Merger Review with Intermediate Buyer Power*

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Abstract

Buyer power features prominently in antitrust cases and debates, particularly as it relates to the potential for a merger among suppliers to harm a buyer. Using a Myersonian mechanism design approach, Loertscher and Marx (2019b) provide a framework for merger review for markets with buyer power, assuming that buyer power is a zero-one variable. In the present paper, we extend this analysis by treating buyer power as a continuous variable (technically, as a Ramsey weight) that ranges from zero to one. This generalization is relevant because, among other reasons, the Ramsey weight can be interpreted as a conduct parameter that can be estimated. Moreover, we establish the robustness of prior results to an alternative way of modelling merger-related cost synergies, and we show that when an acquiring firm’s choice of target is endogenous, its profit-maximizing choice depends on the buyer’s power.

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1 Introduction

Buyer power features prominently in antitrust debates and cases,\(^1\) and merger defenses based on buyer power have been said to sometimes be embraced as if they had *talismanic* power (Steptoe, 1993). Nonetheless, buyer power has proved difficult if not impossible to capture in standard oligopoly models like Cournot or Bertrand simply because these models assume price-taking behaviour on one side of the market. Using a Myersonian mechanism design approach, Loertscher and Marx (2019b) provide a framework for merger review for markets with buyer power, assuming that buyer power is a zero-one variable. In the present paper, we extend this analysis by treating buyer power as a continuous variable (technically, as a Ramsey weight) that ranges from zero to one (Ramsey, 1927). This generalization is relevant, among other reasons, because the Ramsey weight can be interpreted as a conduct parameter that can be estimated.

Moreover, in this paper we adhere to an alternative way of modelling cost synergies,\(^2\) thereby showing that the main insights from Loertscher and Marx (2019b) are robust with respect to the specifics of how cost synergies are captured.\(^3\) As we show, the main comparative statics and the key insights from Loertscher and Marx (2019b) carry over to the more general setting we study here. Here, of course, the quantitative effects will vary continuously with the Ramsey weight.

Conceptually, the present paper is part of an emerging research agenda to further develop incomplete information models in Industrial Organization. This agenda is inspired by Stigler’s arguments (Stigler, 1961, 1964) and uses the mechanism design techniques in the tradition of Myerson (1981) and Myerson and Satterthwaite (1983) to model informational asymmetries. Recent theoretical contributions in this area include the aforementioned paper on buyer power (Loertscher and Marx, 2019b), which defines the designer’s power in the same way as Bulow and Klemperer (1996),\(^4\) Loertscher and Marx (2019a),

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\(^1\)The U.S. *Horizontal Merger Guidelines* (hereafter *Guidelines*), which guide courts in the United States in the evaluation of potential anticompetitive effects of a merger, state (p. 27): “The Agencies consider the possibility that powerful buyers may constrain the ability of the merging parties to raise prices.” Merger guidelines in other jurisdictions provide a similar treatment of buyer power. The European Commission’s *Guidelines on the Assessment of Horizontal Mergers* discuss the possibility that “buyer power would act as a countervailing factor to an increase in market power resulting from the merger” (para. 11), and the Australian Competition and Consumer Commission’s *Merger Guidelines* view “countervailing power” as a competitive constraint that can limit merger harms (paras. 1.4, 5.3, 7.48).

\(^2\)On the effects of allowing the defense of a merger based on merger-related cost synergies on the merger review process, see Lagerlöf and Heidhues (2005).

\(^3\)For an estimation of merger-related synergies in the hard disk drive industry, see Igami and Uetake (2019).

\(^4\)If the designer is the buyer (as in this paper), then designer power is the ability to hold an optimal procurement mechanism, and if the designer is the seller (as in Bulow and Klemperer (1996)), it is the ability to hold an optimal sales mechanism. Specifically, in the setup of Bulow and Klemperer (1996), the optimal sales mechanism can be implemented by first holding an ascending-bid auction and then making
which uses a related setup to define and test for coordinated effects and maverick firms, Loertscher and Marx (2019c), which analyzes the competitive effects of mergers that are combined with mix-and-match divestitures, and Loertscher and Niedermayer (2019), which analyzes optimal transaction fees with incomplete information. Important precursors to this strand of literature are the analyses of mergers and collusion in auction-based markets by McAfee and McMillan (1992), Waehrer (1999), Waehrer and Perry (2003), Blume and Heidhues (2008), Miller (2014), and Froeb et al. (2017). Chae and Heidhues (2004) provide a model in which mergers or alliances among buyers can produce a buyer with enhanced bargaining power. Recent empirical work using Myersonian mechanism design techniques to analyze bargaining include Loertscher and Niedermayer (2018), Larsen (2018), and Larsen and Zhang (2018). Although bargaining power parameters can be estimated in models based on Nash bargaining, see, e.g., Crawford and Yurukoglu (2012), Grennan (2013), Gowrisankaran et al. (2015), Ho and Lee (2017), Ghili (2018), Backus et al. (2018), and Dubois et al. (2018), incomplete information bargaining models offer the ability to capture features such as the tradeoff between surplus extraction and efficiency and the possibility of bargaining breakdown, which do not naturally arise in complete information models.

In Section 2 we describe the setup, and in Section 3 we derive the equilibrium. In Section 4 we present the results. Section 5 provides an extension to a different interpretation of the designer’s objective function. Section 6 concludes.

2 Setup

We extend the setup of Loertscher and Marx (2019b) to allow for intermediate buyer power. The setup has one product and one buyer. In the pre-merger market, there are \( n \geq 2 \) suppliers, indexed by \( 1, \ldots, n \). The buyer has value \( v > c \) for one unit of the product. Each supplier \( i \in \{1, \ldots, n\} \) draws a cost \( c_i \) independently from a continuously differentiable distribution \( G_i \) with support \([c, \bar{c}]\) and density \( g_i \) that is positive on the interior of the support.\(^5\) Each supplier is privately informed about her type, and so the suppliers’ types are unknown to the buyer. All of this is common knowledge.

We let suppliers 1 and 2 be the merging suppliers (except in part of Section 4 where we consider a supplier’s choice of merger partner). We model a merger as allowing the merging suppliers to rationalize production by producing using the lower of their two costs. Thus, given pre-merger costs \( c_1, \ldots, c_n \), in the corresponding post-merger market, the nonmerging suppliers would have the same costs as pre-merger, and the merged entity take-it-or-leave-it offer to the auction winner.

\(^5\)Loertscher and Marx (2019b) offer an extension to asymmetric suppliers but state the results in the body of the paper in terms of symmetric suppliers.
would have cost \( \min \{c_1, c_2\} \). We denote the distribution for the minimum of the pre-merger costs of suppliers 1 and 2 by \( \hat{G}(c) \equiv 1 - (1 - G_1(c))(1 - G_2(c)) \), with density \( \hat{g} \).

Buyers, suppliers, and the designer are risk neutral. The buyer’s payoff is zero if he does not trade and is equal to his value minus the price he pays if he does trade. Similarly, a supplier’s payoff is zero if she does not trade and is equal to the payment she receives minus her cost if she does trade.

The timing of the model is as follows: 1. The buyer announces the mechanism. 2. Suppliers realize costs privately. 3. Suppliers independently and simultaneously decide whether to participate in the mechanism and submit reports. 4. Allocations and transfers are determined by the mechanism. When considering a merger, we assume that the merger occurs at stage 0 and is observed by the buyer and all suppliers. Thus, the profitability of a merger to merging suppliers and the harm from a merger to the buyer are evaluated at the ex ante stage before types are realized.

Define the supplier \( i \)'s pre-merger virtual cost function by

\[
\Gamma_i(c) \equiv c + G_i(c)/g_i(c),
\]

which we assume is increasing.\(^7\) The virtual cost function for the merged entity is

\[
\hat{\Gamma}(c) \equiv c + \hat{G}(c)/\hat{g}(c).
\]

The corresponding weighted virtual costs are for \( \beta \in [0, 1] \),

\[
\Gamma^\beta_i(c) \equiv (1 - \beta)c + \beta \Gamma_i(c) = c + \beta \frac{G_i(c)}{g_i(c)}
\]

(1)

\(^6\)In our setup with one buyer with single-unit demand, the merged entity need only supply one unit and so it seems appropriate to assume that it can supply that unit at the minimum of the costs of the two merging suppliers.

\(^7\)To see the role played by the virtual cost function, note that a monopolist buyer with value \( v \) maximizes its expected payoff by setting a price \( p \) that solves \( \max_p (v - p)G(p) \). This problem has first-order condition \( v = \Gamma(p^*) \), where \( \Gamma(c) \equiv c + G(c)/g(c) \). Given the assumption that the virtual cost function \( \Gamma \) is increasing, the second-order condition is satisfied whenever the first-order condition is. Thus, the buyer’s optimal price equates the buyer’s value with the seller’s virtual cost. Further, denoting the probability of trade by

\[
q(c) = \begin{cases} 
1 & \text{if } c < p^*, \\
0 & \text{otherwise},
\end{cases}
\]

the term \( G(c)/g(c) \) in the virtual cost function can be related to the supplier’s expected surplus or “information rent” because the seller’s expected surplus is

\[
\mathbb{E}_c [(p^* - c) q(c)] = \int_{\xi}^{p^*} (p^* - c) g(c) dc = \int_{\xi}^{p^*} G(c) dc = \int_{\xi}^{p^*} \frac{G(c)}{g(c)} g(c) dc = \mathbb{E}_c \left[ \frac{G(c)}{g(c)} q(c) \right].
\]
and

\[ \hat{\Gamma}^\beta(c) \equiv (1 - \beta)c + \beta \hat{\Gamma}(c) = c + \beta \frac{\hat{G}(c)}{\hat{g}(c)}. \] (2)

By construction, when \( \beta = 0 \), the weighted virtual cost corresponds to the true cost, and when \( \beta = 1 \), to the usual notion of a supplier’s virtual cost. Because we allow the possibility that the densities are zero at \( c \) (and also possibly at \( \bar{c} \), for \( \beta > 0 \), define \( \Gamma_i^\beta(c) = \lim_{c \to c} \Gamma_i^\beta(c) = c \), and similarly for \( \hat{\Gamma}^\beta \). As usual, if the density is zero at the upper support of the distribution, then for \( \beta > 0 \) the weighted virtual cost is infinite at \( \bar{c} \). If \( \Gamma_i^\beta(\bar{c}) \) is finite, then for \( x > \Gamma_i^\beta(\bar{c}) \), define \( \Gamma_i^{\beta-1}(x) \equiv \bar{c} \), and similarly for \( \hat{\Gamma}^\beta \).

We assume that the designer’s objective is to maximize the expected value of

\[ \beta(\text{buyer surplus}) + (1 - \beta)(\text{social surplus}), \] (3)

subject to incentive compatibility and individual rationality. Thus, when \( \beta = 0 \), the objective is the maximization of expected social surplus, and when \( \beta = 1 \), the objective is the maximization of buyer surplus in an environment with one-sided private information. Our parameterization with \( \beta \in [0,1] \) allows the possibility for any intermediate price formation process between the two extremes.

We refer to \( \beta = 0 \) as the case with no buyer power, \( \beta \in (0,1) \) as the case with intermediate buyer power, and \( \beta = 1 \) as the case with full buyer power. We also in places refer to \( \beta \) as the buyer’s power. Readers familiar with Ramsey pricing will recognize that this is the mechanism design analog to Ramsey pricing. Specifically, assume the designer’s objective is maximize expected social surplus subject to the constraint that the buyer’s expected profit is some \( \Pi \geq 0 \) (and subject to incentive compatibility and individual rationality constraints). Letting \( \lambda \) be the solution value of the Lagrange-multiplier associated with the profit constraint for a given \( \Pi \), the solution to the problem (3) is the same as the solution to this Ramsey problem for \( \beta = \lambda/(1 + \lambda) \).

As shown in Loertscher and Marx (2019b, Lemma 1), the allocation and payments in the dominant strategy implementation of the optimal mechanism for a designer whose objective is the expected value of (3), subject to incentive compatibility and individual rationality, corresponds to the equilibrium outcome of a procurement-plus-bargaining procedure defined as follows:

The buyer conducts a discriminatory descending clock auction with supplier-specific reserves for supplier \( i \) of \( r_i \equiv \Gamma_i^{\beta-1}(v) \) and for the merged entity of \( \hat{r} \equiv \hat{\Gamma}^{\beta-1}(v) \). The clock price starts at \( \bar{p} \) sufficiently large that all suppliers are active.\(^8\) As the clock price decreases, suppliers can choose to exit. When a supplier exits, she becomes inactive and remains so. The auction is discriminatory in that activity by a supplier at a clock price

\[^8\text{Technically, } \bar{p} \text{ has to be no less than } \max_i \{\Gamma_i^\beta(\bar{c}), \hat{\Gamma}^\beta(\bar{c})\}.\]
of $p$ obligates the supplier to supply the product at a personalized price of $\Gamma_i^{\beta-1}(p)$ for supplier $i$ and of $\hat{\Gamma}^{\beta-1}(p)$ for the merged entity, should the buyer choose to trade with that supplier. The clock stops when only one active supplier remains, with ties broken by randomization. Then the final active supplier trades if its personalized price is less than its reserve. Otherwise, the buyer makes a take-it-or-leave-it offer to the final active supplier equal to that supplier’s reserve, which the supplier can accept or reject. If the buyer’s offer is accepted, trade occurs at the reserve, and otherwise there is no trade.

This procurement-plus-bargaining procedure has a natural interpretation in terms of supplier-specific discounts. In particular, it is equivalent to an auction in which participation at a clock price of $p$ obligates a supplier to supply the product at a price equal to the clock price $p$ less a supplier-specific discount of $p - \Gamma_i^{\beta-1}(p)$ for supplier $i$ and $p - \hat{\Gamma}^{\beta-1}(p)$ for the merged entity. These discounts are zero in the case of $\beta = 0$ and positive for $\beta > 0$. For distributions with linear virtual cost functions, this interpretation amounts to the requirement that suppliers grant fixed percentage discounts relative to the clock price, with different percentages applying to suppliers with different distributions.

We model the market outcome as the allocation and payments that correspond to the (essentially unique) equilibrium in non-weakly dominated strategies of the procurement-plus-bargaining procedure. In that equilibrium, supplier $i$ with cost $c_i$ remains active as long as the clock price is greater than $\Gamma_i^\beta(c_i)$ and a merged entity with cost $c$ remains active as long as the clock price is greater than $\hat{\Gamma}^\beta(c)$.

The discount required of each supplier in the auction phase is increasing in $\beta$, and the buyer’s take-it-or-leave-it offer in the negotiation phase is decreasing in $\beta$. Thus, the parameter $\beta$ measures the aggressiveness with which the buyer requires discriminatory discounts of the suppliers and the aggressiveness of the buyer’s final take-it-or-leave-it offer. In some cases it will be useful to differentiate between these two components of buyer power. First, there is discrimination power, which is the ability of the buyer to discriminate between suppliers. Second, there is monopsony power, which is the ability of the buyer to commit to cancel a procurement even though there may be gains from trade. A buyer without power exerts neither discrimination power nor monopsony power. For a buyer with power, whether he optimally exerts one or both of these powers depends on the problem at hand. For example, when all suppliers are ex ante symmetric, the buyer does not benefit from exercising his discrimination power, and if the buyer’s value is sufficiently large that it would purchase from a supplier that had the highest possible cost, i.e., $v > \max_{i \in M}\{\Gamma_i^\beta(\bar{c})\}$, then the buyer does not benefit from exercising his monopsony power.

In Section 5, we derive conditions under which one can interpret the bargaining game.

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9 The virtual cost function for seller $i$ is linear if $G_i(c) = x^k$ for some $k > 0$. 

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as an extensive form game in which the buyer has full power with probability $x(\beta)$ and no power with probability $1 - x(\beta)$, where $x(\beta)$ is a monotonic function of $\beta$. Given this monotonicity, all comparative statics derived below continue to hold under this alternative interpretation of buyer power.

Remarks on the complete information analog

It is perhaps useful at this point to consider what the complete information counterpart would be of our model. For example, suppose that each supplier $i$ has a known cost $c_i$. This could arise in the limit of our model as the cost distribution for each supplier $i$ converges to a point mass on $c_i$. Then no buyer power, i.e., $\beta = 0$, corresponds to the case in which the buyer holds a second-price auction, or, equivalently, the suppliers engage in Bertrand competition. In that case, the buyer trades with the supplier with the lowest cost and pays an amount equal to the second-lowest cost. We can incorporate buyer power by letting $\beta$ be the probability that the buyer gets to make a take-it-or-leave-it offer to the supplier of its choice, with the suppliers engaging in Bertrand competition with the complementary probability. Then with probability $\beta$, the buyer makes an offer to the lowest-cost supplier of a price equal to that supplier’s cost. It follows, letting $c(1)$ denote the lowest cost among the suppliers and $c(2)$ denote the second-lowest cost among the suppliers, that the buyer’s expected payoff is

$$\beta(v - c(1)) + (1 - \beta)(v - c(2))$$

and the expected payoff of the lowest-cost supplier is

$$(1 - \beta)(c(2) - c(1)).$$

Thus, in this complete information analog, as the buyer power parameter $\beta$ increases from zero to one, the split of the gains from trade shifts from the lowest-cost supplier to the buyer. For $\beta < 1$, there is a division of the gains from trade between the buyer and the low-cost seller that gives a greater share to the buyer as $\beta$ increases, and for $\beta = 1$, the buyer extracts all of the gains from trade. In our setup with incomplete information, the buyer’s surplus also increases with $\beta$, but the buyer can never extract all of the gains from trade from the suppliers because suppliers always receive at least their information rent. In that sense, a supplier’s private information protects it from full surplus extraction.
3 Equilibrium

In equilibrium, the auction stage of the procurement-plus-bargaining procedure selects the supplier with the lowest weighted virtual type.

Focusing on the pre-merger market, let $k$ be the index of the final active supplier at the auction, and let $j$ be the index of the last supplier to exit the auction. In equilibrium, the auction ends at price $\Gamma_k^\beta(c_j)$ that is greater than or equal to $\Gamma_k^\beta(c_k)$. If $\Gamma_k^\beta(c_j) < \Gamma_k^\beta(c_k)$, then trade occurs at price $\Gamma_k^\beta(c_k)$, and otherwise the buyer offers supplier $k$ a price of $\Gamma_k^\beta(c_j)$. In equilibrium, supplier $k$ rejects the buyer’s offer if it is less than $c_k$ and accepts the offer if it is greater than $c_k$. Thus, in the pre-merger market, when the type vector is $c$ and $\hat{p}$ is the second lowest among $\{\Gamma_1^\beta(c_1), ..., \Gamma_n^\beta(c_n)\}$, the buyer purchases from supplier $k \in \{1, ..., n\}$ if and only if

$$\Gamma_k^\beta(c_k) < \min\{v, \hat{p}\},$$

in which case supplier $k$ receives a payment of $\Gamma_k^\beta(c_k)$.

Writing this slightly differently, the pre-merger allocation rule specifies a quantity for supplier $k$ of

$$q_k^\beta(c) \equiv \begin{cases} 1 & \text{if } v > \Gamma_k^\beta(c_k) = \min_{j \in \{1, ..., n\}} \Gamma_j^\beta(c_j), \\ 0 & \text{otherwise.}\end{cases}$$

(4)

Analogously, in the post-merger market, when the merged entity has cost $c$, the remaining suppliers have costs $c_3, ..., c_n$, and $\hat{p}$ is the second lowest among $\{\hat{\Gamma}^\beta(c), \Gamma_3^\beta(c_3), ..., \Gamma_n^\beta(c_n)\}$, the buyer purchases from supplier $k \in \{3, ..., n\}$ if and only if

$$\Gamma_k^\beta(c_k) < \min\{v, \hat{p}\},$$

in which case supplier $k$ receives a payment of $\Gamma_k^\beta(c_k)$, and the buyer purchases from the merged entity if and only if

$$\hat{\Gamma}^\beta(c) < \min\{v, \hat{p}\},$$

in which case the merged entity receives a payment of $\hat{\Gamma}^\beta(c_k)$. Thus, the post-merger allocation rule specifies a quantity for supplier $k \in 3, ..., n$ of

$$\overline{q}_k^\beta(c) \equiv \begin{cases} 1 & \text{if } v > \Gamma_k^\beta(c_k) = \min_{j \in \{3, ..., n\}} \{\hat{\Gamma}^\beta(c_k), \Gamma_j^\beta(c_j)\}, \\ 0 & \text{otherwise}\end{cases}$$

(5)
and a quantity for the merged entity of

\[ q^\beta(c) \equiv \begin{cases} 
1 & \text{if } v > \hat{\Gamma}^\beta(\min\{c_1, c_2\}) = \min_{j \in \{3, \ldots, n\}} \left\{ \hat{\Gamma}_j^\beta(\min\{c_1, c_2\}), \Gamma_j^\beta(c_j) \right\}, \\
0 & \text{otherwise.} 
\]  

(6)

As noted, when \( \beta = 0 \), the weighted virtual cost is equal to the cost, i.e., \( \Gamma_0(c) = c \) and \( \hat{\Gamma}_0(c) = c \), which implies that the suppliers exit the auction at prices equal to their costs. However, when \( \beta > 0 \), the weighted virtual cost is greater than the cost, i.e., \( \Gamma_\beta(c) > c \) and \( \hat{\Gamma}_\beta(c) > c \), and so suppliers exit at prices above their costs, and the buyer’s take-it-or-leave-it offer is less than his value. Thus, although buyer power increases expected buyer surplus, it induces inefficiency in the allocation both because discriminatory discounts can result in a supplier other than the lowest cost supplier winning the auction and because the buyer’s take-it-or-leave-it offer is below his value, leading to the possibility of no trade even when there are gains from trade. Furthermore, when \( \beta = 1 \), the discriminatory discounts and take-or-leave-it-offer constitute the optimal mechanism for the buyer. Thus, as \( \beta \) increases from zero to one, the procurement-plus-bargaining procedure moves from a nondiscriminatory clock auction with reserve equal to the buyer’s value to an optimal procurement mechanism.

To illustrate, suppose that \( n = 2 \) and that for \( i \in \{1, 2\} \), \( G_i(c) = c^{\alpha_i} \) with \( \alpha_i > 0 \) and support \([0, 1]\), which implies \( \Gamma_i^\beta(c) = \frac{\alpha_i + \beta}{\alpha_i} c \). Focus on the pre-merger market. In this case, the procurement-plus-bargaining procedure requires a fixed percentage discount from buyer \( i \) of

\[ \frac{p - \Gamma_i^{\beta^{-1}}(p)}{p} = \frac{p - p \frac{\alpha_i}{\alpha_i + \beta}}{p} = \frac{\beta}{\alpha_i + \beta}, \]

and the buyer’s take-it-or-leave-it offer when the final auction price is \( \hat{p} \) is \( \frac{\alpha_i}{\alpha_i + \beta} \min\{v, \hat{p}\} \).

Thus, no discounts are required when \( \beta = 0 \), but when \( \beta > 0 \), a larger discount is required from the supplier with the “better” cost distribution (smaller \( \alpha_i \)). In addition, when \( \beta = 0 \), the buyer’s take-it-or-leave-it offer is just the minimum of \( v \) and the final auction price, but when \( \beta > 0 \), the buyer demands a larger discount relative to the minimum of \( v \) and the final auction price the better is the winning supplier’s cost distribution.

Equilibrium trades for symmetric and asymmetric cases are illustrated in Figure 1. The comparison of panels (a) and (b) shows that with symmetric suppliers, an increase in \( \beta \) causes as expansion of the region of no trade. An increase in \( \beta \) reduces the buyer’s

\[ \frac{\alpha_i}{\alpha_i + \beta} \min\{v, \hat{p}\}. \]

\[ \frac{\alpha_i}{\alpha_i + \beta} \min\{v, \hat{p}\}. \]

\[ \frac{\alpha_i}{\alpha_i + \beta} \min\{v, \hat{p}\}. \]

In equilibrium, if \( \Gamma_i^\beta(c_1) < \Gamma_i^\beta(c_2) \), then supplier 1 wins the auction at a clock price of \( \Gamma_2^\beta(c_2) = \frac{\alpha_2 + \beta}{\alpha_2} c_2 \) and so receives a take-it-or-leave-it offer of \( \min \left\{ \frac{\alpha_1}{\alpha_1 + \beta} v, \frac{\alpha_1}{\alpha_1 + \beta} \frac{\alpha_2 + \beta}{\alpha_2} c_2 \right\} \). In the symmetric case in which \( \alpha_1 = \alpha_2 = \alpha \), the buyer purchases from the low-cost supplier whenever \( \min \{c_1, c_2\} \leq \frac{\alpha}{\alpha + \beta} v \) at a price of \( \min \left\{ \frac{\alpha}{\alpha + \beta} v, \max\{c_1, c_2\} \right\} \).
reserve price, causing it to be binding for a larger subset of possible types. Panels (c) and (d) show the case of asymmetric suppliers. In that case, the solid diagonal line that divides the region in which supplier 1 trades from the region in which supplier 2 trades is no longer the 45-degree line. Instead, in addition to the inefficiency associated with the region of no trade, there is also inefficiency because for type realizations in the region between the solid diagonal line and the 45-degree line (dashed diagonal line), supplier 2 trades even though supplier 1 has a lower cost. This occurs because, with asymmetric suppliers, the buyer discriminates against the supplier with the better cost distribution, and in the illustration of Figure 1(c) and (d), that is supplier 1. The comparison of panels (c) and (d) shows that as $\beta$ increases, the region of no trade increases and the region in which the buyer does not purchase from the low-cost supplier increases. Thus, the buyer’s exercise of both his discrimination power and his monopsony power increases as $\beta$ increases.

Insert Figure 1 here.

Because of discrimination by a powerful buyer against a supplier with a “better” cost distribution, a supplier who trades with higher probability in equilibrium in a market with buyer power is not necessarily the supplier who draws her type from the “better” distribution. Even if $G_1(c) > G_2(c)$ for $c \in (\underline{c}, \bar{c})$, so that supplier 1 is the lower cost supplier in the sense of first order stochastic dominance, supplier 2 may have a higher probability of trade when there is buyer power.\footnote{For example, if $G_1(c) = c^2$ and $G_2(c) = c^3$, then for $n = 2$, $\beta = 1$, and $v$ sufficiently large that trade always occurs, supplier 2 has the worse cost distribution but has a probability of trade that is greater than 50%.
}

Thus, in markets with buyer power, one cannot necessarily infer from suppliers’ market shares the ranking of the suppliers’ cost distributions.

As illustrated in Figure 1 for the case of $n = 2$, trade occurs whenever the lowest virtual cost among the suppliers is less than $v$ (i.e., for some supplier $i$, $c_i < \Gamma_i^{\beta^{-1}}(v)$). Thus, before the merger, the probability of trade is

$$
\Pr(\text{trade}) = \Pr\left( \min_{i \in \{1, \ldots, n\}} \{\Gamma_i^{\beta}(c_i)\} < v \right)
$$

and after a merger of suppliers 1 and 2, it is

$$
\Pr(\text{trade}) = \Pr\left( \min_{i \in \{3, \ldots, n\}} \{\tilde{\Gamma}_i^{\beta}(\min\{c_1, c_2\}), \Gamma_i^{\beta}(c_i)\} < v \right).
$$

It follows from standard mechanism design arguments, that expected buyer surplus in the equilibrium of the procurement-plus-bargaining procedure can be written in terms of
virtual costs (rather than weighted virtual costs) and the allocation rule. For the details of these arguments, see Appendix A. Specifically, expected buyer surplus in the pre-merger market is

\[ E_c \left[ \sum_{i=1}^{n} q_i^\beta(c)(v - \Gamma_i(c_i)) \right], \tag{9} \]

and in the post-merger market is

\[ E_c \left[ \sum_{i=3}^{n} \bar{q}_i^\beta(c)(v - \Gamma_i(c_i)) + \bar{q}^\beta(c) \left( v - \hat{\Gamma}(\min\{c_1, c_2\}) \right) \right]. \tag{10} \]

Thus, expected buyer surplus depends on buyer power only through its affect on the allocation.

Letting \( \Delta Q \) be the expected change in the quantity traded as the result of a merger, we have

\[ \Delta Q \equiv E_c \left[ \sum_{i=3}^{n} \bar{q}_i^\beta(c) + \bar{q}^\beta(c) - \sum_{i=1}^{n} q_i^\beta(c) \right]. \]

To analyze merger effects, we must account for the change in the buyer’s procurement procedure as a result of the merger. For this purpose, the following lemma is useful.

**Lemma 1** The distribution of the merged entity dominates the distribution of each of the merging suppliers in terms of the reverse hazard rate: for all \( c \in [\underline{c}, \bar{c}] \),

\[ \frac{\hat{G}(c)}{\hat{g}(c)} > \min \left\{ \frac{G_1(c)}{g_1(c)}, \frac{G_2(c)}{g_2(c)} \right\}, \]

with equality for \( c = \underline{c} \).

**Proof.** See Appendix B.

It follows from Lemma 1 that for any cost draw, the weighted virtual cost of the merged entity is greater than or equal to the minimum of the weighted virtual costs of the merging suppliers: for all \( c \in [\underline{c}, \bar{c}] \),

\[ \hat{\Gamma}^\beta(c) \geq \min \{ \Gamma_1^\beta(c), \Gamma_2^\beta(c) \}, \tag{11} \]

with a strict inequality when \( \beta > 0 \) and \( c > \underline{c} \). Thus, the take-it-or-leave-it offer by the buyer to the merged entity is less than the larger of the offers to the two pre-merger suppliers.
We say that virtual dominance holds if for all \(c_1, c_2 \in [c, \bar{c}]\),

\[
\hat{\Gamma}^\beta (\min \{c_1, c_2\}) \geq \min \{\Gamma_1^\beta (c_1), \Gamma_2^\beta (c_2)\},
\]

with a strict inequality for a positive measure set of costs. Virtual dominance holds, for example, when \(\beta > 0\) and the merging suppliers are symmetric (by Lemma 1).\(^{12}\)

### 4 Merger analysis

In what follows, we consider the effect of buyer power on the competitive effects of a merger, including the effect of buyer power on the reduction in output as a result of a merger, the profitability of a merger for the merging parties, a firm’s optimal choice of acquisition target, and the profitability of entry. In addition, we examine how buyer power affects the gains that accrue to the merging parties as a result of merger-related cost synergies.

**Effects on output**

As we now show, the greater is buyer power, the greater is the reduction in output as a result of a merger.

The buyer’s expected payment and expected surplus depend on buyer power only through the allocation rule, which depends on the ranking of the weighted virtual costs. When virtual dominance is satisfied, \(\hat{\Gamma}^\beta (\min \{c_1, c_2\}) - \min \{\Gamma_1^\beta (c_1), \Gamma_2^\beta (c_2)\}\) is increasing in buyer power, which implies that the increase in weighted virtual costs as a result of a merger is increasing in buyer power. This implies that the quantity reduction as a result of a merger is greater when buyer power is greater.

**Proposition 1** If virtual dominance holds, then the change in quantity as a result of a merger is decreasing in buyer power, i.e., \(\Delta Q\) is decreasing in \(\beta\).

**Proof.** See Appendix B.

The quantity effect described in Proposition 1 implies that the decrease in consumer and social surplus as a result of a merger is greater the more powerful is the buyer.

\(^{12}\)Condition (12) also holds for some asymmetric distributions, e.g., \(G_1(c) = c\) for \(c \in [0, 1]\) and \(G_2(c) = c\) for \(c \in [0, 1/2]\) and \(G_2(c) = 2/3c^2 + 1/3\) for \(c \in (1/2, 1]\). It does not hold, for example, if \(G_1(c) = c\) and \(G_2(c) = c^2\), with support \([0, 1]\), for \(c_1\) sufficiently small and \(c_2\) sufficiently large.
Effects on the merging parties

Turning to the effects of a merger on the merging parties, the literature has recognized the “merger paradox” that a merger in the Cournot model is not profitable for the merging firms unless it is a merger to monopoly (Salant et al. (1983)). In our setup, when the buyer has no or low buyer power, there is no such paradox. However, for sufficiently powerful buyers, it can be the case that an exogenously given merger is not profitable for the merging suppliers. Specifically, if the buyer is sufficiently powerful, then the increased monopsony power that he exerts against the merged entity (in the form of a more aggressive take-it-or-leave-it offer) as a result of the reverse-hazard-rate increasing shift in the distributions can dominate the effect of the merger of eliminating a competing bid.

In general, a merger can cause the merging suppliers to trade at a different price, cause them not to trade when they would have pre-merger, or have no effect. For illustration, consider the case of two symmetric merging suppliers, i.e., a merger to monopoly. In that case, when the effect of the merger is to cause the merging suppliers to trade at a different price than they would have pre-merger, then there are three possibilities. To describe these, let \( p = \Gamma^\beta(v) \), where \( \Gamma^\beta \) is the common weighted virtual cost function for the two suppliers, and let \( \hat{p} = \hat{\Gamma}^\beta(v) \). It follows from Lemma 1 and the assumption of symmetry that for any \( v \in (c, \Gamma^\beta(c)) \), we have \( \hat{p} < p \).

When \( \min\{c_1, c_2\} < \hat{p} < p < \max\{c_1, c_2\} \), the merger reduces the payment to the merging suppliers from \( p \) to \( \hat{p} \); when \( \min\{c_1, c_2\} < \hat{p} < \max\{c_1, c_2\} < p \), it reduces the payment to the merging suppliers from \( \max\{c_1, c_2\} \) to \( \hat{p} \); and when \( \min\{c_1, c_2\} < \max\{c_1, c_2\} < \hat{p} < p \), it increases the payment to the merging suppliers from \( \max\{c_1, c_2\} \) to \( \hat{p} \).

When the effect of the merger is to cause the merging suppliers not to trade when they would have in the pre-merger market, i.e., when \( \hat{p} < \min\{c_1, c_2\} < p \), then the merging suppliers’ surplus falls from either \( p - \min\{c_1, c_2\} \) or \( \max\{c_1, c_2\} - \min\{c_1, c_2\} \) to zero.

If \( \beta = 0 \), then \( p = \hat{p} \), and so only the gain remains, implying that the merger is profitable. This result extends to the case of any number of potentially asymmetric firms (Loertscher and Marx, 2019b, Prop. 6). Continuity then implies that the merger is profitable for all positive but sufficiently small values of \( \beta \).

Although both \( p \) and \( \hat{p} \) are decreasing in \( \beta \), the difference between \( p \) and \( \hat{p} \) is not monotonic in \( \beta \), so we cannot say in general that the change in the expected joint payoff of the merging suppliers as a result of a merger is decreasing in buyer power. But, as shown in Proposition 2, when the buyer does not exert his monopsony power in the pre-merger market, then the change in the expected joint profit of suppliers merging to monopoly is indeed decreasing in buyer power.
Defining the *profitability of a merger* to be the change in the expected joint payoff of the merging suppliers as a result of a merger, and saying that the merger is profitable if this is positive, we have the following result:

**Proposition 2** There exists $\beta > 0$ such that a merger is profitable for all $\beta \in [0, \beta]$. Moreover, for $v$ sufficiently large, a merger to monopoly is profitable for all $\beta \in [0, 1]$. However, for a merger to monopoly by symmetric suppliers, (i) given $\beta > 0$, there exists $\hat{v} \in (c, \bar{v})$ such that for all $v \in (c, \hat{v})$, the merger is not profitable; and (ii) if the buyer does not exert monopsony power in the pre-merger market, i.e., $\Gamma^{\beta^{-1}}(v) = \bar{v}$, then the profitability of the merger is nonincreasing in buyer power (and decreasing for $\hat{\Gamma}^{\beta^{-1}}(v) < \bar{v}$).

**Proof.** See Appendix B.

Proposition 2 provides conditions under which, even with positive buyer power, a merger is profitable, but also provides conditions under which increasing buyer power reduces the profitability of a merger. The proposition implies that a merger to monopoly by symmetric suppliers is profitable for $\beta = 0$, but becomes less profitable as $\beta$ increases and eventually becomes unprofitable if $v$ is sufficiently low. Intuitively, when $v$ is very low, there is little benefit to the merged entity from the merger’s suppression of a bid, but as a result of the merger, the buyer exercises his monopsony power more aggressively, which harms the merged entity.

**Choice of acquisition target**

In addition to affecting incentives to merge, buyer power affects a supplier’s preferred choice of acquisition target.\(^\text{13}\) We assume that an acquisition occurs prior to the realization of costs, implying that the profitability of a merger is evaluated at the ex ante stage. Suppose supplier 1 chooses her target among the rivals $i \in \{2, \ldots, n\}$. Assume that for all $c \in [c, \bar{c}]$, $G_2(c) \geq \ldots \geq G_n(c)$, and neglect the price that supplier 1 has to pay to acquire the target and focus on the joint post-merger profit. How does supplier 1’s preferred target vary with $\beta$ and $G_1$?

For $\beta = 0$, supplier 2 is the preferred target for all $G_1$. To see this, note that acquiring supplier 2 reduces costs the most, maximizes the probability of winning, and eliminates the toughest rival bidder. Moreover, if for all $c \in [c, \bar{c}]$, $G_1(c) \geq G_3(c)$, supplier 1 would be supplier 2’s preferred target, indicating that in the absence of buyer power, we should

\(^{13}\)See Banal-Estañol et al. (2010) for an analysis of a setting in which the target screens prospective acquirers for potential synergies.
expect “positive assortative matching” insofar as the gains from a merger of two suppliers are maximized by a merger between the strongest two suppliers.\(^{14}\)

However, for \(\beta > 0\), there is the additional effect that the stronger is the target’s cost distribution, the greater will be the discrimination faced by the merged entity in the post-merger procurement. As a result, the preferred target can vary with \(\beta\) and with \(G_1\) as we illustrate in Figure 2 for the case of three suppliers. We assume that supplier \(i\) draws her cost from distribution \(G_i(c) = c^{\alpha_i}\) with support \([0, 1]\). Consequently, \(G_2(c) \geq G_3(c)\) for all \(c \in [0, 1]\) is equivalent to \(\alpha_2 \leq \alpha_3\). For Figure 2, we assume specifically that \(\alpha_2 = 1\) and \(\alpha_3 = 2\).

The expected payoff of the merged entity that combines suppliers 1 and 2 is analogous to the expressions for the buyer’s expected payoff given in Section 3 once one notes that the quantity traded by the merged entity that combines suppliers 1 and 2 is one if \(\hat{\Gamma}_{1,2}^\beta(\min\{c_1, c_2\}) < \min\{v, \Gamma_{3}^\beta(c_3)\}\) and zero otherwise, where \(\hat{\Gamma}_{i,j}^\beta\) denotes the weighted virtual cost of the merged entity that combines suppliers \(i\) and \(j\). Thus, the expected payoff of the merged entity if supplier 1 acquires supplier 2 is

\[
E_c[(\hat{\Gamma}_{1,2}^1(\min\{c_1, c_2\}) - \min\{c_1, c_2\}) \cdot 1_{\hat{\Gamma}_{1,2}^\beta(\min\{c_1, c_2\}) < \min\{v, \Gamma_{3}^\beta(c_3)\}}],
\]

and the expected payoff of the merged entity if supplier 1 acquires supplier 3 is

\[
E_c[(\hat{\Gamma}_{1,3}^1(\min\{c_1, c_3\}) - \min\{c_1, c_3\}) \cdot 1_{\hat{\Gamma}_{1,3}^\beta(\min\{c_1, c_3\}) < \min\{v, \Gamma_{3}^\beta(c_2)\}}],
\]

Figure 2(a) assumes \(\alpha_1 = 0.5\), so that supplier 1 is the strongest supplier pre merger. As the figure shows, the merged entity has greater expected surplus when the merger combines supplier 1 with her stronger rival, supplier 2, if buyer power is sufficiently low (\(\beta < 0.7328\)). But if the buyer is sufficiently powerful, then the merged entity has greater expected surplus when the merger involves the weaker rival, supplier 3. In contrast, as shown in Figure 2(b), when \(\alpha_1 = 2\), so that supplier 1 is pre merger at par with supplier 3, then supplier 1 prefers to merge with her stronger rival, supplier 2, even when \(\beta = 1\).

Insert Figure 2 here.

Thus, we have the following proposition:

**Proposition 3** Assuming that for all \(c \in [c, \bar{c}]\), \(G_2(c) \geq \ldots \geq G_n(c)\), then the expected payoff of the merged entity that combines supplier 1 with one of the suppliers 2, ..., \(n\)

\(^{14}\)As observed in Loertscher and Marx (2019b), without buyer power, a merger is equivalent to perfect collusion. Therefore, the fact that, absent buyer power, the gains from a merger are largest for the strongest suppliers is related to the observation made by Loertscher and Marx (2019a) that, without buyer power, the gains from collusion are largest for the strongest suppliers.
is maximized if she merges with supplier 2 when the buyer has no power; however, the acquisition target that maximizes the merged entity’s expected payoff can vary with the buyer’s power and, for positive buyer power, with supplier 1’s cost distribution.

Effects on entry

When evaluating the likely competitive effects of a merger, competition authorities regularly consider whether the merger (together with any price increases that result from the merger) might induce entry into the market and whether such entry might ameliorate any competitive harms from the merger.\(^\text{15}\) In assessing the likelihood of entry, considerations include whether entry is likely to be profitable.\(^\text{16}\)

A merger does not affect the lowest cost among the suppliers in the industry, \(\tilde{c} \equiv \min_{i \in \{1, \ldots, n\}} c_i\). When \(\beta = 0\), regardless of whether a merger occurs, if a potential entrant chooses to enter, it trades if and only if its cost is less than \(\tilde{c}\). Further, when it does trade, it is paid \(\tilde{c}\). Thus, when \(\beta = 0\), the profitability of entry is not affected by a merger (Loertscher and Marx, 2019b, Proposition 7).

However, when there is buyer power, under the virtual dominance condition of (12), a merger increases the weighted virtual cost of the merged entity, \(\hat{\Gamma}^\beta(\min\{c_1, c_2\})\), relative to the lowest weighted virtual cost of the merging suppliers, \(\min\{\Gamma_1^\beta(c_1), \Gamma_2^\beta(c_2)\}\). Further, as we show in the proof of Proposition 1, the difference between these two increases with \(\beta\). This means that, conditional on entry, a merger increases both the probability that the entrant trades and the payment to the entrant if it trades, and the effects are greater the larger is \(\beta\). Thus, a merger increases the profitability of entry, and more so the larger is \(\beta\), and this holds regardless of the entrant’s cost distribution. We summarize with the following result:

**Proposition 4** If virtual dominance holds, then the greater is buyer power, the greater is the (positive) impact of a merger on the expected payoff from entry for any cost distribution for the entrant.

Cost efficiencies

Finally, consider the impact of cost efficiencies on merger effects. For the purposes of analyzing cost synergies, we assume that pre-merger all suppliers are symmetric with

\(^{15}\)As stated in the *Guidelines* (p. 28): “As part of their full assessment of competitive effects, the Agencies consider entry into the relevant market. The prospect of entry into the relevant market will alleviate concerns about adverse competitive effects only if such entry will deter or counteract any competitive effects of concern so the merger will not substantially harm customers.” Related to the likely sufficiency of entry to remedy harms, see the *Guidelines*, Section 9.3.

\(^{16}\)See the *Guidelines*, Section 9.2.
cost distribution $G$ with support $[c, \bar{c}]$.\footnote{The assumption of bounded support is not essential for our cost synergies results. They continue to hold, for example, with $G(c) = 1 - e^{-c}$ with support $[0, \infty)$.} Recall that a merger that endows the merged entity with no cost efficiencies beyond the ability to produce at the lower cost of the two pre-merger suppliers draws its cost from the distribution

$$\hat{G}(c) = 1 - (1 - G(c))^2.$$ 

In this section, we consider the possibility of incremental cost efficiencies for the merged entity beyond simply producing at the lower cost of the two pre-merger suppliers. Thus, we consider the case in which the merged entity’s cost distribution be given by

$$\bar{G}(c) \equiv 1 - (1 - G(c))^s,$$

where $s = 2$ corresponds to the case in which the merged entity’s cost is the minimum of the costs of the two pre-merger suppliers, and $s > 2$ corresponds to the case of incremental cost synergies from the merger.

This implies that the merged-entity’s weighted virtual cost,

$$\Gamma^\beta(c; s) \equiv c + \beta \frac{1 - (1 - G(c))^s}{s(1 - G(c))^{s-1}g(c)},$$

is increasing and unbounded in $s$ for any $c > \underline{c}$ and $\beta > 0$. This then implies that for any $\beta > 0$, the buyer’s optimal reserve for the merged entity, $\Gamma^{\beta^{-1}}(v; s)$, decreases in $s$ and goes to $\underline{c}$ as $s \to \infty$. Thus, cost synergies squeeze the merged entity’s informational rents when the merged entity faces a buyer with power, and more so the greater is the buyer’s power.

This gives us the following result:

**Proposition 5** The greater are merger-related cost synergies, the lower is the merged entity’s expected surplus when the merged entity faces a buyer with power. The decrease is larger the greater is the buyer’s power. That is, the optimal reserve applied to the merged entity is decreasing in $s$, decreasing in $\beta$, and goes to $\underline{c}$ as $s$ goes to $\infty$.

As shown in Proposition 5, cost synergies have a downside for a merged entity who faces a buyer with power because the cost synergies induce the buyer to exert his monopsony power (and his discrimination power) more aggressively, which squeezes the merged entity’s information rents and in the limit, as the extent of cost synergies increases, drives the merged entity’s profit to zero.
5 Alternative interpretation of intermediate buyer power

We now provide an alternative (and largely equivalent) interpretation of the Ramsey parameter $\beta$ as capturing the commitment power of the buyer. Specifically, for the purposes of this section, we assume that suppliers are symmetric in that they all draw their cost types from the same distribution $G$, with virtual type function $\Gamma$. We assume that the buyer starts the descending clock auction at his maximum willingness to pay $r = \min\{v, \pi\}$. And we assume that at the end of the descending clock auction, with probability $x$, the buyer can make a final take-it-or-leave-it offer, and with probability $1 - x$, the buyer has no such opportunity, so that the transaction takes place at the standing clock price. The optimal take-it-or-leave-it offer for the buyer is $\Gamma^{-1}(r)$.

If the buyer can make this offer only with probability $x$, the expected or average offer is

$$x\Gamma^{-1}(r) + (1 - x)r. \tag{13}$$

Because $\Gamma^{\beta^{-1}}(r) \in [\Gamma^{-1}(r), r]$, it follows that $\Gamma^{\beta^{-1}}(r)$ is a weighted average of the lower and the upper bound. Letting $x$ be the weight on the lower bound, we see that the buyer’s expected offer (13) is equal to $\Gamma^{\beta^{-1}}(r)$.\footnote{In general, there is no probability $x$ such that this offer is the same as $\Gamma^{\beta^{-1}}(r)$ for all $r$, so, under this interpretation, one cannot pursue comparative statics with respect to $r$. However, this would be possible if we assume that $G(c) = e^c$ with $c \in [0, 1]$, so that $\Gamma^{-1}(r) = (\beta + \alpha)r/\alpha$ and $x\Gamma^{-1}(r) + (1 - x)r = (\alpha + 1 - x)r/(\alpha + 1)$, which are the same for $x = \beta^{\alpha+1}/\alpha^{\alpha+\beta}$.} Further, because $x$ is increasing in $\beta$, all our results hold with $x$ in lieu of $\beta$, and comparative statics results with respect to $\beta$ also generalize.

If the suppliers are ex ante asymmetric, the analysis does not generalize because the buyer would like to discriminate among the suppliers in the procurement and apply supplier-specific final offers. In this case, there is no single probability $x$ that provides consistency with the alternative interpretation offered here for the symmetric case.

That being said, the idea that buyers may be able to make binding take-it-or-leave-it offers with some probability while being unable to discriminate suggests an alternative form of intermediate buyer power—one according to which the buyer has some some monopsony and no discrimination power. Exploring the implications of this seems a worthwhile exercise for future research.
6 Conclusion

We show that the key results of Loertscher and Marx (2019b) for a powerful buyer who can employ an optimal procurement mechanism are not knife-edge, but rather extend in a continuous manner to a buyer whose power is intermediate between that of a buyer without power and a fully powerful buyer.

This paper contributes to a stream of research motivated by the importance of developing modeling tools that allow for incomplete information in the analysis of mergers. By doing so, one can make precise and then analyze a number of ideas currently in use by competition authorities, including the notion of buyer power, definitions and measurements of coordinated effects and maverick firms, and the role for divestiture-based remedies for merger harm. In particular, one can show that buyer power does not eliminate merger harm; some, but not all, mergers raise concerns of coordinated effects; acquisitions that eliminate a maverick firm need not put a market at risk for coordination; and divestitures that restore the number of firms in a market can, but need not, be sufficient to remedy competitive harms.\(^{19}\)

As this paper indicates, in some settings, the calibration of models used for merger simulation can usefully include the measurement of the power of downstream buyers when evaluating a merger among upstream suppliers. Whereas our focus here is on the power of a single buyer, extensions could allow multiple buyers with varying power. A challenge in such extensions is to allow for multi-unit supply and demand, which we leave for future research. Further topics for future research include the identification of buyer power parameters in empirical analyses and the extension of our alternative interpretation of intermediate buyer power based on the probability that the buyer has commitment power to more general setups with asymmetric suppliers.

\(^{19}\)The buyer power result is shown in this paper and its predecessor (Loertscher and Marx, 2019b). For coordinated effects and mavericks, see Loertscher and Marx (2019a). Divestitures are analyzed in Loertscher and Marx (2019c).
A Appendix: Mechanism design concepts

Take as given a direct mechanism \((q, m)\) mapping suppliers’ reports to a probability of trade and payments, \(q : [c, \overline{c}]^n \to [0, 1]^n\) and \(m : [c, \overline{c}]^n \to \mathbb{R}^n\), where \(q_i(c) \in [0, 1]\) is the probability with which supplier \(i\) trades and \(m_i(c)\) is the payment to supplier \(i\) given cost types \(c\). Let \(\hat{q}_i(z)\) be supplier \(i\)’s expected quantity if it reports \(z\) and all other suppliers report truthfully, 
\[
\hat{q}_i(z) = \mathbb{E}_{c_{-i}}[q_i(z, c)],
\]
and let \(\hat{m}_i(z)\) be supplier \(i\)’s expected payment if it reports \(z\) and the other suppliers report truthfully, 
\[
\hat{m}_i(z) = \mathbb{E}_{c_{-i}}[m_i(z, c)].
\]

Because we assume independent draws, \(\hat{q}_i(z)\) and \(\hat{m}_i(z)\) depend only on supplier \(i\)’s report \(z\) and not on supplier \(i\)’s true type. Suppose that the mechanism satisfies incentive compatibility, i.e., for all \(i \in \{1, \ldots, n\}\) and all \(c, z \in [c, \overline{c}]\), 
\[
U_i(c) \equiv \hat{m}_i(c) - \hat{q}_i(c) \geq \hat{m}_i(z) - \hat{q}_i(z)c,
\]
and interim individual rationality, i.e., for all \(i \in \{1, \ldots, n\}\) and \(c \in [c, \overline{c}], U_i(c) \geq 0\).

As we now show, standard arguments (e.g., Krishna, 2002, Chapter 5.1) imply that in any incentive compatible, interim individually rational mechanism with allocation rule \(q\), the expected payment by the buyer to supplier \(i\) is
\[
\mathbb{E}_c [q_i(c) \cdot \Gamma_i(c_i)].
\]

The expected payoff of supplier \(i\) with type \(c\) that reports \(z\) is \(\hat{m}_i(z) - \hat{q}_i(z)c\). Incentive compatibility implies that
\[
U_i(c) = \max_{z \in [c, \overline{c}]} \{\hat{m}_i(z) - \hat{q}_i(z)c\},
\]
i.e., \(U_i\) is a maximum of a family of affine functions, which implies that \(U_i\) is convex and so absolutely continuous and differentiable almost everywhere in the interior of its domain. In addition, incentive compatibility implies that \(U_i(z) \geq \hat{m}_i(z) - \hat{q}_i(c)z = U_i(c) - \hat{q}_i(c)(z - c)\), which for \(\delta > 0\) implies
\[
\frac{U_i(c + \delta) - U_i(c)}{\delta} \geq -\hat{q}_i(c)
\]
and for \(\delta < 0\) implies
\[
\frac{U_i(c + \delta) - U_i(c)}{\delta} \leq -\hat{q}_i(c),
\]
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so taking the limit as $\delta$ goes to zero, at every point $c$ where $U_i$ is differentiable, $U_i'(c) = -\hat{q}_i(c)$. Because $U_i$ is convex, this implies that $\hat{q}_i(c)$ is nonincreasing. Because every absolutely continuous function is the definite integral of its derivative, we have

$$U_i(c) = U_i(\bar{c}) + \int_c^{\bar{c}} \hat{q}_i(t)dt. \quad (15)$$

This implies that, up to an additive constant equal to the expected payoff of a supplier with the worst type, $U_i(\bar{c})$, which is zero in our case because individual rationality binds for a supplier with the worst type, a supplier’s expected payoff in an incentive compatible direct mechanism depends only on the allocation rule.

Using the definition of $U_i$ in (14), we can rewrite (15) as

$$\hat{m}_i(c) = \hat{q}_i(c)c + \int_c^{\bar{c}} \hat{q}_i(t)dt. \quad (16)$$

Using (16), the expected payment to supplier $i$ is

$$E_c \left[ \hat{m}_i(c_i) \right] = \int_{\bar{c}}^{c} \hat{m}_i(c)g_i(c)dc$$

$$= \int_{\bar{c}}^{c} \left( \hat{q}_i(c)c + \int_c^{\bar{c}} \hat{q}_i(t)dt \right) g_i(c)dc$$

$$= \left( \int_{\bar{c}}^{c} \hat{q}_i(c)cg_i(c)dc + \int_{\bar{c}}^{c} \int_{\bar{c}}^{t} \hat{q}_i(t)g_i(c)dcdt \right)$$

$$= \left( \int_{\bar{c}}^{c} \hat{q}_i(c)cg_i(c)dc + \int_{\bar{c}}^{c} \hat{q}_i(t)G_i(t)dt \right)$$

$$= \left( \int_{\bar{c}}^{c} \hat{q}_i(c) \left( c + \frac{G_i(c)}{g_i(c)} \right) g_i(c)dc \right)$$

$$= \int_{\bar{c}}^{c} \hat{q}_i(c)\Gamma_i(c)g_i(c)dc$$

$$= E_c \left[ \hat{q}_i(c_i) \cdot \Gamma_i(c_i) \right]$$

$$= E_c \left[ q_i(c) \cdot \Gamma_i(c_i) \right]$$

where the first equality uses the definition of the expectation, the second uses (16), the third switches the order of integration, the fourth integrates, the fifth collects terms, the sixth uses the definition of the virtual cost $\Gamma_i$, and the last two equalities use the definition of the expectation.
B Appendix: Proofs

Proof of Lemma 1. Using the definition of \( G \), the inequality in the lemma holds if and only if for all \( c \in (\bar{c}, \overline{c}) \),

\[
1 - \frac{(1 - G_1(c))(1 - G_2(c))}{g_1(c)(1 - G_2(c)) + g_2(c)(1 - G_1(c))} > \min \left\{ \frac{G_1(c)}{g_1(c)}, \frac{G_2(c)}{g_2(c)} \right\}, \tag{17}
\]

Take \( c \in (\bar{c}, \overline{c}) \) and suppose, without loss of generality, that \( \frac{G_1(c)}{g_1(c)} \leq \frac{G_2(c)}{g_2(c)} \). Then we can rewrite (17) as

\[
\frac{G_2(c)}{g_2(c)} > \frac{G_1(c)}{g_1(c)}(1 - G_1(c)),
\]

which holds by the assumptions that \( \frac{G_1(c)}{g_1(c)} \leq \frac{G_2(c)}{g_2(c)} \) and \( c > \bar{c} \). The case \( \frac{G_1(c)}{g_1(c)} \geq \frac{G_2(c)}{g_2(c)} \) follows along the same lines by changing indices. This completes the proof. ■

Proof of Proposition 1. For cost values above the lower support, \( \frac{\partial \hat{\Gamma}(c)}{\partial \beta} > 0 \) and for all \( i \in \{1, ..., n\} \), \( \frac{\partial \Gamma_i(c)}{\partial \beta} > 0 \). Thus, it is clear from (7) and (8) that the probability of trade decreases in \( \beta \) both before and after a merger.

Because the virtual type functions and costs of the nonmerging suppliers are not affected by a merger, the change in the probability of trade as a result of a merger is determined by

\[
\hat{\Gamma}(\min \{c_1, c_2\}) - \min \left\{ \Gamma_1(c_1), \Gamma_2(c_2) \right\} \geq 0, \tag{18}
\]

where the inequality follows from the assumption of virtual dominance (see (12)), which also implies that the inequality is strict for a positive measure set of costs.

In what follows, we show that for \( \beta > 0 \), the derivative of the left side of (18) with respect to \( \beta \) is nonnegative and positive for a positive measure set of costs, which establishes that the decrease in the probability of trade as a result of a merger is greater (a larger decrease) as \( \beta \) increases, which establishes that \( \Delta Q \) is decreasing in \( \beta \).

Letting \( i \in \{1, 2\} \) be such that \( \Gamma_i(c_i) = \min \left\{ \Gamma_1(c_1), \Gamma_2(c_2) \right\} \), then

\[
\frac{\partial}{\partial \beta} \left[ \hat{\Gamma}(\min \{c_1, c_2\}) - \min \left\{ \Gamma_1(c_1), \Gamma_2(c_2) \right\} \right] = \frac{\hat{G}(\min \{c_1, c_2\})}{g(\min \{c_1, c_2\})} - \frac{G_i(c_i)}{g_i(c_i)}.
\]

If for \( i, j \in \{1, 2\}, c_i \leq c_j \) and \( \Gamma_i(c_i) \leq \Gamma_j(c_j) \), then the expression in (19) becomes, for \( \beta > 0 \),

\[
\frac{\hat{G}(c_i)}{g_i(c_i)} - \frac{G_i(c_i)}{g_i(c_i)} = \frac{1}{\beta} \left( \hat{\Gamma}(c_i) - \Gamma_i(c_i) \right) \geq 0,
\]

where the inequality holds by (18), with a strict inequality for an open measure set of
costs. If instead \( c_j < c_i \) and \( \Gamma^\beta_i(c_i) \leq \Gamma^\beta_j(c_j) \), then the expression in (19) becomes, for \( \beta > 0 \),

\[
\frac{\hat{G}(c_j)}{\hat{g}(c_j)} - \frac{G_i(c_i)}{g_i(c_i)} = \frac{1}{\beta} \left( \hat{\Gamma}^\beta(c_j) - \Gamma^\beta_i(c_i) - c_j + c_i \right) > \frac{1}{\beta} \left( \hat{\Gamma}^\beta(c_j) - \Gamma^\beta_i(c_i) \right) \geq 0.
\]

Thus, we have shown that for \( \beta > 0 \), (19) is nonnegative and positive for a positive measure set of costs, completing the proof. ■

Proof of Proposition 2. The result that there exists \( \bar{\beta} > 0 \) such that a merger is profitable for all \( \beta \in [0, \bar{\beta}] \) follows from the text.

The result that for \( v \) sufficiently large, a merger to monopoly is profitable for all \( \beta \in [0, 1] \) follows because as \( v \) goes to infinity, the buyer’s reserve approaches (or equals) \( \bar{c} \), which implies that trade occurs with probability 1 both before and after the merger, but after the merger trade occurs at the higher price of \( \bar{c} \) rather than \( \Gamma^\beta_1 - \Gamma^\beta_2(c_1) \) or \( \Gamma^\beta_2 - \Gamma^\beta_1(c_1) \), and the merged entity’s cost is at least as low as the cost of the trading pre-merger supplier.

We next show the result that for a merger to monopoly by symmetric suppliers, if the buyer does not exert monopsony power in the pre-merger market, then the profitability the merger is nonincreasing in buyer power (and decreasing for \( \hat{\Gamma}^{\beta-1}(v) < c \)). Let \( p \) and \( \hat{p} \) be as defined in the text. If the buyer does not exert monopsony power in the pre-merger market for any \( \beta \in [0, 1] \), then for all \( \beta \in [0, 1] \), we have \( p = \bar{c} \), and so the joint payoff of symmetric pre-merger suppliers does not depend on \( \beta \). Note that \( \hat{p} \) is nonincreasing in \( \beta \). Using integration by parts and \( \hat{G}(c) = 1 - (1 - G(c))^2 = G(c)(2 - G(c)) \), the post-merger expected payoff of the merged entity is

\[
\int_{\bar{c}}^{\hat{p}} (\hat{p} - c) d\hat{G}(c) = (\hat{p} - c)\hat{G}(c) \bigg|_{c=\hat{p}}^c + \int_{\bar{c}}^{\hat{p}} \hat{G}(c) dc = \int_{\bar{c}}^{\hat{p}} \hat{G}(c) dc = \int_{\bar{c}}^{\hat{p}} G(c)(2 - G(c)) dc,
\]

which depends only on \( \beta \) through \( \hat{p} \) and so is nonincreasing in \( \beta \) and decreasing when \( \hat{p} < \bar{c} \), which occurs when \( \hat{\Gamma}^{\beta-1}(v) < \bar{c} \). This completes the proof of this component of the proposition.
Finally, we show that for a merger to monopoly by symmetric suppliers, given $\beta > 0$, there exists $\hat{v} \in (\zeta, \bar{v})$ such that for all $v \in (\zeta, \hat{v})$, the merger is not profitable. We establish the result for the case of $g(\zeta) > 0$. The extension to the case of $g(\zeta) = 0$, which is omitted, is tedious but follows in a straightforward manner by taking additional derivatives along the lines of the proof of Loertscher and Marx (2019b, Prop. 6).

The optimal take-it-or-leave-it offer pre-merger $p(v)$ satisfies $\Gamma^\beta(p(v)) = v$, and the optimal take-it-or-leave-it offer post-merger $p_m(v)$, satisfies $\Gamma^\beta_m(p_m(v)) = v$. For any $v \in (\zeta, \Gamma^\beta(\bar{v}))$, we have

$$p_m(v) < p(v),$$

while for $v = \zeta$ we have $p_m(v) = p(v) = \zeta$.

It is useful to begin with the properties of how the optimal take-it-or-leave-it offers vary with $v$, in particular in the neighborhood of $v \approx \zeta$. Let $\sigma(\zeta) = G(c)/g(c)$. Observe that $\sigma(\zeta) = 0$,

$$\sigma'(c) = 1 - \sigma(c) \frac{g'(c)}{g(c)} \big|_{c=\zeta} = 1,$$

and

$$\sigma''(c) = -\sigma'(c) \frac{g'(c)}{g(c)} - \sigma(c) \frac{g''(c) - (g'(c))^2}{g^2(c)} \big|_{c=\zeta} = -\frac{g'(c)}{g(c)}.$$

The optimal take-it-or-leave-it offer $p(v)$ satisfies

$$p(v) + \beta \sigma(p(v)) = v.$$

Differentiating we get

$$p'(v) = \left. \frac{1}{1 + \beta \sigma'(p(v))} \right|_{v=\zeta} = \frac{1}{1 + \beta}$$

and differentiating once more we get

$$p''(v) = -\frac{\beta \sigma''(p(v))}{(1 + \beta \sigma'(p(v)))^2} \big|_{v=\zeta} = -\frac{\beta \sigma''(\zeta)}{(1 + \beta)^2}.$$

Letting $\sigma_m(s) = \sigma(c) \frac{2 - G(c)}{2(1 - G(c))}$, we have

$$p'_m(v) = \left. \frac{1}{1 + \beta \sigma'_m(p(v))} \right|_{v=\zeta} = \left. p'(v) \right|_{v=\zeta}$$

and differentiating once more

$$p''_m(v) = -\frac{\beta \sigma''_m(p_m(v))}{(1 + \beta \sigma'_m(p_m(v)))^2} \big|_{v=\zeta} = -\frac{\beta \sigma''(\zeta)}{(1 + \beta)^2}.$$
Using
\[
\sigma'_m(c) = \sigma'(c) \frac{2 - G(c)}{2(1 - G(c))} + \sigma(c) \frac{2g(c)}{(2(1 - G(c)))^2}
\]
\[
\left|_{c=\xi} \right. = \sigma'(\xi) = 1.
\]
and
\[
\sigma''_m(c) = \sigma''(c) \frac{2 - G(c)}{2(1 - G(c))} + 2\sigma'(c) \frac{2g(c)}{(2(1 - G(c)))^2} + \sigma(c) \left[ \frac{2 - G(c)}{2(1 - G(c))} \right]'
\]
we get that, at \(c = \xi\),
\[
\sigma''_m(c)|_{c=\xi} = -\frac{g'(\xi)}{g(\xi)} + g(\xi) = \sigma''(\xi)|_{c=\xi} + g(\xi).
\]
Consequently, for any \(\beta > 0\), we have
\[
p''_m(\xi) = -\frac{\beta}{(1 + \beta)^2} [\sigma''(\xi) + g(\xi)] = p''(\xi) - \frac{\beta}{(1 + \beta)^2} g(\xi) < p''(\xi). \tag{20}
\]
Suppose for now that the buyer applies some take-it-or-leave-it offer \(p \leq \xi\). Each pre-merger supplier’s expected profit has the lower bound
\[
\pi(p) = (1 - G(p)) \int_{\xi}^{p} [p - c]dG(c).
\]
This is a lower bound because it only considers the possibility that suppliers trade at the take-it-or-leave-it offer when the competitor has a higher cost draw. (They may also trade at a price equal to the higher cost if that cost is less than \(p\).) Integrating by parts and multiplying by 2, the joint pre-merger profits are at least
\[
\Pi(p) = 2\pi(p) = 2 \int_{\xi}^{p} G(c)dc - 2G(p) \int_{\xi}^{p} G(c)dc.
\]
The derivative \(\Pi'(p)|_{p=\xi}\) is 0; the second derivative, evaluated at \(p = \xi\), is \(\Pi''(p)|_{p=\xi} = 2g(\xi)\).

Given a take-it-or-leave-it offer \(p\), the (exact) post-merger expected profit is
\[
\Pi_m(p) = 2 \int_{\xi}^{p} G(c)dc - \int_{\xi}^{p} G^2(c)dc,
\]
whose first and second derivatives at \(p = \xi\) are the same as those of \(\Pi(p)\).

Define \(f(v) \equiv \Pi(p(v))\) and \(f_m(v) \equiv \Pi_m(p_m(v))\). Then, evaluated at \(v = \xi\), the first time the derivatives of \(f\) and \(f_m\) are different from 0 is at the third derivative, whose
values are

\[ f'''(v)|_{v=\xi} = 2g(\xi)p''(\xi) \]

and

\[ f'''_m(v)|_{v=\xi} = 2g(\xi)p'_m(\xi). \]

By (20), we have

\[ f'''_m(v)|_{v=\xi} < f'''(v)|_{v=\xi}, \]

which proves the result. ■
References


Figures
Figure 1: Equilibrium trades with two suppliers, $v = 1$, and $G_i(c) = c^{\alpha_i}$. Panels a and b show the symmetric case with $\alpha_1 = \alpha_2 = 2$ and $\Gamma \equiv \Gamma_1 = \Gamma_2$. Panels c and d show the symmetric case with $\alpha_1 = 1$ and $\alpha_2 = 4$, implying that $G_1$ is first-order stochastically dominated by $G_2$. The diagonal dashed lines are the 45-degree lines.
Figure 2: Merged entity’s expected surplus as a function of $\beta$ with three suppliers drawing costs from $G_i(c) = c^{\alpha_i}$, where $\alpha_2 = 1$ and $\alpha_3 = 2$ and where the cost distribution for supplier 1 has $\alpha_1 = 0.5$ in panel (a) and $\alpha_1 = 2$ in panel (b). Assumes $v = 50$. 