Online Appendix to accompany “Countervailing Power”
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Online appendix: Axiomatic approach

In this appendix, we provide axiomatic foundations for our price-formation mechanism. Just as the Nash bargaining solution (and cooperative game theory more generally) abstracts away from specific bargaining protocols, our mechanism design based approach does the same. Nash bargaining maps primitives to a bargaining solution that specifies agents’ payoffs, and our approach maps primitives (type distributions of the agents) to agents’ expected payoffs via the unique mechanism that satisfies our axioms.

We take a setup with incomplete information involving independent private types as given and impose axioms on the mechanism that defines the price-formation process. This differs from the existing literature, which imposes axioms on outcomes. In light of the stringent discipline that the incomplete information paradigm imposes, this point of departure is necessary. For example, in light of the impossibility theorem of Myerson and Satterthwaite (1983), asking for efficient outcomes is “fruitless,” as noted by Ausubel et al. (2002).

As we now show, axioms of incentive compatibility, individual rationality, and no deficit, identify a set of feasible mechanisms. Additional axioms of constrained efficiency and symmetry pin down a unique mechanism. Generalizing the efficiency and symmetry axioms allows differential weights on agents’ welfare, analogous to generalized Nash bargaining, where each player’s welfare does not necessarily have the same weight.

Observe that properties like the Payoff Equivalence Theorem are distribution free (or detail free) insofar as they hold for any distributions $F$ and $G_1,...,G_n$ that have compact supports and positive densities on $[v, \bar{v})$ and $(c, \bar{c}]$, respectively. In formulating our axioms, we are therefore guided by the principle that the axioms should make no reference to distributional assumptions and should make no presumptions beyond these foundational assumptions on the setup. That said, in the body of the paper we assume regularity (i.e., that virtual value and cost functions are increasing) in order to avoid the technicalities of ironing. We do the same here, although all results continue to hold when the weighted and unweighted virtual value and cost functions are replaced by their ironed counterparts.

The first three axioms ensure that the price-formation mechanism is feasible in the incomplete information paradigm, which means that beyond satisfying resource constraints, the mechanism satisfies incentive compatibility, individual rationality, and does not run a deficit.\footnote{To simplify the exposition, we only require the mechanisms not to run a deficit in expectation, allowing for the possibility that ex post the mechanisms may run a budget deficit for some realizations. At some...}
Axiom 1: Incentive compatibility: The mechanism is incentive compatible.

Axiom 2: Individual rationality: The mechanism is individually rational.

Axiom 3: No deficit: The mechanism does not run a deficit.

Axioms 1–3 are, obviously, consistent with our price-formation mechanism with any weights \( w \). Axioms 1–3 constrain the incomplete information setup, but they also hold, in a sense, in the Nash bargaining framework (Nash, 1950). In that complete information setup, incentive compatibility is trivially satisfied because the “mechanism” already knows the agents’ types, and participation in Nash bargaining is individually rational because the bargaining outcome gives each agent a payoff of at least its disagreement payoff. In addition, there is no scope for running a deficit. Thus, there is a sense in which Axioms 1–3 are implied by the other aspects and axioms in the Nash bargaining setup.

Our fourth and fifth axioms ensure that social surplus is maximized, conditional on the constraints imposed by the other axioms, and that when that maximizer is not unique, the solution is one that treats the buyer and sellers symmetrically.

Axiom 4: Efficiency: The mechanism maximizes the objective of expected social surplus subject to the conditions of Axioms 1–3.

Axiom 5: Symmetry: Whenever surplus \( K > 0 \) is available to be distributed to agents while still respecting Axioms 1–4, it is distributed equally among the agents.

Axioms 4 and 5 identify a unique mechanism within the class of direct mechanisms that maximize expected social surplus subject to incentive compatibility, individually rationality, and no deficit, namely the one with \( \hat{u}_B(v) = \hat{u}_1(\bar{c}) = \ldots = \hat{u}_n(\bar{c}) = \frac{1}{n+1}{\pi\beta_s} \). This is the price-formation mechanism with weights \( w = 1 \).

Axioms 4 and 5 have clear counterparts in the “efficiency” and “symmetry” axioms that underlie the Nash bargaining solution. The efficiency axiom in Nash bargaining requires efficiency for any realization of types, whereas Axiom 4 requires efficiency subject to the feasibility constraints of the incomplete information paradigm. Axiom 5 requires that the outcome treat the buyer and sellers symmetrically whenever that can be done within the context of the other axioms, which is similar to Nash’s requirement of symmetry.

If \( \pi_{\beta^*} = 0 \), as is the case when \( \beta^* < ! \), then Axioms 1–4 imply that \( \hat{u}_B(v) = \hat{u}_1(\bar{c}) = \ldots = \hat{u}_n(\bar{c}) = \ldots = \)

overhead cost, ideas along the lines of Arrow (1979) and d’Aspremont and Gérard-Varet (1979) can be used to avoid deficits for all type realizations.
\( \hat{u}_n(\bar{c}) = 0 \), and so the symmetry axiom has no additional bite beyond the other axioms. But if \( \pi_{B^*} > 0 \), then the individual rationality constraints are slack, which permits \( \hat{u}_B(\bar{v}) > 0 \) and \( \hat{u}_i(\bar{c}) > 0 \). In this case when \( n = 1 \), Axiom 5 is interpreted to mean that, just like when the individual rationality constraints bind, \( \hat{u}_B(\bar{v}) = \hat{u}_1(\bar{c}) \). This arises when \( \bar{v} > \bar{c} \), in which case all five axioms are satisfied using the posted-price mechanism introduced above with \( p = (\bar{v} + \bar{c})/2 \). Notice the similarity to Nash bargaining here—the posted price is the same price at which a buyer with value \( \bar{v} \) and a seller with cost \( \bar{c} \) would trade under Nash’s axioms and assumptions.

Finally, Nash bargaining specifies, in addition to efficiency and symmetry, axioms of invariance to affine transformations of the utility functions and independence to irrelevant alternatives. In the incomplete information paradigm, the assumption of risk neutrality (and the associated quasilinear preferences) means that invariance to affine transformations of the utility functions is maintained. And a restriction that certain allocations or transfer payments are not permitted does not affect the outcome of the price-formation mechanism as long as the optimal allocation and transfers remain available. Thus, the mechanism satisfies the additional axioms of Nash.

We now state our characterization result.

**Theorem 1.** The price-formation mechanism with weights \( w = 1 \) is the unique direct mechanism satisfying Axioms 1–5.

*Proof of Theorem 1.* When \( w = 1 \), then by definition, the price-formation mechanism maximizes welfare subject to incentive compatibility, individual rationality, and no deficit. Further, because the allocation pins down the agents’ interim expected payoffs up to a constant, the mechanism is unique up to the payoffs of the worst-off types, \( \hat{u}_B(\bar{v}) \) and \( \hat{u}_i(\bar{c}) \), ..., \( \hat{u}_n(\bar{c}) \), but these are uniquely pinned down by the symmetry assumption. ■

We extend our efficiency and symmetry axioms to allow for different bargaining weights for the buyer and sellers, with at least one of the weights being positive, as follows:

**Axiom 4'(w):** Generalized efficiency with weights \( w \): The mechanism maximizes the weighted expected social surplus, \( \mathbb{E}_{w,c}[W(v, c; w)] \), subject to the conditions of Axioms 1–3.

**Axiom 5'(w):** Generalized symmetry with weights \( w \): Whenever surplus \( K > 0 \) is available to be distributed to agents while still respecting Axioms 1–3 and 4'(w), it is distributed with share \( w_B/\bar{w} \) going to the buyer and share \( w_i/\bar{w} \) going to each supplier \( i \), where \( \bar{w} \equiv w_B + w_1 + \ldots + w_n \).

This leads us to the result that the price-formation mechanism is uniquely defined by the
axioms and criteria described above. The proof is similar to that of Theorem 1, but with adjustments for the buyer and seller bargaining weights, and so is omitted.

Theorem 2. The price-formation mechanism with weights \( w \) is the unique direct mechanism satisfying Axioms 1–3, 4'(w), and 5'(w).

References for the online appendix


