Merger Review for Markets with Buyer Power*

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Abstract

We analyze the competitive effects of mergers in markets with buyer power. Using mechanism design arguments, we show that without cost synergies, mergers harm buyers, regardless of buyer power. However, buyer power mitigates the harm to a buyer from a merger of symmetric suppliers. With buyer power, a merger increases incentives for entry, increases investment incentives for rivals, and can increase investment incentives for merging parties. Because buyer power reduces the profitability of a merger, it increases the profitability of perfect collusion relative to a merger. Cost synergies can eliminate merger harm, but also render otherwise profitable mergers unprofitable.

Keywords: unilateral effects, cost efficiencies, innovation, auctions versus negotiations

JEL Classification: D44, D82, L41

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1 Introduction

Buyer power features prominently in the antitrust analysis of mergers. That buyer power will prevent merging suppliers from being able to negotiate higher prices is a frequent merger defense. As one observer put it, buyer power is sometimes embraced by courts as if it had “talismanic power.” Because powerful buyers can withstand upward price pressure from mergers of suppliers, so this appealing argument goes, they are not harmed by such mergers.

In large part because the most commonly used models in merger review, which are based on Cournot or Bertrand competition, do not accommodate powerful buyers, it is often not clear what exactly is meant by buyer power. However, this does not mean that buyer power lacks empirical plausibility. For example, computer manufacturers such as Dell and HP procure components from upstream suppliers using competitive procurements and face-to-face negotiations. Although these buyers might value having the latest generation of a component, they may also be willing to continue to manufacture using a prior generation in the absence of a sufficiently low price for the new generation. Or they may be willing to take a tough negotiating stance with the low-cost supplier, even if it means the possibility of ultimately having to purchase from a higher-cost supplier. As another case in point, oil companies such as Shell, Exxon-Mobil, and BP procure oilfield services for their wells using competitive procurements and negotiations in which they play oilfield services providers off against one another. Likewise, municipalities procuring road improvements, park landscaping, and other city services sometimes cancel procurements in the face of what they view as insufficiently competitive pricing.

The use of competitive procurement processes by the buyers in these examples makes it natural to model buyer power in the context of a procurement. This is the approach we take in this paper. We view the buyer as designing a procurement mechanism in which

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1See, e.g., Steptoe (1993) and Carlton and Israel (2011).
2As stated by Steptoe (1993, p. 494), “Although the strong-buyer defense may be valid in a variety of circumstances, I believe that the courts have sometimes embraced it as if it had talismanic power that cured all doubts about a merger.”
3According to the U.S. Horizontal Merger Guidelines (hereafter U.S. Guidelines) which guide courts in the United States in how to evaluate the potential anticompetitive effects of a merger, “The Agencies consider the possibility that powerful buyers may constrain the ability of the merging parties to raise prices.” (U.S. Guidelines, p. 27) Merger guidelines in other jurisdictions provide a similar treatment of buyer power. The European Commission’s Guidelines on the Assessment of Horizontal Mergers (hereafter EC Guidelines) discuss the possibility that “buyer power would act as a countervailing factor to an increase in market power resulting from the merger.” (para. 11) The Australian Competition and Consumer Commission’s Merger Guidelines view “countervailing power” as a competitive constraint that can limit merger harms. (paras. 1.4, 5.3, 7.48)
4See Kumar et al. (2015, online appendix).
suppliers participate. The suppliers’ costs are their private information, and so the buyer’s mechanism is constrained by incentive compatibility and individual rationality for the suppliers. Similar to Bulow and Klemperer’s (1996) approach to modeling environments with and without a powerful seller, in our model a buyer with no buyer power must rely on the outcome of an efficient auction among potential suppliers, with no ability to further negotiate. In contrast, a powerful buyer can demand discriminatory discounts from suppliers and negotiate with the auction winner, thereby implementing the buyer surplus maximizing mechanism, subject to incentive compatibility and individual rationality.

Our approach captures, in a general way, procurement markets, where purchasing can involve combinations of requests for proposals, auctions, and negotiations. It combines this generality with a disciplined, principled analysis of buyer power, without being subject to a pitfall of complete information models that bargaining is always efficient. Our procurement-based approach allows us to build on insights from the theory of optimal auctions and to exploit results from mechanism design. The framework accounts for market power because each supplier is treated as having a monopoly over its private information. Yet, suppliers are “oligopolistic” in the sense that when there are more of them, the market power of each individual supplier decreases.

We model a merger without cost synergies as allowing the merged entity to produce at a cost equal to the minimum of the costs of the merging suppliers. Using revealed preference arguments, we show that without cost synergies, a merger of suppliers is harmful for a buyer regardless of buyer power, despite the fact that the buyer demands a lower price from the merged entity than it would from symmetric pre-merger suppliers. Thus, although buyer power can deter harmful mergers, it can never eliminate harm if mergers occur.

We also analyze the effect of a merger on incentives to merge, enter, and innovate. Absent buyer power, mergers without synergies are neutral for rivals and potential entrants and profitable for the merging suppliers. With buyer power, a merger of symmetric suppliers is beneficial for rivals and increases the expected profits of entrants. Buyer power does not necessarily render a merger unprofitable for the merging suppliers, but it can, in some cases even for a merger to monopoly. Regarding innovating, we show that the

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5 For an early criticism of complete information model, see, for example, Samuelson (1985, p. 322), who writes: “In pursuit of a preferred agreement, one party may threaten the other and, for credibility’s sake, bind himself to carry out the threat some portion of the time. When he does, efficiency fails. Alternatively, the parties may adopt the standard negotiation bluff, insisting on ultrafavorable (and incompatible) terms of agreement. If agents persist in these demands, a mutually beneficial agreement may be lost. Although proponents of the Coase presumption may regard these actions as irrational, it is no less true that such behavior (e.g., strikes, the carrying out of costly threats) frequently occurs. Moreover, it is unrealistic to suppose that the bargaining setting is one of perfect information.”

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effect on incentives for cost reducing investment depends on buyer power and differs for
the merging and nonmerging suppliers. Without buyer power, a merger does not affect
incentives for investment by nonmerging suppliers and increases incentives for investment
by merging suppliers. With buyer power, a merger increases incentives for investment by
nonmerging suppliers and, in some cases, increases incentives for investment by merging
suppliers.

A natural view is that mergers and perfect collusion are equivalent. Although this
way of thinking is correct without buyer power, it is misleading in its presence. First,
mergers are public events, while collusion happens under the surface. Powerful buyers
can thus be expected to take defensive actions against the increased market power of
a merged entity, but not necessarily against a cartel. Therefore, buyer power makes
collusion more profitable relative to mergers, assuming the collusion is not detected (or
suspected). Moreover, because buyer power induces more aggressive price demands by the
buyer, losing bids above these price demands are more likely with buyer power even with
competitive bidding, making collusive bidding involving deliberately losing bids harder
to detect. Consequently, buyer power makes collusion not only more profitable, but also
more difficult to detect.

This contrasts with Carlton and Israel (2011), who argue that powerful buyers may
actually be harmed more by a merger than buyers without buyer power. They base this
on the possibility that, in the absence of powerful buyers, suppliers could collude to set
monopoly prices, and thus a merger would have no effect. But, so the argument goes, with
powerful buyers, prices would be below monopoly levels before the merger and therefore
increase as a result of a merger. They conclude that “there is no theoretical necessity
that the presence of powerful buyers must always lessen the price effects from a merger”
(Carlton and Israel, 2011, p. 132). Although we do not disagree with the letter of this
statement, we show that the presence of powerful buyers is more likely to invite collusion
than the absence of buyer power and that, in our procurement setting, it is a theoretical
necessity that powerful buyers are less affected by a merger of symmetric suppliers than
those without buyer power. Thus, our results are consistent with empirical evidence
that collusion in, for example, products such as disk drives, LCD panels, and auto parts,
negatively affected seemingly powerful buyers, like Dell, HP, Microsoft, and major auto
manufacturers.

While the results described above may strike one as surprising at first, they have a
clear and simple intuition. Without buyer power, the allocation is efficient before and after
the merger. This explains the neutrality result for rivals. Because a merger eliminates a
bid, the merger harms the buyer regardless of buyer power. Without buyer power, the elimination of a competing bid is the only effect of the merger on the merging suppliers because the allocation is the same before and after merger, which explains the profitability of the merger without buyer power.

In general, the powerful buyer adjusts the allocation rule in response to the merger.\(^6\) In our model of mergers without cost synergies, the merged entity’s cost distribution is dominated in terms of the reverse hazard rate by each merging supplier’s pre-merger distribution. This means that, like the seller in Myerson’s optimal auction who discriminates against strong bidders, a powerful buyer discriminates against the merged entity in its competition with the other suppliers and applies a more aggressive reserve price to the merged entity.\(^7\) But this discrimination and the more aggressive reserve imply that the merger benefits rivals and potential entrants but may not be profitable for the merging suppliers when the buyer is powerful.

It is useful to think of buyer power as consisting of both bargaining power, which captures the ability of the buyer to discriminate among suppliers, and monopsony power, which is the buyer’s ability to set a binding reserve price. In our framework, bargaining and monopsony power do not vary with the merger; however, their optimal exertion does. For example, it may be that the buyer optimally exerts neither of these powers before a merger and optimally exerts both after a merger. These two components of buyer power not only resonate with notions that fare prominently in current antitrust commentary, but also take on a precise meaning within our framework in a way that captures the spirit of the antitrust usage.\(^8\) In addition, the notion of bargaining and monopsony power as components of buyer power proves useful for explaining many of the key results, as will become clear in what follows.

The results summarized above imply that, without cost synergies, mergers are always detrimental to buyers irrespective of their power.\(^9\) This motivates us to extend the analysis

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\(^6\)The change in the optimal allocation rule post merger makes it less obvious why the bid elimination harms the buyer. However, a revealed preference argument shows that the merger harms even the powerful buyer: the buyer could essentially use the post-merger optimal allocation rule before the merger and would be better off as a result of the additional competition between the two merging suppliers. Moving to the optimal pre-merger mechanisms leaves the buyer \textit{a fortiori} better off without the merger.

\(^7\)See McAfee and McMillan (1987) on how optimal ascending auctions involve discrimination in favor of weaker bidders.

\(^8\)The OECD Roundtable on Monopsony and Buyer Power (p. 9) found: “There are two types of buyer power: monopsony power and bargaining power. ... Both types of buyer power result in lower prices, though the lower price obtained from monopsony power is achieved through the act of purchasing less, whereas the lower price obtained from bargaining power is achieved through the threat of purchasing less.”

\(^9\)The classic result of Bulow and Klemperer (1996) that a revenue maximizing seller is better off with
to account for such cost efficiencies. We model cost efficiencies as a commonly known percentage decrease in the cost of the merged entity relative to the minimum cost of the two merging suppliers before the merger. The assumption that this percentage is known is based on cost efficiencies being part of a merger review and therefore information that both the buyer and the merging suppliers have. We show that, as expected, cost efficiencies make mergers unambiguously less harmful to the buyer and, if the efficiencies are large enough, can cause the buyer to welcome the merger. However, arguments based on cost synergies do not constitute a slam-dunk defense: The cost synergies required to make a merger acceptable to the buyer may make it unprofitable to the merging suppliers. Indeed, whether the buyer is powerful or not, the post-merger profit of the merged entity goes to zero as synergies approach 100 percent.\(^{10}\) Although cost synergies reduce costs, they also squeeze suppliers’ informational rents and thereby profits. Eventually, the latter effect dominates.

There is a related literature on merger analysis based on auction models, including Waechrer (1999), Waechrer and Perry (2003), Miller (2014), and Froeb et al. (2017). Waechrer (1999) examines mergers in both asymmetric first-price and second-price auction markets.\(^{11}\) Waechrer and Perry (2003) focus on open auctions and allow the optimal reserve to adjust post merger. Our approach differs in considering optimal procurements with asymmetric bidders and allowing varying buyer power. Miller (2014) considers a procurement setting in which buyers purchase from suppliers of differentiated products using a variant of a second-price auction and develops a stochastic model that can be calibrated to estimate merger effects. Froeb et al. (2017) consider effects of a merger between bidders in an optimal (ascending) auction when bidders draw their values from a family of power-related distributions, where each bidder’s value can be viewed as the maximum of some number of draws from a common distribution. They show that a merger reduces the auctioneer’s expected revenue and that, under certain conditions, a merger to monopoly is not profitable for the merging bidders.

Related work on buyer power and its role in merger analysis includes papers in the

\(^{10}\)As discussed later, the result for a buyer without power is sensitive to whether cost efficiencies affect the upper support of the merged entity’s cost distribution.

\(^{11}\)In addition, Dalkir et al. (2000) examine mergers in asymmetric first-price auctions using simulated equilibrium bidding strategies. Thomas (1999) examines mergers in asymmetric first-price auctions by deriving equilibrium bidding strategies for the binomial cost distribution.
vertical contracting literature in the tradition of Dobson and Waterson (1997), Horn and Wolinsky (1988), and Inderst and Wey (2007). These are complete information models with Nash or other bilateral bargaining protocols, which do not exhibit the tradeoff between efficiency and rent extraction that arises in a procurement-based model such as ours.\textsuperscript{12} In addition, our framework extends to allow multi-product firms that produce complementary products, as shown in the Online Appendix.\textsuperscript{13}

Other authors have considered the effect of a merger on incentives for investment. Motta and Tarantino (2017) show that in a Bertrand oligopoly with differentiated products, absent efficiency gains, a merger lowers total output and as a result lowers total investment in cost reduction. López and Vives (2016) focus on the effect of a symmetric increase in cross-ownership in a symmetric Cournot model with R&D spillovers.\textsuperscript{14} They show that an increase in cross-ownership increases cost-reducing investment for high levels of R&D spillovers but decreases investment for low levels of R&D spillovers.\textsuperscript{15} Our model has no R&D spillovers (investment by one supplier has no effect on other suppliers’ costs), but we focus on the change in cross-ownership resulting from a merger, which necessarily involves asymmetries. Although we do not pursue it here, our approach can also be extended to account for cross-ownership among suppliers along the lines of Lu (2012).

The paper is structured as follows. Section 2 defines the setup. Sections 3 and 4 consider mergers in the absence of cost efficiencies. Section 3 shows that mergers harm buyers. Section 4 analyzes effects on all parties and on their incentives assuming ex ante symmetric suppliers. Section 5 incorporates cost efficiencies. Section 6 briefly discusses the generalization to allow for ex ante asymmetries. Section 7 concludes.

\textsuperscript{12}A related empirical literature is also grounded in models of Nash bargaining, including Crawford and Yurukoglu (2012), Gowrisankaran et al. (2015), and Collard-Wexler et al. (forthcoming).

\textsuperscript{13}O’Brien and Shaffer (2005) consider mergers with multi-product suppliers in a complete information setup with Nash bargaining.

\textsuperscript{14}Nocke and Whinston (2010) and Mermelstein et al. (2015) provide models of sequential mergers in the Cournot setup.

\textsuperscript{15}In the model of López and Vives (2016), when spillovers are high, the dominant effect of the decrease in competition associated with cross-ownership is to allow investing firms to better appropriate the benefits of their investments, increasing incentives for investment. However, when spillovers are low, the dominant effect of increased cross-ownership is a reduction in output and a corresponding reduction in incentives for cost-reducing investment.
2 Setup

In the baseline setup, we consider one product and one buyer.\textsuperscript{16} In the pre-merger market, there are \( n \geq 2 \) suppliers, indexed 1, ..., \( n \). Each supplier \( i \in \{1, ..., n\} \) draws a cost \( c_i \) independently from a continuously differentiable distribution \( G_i \) with support \([c, \bar{c}]\) and density \( g_i \) that is positive on the interior of the support. Each supplier is privately informed about its type, and so the suppliers’ types are unknown to the buyer. The buyer has value \( v > c \) for one unit of the product. All of this is common knowledge.

We refer to the model with \( G_i = G \) for all \( i \) as the symmetric model, and the one where different suppliers have different distributions as the asymmetric model. Section 3 applies to the more general asymmetric model. In Sections 4 and 5, we specialize to the symmetric model for clarity of exposition, highlighting places where the assumption of symmetry is a restriction. In Section 6 we discuss the extent to which our results extend to the asymmetric model.

We let suppliers 1 and 2 be the merging suppliers. Like Farrell and Shapiro (1990), we model a merger as allowing the merging suppliers to rationalize production by producing using the lower of their two costs.\textsuperscript{17} We discuss the possibility of further cost efficiencies from the merger in Section 5. Thus, given pre-merger costs \( c = (c_1, ..., c_n) \), in the corresponding post-merger market, the nonmerging suppliers have the same cost as before the merger, and the merged entity has cost \( c = \min \{c_1, c_2\} \).

We denote the distribution for the minimum of the pre-merger costs of suppliers 1 and 2 by \( \hat{G}(c) \equiv 1 - (1 - G_1(c))(1 - G_2(c)) \), with density \( \hat{g} \). Under symmetry, we have \( \hat{G}(c) = 1 - (1 - G(c))^2 \). Thus, even if all suppliers are ex ante symmetric in the pre-merger market, in the post-merger market they are not because the merged entity draws its cost from a different distribution.\textsuperscript{18}

Buyers and suppliers are risk neutral. A buyer’s payoff is zero if it does not trade and is equal to its value minus the price paid if trade does occur. Similarly, a supplier’s payoff is zero if it does not trade and is equal to the payment it receives minus its cost if it does trade.

\textsuperscript{16}The assumption of one buyer is conservative from the perspective of providing the scenario in which buyer power is most likely to enable a buyer to remedy merger harms.

\textsuperscript{17}This is the approach also taken by Salant et al. (1983), Perry and Porter (1985), Waehrer (1999), Dalkir et al. (2000), and Froeb et al. (1999). This type of merger is equivalent to efficient (observable) collusion as discussed by Mailath and Zemsky (1991) and McAfee and McMillan (1992).

\textsuperscript{18}If one perceives a merger as an acquisition of one firm by another, then the acquiring firm might view itself as more efficient because it draws its cost from \( \hat{G} \) rather than \( G \); however, because the distribution of the minimum cost across all firms is unchanged, it seems appropriate to refer to this as an acquisition/merger without cost efficiencies.
The setup’s merits include the assumption of private information and independently distributed private types.\textsuperscript{19} This neither presumes nor precludes efficiency and results in a tradeoff between efficiency and rent extraction when the buyer is powerful. It also means that the optimal Bayesian mechanism provides a practical benchmark.\textsuperscript{20}

Modelling buyer power as the ability of a buyer with demand for a single unit to commit to an optimal procurement is natural from a methodological perspective.\textsuperscript{21} Although in a setup with single-unit demand there is no scope to meaningfully speak of large and small buyers, there are reasons why a “real-world” large buyer might be more likely to have such commitment power than a small buyer.\textsuperscript{22} For example, large buyers are likely to be around longer, ceteris paribus, so reneging on commitments may be more costly to their long-run reputation. Also, larger buyers are likely to be active in more markets than smaller buyers, so again may obtain more value from building up a reputation and incur more cost from damaging that reputation by violating commitments. Further, a large buyer may be more likely to have the sophistication required to implement an optimal procurement.

The buyer uses a procurement procedure to select a supplier and determine a purchase price. We adapt the approach used by Bulow and Klemperer (1996) to our procurement setup and extend it to accommodate asymmetric suppliers. Bulow and Klemperer (1996) model a seller without power as using a second-price auction and a seller with power as using an ascending auction followed by a take-it-or-leave-it offer by the seller to the auction winner, which is an optimal sales mechanism in their setup. In our procurement context, the case of no buyer power corresponds to a buyer using a descending clock auction with a reserve (or starting price) equal to the minimum of the buyer’s value and the upper

\textsuperscript{19}Our procurement-based model highlights a difference between a setup in which suppliers’ private information is cost versus one in which it is quality. Although the suppliers’ payoffs can be modeled similarly in either case, with private information regarding quality, the buyer’s payoff would depend directly on the trading supplier’s private information. This would induce an element of interdependency in valuations, which in general renders very different (and arguably less practical) mechanisms optimal (see Mezzetti, 2004, 2007).

\textsuperscript{20}The assumption of independently distributed types is made on theoretical grounds and captures the notion that there may be a trade-off between profit and efficiency. With correlated types, no matter how small the degree of correlation, profit maximization and efficiency are in no conflict, as shown by Crémer and McLean (1985, 1988), but this requires mechanisms that involve gambles that do not seem plausible or realistic; see, for example, Kosmopoulou and Williams (1998) and Börgers (2015).

\textsuperscript{21}In principle, the approach of Myerson (1981) extends straightforwardly to settings in which the buyer has demand for multiple units with commonly known values for these, provided the suppliers’ private information remains one-dimensional. However, a merger between two suppliers creates a multi-unit supplier that is most naturally thought of as having two-dimensional private information. If the buyer has demand for one unit only, the problem with multi-dimensional private information is moot.

\textsuperscript{22}For two case studies of buyer power, see Roberts et al. (2011).
bound of the support of the cost distribution, that is, \( \min\{v, \overline{c}\} \).\(^{23}\) This is an efficient mechanism. The case of buyer power corresponds to a buyer using an optimal procurement mechanism, which quite generally (for example, under the assumptions introduced in Section 4) can be implemented as a discriminatory descending clock auction possibly followed by a final take-it-or-leave-it offer by the buyer to the auction winner. The details of this implementation of the optimal procurement mechanism are provided in Section 4 as they are not required in Section 3.

3 Mergers harm buyers

We define buyer surplus as the ex ante expected surplus of the buyer, that is, the buyer’s expected value from obtaining the good minus the buyer’s expected payment.\(^{24}\) We say that a merger harms the buyer if the buyer’s expected surplus is smaller after the merger than before.

We first state a remarkably general result:

**Theorem 1.** A merger without cost synergies harms the buyer.

**Proof.** See the Appendix.

According to Theorem 1, a merger without cost synergies harms the buyer irrespective of whether the buyer has power and independently of any additional assumptions on the sellers’ cost distributions. The theorem has a clear intuition: A merger eliminates a bid, and that harms the buyer.

This intuition is borne out in the proof. The proof is fairly straightforward when the buyer has no power because then the allocation rule is efficient before and after merger, but there is one fewer bid after merger. However, with buyer power, the argument is more subtle and intricate. The mechanism design problems before and after a merger differ, and so does the powerful buyer’s optimal procurement mechanism and allocation rule. Moreover, absent additional assumptions, we do not even know the structure of the optimal mechanisms. Fortunately, these obstacles can be circumvented by taking the buyer’s optimal mechanism after the merger as given. (From Myerson (1981), we know

\(^{23}\)The assumption that the auction starts at a price no greater than \( \overline{c} \) only has an effect when there is only one supplier and \( v > \overline{c} \). With two or more suppliers, Bertrand competition between them prevents prices from rising above \( \overline{c} \). Technically, the assumption means that the individual rationality constraint always binds for a supplier with the highest possible cost draw.

\(^{24}\)A formal definition is provided in Section 4.
that this mechanism exists and can be derived using the techniques developed there.) This mechanism can then be augmented and applied to the pre-merger market as follows: keep the allocation and payment rules for the nonmerging suppliers the same as in the optimal post-merger mechanism and have suppliers 1 or 2 produce in exactly the same instances as the merged entity would under the optimal post-merger mechanism. To make this incentive compatible for suppliers 1 and 2, the augmented mechanism needs to induce competitive bidding between suppliers 1 and 2 for the right to produce. Because this additional bidding increases the buyer’s expected surplus, it follows that the buyer must be better off before the merger—even though the augmented mechanism is not optimal in the pre-merger market, it is available before the merger. Applying the optimal pre-merger mechanism only reinforces the conclusion that a powerful buyer is better off before the merger than after the merger.

4 Analysis of merger effects

As shown in Theorem 1, merger harm to the buyer is general and not eliminated by buyer power. To analyze whether buyer power mitigates merger harm, and various other interactions between buyer power and mergers, we now impose symmetry; that is, assume $G_i = G$ for all $i$, and additional structural assumptions. We comment along the way which of our results do not generalize to pre-merger asymmetric suppliers, at times distinguishing between whether or not the two merging suppliers are symmetric with each other.

We begin by analyzing the effects of a merger on market outcomes assuming no cost efficiencies from the merger or other countervailing effects. Then we consider suppliers’ incentives to merge, enter, or invest in cost-reducing innovation. In Section 5, we allow merger-related cost efficiencies.

Before doing so, we introduce a regularity condition—increasing virtual costs—that renders a powerful buyer’s optimization problem quasiconcave and thereby simplifies the analysis. The mechanism design concepts and results that we introduce, while seemingly of a technical nature, are key to the analysis and are the natural counterparts to standard concepts familiar to industrial organization economists. Throughout this and the following section, we assume that suppliers are symmetric before the merger and that the regularity condition holds.
4.1 Preliminaries

Let us start by considering the optimization problem for a buyer with value \( v \) that makes a take-it-or-leave-it offer to a single supplier that draws its cost from the distribution \( G \):

\[
\max_p (v - p)G(p).
\]

If, for example, \( v \leq \tau \), the optimal price offer is characterized by the first-order condition

\[
0 = g(p) (v - \Gamma(p)),
\]

where

\[
\Gamma(c) \equiv c + \frac{G(c)}{g(c)}
\]

is the supplier’s virtual cost. If \( \Gamma \) is increasing, the second-order condition is satisfied when the first-order condition is, implying that the problem is quasiconcave. Thus, the assumption that \( \Gamma \) is increasing is the optimal-procurement analog to the assumption of decreasing marginal revenue in models of industrial organization. In what follows, we assume that \( G \) is such that both \( \Gamma \) and the merged entity’s virtual cost

\[
\hat{\Gamma}(c) \equiv c + \frac{\hat{G}(c)}{\hat{g}(c)} = c + \frac{G(c)}{g(c)} \frac{2 - G(c)}{2(1 - G(c))},
\]

(1)

are increasing. A sufficient condition for this is that \( G/g \) is nondecreasing. We assume that \( g(\tau) \) is finite,\(^{25}\) but we allow the possibility that \( g \) is zero at the boundaries of the support. We define \( \Gamma(\underline{c}) \equiv \lim_{c \to \underline{c}} \Gamma(c) = \underline{c} \) and analogously for \( \hat{\Gamma} \). For \( x > \Gamma(\tau) \), define \( \Gamma^{-1}(x) \equiv \underline{c} \).

It will be useful to define a single procurement process parameterized by \( \beta \in \{0, 1\} \) that nests the cases with \( \beta = 1 \) and without \( \beta = 0 \) buyer power.\(^{26}\) To do so, for

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\(^{25}\)An implication of this is that \( \lim_{c \to \tau} \hat{\Gamma}(c) = \infty \), and so \( v < \hat{\Gamma}(\tau) \). Thus, the buyer exerts monopsony power – as defined in the introduction (and discussed after Proposition 1 below) – against the merged entity for some cost realizations. The assumption that \( g \) is finite at \( \tau \) can be relaxed at the cost of adding more assumptions or case distinctions to Propositions 2, 6, and 11 as we discuss below.

\(^{26}\)We assume that buyer power itself is not affected by a merger among suppliers, which seems natural if buyer power derives from the size and/or sophistication of the buyer, as suggested by the EC Guidelines (para. 65), or from the ability to vertically integrate upstream or sponsor entry, as suggested by the U.S. Guidelines (p. 27). Further, the EC Guidelines (para. 67) require that buyer power “must also exist and remain effective following the merger” for it to be viewed as moderating merger harm, which is consistent with our assumption. That said, the EC Guidelines raise the possibility that a merger could reduce buyer power “because a merger of two suppliers may reduce buyer power if it thereby removes a credible alternative” (EC Guidelines, para. 67). If anything, our analysis suggests that a buyer’s incentive to become powerful increases following a merger of two of its suppliers (see the discussion on p. 21).
\[ \beta \in \{0, 1\}, \text{ we define } \textit{weighted} \text{ virtual cost functions by} \]

\[ \Gamma_\beta(c) \equiv (1 - \beta)c + \beta \Gamma(c) \quad \text{and} \quad \hat{\Gamma}_\beta(c) \equiv (1 - \beta)c + \beta \hat{\Gamma}(c). \quad (2) \]

By construction, when \( \beta = 0 \), a supplier’s weighted virtual cost corresponds to its true cost, and when \( \beta = 1 \), to its virtual cost. If \( \Gamma_\beta(\tau) \) is finite, then for \( x > \Gamma_\beta(\tau) \), define \( \Gamma^{-1}_\beta(x) \equiv \tau \), and analogously for \( \hat{\Gamma}_\beta \).

**Procurement auctions and mechanisms** We assume that the buyer uses an \textit{auction-plus-final-offer procedure} that consists of an auction phase and a final take-it-or-leave-it offer phase defined as follows: In the \textit{auction phase}, the buyer conducts a (possibly discriminatory) descending clock auction starting from a clock price of \( \min \{v, \tau\} \). As the clock price decreases, participating suppliers can choose to exit. When a supplier exits, the supplier becomes inactive and remains so. The clock stops when only one active supplier remains, with ties broken by randomization. The auction is potentially discriminatory in that activity by supplier \( i \) at a clock price of \( p \) obligates supplier \( i \) to supply the product at the clock price \( p \) less a supplier-specific discount of \( p - \Gamma^{-1}_\beta(p) \) if \( i \) is an independent supplier and \( p - \hat{\Gamma}^{-1}_\beta(p) \) if \( i \) is the merged entity, should the buyer choose to trade with that supplier. In the \textit{final offer phase}, the buyer implements a supplier-specific reserve of \( \Gamma^{-1}_\beta(v) \) for a nonmerged supplier and \( \hat{\Gamma}^{-1}_\beta(v) \) for the merged entity as follows: If the auction price is below the reserve, then trade occurs at the auction price. Otherwise, the buyer offers the reserve, take-it-or-leave-it, to the auction winner. If the offer is accepted, trade occurs at the accepted price, and if it is rejected, there is no trade.

A supplier’s strategy specifies the price at which the supplier exits in the auction phase (as a function of the observed bidding history) and whether to accept or reject any particular final offer from the buyer. Clearly, in the final offer phase, the best response of a supplier with cost \( c \) is to accept price offers greater than \( c \) and reject those less than \( c \). In the auction phase, for a supplier with cost \( c \), remaining active in the auction phase until the price reaches \( \Gamma_\beta(c) \) and then exiting weakly dominates other strategies. If a supplier with cost \( c \) wins at a clock price of \( p = \Gamma_\beta(c) \), then the supplier is obligated to supply at price \( \Gamma^{-1}_\beta(p) = c \). Exiting at a price greater than \( \Gamma_\beta(c) \) causes the supplier to forego the possibility of positive surplus, and remaining active at a price less than \( \Gamma_\beta(c) \) only affects the supplier’s surplus if it causes the supplier to win when it would not have otherwise, and in that case leaves the supplier with nonpositive surplus (negative if trade occurs at the price determined by the auction phase and zero if the buyer makes a final offer that the supplier then rejects).
Thus, by standard auction logic, in the essentially unique equilibrium in non-weakly dominated strategies of the auction-plus-final-offer procedure, each supplier remains active until the clock price reaches its weighted virtual cost and then exits, and if a supplier wins the auction, the supplier accepts a final offer from the buyer if and only if it is greater than or equal to the supplier’s cost.

For \( \beta = 0 \), the auction-plus-final-offer procedure corresponds to a descending clock auction with reserve \( \min \{v, \pi\} \).\(^{27}\) For \( \beta = 1 \), the auction is not discriminatory in the pre-merger market because suppliers are symmetric. However, in the post-merger market, the merged entity is subject to a larger discount off the clock price relative to the nonmerged suppliers because \( \hat{G} \) is dominated by \( G \) in terms of the reverse hazard rate.\(^{28}\) To see this, observe that for all \( c \in [\underline{c}, \overline{c}] \),

\[
\frac{\hat{g}(c)}{\hat{G}(c)} = \frac{g(c)}{G(c)} \frac{2(1 - G(c))}{2 - G(c)} \leq \frac{g(c)}{G(c)},
\]

with a strict inequality for \( c \) in the interior of the support.\(^{29}\) Consequently, for all \( c \in [\underline{c}, \overline{c}] \), we have \( \hat{\Gamma}(c) \geq \Gamma(c) \) and hence, for all \( p, \hat{\Gamma}^{-1}(p) \leq \Gamma^{-1}(p) \), with strict inequalities for \( c \in (\underline{c}, \overline{c}) \) and \( p \in (\underline{\hat{\Gamma}(c)}, \overline{\hat{\Gamma}(\overline{c})}) \), respectively. Thus, with buyer power, the buyer behaves more aggressively towards the merged entity than towards nonmerged suppliers, demanding greater discounts from the merged entity in the auction phase and applying a lower reserve to the merged entity in the final offer phase.

It is useful to relate the auction-plus-final-offer procedure described above to standard concepts from mechanism design theory. By the revelation principle, the outcome of this game can also be achieved as the equilibrium outcome of a direct mechanism that asks each player \( i \) to report its type \( c_i \) to the mechanism and that makes the allocation and payments a function of the collection of reports \( c \). Given an allocation rule \( q(c) = (q_1(c), ..., q_n(c)) \) pre merger and \( \hat{q}(c) = (\hat{q}(c), \hat{q}_3(c), ..., \hat{q}_n(c)) \) post merger, where \( q_i(c), \hat{q}(c), \hat{q}_i(c) \in [0, 1] \), \( \sum_{i=1}^n q_i(c) \leq 1 \), and \( \hat{q}(c) + \sum_{i=3}^n \hat{q}_i(c) \leq 1 \), with values of 1 (0) meaning that a player does (does not) produce, incentive compatibility and binding individual rationality constraints for the least efficient seller types then pin down the interim expected payments of every

\(^{27}\)To see this, recall first that, for \( \beta = 0 \), the weighted virtual cost functions are the identity functions. This means that the auction phase has discounts of 0 and so is not discriminatory. In addition, when \( \beta = 0 \), the supplier-specific reserves are just \( \min \{v, \pi\} \), which is the starting price for the auction phase.

\(^{28}\)Recall that the reverse hazard rate for a distribution \( F(c) \) with density \( f(c) \) is \( f(c)/F(c) \) and that \( F(c) \) dominates \( G(c) \) in terms of the reverse hazard rate if \( f(c)/F(c) \geq g(c)/G(c) \) for all \( c \).

\(^{29}\)Dominance in terms of the reverse hazard rate implies first-order stochastic dominance, that is, \( (3) \) implies \( G(c) \leq \hat{G}(c) \).
Buyer and social surplus  The following is based on standard arguments from mechanism design theory; see, e.g., Krishna (2009). Given incentive compatibility and binding individual rationality constraints for sellers of type $c$, pre-merger expected buyer surplus given allocation rule $q(\cdot)$ is

$$BS_{pre}(q(\cdot)) \equiv E_c \left[ \sum_{i=1}^{n} q_i(c)(v - \Gamma(c_i)) \right],$$

while post merger, expected buyer surplus given allocation rule $\hat{q}(\cdot)$ is

$$BS_{post}(\hat{q}(\cdot)) \equiv E_c \left[ \hat{q}(c) \left( v - \hat{\Gamma}(\min\{c_1, c_2\}) \right) + \sum_{i=3}^{n} \hat{q}_i(c)(v - \Gamma(c_i)) \right].$$

Given the allocation rules, social surplus pre and post merger is given, respectively, as

$$SS_{pre}(q(\cdot)) \equiv E_c \left[ \sum_{i=1}^{n} q_i(c)(v - c_i) \right]$$

and

$$SS_{post}(\hat{q}(\cdot)) \equiv E_c \left[ \hat{q}(c) \left( v - \min\{c_1, c_2\} \right) + \sum_{i=3}^{n} \hat{q}_i(c)(v - c_i) \right].$$

As discussed above, when $\beta = 0$, the auction-plus-final-offer procedure maximizes social surplus. As shown in the proof of the following lemma, when $\beta = 1$, the procedure is optimal for the buyer in the sense of maximizing expected buyer surplus.

**Lemma 1.** The equilibrium outcome of the auction-plus-final-offer procedure corresponds to the allocation and payments in the dominant strategy implementation of the optimal mechanism for a designer whose objective is to maximize the expected value of

$$\beta(\text{buyer surplus}) + (1 - \beta)(\text{social surplus}),$$

subject to incentive compatibility and individual rationality.

**Proof.** See the Appendix.

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30There are transfers that make a mechanism with allocation rule $q(c)$, respectively $\hat{q}(c)$, incentive compatible if and only if for all $i$ and all $c$, $q_i(c)$, respectively $\hat{q}_i(c)$, is nonincreasing in $c_i$; see, e.g., Krishna (2009).
Naturally, central to our analysis is how a merger affects expected buyer surplus, social surplus, price, and quantity, and how these changes depend on buyer power. For given \( \beta \in \{0, 1\} \), denote by \( q^\beta (\cdot) \) and \( \hat{q}^\beta (\cdot) \) the allocation rule implied by the auction-plus-final-offer procedure pre and post merger, respectively. Letting \( BS^\beta_{\text{pre}} \equiv BS_{\text{pre}}(q^\beta (\cdot)) \) and \( BS^\beta_{\text{post}} \equiv BS_{\text{post}}(\hat{q}^\beta (\cdot)) \), we denote by

\[
\Delta BS^\beta \equiv BS^\beta_{\text{post}} - BS^\beta_{\text{pre}}
\]

the difference in expected buyer surplus post and pre merger. From Theorem 1, it follows that

\[
\Delta BS^0 < 0 \quad \text{and} \quad \Delta BS^1 < 0.
\] (5)

Analogously, we denote by

\[
\Delta SS^\beta \equiv E_c \left[ \hat{q}^\beta (c) (v - \min\{c_1, c_2\}) + \sum_{i=3}^n \hat{q}_i^\beta (c) (v - c_i) \right] - E_c \left[ n \sum_{i=1}^n q_i^\beta (c) (v - c_i) \right]
\]

the change in expected social surplus as a result of a merger. We denote by \( \Delta P^\beta \) the change in the buyer’s expected payment as result of a merger, which is given by

\[
\Delta P^\beta \equiv -E_c \left[ \hat{q}^\beta (c) \hat{\Gamma}(\min\{c_1, c_2\}) + \sum_{i=3}^n \hat{q}_i^\beta (c) \Gamma(c_i) \right] + E_c \left[ \sum_{i=1}^n q_i^\beta (c) \Gamma(c_i) \right],
\]

and by \( \Delta Q^\beta \) the expected change in the quantity traded as a result of a merger, that is:

\[
\Delta Q^\beta \equiv E_c \left[ \hat{q}^\beta (c) + \sum_{i=3}^n \hat{q}_i^\beta (c) - \sum_{i=1}^n q_i^\beta (c) \right].
\]

4.2 Effects of a merger on market outcomes

A merger has two key effects on the supply side of the market. First, the number of suppliers is reduced by one. Second, as noted above, the merged entity has a better cost distribution than any of the individual suppliers, that is, \( \hat{G}(c) \geq G(c) \) for all \( c \in [c_1, c_2] \). Because of the change in distribution, the merger affects the auction-plus-final-offer procedure for a powerful buyer, as we discuss below.

We first address the case without buyer power. Because a buyer without buyer power uses an efficient mechanism before and after the merger, the allocation is efficient with
and without the merger. The allocation not being affected by the merger means \( \hat{q}_0(c) = q_0^1(c) + q_0^2(c) \) and, for \( i \in \{3, ..., n\} \), \( q_0^0(c) = q_0^i(c) \). If \( c_1 = \min\{c\} \), then supplier 1 wins in the pre-merger market and receives payment \( \min\{v, c_2, ..., c_n\} \), and the merged entity wins in the post-merger market, but receives the weakly larger payment \( \min\{v, c_3, ..., c_n, \overline{c}\} \).\(^{31}\)

Because the merger eliminates a bid, the only effect of a merger is to increase the buyer’s payment in cases in which the pre-merger outcome would have involved one of the merging suppliers winning and the other one determining the price.

Thus, although the allocation is not affected by the merger when there is no buyer power, the expected payment by the buyer increases.

**Proposition 1.** In the absence of buyer power, a merger results in the same allocation for any realization of costs (implying that \( \Delta Q^0 = 0 \) and \( \Delta SS^0 = 0 \)), and a higher expected payment by the buyer (\( \Delta P^0 > 0 \)).

*Proof.* See the Appendix.

In addition to the effects identified in Proposition 1, by Theorem 1, without buyer power, a merger results in lower expected buyer surplus (\( \Delta BS^0 < 0 \)).

Proposition 1 highlights a contrast with the results of Farrell and Shapiro (1990), who show that under Cournot competition, in the absence of cost synergies, a merger causes the quantity to decrease and the price to increase. In our setup, there can be a price effect without a quantity effect.

Next we analyze the case with buyer power. As mentioned in the introduction, we can decompose buyer power into *bargaining power*, the ability to optimally discriminate among suppliers in the auction phase, and *monopsony power*, the ability of a buyer to commit in the final offer phase to a take-it-or-leave-it offer that is less than \( \min\{v, \overline{c}\} \). A buyer who exerts bargaining power may sometimes purchase from a supplier who does not have the lowest cost, and a buyer who exerts monopsony power may sometimes not purchase, even when the lowest cost is below the buyer’s willingness to pay.

In our framework, bargaining power and monopsony power do not vary with a merger. However, a merger affects the extent to which it is optimal for the buyer to exert its bargaining and monopsony power. Because the pre-merger market is symmetric, a powerful buyer does not exert its bargaining power before the merger. After the merger, the

\(^{31}\)Likewise, if \( c_2 = \min\{c\} \), then supplier 2 wins in the pre-merger market and receives payment \( \min\{v, c_1, c_3, ..., c_n\} \), and the merged entity wins in the post-merger market, but receives the weakly larger payment \( \min\{v, c_3, ..., c_n, \overline{c}\} \), where we add \(\overline{c}\) to this last expression to account for the case of \( n = 2 \).
extent to which a powerful buyer exerts its bargaining power is driven by the presence of rivals and the difference between virtual cost functions $\Gamma$ and $\hat{\Gamma}$. Because $\Gamma(c) < \hat{\Gamma}(c)$ for all $c \in (c, \hat{c})$, the buyer discriminates against the merged entity relative to rivals in the auction phase by requiring larger discounts of the merged entity. Thus, in the presence of rivals, the exertion of bargaining power can cause the merged entity not to trade when supplier 1 or 2 would have traded before the merger, and it can cause the merged entity to be paid less than supplier 1 or 2 would have been paid before the merger. However, in the case of merger to monopoly, bargaining power has no effect—there is no scope for discrimination before the merger because suppliers are symmetric and none after the merger because there is only one supplier.

The extent to which a powerful buyer exerts its monopsony power against a supplier can be measured by the amount by which the buyer’s supplier-specific reserve for that supplier falls below $\tau$. Relative to a buyer with a low willingness to pay, a buyer with a high willingness to pay will exert its monopsony power to a lesser extent, or possibly not at all. In particular, a buyer with a value greater than $\Gamma(\tau)$ never finds it optimal to make a final offer to a nonmerged supplier that might be rejected. Thus, when $\Gamma(\tau)$ is finite, which implies that $\Gamma(\tau) < \hat{\Gamma}(\tau)$, a merger increases the range of buyer values for which the buyer exerts its monopsony power. In addition, because $v < \hat{\Gamma}(\tau)$, a merger increases the extent to which the buyer exerts its monopsony power against the merged entity relative to a nonmerged supplier, as reflected in a more aggressive final offer to the merged entity. The powerful buyer’s increased inclination to exert monopsony power post merger resonates with the popular view that powerful buyers will withstand upward price pressure due to a merger. Nevertheless, as shown in Theorem 1, mergers harm even powerful buyers.

Figure 1 illustrates the mechanics at work. Panel (a) shows the effects of a merger with buyer power and $n = 2$, and panel (b) shows the effects of a merger with buyer power and $n \geq 3$. Both panels assume $v < \Gamma(\tau)$, so that the buyer exerts monopsony power against both merged and nonmerged suppliers. As shown in panel (a), when $n = 2$, there is no bargaining power effect, but as a result of the exertion of monopsony power, a merger reduces the set of types for which trade occurs (corresponding to the vertically hashed area). As shown in panel (b), when $n \geq 3$, the merger not only reduces the set of types

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32 One could quantify the extent to which a powerful buyer exerts monopsony power against a non-merged supplier by $(\tau - \Gamma^{-1}(v))/(\tau - \tau)$ and against the merged entity by $(\tau - \hat{\Gamma}^{-1}(v))/(\tau - \tau)$, both of which vary from 0 (no exertion of monopsony power) to 1 (maximal exertion of monopsony power) as $v$ varies from $\tau$ to $\infty$.

33 This follows from the assumption that $g(\tau)$ is finite, which implies that $\hat{\Gamma}(\tau)$ is infinite.
for which trade occurs due to the additional exertion of monopsony power (represented by the vertically hashed area), but also shifts trade away from the merged entity and towards higher cost nonmerged suppliers due to the exertion of bargaining power. In particular, for types in the horizontally hashed area, one of the merging suppliers trades before the merger, but increased discrimination against the merged entity results in a higher cost nonmerged supplier trading after the merger.

\begin{align*}
\text{(a) Effects of a merger with } n = 2 & \quad \text{(b) Effects of a merger with } n \geq 3 \\
\begin{array}{c}
\text{trade with} \\
\text{supplier 1}
\end{array} & \quad \begin{array}{c}
\text{trade with} \\
\text{nonmerging supplier}
\end{array}
\end{align*}

\begin{align*}
\Gamma^{-1}(v) & \quad \Gamma^{-1}(v) \\
\Gamma^{-1}(v) & \quad \Gamma^{-1}(v)
\end{align*}

Figure 1: Bargaining and monopsony power of a powerful buyer. Panel (a) refers to the case of only two suppliers (merger to monopoly). The vertically hashed area indicates types for which there is trade with the low-cost supplier prior to the merger but no trade after the merger, i.e., the buyer exerts more monopsony power after the merger than before. Panel (b) refers to the case of three or more suppliers. When one of the merging suppliers trades, it is the lower cost of the two merging suppliers, and when one of the nonmerging suppliers trades, it is the lowest cost of the nonmerging suppliers. The vertically hashed area again indicates types for which the buyer exercises additional monopsony power after the merger. The horizontally hashed area indicates types for which after the merger the buyer exerts its bargaining power, trading with a merging supplier prior to the merger, but with a higher-cost nonmerging supplier after the merger. In both panels, the shaded area indicates types for which the buyer exercises its monopsony power both before and after the merger.

We summarize these results as follows:\textsuperscript{34}

**Proposition 2.** With buyer power, a merger results in a weakly lower expected quantity traded ($\Delta Q^1 \leq 0$) (strictly if $n = 2$ or if $n \geq 3$ and $v < \Gamma(\overline{\tau})$) and strictly lower expected social surplus ($\Delta SS^1 < 0$).

In addition to the effects identified in Proposition 2, by Theorem 1, with buyer power, a merger results in lower expected buyer surplus ($\Delta BS^1 < 0$).

The decrease in quantity in Proposition 2 is particularly notable because merger review tends to focus on the price effects of a merger, with little consideration to effects on quantity. For example, after a merger to monopoly, the buyer who exerts monopsony

\textsuperscript{34}If we relax the assumption that $g(\overline{\tau})$ is finite, then when $n = 2$, in order for the results in Proposition 2 for $\Delta Q^1$ and $\Delta SS^1$ to hold with a strict inequality, we require $v < \Gamma(\overline{\tau})$. 

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power against the merged entity may pay a lower price.\textsuperscript{35} Despite this lower offer, which might be perceived as a reduction in the price paid by the buyer, the buyer is strictly worse off as a result of the merger. Thus, in this example, a focus on price effects leads to the wrong conclusion regarding the effects of the merger on the buyer.

The result that with buyer power a merger reduces social surplus does not necessarily generalize to the setting with asymmetric suppliers before the merger. With asymmetries pre merger, the powerful buyer optimally discriminates before the merger, which is socially wasteful, but after the merger may optimally choose not to discriminate or to discriminate less.\textsuperscript{36} Consequently, the result in Proposition 2 that \( \Delta SS^1 < 0 \) does not, without additional restrictions, extend to the case of pre-merger asymmetries among suppliers.

In both our setup with buyer power and that of Farrell and Shapiro (1990), a merger results in a shift in quantity away from the merged entity and towards other suppliers (or towards zero if \( n = 2 \)). In Farrell and Shapiro, the internalization of externalities by the merging suppliers results in a decrease in their equilibrium quantity, and a corresponding increase in the equilibrium quantities of nonmerging suppliers. The resulting decrease in total quantity implies an increase in price. In contrast, in our setup with buyer power, a merger causes the buyer to protect itself from the increased market power of the merged entity by exerting its bargaining power to more aggressively discriminate against the merged entity relative to rivals and by exerting its monopsony power to make a more aggressive final offer to the merged entity. This response by the buyer results in a decrease in quantity for the merged entity, both due to a greater probability of no trade at all, and a shift towards trade with nonmerged suppliers. But, despite the buyer’s more aggressive stance towards the merged entity, the loss of competition causes a decrease in the buyer’s expected surplus.

Carlton and Israel (2011) argue that because the threat from a buyer’s outside option—for example, due to vertical integration that allows the buyer to produce in-house—is not diminished by a merger of suppliers, a merger of suppliers may have little effect on prices. In contrast, Proposition 2 shows that even when buyer power is derived from an outside option that leaves the buyer with the same willingness to pay (equal to \( v \)) before the merger and after the merger, the harmful effects of the merger on the buyer are not

\textsuperscript{35}In the example of merger to monopoly, the post-merger final offer \( \hat{\Gamma}^{-1}(v) \) is the transaction price. For \( v \) sufficiently small, \( \Gamma^{-1}(v) \) is a good proxy for the transaction price before the merger, but because it is higher, the transaction is both more likely and more expensive before the merger. On balance, the buyer is made worse off by the merger.

\textsuperscript{36}For example, if \( G_j = G \) for all \( j \geq 3 \) and \( G_1 = G_2 = 1 - \sqrt{1 - G} \), the buyer favors suppliers 1 and 2 before the merger and does not discriminate post merger because \( \hat{G} = G \).
eliminated. Moreover, as argued above, price effects are not a reliable indicator of the harm a merger inflicts on a buyer.

Propositions 1 and 2 also provide an interesting contrast with the classic result from the auctions literature due to Bulow and Klemperer (1996) that says that a seller is better off with \( n + 1 \) bidders drawing their types from identical distributions and no power than having only \( n \) bidders and maximal power. This suggests the following thought experiment in our setting: Would the buyer prefer facing the merged entity and having buyer power to having no merger and no buyer power? Intuition based on Bulow and Klemperer might suggest that the buyer is better off with no merger and no buyer power. However, this intuition is wrong in general because, in a nutshell, a merger only eliminates a losing bid, not a cost draw, which is the key difference relative to the Bulow and Klemperer thought experiment.

If a powerful buyer’s value is sufficiently large that it does not exert its monopsony power, then a Bulow-Klemperer-like result holds because, in that case, the powerful buyer and the powerless buyer have the same pre-merger payoff (both buy from the low-cost supplier at a price equal to the second-lowest cost). Thus, for \( v \) sufficiently large, \( BS_{0 \text{pre}}^0(v) = BS_{1 \text{pre}}^1(v) > BS_{1 \text{post}}^1(v) \), where we add the argument \( v \) to highlight the dependence of buyer surplus on \( v \) and where the inequality follows from Theorem 1. Intuitively, if a powerful buyer does not exert its monopsony power, then it is left with only its bargaining power, which is less valuable than having an additional bid.\(^{37}\)

However, as shown in the proof of Proposition 3, for \( v \) sufficiently small that the powerful buyer exerts its monopsony power, the opposite of the Bulow-Klemperer result holds: buyer power dominates competition in the sense that \( BS_{0 \text{pre}}^0(v) < BS_{1 \text{post}}^1(v) \).

**Proposition 3.** There exist \( v' \) and \( v'' \) with \( \underline{v} < v' \leq v'' \) such that for all \( v > v'' \), \( BS_{0 \text{pre}}^0(v) > BS_{1 \text{post}}^1(v) \), and for all \( v \in (\underline{v}, v') \), \( BS_{0 \text{pre}}^0(v) < BS_{1 \text{post}}^1(v) \).

**Proof.** See the Appendix.

As we show below, buyer power mitigates some harmful effects of a merger but exacerbates others. Propositions 1 and 2 imply that \( 0 = \Delta Q^0 \geq \Delta Q^1 \) (with a strict inequalities in the cases described in Proposition 2) and that \( 0 = \Delta SS^0 > \Delta SS^1 \). Thus, quantity and social surplus effects are exacerbated by buyer power. In contrast, when the merging

\(^{37}\)In the asymmetric setup, the Bulow-Klemperer-like result that \( BS_{0 \text{pre}}^0(v) > BS_{1 \text{post}}^1(v) \) does not hold even for large \( v \). To see this, suppose that \( n = 2 \) and that distribution \( G_1 \) has almost all of its probability weight near \( \underline{v} \) and \( G_2 \) has almost all of its probability weight near \( \overline{v} \). Then for \( v \) sufficiently large, \( BS_{0 \text{pre}}^0(v) \) is close to \( v - \overline{v} \), but \( BS_{1 \text{post}}^1 \) is close to \( v - \underline{v} \).
suppliers are symmetric, buyer power mitigates the effect of a merger on expected buyer surplus, as shown in the proof of the following proposition:

**Proposition 4.** $0 = \Delta Q^0 \geq \Delta Q^1$, $0 = \Delta SS^0 > \Delta SS^1$, and $0 > \Delta BS^1 > \Delta BS^0$.

**Proof.** See the Appendix.

The effect of buyer power on expected buyer surplus described in Proposition 4 resonates with a line of thinking that portrays buyer power as a countervailing force: While all buyers are harmed by mergers without cost synergies (Theorem 1), buyer power mitigates harm. This insight generalizes to settings with asymmetric suppliers, provided the merging suppliers are symmetric. However, if the merging suppliers are asymmetric, it may be that merger harm is greater with buyer power, i.e., $0 > \Delta BS^0 > \Delta BS^1$, because the merger deprives the powerful buyer of the ability to discriminate between the merging suppliers.\(^{38}\)

We model buyer power as an exogenously given attribute of a buyer. Alternatively, one could take the view that a buyer can choose to exercise buyer power at some cost. It then follows from the above analysis that a merger between symmetric suppliers increases the value of exercising this power. To see why, note that the value of buyer power is $BS^1_{\text{pre}} - BS^0_{\text{pre}}$ pre merger and $BS^1_{\text{post}} - BS^0_{\text{post}}$ post merger. From Proposition 4, we know that $\Delta BS^1 > \Delta BS^0$, which is equivalent to $BS^1_{\text{post}} - BS^0_{\text{post}} > BS^1_{\text{pre}} - BS^0_{\text{pre}}$. In such a framework, rather than being a countervailing power to a supply-side merger, buyer power would be a reaction to a merger, potentially raising new issues for (dynamic) merger review.

Another relevant question is how the number of rivals affects the harm from mergers. Competition authorities typically view the presence of large numbers of nonmerging suppliers as a factor that mitigates the harm from a merger.\(^{39}\) We now show that this is indeed the case in our setup. Without buyer power, the monotonicity result is almost immediate because the allocation rule does not change with the merger. Because of the revenue equivalence theorem (see, for example, Myerson (1981), Krishna (2002) or Börgers (2015)), this implies $\Delta BS^0 = \int_{c}^{c_{\text{min}}[v,\Gamma]} (\Gamma(c) - \hat{\Gamma}(c))(1 - G(c))^{n-2}d\hat{G}(c)$. This is negative\(^{38}\)To fix ideas, consider a case with $n = 2$, $G_1 \neq G_2$, and $v, \Gamma_1(c)$, and $\Gamma_2(c)$ such that before the merger the powerful buyer only exerts its bargaining power. Before the merger, trade occurs with probability 1 regardless of buyer power, but the powerful buyer is better off than the buyer without buyer power because its expected payment is smaller. After the merger, the powerful buyer can no longer discriminate and obtains approximately the same payoff as the buyer without power if $v$ is large, whence $\Delta BS^0 > \Delta BS^1$ follows.\(^{39}\)For models of vertical integration in which harm increases with the competitiveness of the industry, see Riordan (1998) and Loertscher and Reisinger (2014).
because $\Gamma(c) < \hat{\Gamma}(c)$ for $c > \underline{c}$, increasing in $n$ because $(1 - G(c))^{n-2}$ decreases in $n$, and goes to 0 as $n$ goes to infinity because for all $c \in (\underline{c}, \overline{c}]$, $(1 - G(c))^{n-2}$ converges uniformly to zero.

With buyer power, matters are complicated by the fact that a merger changes the extent to which the buyer optimally exerts its bargaining and monopsony power. In other words, with buyer power a merger changes the optimal allocation rule. If the powerful buyer used the same allocation rule before and after the merger, the change in its expected surplus would be

$$\Delta BS^1 \equiv \int_{\underline{c}}^{\Gamma^{-1}(v)} (\Gamma(c) - \hat{\Gamma}(c))(1 - G(c))^{n-2} d\hat{G}(c),$$

which is negative, increasing in $n$, and goes to 0 as $n$ goes to infinity for the same reasons as without buyer power. Because the powerful always has the option to keep the allocation rule the same after merger, it follows that $\Delta BS^1$ is a lower bound for $\Delta BS^1$, i.e., $\Delta BS^1 \leq \Delta BS^1$.

**Proposition 5.** With no buyer power, buyer harm decreases in $n$, i.e., $\Delta BS^0$ increases in $n$, and goes to zero as $n$ goes to infinity, i.e., $\lim_{n \to \infty} \Delta BS^0 = 0$. With buyer power, for any $n$, $\Delta BS^1 \leq \Delta BS^1 \leq 0$. Moreover, $\Delta BS^1$ increases in $n$ and goes to zero as $n$ goes to infinity. Consequently, buyer harm goes to zero as $n$ goes to infinity. Likewise, harm to social surplus goes to zero as $n$ goes to infinity. That is, $\lim_{n \to \infty} \Delta BS^1 = 0$ and $\lim_{n \to \infty} \Delta SS^1 = 0$.

**Proof.** See the Appendix.

With buyer power, an increase in $n$ monotonically increases the post-merger expected surplus of a powerful buyer that does not adjust the allocation rule in the wake of the merger. Because the probabilities of procuring from suppliers 1 and 2 before the merger and from the merged entity after the merger decrease in $n$, it follows that $\Delta BS^1$ increases in $n$. Of course, the powerful buyer can do better by adjusting the allocation rule and exerting monopsony and bargaining power after the merger. However, the effectiveness of the exertion of these powers decreases in $n$ exactly because the buyer becomes less likely to procure from the merging suppliers. Thus, we cannot preclude the possibility that, at

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40 In the asymmetric setup, the corresponding lower bound for $\Delta BS^1$ is only monotonically increasing (as the set of nonmerging suppliers expands from any given set to a superset of that set) under the assumption that the two merging suppliers are symmetric. Asymmetries among the nonmerging suppliers play no role.
some point, \( \Delta BS^1 \) decreases in \( n \), although we know that its lower bound increases in \( n \). That said, numerical calculations show that \( \Delta BS^1 \) is monotonically increasing in \( n \) for a range of distributions.\(^{41}\)

### 4.3 Merger-related incentives

As we show, the incentives to merge, enter, and innovate are affected by buyer power.

**Incentives to merge**

A well-known result for mergers in the Cournot model is that a merger without cost synergies always benefits rival firms, but is not profitable for the merging firms unless it is a merger to monopoly (see Salant et al., 1983).\(^{42}\) As we now show, in our procurement model, things are starkly different.

As already foreshadowed by the preceding analysis, without buyer power, mergers are always profitable for the merging suppliers and neutral for the rivals. In the absence of buyer power, a merger either does not change or increases the joint surplus of the merging suppliers. The increase occurs when, before the merger, one of the merging suppliers wins and is paid the cost of the other merging supplier, which occurs with positive probability. Then, after the merger, the merged entity wins but receives a larger payment. In other cases, no supplier’s quantity or payment is affected by the merger.

As we now show, with buyer power, mergers always benefit rivals but may or may not be profitable for the merging suppliers. As recognized by, e.g., Condorelli and Szentes (2017), in a trade environment with private information, an agent may have an incentive to take an action that increases the inefficiency of the outcome in order to increase the agent’s information rent. A similar effect occurs in our model with buyer power. With buyer power, when suppliers merge, they increase the inefficiency of the outcome and create an entity with a larger information rent than either one individually. But, because either effect can dominate, suppliers facing a powerful buyer may or may not benefit from a merger.\(^{43}\)

\(^{41}\)We have been unable to construct an example in which \( \Delta BS^1 \) is not monotonically increasing in \( n \).

\(^{42}\)Deneckere and Davidson (1985) show that this merger paradox can be resolved in a model with differentiated products, although in their model mergers remain more beneficial for outsiders than for the merging firms. Perry and Porter (1985) show that the paradox can be resolved in a model with homogeneous products when firms have constant returns to scale production functions where one input is fixed in the short run, which implies strictly increasing marginal costs.

\(^{43}\)In contrast, in Condorelli and Szentes (2017), the effect of the larger information rent always dominates because in their model the agent with private information chooses the distribution from which the agent’s private information is drawn and, in equilibrium, chooses a distribution that increases the agent’s
In particular, with buyer power a merger to monopoly is not profitable if the buyer has sufficiently strong incentives to exert monopsony power, but is profitable if the buyer does not exert its monopsony power. If the buyer does not exert its monopsony power, then before the merger, the lower-cost merging supplier wins and is paid the second-lowest cost. After the merger, the merged entity wins and is paid $c_7$, which is larger, so the merger increases the joint surplus of the merging suppliers. In contrast, when the merged entity faces rivals, a powerful buyer’s optimal exertion of its bargaining power can be sufficient to deter a merger.\textsuperscript{44} If one of the merging suppliers, say supplier 1, wins before the merger, it is paid $\min\{c_2, \ldots, c_n\}$. After the merger, the exertion of bargaining power by the buyer may cause the supplier not to win or to be paid less when the supplier does win.

In the following proposition, a merger being profitable means that the expected post-merger profit of the merged entity exceeds the sum of the expected profits of the merging suppliers before the merger, while a merger benefitting rivals means that the nonmerging suppliers are better off after the merger than before the merger. A merger is called neutral for rivals if it does not affect the profits of nonmerging suppliers.

**Proposition 6.** With no buyer power, a merger is profitable for the merging suppliers and neutral for nonmerging suppliers. With buyer power, any merger, whether it is profitable or not, benefits the nonmerging suppliers. With buyer power and $v$ sufficiently large, a merger to monopoly is profitable. With buyer power and $v$ sufficiently small, a merger to monopoly is not profitable. Likewise, with buyer power and $n$ sufficiently large, a merger is not profitable.

*Proof.* See the Appendix.

The intuition for why with buyer power a merger to monopoly is not profitable for $v$ sufficiently small is straightforward. With buyer power, a merger has two effects. It eliminates a competing bid, which is good for the merging suppliers, and makes it more likely that the buyer exerts its bargaining and monopsony power against the merged entity, which is bad for the merged entity. When $v \to c$, the buyer strongly exerts its monopsony power, so the elimination of the competing bid is second-order—both with and without

\textsuperscript{44}For example, if $n = 3$ and suppliers’ types are drawn from $U[0, 1]$, and if $v \geq 1$, then the pre-merger expected joint surplus of the two merging suppliers is greater than the expected surplus of the merged entity even if we restrict the buyer not to exert monopsony power, i.e., the buyer can discriminate in the auction phase but not make a final offer.
the merger, the price of any transaction is almost surely determined by the buyer’s final offer. Because this offer becomes more aggressive after the merger, the negative effect dominates.

A similar logic drives the result that in sufficiently competitive environments, mergers in the presence of buyer power are not profitable. The lowest cost draw by any of the nonmerged suppliers, denoted \( c(1) \), provides the buyer with an outside option. Before the merger, the buyer pays a merging supplier at most \( c(1) \), but after the merger at most \( \hat{\Gamma}^{-1}(\Gamma(c(1))) \), where \( c(1) > \hat{\Gamma}^{-1}(\Gamma(c(1))) \) for \( c(1) > \zeta \). As \( n \) goes to infinity, \( c(1) \to \zeta \) almost surely, which is akin to \( v \to \zeta \) for fixed \( n \), so in the limit these bounds determine the price that a winning merged entity receives, with that price being lower post merger.

Proposition 6 has a number of implications. First, for powerful buyers, collusion may be a greater concern than mergers. To see this, note that collusion between two suppliers has a similar effect to a merger in that it eliminates a competing supplier. However, if collusion is not detected or anticipated by the buyer, it has no effect on the procurement mechanism. Thus, collusion always increases the expected payoff of the colluding suppliers, even when a merger is not profitable.

Second, in the absence of buyer power, when \( v > \zeta \), perfect collusion between two suppliers requires the second bid to be equal to \( \zeta \) (to guarantee that it never binds). Such a bid potentially raises suspicions. However, with buyer power and a buyer who exerts monopsony power, perfect collusion only requires the second bid to be greater than or equal to the buyer’s final offer, which is less suspicious because cost realizations in this range occur with positive probability. Thus, buyer power makes collusion not only more profitable relative to a merger, but also more difficult to detect (relative to the case without buyer power).

Third, the broad insight of Proposition 5 is that an increase in the number of rivals decreases the buyer’s harm from the merger. To that, Proposition 6 adds the insight that a larger number of suppliers also makes the merger less profitable for the merging suppliers. Thus, arguments that a merger is profitable and that large numbers of nonmerging suppliers mitigate harms may be incompatible when the buyer is powerful.

Fourth, with buyer power, even a merger to monopoly may not be profitable. In this sense, a powerful buyer is better at deterring mergers than at eliminating harmful merger effects.
Incentives to enter

When evaluating the likely competitive effects of a merger, competition authorities regularly consider whether the merger (together with any price increases that result from the merger) might induce entry into the market and whether the possibility of such entry might alleviate concerns.\footnote{As stated in the \textit{U.S. Guidelines} (p. 28): “As part of their full assessment of competitive effects, the Agencies consider entry into the relevant market.” Related to the likely sufficiency of entry to remedy harms, see the \textit{U.S. Guidelines}, Section 9.3 and \textit{EC Guidelines}, Section VI.} In assessing the likelihood of entry, considerations include, among other things, whether entry is likely to be profitable.\footnote{See the \textit{U.S. Guidelines}, Section 9.2 and \textit{EC Guidelines}, Section VI.}

We say that a merger potentially induces entry if the expected profit of an entrant is greater after the merger than before the merger, and we say that a merger does not induce entry if the expected profit of an entrant is no greater after the merger than before the merger.

Because market shares are not affected by a merger without buyer power, the profitability of entry is also not affected by a merger. Consequently, absent buyer power a merger does not induce entry. In contrast, with buyer power, a merger induces the buyer to discriminate against the merging suppliers. This shifts market shares to the rivals and therefore increases the profitability of entry. Summarizing, we have the following result.

\textbf{Proposition 7.} \textit{In the absence of buyer power, a merger is neutral for nonmerging suppliers and so does not induce entry, but with buyer power, a merger increases the expected payoff from entry and so potentially induces entry.}

Incentives to innovate

To analyze a merger’s effect on incentives to innovate by investing in cost-reducing technologies, we stipulate that, prior to its cost realization, each supplier can make an investment that induces a first-order stochastic dominance shift of its cost distribution.\footnote{For an equilibrium analysis of vertical integration in a procurement model with first-price auctions and investments that shift the support of the cost distributions, see Loertscher and Riordan (forthcoming).} Specifically, letting $G_I$ denote a supplier’s cost distribution after investment, we assume that $G_I(c) \geq G(c)$ for all $c \in [\underline{c}, \overline{c}]$. In our analysis of investment incentives, we assume that investments are not observable by the buyer, which allows us to keep the buyer’s mechanism fixed as we change investments. This approach has the benefit of parsimony and detail freeness—we do not have to impose assumptions on how exactly investment affects the cost distribution or on the cost of investment. It also appears to correspond to the rewards for innovation that antitrust authorities are likely to evaluate.
When discussing the incentives to innovate, it is useful to think of each supplier $i$ pre-merger as corresponding to a “plant” and accordingly of investments (and cost distributions) as being plant-specific. This permits us to evaluate the merging suppliers’ incentives to innovate per plant.

Let $\pi_i^\beta(c)$ be supplier $i$’s expected profit before the merger when its cost is $c$ and the buyer’s power is $\beta$, and let $\hat{\pi}_i^\beta(c)$ be the expected profit after the merger of supplier $i$ in the same contingency, where for $i \in \{1, 2\}$, this is expected profit per plant. We say that for a given $\beta$, supplier $i$’s incentives to invest increase with the merger if

$$\int_c^\infty \pi_i^\beta(c)[dG_I(c) - dG(c)] > \int_c^\infty \pi_i^\beta(c)[dG_I(c) - dG(c)],$$

and we say that a merger is neutral (decreases incentives to invest) for supplier $i$ if

$$\int_c^\infty [\hat{\pi}_i^\beta(c) - \pi_i^\beta(c)]dG_I(c) = (<) \int_c^\infty [\hat{\pi}_i^\beta(c) - \pi_i^\beta(c)]dG(c).$$

Because we assume that $G$ first-order stochastically dominates $G_I$, it follows that a merger increases (decreases) incentives to invest for $i$ if $\hat{\pi}_i^\beta(c) - \pi_i^\beta(c)$ decreases (increases) in $c$ and is neutral for $i$ if $\hat{\pi}_i^\beta(c) - \pi_i^\beta(c)$ is constant. In what follows, our focus is thus naturally on the sign of the derivative of the expression $\hat{\pi}_i^\beta(c) - \pi_i^\beta(c)$.

Let us first consider the incentive effects of the rivals of the merging suppliers, that is, suppliers $i = 3, \ldots, n$, beginning with the case without buyer power.

By the revenue (or payoff) equivalence theorem, we know that in any incentive compatible mechanism the interim expected payoff of supplier $i$ when its cost is $c \in [\underline{c}, \overline{c}]$ is $\int_c^\overline{c} q_i(x)dx$ plus a constant (which under the assumptions we impose is 0), where $q_i(x)$ is the probability that $i$ produces, which is determined by the allocation rule.\footnote{The constant is the interim expected payoff of supplier $i$ when its cost is $\overline{c}$. This payoff is 0 because the buyer, for whom $\overline{c}$ is known, does not accept bids above $\overline{c}$ (see footnote 23).}

Before the merger, the probability that supplier $i$ with cost $c$ produces is $(1 - G(c))^{n-1}$. Thus, for all $c \leq \min\{v, \overline{c}\}$,

$$\pi_i^0(c) = \int_c^{\min\{v, \overline{c}\}} (1 - G(x))^{n-1}dx,$$

and $\pi_i^0(c) = 0$ for all larger $c$. Without buyer power, $(1 - G(c))^{n-1}$ is also the probability that supplier $i$ with cost $c$ produces after the merger, so we have $\hat{\pi}_i^0(c) = \pi_i^0(c)$ for all $c$.
and all $i \in \{3, ..., n\}$. Thus, without buyer power, a merger is neutral for rivals’ incentives to invest.

With buyer power, the arguments are similar. For all $c \leq \Gamma^{-1}(v)$ (recall that we define $\Gamma^{-1}(v)$ so that $\Gamma^{-1}(v) \leq \bar{c}$) and all $i \in \{3, ..., n\}$,

$$\pi^1_i(c) = \int_c^{\Gamma^{-1}(v)} (1 - G(x))^{n-1} dx,$$

and

$$\hat{\pi}^1_i(c) = \int_c^{\Gamma^{-1}(v)} (1 - G(\hat{\Gamma}^{-1}(\Gamma(x))))(1 - G(x))^{n-3} dx,$$

and, of course, $\pi^1_i(c) = \hat{\pi}^1_i(c)$ for all larger $c$. It follows that for all $c \leq \Gamma^{-1}(v)$, supplier $i$’s post-merger expected payoff is more sensitive to its cost than its pre-merger expected payoff: $\hat{\pi}^1_i(c) \leq \pi_i^1(c) < 0$, where the inequality is strict for all $c \in (\bar{c}, \Gamma^{-1}(v))$. Thus, with buyer power, a merger increases rivals’ incentives to invest.

Consider now the merging suppliers’ incentives to invest. Without buyer power, for $i \in \{1, 2\}$, $\pi^0_i(c)$ is as defined in (9). After the merger, for $i \in \{1, 2\}$ and $c \leq \min\{v, \bar{c}\}$,

$$\hat{\pi}^0_i(c) = \int_c^{\min\{v, \bar{c}\}} (1 - G(c))(1 - G(x))^{n-2} dx,$$

and $\hat{\pi}^0_i(c) = 0$ for all larger $c$. Observing that, for $i \in \{1, 2\}$ and $c \leq \min\{v, \bar{c}\}$,

$$\hat{\pi}^0_i(c) - \pi^0_i(c) = -\frac{g(c)}{1 - G(c)} \hat{\pi}^0_i(c) \leq 0,$$

with a strict inequality for all $c \in [\bar{c}, \min\{v, \bar{c}\})$, unless $g(\bar{c}) = 0$, and $\hat{\pi}^0_i(c) - \pi^0_i(c) = 0$ otherwise, it follows that without buyer power a merger increases the merging suppliers’ incentives to invest per plant.

Finally, we address the merging suppliers’ incentives to invest in the presence of buyer power, which is the only case for which the merger-related change in incentives cannot be signed. Before the merger, for $i \in \{1, 2\}$, $\pi^1_i(c)$ is as defined in (9). After the merger, for $c \leq \hat{\Gamma}^{-1}(v)$ and $i \in \{1, 2\}$,

$$\hat{\pi}^1_i(c) = \int_c^{\hat{\Gamma}^{-1}(v)} (1 - G(c))(1 - G(\hat{\Gamma}^{-1}(\hat{\Gamma}(x))))^{n-2} dx,$$
implying, for $c \leq \hat{\Gamma}^{-1}(v)$,

$$\hat{\pi}^I_i(c) - \pi^I_i(c) = -(1 - G(c)) \left[ (1 - G(\Gamma^{-1}(\hat{\Gamma}(c))))^{n-2} - (1 - G(c))^{n-2} \right] - \frac{g(c)}{1 - G(c)} \hat{\pi}_i^I(c),$$

while for $c \in (\hat{\Gamma}^{-1}(v), \Gamma^{-1}(v))$, we have $\hat{\pi}^I_i(c) - \pi^I_i(c) = -\pi^I_i(c) > 0$. Thus, with buyer power, the effects of the merger on the merging suppliers’ incentives to invest cannot be signed without imposing additional assumptions.

Figure 2: Effect of a merger on incentives to invest for a merging supplier. Panel (a): Pre-merger profit of a merging supplier (solid line) and post-merger profit of the merged entity at one plant (dashed line) as a function of cost. The merger is profitable for the merging suppliers in expectation. Panel (b): Pre-merger cost distribution $G$ and two possible post-investment cost distributions, one such that a merger increases incentives for a merging supplier to invest (dashed line, $G_I(c) = \sqrt{c}$ for $c \in [0, 1]$) and one such that a merger decreases incentives for a merging supplier to invest (solid line, $G_I(c) = c$ for $c \in [0, 1/4]$ and $G_I(c) = (1 + 24c^2 - 16c^3)/9$ for $c \in (1/4, 1]$). Both panels assume $\beta = 1$, $n = 2$, $v = 2.1$, and $G$ uniform on $[0, 1]$.

Figure 2(a) provides an illustration for a case in which the buyer exerts monopsony power and the merger is profitable for the merging suppliers. Because $\hat{\pi}^I_i(c)$ and $\pi^I_i(c)$ have an interior point of intersection, whether (7) holds depends on the details of the model, such as the way in which investments shift the distribution. Figure 2(b) shows two possible functions for $G_I$. The dashed function shifts substantial mass to the lower part of the support, where $\hat{\pi}^I_i(c) > \pi^I_i(c)$, implying that a merger increases incentives to invest. In contrast, the solid function only shifts probability mass for relatively high costs, where $\hat{\pi}^I_i(c) < \pi^I_i(c)$, implying that a merger reduces incentives to invest.

The example of Figure 2 shows that even when a merger is profitable, the merging suppliers’ incentives to invest can go either up or down as a result of the merger. This contrasts with results for investment in complete information models of mergers, where
incentives to invest increase or decrease with a supplier’s equilibrium quantity, implying that a merger increases incentives to invest for nonmerging suppliers and decreases them for merging suppliers (see, e.g., Motta and Tarantino, 2017).

Combining the example in Figure 2 with the preceding results, we have the following:

**Proposition 8.** Without buyer power, a merger is neutral for nonmerging suppliers’ incentives to invest and increases the merging suppliers’ incentives to invest. With buyer power, a merger increases incentives to invest for nonmerging suppliers but can either increase or decrease incentives for the merging suppliers.

## 5 Cost efficiencies

By Theorem 1, absent cost efficiencies, any merger is detrimental to the buyer regardless of buyer power. Cost efficiencies are thus necessary for mergers not to harm the buyer.

To model cost efficiencies, we now assume that the merged entity’s cost is reduced by a multiplicative factor \(1 - s\), where \(s \in [0, 1]\) measures the strength of cost synergies. Thus, given \(c_1\) and \(c_2\), the merged entity’s cost is \((1 - s) \min\{c_1, c_2\}\). Because likely cost efficiencies are part of a merger review and therefore information that both the buyer and the merging suppliers share, we assume that \(s\) is commonly known. Moreover, these synergies are typically expressed as percentages, which motivates us to model them as multiplicative. For simplicity, we assume \(c = 0\). Given this model of cost synergies, the distribution of costs after the merger for the merged entity with cost synergies \(s\) is, for \(c \in [0, (1 - s)\overline{c}]\),

\[
\overline{G}(c) \equiv \hat{G}(c/(1 - s)),
\]

with density \(\overline{g}(c) \equiv \hat{g}(c/(1 - s))/(1 - s)\). Let \(\Delta BS^\beta(s)\) and \(\Delta SS^\beta(s)\) be the analogs to \(\Delta BS^\beta\) and \(\Delta SS^\beta\) for the case with cost efficiencies \(s\), which implies that \(\Delta BS^\beta(0) \equiv \Delta BS^\beta\) and \(\Delta SS^\beta(0) = \Delta SS^\beta\).

There are, of course, many alternative ways to model cost efficiencies, and while we believe that our approach is a sensible one, we do not claim that it is the only way.

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49If we relax the assumption that \(g(\overline{c})\) is finite, then it is possible to have \(v \geq \hat{\Gamma}(\overline{c})\), in which case a merger increases incentives to invest for merging suppliers when the buyer is powerful. It follows that the exertion of monopsony power is necessary for a merger to decrease incentives for the merging suppliers to invest in cost reduction.

50The *EC Guidelines* state that, to be considered, cost efficiencies must be “verifiable.” (*EC Guidelines*, para. 78) The *U.S. Guidelines* (p. 30) state that “it is incumbent upon the merging firms to substantiate efficiency claims so that the Agencies can verify by reasonable means the likelihood and magnitude of each asserted efficiency....”
to capture cost efficiencies nor that the results we derive below hold robustly across all possible alternatives. With that in mind, we take it as a given in what follows that our statements are conditional on our setup and on our way of incorporating cost efficiencies.

Cost synergies improve the merged entity’s distribution and, therefore, increase the set of trades that are beneficial to the buyer. The buyer would benefit from cost synergies if it used exactly the same procurement mechanism as without cost synergies. Consequently, the buyer must be better off with cost synergies if it adjusts its mechanism to account for those synergies. In addition, it is straightforward to show that post-merger social surplus also increases with cost synergies.

Recall that without cost synergies, a powerful buyer is harmed less by a merger of symmetric suppliers than a buyer without power, i.e., $0 > \Delta BS^1(0) > \Delta BS^0(0)$. This raises the question whether the presence of buyer power means that a lower level of cost synergies is required to offset merger harm than without buyer power. Interestingly, this is not necessarily the case as shown next. For a level of synergies $s$ sufficiently close to zero, $\Delta BS^1(s) > \Delta BS^0(s)$ follows from Proposition 4 and continuity. But when $s = 1$, the post-merger buyer has payoff $v$ regardless of buyer power, while the pre-merger buyer has higher expected surplus with buyer power than without, implying that $\Delta BS^0(1) > \Delta BS^1(1)$.

We summarize in the following proposition:

**Proposition 9.** With cost synergies, the harm to the buyer and to social surplus from a merger decreases with cost synergies, i.e., $\Delta BS^0(s), \Delta BS^1(s), \Delta SS^0(s),$ and $\Delta SS^1(s)$ are increasing in $s$ (strictly so with the exception of $\Delta BS^0(s)$ for $n = 2$ and $v < (1 - s)c$).

Moreover, for $s$ sufficiently close to zero, $0 > \Delta BS^1(s) > \Delta BS^0(s)$, and for $s$ sufficiently close to one, $\Delta BS^0(s) > \Delta BS^1(s) > 0$.

The second part of Proposition 9 implies that merging suppliers cannot argue that, for example, one could alternatively model cost synergies $s \geq 2$ as resulting in a cost distribution for the merged entity of $G(c) = 1 - (1 - G(c))^s$ on $[c, \overline{c}]$, where $s = 2$ corresponds to the case of no cost synergies. In this formulation, our result for the case of buyer power that cost efficiencies reduce the merged entity’s information rents continues to hold. This follows because the virtual cost function for the merged entity is increasing and unbounded in $s$ for any $c > \underline{c}$, which implies that a powerful buyer’s optimal reserve for the merged entity goes to $\underline{c}$ as $s$ goes to $\infty$. However, for a buyer without power, cost synergies in this formulation do not reduce the upper support of the cost distribution and so do not affect the buyer’s reserve. Thus, according to this specification, without buyer power mergers with cost efficiencies are always profitable.

The case of a buyer with no buyer power, $n = 2$, and $v < \overline{c}$ is an exception insofar as this buyer’s post-merger payoff is 0 for all $s \leq 1 - v/\overline{c}$ because all trade occurs at a price equal to $v$; however, when $s$ exceeds this threshold, trade occurs with probability 1 at the price $(1 - s)\overline{c} < v$, implying that sufficiently large cost synergies benefit even a powerless buyer with a small value.
in general, the cost efficiencies required to eliminate harm from a merger are lower when they face powerful buyers than when they face weak buyers.\textsuperscript{53}

Another interesting aspect of Proposition 9 is that it has different implications for merger policy depending on whether one adopts a buyer or a social surplus perspective as summarized in the following corollary:

**Corollary 1.** *In the absence of buyer power, a competition authority using a social surplus standard is more permissive than one using a buyer surplus standard. Formally, $\Delta SS^0(s) \geq 0$ for all $s \in [0, 1]$, while $\Delta BS^0(s) \geq 0$ if and only if $s \in [s^*, 1]$ with $s^* > 0$. 

Proof. See the Appendix."

The result of Corollary 1 does not necessarily hold when the buyer is powerful. Except in the special case of two symmetric suppliers and $v$ sufficiently large, $\Delta SS^1(0) < 0$, and depending on distributions, as $s$ increases, either $\Delta SS^1(s)$ or $\Delta BS^1(s)$ can be greater.\textsuperscript{54}

**Effect on the merged entity of cost synergies with no buyer power**

Next, we analyze the effects of cost synergies on the merged entity’s expected surplus, and thereby on the incentives to merge. We begin with the case without buyer power.

In the absence of cost efficiencies, a buyer without buyer power holds a descending auction with a maximum acceptable bid of $\min\{v, \overline{c}\}$. With cost efficiencies, the maximum acceptable bid by the merged entity, denoted $p^0(s)$, is

$$p^0(s) \equiv \min\{v, (1 - s)\overline{c}\},$$

recognizing that the highest possible cost draw for the merged entity is now $(1 - s)\overline{c}$. This fact has an immediate implication that may not have been fully anticipated: Cost efficiencies do not only reduce the merged entity’s cost but also cost uncertainty and thereby the merged entity’s information rents. Letting $\hat{\Pi}(s)$ denote the merged entity’s expected surplus with cost synergies $s$, we have the following result:

\textsuperscript{53}As with Proposition 4, the result that for $s$ sufficiently close to zero, $0 > \Delta BS^1(s) > \Delta BS^0(s)$, does not necessarily generalize to asymmetric suppliers. However, it does if the two merging suppliers are symmetric.

\textsuperscript{54}It is straightforward to find specifications with powerful buyers in which there is a range of cost synergies $s$ such that $\Delta SS^1(s) < 0 < \Delta BS^1(s)$. This occurs, for example if $n = 2$, $v = 1$, and suppliers draw their costs from the uniform distribution on $[0, 1]$. In this case, a buyer surplus standard is more permissive in terms of allowing mergers than a social surplus standard. The opposite result holds in the same example with $v = 3$, in which case there is a range of cost synergies $s$ such that $\Delta BS^1(s) < 0 < \Delta SS^1(s)$.\textsuperscript{32}
Proposition 10. With cost synergies, no buyer power, and two suppliers, the following holds:

(i) if $v < \overline{c}$, then the merged entity’s expected surplus increases and then decreases as cost synergies increase; that is, there exists $\hat{s} \in (0, 1)$ such that $\hat{\Pi}(s)$ increases in $s$ for $s < \hat{s}$ and decreases in $s$ for $s > \hat{s}$;

(ii) if $v \geq \overline{c}$, then the merged entity’s expected surplus decreases as cost synergies increase; that is, $\hat{\Pi}(s)$ decreases in $s$ for all $s \in [0, 1]$;

(iii) the merged entity’s expected surplus is zero when $s = 1$; that is, $\hat{\Pi}(1) = 0$.

Proof. See the Appendix.

Proposition 10 implies that for merging suppliers, cost synergies are, at best, a mixed blessing. While they unambiguously increase buyer surplus (Proposition 9) and therefore make mergers more likely to be approved, they eventually (if $v < \overline{c}$) or universally (if $v \geq \overline{c}$) decrease post-merger expected surplus for the merging suppliers. The intuition is straightforward. Although synergies decrease production costs, which is good for the merging suppliers, they also reduce information rents, which is bad for them. As synergies increase, the latter, detrimental effect dominates.

Effect on the merged entity of cost synergies with buyer power

With buyer power, the payment that the merged entity receives is constrained by a powerful buyer’s ability to make a final offer. In the presence of cost synergies, the buyer’s supplier-specific reserve for the merged entity is

$$p^1(s) \equiv (1 - s)\hat{\Gamma}(c/(1 - s)).$$

Thus, the price faced by the merged entity, in the absence of rivals, is $p^1(s)$, which decreases in $s$ for $s \in [0, 1)$. Because $p^1(s)$ is bounded above by $(1 - s)\overline{c}$, it follows that $p^1(1) = 0$. Thus, we have the following result.

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55The virtual cost for the merged entity with cost type $c$ and cost synergies $s$ is $(1 - s)\hat{\Gamma}(c/(1 - s))$, implying a reserve $p^1(s)$ defined by $(1 - s)\hat{\Gamma}(p^1(s)/(1 - s)) = v$.

56Our assumption that $g(\overline{c})$ is finite implies a decreasing price elasticity of supply $\hat{\varepsilon}(c) \equiv c/\hat{\sigma}(c)$, where $\hat{\sigma}(c) \equiv \hat{G}(c)/\hat{g}(c)$ is the reverse hazard rate without cost efficiencies. If we relax the assumption that $g(\overline{c})$ is finite, then the effect of $s$ on the price faced by the merged entity, in the absence of rivals, depends on the level of synergies and whether the elasticity is constant or increasing. changes. Letting $\hat{s} = \max\{0, (\hat{\Gamma}(\overline{c}) - v)/\hat{\Gamma}(\overline{c})\}$, if $\hat{\varepsilon}'(c) = 0$, then $p^1(s)$ does not vary with $s$ for $s \in [0, \hat{s})$ and decreases linearly in $s$ for $s \in (\hat{s}, 1]$. If $\hat{\varepsilon}'(c) > 0$, then $p^1(s)$ increases in $s$ for $s \in [0, \hat{s})$ and decreases linearly in $s$ for $s \in (\hat{s}, 1]$. 

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Proposition 11. With cost synergies, with buyer power and two suppliers, the price faced by the merged entity is 0 if \( s = 1 \) and otherwise decreases in \( s \).

As shown in Proposition 11, the basic intuition from Proposition 10 carries over to the case with buyer power. Cost synergies squeeze informational rents and thereby profit margins. In the extreme, the surplus of the merged entity goes to zero because the merged entity is left with no private information.

Propositions 10 and 11 have the following corollary:

Corollary 2. With cost synergies, the expected surplus of the merged entity is maximized at a level of cost synergies strictly less than one.

Two tensions thus emerge from this analysis for cost synergies (and buyer power) as a merger defense. First, buyer power requires (weakly) larger synergies for the merger to be profitable. This is immediate because without buyer power, the merger is already profitable without synergies. Thus, merging suppliers who invoke buyer power as a countervailing effect may also have to make a case for why there are cost synergies that are large enough to outweigh the buyer power effects. Second, synergies are a double-edged sword for merging suppliers. Although cost synergies reduce production costs and make a merger more likely to be acceptable for the buyer, they also reduce information rents and, eventually, profits, implying that post-merger profits are maximized at an intermediate level of synergies.

The above analysis rests on the assumption that there are only two suppliers, so that the merger being considered is a merger to monopoly. With rival suppliers after the merger, business stealing from rivals is an additional effect of cost synergies. The main effect of this will be that without buyer power mergers harm rivals and with buyer power mergers do not necessarily benefit rivals. The result that cost synergies drive the merged entity’s profits to zero when buyer power is sufficiently strong is robust to the presence of rivals because it only depends on the fact that synergies increase the merged entity’s virtual cost function.

6 Generalization

Having highlighted along the way in Sections 4 and 5 which of our results do not necessarily extend to setups with suppliers that are asymmetric before the merger, we now briefly discuss the many results that do generalize to pre-merger asymmetries, and explain why they do so. We begin with the case without buyer power.
Without buyer power, Propositions 1–11 generalize to the setup with asymmetric pre-merger suppliers because a buyer with no buyer power does not discriminate among suppliers.\(^\text{57}\) Consequently, for a given realization of costs, asymmetries among suppliers have no effect on the market outcome.

With buyer power, some of our results make use of the fact that when \(G_1 = G_2\), the virtual cost of merged entity is greater than the lower of the two virtual costs of the merging suppliers for interior costs. Generalizing this to account for asymmetries between the merging suppliers, we say that virtual dominance holds, if for all \(c_1, c_2 \in [c, \bar{c}]\),

\[
\hat{\Gamma} \left( \min \{c_1, c_2\} \right) \geq \min \{\Gamma_1(c_1), \Gamma_2(c_2)\},
\]

with a strict inequality for a positive measure set of costs. Virtual dominance holds, for example, when the merging suppliers are symmetric, i.e. \(G_1 = G_2\), and for certain forms of asymmetries between \(G_1\) and \(G_2\).\(^\text{58}\) This is why symmetry of the merging suppliers is sufficient for all the results that rely on \(\hat{\Gamma}(c) \geq \Gamma(c)\) to generalize to pre-merger asymmetries among the nonmerging suppliers, i.e., settings in which \(G_3, \ldots, G_n\) are not restricted beyond having increasing virtual cost functions.

With the four exceptions discussed along the way and discussed in further detail below, the results with buyer power continue to hold as long virtual dominance holds. That said, although virtual dominance is sufficient for almost all of our results, it is not necessary. For example, if \(G_1(c) = c\) and \(G_2(c) = c^2\), both with support \([0, 1]\), then for \(c_1\) sufficiently small and \(c_2\) sufficiently large, virtual dominance does not hold,\(^\text{59}\) but numerical calculations show that with buyer power, merger effects are as described in Proposition 2. In particular, with buyer power, the expected quantity traded and expected social surplus are reduced by a merger to monopoly.

The four exceptions are: \((i)\) the result in Proposition 2 (and also Proposition 4) that \(\Delta SS^3 < 0\) no longer holds in the asymmetric setup as discussed there;\(^\text{60}\) \((ii)\) the Bulow-
Klemperer-like result in Proposition 3 that for sufficiently large $v$, $BS^0_{\text{pre}}(v) > BS^1_{\text{post}}(v)$ no longer holds in the asymmetric setup as noted prior to the proposition; (iii) the results in Propositions 4 and 9 that $\Delta BS^1 > \Delta BS^0$ in the absence of cost synergies or for cost synergies sufficiently close to zero do not hold in general, but do hold if the two merging suppliers are symmetric as discussed after Proposition 4; and (iv) the result in Proposition 5 that the lower bound for $\Delta BS^1$ is monotonically increasing (as the set of nonmerging suppliers expands from any given set to a superset of that set) requires the assumption that the two merging suppliers are symmetric.

7 Conclusion

We provide a framework for analyzing markets with buyer power. The framework captures procurement-based price formation and explicitly incorporates buyer power. We show that mergers without cost efficiencies harm buyers and that powerful buyers are harmed less than those without power by mergers of symmetric suppliers. Without buyer power, mergers are always profitable for the merging suppliers and neutral for their rivals. In contrast, with buyer power, mergers fairly generally benefit rivals, and even a merger to monopoly may not be profitable for the merging suppliers. Merger effects and incentives to merge are smaller the greater is the number of competing suppliers.

Absent buyer power, a merger increases the incentives to invest in cost reduction for the merging suppliers and does not affect the incentives of their rivals. In contrast, with buyer power, a merger increases rivals’ incentives to invest and can increase or decrease the incentives for the merging suppliers.

In an extension, we allow for cost efficiencies, which reduce and ultimately eliminate merger harm to the buyer. Because cost efficiencies not only reduce merging suppliers’ costs, but also their information rents, cost efficiencies do not necessarily benefit merging suppliers.

Interpreting buyer power as consisting of bargaining power and monopsony power, which respectively represent the buyer’s abilities to discriminate among suppliers and to credibly set a reserve price, connects our analysis to notions that feature prominently in concurrent antitrust debates. It also proves useful in understanding the various effects of mergers with buyer power. While a powerful buyer has bargaining and monopsony power, the extent to which it exerts its monopsony power decreases in its value and may be zero when its value becomes large enough. The exertion of monopsony power is necessary but not sufficient for a merger to have quantity effects and to decrease incentives for the
merging suppliers to invest in cost reduction, and for buyer power to deter merger to monopoly.

The exertion of bargaining power is sufficient for a merger that is not a merger to monopoly to shift production away from the merged entity towards their rivals. Our model thus has the empirical prediction that rivals’ market shares expand after a merger of symmetric suppliers without cost efficiencies in the presence of buyer power and stay the same without it.

As some of our results depend on whether the merging suppliers are symmetric, an interesting but also challenging issue for future research is to determine which firms have the most to gain from merging, asymmetric or symmetric ones, and the extent to which the answer depends on buyer power and on the characteristics of the nonmerging suppliers.

Another interesting avenue for future research is to extend our model to consider a market with multiple buyers and multiple sellers. Further, to introduce buyer size into the model, one could consider buyers with multi-unit demand that could be met either through single sourcing or multi-sourcing. However, the challenge with multiple buyers (even with single-unit demand) or a single buyer with multi-unit demand is that a merged entity will have multi-unit supply (see also footnote 21). For the Myersonian approach to remain tractable, alternative modeling assumptions are required to make the merged entity’s private information one dimensional.

It may also be valuable in future research to explore the implications of our results for merger screens. Preliminary screens for merger effects, such as measures of upward pricing pressure or the change in the Herfindahl-Hirschman Index, have proven useful to antitrust authorities (see, e.g., Miller et al., 2016). Our model, combined with parametric assumptions on the family of cost functions, could provide the foundation for a preliminary screen for mergers in procurement markets by allowing one to calculate, as a function of market shares and the extent to which the buyer’s value exceeds suppliers’ maximum cost, the level of cost synergies required to eliminate harm to the buyer. The preliminary screen would then be based on a comparison between established thresholds for reasonable

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61 We take a step in this direction in the Online Appendix, where we extend our model to allow multiple products that are perfect complements for the buyer. In the extension, we allow the possibility that some suppliers produce all of the complementary products, while others only produce a subset. In this setup, we are able to account for cost synergies associated with multi-product suppliers, demand side complementarities that cause a buyer to prefer to single source for multiple products, and the interaction of these with buyer power.

62 For example, one could assume that a merger between two suppliers with single-unit capacities leads to a supplier with a capacity of two and a constant marginal cost drawn from the distribution that is a mixture of the distribution of the lowest and the highest draw from the two independent distributions, with the mixture commonly known and representing potential cost efficiencies.
merger-related cost synergies and the empirically estimated or calibrated level required to eliminate harm for the proposed transaction derived from our model.
A Appendix: Proofs

Proof of Theorem 1. We first consider the case with buyer power. By the Revelation Principle, we can focus on incentive compatible direct revelation mechanisms. Our assumptions of compact support and positive density on the interior of the support imply that an optimal mechanism—that is, an incentive compatible and individually rational mechanism that maximizes the buyer’s ex ante expected surplus—exists and can be derived by applying the techniques of Myerson (1981) to a procurement problem. Further, the dominant strategy implementation of an optimal mechanism is unique and involves threshold payments, which we define formally shortly.

Let \((\hat{q}, \hat{m}): [c, \overline{c}]^{n-1} \to [0, 1]^{n-1} \times R^{n-1}\) be the optimal mechanism, where \(\hat{q} = (\hat{q}_3, ..., \hat{q}_n)\) is the allocation rule that specifies the probabilities \(\hat{q}_i(c)\) with which sellers \(i = 3, ..., n\) produce if the reports are \(c = (c, c_3, ..., c_n)\), with \(\hat{q}(c)\) specifying the probability that the merged entity produces when the reports are \(c\). The allocation rule permits incentive compatible implementation if and only if \(\hat{q}_i(c)\) is nonincreasing in \(c_i\) and \(\hat{q}(c)\) is nonincreasing in the merged entity’s reported cost \(c\). In the dominant strategy implementation, the threshold payments \(\hat{m} = (\hat{m}_3, ..., \hat{m}_n)\) are such that for any \(i\) and \(c\) such that \(\hat{q}_i(c) = \overline{q} > 0\), \(\hat{m}_i(c) = \sup \{x \in [c_i, \overline{c}] : \hat{q}_i(x, c_{-i}) = \overline{q}\}\), where \(c_{-i}\) are the reports of all suppliers other than \(i\) and \(\hat{q}_i(c) = 0\) if \(\hat{q}_i(c) = 0\), and, for the merged entity, for any \(c\) such that \(\hat{q}(c) = \overline{q} > 0\), \(\hat{m}(c) = \sup \{x \in [c, \overline{c}] : \hat{q}(x, c_3, ..., c_n) = \overline{q}\}\) and \(\hat{m}(c) = 0\) if \(\hat{q}(c) = 0\). (The expected payoff of a supplier with cost \(c_0\) who trades with probability \(q_0\) and who receives the threshold payment \(m_0\) when it trades is \(q_0(m_0 - c_0)\).) Denote the expected buyer surplus this mechanism generates by \(BS_{post}^1(\hat{q}, \hat{m})\).

Next, we apply that mechanism to the pre-merger market in the following sense: Let \((q, m): [c, \overline{c}]^n \to [0, 1]^n \times R^n\), where for \(i, j \in \{1, 2\}\) and \(j \neq i\),

\[
q_i(c) = \begin{cases} 
\hat{q}(\min \{c_1, c_2\}, c_3, ..., c_n), & \text{if } c_i < c_j \\
0, & \text{otherwise}
\end{cases}
\]

and

\[
m_i(c) = \begin{cases} 
\sup \{x \in [c_i, \overline{c}] : q_i(x, c_{-i}) = q_i(c)\}, & \text{if } q_i(c) > 0 \\
0, & \text{otherwise},
\end{cases}
\]

where \(c_{-i} = (c_j, c_3, ..., c_n)\), and for \(i \in \{3, ..., n\}\), \(q_i(c) = \hat{q}_i(\min \{c_1, c_2\}, c_3, ..., c_n)\) and \(m_i(c) = \hat{m}_i(\min \{c_1, c_2\}, c_3, ..., c_n)\). Then \((q, m)\) is incentive compatible and individually rational in the pre-merger market, generating an ex ante expected buyer surplus that we denote \(BS_{pre}^1(q, m)\). Because for \(i \in \{1, 2\}\), \(m_i(c) \geq \hat{m}(c)\), with strict inequality for a set
of realizations that has positive measure, $BS^1_{pre}(q,m) > BS^1_{post}(\hat{q},\hat{m})$. Of course, $(q,m)$ will typically not be the optimal pre-merger mechanism, reinforcing the conclusion that the buyer is strictly better off before the merger than after the merger.

Without buyer power, the same arguments apply, with the allocation rule $\hat{q}$ being given by the efficient allocation rule.

Proof of Lemma 1. Focus on the pre-merger market. Analogous arguments apply to the post-merger market. Standard mechanism design arguments imply that (4) is maximized, subject to incentive compatibility and individual rationality, if trade occurs between the buyer and the supplier with the lowest weighted virtual cost if and only if that weighted virtual cost is less than or equal to $v$. If no supplier has a weighted virtual cost less than or equal to $v$, then there is no trade. Because suppliers remain active up to their weighted virtual costs in the essentially unique equilibrium in non-weakly dominated strategies of the auction-plus-final-offer procedure, the auction-plus-final-offer procedure implements this outcome.

Proof of Proposition 1. Assume no buyer power. Trade occurs pre-merger if and only if $\min_{i \in \{1,...,n\}} c_i \leq v$, which is also the condition for trade post-merger. Thus, the probability of trade is not affected by a merger. However, the expected payment by the buyer increases as a result of the merger because the elimination of a supplier increases the expected threshold payments: from a pre-merger payment conditional on trade of $\min \{v, 2^{nd}\{c_1, ..., c_n\}\}$ to a post-merger payment conditional on trade of $\min\{v, 2^{nd}\{\min\{c_1, c_2\}, c_3, ..., c_n, \tau\}\}$, where the operator $2^{nd}$ selects the second-lowest element of a set. Because the quantity is unchanged but expected payment is greater, the expected buyer surplus decreases as a result of a merger.

Proof of Proposition 3. The result that $BS^0_{pre}(v) > BS^1_{post}(v)$ for $v$ sufficiently large follows from the text. Here we prove that $BS^0_{pre}(v) < BS^1_{post}(v)$ for $v$ sufficiently small. For the proof, it will be useful to define notation for the lowest derivative of $g$ that is nonzero at $\zeta$. Specifically, define $k \equiv \min\{i \in \{0,1,2,...\} \mid g^{(i)}(\zeta) \neq 0\}$, so that $g^{(0)}(\zeta) = ... = g^{(k-1)}(\zeta) = 0$ and $g^{(k)}(\zeta) > 0$ (the value of $k$ is well defined because $g$ is assumed positive on the interior of the support). The case with $g(\zeta) > 0$ corresponds to the case with $k = 0$.

Let $H$ be the distribution of the second-lowest of the $n$ cost draws, with density $h$, and note that $h(y) = n(n-1)(1 - G(y))^{n-2}G(y)g(y)$ and $h(\zeta) = 0$. Pre merger with no
buyer power, the buyer pays \( \min \{c(2), v\} \), where \( c(2) \) is the second-lowest cost, so

\[
BS_{\text{pre}}^0(v) = E \left[ v - c(2) \mid c(2) < v \right] \Pr \left( c(2) < v \right) = \int_{\min\{v, c(2)\}}^{\min\{v, \bar{v}\}} (v - y) dH(y).
\]

Thus, for \( v < \bar{v} \), \( BS_{\text{pre}}^0(v) = \int_{\min\{v, c(2)\}}^{\min\{v, \bar{v}\}} (v - y) dH(y) \), which is zero at \( v = \bar{c} \). Differentiating again, we get, for \( v < \bar{v} \), \( BS_{\text{pre}}^0(v) = h(v) \), which is also zero at \( v = \bar{c} \). More generally, letting \( BS_{\text{pre}}^{0(j)} \) denote the \( j \)-th derivative of \( BS_{\text{pre}}^0 \),

\[
BS_{\text{pre}}^{0(k+2)}(\bar{c}) = h^{(k)}(v) = 0.
\]

Turning to the post-merger market, let \( \hat{H} \) be the distribution of the lowest cost among \( c_3, \ldots, c_n \), with density \( \hat{h} \). (To allow for the case of \( n = 2 \), in what follows set \( \hat{H}, \hat{h}, \) and all their derivatives equal to zero.) Thus, \( \hat{H}(c) = 1 - (1 - G(c))^{n-2} \) and \( \hat{h}(c) = (n - 2)(1 - G(c))^{n-3}g(c) \). Letting \( \hat{\hat{c}}_3 \) denote the lowest cost among \( c_3, \ldots, c_n \) and \( \hat{c} \) denote \( \min \{c_1, c_2\} \),

\[
BS_{\text{post}}^{1}(v) = E \left[ v - \min\{\Gamma^{-1}(\hat{\hat{c}}_3), \Gamma^{-1}(\hat{c})\} \mid \Gamma(\hat{\hat{c}}_3) \leq \min\{\Gamma(\hat{\hat{c}}_3), v\} \right] \Pr(\Gamma(\hat{\hat{c}}_3) \leq \min\{\Gamma(\hat{\hat{c}}_3), v\})
\]

\[
+ E \left[ v - \min\{\Gamma^{-1}(\Gamma(\hat{\hat{c}}_3)), \Gamma^{-1}(\hat{c})\} \mid \Gamma(\hat{\hat{c}}_3) \leq \min\{\Gamma(\hat{\hat{c}}_3), v\} \right] \Pr(\Gamma(\hat{\hat{c}}_3) \leq \min\{\Gamma(\hat{\hat{c}}_3), v\}),
\]

which we can write as

\[
BS_{\text{post}}^{1}(v) = \int_{\min\{v, \hat{\hat{c}}_3\}}^{\min\{v, \hat{\hat{c}}_3\}} \int_{\min\{v, \hat{c}\}}^{\min\{v, \hat{\hat{c}}_3\}} \left( v - \Gamma^{-1}(\hat{\hat{c}}_3) \right) d\hat{H}(\hat{\hat{c}}_3) d\hat{G}(\hat{\hat{c}})
\]

\[
+ \int_{\min\{v, \hat{\hat{c}}_3\}}^{\min\{v, \hat{\hat{c}}_3\}} \int_{\min\{v, \hat{c}\}}^{\min\{v, \hat{\hat{c}}_3\}} \left( v - \Gamma^{-1}(\hat{\hat{c}}_3) \right) d\hat{H}(\hat{\hat{c}}_3) d\hat{G}(\hat{\hat{c}})
\]

\[
+ \int_{\min\{v, \hat{\hat{c}}_3\}}^{\min\{v, \hat{\hat{c}}_3\}} \int_{\min\{v, \hat{\hat{c}}_3\}}^{\min\{v, \hat{\hat{c}}_3\}} \left( v - \Gamma^{-1}(\hat{\hat{c}}_3) \right) d\hat{G}(\hat{\hat{c}}) d\hat{H}(\hat{\hat{c}}_3)
\]

\[
+ \int_{\min\{v, \hat{\hat{c}}_3\}}^{\min\{v, \hat{\hat{c}}_3\}} \int_{\min\{v, \hat{\hat{c}}_3\}}^{\min\{v, \hat{\hat{c}}_3\}} \left( v - \Gamma^{-1}(\hat{\hat{c}}_3) \right) d\hat{G}(\hat{\hat{c}}) d\hat{H}(\hat{\hat{c}}_3).
\]

Taking the derivative with respect to \( v \), one can show that \( BS_{\text{post}}^{1}(\bar{c}) = 0 \). Differentiating
again and using $\Gamma^{-1}(c) = \hat{\Gamma}^{-1}(c) = \frac{1}{2}$, one can show that
\[
BS_{\text{post}}^{1+}(c) = 2\hat{\Gamma}^{-1}(c) \left( 1 - \hat{\Gamma}^{-1}(c) \right) \hat{g}(\hat{\Gamma}^{-1}(c)) + 2\Gamma^{-1}(c) \left( 1 - \Gamma^{-1}(c) \right) \hat{h}(\Gamma^{-1}(c))
\]
\[
= \frac{1}{2} \left( \hat{g}(c) + \hat{h}(c) \right)
\]
\[
= \frac{n}{2} g(c),
\]
where the final equality uses $\hat{g}(c) = 2g(c)$ and $\hat{h}(c) = (n-2)g(c)$. It follows that if $k = 0$ so that $g(c) > 0$, then $BS_{\text{post}}^{1}(v)$ increases faster with $v$ at $v = c$ than does $BS_{\text{pre}}^{0}(v)$, implying that there exists $v' > c$ such that for all $v \in (c, v')$, $BS_{\text{post}}^{1}(v) > BS_{\text{pre}}^{0}(v)$.

More generally,
\[
BS_{\text{post}}^{1(k+2)}(c) = \frac{k + 2}{2^{k+2}} \left( \hat{g}^{(k)}(c) + \hat{h}^{(k)}(c) \right)
\]
\[
= \frac{k + 2}{2^{k+2}} \left( 2g^{(k)}(c) + (n-2)g^{(k)}(c) \right)
\]
\[
= \frac{k + 2}{2^{k+2}} n g^{(k)}(c),
\]
which is positive by the definition of $k$. Thus, all derivatives of $BS_{\text{pre}}^{0}(v)$ at $v = c$ up to and including the $k + 2$-nd derivative are zero, and all derivatives of $BS_{\text{post}}^{1}(v)$ at $v = c$ up to the $k + 2$-nd derivative are zero, but the $k + 2$-nd derivative is positive. Because $BS_{\text{post}}^{1}(v)$ increases faster than $BS_{\text{pre}}^{0}(v)$ at $v = c$, it follows that there exists $v' > c$ such that for all $v \in (c, v')$, $BS_{\text{post}}^{1}(v) > BS_{\text{pre}}^{0}(v)$. ■

**Proof of Proposition 4.** The results that $0 = \Delta Q^0 \geq \Delta Q^1$ and $0 = \Delta SS^0 > \Delta SS^1$ follow from Propositions 1 and 2. The result that $\Delta BS^1$ and $\Delta BS^0$ are negative follows from Theorem 1. We show that $\Delta BS^1 > \Delta BS^0$. To do so, it is useful to consider a range of buyer power $\beta \in [0, 1]$. Suppose temporarily that a buyer with buyer power $\beta$ uses virtual cost $\Gamma_{\beta}$ to evaluate all of the pre-merger and post-merger suppliers (in our model the buyer with buyer power uses $\hat{\Gamma}$ to evaluate the merged entity). Let $x \equiv \min \{c_3, \ldots, c_n, \overline{c}\}$ and $F$ be the distribution of $x$. For such a buyer, the probability of trade is not affected by the merger, and the payment is affected only when $\min\{c_1, c_2\} < \max\{c_1, c_2\} < \min\{\Gamma_{\beta}^{-1}(v), x\}$, in which case the buyer pays $\min\{\Gamma_{\beta}^{-1}(v), x\}$ instead of $\max\{c_1, c_2\}$. Thus, the expected change in buyer surplus as a result of a merger is
\[
2E[c] \max\{c_1, c_2\} - \min\{\Gamma_{\beta}^{-1}(v), x\} | \min\{c_1, c_2\} < \max\{c_1, c_2\} < \min\{\Gamma_{\beta}^{-1}(v), x\}] \\
\cdot \Pr \left( \min\{c_1, c_2\} < \max\{c_1, c_2\} < \min\{\Gamma_{\beta}^{-1}(v), x\} \right),
\]

42
which we can write as
\[
2 \int_{\xi}^{v} \int_{\xi}^{\min\{\Gamma^{-1}_\beta(v), \bar{x}\}} \int_{\xi}^{c_2} (c_2 - \min\{\Gamma^{-1}_\beta(v), x\})dG(c_1)dG(c_2)dF(x).
\] (11)

Differentiating (11) with respect to $\beta$, we get:
\[
-2 \frac{\partial \Gamma^{-1}_\beta(v)}{\partial \beta} \int_{\Gamma^{-1}_\beta(v)}^{v} \int_{\Gamma^{-1}_\beta(v)}^{\Gamma^{-1}_\beta(v)} \int_{\xi}^{c_2} dG(c_1)dG(c_2)dF(x) \geq 0,
\]
where the inequality follows because $\frac{\partial \Gamma^{-1}_\beta(v)}{\partial \beta} \leq 0$. Thus, for such a buyer, the change in surplus as a result of a merger is weakly greater (closer to zero) when $\beta = 1$ than when $\beta = 0$. This implies that for a buyer who uses virtual cost $\hat{\Gamma}_1$ for the merged entity, $\Delta BS^1 > \Delta BS^0$. □

**Proof of Proposition 5.** We first consider the case with no buyer power. Let $L_{n-2}(c) \equiv 1 - (1 - G(c))^{n-2}$ be the distribution of the lowest cost draw of the $n - 2$ rivals of the merging suppliers. The buyer’s expected surplus before the merger is

\[
BS^0_{pre} = \int_{\xi}^{v} (v - \Gamma(c))(1 - L_{n-2}(c))d\hat{G}(c) + \int_{\xi}^{v} (v - \Gamma(c))(1 - \hat{G}(c))dL_{n-2}(c).
\]

After the merger, the buyer’s expected surplus is

\[
BS^0_{post} = \int_{\xi}^{v} (v - \hat{\Gamma}(c))(1 - L_{n-2}(c))d\hat{G}(c) + \int_{\xi}^{v} (v - \Gamma(c))(1 - \hat{G}(c))dL_{n-2}(c).
\]

Taking the difference, we obtain

\[
\Delta BS^0 \equiv BS^0_{post} - BS^0_{pre} = \int_{\xi}^{v} (\Gamma(c) - \hat{\Gamma}(c))(1 - L_{n-2}(c))d\hat{G}(c).
\]

Because $\Gamma(c) - \hat{\Gamma}(c) < 0$ for any $c > \xi$ and $1 - L_{n-2}(c)$ decreases in $n$, $\Delta BS^0$ increases in $n$. Because for all $c \in (\xi, \bar{c}]$, $1 - L_{n-2}(c)$ converges uniformly to zero as $n$ goes to infinity, it follows that $\lim_{n \to \infty} \Delta BS^0 = 0$. Further, as noted, $\lim_{n \to \infty} \Delta BS^1 = 0$ and $\Delta BS^1 \leq \Delta BS^1$, which imply that $\lim_{n \to \infty} \Delta BS^1 = 0$. With buyer power, social surplus
before the merger is

$$SS_{pre}^1 = \int_{\underline{c}}^{\Gamma^{-1}(v)} (v-c)(1-L_{n-2}(c))d\hat{G}(c) + \int_{\underline{c}}^{\Gamma^{-1}(v)} (v-c)(1-\hat{G}(c))dL_{n-2}(c)$$

and after the merger is

$$SS_{post}^1 = \int_{\underline{c}}^{\hat{\Gamma}^{-1}(v)} (v-c)(1-L_{n-2}(\Gamma^{-1}(\hat{\Gamma}(c))))d\hat{G}(c) + \int_{\underline{c}}^{\Gamma^{-1}(v)} (v-c)(1-\hat{G}(\Gamma^{-1}(\Gamma(c))))dL_{n-2}(c).$$

Taking the difference and using \(\lim_{n\to\infty} 1 - L_{n-2}(c) = 0\) for \(c > \underline{c}\), we have

$$\lim_{n\to\infty} \Delta SS^1 = \lim_{n\to\infty} \int_{\underline{c}}^{\Gamma^{-1}(v)} (v-c)(\hat{G}(c) - \hat{G}(\Gamma^{-1}(\Gamma(c))))dL_{n-2}(c).$$

Writing out the expression for \(dL_{n-2}(c)\) and using the fact that for \(a \in [0,1)\), \(na^n\) converges uniformly to zero as \(n\) goes to infinity, we have

$$\lim_{n\to\infty} \Delta SS^1 = \lim_{n\to\infty} \int_{\underline{c}}^{\Gamma^{-1}(v)} (v-c)(\hat{G}(c) - \hat{G}(\Gamma^{-1}(\Gamma(c))))(n - 2)(1 - G(c))^n g(c)dc = 0,$$

which completes the proof. ■

**Proof of Proposition 6.** Results without buyer power and results related to effects on the nonmerging suppliers follow straightforwardly from the text. The result that with buyer power and \(v\) sufficiently large, a merger to monopoly is profitable follows because as \(v\) goes to infinity, the buyer’s reserve approaches (or equals) \(\tau\), which implies that trade occurs with probability 1 both before and after the merger, but after the merger at the higher price of \(\tilde{c}\) rather than \(\max \{c_1, c_2\}\).

We next show that with buyer power and \(v\) sufficiently close to \(\underline{c}\), a merger to monopoly is not profitable. Suppose \(n = 2\). Define \(p(v)\) and \(\hat{p}(v)\) to be the buyer’s optimal final offer as a function of \(v\) in the pre-merger and post-merger markets, respectively. Thus, \(\Gamma(p(v)) = v\) and \(\hat{\Gamma}(\hat{p}(v)) = v\), implying that \(p(\underline{c}) = \hat{p}(\underline{c}) = \underline{c}\). Define \(\Pi(p)\) and \(\hat{\Pi}(p)\) to be the joint expected profit of the two merging suppliers as a function of the final offer \(p\) in the pre-merger and post-merger markets, respectively. Before the merger, given a final offer \(p\), the joint expected profit of the two merging suppliers is

$$\Pi(p) = \int_{\underline{c}}^{p} (p - c)d\hat{G}(c) - 2\int_{\underline{c}}^{p} \int_{\underline{c}}^{p} (p - y)dG(y)dG(c),$$

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where the first term is the joint profit of the two suppliers if they do not compete against each other and are always paid the price \( p \), producing at the lowest cost. From that we subtract the lost profits due to having competing bids, where the winner is paid \( y \leq p \) instead of \( p \). Changing the order of integration in the double integral, we obtain

\[
\Pi(p) = \int_{c}^{p} (p - c)d\hat{G}(c) - 2 \int_{c}^{p} (p - y)G(y)dG(y) = \int_{c}^{p} G(y)(2 - 2G(y))dy.
\]

The first term in the original expression for \( \Pi(p) \) is the post-merger expected profit given a final offer \( p \):

\[
\hat{\Pi}(p) = \int_{c}^{p} (p - c)d\hat{G}(c) = \int_{c}^{p} G(y)(2 - G(y))dy.
\]

Clearly, \( \Pi(p(c)) = \hat{\Pi}(\hat{p}(c)) = 0 \). We show that \( \Pi(p(v)) \) increases faster than \( \hat{\Pi}(\hat{p}(v)) \) at \( v = c \), implying by continuity that \( \Pi(p(v)) > \hat{\Pi}(\hat{p}(v)) \) for \( v \) in a neighborhood to the right of \( c \). Specifically, letting \( f(v) \equiv \Pi(p(v)) \) and \( \hat{f}(v) \equiv \hat{\Pi}(\hat{p}(v)) \), we look at the derivatives of \( f(v) - \hat{f}(v) \), evaluated at \( v = c \), and show that the “first time” the derivatives differ, the derivative is positive, i.e., if \( j = \min \{ i \in \{ 1, 2, \ldots \} \mid f^{(i)}(c) \neq \hat{f}^{(i)}(c) \} \), then \( f^{(j)}(c) - \hat{f}^{(j)}(c) > 0 \), where \( f^{(j)} \) denotes the \( j \)-th derivative of \( f \). The index \( j \) is well defined because \( f(c) = \hat{f}(c) \) and, as argued in the text, for \( v \) sufficiently large, \( f(v) \neq \hat{f}(v) \). The challenge in showing that \( f^{(j)}(c) - \hat{f}^{(j)}(c) > 0 \) arises because \( p(v) \) increases faster than \( \hat{p}(v) \) at \( v = c \) (because \( \Gamma^{-1}(c) > \hat{\Gamma}^{-1}(c) \) for all \( c > c \)), but \( \Pi(p) \) increases more slowly than \( \hat{\Pi}(p) \) at \( c \). This is illustrated in Figure 3(a). We show that the effect of the change in final offers dominates the effect of the suppression of a bid when \( v \) is close to \( c \) as illustrated in Figure 3(b).

![Figure 3: Illustration of \( p, \hat{p}, \Pi, \) and \( \hat{\Pi} \) for the case of \( G(x) = x^2 \) with support \([0, 1]\).](image)

In order to illustrate the logic of the proof, we begin by considering the case with \( g(c) > 0 \). First, consider the derivatives of \( \Gamma \) and \( \hat{\Gamma} \). As always, \( \Gamma(c) = \hat{\Gamma}(c) = c \). When
$g(\xi) > 0$, $\Gamma'(\xi) = \hat{\Gamma}'(\xi) = 2$, $\Gamma''(\xi) = -\frac{\ddot{g}(\xi)}{g(\xi)}$, and $\hat{\Gamma}''(\xi) = -\frac{\ddot{g}(\xi)}{g(\xi)}$, so that
\[\hat{\Gamma}''(\xi) - \Gamma''(\xi) = g(\xi).\] (12)

Using the definitions of $p$ and $\hat{p}$ and the equality of $\Gamma$ and $\hat{\Gamma}$ and their first derivatives at $\xi$, we have
\[p'(\xi) = \hat{p}'(\xi) = \frac{1}{\Gamma'(\xi)} > 0\] (13)
and
\[p''(\xi) - \hat{p}''(\xi) = \left(\frac{p'(\xi)}{\Gamma'(\xi)}\right)^2 \left(\hat{\Gamma}''(\xi) - \Gamma''(\xi)\right).\] (14)

In preparation for generalizing this beyond the case with $g(\xi) > 0$, it will be useful to state these expressions in terms of the lowest derivative of $g$ that is nonzero at $\xi$. Specifically, define $k \equiv \min\{i \in \{0, 1, 2, \ldots\} | g^{(i)}(\xi) \neq 0\}$, so that $g^{(0)}(\xi) = \ldots = g^{(k-1)}(\xi) = 0$ and $g^{(k)}(\xi) > 0$ (the value of $k$ is well defined because $g$ is assumed positive on the interior of the support). The case with $g(\xi) > 0$ corresponds to the case with $k = 0$. For the general case, equations (12), (13), and (14) can be stated as:
\[\hat{\Gamma}^{(k+2)}(\xi) - \Gamma^{(k+2)}(\xi) = \frac{\Gamma''(\xi)}{2} g^{(k)}(\xi),\] (15)
\[p'(\xi) = \hat{p}'(\xi) > 0,\] (16)
and
\[p^{(k+2)}(\xi) - \hat{p}^{(k+2)}(\xi) = \left(\frac{p'(\xi)}{\Gamma'(\xi)}\right)^{k+2} \left(\hat{\Gamma}^{(k+2)}(\xi) - \Gamma^{(k+2)}(\xi)\right).\] (17)

Turning to the functions $\Pi$ and $\hat{\Pi}$, note that $\Pi'(\xi) = \hat{\Pi}'(\xi) = 0$. In addition, when $g(\xi) > 0$, then
\[\Pi''(\xi) = \hat{\Pi}''(\xi) = 2g(\xi)\] (18)
and
\[\Pi'''(\xi) - \hat{\Pi}'''(\xi) = -2 \left(g(\xi)\right)^2.\] (19)

In the general case, equations (18) and (19) can be stated as:
\[\Pi^{(k+2)}(\xi) = \hat{\Pi}^{(k+2)}(\xi) = 2g^{(k)}(\xi)\] (20)
and
\[\Pi^{(2k+3)}(\xi) - \hat{\Pi}^{(2k+3)}(\xi) = -(k + 1)(k + 2) \left(g^{(k)}(\xi)\right)^2.\] (21)

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Now for the case of \( g(c) > 0 \), consider the derivatives of \( f(v) - \hat{f}(v) \), evaluated at \( v = c \). When evaluated at \( c \), the first derivative is zero because the first derivatives of \( \Pi \) and \( \hat{\Pi} \) are equal and the first derivatives of \( p \) and \( \hat{p} \) are equal. The second derivative of \( f - \hat{f} \) is also zero because the first and second derivatives of \( \Pi \) and \( \hat{\Pi} \) are equal, the first derivatives of \( p \) and \( \hat{p} \) are equal, and the second derivatives of \( p \) and \( \hat{p} \), which differ, only enter in terms involving \( \Pi' \) or \( \hat{\Pi}' \), which are zero. At the third derivative, we have a difference:

\[
f'''(c) - \hat{f}'''(c) = (\Pi'''(c) - \hat{\Pi}'''(c))(p'(c))^3 + 3\Pi'(c)''(c)(p''(c) - \hat{p}''(c))\]

where the second equality uses (14), (18), and (19), the third equality uses (12), and the fourth equality rearranges. The inequality uses the assumption that \( g(c) > 0 \) and the result that \( p'(c) > 0 \), which is stated in (13). This completes the proof for the case in which \( g(c) > 0 \).

In the general case, when \( g(c) = g'(c) = ... = g^{(k-1)}(c) = 0 \) and \( g^{(k)}(c) > 0 \), the lowest derivative of \( f - \hat{f} \) that is nonzero at \( c \) is the \( 2k + 3 \)-rd derivative (below we suppress the argument \( c \) for readability and use \( \binom{a}{b} \) to denote the binomial coefficient \( a!/(b! (a-b)!))\):

\[
f^{(2k+3)}(c) - \hat{f}^{(2k+3)}(c) = (\Pi^{(2k+3)}(c) - \hat{\Pi}^{(2k+3)}(c))(p^{(k+3)} + (\binom{k+3}{k+2} \Pi^{(k+2)} \cdot (p')^{k+1} (p^{(k+2)} - \hat{p}^{(k+2)}))\]

where the second equality uses (17), (20), and (21), the third equality uses (15), and the inequality uses the definition of \( k \), the positivity of \( p'(c) \) as stated in (16), and the result
that for all \( k \in \{0, 1, \ldots \} \),

\[
0 < -(k + 1)(k + 2) + \frac{(2k + 3)!}{(k + 2)! (k + 1)!},
\]

which completes the proof that a merger to monopoly is not profitable for \( v \) sufficiently small.

The last part of the proposition holds by similar logic. Denote by \( c_{(1)} \) the lowest cost draw by any of the nonmerging suppliers. When \( n > 2 \), the price to a merging supplier as a result of the auction phase is at most \( c_{(1)} \) pre-merger and \( \hat{\Gamma}^{-1}(\Gamma(c_{(1)})) \) post-merger. As \( n \) goes to infinity, \( c_{(1)} \to \zeta \) almost surely and \( \hat{\Gamma}^{-1}(\Gamma(c_{(1)})) \to \zeta \) almost surely, but for all \( c_{(1)} > \zeta \), \( c_{(1)} > \hat{\Gamma}^{-1}(\Gamma(c_{(1)})) \). Thus, in the limit as \( n \) goes to infinity, with probability 1 a winning merging firm is paid \( c_{(1)} \) pre-merger and \( \hat{\Gamma}^{-1}(\Gamma(c_{(1)})) \) post merger, where the post-merger payment is lower. ■

Proof of Corollary 1. Without buyer power, we know from Proposition 4 and Theorem 1 that a merger is neutral for social surplus but reduces buyer surplus, \( \Delta SS^0(0) = 0 > \Delta BS^0(0) \). Using the results of Proposition 9, it follows that there exists \( s^* \in (0, 1) \) such that for cost synergies \( s \in [0, s^*) \), \( \Delta BS^0(s) < 0 < \Delta SS^0(s) \) and for cost synergies \( s \in (s^*, 1] \), \( 0 < \Delta BS^0(s) \) and \( 0 < \Delta SS^0(s) \). ■

Proof of Proposition 10. Consider first \( (1 - s)\overline{\pi} \leq v \) (a sufficient condition for which is obviously \( \overline{\pi} \leq v \)). Then there will always be trade post merger and the buyer pays \( (1 - s)\overline{\pi} \). Consequently,

\[
\hat{\Pi}(s) = \int_0^{(1-s)\overline{\pi}} ((1-s)\overline{\pi} - c)\overline{g}(c)dc
= \int_0^{(1-s)\overline{\pi}} (\overline{\pi} - c/(1-s))\hat{g}(c/(1-s))dc.
\]

Making the change of variables with \( y \equiv c/(1-s) \), which implies \( dc = (1-s)dy \), we obtain

\[
\hat{\Pi}(s) = (1-s) \int_0^{\overline{\pi}} (\overline{\pi} - y)\hat{g}(y)dy = (1-s)\hat{\Pi}(0),
\]

implying, for \( (1-s)\overline{\pi} \leq v \),

\[
\frac{\partial \hat{\Pi}(s)}{\partial s} = -\hat{\Pi}(0) < 0 \text{ and } \hat{\Pi}(1) = 0.
\]
Assume next that \( \overline{c} > v \), which means there is a \( \hat{s} \in (0, 1) \) such that \( (1 - s)\overline{c} < v \) if and only of \( s > \hat{s} \equiv 1 - v/\overline{c} \). For all \( s \leq \hat{s} \), the buyer will offer \( v \) whenever there is trade. Consequently, for \( s \leq \hat{s} \),

\[
\hat{\Pi}(s) = \int_0^v (v - c)g(c)dc = \frac{1}{1 - s} \int_0^v (v - c)\hat{g}(c/(1 - s))dc.
\]

Making the change of variables \( y \equiv c/(1 - s) \) again, we obtain

\[
\hat{\Pi}(s) = \int_0^{v/(1 - s)} (v - (1 - s)y)\hat{g}(y)dy,
\]

whose derivative is

\[
\frac{\partial \hat{\Pi}(s)}{\partial s} = \int_0^{v/(1 - s)} y\hat{g}(y)dy > 0.
\]

For \( s > \hat{s} \), the analysis is as in the case with \( \overline{c} \leq v \), with the result that \( \frac{\partial \hat{\Pi}(s)}{\partial s} < 0 \). \( \blacksquare \)
References


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