth models provide a natural environment for general equilibrium asset pricing.

Similarly, the paper is closely related to recent contributions examining the link between technological innovation and asset pricing (Garleanu, Panageas and Yu (2009), Garleanu, Kogan and Panageas (2009), Pastor and Veronesi (2009)). While these models generate similar dynamics as our paper for key macroeconomic variables, such as consumption growth, they focus on adoption of new technologies. In this sense, because we focus on the creation of new technologies, our approach is complementary. Moreover, in these aforementioned models of technology adoption, the arrival of new technologies, and therefore the growth rate dynamics, follow exogenously specified processes, whereas we endogenize these growth dynamics. Additionally, another key difference is that these papers use preference specifications such that these growth rate dynamics are not priced. Lin (2009) examines the link between endogenous technological change and the cross-section of returns in a partial equilibrium model.

Methodologically, our paper is a variation of recent contributions seeking to link the endogenous growth literature and the business cycle literatures (Jones, Manuelli, Siu, Stacchetti (2004), Jones, Manuelli, Siu (2004), Jones and Manuelli (2005)). Within this literature we build on the recent contributions identifying medium term business cycles (Comin and Gertler (2007), Comin, Gertler and Santacreu (2009)). One way of interpreting our results is that they suggest that this macroeconomic notion of medium term cycles is closely linked to the finance notion of long-run risks. Comin et al. in turn build on the seminal contributions of Romer (1990). In these models endogenous growth is generated by increasing the number of intermediate goods used in the final good production, hence the moniker, expanding-variety models. This endogenous growth mechanism was first initiated successfully by Romer. In this context, the paper is also related to a recent set of papers by Bilbiie, Ghironi and Melitz (2007, 2008) who explore the business cycle implications of endogenous product variety. While these papers do address how creation and adoption of new technologies affect asset prices, they do not consider risk premia and the dynamics of consumption growth, which are
central to asset pricing and which are the main contributions of our paper.

The paper is structured as follows. In section 2 we describe our benchmark model. In section 3 we qualitatively explore the growth and productivity processes arising in equilibrium and details their links with the real business cycle model. We examine its quantitative implications for productivity, macroeconomic quantities and asset prices in section 4, along with a number of empirical predictions. In section 5 we provide two extensions the benchmark model, one motivated by the asset pricing literature, the other by the empirical evidence on R&D. Section 6 concludes.

2 Model

We start by describing out benchmark endogenous growth model. The model is a stochastic version of the seminal work in Romer (1990), to which we add capital accumulation subject to convex adjustment costs and assume that households have Epstein-Zin recursive preferences. Additionally, to highlight the propagation mechanism, we compare the endogenous growth model with a standard neoclassical growth model with labor augmenting technology that has the same aggregate production function and capital adjustment costs as our benchmark model. In the following, we will refer to our benchmark model as the ENDO model and refer to the neoclassical growth model as the EXO model.

2.1 Benchmark Endogenous Growth Model (ENDO)

In our benchmark model, rather than assuming exogenous technological progress, growth arises through firms’ R&D investment. R&D investment leads to creation of new patents or intermediate goods used in the production of a consumption good. An increasing number of intermediate goods is the ultimate source of sustained growth, hence the model is a version of an expanding-variety model of endogenous growth.

The model features a representative final good firm which produces the single consumption good and behaves competitively. The production of the consumption good requires capital, labor and a composite intermediate good. Furthermore production of the final good is subject to stationary exogenous shocks. Intermediate goods are produced by a continuum of monopolistic producers. As in Romer (1990) introduction of new intermediate goods is the ultimate source of sustained productivity growth. Creation of new intermediate goods depends on research and development activity. We assume that the representative household has Epstein-Zin preferences, whose consumption and savings problem is fairly standard.

**Household** The representative household has Epstein-Zin preferences defined over consumption:

\[ U_t = \left\{ (1 - \beta)C_t^{\frac{1-\gamma}{\sigma}} + \beta(E_t(U_{t+1}^{1-\gamma}))^{\frac{\sigma}{1-\gamma}} \right\}^{\frac{1}{1-\gamma}} , \]
where $\gamma$ is the coefficient of relative risk aversion, $\psi$ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$. When $\psi \neq \frac{1}{\gamma}$, the agent cares about news regarding long-run growth prospects. In the long-run risks literature, the parametrization $\psi > \frac{1}{\gamma}$ is assumed, that is, the agent has a preference for early resolution of uncertainty, so that the agent dislikes shocks to long-run expected growth rates.

The household maximizes utility by participating in financial markets and by supplying labor. Specifically, the household can take positions $Z_t$ in the stock market, which pays an aggregate dividend $D_t$, and in the bond market, $B_t$. Accordingly, the budget constraint of the household becomes

$$C_t + Q_t Z_t + B_{t+1} = W_t L_t + (Q_t + D_t) Z_t + R_t B_t$$

where $Q_t$ is the stock price, $R_t$ is the gross risk free rate, $W_t$ is the wage and $L_t$ denotes hours worked.

As described above, the production side of the economy consists of several sectors, so that the aggregate dividend can be further decomposed into the individual payouts of these sectors, in a way to be described below.

As usual, the setup implies that the stochastic discount factor in the economy is given by

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left[ \frac{\mathbb{E}_t(U_{t+1}^{\gamma-1/\psi})}{U_t^{\gamma-1/\psi}} \right]$$

(1)

**Final Goods Sector** There is a representative firm that uses capital $K_t$, labor $L_t$ and a composite of intermediate goods $G_t$ to produce the final (consumption) good. Also, assume that the final goods firm owns the capital stock and has access to the CRS production technology

$$Y_t = (K_t^{\alpha} (\Omega_t L_t)^{1-\alpha})^{1-\xi} G_t^\xi$$

(2)

where the composite $G_t$ is defined according to the CES aggregator,

$$G_t = \left[ \int_0^{N_t} X_{i,t}^\xi \, di \right]^{1/\nu}$$

and $X_{i,t}$ is intermediate good $i \in [0, N_t]$, where $N_t$ is the measure of intermediate goods in use at date $t$. Furthermore, $\alpha$ is the capital share, $\xi$ is the intermediate goods share, and $\nu$ is the elasticity of of substitution between the intermediate goods. Note that $\nu > 1$ is assumed so that increasing the variety of intermediate goods raises the level of productivity in the final goods sector. This property is crucial for sustained growth. The productivity shock $\Omega_t$ is assumed to follow a stationary Markov process. Because of the stationarity of the forcing process, sustained growth will arise endogenously from the development of new intermediate
goods. We will describe the research and development process below.

Define dividends of the final goods firm as

\[ D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{t,i} X_{i,t} \, di \]

where \( I_t \) is capital investment, \( W_t \) is the wage rate, and \( P_{t,i} \) is the price per unit of intermediate good \( i \), which the final goods firm takes as given. The last term captures the costs of buying intermediate goods at time \( t \).

In line with the literature on production-based asset pricing, we assume that investment is subject to capital adjustment costs. The capital stock then evolves as

\[ K_{t+1} = (1 - \delta) K_t + \Lambda \left( \frac{I_t}{K_t} \right) K_t \]

where the capital adjustment cost function \( \lambda(\cdot) \) is specified as in Jermann (1998)

\[ \Lambda \left( \frac{I_t}{K_t} \right) \equiv \frac{\alpha_1}{1 - \frac{\zeta}{\zeta}} \left( \frac{I_t}{K_t} \right)^{1 - \frac{1}{\zeta}} + \alpha_2 \]

The parameter \( \zeta \) represents the elasticity of the investment rate; in particular the limiting cases \( \zeta \to 0 \) and \( \zeta \to \infty \) represent infinitely costly adjustment and frictionless adjustment, respectively. The parameters \( \alpha_1 \) and \( \alpha_2 \) are set so that there are no adjustment costs in the deterministic steady state. Specifically,

\[ \alpha_1 = (\Delta N_{ss} - 1 + \delta) \frac{1}{\zeta} \]
\[ \alpha_2 = \frac{1}{\zeta - 1} (1 - \delta - \Delta N_{ss}) \]

Taking the stochastic discount factor \( M_t \) as given, the firm’s problem is to choose investment, labor and intermediate goods input to maximize shareholder’s wealth. This can be formally stated as

\[ \max_{\{I_t, L_t, K_{t+1}, X_{i,t}\}_{t \geq 0, i \in [0, N_t]}} E_0 \left[ \sum_{t=0}^{\infty} M_t D_t \right] \]

subject to

\[ D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{t,i} X_{i,t} \, di \]
\[ K_{t+1} = (1 - \delta) K_t + \Lambda \left( \frac{I_t}{K_t} \right) K_t \]
where $\delta$ is the depreciation rate of capital. Accordingly, denoting the multiplier on the capital accumulation constraint by $q_t$, the Lagrangian for the firm’s problem is

$$\mathcal{L} = E_0 \left[ \sum_{t=0}^{\infty} M_t \left\{ Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} \, di + q_t \left( (1 - \delta) K_t + \Lambda \left( \frac{I_t}{K_t} \right) K_t - K_{t+1} \right) \right\} \right]$$

with corresponding first order conditions are:

$$q_t = \frac{1}{\Lambda_t}$$
$$W_t = (1 - \alpha)(1 - \xi) \frac{Y_t}{L_t}$$
$$1 = E_t \left[ M_{t+1} \left\{ \frac{1}{q_t} \left( \alpha(1 - \xi) \frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta) - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} \Lambda_{t+1} \right) \right\} \right]$$
$$P_{i,t} = \left( K_t^\alpha (\Omega_t L_t) \right)^{1-\alpha} \frac{1}{\nu} \left[ \int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} \, di \right]^{\nu-1} \frac{1}{\nu} X_{i,t}^{\frac{1}{\nu}-1}$$

where $\Lambda_t \equiv \Lambda \left( \frac{I_t}{K_t} \right)$ and $\Lambda'_t \equiv \Lambda' \left( \frac{I_t}{K_t} \right)$.

**Intermediate Goods Sector** Intermediate goods producers have monopoly power. Given the demand schedules set by the final good firm, monopolists producing the intermediate goods set the prices in order to maximize their profits. Intermediate goods producers transform one unit of the final good in one unit of their respective intermediate good. In this sense production is “roundabout” in that monopolists take final good as given as they are tiny themselves. This fixes the marginal cost of producing one intermediate good at unity.

The first-order condition with respect to $X_{i,t}$ implicitly gives the demand schedule for intermediate good $i$ as a function of the price $P_{i,t}$. The local monopolist producing $X_{i,t}$ takes the demand schedule $X_{i,t}(P_{i,t})$ as given and produces at unit cost using the final good input. Thus, the monopolist solves the following static profit maximization problem each period

$$\max_{P_{i,t}} \Pi_{i,t} \equiv P_{i,t} \cdot X_{i,t}(P_{i,t}) - X_{i,t}(P_{i,t})$$

The monopolistically competitive characterization of the intermediate goods sector a la Dixit and Stiglitz (1977) results in the symmetric industry equilibrium conditions

$$X_{i,t} = X_t$$
$$P_{i,t} = P_t = \nu$$
That is, each intermediate goods producer produces the same amount and charges a markup \( \nu > 1 \) over marginal cost. Substituting these two equilibrium conditions into the definition for \( G_t \) and the F.O.C. w.r.t. \( X_{i,t} \) yields:

\[
G_t = N_t^\nu X_t \tag{3}
\]

\[
X_t = \left( \frac{\xi}{\nu} \left( K_t^\alpha (\Omega_t L_t)^{1-\alpha} \right)^{1-\xi} N_t^{\nu \xi - 1} \right)^{\frac{1}{1-\xi}} \tag{4}
\]

and hence

\[
\Pi_t = (\nu - 1) X_t
\]

Consequently, the intermediate good input and hence monopoly profits are procyclical.

The value of owning exclusive rights to produce intermediate good \( i \) is equal to the present discounted value of the current and future monopoly profits

\[
V_{i,t} = \Pi_{i,t} + \phi E_t[M_{t+1}V_{i,t+1}]
\]

where \( \phi \) is the survival rate of an intermediate good. Imposing the symmetric equilibrium conditions, we can drop the \( i \) subscript and write

\[
V_t = \Pi_t + \phi E_t[M_{t+1}V_{t+1}]
\]

Again, given the procyclicality of profits, this implies that the values of patents and hence the payoff to innovation is procyclical.

**R&D Sector** Innovators develop intermediate goods for the production of final output. They do so by conducting research and development, using the final good as input at unit cost. For simplicity, we assume that households can directly invest in research and development. They develop new intermediate goods, whose patents can then be sold in the market for intermediate goods patents. A new intermediate goods producer will buy the new patent. Assuming that this market is competitive, the price of a new patent will equal the value of the new patent to the new intermediate goods producer.

The R&D sector develops new intermediate goods and sells them to firms in the intermediate goods sector. It has access to linear technology and uses the final good as input to produce new varieties. Specifically, the
law of motion for the measure of intermediate goods \( N_t \) is

\[ N_{t+1} = \vartheta_t S_t + \phi N_t \]

where \( S_t \) denotes R&D expenditures (in terms of the final good) and \( \vartheta_t \) represents the productivity of the R&D sector that is taken as exogenous by the R&D sector. In similar spirit as Comin and Gertler (2006), we assume that this technology coefficient captures a congestion externality effect so that higher R&D intensity leads to lower productivity in the innovation sector

\[ \vartheta_t = \frac{\chi \cdot N_t}{S_t^{1-\eta} N_t^\eta} \]

where \( \chi > 0 \) is a scale parameter and \( \eta \in [0, 1] \) is the elasticity of new intermediate goods with respect to R&D. Since there is free entry into the R&D sector, the following break-even condition must hold:

\[ E_t[M_{t+1}V_{t+1}](N_{t+1} - \phi N_t) = S_t \]

This says that the expected sales revenues equals costs. This condition can be equivalently formulated, at the margin, as

\[ \frac{1}{\vartheta_t} = E_t[M_{t+1}V_{t+1}] \]

which states that marginal cost equals expected marginal revenue.

**Market Clearing** Final output is used for consumption, investment in physical capital, factor input used in the production of intermediate goods, and R&D:

\[ Y_t = C_t + I_t + N_t X_t + S_t \]

Alternatively, this can be written as

\[ Y_t = C_t + I_t + N_t^{1-\nu} G_t + S_t \]

where the term \( N_t^{1-\nu} G_t \) captures the costs of intermediate goods production. Given that \( \nu > 1 \) reflecting monopolistic competition, it follows that increasing product variety increases the efficiency of intermediate goods production, as the costs fall as \( N_t \) grows.

Furthermore, since the agent has no disutility for labor, she will supply her entire endowment, which is
Forcing Process We introduce uncertainty into the model by means of an exogenous shock \( \Omega_t \) to the level of technology, that is assumed to evolve as

\[
\begin{align*}
\Omega_t &= e^{a_t} \\
a_t &= \rho a_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2)
\end{align*}
\]

Note first that this process is strictly stationary, so that sustained growth in the model will not arise through exogenous trend growth in exogenous productivity, but endogenously. Second, while formally, \( \Omega_t \) resembles labor augmenting technology, it does not represent measured TFP in our setting. Rather, measured TFP in the model can be decomposed in an exogenous component, driven by \( \Omega_t \), and an endogenous component which is driven by the accumulation of intermediate goods and hence innovation, which is also the source of sustained growth. We discuss the dynamics of productivity in detail in section 3.

2.2 Exogenous Growth Model (EXO)

To contrast with our benchmark endogenous growth model (ENDO), we consider the neoclassical growth model with exogenous labor augmenting technology. We will examine the connection between the two models in detail in the following section. Section 3 will present qualitative results, while section 4 examines the models quantitatively.

Specifically, rather than growth being endogenously determined through accumulation of intermediate goods \( N_t \), we instead exogenously specify the evolution of the corresponding \( \tilde{N}_t \) as an exponential time trend. Furthermore, there is no more role of innovation and production will simply be a one-sector representative firm that uses only capital and labor as factor inputs. This specification of production and technology for the EXO model is standard in the real business cycle (RBC) literature. The capital accumulation equation and capital adjustment cost function are exactly the same as in the ENDO model. Furthermore, the household’s problem is unchanged, we will simply describe the production sector.

Production A representative firm produces the final (consumption) good using capital \( K_t \) and labor effort \( L_t \) with constant returns to scale technology

\[
Y_t = K_t^\alpha (\tilde{Z}_t L_t)^{1-\alpha}
\]
where productivity $\tilde{Z}_t$ is specified as a trend stationary process

$$\tilde{Z}_t = A\Omega_t \tilde{N}_t$$

$$\tilde{N}_t = e^{\mu t}$$

where the constant $A \equiv \left(\frac{\xi}{\rho}\right)^{\alpha - \left(\frac{\xi}{\rho}\right)} > 0$ and the productivity shock $\Omega_t$ are specified exactly as in the ENDO model:

$$\Omega_t = e^{a_t}$$

$$a_t = \rho a_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2)$$

Dividends are defined as

$$D_t = Y_t - W_t L_t - I_t$$

where $I_t$ is capital investment and $W_t$ is the wage rate. The capital stock evolves as

$$K_{t+1} = (1 - \delta)K_t + \Lambda \left(\frac{I_t}{K_t}\right)K_t$$

Taking the pricing kernel $M_t$ as given, the firm’s problem is to maximize shareholder’s wealth, which can be formally stated as

$$\max_{\{I_t, L_t, K_{t+1}\}} E_0 \left[ \sum_{t=0}^{\infty} M_t D_t \right]$$

subject to

$$D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{t,i} X_{i,t} \, di$$

$$K_{t+1} = (1 - \delta)K_t + \Lambda \left(\frac{I_t}{K_t}\right)K_t$$
with corresponding first-order conditions are:

\[
q_t = \frac{1}{\Lambda_t'}
\]

\[
W_t = (1 - \alpha) \frac{Y_t}{K_t}
\]

\[
1 = E_t \left[ M_{t+1} \left\{ \frac{1}{q_t} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta) - \frac{I_{t+1}}{Y_{t+1}} + q_{t+1} \Lambda_{t+1} \right) \right\} \right]
\]

where \( \Lambda_t = \Lambda \left( \frac{I_t}{K_t} \right) \) and \( \Lambda_t' = \Lambda' \left( \frac{I_t}{K_t} \right) \).

Output is used for consumption and investment:

\[
Y_t = C_t + I_t
\]

We now turn to a direct comparison of the two models, ENDO and EXO. Qualitatively, the respective dynamics are driven by different productivity processes, as we discuss in section 3. The remaining sections provide quantitative results.

3 Equilibrium Growth and Productivity

In our benchmark model, sustained growth is an equilibrium phenomenon resulting from agents’ decisions. Moreover, these decisions generate growth rate and productivity dynamics contrasting with those implied by more standard macroeconomic frameworks. These dynamics will be reflected in asset prices. In this section we describe these patterns qualitatively, and contrast them with the exogenous growth specification (EXO) described above. We will provide empirical evidence supporting these patterns and a quantitative analysis in the next section.

First, it is convenient to represent the aggregate production function in the ENDO model in a form that permits straightforward comparison with the EXO specification. To that end, note that using the equilibrium conditions derived above, final output can be rewritten as follows:

\[
Y_t = \left( \frac{\xi}{\nu} \right)^{\frac{\delta}{\alpha}} K_t^{\alpha} \left( \Omega_t L_t \right)^{1-\alpha} N_t^{\frac{\nu - \delta}{\alpha}}
\]

For sustained growth to obtain in this setting we need to impose a parametric restriction. Technically, to ensure balanced growth, we need the aggregate production function to be homogeneous of degree one in the
accumulating factors $K_t$ and $N_t$. Thus, the following parameter restriction needs to be satisfied:

$$\alpha + \frac{\nu \xi - \xi}{1 - \xi} = 1$$

Imposing this restriction, we have a production function that resembles the standard neoclassical one with labor augmenting technology:

$$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}$$

where total factor productivity (TFP) is

$$Z_t \equiv \bar{A} \Omega_t N_t$$

and $\bar{A} \equiv \left( \frac{\xi}{\bar{\nu}} \right)^{\frac{\xi}{1-\alpha(1-\xi)}} > 0$ is a constant. From the specification of the EXO model, we recall that

$$\tilde{Z}_t = \bar{A} \Omega_t \tilde{N}_t$$

$$\tilde{N}_t = e^{\mu t}$$

Hence, the fundamental difference between the ENDO model and the canonical real business cycle (RBC) framework is that the trend component of the TFP process, $N_t$, is endogenous and fluctuates in the ENDO model but exogenous and deterministic in the RBC model. This is naturally reflected in the dynamics of productivity growth rates in the respective models. Clearly, for log productivity growth rates we have

$$\Delta z_t = \Delta n_t + \Delta a_t$$

in the ENDO model, and similarly

$$\Delta \tilde{z}_t = \mu + \Delta a_t$$

in the EXO model, where lowercase letters denote logs. Realistically, $a_t$ will be a fairly persistent process, so that we can write

$$\Delta z_t \approx \Delta n_t + \epsilon_t$$

$$\Delta \tilde{z}_t \approx \mu + \epsilon_t$$
Accordingly, while in the EXO model productivity growth will be roughly iid, it will inherit a second component in the benchmark model which depends on the accumulation of patents. Clearly, in the model, the accumulation of patents is determined by the dynamics of R&D. Therefore, qualitatively and quantitatively, the dynamics of productivity growth reflect the dynamics of innovation.

To see this, rewrite the growth rate of productivity, $\Delta Z_t$, as $\Delta Z_t = \Delta N_t \cdot \Delta \Omega_t$. Given a realistically persistent calibration of $\{\Omega_t\}$ in logs, we have $\Delta \Omega_t \approx e^{\epsilon_t}$. On the other hand, given the accumulation of $N_t$ as $N_t = \vartheta_t \cdot S_{t-1} + \phi N_{t-1}$, the growth rate of patents becomes $\Delta N_t = \vartheta_t \cdot \tilde{S}_{t-1} + \phi$, where we set

$$\tilde{S}_t = \frac{S_t}{N_t}$$

We will refer to $\tilde{S}_t$ as the R&D intensity. Accordingly, we find $\Delta Z_t \approx (\vartheta_t \cdot \tilde{S}_{t-1} + \phi)(e^{\epsilon_t})$. Thus,

$$E_t[\Delta Z_{t+1}] \approx E_t \left[ (\vartheta_t \cdot \tilde{S}_{t-1} + \phi)(e^{\epsilon_{t+1}}) \right]$$

$$= (\vartheta_t \cdot \tilde{S}_{t-1} + \phi)E_t [e^{\epsilon_{t+1}}]$$

$$\approx \vartheta_t \cdot \tilde{S}_{t-1} + \phi$$

Thus, while in the EXO model, expected productivity growth is approximately constant, the ENDO model exhibits variation in expected growth driven by the R&D intensity. Empirically, the R&D intensity is a fairly persistent and volatile process. We will therefore expect productivity growth in the ENDO model to exhibit substantial low-frequency variation. In other words, this difference between the models leads to a very different propagation of the productivity shock $a_t$; namely, the ENDO model generates low- and high-frequency cycles whereas the RBC model only generates high-frequency cycles. We provide empirical and quantitative evidence on this channel in the next section.

The equilibrium productivity growth dynamics implied by the model resemble closely those specified by Croce (2008). Croce specifies productivity to contain an iid component as well as a small persistent component. He refers to that latter component as long-run productivity risk. While he exogenously specifies these dynamics, we show that such long-run productivity risk arises naturally endogenously in a setting with endogenous growth and that it is linked with innovation.

4 Quantitative Results

In this section we explore the quantitative implications of the model using simulations. We use perturbation methods to solve the model. To account for risk premia and potential time variation in them, we use a higher order approximation around the stochastic steady state (see Caldara, Fernandez-Villaverde, Rubio-Ramirez,
The next section describes our calibration.

4.1 Calibration

In this section, we present the benchmark calibration used to assess the quantitative implications of the endogenous growth model (ENDO). The elasticity of intertemporal substitution $\psi$ and the coefficient of relative risk aversion $\gamma$ are set to standard values in the long-run risks literature.\(^2\) Note that in this parametrization, $\psi > \frac{1}{\gamma}$, which implies that the agent dislikes shocks to expected growth rates and is particularly important for generating a sizeable risk premium in this setting. The subjective discount factor $\beta$ has a large impact on the level of the riskfree rate and the growth rate, which are tightly linked variables. Thus, $\beta$ is set to be consistent with both the level of the riskfree rate and average growth rate. The scale parameter $\chi$ is calibrated to help match balanced growth evidence. The depreciation rate of capital $\delta$ and capital share $\alpha$ are set to standard values in the real business cycle literature. The capital adjustment cost parameter $\zeta$ is set to match the relative volatility of consumption to output. The markup in the intermediate goods sector $\xi$ and the elasticity of new intermediate goods with respect to R&D $\eta$ are set to values similar to Comin and Gertler (2006).\(^3\) Since the variety of intermediate goods can be interpreted as the stock of R&D (a directly observable quantity), we can then interpret one minus the survival rate $\phi$ as the depreciation rate of the R&D stock. Hence, we set the parameter $\phi$ to match the depreciation rate the R&D stock assumed by the BLS. The persistence parameter $\rho$ of the productivity shocks \(a_t \equiv \log(\Omega_t)\) is calibrated to match the first autocorrelation of R&D intensity, which is the key driver of expected growth rates. Furthermore, this value for $\rho$ allows us to be consistent with the first autocorrelations of the key quantity growth rates and productivity growth.\(^4\) The volatility parameter $\sigma$ is set to match output growth volatility. In addition, for the exogenous growth model (EXO) we keep the common parameters the same to facilitate the comparison of the economic mechanisms between the two models. The one parameter that is in the EXO model but not the ENDO model is the trend growth parameter $\mu$, which is set to match average output growth.

\(^2\)See Bansal and Yaron (2004).

\(^3\)Note that $\eta$ is within the range of panel and cross-sectional estimates from Griliches (1990) and Comin and Gertler (2006).

\(^4\)To provide further discipline on the calibration of $\rho$, note that Since the ENDO model implies the TFP decomposition, $\Delta z_t = \Delta a_t + \Delta n_t$, we can project log TFP growth on log growth of the R&D stock to back out the residual $\Delta a_t$. The autocorrelations of the extracted residual $\Delta a_t$ show that we cannot reject that it is white noise. Hence, in levels, it must be the case that $a_t$ is a persistent process to be consistent with this empirical evidence. In our benchmark calibration, the annualized value of $\rho$ is .95.
4.2 Productivity Dynamics

Many of the key implications of the benchmark model can be understood by looking at the endogenous dynamics of total factor productivity (TFP) growth, $\Delta Z_t$, which we outlined in section 3:

$$E_t[\Delta Z_{t+1}] \approx \theta_t \cdot \hat{S}_t + \phi$$

Qualitatively therefore, the dynamics of TFP are driven by the endogenous movements in R&D. This is in sharp contrast to the EXO specification, where productivity growth is roughly iid.

Quantitatively, the implications of the model will thus depend on the ability of our calibration to match basic stylized facts about R&D activity and innovation. As table 3 documents, the model is broadly consistent with volatilities and autocorrelations of R&D investment, the stock of R&D and R&D intensities in the data. Crucially, as in the data, the R&D intensity is a fairly volatile and persistent process. Specifically, we match its annual autocorrelation of 0.93.

The above decomposition of the expected growth rate of TFP therefore suggests a highly persistent component in TFP growth. Table 4 confirms this prediction, both in the data as well as in the model. While uncovering the expected growth rate of productivity as a latent variable in the data (as in Croce (2010)) suggests an annual persistence coefficient of 0.93, our model closely matches this number with a persistence coefficient of 0.95. Moreover, the volatilities of expected TFP growth rates in the data and in the model roughly match. Naturally, matching the persistence properties of TFP in the data is important in a production economy, as this will affect the dynamics of macroeconomic variables. Note that in contrast to our benchmark model, the EXO specification implies that TFP growth is roughly iid, in contrast to the empirical evidence.

Qualitatively, the above decomposition suggests that the R&D intensity should track productivity growth, and given the persistence of the R&D intensity, expected productivity growth rather well. Figure 1 visualizes this pattern for expected productivity growth, using a simulated sample path. For realized growth rates, this is confirmed in figures 2 and 3, both in the model as well as in the data. The plots visualize the small, but persistent component in TFP growth induced by equilibrium R&D activity.

Accounting for an endogenous persistent component in productivity growth helps our benchmark model match observed productivity dynamics relative to the exogenous specification, as table 4 documents. While the benchmark model roughly matches the annual autocorrelation of productivity growth, the EXO model is an order of magnitude off. The latter is not surprising, given that productivity growth is roughly iid in that specification. The persistent component ENDO specification also accounts for a fairly volatile conditional mean of productivity growth as well as higher long-term volatilities than in the iid case.

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On the other hand, from an empirical point of view, these results suggest, that R&D activity, and especially the R&D intensity should forecast productivity growth rather effectively. We confirm this prediction in table 6, which documents results from projecting productivity growth on R&D intensity, over several horizons. In the data, R&D intensity forecasts productivity growth over several years, significantly, and with $R^2$’s increasing with horizons. Qualitatively, the model replicates this pattern rather well.

The intuition for these results comes from the endogenous R&D dynamics that the model generates. This can be readily gleaned from the impulse responses to an exogenous shock displayed in figure 4. It exhibits responses of quantities in the patents sector. Crucially, after a shock profits rise persistently. Intuitively, a positive shock in the final goods sector raises the demand $X_t$ for intermediate goods, and with $\Pi_t = (\nu-1)X_t$, this translates directly into higher profits. Naturally, given persistently higher profits, the value of a patent goes up, as shown in the third panel. Then in turn, as the payoff to innovation is the value of patents, this triggers a persistent increase in the R&D intensity. This yields the persistent endogenous component in productivity growth displayed above. Crucially, the exogenous shock has two effects. It immediately raises productivity of the final output firm, leading to standard fluctuations at business cycle frequency, but it also induces more R&D which will be reflected in more patents with a lag. Importantly, the increases in R&D are persistent, leading to fluctuations at lower frequencies. Intuitively, in this setting, an exogenous shock to the level of productivity endogenously generates a persistent shock to the growth rate of the economy, or in other words, to growth waves.

4.3 Consumption Dynamics and Endogenous Long-Run Risk

In the previous section, we documented that the benchmark model has rich implications for the dynamics of TFP, which will naturally be reflected in the quantity dynamics of our production economy. With a view towards asset pricing, we focus on the implications for consumption dynamics in this section. In particular, we examine the dynamics of expected consumption growth that the model generates. While Bansal and Yaron (2004) have shown in an endowment economy that persistent variation in expected consumption growth coupled with recursive preferences can generate substantial risk premia in asset markets, the empirical evidence regarding this channel is still controversial. In this light, providing theoretical evidence in production economies supporting the mechanism would be reassuring. While several papers have considered how such long run risks can arise endogenously in production economies (Kaltenbrunner and Lochstoer (2008), Croce (2008), Campanale, Castro and Clementi (2008)), these studies operate in versions of the real business cycle model (proxied by the EXO specification here) and typically do not generate sufficient endogenous risks to match asset market statistics.

Table 7 documents basic properties of consumption growth in the model. While the model is calibrated
to match the volatility of consumption growth (as reported in table 2), it also roughly replicates its annual autocorrelation. This is in sharp contrast to the EXO specification, where consumption growth is barely autocorrelated. This is somewhat symptomatic for the standard real business cycle, which typically fails to match the observed autocorrelation of growth rates, or, in other words, lacks propagation (Cogley and Nason, 1995). More importantly, the table also documents that the benchmark model produces substantial variation in expected consumption growth, considerably more than the EXO specification. On the other hand, the impulse response in figure 10 shows that this variation is also very persistent, again in sharp contrast to the EXO model, suggesting that the benchmark model generates quantitatively significant long run risks in consumption growth. This will be reflected in the model’s asset pricing implications, to be discussed below. Note that while Bansal and Yaron (2004) in an endowment economy setting and Croce (2008) in a production economy generate long run risks by introducing independent, persistent shocks to consumption and productivity growth respectively, in our model fluctuations in realized consumption growth and persistent variation in expected consumption growth are due to one source of exogenous uncertainty only. Hence, the model translates this disturbance in substantial low frequency movements in consumption growth, or, in other words, provides a strong mechanism to propagate this shock. This is also reflected in the substantial long-term volatilities that consumption growth exhibits in the model, as reported in the table.

Naturally, persistence in expected consumption growth is just a reflection of persistent dynamics in productivity growth. Hence, our model suggests that the dynamics of innovation is ultimately the source of long run risk in consumption growth. Innovation generates persistent productivity gains, which are reflected in long growth waves in consumption. Figure 5 visualizes how closely expected consumption growth tracks the R&D intensity, which is the key determinant of expected productivity growth, in a sample path simulated from the model. Empirically, this suggests that measures related to innovation, and the R&D intensity in particular, should have forecasting for consumption growth. We verify this in table 8, which reports results from projecting future consumption growth over various horizons on the R&D intensity. Empirically, the R&D intensity predicts future consumption growth over horizons up to 5 years, with significant point estimates, and $R^2$’s increasing with horizons. Qualitatively, the model reflects this pattern reasonably well. This gives empirical support to the notion of innovation-driven low frequency variation in consumption growth.

Using statistical techniques, Bansal, Kiku and Yaron (2007), construct series for expected consumption growth in the data. Notably, the correlation between the R&D intensity and their model-free series is about 70%, which suggests a macroeconomic foundation for predictability in consumption growth.

Another way of capturing low frequency variation in consumption growth is by using appropriate filters. Here we follow Comin and Gertler (2006) who identify high, medium and low frequency cycles respectively
in quantities using bandpass filters (Christiano and Fitzgerald (2003)). In their spirit, we identify high frequency or business cycle movements with a bandwidth of 2 to 32 quarters, medium frequency components with a bandwidth of 32 to 200 quarters, and low frequency movements with a bandwidth of 200 to 400 quarters. Interestingly, there is a close correspondence between Comin and Gertler’s macroeconomic notion of a medium term cycle and the finance notion of long-run risks in consumption growth. This is visualized in figure 6, which plots consumption growth and its medium term component in a simulated sample path as well as in the data.

### 4.4 Fluctuations and Propagation

While consumption dynamics are important for asset pricing, endogenous persistent variation in expected productivity growth suggests a propagation mechanism for quantities more generally, which, as alluded to above, standard macro models typically lack. We therefore now turn to a more systematic discussion of the macroeconomic implications of the model. Table 2 reports basic statistics implied by the model. Concentrating on the quantities for the moment, the table shows that the model is reasonably consistent with basic quantity statistics. In particular, our benchmark model does just as well as the EXO model, which is a standard real business cycle model. One way of interpreting this finding is by saying that the ENDO model generates high-frequency dynamics or business cycle statistics in line with the canonical real business cycle model. On the other hand, both specifications predict investment to be too smooth. This is a general difficulty with production models with recursive preferences.

While table 2 suggests that our benchmark endogenous growth model performs equally well as the real business cycle model when it comes to basic quantity statistics, the endogenous productivity dynamics of the model lead to a strong propagation mechanism absent in the latter model, as we explore now.

Table 9 reports autocorrelations of basic growth rates, in the data, as well as in the ENDO and the EXO model. Note first that while all growth rates exhibit considerable positive autocorrelation at annual frequencies, the corresponding persistence implied by the EXO model is virtually zero, and sometimes even negative. This is one of the main weaknesses of the real business cycle model (as pointed out e.g. in Cogley and Nason (1995)). Quite in contrast, our ENDO model generates substantial positive autocorrelation in all quantities, sometimes qualitatively and sometimes quantitatively close to their data counterparts. Note that the exogenous component of productivity is the same in both model. Accordingly, the ENDO model possesses a strong propagation mechanism induced by the endogenous component of productivity, e.g. by the R&D.

The intuition for this endogenous propagation is of course simple, and tightly linked to the dynamics of

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5In a companion paper, Croce, Kung and Schmid (2010), we show how to overcome this problem by explicitly modeling financial frictions. Therefore, we do not pursue this further in the present paper.
TFP documented in the previous section. To the extent that innovation induces a persistent component in productivity, this will be reflected in quantity dynamics. Recall however, that the TFP dynamics implied by the model is consistent with the empirical evidence. As for consumption growth, this suggests that the driver of expected productivity growth, namely the R&D intensity, should forecast quantity growth. This is verified in table 12 for output growth.

The propagation mechanism implies that macroeconomic quantities display markedly different behavior at different frequencies, in other words, it implies a rich intertemporal distribution of growth rates. This can be seen from tables 10 and 11, which capture the intertemporal distribution of growth rates. While the implied volatilities of growth rates of the EXO and ENDO model are basically undistinguishable at short horizons, in the ENDO model they grow fairly quickly over longer horizons. Another way of interpreting this finding is that the ENDO model generates significant dynamics at medium and lower frequencies, while the EXO model does not. Empirically, this can be captured by looking at medium and low frequency components of growth rates. The correspondence of high and medium term movements in output growth in both the data and the model is visualized in figure 7.

Another implication of the model is that it generates cash flow dynamics in line with the empirical evidence. First of all, it generates heavily procyclical profits. This can be seen from figure 4. This is in line with recent work on expanding variety models in Bilbiie, Ghironi, Melitz (2007), but typically presents a challenge to macro models. In our setting, this is driven by the procyclical demand for intermediate goods. Second, the model generates a persistent component in dividend growth. This can be seen in table 10, which documents considerable volatility in conditional expected dividend growth, which implied substantial variation in the conditional mean of cash flow growth. This is visualized in figure 10. Again, this is in stark contrast to the exogenous growth specification. This will be important from an asset pricing perspective, as only the benchmark model generates sufficient long-run uncertainty about dividend growth.

One way of summarizing the results of this section is by saying that while matching business cycle statistics well, the benchmark endogenous growth model generates substantial movements in quantities at medium and lower frequencies, in contrast to the real business cycle model. In other words, the endogenous growth model exhibits a strong propagation mechanism absent in the real business cycle model.

4.5 Asset Pricing Implications

In order to use our model to shed some light on the link between macroeconomic risk and growth via asset price data, we must first make sure that the model is quantitatively consistent with basic asset market statistics. As we will show shortly, the quantity dynamics discussed in the previous section will be key in generating high risk premia roughly in line with historical data. Since we assume that the agent has Epstein-
Zin utility with a preference for an early resolution of uncertainty, this implies that not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth.

Table 2 reports asset market statistics along with the quantity statistics referred to earlier, both for the benchmark model and the exogenous growth specification. The benchmark model is quantitatively broadly consistent with basic asset price data: It generates a low and smooth risk-free rate and a sizeable equity premium. The equity premium is in excess of 4%. The volatility of the equity premium is a close to 7% annually, which seems to fall dramatically short of the historical volatility of the market return. However, when comparing the volatility in the model to the fraction of total volatility explained by productivity, as reported by Ai, Croce and Li (2010), then the model implication is reasonable.

It is instructive to compare the asset pricing implications of the benchmark model with those of the exogenous growth specification. While, as discussed previously, the quantity implications of the models are similar at high frequencies, the price implications are radically different. As can be seen from the table, the risk free rate is counterfactually high in the exogenous growth specification, and the equity premium is close to zero and only a tiny fraction of what obtains in the benchmark model. These differences are intimately connected to differences in medium-term dynamics that the two models generate, as reported in tables 10 and 11. Intuitively, in the setting with exogenous growth, expected growth rates are roughly constant (as in the real business cycle model), therefore diminishing households’ precautionary savings motive. In such a setting, households want to borrow against their future income, which in equilibrium can only be prevented by a prohibitively high interest rate. In the endogenous growth setting, however, taking advantage of profit opportunities in the intermediate goods sector leads to long and persistent swings in aggregate growth rates, and higher volatility over longer horizons. In this context, households optimally save for low growth episodes, leading to a lower interest rate in equilibrium. On the other hand, the model also generates a substantial equity premium. This means that in equilibrium, dividends are risky. The reason is, as discussed above, the cash flows naturally inherit a persistent component from the endogenous component of productivity. These cash flow dynamics not only affect risk premia, but naturally also asset market valuations. In particular, the model roughly matches the dynamics of stock market values, in the data, as measured by Tobin’s Q. This can be seen from tables 2 and 9. Not only does the model generate considerably higher stock market volatility than its EXO counterpart, it also matches its autocorrelation. This suggests that innovation-driven cycles rationalize long-term movements in stock market valuations. These effects can also be seen in figure 10. Figure 10 shows conditional expected growth rates for macro variables. Specifically, the figure documents that following a productivity shock expected growth rates respond strongly in a persistent fashion in the ENDO model whereas in the EXO model expected growth rates are virtually unresponsive to the shock.
Therefore innovations to realized consumption and dividend growth are coupled with innovations to expected
growth, both of which are priced when agents have Epstein-Zin utility with a preference for early resolution
of uncertainty. Therefore bad shocks are simultaneously bad shocks for the long run, thus rendering equity
claims very risky. This can be seen in figure 11. While qualitatively in both benchmark and exogenous
growth model the responses of prices to a shock go in the same directions, quantitatively the effects are
much more pronounced in the former, again owing to the increased persistence and long-term volatility that
it displays. Another way of understanding the asset pricing implications of the models is to recall that the
equity premium is $E[r_d - r_f] \approx -cov(m, r_d)$. This implies that equity must offer the higher a premium, the
more equity returns and the discount factor move in opposite directions. We can see from figure 10 that the
benchmark model displays stronger co-movements of equity returns and discount factor, leading to a higher
equity premium. Another implication that the figure suggests is that the model generates a negative term
premium. As the interest rate is procyclical, this means that after a bad shock interest rates fall and hence
long-maturity bond prices rise, making longer maturity bonds effectively good hedges against shocks, which
will be reflected in a negative term premium. While this may be counterfactual when applied to the nominal
the structure, there is evidence that the real term structure is actually downward sloping.

Taken together, these results suggest that an endogenous growth model with recursive preferences is
a natural environment to understand asset pricing in a general equilibrium setting. More specifically, the
mechanisms that allow the model to generate high risk premia, namely long persistent swings in growth
rates coupled with recursive preferences, are exactly those that Bansal and Yaron (2004) specify exogenously
and refer to as long-run risks. While the dynamics they specify as somewhat hard to detect in the data,
they are a natural implication of agents’ optimal innovation and R&D decisions in our model. The model
suggests that these effects are quantitatively significant. This also suggests that the growth rate dynamics
that Bansal and Yaron specify naturally endogenously arise in a wide class of stochastic endogenous growth
models, making them a very natural way to link asset prices to long-term growth prospects.

4.6 Medium-Term Comovement

So far, we have discussed how the benchmark model generates fluctuations in quantities and prices at various
frequencies. However, the model also has interesting and realistic implications for comovement between prices
and quantities at medium frequencies. This is displayed in figures 8, 12 and 13.

Figure 8 reveals that the model replicates the medium term comovements between productivity and
quantities in the data. This is noteworthy because it reveals the significant variation macro data exhibit at
medium frequencies and the significant comovement between productivity and quantities, which is mirrored
by the ENDO model.
Figure 13 shows the close match between the price-dividend ratio and productivity growth in the data and the benchmark model at medium frequencies. This strongly suggests productivity-driven slow movements in asset market valuations in the data. In the model, these movements are driven by variation in expected cash flows, induced by time variation in R&D intensity. This is because risk premia in the benchmark model are essentially constant. While there is evidence for time-variation in expected cash flows as discussed above, time variation in price-dividend ratios is often related to time-variation in risk premia. To allow for such predictability, we extend our model below to account for stochastic volatility.

At medium frequencies we also find strong cross-correlations between stock returns and consumption growth. This is displayed in figure 12, indicating the lag-lead structure between returns and consumption growth. In the data and at medium frequencies, returns lead consumption growth by several quarters and the lead correlations die away more slowly (relative to the lag correlations). In other words, lower-frequency movements in returns contain important information regarding long-run movements in future growth. The ENDO model replicates this feature whereas the EXO model does not. This important divergence between the two models is due to the fact that in the ENDO model, growth rates contain a predictable component, which is absent in the EXO model, that is a key determinant of asset prices. In sum, the benchmark model is able reconcile the medium-term relationship between returns and growth that the neoclassical growth model fails to produce.

4.7 Sensitivity

Tables 13, 14 and 15 provide some sensitivity analysis with respect to preference and technology parameters that further illustrate the mechanisms at work in the model. Table 13 further contrasts a variant of the EXO model with the benchmark model. More specifically, the variant EXO-AC sets capital adjustment costs in the exogenous growth model to a level which allows the model to exactly match the volatility of consumption growth. This results in lower adjustment costs and yields higher investment volatility, resulting in higher autocorrelation in consumption growth. However, the persistence in consumption growth and the volatility of the conditional mean of consumption growth and therefore the equity premium is still an order of magnitude lower than in the benchmark model. This further documents that the equity premium in the consequence of the intertemporal distribution of consumption growth. This is also documented in the second column of the table, which reports results from setting $\psi = \frac{1}{7}$, that is effectively from assuming CRRA preferences. Here, in spite of higher short-run consumption risk, the equity premium is negligible.

Table 14 reports sensitivity with respect to the key preference parameters, risk aversion and intertemporal elasticity of substitution. Consider first varying risk aversion. Consistent with the results in Tallarini
(2000), varying risk aversion barely affects standard business cycle statistics, that is, second moments. In contrast, it has dramatic effects on the risk-free rate, by raising the precautionary savings motive, and the equity premium. On the other, relative to Tallarini, the benchmark endogenous growth model exhibits a new effect, namely sensitivity of the average growth rate relative to the risk aversion. Specifically, raising risk aversion fosters growth. This has a simple intuition: Raising risk aversion increases the precautionary savings motive, resulting in higher investment and hence higher growth. In contrast, raising the IES affects all moments of both prices and quantities. A higher propensity to substitute over time increases the response of investment to productivity and expected productivity growth and accordingly its volatility. In turn this smooths consumption growth and increases its persistence. This raises the volatility of the conditional mean of consumption growth and hence lowers the risk-free rate and increases the equity premium.

Table 15 illustrates the effects of varying adjustment costs and persistence of the exogenous shock. Raising adjustment costs renders marginal q more sensitive to shocks, and hence increases return volatility and the equity premium. On the other hand, it lowers average growth as it impedes capital accumulation. The table also illustrates the importance of persistence for both prices and quantities.

5 Extensions

In this section, we provide two extensions to our benchmark model, one motivated by the asset pricing literature, and the other by the empirical evidence on R&D. First, we introduce stochastic volatility into the model, that is, we consider the possibility that productivity may exhibit time-varying volatility (see e.g. Croce (2010) for empirical evidence). This is motivated by the benchmark long-run risk model by Bansal-Yaron which entails stochastic volatility in consumption growth, and extends it to a production economy with endogenous growth. Second, we consider the possibility that productivity in the R&D sector may be subject to shocks, and examine the quantitative implications of this assumption in the context of our model.

5.1 Stochastic Volatility (ENDO-SV)

We now extend our benchmark model to allow for stochastic volatility. We do this for several reasons. A number of papers have recently pointed to the importance of volatility shocks for macroeconomic dynamics (Justiniano and Primiceri (2008), Bloom (2009), Fernandez-Villaverde and Rubio-Ramirez (2011)). On the other hand, from an asset pricing perspective, starting from Bansal and Yaron (2004) the long-run risk literature has usually appealed to stochastic volatility. As our model generates long-run risks endogenously, it is interesting to see whether the stochastic volatility intuition carries over to an endogenous growth setting.
For the model ENDO-SV, modify the forcing process so that volatility is time-varying:

\[
\Omega_t = e^{\alpha_t}
\]
\[
a_t = \rho a_{t-1} + \sigma_{t-1} \epsilon_t
\]
\[
\sigma_t^2 = \hat{\sigma}^2 + \lambda_1 (\sigma_{t-1}^2 - \hat{\sigma}^2) + \sigma_{\epsilon} \epsilon_t
\]

where \(\epsilon_t, \epsilon_t \sim N(0,1)\). The calibration is reported in table 16, with summary statistics displayed in table 17.

Our main qualitative results are summarized in figure 14, where we report the responses of quantities and prices to a volatility shock. To interpret the results, first note that a volatility shock does not alter effective productivity, but only its distribution. As actual disposable resources are unchanged after the shock, the responses of consumption and investment (including R&D) go in opposite directions. Consistent with the earlier discussions, the strong precautionary savings motive in our calibration leads to an increase in R&D investment and hence to a persistent increase in the number of patents or intermediate goods. Accordingly, this implies a positive conditional relationship between volatility and growth. However, this implies a fall in consumption upon impact. Similarly, given the precautionary savings motive, the risk-free rate falls, and expected excess returns go up. The latter observation suggests that, as could be expected, introducing stochastic volatility leads to time-variation in risk premia and hence, at least qualitatively, to predictability. Will it necessarily lead to a higher unconditional equity premium? The answer is no, and the reason lies in the endogenous consumption dynamics of the model. While, as can be seen in the figure, a volatility shock leads a fall in realized consumption growth, it also leads to an increase in expected consumption growth. Hence, a volatility shock is a bad shock for the short-run, but a good shock for the long-run in the language of Kaltenbrunner and Lochstoer (2008). As both of these effects are priced with Epstein-Zin preferences, with opposite signs, they tend to offset each other and the overall effect is small. Hence, in a production economy setting, with mean reverting volatility, the unconditional effects of introducing stochastic volatility are small. Quantitatively, as can be seen from table 17, the equity premium actually falls. This is in stark contrast to the specification in Bansal and Yaron (2004), where stochastic volatility accounts for a considerable fraction of the unconditional equity premium. Hence, accounting for the endogeneity of consumption is important when assessing the impact of stochastic volatility on equilibrium asset prices. One interesting alternative would be to follow Justiniano and Primiceri (2008) who assume volatility shocks to be permanent.

One interesting implication of introducing volatility shocks can be seen from table 17, namely the positive autocorrelation of returns, which was virtually absent in the benchmark model. While quantity growth rates could be predicted in the latter model, it essentially did not feature return predictability. This is different in the present extension with time-varying risk, as we explore in the next table. As reported in table 18,
and consistent with the empirical evidence, P/D ratios predict returns negatively, and increasingly so with increasing horizon as measured by the $R^2$. As a matter of fact, the population $R^2$ reported demonstrate considerable explanatory power for the P/D ratio in the model.

5.2 Stochastic R&D Productivity

For simplicity, the benchmark model exhibited deterministic productivity in the R&D sector. However, both anecdotal and empirical evidence point to substantial uncertainty about the returns to R&D. In this section we extend the benchmark model to account for stochastic productivity in the innovation sector. Within the context of the model, this provides a natural way of introducing 'news shocks', that is, shocks to expected productivity that are uncorrelated with current productivity. Note that while our benchmark model featured shocks to expected productivity, these were perfectly correlated with current productivity in this setting with a single exogenous disturbance.

Specifically, we modify the congestion externality term to include a stationary shock $f_t$:

$$\vartheta_t = \frac{e^f_t \chi \cdot N_t}{\bar{S}^{1-\eta} \theta N_t^{\eta}}$$

$$f_t = \rho_f f_{t-1} + \sigma_f \epsilon_{f,t}$$

where $\epsilon_{f,t} \sim N(0, 1)$. Table 19 reports the calibration. Table 20 reports summary statistics, while figure 15 displays impulse responses, which are also our main qualitative results. To start with, while by construction the R&D shock does not affect current productivity, it does, as shown in lowest panel of figure 15, affect expected productivity growth. Indeed, on impact expected productivity growth rises on impact. Intuitively, higher productivity in the R&D sector increases R&D expenditures, resulting in a higher R&D intensity. On the other hand, as discussed beforehand, a higher R&D intensity is reflected in a rise in expected productivity growth through the endogenous component of TFP. In this sense, a R&D shock can be thought of as a news shock. Note however that, relative to a technology shock, the responses are quantitatively small.

As realized productivity does not move on impact, disposable resources are unchanged, so that consumption, investment and R&D move in different directions. In particular, as discussed R&D goes up, given expected productivity gains, investment rises as well, so that consumption actually falls on impact. To what extent these dynamics actually reflect the data is unclear, as the empirical evidence on news shocks is still controversial and sensitive to the identification schemes for them. While Beaudry and Portier (2006) find that a news shock leads to positive comovement of quantities, Barsky and Sims (2010) find that consumption rises, while investment and labor fall. Labor supply likely plays a major role in these dynamics, that we have abstracted from completely. We leave this for future research.
On the quantitative side, table 20 documents that accounting for news shocks originating in the R&D sector may have interesting implications for asset pricing. In particular, since the R&D shocks effectively acts a persistent expected productivity shock, it overall raises persistent variation in expected growth rates. This is reflected in a higher volatility of the conditional mean of consumption growth, thus generating more long run risk. Therefore, the equity premium increases. It would be interesting to examine the asset pricing implications of a model driven solely by R&D shocks. We leave this for future research as well.

6 Conclusion

We provide a quantitative analysis of a stochastic model of endogenous growth where households have recursive Epstein-Zin preferences. In the model, innovation and investment in research and development is the ultimate source of sustained growth. In the data, R&D is quite volatile, persistent and cyclical. Our model then predicts that these movements in the sources of growth will be reflected in the dynamics of the aggregate economy, leading to long-term cycles and persistent growth waves. More precisely, the model predicts a small, but persistent innovation-driven endogenous component in productivity growth leading to long and persistent swings in macroeconomic quantities. Therefore, in spite of being a driven by a single exogenous shock, the model generates significant cycles at high and low frequencies generated by the endogenous response of innovation to the shock. In other words, the innovation process generates a strong propagation mechanism absent in standard macroeconomic models. Empirically, we find strong support for innovation driven medium and low frequency fluctuations in aggregate growth rates in the data.

Under the assumption of recursive preferences these quantity dynamics have strong implications for asset prices and risk premia. With such a preference specification agents are very averse to the low-frequency variation in expected growth rates that the model generates, which yields high risk premia in asset markets and a low risk and stable free rate. As such the model provides a macroeconomic foundation for long run risks in asset markets, as pioneered by Bansal and Yaron (2004), and suggests that a strong propagation mechanism in macro models and high risk premia are inherently linked. Moreover, the model predicts innovation and productivity driven low frequency movements in stock market values and price-dividend ratios, in line with the empirical evidence.

In short, our model implies that there are tight links between macroeconomic risk, growth and risk premia in asset markets and hence suggests that stochastic models of endogenous growth are a useful framework for quantitative macroeconomic modeling and asset pricing.
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Appendix A. Data

Annual and quarterly data for consumption, capital investment, and GDP are from the Bureau of Economic Analysis (BEA). Annual data on private business R&D investment is from the survey conducted by National Science Foundation (NSF). Annual data on the stock of private business R&D is from the Bureau of Labor Statistics (BLS). Annual productivity data is obtained from the BLS and is measured as multifactor productivity in the private nonfarm business sector. The sample period is for 1953-2008, since R&D data is only available during that time period. Consumption is measured as expenditures on nondurable goods and services. Capital investment is measured as private fixed investment. Output is measured as GDP. The variables are converted to real using the Consumer Price Index (CPI), which is obtained from the Center for Research in Security Prices (CRSP).

Monthly nominal return and yield data are from CRSP. The real market return is constructed by taking the nominal value-weighted return on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) and deflating it using the CPI. The real risk-free rate is constructed by using the nominal average one-month yields on treasury bills and taking out expected inflation.\(^6\) Aggregate market and book value are from the Flow of Funds account.

\(^6\)We model the monthly time series process for inflation using an AR(4).
This table reports the benchmark quarterly calibration used for the endogenous growth (ENDO) and exogenous growth (EXO) models.
Table 2: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Moments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[\Delta y]$</td>
<td>1.90%</td>
<td>1.90%</td>
<td>1.90%</td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>1.62%</td>
<td>1.21%</td>
<td>2.58%</td>
</tr>
<tr>
<td>$E[r_d^* - r_f]$</td>
<td>5.57%</td>
<td>4.10%</td>
<td>0.19%</td>
</tr>
<tr>
<td><strong>Second Moments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta c}/\sigma_{\Delta y}$</td>
<td>0.61</td>
<td>0.61</td>
<td>1.13</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}/\sigma_{\Delta c}$</td>
<td>4.38</td>
<td>1.85</td>
<td>0.65</td>
</tr>
<tr>
<td>$\sigma_{\Delta s}/\sigma_{\Delta y}$</td>
<td>2.10</td>
<td>1.64</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{\Delta z}/\sigma_{\Delta y}$</td>
<td>1.22</td>
<td>1.52</td>
<td>1.54</td>
</tr>
<tr>
<td>$\sigma_{\Delta c}$</td>
<td>1.42%</td>
<td>1.42%</td>
<td>2.58%</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.67%</td>
<td>0.30%</td>
<td>0.09%</td>
</tr>
<tr>
<td>$\sigma_{r_d-r_f}$</td>
<td>14.98%</td>
<td>6.35%</td>
<td>4.10%</td>
</tr>
<tr>
<td>$\sigma_Q$</td>
<td>12.85%</td>
<td>16.46%</td>
<td>5.19%</td>
</tr>
</tbody>
</table>

This table presents annual first and second moments from the endogenous growth (ENDO) model, the exogenous growth (EXO) model, and the data. The models are calibrated at a quarterly frequency and the moments are annualized. Since the equity risk premium from the models is unlevered, we follow Boldrin, Christiano, and Fisher (2001) and compute the levered risk premium from the model as: $r_{d,t+1}^* - r_{f,t} = (1 + \kappa)(r_{d,t+1} - r_{f,t})$, where $r_d$ is the unlevered return and $\kappa$ is the average aggregate debt-to-equity ratio, which is set to $\frac{2}{3}$. Annual macro data are obtained from the BEA, BLS, and NSF. Monthly return data are from CRSP and the corresponding sample moments are annualized. The data sample is 1953-2008.
### Table 3: Innovation Dynamics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO</th>
<th>(\sigma_{\Delta s})</th>
<th>4.89%</th>
<th>3.82%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(AC1(\Delta s))</td>
<td>0.21</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(AC1(\Delta n))</td>
<td>0.90</td>
<td>0.94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(AC1(S/N))</td>
<td>0.93</td>
<td>0.93</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table reports summary statistics for innovation-related variables. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual macro data are from the BEA, BLS, and NSF.

### Table 4: Expected Productivity Growth Rate Dynamics

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>ENDO</th>
<th>(\rho_{\tilde{x}})</th>
<th>0.93</th>
<th>0.95</th>
<th>(\sigma(\tilde{x}))</th>
<th>1.10%</th>
<th>1.20%</th>
</tr>
</thead>
</table>

This table reports the annual persistence and standard deviation of the expected growth rate component of productivity growth from the data and from the endogenous growth (ENDO) model. The estimates are taken from Croce (2010), where the expected growth rate component of productivity \(\tilde{x}_{t-1}\) is a latent variable that is assumed to follow an AR(1). In contrast, in the ENDO model the expected growth rate component is the growth rate of the variety of intermediate goods \(\Delta n_t\), an endogenous structural variable of the model. In particular, since the shock \(\Omega_t\) is persistent, log productivity growth can be written approximately as \(\Delta z_t = \tilde{x}_{t-1} + \epsilon_t\), where \(\tilde{x}_{t-1} \equiv \Delta n_t\) and \(\epsilon_t\) is an iid disturbance. The ENDO model endogenously generates a productivity process that is the same as the exogenous specification of Croce (2010), which is supported empirically.

### Table 5: Productivity Dynamics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
<th>(AC1(\Delta z))</th>
<th>0.09</th>
<th>0.11</th>
<th>-0.020</th>
<th>(\sigma(E_t[\Delta z_{t+1}]))</th>
<th>0.38%</th>
<th>0.27%</th>
<th>(\sigma(\Delta z(5)))</th>
<th>9.29%</th>
<th>7.45%</th>
<th>(\sigma(\Delta z(10)))</th>
<th>15.79%</th>
<th>9.92%</th>
<th>(\sigma(\Delta z(20)))</th>
<th>25.24%</th>
<th>12.67%</th>
</tr>
</thead>
</table>

This table reports summary statistics for productivity growth: Annual autocorrelation, volatility of the conditional mean, and 5, 10 and 20 year volatilities. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual macro data are from the BEA, BLS, and NSF.
Table 6: Productivity Growth Forecasts

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\beta}$</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.014</td>
<td>0.009</td>
</tr>
<tr>
<td>2</td>
<td>0.031</td>
<td>0.015</td>
</tr>
<tr>
<td>3</td>
<td>0.049</td>
<td>0.024</td>
</tr>
<tr>
<td>4</td>
<td>0.069</td>
<td>0.032</td>
</tr>
<tr>
<td>5</td>
<td>0.091</td>
<td>0.041</td>
</tr>
</tbody>
</table>

This table presents productivity growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons (k) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, $\Delta z_{t+1} + \cdots + \Delta z_{t+k-1,t+k} = \alpha + \beta s_t + \nu_{t,t+k}$. In the data, the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Multifactor productivity data and R&D stock data are from the BLS. R&D flow data are from the NSF.
Table 7: Consumption Dynamics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC1(Δc)</td>
<td>0.40</td>
<td>0.46</td>
<td>-0.002</td>
</tr>
<tr>
<td>σ(Ex[Δct+1])</td>
<td>0.51%</td>
<td>0.17%</td>
<td></td>
</tr>
<tr>
<td>σΔc(5)</td>
<td>6.63%</td>
<td>5.63%</td>
<td></td>
</tr>
<tr>
<td>σΔc(10)</td>
<td>11.97%</td>
<td>7.70%</td>
<td></td>
</tr>
<tr>
<td>σΔc(20)</td>
<td>21.18%</td>
<td>10.22%</td>
<td></td>
</tr>
</tbody>
</table>

This table reports summary statistics for consumption growth: Annual autocorrelation, volatility of the conditional mean, and 5, 10 and 20 year volatilities. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual macro data are from the BEA, BLS, and NSF.

Table 8: Consumption Growth Forecasts

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\beta}$</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.017</td>
<td>0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.034</td>
<td>0.012</td>
</tr>
<tr>
<td>3</td>
<td>0.048</td>
<td>0.017</td>
</tr>
<tr>
<td>4</td>
<td>0.062</td>
<td>0.023</td>
</tr>
<tr>
<td>5</td>
<td>0.077</td>
<td>0.030</td>
</tr>
</tbody>
</table>

This table presents consumption growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons (k) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, $\Delta c_{t+1} + \cdots + \Delta c_{t+k-1, t+k} = \alpha + \beta \hat{s}_t + \nu_{t+k}$. In the data the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Consumption data is from the BEA, R&D flow data is from the NSF, and R&D stock data is from the BLS.
Table 9: First Autocorrelations

<table>
<thead>
<tr>
<th>AC1(Δz)</th>
<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.09</td>
<td>0.11</td>
<td>-0.02</td>
</tr>
<tr>
<td>AC1(Δc)</td>
<td>0.40</td>
<td>0.46</td>
<td>-0.002</td>
</tr>
<tr>
<td>AC1(Δy)</td>
<td>0.37</td>
<td>0.21</td>
<td>0.001</td>
</tr>
<tr>
<td>AC1(Δi)</td>
<td>0.25</td>
<td>0.14</td>
<td>0.012</td>
</tr>
<tr>
<td>AC1(Q)</td>
<td>0.95</td>
<td>0.96</td>
<td>0.89</td>
</tr>
</tbody>
</table>

This table reports first autocorrelations of annual variables. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual macro data are from the BEA, BLS, and NSF.

Table 10: Volatility of Expected Growth Rates

<table>
<thead>
<tr>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ(E[Δzt+1])</td>
<td>0.38%</td>
</tr>
<tr>
<td>σ(E[Δct+1])</td>
<td>0.51%</td>
</tr>
<tr>
<td>σ(E[Δyt+1])</td>
<td>0.42%</td>
</tr>
<tr>
<td>σ(E[Δit+1])</td>
<td>0.37%</td>
</tr>
<tr>
<td>σ(E[Δdt+1])</td>
<td>0.92%</td>
</tr>
</tbody>
</table>

This table reports annualized volatilities of expected growth rates from the endogenous growth (ENDO) and exogenous growth (EXO) models.

Table 11: Volatility of High-, Medium-, and Low-Frequency Components

<table>
<thead>
<tr>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>ENDO</td>
<td>EXO</td>
</tr>
<tr>
<td>σΔy</td>
<td>1.95%</td>
<td>2.17%</td>
</tr>
<tr>
<td></td>
<td>0.69%</td>
<td>0.62%</td>
</tr>
<tr>
<td></td>
<td>0.49%</td>
<td>0.30%</td>
</tr>
</tbody>
</table>

This table reports the annualized volatilities of high-, medium-, and low-frequency components of output growth from the data and from the ENDO and EXO models. The bandpass filter from Christiano and Fitzgerald (2003) is used to isolate the components of the various frequencies. The high-frequency component is defined as a bandwidth of 2 to 32 quarters. The medium-frequency component is defined as a bandwidth of 32 to 200 quarters. The low-frequency component is defined as a bandwidth of 200 to 400 quarters. Quarterly output data is from the BEA.
This table presents output growth forecasting regressions from the data and from the endogenous growth (ENDO) model for horizons \((k)\) of one year to five years. Specifically, we project real per capita output growth on log R&D intensity, \(\Delta c_{t+1} + \cdots + \Delta c_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}\). In the data, the regression is estimated via OLS with Newey-West standard errors with \(k - 1\) lags. The model regression results correspond to the population values. Overlapping annual observations are used. Output data is from the BEA, R&D flow data is from the NSF, and R&D stock data is from the BLS.

<table>
<thead>
<tr>
<th>Horizon ((k))</th>
<th>Data</th>
<th>ENDO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\hat{\beta})</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.020</td>
<td>0.013</td>
</tr>
<tr>
<td>2</td>
<td>0.046</td>
<td>0.022</td>
</tr>
<tr>
<td>3</td>
<td>0.068</td>
<td>0.029</td>
</tr>
<tr>
<td>4</td>
<td>0.089</td>
<td>0.041</td>
</tr>
<tr>
<td>5</td>
<td>0.114</td>
<td>0.051</td>
</tr>
</tbody>
</table>
Table 13: Sensitivity Analysis

<table>
<thead>
<tr>
<th>First Moments</th>
<th>ENDO</th>
<th>ENDO-CRRA</th>
<th>EXO</th>
<th>EXO-AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Delta y]$</td>
<td>1.90%</td>
<td>0.51%</td>
<td>1.90%</td>
<td>1.90%</td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>1.21%</td>
<td>2.44%</td>
<td>2.58%</td>
<td>2.60%</td>
</tr>
<tr>
<td>$E[r^*_d - r_f]$</td>
<td>4.10%</td>
<td>0.61%</td>
<td>0.19%</td>
<td>1.44%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second Moments</th>
<th>ENDO</th>
<th>ENDO-CRRA</th>
<th>EXO</th>
<th>EXO-AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\Delta c}/\sigma_{\Delta y}$</td>
<td>0.61</td>
<td>1.28</td>
<td>1.13</td>
<td>0.63</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}/\sigma_{\Delta c}$</td>
<td>1.85</td>
<td>0.29</td>
<td>0.65</td>
<td>2.86</td>
</tr>
<tr>
<td>$\sigma_{\Delta s}/\sigma_{\Delta y}$</td>
<td>1.64</td>
<td>0.82</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{r_f}$</td>
<td>0.30%</td>
<td>0.42%</td>
<td>0.09%</td>
<td>0.10%</td>
</tr>
<tr>
<td>$\sigma_{r_d - r_f}$</td>
<td>6.35%</td>
<td>2.07%</td>
<td>4.10%</td>
<td>1.43%</td>
</tr>
<tr>
<td>$AC(\Delta c)$</td>
<td>0.46</td>
<td>0.06</td>
<td>-0.002</td>
<td>0.14</td>
</tr>
<tr>
<td>$\sigma(E_t[\Delta c_{t+1}])$</td>
<td>0.51%</td>
<td>0.04%</td>
<td>0.17%</td>
<td>0.18%</td>
</tr>
</tbody>
</table>

This table compares key summary statistics from the benchmark ENDO and EXO models with alternative specifications. ENDO-CRRA is the same as the ENDO model but with the one modification that $\psi = \frac{1}{\gamma}$, so that the Epstein-Zin preferences collapse to CRRA utility. All the other parameters are kept the same as the benchmark calibration. EXO-AC is the same as the EXO model with the one modification that the adjustment cost parameter $\zeta$ is set to match consumption growth volatility, which corresponds to a value of 5.1. All other parameters are kept the same as the benchmark calibration.

Table 14: Sensitivity Analysis: Preference Parameters

<table>
<thead>
<tr>
<th>First Moments</th>
<th>ENDO</th>
<th>$\gamma = 4$</th>
<th>$\gamma = 18$</th>
<th>$\psi = 0.5$</th>
<th>$\psi = 2.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Delta y]$</td>
<td>1.90%</td>
<td>1.89%</td>
<td>2.02%</td>
<td>0.86%</td>
<td>2.38%</td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>1.21%</td>
<td>2.11%</td>
<td>0.06%</td>
<td>2.23%</td>
<td>0.87%</td>
</tr>
<tr>
<td>$E[r^*_d - r_f]$</td>
<td>4.10%</td>
<td>1.55%</td>
<td>7.55%</td>
<td>1.28%</td>
<td>5.06%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second Moments</th>
<th>ENDO</th>
<th>$\gamma = 4$</th>
<th>$\gamma = 18$</th>
<th>$\psi = 0.5$</th>
<th>$\psi = 2.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\Delta c}/\sigma_{\Delta y}$</td>
<td>0.61</td>
<td>0.61</td>
<td>0.62</td>
<td>1.09</td>
<td>0.52</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}/\sigma_{\Delta c}$</td>
<td>1.85</td>
<td>1.86</td>
<td>1.83</td>
<td>0.57</td>
<td>2.37</td>
</tr>
<tr>
<td>$\sigma_{\Delta s}/\sigma_{\Delta y}$</td>
<td>1.64</td>
<td>1.63</td>
<td>1.66</td>
<td>1.11</td>
<td>1.73</td>
</tr>
<tr>
<td>$\sigma_{r_f}$</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.43%</td>
<td>2.61%</td>
<td>1.21%</td>
</tr>
<tr>
<td>$\sigma_{r_d - r_f}$</td>
<td>6.35%</td>
<td>6.36%</td>
<td>6.36%</td>
<td>3.41%</td>
<td>6.95%</td>
</tr>
<tr>
<td>$AC(\Delta c)$</td>
<td>0.46</td>
<td>0.47</td>
<td>0.46</td>
<td>0.07</td>
<td>0.62</td>
</tr>
<tr>
<td>$\sigma(E_t[\Delta c_{t+1}])$</td>
<td>0.51%</td>
<td>0.52%</td>
<td>0.52%</td>
<td>0.19%</td>
<td>0.59%</td>
</tr>
</tbody>
</table>

This table compares key summary statistics from the benchmark calibration of the endogenous growth model (ENDO) with other calibrations that vary the preference parameters, risk aversion $\gamma$ and the elasticity of intertemporal substitution $\psi$, one at a time while holding all other parameters fixed at the benchmark calibration. In the benchmark calibration for ENDO, $\gamma = 10$ and $\psi = 1.85$. The models are calibrated at a quarterly frequency and the summary statistics are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).
This table compares the key summary statistics from the benchmark calibration of the endogenous growth (ENDO) model with other calibrations that vary the capital adjustment cost parameter $\xi$ and the persistence parameter $\rho$ of the shock $\log(\Omega_t)$ one at a time while holding all other parameters fixed at the benchmark calibration. In the benchmark calibration for ENDO, $\xi = 0.70$ and $\rho^4 = .95$. The models are calibrated at a quarterly frequency and the summary statistics are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).
Table 16: Calibration of Stochastic Volatility Process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\sigma}$</td>
<td>Mean of $\sigma$</td>
<td>1.75%</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Autocorrelation of $\sigma$</td>
<td>0.98</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>Volatility of shock $e$</td>
<td>$8 \times 10^{-4}$%</td>
</tr>
</tbody>
</table>

This table reports the quarterly calibration of the stochastic volatility process for the ENDO-SV model.

Table 17: Annualized Summary Statistics: Stochastic Volatility

<table>
<thead>
<tr>
<th></th>
<th>ENDO</th>
<th>ENDO-SV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[\Delta y]$</td>
<td>1.90%</td>
<td>1.90%</td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>1.21%</td>
<td>1.24%</td>
</tr>
<tr>
<td>$E[r^*_d - r_f]$</td>
<td>4.10%</td>
<td>3.94%</td>
</tr>
<tr>
<td><strong>Second Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta c}/\sigma_{\Delta y}$</td>
<td>0.61</td>
<td>0.64</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}/\sigma_{\Delta c}$</td>
<td>1.85</td>
<td>1.72</td>
</tr>
<tr>
<td>$\sigma_{\Delta s}/\sigma_{\Delta c}$</td>
<td>1.64</td>
<td>1.59</td>
</tr>
<tr>
<td>$\sigma_{\Delta c}$</td>
<td>1.42%</td>
<td>1.49%</td>
</tr>
<tr>
<td>$\sigma_{r_f}$</td>
<td>0.30%</td>
<td>0.32%</td>
</tr>
<tr>
<td>$\sigma_{r_d-r_f}$</td>
<td>6.35%</td>
<td>6.17%</td>
</tr>
<tr>
<td>$ACF_1(\Delta c)$</td>
<td>0.46</td>
<td>0.43</td>
</tr>
<tr>
<td>$\sigma(E[r_{t+1}])$</td>
<td>0.51</td>
<td>0.51</td>
</tr>
</tbody>
</table>

This table presents annual first and second moments from the endogenous growth (ENDO) model and endogenous growth with stochastic volatility (ENDO-SV) model, and the data. The models are calibrated at a quarterly frequency and the summary statistics are annualized. Annual macro data are obtained from the BEA and NSF. Monthly return data are from CRSP and the corresponding sample moments are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).
### Table 18: Risk Premia Forecasting Regressions

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO-SV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\beta}$</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>-0.113</td>
<td>0.042</td>
</tr>
<tr>
<td>2</td>
<td>-0.194</td>
<td>0.079</td>
</tr>
<tr>
<td>3</td>
<td>-0.247</td>
<td>0.097</td>
</tr>
<tr>
<td>4</td>
<td>-0.290</td>
<td>0.106</td>
</tr>
<tr>
<td>5</td>
<td>-0.371</td>
<td>0.122</td>
</tr>
</tbody>
</table>

This table presents return forecasting regressions from the data and from the endogenous growth model with stochastic volatility for horizons (k) of one to five years with overlapping quarterly data. Specifically, we project risk premia on the log price-dividend ratio, $r_{t+1}^{ex} + \cdots r_{t+k-1,t+k}^{ex} = \alpha + \beta(p_t - d_t) + \nu_{t,t+k}$. In the data, the regression is estimated via OLS with Newey-West standard errors with $4 \times k - 1$ lags. The model regression results correspond to the population values. Return, price and dividend data are from CRSP.
Table 19: Calibration of R&D Shock $f_t$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_f$</td>
<td>Standard deviation of shock $\epsilon_f$</td>
<td>1.00%</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Autocorrelation of $\sigma$</td>
<td>0.96</td>
</tr>
</tbody>
</table>

This table reports the quarterly calibration of the shock to R&D productivity for the ENDO-RD model.

Table 20: Annualized Summary Statistics: Stochastic R&D Productivity

<table>
<thead>
<tr>
<th></th>
<th>ENDO</th>
<th>ENDO-RD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[\Delta y]$</td>
<td>1.90%</td>
<td>2.50%</td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>1.21%</td>
<td>1.22%</td>
</tr>
<tr>
<td>$E[\hat{r}_d - r_f]$</td>
<td>4.10%</td>
<td>5.29%</td>
</tr>
<tr>
<td><strong>Second Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta c} / \sigma_{\Delta y}$</td>
<td>0.61</td>
<td>0.89</td>
</tr>
<tr>
<td>$\sigma_{\Delta i} / \sigma_{\Delta c}$</td>
<td>1.85</td>
<td>1.33</td>
</tr>
<tr>
<td>$\sigma_{\Delta s} / \sigma_{\Delta y}$</td>
<td>1.64</td>
<td>1.82</td>
</tr>
<tr>
<td>$\sigma_{\Delta c}$</td>
<td>1.42%</td>
<td>2.12%</td>
</tr>
<tr>
<td>$\sigma_{r_f}$</td>
<td>0.30%</td>
<td>0.47%</td>
</tr>
<tr>
<td>$\sigma_{r_d-r_f}$</td>
<td>6.35%</td>
<td>6.73%</td>
</tr>
<tr>
<td>$ACF_1(\Delta c)$</td>
<td>0.46</td>
<td>0.39</td>
</tr>
<tr>
<td>$\sigma(E_t[\Delta c_{t+1}])$</td>
<td>0.51</td>
<td>0.87</td>
</tr>
</tbody>
</table>

This table presents annual first and second moments from the endogenous growth (ENDO) model and endogenous growth with stochastic R&D productivity (ENDO-RD) model, and the data. The models are calibrated at a quarterly frequency and the summary statistics are annualized. Annual macro data are obtained from the BEA and NSF. Monthly return data are from CRSP and the corresponding sample moments are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).
This figure plots expected log productivity growth $E_t[\Delta z_{t+1}]$ (thick line) and R&D intensity $\frac{S_t}{N_t}$ (thin line) from the ENDO model for a sample simulation of 200 quarters. In the model, R&D intensity is the key determinant of expected growth rates.
The left panel plots demeaned log consumption growth $\Delta c_t$ (thin line) with R&D intensity $S_t N_{t-1}$ (thick bold line) from the ENDO model for a sample simulation of 200 quarters. The right panel plots demeaned log output growth $\Delta y_t$ (thin line) with R&D intensity $S_t N_{t-1}$ (thick bold line) from the ENDO model for a sample simulation of 200 quarters. In the model, R&D intensity is the key determinant of expected growth rates.

The left panel plots demeaned log consumption growth $\Delta c_t$ (dashed line) with R&D intensity $S_t N_{t-1}$ (bold line) from the data. The right panel plots demeaned log output growth $\Delta y_t$ (dashed line) with R&D intensity $S_t N_{t-1}$ (bold line) from the data. Annual data on aggregate output and consumption is from the BEA. Annual data on R&D expenditures are from the NSF and data on R&D stocks are from the BLS. In the model, R&D intensity is the key determinant of expected growth rates.
This figure shows quarterly log-deviations from the steady state for the ENDO model. All deviations are multiplied by 100.
This figure plots expected log consumption growth $E_t[\Delta c_{t+1}]$ (thick line) and R&D intensity $\frac{S_t}{N_t}$ (thin line) from the ENDO model for a sample simulation of 200 quarters. In the model, R&D intensity is the key determinant of expected growth rates.
Figure 6: Consumption Growth with Medium-Frequency Component

The left panel plots the demeaned log consumption growth rate (thin line) with the medium-frequency component (thick line) from the data. The right panel plots the demeaned log consumption growth rate (thin line) with the medium-frequency component (thick line) from the ENDO model. The medium-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (2003) and selecting a bandwidth of 32 to 200 quarters. Quarterly consumption data are obtained from the BEA.
The left panel plots the demeaned log output growth rate (thin line) with the medium-frequency component (thick line) from the data. The right panel plots the demeaned log output growth rate (thin line) with the medium-frequency component (thick line) from the ENDO model. The medium-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (2003) and selecting a bandwidth of 32 to 200 quarters. Quarterly output data are obtained from the BEA.

This figure plots the medium-frequency growth components for productivity (dashed line), output (thin line), and consumption (bold line). The left panel corresponds to a sample simulation from the ENDO model and the right panel corresponds to the data. The medium-frequency component is obtained by applying the bandpass filter from Christiano and Fitzgerald (2003) to annual data and selecting a bandwidth of 8 to 50 years. Annual data on GDP and consumption are from the BEA and annual productivity data are from the BLS.
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.
Figure 10: Expected Growth Rates

This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.
The left panel plots cross-correlations of the medium-frequency component of the equity return and the medium-frequency component of consumption growth for the ENDO (bold line) and EXO (dashed line) models: \( \text{corr}(r_{d,t}, \Delta c_{t+k}) \). The right panel plots the same cross-correlations from the data. The medium-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (2003) and selecting a bandwidth of 32 to 200 quarters. Quarterly consumption data is obtained from the BEA. Monthly return data is obtained from CRSP and then compounded to a quarterly frequency.

This figure plots the medium-frequency components for productivity growth (bold line) and for the price-dividend ratio (thin line). The left panel corresponds to a sample simulation from the ENDO model and the right panel corresponds to the data. The medium-frequency component is obtained by applying the bandpass filter from Christiano and Fitzgerald (2003) to annual data and selecting a bandwidth of 8 to 50 years. The correlation between the two series is 0.46 in the data and 0.67 in the model. Annual data on productivity are from the BLS and price-dividend data are from CRSP.
This figure shows quarterly log-deviations from the steady state from a one standard deviation shock to volatility for the endogenous growth model with stochastic volatility. All deviations are multiplied by 100.
This figure shows quarterly log-deviations from the steady state from a one standard deviation shock to R&D productivity in the endogenous growth model. All deviations are multiplied by 100.