Countervailing Power*

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Abstract

Countervailing power arguments hypothesize that mergers on one side of the market increase social surplus by offsetting power on the other. Despite its popular appeal, countervailing power has met with skepticism and been difficult to formalize. We provide a model with a price-formation process whose efficiency depends on agents’ bargaining weights. Keeping these fixed, mergers never increase and sometimes decrease social surplus. However, mergers that level the playing field by equalizing bargaining weights can increase social surplus. Moreover, our model with endogenous efficiency of price formation sheds new light on vertical integration, investment incentives, bargaining breakdown, and bargaining externalities.

Keywords: price formation, bargaining power, productive power, vertical integration, investment incentives

JEL Classification: D44, D82, L41

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1 Introduction

The concept of countervailing power features prominently in antitrust debates. While the idea that increasing market power on one side of the market to countervail existing market power on the other side has appeal, the concept of countervailing power has been controversial since its beginning, partly because formalizing it has proved challenging. According to the New Palgrave Dictionary, the reason is that “it is difficult to model bilateral monopoly or oligopoly, and there exists no single canonical model” [Snyder (2008, p. 1188)]. Obviously, for countervailing power to increase social surplus requires a model with an explicit price-formation process in which bargaining powers affect not only the division but also the size of social surplus.

In this paper, we provide a model that has precisely these features. We consider a procurement problem with one buyer and one or more suppliers in which private information pertains to both sides of the market. We assume that the buyer’s value and the suppliers’ costs are independent draws from continuous distributions with compact supports. The distributions are common knowledge, but the realized value and costs are the buyer’s and the suppliers’ private information. We model the price-formation process as an incentive-compatible, individually-rational mechanism that maximizes the weighted sum of expected buyer and supplier, subject to a no-deficit constraint. The weights in the objective reflect the relative bargaining powers of the buyer and the suppliers, and so we refer to them as the agents’ bargaining weights. These bargaining weights affect the division of the social surplus when ex post efficient trade is possible, and whether or not ex post efficient trade is possible. As we show, our price-formation mechanism encompasses Chatterjee and Samuelson’s (1983) k-double auction for the bilateral trade setup with uniformly distributed types, and, more generally, the outcomes of our mechanism are equilibrium outcomes of an extensive-form game in which a fee-setting broker runs a competitive procurement for a buyer. In addition, our approach has axiomatic foundations, which we describe in the online appendix.

A key insight that arises from our price-formation process is that typically ex post efficiency is impossible both when the buyer has all the bargaining power and when the suppliers have all the bargaining power. Social surplus is maximized with symmetric bargaining weights. As we show, this has implications for the analysis of both horizontal and vertical mergers, as well as investment incentives and bargaining externalities.

We turn first to results pertaining to horizontal and vertical mergers. We show that, while a horizontal merger between suppliers that does not affect the bargaining weights never improves social surplus and always makes achieving the first-best more difficult, a horizontal

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1As [Carlton and Israel (2011, p. 128)] put it: “For changes in bargaining outcomes due to a buyer merger to create a true efficiency, it must be that, post-merger, the parties are better able to arrive at an optimal nonlinear price schedule, perhaps due to lower transactions costs, which moves output closer to the competitive level.”

2For analysis of fee-setting intermediaries, see [Loertscher and Niedermayer (2019)].
merger that “levels the playing field” by equalizing bargaining weights can improve social surplus. Indeed, such a merger can make the first-best possible when, prior to the merger and the change in bargaining weights, the first-best was not achievable because the price-formation process was too strongly tilted towards the buyer. To develop a sense for how countervailing power is obtained, it is useful to distinguish between an agent’s productive power (or strength) and its bargaining power. Productively stronger buyers have, or tend to have, higher values, and productively stronger suppliers have, or tend to have, lower costs. In contrast, an agent’s bargaining power captures its ability to bias the price-formation process in its favor. While, empirically, productive power and bargaining power may be correlated, conceptually, they are distinct and independent. As a case in point, both business and leisure air travellers are price-takers and hence have the same amount of bargaining power, that is, none. However, business passengers are productively stronger, which is why they are charged higher prices. We show that changes in bargaining power are necessary, without being sufficient, for there to be countervailing power. We identify necessary conditions for a countervailing power defense of a merger, and we discuss evidence that would support the inference that these conditions hold, including the potential role for data on bargaining breakdown.

Related to vertical mergers, we show that vertical integration between the buyer and a supplier can create a bilateral trade problem à la Myerson and Satterthwaite (1983) in which the first-best is impossible when it was possible prior to integration. This occurs, for example, when vertical integration leaves only one independent supplier in the market and when the buyer’s lowest possible value before integration exceeds the suppliers’ highest possible cost. In situations like these, vertical integration is thus socially harmful. It is so in ways and for reasons that are absent when the efficiency of the price-formation process is exogenously fixed. Of course, vertical integration also eliminates a bilateral trade problem, namely that within the newly created entity. Therefore, the social surplus effects of vertical integration can go either way. Importantly, we show that under appropriate assumptions, the likely effects of vertical integration can be estimated using pre-integration data. Although our analysis does not imply that vertical integration is universally bad, it does show that a presumption that vertical integration improves social surplus is not warranted.

In addition to our results for mergers, by extending our model to include investment, we also shed new light on incentives to invest. These incentives are at the center stage of current debates in antitrust and have been at the heart of the theory of the firm for more than thirty

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3For example, Snyder [1996] models a buyer’s bargaining power in terms of its ability to accumulate a backlog of unfilled orders, as distinct from its willingness to pay for the orders.

years. We assume that investments improve distributions in the sense of first-order stochastic dominance shifts without affecting the supports and that, as in the theory of the firm, investments are not contractible, which implies that the price-formation process does not vary with investments. In this setup, we show that the equilibrium investments are efficient if and only if the price-formation process is efficient. This is the opposite of the result obtained in the theory of the firm, where efficient price formation (for example, Nash bargaining) with complete information induces hold up and thereby inefficient investments. The privacy of information in incomplete information models protects agents against hold up. To understand the intuition, recall that with incomplete information, and the requirement of incentive-compatible prices, in order to ensure an efficient allocation, every agent has to be paid its social marginal product (see, e.g., Green and Laffont 1977; Holmström 1979). But this is exactly the condition that has to be satisfied for investments to be efficient. Hence, with incomplete information, investments are efficient if and only if the price-formation process induces the ex post efficient allocation. Beyond highlighting another fundamental difference to complete information models, this analysis allows us to connect market structure, which affects the efficiency of the price-formation process, with the efficiency of investment.

Finally, in an extension that allows for multi-object demand by the buyer and for supplier-specific preferences, we analyze bargaining externalities. We provide conditions under which an increase in the buyer’s preference for one supplier’s product increases the payoffs for all of the suppliers. This occurs when an increase in a buyer’s preference for a supplier improves the efficiency of the overall price-formation process to the benefit of all suppliers.

The remainder of the paper is structured as follows. In Section 2, we introduce the setup. In Section 3, we derive a price-formation process that incorporates bargaining weights. As we show, our mechanism coincides with Chatterjee and Samuelson’s (1983) $k$-double auction for the case of one supplier and uniformly distributed types and, more generally, is an equilibrium outcome of a procurement game involving a fee-setting broker. In Section 4, we derive results for horizontal and vertical mergers. In Section 5, we consider extensions that allow investment and multi-object demand with supplier-specific preferences. In Section 6, we discuss related literature, and in Section 7, we provide concluding remarks. The formal mechanism design results and longer proofs are relegated to appendices. An axiomatic approach to our price-formation mechanism is contained in the online appendix.

## 2 Setup

We consider a procurement setup with $n$ suppliers indexed by $i \in \mathcal{N} \equiv \{1, \ldots, n\}$, each with the capacity to produce one unit of the good, and one buyer, indexed by $B$, with demand for one unit.

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5As we discuss, a similar result holds for investment by the suppliers in quality.
The buyer draws its value $v$ independently from a distribution $F(v)$ with support $[\underline{v}, \overline{v}]$ and density $f(v)$ that is positive for all $v \in [\underline{v}, \overline{v}]$. Supplier $i$ draws its cost $c_i$ independently from distribution $G_i(c_i)$ with support $[\underline{c}_i, \overline{c}_i]$ and density $g_i(c_i)$ that is positive for all $c_i \in (\underline{c}_i, \overline{c}_i)$. We assume that $F$ and $G_1, \ldots, G_n$ are independent and common knowledge, while the realized value $v$ and the realized costs $c_1, \ldots, c_n$ are the private information of the buyer and individual suppliers, respectively. To save on notation, we ignore ties among the agents’ types. Consistent with the literature, we model a merger between two suppliers as creating a merged entity’s whose cost is drawn from the distribution of the minimum of two independent cost draws, one from each of the two merging suppliers’ distributions, i.e., a merged entity formed from suppliers $i$ and $j$ draws its cost $c$ from the distribution $\hat{G}(c) \equiv 1 - (1 - G_i(c))(1 - G_j(c))$.

The buyer and the suppliers have quasilinear preferences. The expected payoff of supplier $i$ with cost $c_i$ when producing the good with probability $q_i$ and receiving the expected monetary transfer $m$ is

$$\hat{u}_i(c_i; m, q_i) = m - c_i q_i. \quad (1)$$

The expected payoff of a buyer with value $v$ when receiving the object with probability $q$ and making the expected monetary payment $m$ is

$$\hat{u}_B(v; m, q) = v q - m. \quad (2)$$

Under ex post efficiency (and ignoring ties), trade occurs between the buyer and supplier $i$ if and only if $v - c_i > \max_{j \neq i} \{0, v - c_j\}$. The problem is trivial if $\overline{v} \leq \underline{c}$ because then it is never ex post efficient to have trade with any supplier. Therefore, from now on, we assume that $\overline{v} > \underline{c}$.

Because we allow both the buyer’s value and the suppliers’ costs to be random variables whose realizations are the agents’ private information, the setup is symmetric with respect to the privacy of information, with the important consequence that ex post efficiency need not be possible. Indeed, for $n = 1$, our setup encompasses the classic Myerson-Satterthwaite (1983) setting, where, as they show, ex post efficient trade is impossible if and only if $\overline{v} < \underline{c}$. We refer to the case with $\overline{v} < \underline{c}$ as the case with overlapping supports and the case with $\overline{v} \geq \underline{c}$ as the case of nonoverlapping supports. With one supplier and nonoverlapping supports, we obtain ex post efficient trade even under incomplete information, but differences with a complete information setup remain. For example, the predictions of the models are
starkly different when trade is preceded by noncontractible investments, as in the extension of Section 5.1, because the privacy of information in our setting provides protection against hold-up.

We denote the buyer’s virtual value function by $\Phi(v) \equiv v - \frac{1 - F(v)}{f(v)}$ and supplier $i$’s virtual cost function by $\Gamma_i(c) \equiv c + \frac{G_i(c)}{g_i(c)}$. We assume that the virtual value and virtual cost functions are increasing. For $a \in [0, 1]$, we define the $a$-weighted virtual value function by $\Phi_a(v) \equiv v - (1 - a)\frac{1 - F(v)}{f(v)}$ and the $a$-weighted virtual cost function for supplier $i$ by $\Gamma_{i,a}(c) \equiv c + (1 - a)\frac{G_i(c)}{g_i(c)}$. Observe that monotonicity of $\Phi(v)$ and $\Gamma_i(c)$ implies that $\Phi_a(v)$ and $\Gamma_{i,a}(c)$ are also monotone.

Although we focus on a procurement setting with one buyer and one or more suppliers, all of our results extend with the appropriate adjustments to a sales auction with one supplier and one or more buyers. In addition, as we show in Section 5.2, the setup extends to allow for multi-unit demand and supplier-specific preferences.

3 Price-formation process

At the heart of essentially any economic model of exchange are assumptions that govern the price-formation process. For example, oligopoly models specify a mapping from firms’ actions to prices, and models based on Nash bargaining specify a mapping from preferences to trades and transfer payments. As mentioned in the introduction, in this paper we hew to this tradition by working with a given price-formation process. We depart from this tradition by introducing a price-formation process that allows for bargaining weights and that has neither the shortcoming of standard oligopoly models that buyers are price takers, nor the problem of Nash bargaining that outcomes are efficient by assumption.

In what follows, we first define the price-formation mechanism in Section 3.1. Then we provide an illustration in Section 3.2 and discuss implementation in Section 3.3.

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7If $f(v) = 0$, define $\Phi(v)$ to be the limit of $\Phi(v)$ as $v$ approaches $v$ from below, and if $g(c) = 0$, define $\Gamma(c)$ to be the limit of $\Gamma(c)$ as $c$ approaches $c$ from above.

8The assumption of increasing virtual type functions can be relaxed through the use of “ironing.”

9This departs from standard notation in that the coefficient on the hazard rate term is $1 - a$ rather than $a$, but because we will be introducing bargaining weights, this modification is useful.

10For evaluating the merits of a countervailing power argument, the standard oligopoly models of Cournot and Bertrand are “dead on arrival” because one side of the market—typically, buyers—is characterized by price-taking behavior and hence has no bargaining or market power.

11The assumption of efficiency embedded in generalized Nash bargaining preempts any social surplus increasing effects of changes in the bargaining weights because the outcome is efficient both before and after the change.
3.1 Definition

We model the price-formation process as a mechanism that maps agents’ types into prices and probabilities of trading. Specifically, we assume that the price-formation process is a direct mechanism \(\langle Q, M \rangle\) that maps the buyer’s and suppliers’ types to quantities (or probabilities of trade) for the buyer and suppliers, \(Q : [v, \bar{v}] \times [c, \bar{c}]^n \rightarrow [0, 1]^{n+1}\), where feasibility requires

\[
Q_B(v, c) \leq \sum_{i \in \mathcal{N}} Q_i(v, c),
\]

and transfers for the buyer and suppliers, \(M : [v, \bar{v}] \times [c, \bar{c}]^n \rightarrow \mathbb{R}^{n+1}\). (Of course, excess production will never be optimal, so (3) will hold with equality.)

This price-formation mechanism is required to satisfy incentive compatibility, individual rationality, and no deficit. A direct mechanism is incentive compatible if it is in the best interest of every agent to report its type truthfully to the mechanism and is individually rational if each agent, for every possible type, is weakly better off participating in the mechanism than walking away, where we normalize the payoffs of not trading and of walking away—that is, the value of the outside option—to zero. A direct mechanism has no deficit if the expected payment from the buyer is greater than or equal to the sum of the expected payments to the suppliers. For formal definitions, see Appendix A.1.

Fix a mechanism \(\langle Q, M \rangle\) and type realizations \((v, c)\). Then for the case of an allocation rule \(Q\) with \(Q_i(v, c) \in \{0, 1\}\) for all \(i \in \mathcal{N}\), the buyer’s ex post surplus as a function of the type realizations is

\[
U_B(v, c) \equiv \hat{u}_B(v; M_B(v, c), Q_B(v, c)) = vQ_B(v, c) - M_B(v, c),
\]

where \(\hat{u}_B\) is the function introduced in (2). Similarly, the ex post surplus of supplier \(i\) is given by

\[
U_i(v, c) \equiv \hat{u}_i(c; M_i(v, c), Q_i(v, c)) = M_i(v, c) - c_iQ_i(v, c),
\]

where \(\hat{u}_i\) is the function introduced in (1). The budget surplus generated by the mechanism

\[\text{12}\]Other than being explicit about the nature of the price-formation process, there is nothing special to this approach because any equilibrium allocation has these properties, regardless of whether the model has complete or incomplete information. However, for complete information models, the dependence on agents’ types is often degenerate insofar as each agent has only one (known) type.

\[\text{13}\]In our independent private values setting, any Bayesian incentive compatible and interim individually rational mechanism can be implemented with dominant strategies and ex post individual rationality. By construction, it yields the same interim and hence ex ante expected payoffs and revenue. Thus, while we formally state our assumptions in Appendix A.1 in terms of Bayesian incentive compatibility and interim individual rationality, one could also use the ex post versions of those constraints.

\[\text{14}\]To simplify the exposition, we only require that the mechanism does not run a deficit in expectation, allowing for the possibility that ex post the mechanism may run a budget deficit for some realizations. At some overhead cost, ideas along the lines of \cite{Arrow1979} and \cite{Aspremont1979} or, alternatively, \cite{Cremer1985} can be used to avoid deficits for all type realizations.
is
\[ R(v, c) \equiv M_B(v, c) - \sum_{i \in N} M_i(v, c), \]
and the welfare or social surplus generated by the mechanism is
\[ W(v, c) \equiv \sum_{i \in N} (v - c_i)Q_i(v, c). \]

Bargaining power

To capture bargaining power, we endow the agents with bargaining weights
\[ w = (w_B, w_1, \ldots, w_n), \]
where \( w_B \in [0, 1] \) is the buyer’s bargaining weight and \( w_i \in [0, 1] \) is supplier \( i \)'s bargaining weight. We assume that at least one agent’s bargaining weight is positive. We define weighted welfare with bargaining weights \( w \) to be
\[ W(v, c; w) \equiv w_B U_B(v, c) + \sum_{i \in N} w_i U_i(v, c). \]

We evaluate market outcomes in the usual way according to the expected value of welfare, \( E_{v,c}[W(v, c)] \).

A mechanism is a first-best mechanism if it maximizes \( E_{v,c}[W(v, c)] \) subject to incentive compatibility and individual rationality, and it is a second-best mechanism if it maximizes that objective with the additional constraint of no deficit, \( E_{v,c}[R(v, c)] \geq 0 \). The first-best and second-best quantities or outcomes are then the quantities or outcomes that arise in the first-best and second-best mechanisms, respectively.

Price-formation mechanism with bargaining weights

The price-formation mechanism with bargaining weights \( w \) is defined as the mechanism that maximizes expected weighted welfare, \( E_{v,c}[W(v, c; w)] \), subject to incentive compatibility, individual rationality, and no deficit. Notice that, because we evaluate outcomes according to expected welfare \( E_{v,c}[W(v, c)] \), the bargaining weights \( w \) are indeed only bargaining weights, that is, they do not affect how outcomes are evaluated, although they affect the distribution of social surplus and, as we will see, sometimes the size of social surplus. Bargaining weights of \( w = (1, 0) \), for example, imply that the buyer has all the bargaining power, while bargaining weights \( w = (0, 1) \) imply that the suppliers have all the bargaining power\(^{15}\).

\(^{15}\)Because there are multiple suppliers, the notion of suppliers having all the bargaining deserves brief elaboration. It means that the mechanism produces the same outcome as one in which a single supplier with cost equal to the minimum of the costs of all the suppliers makes a take-it-or-leave-it offer to the buyer.
The Lagrangian associated with the problem of choosing \((Q, M)\) to maximize \(E_{v,c}[W(v,c;w)]\) subject to no deficit can be written as

\[
E_{v,c}[W(v,c;w) + \rho R(v,c)],
\]

where \(\rho\) is the multiplier on the no-deficit constraint. Using the mechanism design techniques described in Appendix A.1, we can use incentive compatibility to write this as an expression that has one term involving fixed payments to the worst-off types and another term given by

\[
E_{v,c}\left[\sum_{i \in N} [w_B(v - \Phi(v)) + w_i(\Gamma_i(c_i) - c_i) + \rho(\Phi(v) - \Gamma_i(c_i))Q_i(v,c)]\right],
\]

where \(\rho\) is the multiplier on the no-deficit constraint, i.e., the shadow price of budget surplus. Because any budget surplus can be reallocated to the agents through fixed payments, that shadow price is at least \(\max w\). In addition, because a positive expected budget surplus is always possible given our assumption that \(v > c\), the shadow price is finite. Further, defining \(Q_\rho\) to maximize (5) pointwise given \(\rho\), the optimum is characterized by the smallest \(\rho\) greater than or equal to \(\max w\) such that the no-deficit constraint is satisfied at \(Q_\rho\).\(^{16}\)

An immediate implication of this definition is that when \(w = 1\), the price-formation mechanism delivers the second-best quantities. Indeed, any symmetric bargaining weights \(w = (w, ..., w)\) deliver second-best quantities. As an illustration, consider a bilateral trade problem, i.e., assume \(n = 1\). As mentioned, efficient trade is impossible if and only if the supports overlap, i.e., \(\nu < \tau\). Because bargaining weights of \(w = (1, 1)\) yield the second-best outcome, this implies that \(\beta^*(1,1) < 1\) holds if and only if the supports overlap. With nonoverlapping supports, i.e., \(\nu \geq \tau\), the incentive-compatibility and individual-rationality constraints can be satisfied by charging the buyer \(\nu\) and paying the supplier \(\tau\), generating a surplus of \(\nu - \tau \geq 0\), which can then be shared between the buyer and the supplier (e.g., according to their bargaining weights). Note also that for \(w = (1, 0)\), we obtain the buyer’s optimal mechanism. Because the buyer is essentially the residual claimant and so prefers not to trade rather than accepting a deficit, this mechanism does not run a deficit, implying that \(\beta^*(1,0) = 1\). Similarly, for \(w = (0, 1)\), we have the suppliers’ optimal mechanism, which for

\(^{16}\)This follows by the same arguments that were first developed in the working paper version of Gresik and Satterthwaite (1989) and that were first used in published form in Myerson and Satterthwaite (1983).

\(^{17}\)While we do not pursue this here, our approach generalizes directly to the requirement that the mechanism needs to generate a budget surplus of \(K \in \mathbb{R}\) which is not more than the maximum budget surplus that any incentive-compatible, individually-rational mechanism can generate. The second-best mechanism that generates \(K\), but otherwise maximizes the same objective, is then characterized by a mechanism with an allocation rule \(Q_{i,\beta_K(w)}(v,c;w)\) as defined in Lemma \([\ref{lemma:second_best}]\) where \(\beta_K(w)\) is a decreasing function of \(K\). Interpreted in this way, we have \(\beta^*(w) = \beta_0(w)\).
analogous reasons does not run a deficit either, implying that $\beta^*(0,1) = 1$.

Ignoring ties, which occur with probability zero, the allocation rule for the price-formation mechanism is given in the following lemma:

**Lemma 1.** The allocation rule of the price-formation mechanism with bargaining weights $w$ is, for $i \in \mathcal{N}$,

$$Q_{i,\beta^*(w)}(v, c; w) \equiv \begin{cases} 1 & \text{if } \Phi_{i,w}^{\beta^*(w)}(v) \geq \nabla_{i,w}^{\beta^*(w)}(c_i) = \min_{j \in \mathcal{N}} \Gamma_{j,w}^{\beta^*(w)}(c_j), \\ 0 & \text{otherwise.} \end{cases}$$

**Proof.** See Appendix B.

An implication of Lemma 1 is that the probability of trade, and hence social surplus, are increasing in $\beta^*(w)$.

We are left to augment the allocation rule of Lemma 1 with a consistent payment rule. By the payoff equivalence theorem (see, e.g., Myerson [1981], Krishna [2002], Börgers [2015]),18 the interim expected payoff of an agent is pinned down by the allocation rule and incentive compatibility up to a constant that is equal to the interim expected payoff of the worst-off type for that agent. Incentive compatibility implies further that $v$ and $c$ are the worst-off types of the buyer and suppliers, respectively.19 Thus, to complete the definition of the price-formation mechanism, all that remains to be done is to define these constants.

By standard mechanism design arguments (see Appendix A.1), the expected budget surplus for the mechanism with the allocation rule in Lemma 1, not including the constants reflecting payments to worst-off types, can be written in terms of the allocation rule and virtual types as follows:

$$\pi_{\beta^*} \equiv \sum_{i \in \mathcal{N}} \mathbb{E}_{v,c}[\left(\Phi(v) - \nabla_i(c_i)\right) \cdot Q_{i,\beta^*}(v, c; w)].$$

(6)

Of course, if $\beta^*(w) < 1/\max w$, then the no-deficit constraint binds, and it must be that $\pi_{\beta^*(w)} = 0$, in which case the question of how to allocate the budget surplus is moot. In contrast, when $\beta^*(w) = 1/\max w$, the no-deficit constraint need not bind and so $\pi_{\beta^*(w)} > 0$ is possible. In this case, weighted welfare is maximized when $\pi_{\beta^*(w)}$ is allocated among the buyer and suppliers with bargaining weight equal to $\max w$. In the case of multiple such agents, we assume an equal division. Specifically, letting $\hat{u}_B(v) \equiv \mathbb{E}_c[U_B(v, c)]$ and $\hat{u}_i(c_i) \equiv \mathbb{E}_{v,c,i}[U_i(v, c)]$ for $i \in \mathcal{N}$, and letting $m$ denote the number of agents with bargaining

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18 Sometimes, the payoff equivalence theorem is also referred to as the revenue equivalence theorem. However, revenue equivalence is an implication of the payoff equivalence theorem, so the alternative label is somewhat loose.

19 That is, for any mechanism satisfying incentive compatibility, $\underline{v} \in \arg \min_{v \in [v, \bar{v}]} \mathbb{E}_c[U_B(v, c)]$ and $\bar{v} \in \arg \min_{v \in [v, \bar{v}]} \mathbb{E}_{v,c}[U_i(v, c)]$. 

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weight equal to \( \max \sigma \), the price-formation mechanism with bargaining weights \( \sigma \) has

\[
\hat{u}_B(v) = \begin{cases} \frac{\pi^{\beta^*(\sigma)}}{m} & \text{if } w_B = \max \sigma, \\ 0 & \text{otherwise}, \end{cases} \quad \text{and} \quad \hat{u}_i(c) = \begin{cases} \frac{\pi^{\beta^*(\sigma)}}{m} & \text{if } w_i = \max \sigma, \\ 0 & \text{otherwise}. \end{cases} \tag{7}
\]

The outcome of the price-formation process that we consider is then given by the expected buyer and supplier payoffs from the price-formation mechanism, which, following the mechanism design techniques described in Appendix A.1, we can write as follows:

**Proposition 1.** The price-formation mechanism with bargaining weights \( \sigma \) generates expected payoffs

\[
u_B(\sigma) = \hat{u}_B(v) + \mathbb{E}_v \left[ \sum_{i \in \mathcal{N}} \int_v^\infty \mathbb{E}_c \left[ Q_{i,\beta^*(\sigma)}(x, c; \sigma) \right] dx \right]
\]

and, for \( i \in \mathcal{N} \),

\[
u_i(\sigma) = \hat{u}_i(c) + \mathbb{E}_{c_i} \left[ \int_{c_i}^{\infty} \mathbb{E}_{v,c_{-i}} \left[ Q_{i,\beta^*(\sigma)}(v, x, c_{-i}; \sigma) \right] dx \right],
\]

with \( \hat{u}_B(v) \) and \( \hat{u}_i(c) \) given by (7).

We illustrate the set of expected payoff profiles generated by the price-formation process for admissible bargaining weights in Figure 1, which shows a contour plot for the case of 2 suppliers and types that are uniformly distributed on \([0, 1]\).

![Figure 1: Feasible payoff profiles for the case of 2 suppliers for all \( \sigma \in [0, 1]^3 \) such that \( \min \sigma > 0 \). Assumes that types are uniformly distributed on \([0, 1]\). Values for \( u_1(\sigma) \) and \( u_2(\sigma) \) are shown on the axes as indicated, with contours shown for \( u_B(\sigma) \).](image)
### 3.2 Illustration

To illustrate, consider the bilateral trade problem of Myerson and Satterthwaite and assume, for concreteness, that $\nu = \zeta = 0$ and $\overline{\nu} = \overline{\zeta} = 1$\textsuperscript{20}. The price-formation mechanism with $w = (1, 0)$ is the buyer-optimal mechanism, which consists of a buyer of type $v$ making the take-it-or-leave-it offer $\Gamma^{-1}(v)$ to the supplier. For example, when the agents’ types are uniformly distributed, the buyer offers $v/2$, yielding $u_B(1, 0) = 1/12$ and $u_1(1, 0) = 1/24$. Conversely, for $w = (0, 1)$, we have the supplier-optimal mechanism. In this mechanism, the supplier with cost $c$ makes the take-it-or-leave-it offer $\Phi^{-1}(c)$. For uniformly distributed types, this is $(c + 1)/2$, yielding $u_1(0, 1) = 1/12$ and $u_B(0, 1) = 1/24$. These two points are illustrated as dots in Figure 2(a). One can, of course, use randomized take-it-or-leave-it offers with the buyer (resp. supplier) making the offer with some probability. Thus, any convex combination between $(u_B(1, 0), u_1(1, 0))$ and $(u_B(0, 1), u_1(0, 1))$ is achievable, as indicated by the dashed line in Figure 2(a).

![Figure 2](image)

**Figure 2**: Panel (a): Buyer and supplier payoffs for randomized take-it-or-leave-it offers and for the $k$-double auction with $k = 1/2$. Panel (b): Payoffs in the $k$-double auction for all $k \in [0, 1]$. Both panels assume that there is one supplier and that the buyer’s value and the supplier’s cost are uniformly distributed on $[0, 1]$.

However, one can in general do better than using randomized take-it-or-leave-it offers by allowing the allocation rule to vary with the bargaining weights beyond just being a linear combination of the extremes\textsuperscript{21}. Indeed, as is evident from Lemma 1, this is what happens

\textsuperscript{20}Of course, the assumption of identical supports imposes some restrictions. Given this assumption, setting $\nu = 0$ and $\overline{\nu} = 1$ is then an innocuous normalization.

\textsuperscript{21}It is perhaps worth noting that this reasoning is essentially the same as that invoked by Samuelson.
in our price-formation mechanism, whose allocation rule depends on the bargaining weights, and this is also the case for the \( k \)-double auction of Chatterjee and Samuelson (1983). To see this, recall that given \( k \in [0, 1] \), the buyer and supplier in a \( k \)-double auction simultaneously submit bids \( p_B \) and \( p_S \), and trade occurs at the price \( kp_B + (1-k)p_S \) if and only if \( p_B \geq p_S \). By construction, the \( k \)-double auction never incurs a deficit. If the agents’ types are uniformly distributed on \([0, 1]\), then the linear Bayes Nash equilibrium of the \( k \)-double auction results in the following allocation\(^\text{22}\):

\[
\tilde{Q}_k(v, c) \equiv \begin{cases} 
1 & \text{if } v \geq \frac{c + k}{2} + \frac{1-k}{2}, \\
0 & \text{otherwise}.
\end{cases}
\]

The \( k \)-double auction reduces to take-it-or-leave-it offers for \( k \in \{0, 1\} \) but for \( k = 1/2 \), the expected payoffs are 9/128 for each agent, yielding expected social surplus of 9/64. This is larger than the expected social surplus of 1/8 under take-it-or-leave-it offers, as illustrated in Figure 2(b), which shows equilibrium payoffs for the \( k \)-double auction as \( k \) varies from zero to one. Indeed, Myerson and Satterthwaite (1983) were the first to observe that for \( k = 1/2 \) and uniformly distributed types, the \( k \)-double auction yields the second-best outcome.

As we now show, our price-formation mechanism includes the \( k \)-double auction described above as a special case. First note that for the case of one-supplier, the allocation \( Q_{i,\beta^*}(w)(v, c; w) \) is the same for all \( w \) with the same bargaining differential \( \Delta \) defined by

\[
\Delta \equiv \frac{w_B - w_1}{\max\{w_B, w_1\}} \in [-1, 1].
\]

Thus, we can write the allocation rule as functions of \( \Delta \) rather than of the bargaining weights, denoted by \( Q_i^*(v, c; \Delta) \).\(^\text{23}\)

Further, assuming uniformly distributed types, as we show below, for a given \( k \in [0, 1] \), there exists \( \Delta_k \in [-1, 1] \) such that the outcome in the \( k \)-double auction is the same as in the price-formation mechanism with bargaining parameter \( \Delta_k \). Conversely, for any \( \Delta \in [-1, 1] \), there exists \( k_\Delta \) such that the outcome in the \( k_\Delta \)-double auction is the same as in the price-formation mechanism with bargaining parameter \( \Delta \). For example, trade occurs in the price-formation mechanism with bargaining parameter \( \Delta = 1 \) if and only if \( v \geq 2c \), which is the same condition for trade as in the \( k \)-double auction with \( k = 1 \).

To provide some details for the result that the price-formation mechanism nests the \( k \)-double auction for the case of one supplier and uniformly distributed types, note that in this

\(^{1949}\) to demonstrate that with constant returns to scale, the production possibility frontier is concave.

\(^{22}\)In the linear Bayes Nash equilibrium, a buyer of type \( v \) bids \( p_B(v) = (1-k)v/(2(1+k)) + v/(1+k) \) and a supplier with cost \( c \) bids \( p_S(c) = (1-k)/2 + c/(2-k) \).

\(^{23}\)For \( k = 1 \), \( p_B(v) = v/2 \) and \( p_S(c) = c \), and for \( k = 0 \), \( p_B(v) = v \) and \( p_S(c) = (c+1)/2 \).

\(^{24}\)If \( \Delta \in [-1, 0] \), then \( Q_i^*(v, c; \Delta) \equiv Q_{i,\beta^*(1+\Delta,1)}(v, c; 1+\Delta, 1) \) and if \( \Delta \in (0, 1] \), then \( Q_i^*(v, c; \Delta) \equiv Q_{i,\beta^*(1,1-\Delta)}(v, c; 1, 1-\Delta) \).
case the weighted virtual value and cost functions are linear in the types. This allows one
to derive an analytic expression for $\pi_\beta$, which, solving $\pi_\beta = 0$ for $\beta$, produces an analytic
expression for $\beta^*(\Delta)$ for the case of $\max\{w_B, w_1\} = 1$. Specifically, $\beta^*(\Delta) = 1$ if $\Delta \in \{-1, 1\}$,
and otherwise

$$
\beta^*(\Delta) = \frac{4 - 2|\Delta| - 2\sqrt{1 - |\Delta| + \Delta^2}}{3(1 - |\Delta|)}.
$$

(8)

It is then straightforward to derive, for a given $\Delta$, the conditions on $(v, c)$ such that the
quantity traded $Q_1^*(v, c; \Delta)$ is equal to one. Equating this condition with the condition for
trade in the $k$-double auction allows one to identify the relation between $\Delta$ and $k$:

$$
\Delta_k \equiv \frac{1 - 2k}{k^2 - \max\{1, 2k\}}.
$$

(9)

We then have the following result:

**Proposition 2.** For the case of one supplier and types that are uniformly distributed on $[0, 1]$, the
price-formation mechanism with bargaining differential $\Delta_k$ is equivalent to the $k$-double
auction.

*Proof.* See Appendix B.

As intuition for the relation between the $k$-double auction and the price-formation mecha-
nism, note that a $k$-double auction with $k = 1$ essentially gives all the bargaining power to
the buyer because trade, when it occurs, occurs at the buyer’s bid. This is the same as in
the price-formation mechanism with $\Delta = 1$, which is the optimal procurement mechanism
for the buyer. It follows then that the sets of types that result in trade are the same. At the
other extreme, a $k$-double auction with $k = 0$ essentially gives all the bargaining power to
the seller, which is the same as in the price-formation mechanism with $\Delta = -1$. In between,
a $1/2$-double auction balances the bargaining powers of the two agents, which is also the
case in the price-formation mechanism with bargaining parameter $\Delta = 0$. Equalization of
bargaining power increases expected social surplus in our price-formation mechanism just as
it does in the $k$-double auction.

### 3.3 Implementation

In many cases, economists have achieved greater comfort with models of price-formation pro-
cesses when the literature has shown that there exists a noncooperative game that, at least
under some assumptions, has as an equilibrium outcome that is the same as the outcome
delivered by the model under consideration. Indeed, this comfort often extends well beyond
the narrow confines of the foundational game. For example, to support the model of perfectly
competitive markets, one might view price-taking buyers and sellers as submitting demand
and supply schedules to a (fictitious) Walrasian auctioneer who then sets market clearing pricing. Similarly, in the Cournot model, one might view firms as submitting quantities to an auctioneer or market maker who sets the market clearing price. Under assumptions on the alternation of offers and taking the limit as the time between offers goes to zero, Rubinstein bargaining delivers Nash bargaining outcomes (Rubinstein, 1982); and under additional assumptions, including conditions on firms’ marginal contributions and passive beliefs, the limit of an alternating-offers game approximates Nash-in-Nash outcomes (Collard-Wexler et al., 2019).

In light of this, it is perhaps useful to note that, as shown in Proposition 2 above, for the case of one supplier and uniformly distributed types, the \( k \)-double auction of Chatterjee and Samuelson (1983) provides an extensive-form game that delivers the same outcomes as our price-formation mechanism. In addition, in the online appendix, we show that our approach has axiomatic foundations analogous to those that underpin Nash bargaining.

As we discuss now, for general distributions and any number of suppliers, the outcome of our price-formation mechanism arises in equilibrium in an extensive-form game involving a buyer, sellers, and a fee-setting broker. Building on the model of Loertscher and Niedermayer (2019), we define the fee-setting extensive-form game to have one buyer, \( n \geq 1 \) suppliers, and an intermediary that facilitates the buyer’s procurement of inputs from the suppliers and that charges the buyer a fee for its service. The buyer’s value and the suppliers’ costs are not known by the intermediary, although the intermediary does know the distributions \( F \) and \( G_1, \ldots, G_n \) from which those types are independently drawn. The timing is as follows: 1. the intermediary announces (and commits to) a discriminatory clock auction, which we define below, and fee schedule \( (\sigma_1, \ldots, \sigma_n) \), where \( \sigma_i \) maps the price \( p \) paid by the buyer to supplier \( i \) to the fee \( \sigma_i(p) \) paid by the buyer to the intermediary, should the buyer purchase from supplier \( i \); 2. the buyer sets a reserve \( r \) for the auction; 3. the intermediary holds the auction with reserve \( r \), which determines the winning supplier, if any, and the payment to that supplier; 4. given winner \( i \) and payment \( p \), supplier \( i \) provides the good to the buyer, and the buyer pays \( p \) to supplier \( i \) and \( \sigma_i(p) \) to the intermediary. If no supplier bids below the reserve, then there is no trade and no payments are made, including no payment to the intermediary.

As just mentioned, the intermediary uses a discriminatory clock auction with reserve \( r \). Because this is a procurement, it is a descending clock auction, with the clock price starting at the reserve \( r \) and descending from there. As in any standard clock auction, as the clock price changes, participants choose when to exit, and when they exit, they become inactive and remain so. The clock stops when only one active bidder remains, with ties broken by randomization. A discriminatory clock auction specifies supplier-specific discounts off the

\[25\] Microfoundations of the Cournot model along the lines of Kreps and Scheinkman (1983), while dispensing with the assumption of an auctioneer, maintain the assumption of an exogenously given price-formation process by postulating that firms first choose capacities and then prices.
final clock price \((\delta_1, ..., \delta_n)\), where \(\delta_i\) maps the clock price to supplier \(i\)'s discount—activity by supplier \(i\) at a clock price of \(\hat{p}\) obligates supplier \(i\) to supply the product at the price \(\hat{p} - \delta_i(\hat{p})\). By the usual clock auction logic, in the essentially unique equilibrium in non-weakly-dominated strategies, supplier \(i\) with cost \(c_i\) remains active in the auction until the clock price reaches \(\hat{p}\) such that \(\hat{p} - \delta_i(\hat{p}) = c_i\), and then supplier \(i\) exits. We assume that the suppliers follow these strategies.

Turning to the incentives of the buyer and intermediary, the buyer chooses the reserve to maximize its expected payoff, and the intermediary chooses the auction discounts and the fee structure to maximize the expected value of its objective. To allow for the possibility that the intermediary has an interest in promoting the surplus of the agents, we assume that the intermediary’s objective is to maximize expected weighted welfare \(W(v, c; w)\) subject to no deficit, where we refer to \(w\) in this context as intermediary preference weights.

As we show in the following proposition, the outcome of our price-formation mechanism arises as a Bayes Nash equilibrium of this game:

**Proposition 3.** The outcome of the price-formation mechanism with bargaining weights \(w\) is a Bayes Nash equilibrium outcome of the fee-setting extensive-form game with intermediary preference weights \(w\).

*Proof. See Appendix B.*

Thus, the fee-setting extensive-form game, in which a fee-setting intermediary procures an input for the buyer from competing suppliers, provides a microfoundation for the price-formation mechanism. Reminiscent of Crémer and Riordan (1985), the sequential nature of the game allows an equilibrium that is Bayesian incentive compatible for one agent, the buyer, and dominant-strategy incentive compatible for the other agents, the suppliers. The equilibrium of our fee-setting game satisfies ex post individual rationality for both the buyer and suppliers, but only balances the intermediary’s budget in expectation. In contrast, in Crémer and Riordan (1985), the budget is balanced ex post, but only the interim individual rationality constraint is satisfied for the buyer.

To illustrate the fee-setting extensive-form game, we return to the Chatterjee-Samuelson setup with one supplier and uniformly distributed types, where the price-formation mechanism coincides with a \(k\)-double auction. In that case, conditional on trade and a payment \(p\) to the supplier, the buyer’s equilibrium fee to the intermediary is linearly decreasing in \(p\) and given by\(^{26}\)

\[
\sigma(p; \Delta) = \frac{1 - \beta^*(\Delta) (1 + \Delta \cdot 1_{\Delta<0})}{2 - \beta^*(\Delta) (1 + \Delta \cdot 1_{\Delta<0})} (1 - p) - \frac{\beta^*(\Delta) (1 - \Delta \cdot 1_{\Delta>0})}{4 - 2\beta^*(\Delta) (1 + \Delta \cdot 1_{\Delta<0})} p,
\]

\(^{26}\)Because \(\Delta = w_B - w_S\) and the larger of the two weights is is equal to 1, we have \(w_B = 1 + \Delta \cdot 1_{\Delta<0}\) and \(w_S = 1 - \Delta \cdot 1_{\Delta>0}\), which are expressions that appear in the definition of \(\sigma(p; \Delta)\).
where $\beta^*(\Delta)$ is defined in (8). For example, if $\Delta = 1$, then using $\beta^*(1) = 1$, we have $\sigma(p; 1) = 0$, which implies that when trade occurs, which for this case is when $2c \leq v$, the buyer pays $\frac{v}{2}$ to the seller and nothing to the intermediary. This is the same outcome as in the $k$-double auction with $k = 1$, in which case the buyer bids $\frac{v}{2}$, the seller bids $c$, and the buyer’s bid determines the amount paid conditional on trade. For $\Delta \in [-1, 1)$, the expected buyer and supplier payoffs are the same in the fee-setting extensive-form game as in the $k_\Delta$-double auction.

4 Merger review

In this section, we analyze horizontal mergers and vertical integration.

4.1 Horizontal mergers

As mentioned above, we model a horizontal merger between suppliers as creating a merged entity whose cost is drawn from the distribution of the minimum of two independent cost draws, one from each of the merging suppliers’ distributions. We focus on a merger of two suppliers that are symmetric in terms of both their distributions and their bargaining weights. It is then natural to begin by considering the case in which the merged entity has the same bargaining weight as the two merging suppliers, although when we consider countervailing power, we will allow the merged entity to have a larger bargaining weight. Further, as mentioned above, the merged entity draws its cost from the distribution of the minimum of independent cost draws from the merging suppliers’ distributions, so for the case of symmetric merging suppliers with distribution $G_i$ and virtual cost function $\Gamma_i$, the merged entity draws is cost from the distribution $\hat{G}(c) \equiv 1 - (1 - G(c))^2$, with associated virtual cost $\hat{\Gamma}(c) \equiv c + \hat{G}(c)/\hat{g}(c)$. It is straightforward to show that for all $c_1, c_2 \in (\underline{c}, \bar{c})$,

$$\hat{\Gamma}(\min\{c_1, c_2\}) > \min\{\Gamma_i(c_1), \Gamma_i(c_2)\},$$

and similarly for the weighted virtual costs: for all $c_1, c_2 \in (\underline{c}, \bar{c})$ and $a \in [0, 1)$,

$$\hat{\Gamma}_a(\min\{c_1, c_2\}) > \min\{\Gamma_{i,a}(c_1), \Gamma_{i,a}(c_2)\},$$

and

$$\hat{\Gamma}_1(\min\{c_1, c_2\}) = \min\{\Gamma_{i,1}(c_1), \Gamma_{i,1}(c_2)\}.$$

\footnote{For example, if $\Delta = -1$, then using $\beta^*(-1) = 1$, we have $\sigma(p; -1) = \frac{1}{2} - \frac{3}{4}p$, which implies trade occurs when $c \leq 2v - 1$, in which case the buyer pays $2v - 1$ to the seller and $\sigma(2v - 1; -1) = \frac{5}{4} - \frac{3}{2}v$ to the intermediary. Thus, the seller’s expected payoff is $1/12$, the buyer’s expected payment to the intermediary is zero, and the buyer’s expected payoff is $1/24$, just as in the $k$-double auction with $k = 0$.}
An implication of this is that a merged entity trades less often than the lower cost of the two merging suppliers.

**Productive power effects**

We begin by holding bargaining power fixed and considering a merger between suppliers 1 and 2 with common bargaining weight $w_1$ and distribution $G_1$. We focus on the case in which $\max w = 1$. Denote by $\beta^*$ the budget parameter in the pre-merger market and by $\hat{\beta}^*$ the budget parameter in the post-merger market, where for now we drop the dependence of $\beta^*$ and $\hat{\beta}^*$ on the bargaining weights. Given the argument above, if $w_1 \beta^* < 1$, then for all $c_1, c_2 \in (c, \bar{c})$,

$$\hat{\Gamma}_{w_1 \beta^*}(\min\{c_1, c_2\}) > \min\{\Gamma_{1,w_1 \beta^*}(c_1), \Gamma_{1,w_1 \beta^*}(c_2)\},$$

with equality when $w_1 \beta^* = 1$.

If the pre-merger market is characterized by $\beta^* < 1$, then absent any adjustment to $\beta^*$, the merger results in an increased virtual cost for the merged entity relative to the lower of the virtual costs of the pre-merger suppliers, which reduces the probability of trade by the merging suppliers and reduces revenue to the mechanism. Because $\beta^* < 1$ implies that the pre-merger mechanism’s expected revenue is zero, the reduction pushes expected revenue below zero, implying that a decrease in $\beta$ is required to avoid a deficit. Thus, $\hat{\beta}^* < \beta^*$, further reducing the probability of trade. It follows that given a pre-merger market with $\beta^* < 1$, a merger reduces both the buyer’s expected surplus and expected social surplus.

Rivals to the merging suppliers benefit from the increase in the merging suppliers’ virtual cost; however, the decrease in $\beta^*$ and associated overall reduction in trade harms the rivals, leaving the overall effect of the merger on rivals ambiguous. Interestingly, the effect of a merger on the merging parties is also ambiguous and depends on which one dominates: the benefit from the reduction in competition and associated increased payments conditional on trade, or the cost associated with the reduced probability of trade.

In contrast, in the extreme case in which $w_1 \beta^* = 1$, the virtual cost for the merged entity is simply the minimum of the costs of the merging suppliers, and so the merger does not affect market outcomes: for the buyer and rivals, the probability of trade is not affected, and for the merged entity, the probability of trade is equal to the sum of the pre-merger probabilities of trade by the merging suppliers. As a result, the mechanism’s budget surplus is not affected, so $\beta^* = \hat{\beta}^* = 1$, and the merger is neutral for all agents.

In the remaining case with $w_1 < 1$ and $\beta^* = 1$, a merger reduces trade by the merging suppliers, which harms the buyer. The mechanism’s budget surplus is weakly reduced, so $\hat{\beta}^* \leq \beta^*$. As before, effects on rivals and the merging suppliers depend on whether $\hat{\beta}^*$ is strictly below $\beta^*$, and so the effect on rivals and the merging suppliers is ambiguous.

We summarize with the following proposition:
Proposition 4. Considering only productive power effects, a merger of symmetric suppliers with bargaining weight $w_1$ is neutral if $w_1 \beta^*(w) = 1$, and otherwise reduces the buyer’s expected surplus and expected social surplus.

Proposition 4 tells us that, except in extreme cases, the productive power effects of a merger of symmetric suppliers harm the buyer and society. However, bargaining power effects may go in the opposite direction, at least as far as society is concerned. There are two components to this. First, as discussed in the previous section, an equalization of bargaining power increases expected social surplus, so increases in supplier power can countervail the negative effects of a more powerful buyer, even absent a merger. Second, as shown in Proposition 4, the productive power effects of mergers typically reduce expected social surplus, so increases in suppliers’ bargaining power can countervail (balancing or even reversing) this negative effect of mergers. We address countervailing power in detail in what follows.

Countervailing power

The question of whether a merger can be defended on the grounds that it endows merging parties with countervailing power that somehow “levels the playing field” features prominently in concurrent antitrust debates and cases. For example, in a merger context, the Australian government’s 1999 (now superseded) guidelines stated: “If pre-merger prices are distorted from competitive levels by market power on the opposite side of the market, a merger may actually move prices closer to competitive levels and increase market efficiency. For example, a merger of buyers in a market may create countervailing power which can push prices down closer to competitive levels” (ACCC, 1999, para. 5.131). Ho and Lee (2017) find evidence of countervailing power, estimating that mergers among insurers increase the insurers’ bargaining power in negotiations with hospitals. Based on an analysis of hundreds of mergers, Bhattacharyya and Nain (2011, p. 99) find outcomes that are “consistent with the creation of buyer power through downstream consolidation to countervail upstream market power.”

Despite the relevance of the issue, a major obstacle to analyzing the effects of countervailing power in existing modeling approaches is that these take the efficiency of the price-formation process as given. This is true for all oligopoly models, in which agents on one side of the market (typically buyers) are assumed to be price-takers. It also applies to the randomized take-it-or-leave-it offers model that give rise to the straight line in Figure 2(a). In contrast, as illustrated in Figure 2(b), if the equalization of bargaining weights changes the price-formation process from, for example, one based on take-it-or-leave-it offers to one based on a $k$-double auction with $k = 1/2$, then a change in bargaining weights has an impact on social surplus. The price-formation mechanism that we study captures the effects
of bargaining weights on social surplus because the efficiency of the mechanism varies with bargaining weights.

For the purposes of this discussion of countervailing power, let us focus on the case in which suppliers have symmetric bargaining weights, that is, $w_1 = \ldots = w_n$. In this case, as above, payoffs are pinned down by the bargaining differential $\Delta = w_B - w_1$, and so we can write the agents’ expected surpluses as functions of $\Delta$. Letting $u_S(\Delta) \equiv \sum_{i \in N} u_i(\Delta)$, we define the William’s frontier (identified for a special case in Williams (1987)) as

$$F \equiv \{(u_S(\Delta), u_B(\Delta)) \mid \Delta \in [-1, 1]\},$$

with associated mapping $\omega : [u_S(1), u_S(-1)] \to [u_B(-1), u_B(1)]$ defined by

$$\omega(u) = \max\{y \mid (u, y) \in F\}.$$

As discussed above, for the special case of $n = 1$ and uniformly distributed types, the Williams frontier coincides with the payoff frontier for the $k$-double auction, which is depicted in Figure 2(b). As shown there, the frontier is concave to the origin. As we now show, the concavity of the Williams frontier holds generally:

**Proposition 5.** The Williams frontier is concave to the origin, i.e., the function $\omega(\cdot)$ is strictly decreasing and concave.

**Proof.** See Appendix B.

As foreshadowed in footnote 21, the concavity of the Williams frontier follows a logic that is reminiscent of Samuelson’s (1949) argument that the production possibility frontier is concave under constant returns to scale. The price-formation mechanism could randomize over the mechanism that is optimal for the buyer and the mechanism that is (jointly) optimal for suppliers. Hence, any linear combination between $(u_S(1), u_B(1))$ and $(u_S(-1), u_B(-1))$ can be achieved. This corresponds to the thought experiment of Samuelson whereby equal proportions of all inputs are shifted from one sector to another, with which, in his words, a “neophyte bureaucrat might be satisfied” (pp. 184–185). However, in general, one can do better by reoptimizing.

Intuitively, if the second-best mechanism, which we obtain when bargaining weights are equal, does not have a budget surplus, then movement away from that second-best mechanism in a direction that favors one agent over another cannot simply shift payments away from disfavored agents and towards the favored agent because individual rationality would be violated for the disfavored agents. The allocation itself must shift, and any shift away from the second-best allocation reduces welfare.

Building on this, the concavity of the Williams frontier has the following implication:
Corollary 1. A change in bargaining weights that moves them closer to symmetry, i.e., that moves $\Delta$ closer to zero, weakly increases social surplus: if $\Delta' < \Delta \leq 0$ or $0 \leq \Delta < \Delta'$, then

$$u_B(\Delta') + u_S(\Delta') \leq u_B(\Delta) + u_S(\Delta) \leq u_B(0) + u_S(0).$$

Further, if a merger leads to an equalization of bargaining power in the post-merger market, then the merger combined with the equalization of bargaining power increases efficiency if the bargaining power effects outweigh the effects of consolidation. As an example, Figure 3(a) shows a case in which a merger reduces social surplus if the buyer has all the bargaining power both before and after the merger, but a merger increases social surplus if the buyer and suppliers’ bargaining weights are equalized after the merger. Indeed, Figure 3(b) provides an example in which countervailing power restores full efficiency to the post-merger market—specifically, if the buyer has all the bargaining weight prior to the merger, then the pre-merger outcome is not fully efficient, but with symmetric bargaining weights in the post-merger market, the outcome is fully efficient.

![Figure 3](image_url)

Figure 3: Williams frontiers for 2 pre-merger suppliers (blue) and 1 post-merger supplier (orange). The bargaining power differential for the pre-merger market is denoted by $\Delta$ and for the post-merger market by $\hat{\Delta}$. Panel (a) assumes that buyer’s and pre-merger suppliers’ types are uniformly distributed on $[0, 1]$. Panel (b) assumes that the pre-merger suppliers’ costs are uniformly distributed on $[0, 1]$ and that the buyer’s value is uniformly distributed on $[1.25, 2.25]$. The pre-merger market is fully efficient for $\Delta \in [-0.19, 0.12]$, and the post-merger market is fully efficient for $\hat{\Delta} \in [-0.16, 0]$, as indicated by thick portions of the blue and orange lines, respectively, that coincide with the dashed line.

We summarize with the following result:
Corollary 2. A merger combined with an equalization of bargaining weights is no more harmful to expected social surplus than the same merger with no change in bargaining weights and, in some settings, increases expected social surplus.

Necessary conditions for a countervailing power defense

Corollary 2 raises the possibility of a countervailing power defense for a merger. If a merger only involves productive power effects, then it reduces social surplus whenever the first-best is not possible. In contrast, if the merger causes bargaining weights to shift in favor of the suppliers, then the merger may improve expected social surplus despite the adverse productive power effects. Of course, a merger with countervailing power is doubly bad for the buyer—competition among suppliers is reduced and the remaining suppliers have increased bargaining power. Thus, merger review based on a buyer-surplus standard would never be moved by a countervailing power defense. In contrast, merger review based on a social-surplus standard may well be.

Our analysis allows us to identify necessary conditions for a countervailing power defense. First, as just mentioned, the objective of the merger review would need to include the promotion of social surplus, and not just buyer surplus. Second, the buyer would need to have greater bargaining power than the suppliers prior to the merger, so that movement towards the equalization of bargaining power is possible. Third, the buyer would need to retain at least some bargaining power following the merger—buyer power would need to diminish, but not vanish—so that society is not simply trading a dominant buyer for dominant sellers.

This brings to mind the discussion in the EC merger guidelines saying that “it is not sufficient that buyer power exists prior to the merger, it must also exist and remain effective following the merger. This is because a merger of two suppliers may reduce buyer power if it thereby removes a credible alternative” (EC Guidelines, para. 67). Although it is not clear to what extent our notion of bargaining weights coincides with the EC’s notion of buyer power, our conclusions are consistent in that the buyer must have power before a merger and retain at least some power after a merger in order for a countervailing power defense to make economic sense.

Evidence required for countervailing power defense

The necessary conditions for a countervailing power defense raise the question of how one would ascertain that a buyer has bargaining power. Evidence consistent with buyer power

\[28\] The EC merger guidelines state, “Countervailing buyer power in this context should be understood as the bargaining strength that the buyer has vis-à-vis the seller in commercial negotiations due to its size, its commercial significance to the seller and its ability to switch to alternative suppliers” (EC Guidelines, para. 64).
but inconsistent with the absence of buyer power includes: (i) the buyer uses procurement methods that result in suppliers other than the lowest-cost supplier winning, such as handicaps for certain suppliers; (ii) the buyer purchases in separate markets and the distribution of the reserve prices is different across the two markets\(^{29}\) (iii) one observes with positive probability ties in procurement outcomes and randomization over winners.\(^{30}\)

4.2 Vertical integration

Turning to vertical mergers, we now analyze vertical integration between the buyer and one of the suppliers. We assume that the integrated entity can efficiently solve its internal agency problem. This assumption is standard and can be rationalized on the grounds that integration slackens the individual rationality constraints within the integrated entity.

The price-formation mechanism following vertical integration between the buyer and supplier \(i\) is as before, but with the vertically integrated firm acting as a buyer with value \(y = \min\{v, c_i\}\), whose distribution is \(1 - (1 - F(y))(1 - G_i(y))\), and attempting to procure from one of the nonintegrated suppliers. If there is no trade between the vertically integrated firm and the nonintegrated suppliers, then the integrated firm has payoff equal to \(\max\{0, v - c_i\}\) due to internal sourcing, and the nonintegrated suppliers have payoffs of zero.

Consider first a bilateral trade setting with overlapping supports before integration (i.e., \(n = 1\) and \(v < \tau\)). Because ex post efficient trade is impossible when the buyer and supplier are independent entities, it follows immediately from our assumption that the integrated entity can resolve the internal agency problem that vertical integration can increase social surplus. It does so by essentially eliminating a Myerson-Satterthwaite problem. We state this as follows:

**Proposition 6.** With one supplier and overlapping supports, vertical integration increases social surplus (to the first-best).

By Proposition 6 vertical integration can eliminate a Myerson-Satterthwaite problem. However, it can also create one, as we show now.

The assumption of nonoverlapping supports implies that prior to vertical integration, ex post efficient trade is possible. Hence, vertical integration cannot possibly increase social surplus. This leaves the question of whether vertical integration could be neutral. The following proposition shows that the answer is negative.

---

\(^{29}\) A buyer without power would optimally set a reserve equal to its value, so even if suppliers in the different markets draw their types from different distributions, the distribution of reserves would be the same across the markets as long as the buyer’s values for the goods in the markets are drawn from the same distribution.

\(^{30}\) For a buyer with power, this outcome arises when suppliers draw their costs from distributions that are identical but do not satisfy regularity, that is, their virtual costs are not monotone and so the optimal mechanism involves “ironing.” For a buyer without power, regardless of regularity, one would expect the buyer to purchase from the lowest-cost supplier.
Proposition 7. With two or more suppliers and nonoverlapping supports, vertical integration decreases social surplus.

Proof. After integration between the buyer and supplier $i$, the buyer’s willingness to pay is the cost realization of the integrated supplier, that is, $c_i$, whose support is $[c, \bar{c}]$. Thus, we have a generalized Myerson-Satterthwaite problem (generalized insofar as there is one buyer but $n - 1 \geq 1$ suppliers). For this setting, impossibility of efficient trade obtains (see, e.g., Delacrétaz et al. 2019). ■

Proposition 7 provides a clear-cut case in which vertical integration is harmful from the perspective of society. This result, as well as the result in Proposition 6, is robust in that it does not depend on specific assumptions about distributions or beliefs of agents. Indeed, because there is always a dominant strategy implementation of the price-formation mechanism, beliefs play no role. Moreover, we obtain social surplus decreasing vertical integration without imposing any restrictions on the contracting space and without invoking exertion of market power by any player (above and beyond requiring individual-rationality and incentive-compatibility constraints to be satisfied). These are noticeable differences relative to the post-Chicago school literature on vertical contracting and integration, whose predictions rely on assumptions about beliefs, feasible contracts, and/or market power.  

At the heart of both Proposition 3 and Proposition 4 is the fact that the efficiency of the price-formation process is endogenous in our setting. (Of course, our results do rely, inevitably, on support assumptions.) The elimination of a Myerson-Satterthwaite problem through vertical integration is the incomplete-information analogue to the classic double mark-up problem. In contrast to the classical literature, however, there is now a new effect, namely that the market with the remaining suppliers becomes less efficient. Further, it is possible for this latter effect to dominate so that the market as a whole is made less efficient as a result of vertical integration.

Connecting bargaining breakdown with vertical integration

The framework discussed here points to new aspects of procurement data that can be used to inform models of the price-formation process. Specifically, a pervasive feature of real-world bargaining is that negotiations often break down. Anecdotal examples range from the U.S. government shut down, to the British coal miners’ and the U.S. air traffic controllers’ strikes in the 1980s, to failures to form coalition governments in countries with proportional representation.

For an overview, see Riordan (2008). On the sensitivity of complete information vertical contracting results to assumptions of “symmetric,” “passive,” and “wary” beliefs see, e.g., McAfee and Schwartz (1994).

This occurs, for example, with $n = 2$ and symmetric bargaining weights if $F$ is uniform on $[0, 1]$ and for $i \in \{1, 2\}$, $G_i(c) = c^{1/10}$, also with support $[0, 1]$. Then vertical integration causes social surplus to decrease from 0.4827 to 0.4815.
representation, to, possibly, Brexit.\footnote{Providing systematic evidence of bargaining breakdown, \cite{backus2020} analyze a data set covering 25 million observations of bilateral negotiations on eBay and find a breakdown probability of roughly 55 percent.}

In the price-formation mechanism that we study, negotiations break down on the equilibrium path for two reasons. First, it may be that the buyer’s value is below the supplier’s cost, but because of private information, the two parties do not know this before they sit down at the negotiating table. Second, by the Myerson-Satterthwaite theorem, even if the buyer’s value exceeds the supplier’s cost, the constraints imposed by incentive compatibility, individual rationality, and no deficit may prevent ex post efficient trade from taking place.

![Figure 4: Probability of no trade as a function of $\kappa$ assuming that $F(v) = 1 - (1 - v)^{1/\kappa}$ and $G_1(c) = c^{1/\kappa}$, both with support $[0, 1]$, and assuming that $w = 1$.](image)

Under the assumption that negotiations are the outcome of our price-formation mechanism with symmetric bargaining weights (or some other assumption on bargaining weights), one can use observed frequencies of negotiation breakdowns to back out the parameters of the distributions from which the buyer and the supplier draw their types. To illustrate, assume that the buyer’s value $v$ is drawn from the distribution $F(v) = 1 - (1 - v)^{1/\kappa}$ and the supplier’s cost $c$ is drawn from the distribution $G(c) = c^{1/\kappa}$, whose supports are $[0, 1]$, where $\kappa \in [0, \infty)$ has the interpretation of a “capacity.” Figure 4 plots the probability that negotiations break down as a function of $\kappa$ under the assumptions stated. For example, if, as in the data set of \cite{backus2020}, 55 percent of all negotiations break down, eyeballing the figure indicates that $\kappa$ must be around 1.5.\footnote{As described by \cite{crawford2014}, there are regular blackouts of broadcast television stations on cable and satellite distribution platforms due to the breakdown of negotiations over the terms for retransmission of the broadcast signal.}

\footnote{More precisely, for the case considered, a breakdown probability of 55 percent corresponds to $\kappa = 1.6090$.} Rather than treating negotiation breakdowns as measurement error, which is difficult to justify if breakdown occurs more than fifty percent of the time in 25 million observation, the frequency of those breakdowns is valuable information that can be used for estimation in the incomplete information framework.
To illustrate how initial market conditions, particularly the probability of breakdown, can effect the efficiency consequences of vertical integration, consider a pre-integration market with two suppliers and one buyer. Parameterize the type distributions so that \( G_i(c) = c^{1/\kappa_i} \) for \( \kappa_1, \kappa_2 > 0 \) and \( F(v) = 1 - (1 - v)^{1/\kappa} \) for \( \kappa > 0 \), and assume symmetric bargaining weights. As an identifying assumption, assume that suppliers’ distributional parameters are equal to one on average (alternatively one might use, e.g., margin data for identification). We show in Figure 5(a) the results of the calibration of these parameterized distributions given data on supplier market shares and the probability of bargaining breakdown.

(a) Calibration of distributions to data

<table>
<thead>
<tr>
<th>mkt shares</th>
<th>Pr(breakdown)</th>
<th>((\kappa_1, \kappa_2, \kappa))</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-50</td>
<td>10%</td>
<td>(1, 1, 11)</td>
</tr>
<tr>
<td>50-50</td>
<td>30%</td>
<td>(1, 1, 3)</td>
</tr>
<tr>
<td>50-50</td>
<td>55%</td>
<td>(1, 1, 1)</td>
</tr>
</tbody>
</table>

(b) Change in social surplus following vertical integration

Figure 5: Interaction between breakdown probabilities and the effects of vertical integration on social surplus. Panel (a): Calibration of distributional parameters based on market shares and breakdown probabilities assuming one buyer and two suppliers with \( F(v) = 1 - (1 - v)^{1/\kappa} \) and for \( i \in \{1, 2\} \), \( G_i(c) = c^{1/\kappa_i} \), and assuming symmetric bargaining weights and that the suppliers’ distributional parameters are equal to 1 on average. Panel (b): Change in social surplus as a result of vertical integration based on the analogous calibration to that of Panel (a) with symmetric suppliers, but varying the probability of breakdown in the pre-integration market (“pre-VI Pr(bd)” denotes the pre-integration probability of breakdown).

Now consider the effect on social surplus of vertical integration assuming that there are two pre-integration suppliers with equal market shares and assuming that bargaining weights are symmetric both before and after integration. As illustrated in Figure 5(b), in markets where before integration the probability of breakdown is low, the change in social surplus from vertical integration is negative. In that case, the reduced efficiency of price formation with the independent supplier dominates the gain in efficiency associated with internal transactions, and vertical integration reduces social surplus. In contrast, when the probability of breakdown is high prior to integration, then the increased efficiency of internal transactions dominates, and social surplus increases as a result of vertical integration.

As this shows, whether vertical integration can be expected to increase or decrease social surplus can be related to the probability of bargaining breakdown in the market prior to vertical integration. This suggests an incremental role for data on bargaining breakdown beyond its usefulness in the calibration of models of bargaining under incomplete information.
5 Extensions

In this section, we extend the model to include investment and derive results that highlight the tight connection between the efficiency of the price-formation process and the incentives to invest. In addition, we extend the model to allow for multi-object demand by the buyer and supplier-specific preferences and show that in that case, bargaining externalities naturally arise.

5.1 Investment

Investment incentives feature prominently, and at times controversially, in concurrent policy debates.\footnote{For example, related to the 2017 Dow-DuPont merger, the U.S. DOJ’s “Competitive Impact Statement” identifies reduced innovation as a key concern (https://www.justice.gov/atr/case-document/file/973951/download, pp. 2, 10, 15, 16).} They have been at center stage in the theory of the firm since the seminal works of Grossman and Hart (1986) and Hart and Moore (1990) (G-H-M hereafter). To account for the possibility of investment by the buyer and the suppliers, we extend our model, adding investment as an action taken by each agent prior to the realization of private information, where investment improves the agent’s type distribution.

We show that the results under incomplete information differ starkly from those obtained in the G-H-M literature. This literature stipulates complete information and efficient bargaining and, as a consequence, obtains hold-up and inefficient investment. In contrast, in our setting, incomplete information protects agents from hold-up, and investments are efficient if and only if price formation is efficient.

Intuitively, if the price-formation mechanism implements the first-best allocation, each agent is paid its marginal contribution to social welfare. By the usual Vickrey-Clarke-Groves logic, this makes truthfully reporting one’s type a dominant strategy for every agent and aligns each agent’s objective with the planner’s at the allocation stage. Anticipating that this will be the case once types are realized, each agent’s incentives are also aligned with the planner’s at the investment stage because each agent’s and the planner’s reward from investment are the same.

For the purposes of this analysis, we assume that $w$ is symmetric with $w = (w, \ldots, w)$. In this case, $\beta^* = 1/w$ implies that the price-formation mechanism is efficient, i.e., trade occurs if and only if $v \geq \min_{i \in \mathcal{N}} c_i$.

To incorporate the possibility of investment by the agents to improve their type distributions, we suppose that the buyer and each supplier can improve (or more generally change) its type distribution by investing $e_B$ at cost $\Psi_B(e_B)$ for the buyer and $e_i$ at cost $\Psi_i(e_i)$ for supplier $i \in \mathcal{N}$. The resulting type distributions are denoted by $F(v, e_B)$ and by $G_i(c, e_i)$ with densities $f(v, e_B)$ and $g_i(c, e_i)$, respectively. We assume that the supports of the dis-
tributions are fixed and not affected by investment, and we assume that the investment cost functions and distributions are sufficiently well behaved that optimality is characterized by first-order conditions. Consistent with G-H-M, we assume that investments are not contractible. Thus, the price-formation mechanism depends only on equilibrium investment levels and not on realized investment levels. One implication of this is that the payments to the worst-off types, \( \hat{u}_B(v) \) and \( \hat{u}_i(c) \), are not affected by actual investments. We suppose that the buyer and supplier first simultaneously make their investments and then price formation takes place.

The social planner’s problem is to solve

\[
\max_{e} \mathbb{E}_v, e \left[ \left( v - \min_{i \in N} c_i \right) \cdot 1_{v > \min_{i \in N} c_i} \right] - \Psi_B(e_B) - \sum_{i \in N} \Psi_i(e_i). \tag{10}
\]

We denote by \( \bar{e} = (\bar{e}_B, \bar{e}_1, ..., \bar{e}_n) \) a solution to (10).

Let \( e_S \equiv (e_i)_{i \in N} \) and \( e_{S \setminus \{i\}} \equiv (e_j)_{j \in N \setminus \{i\}} \) be vectors of suppliers’ investments and denote by

\[
L(x, e_S) \equiv 1 - \Pi_{i \in N}(1 - G_i(x, e_i)) \quad \text{and} \quad L_{-i}(x, e_{S \setminus \{i\}}) \equiv 1 - \Pi_{j \in N \setminus \{i\}}(1 - G_j(x, e_i))
\]

the distributions of the minimum of all the suppliers’ cost draws and of the minimum of all of supplier \( i \)’s rivals’ cost draws, respectively. Using integration by parts, the social planner’s problem in (10) can be rewritten as

\[
\max_e \int_{x}^{v} L(x, e_S)(1 - F(x, e_B))dx - \Psi_B(e_B) - \sum_{i \in N} \Psi_i(e_i).
\]

Under our assumptions, the social planner’s optimal investments \( \bar{e} \) are thus characterized by

\[
- \int_{x}^{v} L(x, \bar{e}_S) \frac{\partial F(x, e_B)}{\partial e_B} \bigg|_{e_B = \bar{e}_B} dx = \Psi_B'(\bar{e}_B), \tag{11}
\]

and for each \( i \in N \)

\[
\int_{x}^{v} \frac{\partial G_i(x, e_i)}{\partial e_i} \bigg|_{e_i = \bar{e}_i} (1 - F(x, \bar{e}_B))(1 - L_{-i}(x, \bar{e}_{S \setminus \{i\}}))dx = \Psi_i'(\bar{e}_i). \tag{12}
\]

\(^{36}\)For the case of nonoverlapping supports, this can be written as

\[
\int_{x}^{v} (1 - F(x, e_B))dx + \int_{x}^{v} L(x, e_S)dx + v - \bar{e} - \Psi_B(e_B) - \sum_{i \in N} \Psi_i(e_i).
\]

This implies that with nonoverlapping supports the problems of optimizing the buyer’s investment and the suppliers’ investments are separable.
Although we do not require these specific assumptions, simple and natural conditions for the first-order conditions to be satisfied are that the investments induce first-order stochastic dominance shifts in the sense that $\frac{\partial F(x,e_B)}{\partial e_B} < 0$ and $\frac{\partial G_i(x,e_i)}{\partial e_i} > 0$ and that each $\Psi_i$ is nondecreasing.

We now show that investments $e$ are a Nash equilibrium of a simultaneous investment game played by the agents when $e$ is such that $\beta^* = 1/w$. (To make the dependence of $\beta^*$ on investments explicit, we write $\beta^*_e$.)

Assuming that $\beta^*_e = 1/w$, the price-formation mechanism based on $\beta^*_e$ induces trade if and only if $v > \min_{i \in \mathcal{N}} c_i$, and the buyer is paid its threshold type. This is the lowest type in $[v, \bar{v}]$ that the buyer could report and still trade, which is $\max\{v, \min_{i \in \mathcal{N}} c_i\}$. Thus, the buyer’s problem is to choose $e_B$ to maximize

$$\max_{e_B} \mathbb{E}_{v,e} \left[ \left( v - \max_{i \in \mathcal{N}} \{v, \min_{i \in \mathcal{N}} c_i\} \right) \cdot 1_{v > \max_{i \in \mathcal{N}} \{v, \min_{i \in \mathcal{N}} c_i\}} - \Psi_B(e_B) + \hat{u}_B(v) \right].$$

Analogously, supplier $i$’s threshold type is $\min_{j \in \mathcal{N} \setminus \{i\}} \{v, c_j, \bar{v}\}$ and so the problem for supplier $i$ is to choose $e_i$ to maximize

$$\max_{e_i} \mathbb{E}_{v,e} \left[ \left( \min_{j \in \mathcal{N} \setminus \{i\}} \{c_j, v, \bar{v}\} - c_i \right) \cdot 1_{c_i < \min_{j \in \mathcal{N} \setminus \{i\}} \{c_j, v\}} - \Psi_i(e_i) + \hat{u}_i(\bar{v}) \right].$$

Integrating by parts, the problems above can be written as

$$\max_{e_B} \int_{v}^{\bar{v}} L(x, e_S) (1 - F(x, e_B)) dx - \Psi_B(e_B) + \hat{u}_B(v),$$

and

$$\max_{e_i} \int_{c_i}^{\bar{v}} G_i(x, e_i) (1 - F(x, e_B)) (1 - L_i(x, e_S \setminus \{i\})) dx - \Psi_i(e_i) + \hat{u}_i(\bar{v}),$$

whose first order conditions are the same as (11) and (12). Moreover, because the conditions for a Nash equilibrium are less restrictive than those for a social optimum (as there are no cross-partial from $i$ to $j$ to worry about in Nash equilibrium), the fact that (11) and (12) characterize a social optimum implies that they also characterize a Nash equilibrium. In other words, $\bar{e}$ is a Nash equilibrium outcome when $\beta^*_e = 1/w$.

In contrast, as we show in the proof of Proposition 8 when $\beta^*_e < 1/w$, $\bar{e}$ is not a Nash equilibrium. In that case, the price-formation mechanism induces trade if and only if $\Phi_{w,\beta^*_e}(v, e_B) \geq \min_{i \in \mathcal{N}} \Gamma_{i,w,\beta^*_e}(c_i, e_i)$, whereas the social planner’s objective is trade if and only if $v \geq \min_{i \in \mathcal{N}} c_i$. As a result, the agents’ objectives and hence payoff maximizing investments differ from those of the social planner.

Thus, we have the following result:

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37 For $n \geq 2$, we can write this more simply as $\min_{j \in \mathcal{N} \setminus \{i\}} \{v, c_j\}$. 

28
**Proposition 8.** For symmetric bargaining weights \( w = (w, \ldots, w) \), efficient investments \( e \) are a Nash equilibrium of the simultaneous investment game if and only if \( \beta^*_e = 1/w \).

**Proof.** See Appendix B.

As shown in Proposition 8 when \( \beta^*_e = 1/w \), the buyer’s optimal choice of \( e_B \) and supplier \( i \)'s optimal choice of \( e_i \) are identical to the choices that the social planner would make. In other words, efficient price formation implies efficient investments. Intuitively, given that the allocation rule is efficient and involves full trade, each agent is the residual claimant to the surplus that its investment generates. Private information about its type protects the agent from hold-up.

Proposition 8 contrasts with the hold-up that arises in the G-H-M literature. Accordingly, the implications for institutional design differ sharply between incomplete information models in which the efficiency properties of price formation are endogenous and complete information models that assume efficient bargaining. In the latter, the social planner would aim to align, say, property rights with how the agents’ investments affect social surplus. In contrast, in incomplete information models, the planner would choose designs that render price formation efficient. Once price formation is efficient, efficient investments follow.

Hatfield et al. (2018) provide an equivalence result between efficient dominant-strategy mechanisms under incomplete information and efficient investments, which is obviously tightly related to Proposition 8. Efficient dominant strategy mechanisms are equivalent to the Vickrey-Clarke-Groves (VCG) mechanism, and with independent private values, there is a well-known equivalence between Bayesian incentive compatibility and dominant strategy incentive compatibility (see e.g. Gershkov et al., 2013). In this way, Proposition 8 connects to the equivalence result of Hatfield et al. (2018) and to earlier work by Milgrom (1987) and Rogerson (1992). However, the no-deficit constraint in our setting implies that the VCG mechanism is not admissible when we have overlapping supports.

As is perhaps clear from the analysis above, the efficiency result of Proposition 8 continues to hold if instead of investments in cost reduction, each supplier can invest in the “quality” of its product. Specifically, suppose that when supplier \( i \) makes investment \( \theta_i \geq 0 \) in the quality of its product, the buyer then has value \( \theta_i v \) for supplier \( i \)'s product. In this setup, both the planner and supplier \( i \) only value supplier \( i \)'s investment when the buyer trades with supplier \( i \). Because the VCG mechanism gives supplier \( i \) its social marginal product, accounting for

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38 In a setup where efficient bargaining is possible because of shared ownership (rather than the absence of any allocation-relevant private information), Schmitz (2002, p. 176) notes that “Intuitively, ... a party’s ex ante expected utility from an ex post efficient mechanism is (up to a constant) equal to the total expected surplus, so that each party is residual claimant on the margin from his or her point of view.”

39 Related to this, Lauermann (2013) considers a dynamic search model and finds that it is easier/possible to converge to Walrasian efficiency with private information, but without private information, hold up prevents convergence to efficiency. These results are consistent with ours when one interprets search as investment.
the investment \( \theta_i \), efficient investment levels continue to be a Nash equilibrium. This result contrasts with that of Che and Hausch (1999), who study a contracting setup in which investments by suppliers in cost reduction are efficient, but investments by suppliers that benefit the buyer need not be. Importantly, however, there is no incomplete information at the price-formation stage in their model.

**Market structure and the efficiency of investment**

Armed with Proposition 8, we can analyze the effect of a change in market structure, such as vertical integration, on the efficiency of investment. For the results here, assume that agents have symmetric bargaining weights, \( w = (w, \ldots, w) \). With one supplier in the pre-integration market and overlapping supports, we have \( \beta^*_{\bar{e}} < 1/w \), implying that equilibrium investments are inefficient. But, by assumption, trade, and thus investments, are efficient after vertical integration. Thus, with overlapping supports, vertical integration promotes efficient investment insofar as there is an equilibrium with efficient investments after integration but not before. In contrast, with two or more suppliers and nonoverlapping supports, price formation is efficient (\( \beta^*_{\bar{e}} = 1/w \)) without vertical integration, and so investments are efficient without vertical integration. But following vertical integration, the price-formation process has \( \beta^*_{\bar{e}} < 1/w \), and so investments are no longer efficient. Thus, with two or more suppliers and nonoverlapping supports, vertical integration disrupts efficient investment insofar as there is no equilibrium with efficient investments after integration whereas there was one before integration.

**Corollary 3.** Assuming symmetric bargaining weights, with \( n = 1 \) and overlapping supports, vertical integration promotes efficient investment; but with \( n \geq 2 \) and nonoverlapping supports, vertical integration disrupts efficient investment.

---

40In addition, the vertically integrated firm’s objective is further differentiated from the social planner’s objective because it may trade internally. Specifically, letting \( \Phi_\alpha(c,e) \) denote the vertically integrated firm’s weighted virtual value function associated with type \( c \) drawn from \( G_1(\cdot, e) \) and \( \Phi^{-1}_\alpha(x,e) \) denote the inverse with respect to the first argument, the vertically integrated firm’s objective when the mechanism assumes efficient investments is (dropping the subscript \( \bar{e} \) on \( \beta^* \) to ease notation):

\[
E_{v,e} \left[ \max \{0, v - c_1\} \cdot 1_{c_1 < \Phi^{-1}_{\beta^*}(\min_{i \in N \setminus \{1\}} \Gamma_i, \beta^*(c_i, \bar{e}_i), \bar{e}_B) \} + \left(v - \Phi^{-1}_{\beta^*}(\min_{i \in N \setminus \{1\}} \Gamma_i, \beta^*(c_i, \bar{e}_i), \bar{e}_B) \right) \cdot \right. 1_{c_1 > \Phi^{-1}_{\beta^*}(\min_{i \in N \setminus \{1\}} \Gamma_i, \beta^*(c_i, \bar{e}_i), \bar{e}_B) \} - \Psi_B(e_B) - \Psi_1(e_1),
\]

which for \( \beta^*_{\bar{e}} < 1/w \), implies first-order conditions for the vertically integrated firm’s optimal \( e_B \) and \( e_1 \) that differ from the corresponding first-order conditions in the planner’s problem. The inefficiency relates to the difficulties that arise in principal-agent problems when agents’ actions are multi-dimensional (e.g., Holmström and Milgrom 1991).
5.2 Bargaining externalities

To allow for and investigate bargaining externalities, we now return to the model without investment, but generalize it to allow the buyer to have preferences over suppliers and to have demand for $D \in \{1, 2, \ldots\}$ objects. To this end, we let $\theta = (\theta_1, \ldots, \theta_n)$ be a commonly known vector of taste parameters of the buyer, with the meaning that the value to the buyer of trade with supplier $i$ when the buyer’s type is $v$ is $\theta_i v$. Assuming $D \geq n$ for the moment, the buyer’s payoff when receiving the object from supplier $i$ with probability $q_i$ and making the payment $p$ is $\sum_{i \in N} \theta_i v q_i - p$. Adjusting for the buyer’s maximum demand $D$, under (ex post) efficiency, trade should occur between the buyer and supplier $i$ if and only if $\theta_i v - c_i$ is positive and among the $D$ highest values of $(\theta_j v - c_j)_{j \in N}$. The problem is trivial if $\max_{i \in N} \theta_i v > c$ because then it is never ex post efficient to have trade with any supplier. From now on, we therefore assume that $\max_{i \in N} \theta_i v > c$.

This setup encompasses (i) differentiated products by letting the supplier-specific taste parameters differ; (ii) a one-buyer version of the Shapley and Shubik (1972) model by setting $D = 1$; and (iii) a version of the Shapley-Shubik model in which the buyer has demand for multiple products of the suppliers by setting $D > 1$. For an extension of the one-to-many setup that encompasses additional models, see Appendix C.

In order to define the price-formation mechanism for the generalized setup, we define the virtual surplus $\Lambda_{i, \beta}$ associated with trade between the buyer and supplier $i$, accounting for the agents’ bargaining weights and $\beta$:

$$\Lambda_{i, \beta}(v, c; w) \equiv \theta_i \Phi_{wB}^{\beta}(v) - \Gamma_{i, w, \beta}(c).$$

Let $\Lambda_{\beta}(v, c; w) \equiv (\Lambda_{1, \beta}(v, c_1; w_B, w_1), \ldots, \Lambda_{n, \beta}(v, c_n; w_B, w_n))$, and let $\Lambda_{\beta}(v, c; w)_{(D)}$ denote the $D$-th highest element of $\Lambda_{\beta}(v, c; w)$. As before, in order to save notation, we ignore ties. Then, defining $\beta^*(w)$ analogously to before, we have:

**Lemma 2.** In the generalized setup, the price-formation mechanism has the allocation rule

$$Q_{i, \beta^*(w)}(v, c; w) \equiv \begin{cases} 1 & \text{if } \Lambda_{i, \beta^*(w)}(v, c; w_B, w_i) \geq \max\{0, \Lambda_{\beta^*(w)}(v, c; w)_{(D)}\}, \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

**Proof.** See Appendix B.

We can now use this generalized setup to consider bargaining externalities between suppliers.

In Table 1 we consider the case of one buyer and two suppliers with symmetric bargaining weights. Assuming that $F$, $G_1$, and $G_2$ are the uniform distribution on $[0, 1]$, and assuming that $\theta_2 = 1$, we allow the buyer’s preference for supplier 1, $\theta_1$, and the buyer’s total demand,
Table 1: Outcomes for one-to-many price formation for the case of one buyer and two suppliers with symmetric \( w \), types that are uniformly distributed on \([0, 1]\), and \( \theta_2 = 1 \). The values of \( D \) and \( \theta_1 \) vary as indicated in the table.

<table>
<thead>
<tr>
<th></th>
<th>( D = 1 )</th>
<th>( D = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>( \beta^* )</td>
<td>( u_B )</td>
</tr>
<tr>
<td>1</td>
<td>0.73 0.76</td>
<td>0.67 0.72</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As shown in Table 1 focusing on the case with \( D = 1 \), an increase in the buyer’s preference for supplier 1 from \( \theta_1 = 1 \) to \( \theta_1 = 2 \) benefits supplier 1 (\( u_{1,S} \) increases) but harms supplier 2 (\( u_{2,S} \) decreases). The increase in the buyer’s preference for supplier 1 means that supplier 2 is less likely to trade. As a result, supplier 2 is harmed by the increase in the buyer’s preference for supplier 1.

When \( D = 2 \), the results differ. Supplier 1 again benefits from being preferred by the buyer, but in this case supplier 2 also benefits, albeit less than supplier 1. The increase in the buyer’s value from trade with supplier 1 means that the value of \( \beta^* \) is increased, so supplier 2 trades more often. As a result of the change from \( \theta_1 = 1 \) to \( \theta_1 = 2 \), both \( u_{1,S} \) and \( u_{2,S} \) increase.

The effect that we observe in this example when \( D = 2 \) is general in the sense that it holds whenever \( n = D \). In this case, the probability that supplier \( i \) trades, \( \Pr(\theta_i \Phi_{wB\beta^*}(v) \geq \Gamma_{i,w,\beta^*}(c_i)) \), does not depend on the preference parameters of the other suppliers except through their effect on \( \beta^* \). Because an increase in a rival supplier’s preference parameter causes an increase in \( \beta^* \), it increases the probability of trade and so benefits the supplier. Thus, we have the following result:

**Proposition 9.** If \( n = D \), then an increase in the preference parameter for one supplier increases the payoffs for all suppliers.

The result of Proposition 9 no longer holds when \( n > D \), as shown in the example of Table 1 with \( D = 1 \). In that case, even though the increase in \( \theta_1 \) from 1 to 2 increases \( \beta^* \), which benefits rivals, it also reduces the probability that rivals are among the set of at-most \( D \) suppliers that trade.
6 Related literature

George Stigler provided early and forceful arguments that private information held by economic agents is a major obstacle to achieving efficient outcomes, noting that “important aspects of economic organization take on a new meaning when they are considered from the viewpoint of the search for information” (Stigler 1961, p. 213). While Stigler emphasized price dispersion and the problem of uncertainty about price cuts faced by cartels (Stigler 1964), the relevance of private information in connection to prices applies, of course, generally.

Viewed from this angle, we use the Myersonian mechanism design machinery (Myerson 1981) to elicit—search for, as it were—agents’ private information and determine prices. Indeed, our framework builds on the bilateral trade model of Myerson and Satterthwaite (1983), augmented by bargaining weights and possibly multiple suppliers, and thereby combines elements of Myerson and Satterthwaite (1983), Williams (1987), and Gresik and Satterthwaite (1989). Specifically, our procurement model allows for multiple suppliers without imposing restrictions on the supports of the buyer’s value and the suppliers’ costs other than assuming that all cost distributions have the same support.

We generalize Williams’ approach of maximizing an objective that assigns differential weights in a bilateral trade problem by allowing for multiple agents. In light of the quote from New Palgrave Dictionary in the introductory paragraph, our paper reinterprets Myerson and Satterthwaite (1983) as a bilateral monopoly problem and shows that it is tractable and has all the required features.

In particular, the independent private values setting with continuous distributions has the virtue that, for a given objective, the mechanism that maximizes this objective, subject to incentive-compatibility, individual-rationality, and no-deficit constraints, is well defined and pinned down (up to a constant in the payments) by the allocation rule, which is unique. Of particular interest to industrial organization and antitrust economics, it also has the feature that, quite generally, there is a tradeoff between allocating efficiently and extracting rents. This tradeoff is at the heart of both industrial organization and Myerson’s optimal auction. This tradeoff is the reason why the Williams frontier is typically not identical to the 45-degree line and, therefore, the basis from which the possibility of social surplus increasing countervailing power emerges.

Privacy of information endows economic agents with information rents and thereby protects them from hold up, as discussed in our analysis of investments. Even without investment, this protection implies, for example, that first-degree price discrimination is not possible. Rather than being an assumption, the impossibility of first-degree price discrim-

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41 While Gresik and Satterthwaite (1989) also allow for multiple buyers, they restrict attention to identical cost distributions. In that regard, our setup thus shares similarities with the optimal auction setting of Myerson (1981), with the important difference that our setup has two-sided private information.

42 For experimental results consistent with our price-formation mechanism, see Valley et al. (2002, Fig. 3.A). See Larsen (2020) on the first-best and second-best frontiers for wholesale used cars.
ination is an implication in this setup. Moreover, the aforementioned assumptions are essentially the only assumptions that permit a tractable approach that maintain the basic tradeoff between profit and social surplus.

Snyder (2008) notes that the concept of countervailing power has been the subject of controversies ever since it was introduced by Galbraith (1952). As we have shown, the possibility of countervailing power arises naturally in an incomplete information setup exactly because of its inherent tradeoff between efficiency and rent extraction. In this regard, one important contribution of the present paper is the observation that buyer power and countervailing power are distinct things: making a buyer powerful by increasing its bargaining power has socially desirable countervailing effects if the increase in buyer power levels the playing field by making all bargaining weights more equal; if the change moves bargaining weights in the opposite directions, matters worsen.

With regard to vertical integration, our model has the feature that, because the efficiency of the price-formation process is endogenous, vertical integration can decrease social surplus. That is, in this setting there is no basis for the presumption that vertical integration increases social surplus. Related to Judge Leon’s recent ruling in the AT&T-Time-Warner merger, our model has thus the property that vertical integration can be detrimental to social surplus without relying on complete information bargaining.

The tight connection between incentives for efficient investment and incentives for efficiency in incomplete information models has its roots in the seminal works of Vickrey (1961), Clarke (1971), and Groves (1973) and the subsequent uniqueness results of Green and Laffont (1977) and Holmström (1979). Essentially, dominant strategy incentive compatibility under incomplete information requires each agent to be a price taker, and efficiency then

43Likewise, setting a uniform market clearing price is the optimal mechanism for a monopoly with constant marginal costs facing a continuum of buyers, so under these conditions uniform pricing is a conclusion rather than an assumption; if the monopoly has increasing marginal costs and the revenue function that it faces is not concave, then setting non-market-clearing prices may be optimal (see Loertscher and Muir, 2019).

44Dropping the assumption of risk neutrality, Maskin and Riley (1984) and Matthews (1984) show that optimal mechanisms depend on the nature of risk aversion, are not easily characterized, and, among other things, may require payments to and/or from losers. Without independence, as foreshadowed by Myerson (1981), Crémer and McLean (1985, 1988) show that there is no tradeoff between profit and social surplus. Without private values, additional and, therefore, in some sense arbitrary, restrictions may be required to maintain tractability and/or the tradeoff between profit and social surplus (Mezzetti, 2004, 2007). Notwithstanding recent progress, with multi-dimensional private information and multiple agents, the optimal mechanism is not known (see, e.g., Daskalakis et al., 2017). With discrete types, there is no payoff equivalence theorem. In other words, the mechanism is not pinned down by the allocation rule.

45Loertscher and Marx (2019) consider mergers in a procurement setup with one-sided private information, assuming that either bargaining weights are equal or that the buyer has all the bargaining power, and that bargaining weights do not change with a merger (with the exception of noting that a merger increases a buyer’s incentive to acquire bargaining power). Because bargaining weights are fixed, countervailing power does not arise.

46For an overview of the literature on the competitive effects of vertical integration, see Riordan and Salop (1995). As described there, the literature has taken the view that most vertical mergers lead to some efficiencies.
further requires this price to be equal to the agent’s social marginal product (or cost). But, as demonstrated by [Milgrom (1987), Rogerson (1992), Hatfield et al. (2018), and Loertscher and Riordan (2019)], this is precisely the set of conditions that have to be satisfied for incentives for investment to be aligned with efficiency. In contrast, in complete information models in the tradition of Grossman and Hart (1986) and Hart and Moore (1990), efficient bargaining typically creates hold up, which is an impediment to efficient investment. The intuition behind these differences is that, as mentioned, privacy of information provides protection against hold up. Following the Dow-DuPont merger decision, there has been a recent upsurge of interest in industrial organization relating to market structure and the incentives to invest (see, e.g., Federico et al., 2017, 2018; Jullien and Lefouili, 2018; Loertscher and Marx, 2019). Thus, our results pertaining to mergers and vertical integration and investment relate to this strand of literature as well.

Partly based on the availability of large scale microlevel datasets, there has been an even larger upsurge of interest in bargaining (see, for example, Larsen, 2020; Backus et al., 2020, 2019; Zhang et al., 2019). Bargaining has also come to the forefront of the empirical IO literature, in particular in analyses of bundling and vertical integration such as Crawford and Yurukoglu (2012) and Crawford et al. (2018). Collard-Wexler et al. (2019) provide a recent theoretical foundation for the widely used Nash-in-Nash bargaining model. Ho and Lee (2017) apply this framework to the question of countervailing power by insurers when negotiating with hospitals and find evidence that consolidation among insurers improves their bargaining position vis-à-vis hospitals. Our paper contributes to this literature by showing, among other things, that in incomplete information models, bargaining breakdown occurs on the equilibrium path and that the probability of breakdown can, under suitable assumptions, be used to estimate distributions.

7 Conclusions

We analyze a procurement setup with incomplete information that pertains to both sides of the market in which price formation and its efficiency properties are endogenous and depend, among other things, on the bargaining power of the buyer and the suppliers. Social surplus increasing countervailing power and socially harmful vertical integration arise naturally in this setting. We also examine the relation between the efficiency of the price-formation process and the incentives to invest, which differs fundamentally from what obtains in complete information models that are based on the assumption that efficient trade is always possible, and we illustrate the effects of bargaining externalities.

Our paper shows that an economic agent’s strength or weakness has two dimensions that are, conceptually, independent. The first one, which may be thought of as the agent’s productive strength or power, refers to the agent’s productivity. Is the agent likely to have
a high value if it is a buyer or a low cost if it is a supplier? The second dimension captures the agent’s bargaining power, that is, its ability (or inability) to affect the price-formation process in its favour. For example, a buyer who has all the bargaining power facing a single supplier makes higher or lower take-it-or-leave-it offers depending on the realization of its value, and on average these offers will be higher if the buyer’s distribution is stronger, say, in the sense of stochastic dominance. What is indicative of the relative bargaining powers is then not so much the level of prices but rather the price-formation process itself. For example, in a bilateral trade setting, if the buyer (supplier) always makes the price offer, one would conclude that the buyer (supplier) has all the bargaining power, indicating that there is scope for countervailing power. In contrast, if the buyer and supplier participate in a $k$-double-auction with $k = 1/2$, this may be indicative of equal bargaining powers, suggesting that there is no scope for welfare increasing countervailing power.

Avenues for future research are many. For example, one could augment the setup to have multiple buyers and multiple suppliers, which may give rise to a raising rivals’ costs effect of vertical integration. More fundamentally, developing a better understanding of what determines bargaining power would add considerable value. Hopefully, the distinction between productive strength and bargaining power brought to light in the present paper will prove useful in that regard.
A Appendix: Mechanism design foundations

In this appendix, we first define and develop the mechanism design concepts relevant for our analysis (Appendix A.1) and then apply these concepts to derive the Myerson-Satterthwaite impossibility result (Appendix A.2).

A.1 Concepts and derivations

For ease of exposition, in this appendix we assume that $n = 1$. The extension to $n > 1$ is straightforward.

Take as given a direct mechanism $\langle Q, M_B, M_S \rangle$ mapping buyer and supplier reports to a probability of trade and payments, $Q : [v, \overline{v}] \times [c, \overline{c}] \rightarrow [0, 1]$ and $M_B, M_S : [v, \overline{v}] \times [c, \overline{c}] \rightarrow \mathbb{R}$, where, given reports $v$ and $c$, $Q(v, c) \in [0, 1]$ is the probability with which the supplier trades with the buyer, $M_B(v, c)$ is the payment from the buyer to the mechanism, and $M_S(v, c)$ is the payment from the mechanism to the supplier. By the Revelation Principle, the focus on direct mechanisms is without loss of generality.

Let $\hat{q}_B(z)$ be the buyer’s expected quantity if it reports $z$ and the supplier reports truthfully, and let $\hat{m}_B(z)$ be the buyer’s expected payment if it reports $z$ and the supplier reports truthfully:

$$\hat{q}_B(z) = \mathbb{E}_c [Q(z, c)] \quad \text{and} \quad \hat{m}_B(z) = \mathbb{E}_c [M_B(z, c)].$$

Define $\hat{q}_S$ and $\hat{m}_S$ analogously, where $\hat{m}_S$ is the expected payment to the supplier. Because we assume independent draws, for $i \in \{B, S\}$, $\hat{q}_i(z) \text{ and } \hat{m}_i(z)$ depend only on the report $z$ and not on the reporting agent’s true type. The expected payoff of a buyer with type $v$ that reports $z$ is then $\hat{q}_B(z)v - \hat{m}_B(z)$, and the expected payoff of a supplier with type $c$ that reports $z$ is $\hat{m}_S(z) - \hat{q}_S(z)c$.

Key constraints

The mechanism is incentive compatible for the buyer if for all $v, z \in [v, \overline{v}]$,

$$\hat{u}_B(v) \equiv \hat{q}_B(v) - \hat{m}_B(v) \geq \hat{q}_B(z)v - \hat{m}_B(z),$$

and is incentive compatible for the supplier if for all $c, z \in [c, \overline{c}]$,

$$\hat{u}_S(c) \equiv \hat{m}_S(c) - \hat{q}_S(c)c \geq \hat{m}_S(z) - \hat{q}_S(z)c.$$  \hspace{1cm} (15)

Individual rationality is satisfied for the buyer if for all $v \in [v, \overline{v}]$, $\hat{u}_B(v) \geq 0$, and for the supplier if for all $c \in [c, \overline{c}]$, $\hat{u}_S(c) \geq 0$. The mechanism satisfies the no-deficit condition if

$$\mathbb{E}_{v, c} [M_B(v, c) - M_S(v, c)] \geq 0.$$
Interim expected payoffs

Standard arguments (see, e.g., Krishna, 2002, Chapter 5.1) proceed as follows:

Focusing on the buyers, incentive compatibility implies that

\[ \hat{u}_B(v) = \max_{z \in [v, \bar{v}]} \{ \hat{q}_B(z) v - \hat{m}_B(z) \}, \]

i.e., \( \hat{u}_B \) is a maximum of a family of affine functions, which implies that \( \hat{u}_B \) is convex and so absolutely continuous and differentiable almost everywhere in the interior of its domain. In addition, incentive compatibility implies that \( \hat{u}_B(z) \geq \hat{q}_B(v) z - \hat{m}_B(v) = \hat{u}_B(v) + \hat{q}_B(v)(z-v) \), which for \( \varepsilon > 0 \) implies

\[ \frac{\hat{u}_B(v+\varepsilon) - \hat{u}_B(v)}{\varepsilon} \geq \hat{q}_B(v) \]

and for \( \varepsilon < 0 \) implies

\[ \frac{\hat{u}_B(v+\varepsilon) - \hat{u}_B(v)}{\varepsilon} \leq \hat{q}_B(v), \]

so taking the limit as \( \varepsilon \) goes to zero, at every point \( v \) where \( \hat{u}_B \) is differentiable, \( \hat{u}_B'(v) = \hat{q}_B(v) \).

Because \( \hat{u}_B \) is convex, this implies that \( \hat{q}_B(v) \) is nondecreasing. Because every absolutely continuous function is the definite integral of its derivative,

\[ \hat{u}_B(v) = \hat{u}_B(v) + \int_v^\bar{v} \hat{q}_B(t) dt, \]

which implies that, up to an additive constant, a buyer’s expected payoff in an incentive-compatible direct mechanism depends only on the allocation rule. By an analogous argument, \( \hat{u}_S'(c) = -\hat{q}_S(c) \), \( \hat{q}_S(c) \) is nonincreasing, and

\[ \hat{u}_S(c) = \hat{u}_S(\bar{v}) + \int_c^{\bar{v}} \hat{q}_S(t) dt. \]

Mechanism budget surplus

Using the definitions of \( \hat{u}_B \) and \( \hat{u}_S \) in (14) and (15), we can rewrite these as

\[ \hat{m}_B(v) = \hat{q}_B(v) v - \int_v^\bar{v} \hat{q}_B(t) dt - \hat{u}_B(v) \quad (16) \]

\[ ^{47} \text{A function } h : [\underline{v}, \bar{v}] \rightarrow \mathbb{R} \text{ is absolutely continuous if for all } \varepsilon > 0 \text{ there exists } \delta > 0 \text{ such that whenever a finite sequence of pairwise disjoint sub-intervals } (v_k', v_k) \text{ of } [\underline{v}, \bar{v}] \text{ satisfies } \sum_k (v_k' - v_k) < \delta, \text{ then } \sum_k |h(v_k') - h(v_k)| < \varepsilon. \text{ One can show that absolute continuity on compact interval } [a, b] \text{ implies that } h \text{ has a derivative } h' \text{ almost everywhere, the derivative is Lebesgue integrable, and that } h(x) = h(a) + \int_a^x h'(t) dt \text{ for all } x \in [a, b]. \]
and

\[ \hat{m}_S(c) = \hat{q}_S(c)c + \int_c^\infty \hat{q}_S(t)dt + \hat{u}_S(\bar{c}). \]  

(17)

The expected payment by the buyer is then

\[ \mathbb{E}_v [\hat{m}_B(v)] = \int_v^\infty \hat{m}_B(v)f(v)dv \]

\[ = \int_v^\infty (\hat{q}_B(v)v - \int_v^t \hat{q}_B(t)dt) f(v)dv - \hat{u}_B(v) \]

\[ = \left( \int_v^\infty \hat{q}_B(v)v f(v)dv - \int_v^\infty \int_t^\infty \hat{q}_B(t) f(v)dvdt \right) - \hat{u}_B(v) \]

\[ = \left( \int_v^\infty \hat{q}_B(v)v f(v)dv - \int_v^\infty \hat{q}_B(t) (1 - F(t)) dt \right) - \hat{u}_B(v) \]

\[ = \int_v^\infty \hat{q}_B(v) \left( v - \frac{1 - F(v)}{f(v)} \right) f(v)dv - \hat{u}_B(v) \]

\[ = \int_v^\infty \hat{q}_B(v) \Phi(v) f(v)dv - \hat{u}_B(v) \]

\[ = \mathbb{E}_v [\hat{q}_B(v) \Phi(v)] - \hat{u}_B(v), \]

where the first equality uses the definition of the expectation, the second uses (16), the third switches the order of integration, the fourth integrates, the fifth collects terms, the sixth uses the definition of the virtual value \( \Phi \), and the last equality uses the definition of the expectation. Similarly, using (17), the expected payment to the supplier is

\[ \mathbb{E}_c [\hat{m}_S(c)] = \int_c^\infty \hat{m}_S(c)g(c)dc = \mathbb{E}_c [\hat{q}_S(c) \Gamma(c)] + \hat{u}_S(\bar{c}). \]

Thus, imposing \( Q_B(v, c) = Q_S(v, c) \), we have the result that in any incentive-compatible, interim individually-rational direct mechanism \( \langle Q, M_B, M_S \rangle \), the mechanism’s expected budget surplus is

\[ \mathbb{E}_{v,c} [(\Phi(v) - \Gamma(c)) Q(v, c)] - \hat{u}_B(v) - \hat{u}_S(\bar{c}). \]

As mentioned, it is straightforward to extend these results to the case of \( n > 1 \).

A.2 Myerson-Satterthwaite impossibility result

For the purpose of making the paper self-contained, we provide a statement and proof of the impossibility theorem of Myerson and Satterthwaite (1983). Under the IPV assumptions and the assumption that \( v < \bar{c} \), Myerson and Satterthwaite (1983) show that there is no mechanism satisfying incentive compatibility and individual rationality that allocates ex
post efficiently and that does not run a deficit. Their result depends on $v < \bar{v}$ because, without this assumption, ex post efficiency subject to incentive compatibility and individual rationality can easily be achieved without running a deficit. For example, the posted price mechanism that has the buyer pay $p = (v + \bar{v})/2$ to the supplier achieves this.

By now, the proof of this result can be provided in a couple of lines (see, e.g., Krishna, 2002). Consider the dominant strategy implementation in which the buyer pays $p_B = \max\{c, v\}$ and the supplier receives $p_S = \min\{v, \bar{v}\}$ whenever there is trade, and no payments are made otherwise. Notice that $\hat{u}_B(v) = 0 = \hat{u}_S(\bar{v})$. Thus, the individual rationality constraints are satisfied. Further, notice that $p_B - p_S \leq 0$, with a strict inequality for almost all type realizations. This implies that the mechanism runs a deficit in expectation. By the payoff equivalence theorem, any other ex post efficient mechanism satisfying incentive compatibility and individual rationality will run a deficit of at least that size (and a larger one if one or both of the individual rationality constraints are slack).

To see how this impossibility result rests on the assumption $v < \bar{v}$, assume to the contrary that $v \geq \bar{v}$. Then the mechanism described above continues to satisfy incentive compatibility and individual rationality, but for all type realizations $p_B = v \geq \bar{v} = p_S$, which implies that the mechanism does not run a deficit.

B Appendix: Proofs

Proof of Lemma 1 The price-formation mechanism with bargaining weights $w$ maximizes

$$\mathbb{E}_{v,c} \left[ \frac{\beta}{1 + \beta} W(v, c; w) + \frac{1}{1 + \beta} R(v, c) \right],$$

subject to incentive compatibility and individual rationality, and with the largest $\beta \in [0, 1]$ such that the no-deficit condition holds. Using the mechanism design concepts from Ap-
Appendix A for a given $\beta$, the allocation rule maximizes

$$
\mathbb{E}_{v,c} \left[ \frac{\beta}{1 + \beta} \sum_{i \in N} (w_B(v - \Phi(v)) + w_i(\Gamma_i(c_i) - c_i))Q_i(v, c) + \frac{\beta}{1 + \beta} w_B\hat{u}_B(v) + \frac{\beta}{1 + \beta} \sum_{i \in N} w_i\hat{u}_i(\bar{c}) \right] \\
= \frac{1}{1 + \beta} \mathbb{E}_{v,c} \left[ \sum_{i \in N} \left( \frac{\beta}{1 + \beta} w_Bv + \frac{1 - w_B}{1 + \beta} \Phi(v) - \frac{\beta}{1 + \beta} w_i\hat{c}_i - \frac{1 - w_i}{1 + \beta} \Gamma_i(c_i) \right) Q_i(v, c) \right] \\
- \frac{1 - \beta w_B}{1 + \beta} \hat{u}_B(v) - \sum_{i \in N} \frac{1 - \beta w_i}{1 + \beta} \hat{u}_i(\bar{c}) \\
= \frac{1}{1 + \beta} \mathbb{E}_{v,c} \left[ \sum_{i \in N} \left( \frac{\beta}{1 + \beta} w_B(v) - \Gamma_i(c_i) \right) Q_i(v, c) \right] \\
- \frac{1 - \beta w_B}{1 + \beta} \hat{u}_B(v) - \sum_{i \in N} \frac{1 - \beta w_i}{1 + \beta} \hat{u}_i(\bar{c}),
$$

which is maximized with $Q_i(v, c) = 1$ when

$$
\Phi_{w_B}(v) > \Gamma_{i,w_i}(c_i) = \min_{j \in N} \Gamma_{j,w_j}(c_j),
$$

and zero otherwise. And $\beta$ is the largest value in $[0, 1]$ such that there exist payments to the worst-off types such that the no-deficit condition holds, i.e., such that

$$
\mathbb{E}_{v,c} \left[ \sum_{i \in N} \left( \Phi(v) - \Gamma_i(c_i) \right) \cdot 1_{\Phi(v) > \Gamma_{i,w_i}(c_i) = \min_{j \in N} \Gamma_{j,w_j}(c_j)} \right] \geq 0.
$$

Because this expression is satisfied strictly for $\beta = 0$, it follows that the largest $\beta$ such that the inequality is satisfied is strictly positive. $\blacksquare$

Proof of Proposition 2 Note that $\beta^*(w)$ is such that $\pi_{\beta^*(w)} = 0$, where

$$
\pi_{\beta} = \mathbb{E}_{v,c} \left[ (\Phi(v) - \Gamma(c)) \cdot Q_{1,\beta}(v, c; w) \right] \\
= \int_{\frac{1 - w_B}{2 - w_B}}^{w_B} \int_0^{\frac{v - (1 - w_B)\beta(1-v)}{2 - w_B \beta}} (2v - 1 - 2c) dcdv,
$$

41
where the second equality uses the expression for $Q_{1,\beta}(v, c; w)$ from Lemma 1. Solving this and assuming that $\max\{w_B, w_S\} = 1$, we get

$$\beta^*(w) = \begin{cases} 
\frac{2w_B + 2w_S - 2\sqrt{w_B^2 - w_B w_S + w_S^2}}{3w_B w_S} & \text{if } 0 < \min\{w_B, w_S\}, \\
1 & \text{if } 0 = \min\{w_B, w_S\}.
\end{cases}$$

Making the substitution $\Delta = w_B - w_S$ and writing $\beta^*$ as a function of $\Delta$, we have $\beta^*(\Delta) = 1$ for $\Delta \in \{-1, 1\}$ and otherwise $\beta^*(\Delta)$ is given by (8). Substituting the expression for $\beta^*(\Delta)$ into the expression derived from Lemma 1 for $Q_{1,\beta^*(\Delta)}(v, c; 1 + \Delta, 1)$ if $\Delta \in [-1, 0]$ and for $Q_{1,\beta^*(\Delta)}(v, c; 1, 1 - \Delta)$ if $\Delta \in (0, 1)$, we have

$$Q^*_1(v, c; \Delta) = \begin{cases} 
1 & \text{if } \Delta \in (-1, 0] \text{ and } v \geq \frac{2c(\sqrt{\Delta^2 + \Delta + 1} + 2\Delta + 1) + (2\sqrt{\Delta^2 + \Delta + 1} - 2\Delta - 1)(\Delta + 1)}{2(\Delta + 1)(\sqrt{\Delta^2 + \Delta + 1} - \Delta + 1)}, \\
\frac{2c(\sqrt{\Delta^2 - \Delta + 1} + 1 - \Delta) + 2\sqrt{\Delta^2 - \Delta + 1} - 2\Delta - 1}{2(\sqrt{\Delta^2 - \Delta + 1} - 2\Delta + 1)}, & \text{if } \Delta \in [0, 1) \text{ and } v \geq 2c, \\
\frac{\sqrt{\Delta^2 + \Delta + 1} + 2\Delta + 1}{2c} & \text{if } \Delta = 1 \text{ and } v \geq 2c, \\
0 & \text{if } \Delta = -1 \text{ and } v \geq \frac{c + 1}{2}, \\
0 & \text{otherwise}.
\end{cases}$$

It then follows that $Q^*_1(v, c; \Delta_k) = \tilde{Q}_k(v, c)$ if $\Delta_k$ is as given in (9), which completes the proof.

**Proof of Proposition 3.** Consider the Bayes Nash equilibrium of the fee-setting game with intermediary preference weights $w$. Suppose that the intermediary sets auction discounts relative to the clock price $\hat{p}$ of $\delta_i(\hat{p}) = \hat{p} - \Gamma^{-1}_{i, w_i \beta^*(w)}(\hat{p})$ and a fee schedule given by, for all $i \in \mathcal{N}$,

$$\sigma_i(p) = \Phi^{-1}_{w_B \beta^*(w)}(\Gamma_{i, w_i \beta^*(w)}(\Gamma^{-1}_i(p))) - p,$$

and suppose that the buyer sets a reserve of $\Phi_{w_B \beta^*(w)}(v)$. Then, given our assumption that each supplier $i$ follows its weakly dominant strategy of remaining active until a clock price $\hat{p}$ such that $\hat{p} - \delta_i(\hat{p}) = c_i$, supplier $i$ remains active until a price of $\Gamma_{i, w_i \beta^*(w)}(c_i)$, and so supplier $i$ wins if and only if

$$\Gamma_{i, w_i \beta^*(w)}(c_i) = \min_{j \in \mathcal{N}} \Gamma_{j, w_j \beta^*(w)}(c_j) \leq \Phi_{w_B \beta^*(w)}(v),$$

which is the optimal outcome for the intermediary by Lemma 1. In equilibrium, if supplier $i$ wins the auction, then the auction ends with a clock price of

$$\hat{p} \equiv \min_{j \in \mathcal{N} \setminus \{i\}} \{\Phi_{x_B}(v), \Gamma_{j, w_j \beta^*(w)}(c_j)\}.$$
and the buyer makes a payment \( p = \hat{p} - \delta_i(\hat{p}) \) to supplier \( i \) and \( \sigma_i(p) \) to the intermediary.

To summarize, given the suppliers’ optimal bidding strategies and a reserve set by the buyer of \( \Phi_{w_B\beta^*(w)}(v) \), the intermediary’s choice of auction format and fee schedule are optimal because they result in the outcome that maximizes the weighted objective subject to no deficit. It remains to show that the buyer’s best response to the intermediary’s auction format and fee schedule is to choose a reserve of \( \Phi_{w_B\beta^*(w)}(v) \).

Let \( x_B \equiv w_B\beta^*(w) \) and \( x_i \equiv w_i\beta^*(w) \). Define the distribution of supplier \( i \)'s weighted virtual type \( \Gamma_{i,x_i}(c_i) \) by

\[
\tilde{G}_{i,x_i}(z) = G_i(\Gamma_{i,x_i}^{-1}(z)),
\]

and, letting \( x \equiv (x_1, \ldots, x_n) \), define the distribution of the minimum of the weighted virtual types of suppliers other than \( i \) by

\[
\tilde{G}_{-i,x}(z) = 1 - \prod_{j \in N \setminus \{i\}} (1 - \tilde{G}_{j,x_j}(z)).
\]

The expected payment by the buyer to the suppliers given the reserve \( r \) can be written as

\[
\sum_{i \in N} \mathbb{E} \left[ \Gamma_i(c_i) \cdot 1_{\Gamma_i,c_i} \leq \max_{j \neq i} \{r, \Gamma_j,c_j \} \right]
= \sum_{i \in N} \int_{\Gamma_{i,x_i}^{-1}(r)} \int_{\Gamma_i,c_i} \Gamma_i(c_i) d\tilde{G}_{-i,x}(z) dG_i(c_i)
= \sum_{i \in N} \int_{\Gamma_{i,x_i}^{-1}(r)} \Gamma_i(c_i)(1 - \tilde{G}_{-i,x}(\Gamma_{i,x_i}(c_i))) dG_i(c_i)
= \sum_{i \in N} \int_{\Gamma_{i,x_i}^{-1}(r)} y \left[ 1 - \tilde{G}_{-i,x}(\Gamma_{i,x_i}(\Gamma_i^{-1}(y))) \right] \frac{g_i(\Gamma_i^{-1}(y))}{\Gamma_i'(\Gamma_i^{-1}(y))} dy,
\]

where the final equality uses the change of variables \( y = \Gamma_i(c_i) \). Thus, the buyer’s problem is

\[
\max_r \sum_{i \in N} \left( \int_{\Gamma_{i,x_i}^{-1}(r)} (v - y - \sigma_i(y)) \frac{1 - \tilde{G}_{-i,x}(\Gamma_{i,x_i}(\Gamma_i^{-1}(y)))}{\Gamma_i'(\Gamma_i^{-1}(y))} g_i(\Gamma_i^{-1}(y)) dy \right),
\]

which has first-order condition

\[
0 = \sum_{i \in N} \Gamma_i'(\Gamma_{i,x_i}^{-1}(r)) \Gamma_{i,x_i}^{-1}(r) \left( (v - \Gamma_i(\Gamma_{i,x_i}^{-1}(r)) - \sigma_i(\Gamma_i(\Gamma_{i,x_i}^{-1}(r)))) \right) \frac{(1 - \tilde{G}_{-i,x}(r)) g_i(\Gamma_{i,x_i}^{-1}(r))}{\Gamma_i'(\Gamma_{i,x_i}^{-1}(r))}
= \sum_{i \in N} \Gamma_i'(\Gamma_{i,x_i}^{-1}(r)) \Gamma_{i,x_i}^{-1}(r) \left( v - \Phi_{x_B}^{-1}(r) \right) \frac{(1 - \tilde{G}_{-i,x}(r)) g_i(\Gamma_{i,x_i}^{-1}(r))}{\Gamma_i'(\Gamma_{i,x_i}^{-1}(r))},
\]

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where the second equality uses the definition of the fee schedule $\sigma$. Given our assumptions, the second-order condition is satisfied when the first-order condition is, and so the buyer’s problem is solved by $r = \Phi_{x_B}(v) = \Phi_{w_B\beta^*(v)}$, which completes the proof. ■

Proof of Proposition 5 To begin, note that each point on the frontier is associated with a unique bargaining parameter $\Delta = w_B - w_1 = ... = w_B - w_n$. Given bargaining parameter $\Delta$, we denote the associated point on the frontier as $(u_S(\Delta), u_B(\Delta))$, where $u_S$ is the sum of all supplier payoffs.

We first show that $\omega$ is strictly decreasing. If it is not strictly decreasing, then there are two points on the frontier, indexed by $\Delta$ and $\Delta'$, where expected social surplus is strictly greater at the point indexed by $\Delta'$ and expected surplus for both the buyer and the suppliers is weakly greater at the point indexed by $\Delta'$. But then weighted welfare must not have been maximized at the point indexed by $\Delta$ because total surplus could be increased while still satisfying all the constraints and some of that additional surplus could be allocated to one or more of the agents with positive bargaining weight when $w_B - w_S = \Delta$. This completes the proof that $\omega$ is strictly decreasing.

Now turn to the question of concavity. As illustrated in the figure below, suppose that the Williams frontier is not concave.

Then there exist points on the frontier, which we denote by their associated buyer bargaining parameters $\Delta$, $\Delta'$, and $\Delta''$, with $\Delta > \Delta' > \Delta''$, such that $(u_S(\Delta) + u_S(\Delta''))/2 > u_S(\Delta')$ and $(u_B(\Delta) + u_B(\Delta''))/2 > u_B(\Delta')$. Let $\eta(\Delta)$ be the price-formation mechanism associated with bargaining parameter $\Delta$. Then consider the mechanism that is a 50-50 mixture of $\eta(\Delta)$ and $\eta(\Delta'')$. By the construction of the price-formation process, the expected budget surplus to the mechanism in $\eta(\Delta)$, $\eta(\Delta')$, $\eta(\Delta'')$ is zero. The expected weighted welfare under $\eta(\Delta')$
satisfies
\[
\Delta' u_B(\Delta') + \sum_{i \in N} \frac{1 - \Delta'}{n} u_i(S_n(\Delta')) < \Delta' u_B(\Delta) + u_B(\Delta'') + \sum_{i \in N} \frac{1 - \Delta'}{n} u_i(\Delta') + u_i(\Delta''),
\]
where the right side is the expected weighted welfare given \( \Delta' \) under the mechanism that is a 50-50 mixture of \( \eta(\Delta) \) and \( \eta(\Delta'') \), which since the no-deficit condition is satisfied, contradicts the optimality of \( \eta(\Delta') \). This contradiction completes the proof. 

Proof of Proposition 8 Suppose that the price-formation process is the one associated with efficient investments \( \bar{e} \). Then the price-formation mechanism has trade if and only if
\[
\Phi_{\beta^*}(v, e_B) > \min_{i \in N} \Gamma_{i, \beta^*}(c_i, e_i),
\]
where \( \Phi_{\beta^*}(v, e_B) \) is the distribution of \( \Phi_{\beta^*}(\min_{i \in N} \Gamma_{i, \beta^*}(c_i, e_i), e_B) \). Thus, the buyer’s first-order condition is
\[
- \int_{v}^{\bar{v}} \frac{\partial F(x, e_B)}{\partial e_B} dx = \Psi'_B(e_B),
\]
which differs from (11) because \( \beta^* < 1/w \) implies that \( \Phi_{\beta^*}(v, e_B) \) differs from \( L(x, e_S) \) for all \( x \) in an open subset of \( (v, \bar{v}) \), which itself must be an open set because \( \beta^* < 1/w \) implies that \( \bar{v} < \bar{e} \). 

Proof of Lemma 2 The extension to allow supplier specific quality parameters follows by analogous arguments to Lemma 1 noting that the buyer’s value for supplier \( i \)’s good is \( \theta_i v \), which has distribution \( \hat{F}(x) \equiv F(x/\theta_i) \) on \( [\theta_i v, \theta_i \bar{v}] \) with density \( \hat{f}(x) = \frac{1}{\theta_i} f(v/\theta_i) \). Thus, the virtual type when the buyer’s value is \( v \) is
\[
\theta_i v - \frac{1 - \hat{F}(\theta_i v)}{\hat{f}(\theta_i v)} = \theta_i v - \theta_i \frac{1 - F(v)}{f(v)} = \theta_i \Phi(v).
\]
Thus, the parameter \( \theta_i \) “factors out” of the virtual type function.

The extension to multi-object demand follows by standard mechanism design arguments.
Appendix: Generalization

Let $\mathcal{P}$ be the set of subsets of $\mathcal{N}$ with no more than $D$ elements (including the empty set) and let $\theta = \{\theta_X\}_{X \in \mathcal{P}}$ be a commonly known vector of taste parameters of the buyer satisfying the “size-dependent discounts” condition of Delacrétaz et al. (2019). Specifically, let there be supplier-specific preferences $\{\hat{\theta}_i\}_{i \in \mathcal{N}}$ and size-dependent discounts $\{\delta_i\}_{i \in \mathcal{N}}$ with $0 = \delta_0 = \delta_1 \leq \delta_2 \leq \cdots \leq \delta_n$ such that for all $X \in \mathcal{P}$, $\theta_X = \sum_{i \in X} \hat{\theta}_i - \delta_{|X|}$. Thus, the buyer’s value for purchasing from suppliers in $X \in \mathcal{P}$ when its type if $v$ is $\theta_X v$, which depends on the buyer’s value, the buyer’s preferences for standalone purchases from the suppliers in $X$, and a discount that depends on the total number of units purchased. Note that $\theta_\emptyset = 0$, so that the value to the buyer of no trade is zero.

This setup encompasses (i) the homogeneous good model with constant marginal value or decreasing marginal value by setting $\hat{\theta}_i = \theta$ for some common $\theta$ and for $i \in \mathcal{N}$, $\delta_i$ either all zero for constant marginal value or increasing in $i$ for decreasing marginal value; (ii) differentiated products by letting $\hat{\theta}_i$ differ by $i$ and setting all $\delta_i$ to zero; (iii) a one-buyer version of the Shapley-Shubik model by setting $D = 1$; and (iv) a version of the Shapley-Shubik model in which the buyer has demand for multiple products of the suppliers by setting $D > 1$.

Define

$$X^*_\beta(v, c) \in \arg\max_{X \in \mathcal{P}} \theta_X \Phi_\beta(v) - \sum_{i \in X} \Gamma_i, \beta(c_i),$$

i.e., $X^*_\beta(v, c)$ is the set of trading partners for the buyer that maximizes the difference between the ironed $\beta$-weighted virtual value, scaled by $\theta_X \Phi_\beta(v)$, and the ironed $\beta$-weighted virtual costs of the trading partners. We then define $\beta^*$ to be the largest $\beta \in [0, 1]$ such that

$$\mathbb{E}_{v, c} \left[ \theta_X \Phi_\beta(v) - \sum_{i \in X^*_\beta(v, c)} \Gamma_i(c_i) \right] = 0.$$

Given the type realization $(v, c)$, the one-to-many $\beta^*$-mechanism induces trade between the buyer and suppliers in $X^*_\beta(v, c)$. The expected payoff of the buyer is

$$\mathbb{E}_v \left[ \hat{u}_B(v) + \int_v^\infty \sum_{X \in \mathcal{P}} \theta_X \Pr_c \left( X \in X^*_\beta(x, c) \right) \, dx \right],$$

and the expected payoff of supplier $i$ is

$$\mathbb{E}_{c_i} \left[ \hat{u}_i(c) + \int_{c_i}^\infty \Pr \left( i \in X^*_\beta(v, x, c_{-i}) \right) dx \right].$$
References


