Leniency, Profiling and Reverse Profiling in Multi-Product Markets:
Strategic Challenges for Competition Authorities

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I. INTRODUCTION

Antitrust leniency policies have played an important role in the detection of collusive conduct by firms. Basic leniency policies allow a firm to avoid having to pay fines if it co-operates with the competition authority and facilitates the prosecution of its co-conspirators. Although the use of leniency by competition authorities is widespread, it can be augmented in a variety of ways, including special treatment for recidivists, ringleaders, firms that have coerced others to participate, firms that report collusive conduct in relation to additional products while being prosecuted for collusion in relation to one product, and firms that are found to have colluded in relation to additional products after having denied such conduct during earlier investigations of collusion. The effects of these types of adjustments on the incentives for firms to apply for leniency can be complex, and competition authorities must therefore be cautious and strategic in their use of these tools.

In this chapter, we highlight the significance of profiling—and, more specifically, what we call reverse profiling—as a supplement to a leniency policy. Profiling and reverse profiling are policies directed at multi-product firms that have been investigated for collusion in relation to one product. Profiling means that the probability with which the firm’s other products are investigated is higher if the firm is found to have colluded in the first product than if it is not. By reverse profiling we mean the opposite policy, under which the probability with which the multi-product firm’s other products are investigated is lower if the firm is found to have colluded in the first product than if it is not. Profiling and reverse profiling
are particularly relevant in light of evidence that multi-product firms sometimes collude in multiple product offerings.²

If a firm is found to have colluded in relation to other products, it is to be expected of course that fines may be increased for recidivism. Yet despite evidence of recidivism,³ competition authorities such as the United States Department of Justice (DOJ) and the European Commission do not have clear policies on how they will approach the investigation of additional products of firms with a product that has been investigated for an anti-trust violation. There is reason to believe that the DOJ at least pursues a practice of profiling firms. Scott Hammond, who served for eight years as the head of criminal enforcement at the DOJ’s Antitrust Division, has said that ‘The [Antitrust] Division [of the DOJ] will target its proactive efforts in industries that are adjacent to markets where cartel activity is suspected or that involve some overlap of players from other cartels.’⁴

We examine the effects of profiling and reverse profiling on the detection of collusion by multi-product firms, which may be colluding in more than one of their product offerings. In particular, we consider an economic model of leniency in an environment with multi-product colluders, and we analyse the incentives created by profiling and by reverse profiling. To do so, we adapt a model of leniency that we developed with Robert C Marshall in other work. The mathematical details can be found in the appendices to this chapter.

When it comes to profiling, there is ultimately a trade-off. On the one hand, profiling can improve cartel detection if a firm that is colluding in relation to one product is more likely to be colluding in relation to its other products. On the other hand, if a firm knows that applying for leniency in relation to one product (and thereby facilitating the successful prosecution of the cartel in that product) will cause the competition authority to aggressively investigate the firm’s other products, that will reduce incentives for the firm to apply for leniency in the first place. Profiling can therefore reduce leniency applications and cartel detection.

Because competition authorities regularly identify multi-product firms as having engaged in collusion, any policy of profiling or reverse profiling will be triggered on a regular basis. Any such policy requires careful thought because the competition authority’s stance on profiling affects the incentives for firms to apply for leniency in the first place, and thus the overall effectiveness of the leniency policy itself. We show that, counterintuitively, a competition authority may want to use reverse profiling instead of profiling. Specifically, a competition authority may want to commit to a policy of decreased investigation of a firm’s other products if the firm is found to have colluded in relation to one product and, conversely, increased investigation if the firm is not found to have colluded. Our intuition is that when a firm knows that being found to have colluded in relation to the first product will not put its profits in other products at greater risk, it has a greater incentive to apply for leniency in relation to the first product. Thus, reverse profiling increases detection in relation to the

² ‘Recidivist cartel activity has become increasingly common, especially among companies involved in the multitude of electronics components cases prosecuted by the Antitrust Division’: J Sisco, ‘Executives Could Suffer When DoJ Targets Repeat Cartel Offenders—ABA Panel’ Policy and Regulatory Report (13 May 2014) 1.
³ See, eg, E Combe and C Monnier ‘Cartels in Europe: Main Features’ [2010] (2) Concurrences 23.
first product. We show that this effect can outweigh any reductions in detection in relation to the second product.

In Section II, we review evidence on multi-product collusion. In Section III, we briefly discuss related literature. In Section IV, we present a basic leniency model, which is based on the model we developed elsewhere together with Robert C Marshall. In Section V, we discuss the model’s implications for profiling. Section VI concludes.

II. BACKGROUND

As we have described in more detail elsewhere, European Commission decisions in cases in which a cartel was detected typically record the leniency discounts received by cartel participants, and in many cases, one firm receives a 100 per cent reduction in its fine as a result of the leniency policy. Indeed, between 2001 and 2012, a firm received a 100 per cent reduction in its fine as a result of the leniency policy in 55 (54 per cent) of the 101 cartels involved in these cases. These firms (and the cartels in which they participated) spanned a wide variety of products and services.

Based on this same set of European Commission decisions, for this chapter we have identified the firms involved in the cartels, how many cartels each firm was involved in, and whether it ever received a 100 per cent reduction in its fine based on leniency. We found that 322 firms were involved in only one cartel, 74 firms were involved in two or more cartels and eight of those firms were involved in five or more cartels. Of the firms involved in only one cartel, 7 per cent received leniency. Of the firms involved in two or more cartels, 28 per cent received leniency in at least one cartel. We have graphed this data in Figure 1.

Based on the number of firms engaged in multiple cartels, it is reasonable to infer that competition authorities must regularly face choices about whether and how to allocate resources to investigations of the other products of firms that have been found to have engaged in collusion. As the firms involved in multiple cartels tend to be larger, one might expect that larger firms are the key leniency applicants, and that the policy choice about the allocation of investigative resources ought to be driven more by the size of firms than by their history of collusion. The data we present in Figure 2, however, suggests that size is not a fundamental factor in determining the identity of leniency applicants.

In order to obtain a measure of the relative size of the firms in a cartel, we have used the ‘basic amount’ of the fine that is provided in many European Commission decisions. When basic amounts are provided, they are provided for all the cartel participants, including any firm that ultimately receives a 100 per cent discount on that fine based on leniency and

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6 See ibid.

7 Some European Commission decisions apply to more than one cartel. For example, the decision in Vitamins deals with cartels in multiple vitamin product offerings, with a separate application of the leniency policy for each product. See Vitamins (Case COMP/E-1/37512) Commission Decision 2003/2/EC [2003] OJ L6/1.

8 See Marx, Mezzetti and Marshall (n 5) Table 1.
so pays nothing. The basic amount reflects, among other things, the firm’s relevant sales during the infringement period, which is not to exceed 10 per cent of the firm’s worldwide turnover in the preceding year. Figure 2 compares the basic amount of the fine for firms receiving a 100 per cent leniency discount with the minimum, maximum and average fines for the cartel. As shown in Figure 2, any firm (whether small, medium or large) can potentially receive leniency. In a little over half of the cases in which a firm received a 100 per cent leniency discount, the firm receiving that discount was above the average size for firms in that cartel, as measured by the basic fine amount.

9 See LCD—Liquid Crystal Displays (Case COMP/39,309) Commission Decision C(2010) 8761 final, para 378. According to the Guidelines on fines, the basic amount of the fine consists, first, of an amount of between 0% and 30% of a company’s relevant sales during the infringement period, according to the degree of gravity of the infringement. Second, an additional amount of between 15% and 25% of the value of a company’s relevant sales can be added, irrespective of duration, in order to deter horizontal price fixing agreements (the so-called ‘entry fee’). The resulting basic amount can then be increased or reduced for each company if either aggravating or mitigating circumstances are retained. Should the ensuing amount of the fine exceed 10% of the world-wide turnover of an undertaking concerned in the preceding business year, the fine must be reduced to that percentage, pursuant to Article 23 of Regulation (EC) No 1/2003. That amount can still be reduced in accordance with the Leniency Notice, where applicable.
Antitrust leniency has been extensively studied by theoretical economists. Most of the papers in the literature have followed the repeated games approach and focused on the problem of self-enforcement of a cartel structure. A feature of these models is that


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collusive behaviour needs neither communication nor inter-firm transactions to be sustained.\footnote{12} Harrington has taken a different approach to the literature, focusing on repeated games by considering a signaling game involving firms in a cartel that has ended,\footnote{13} which makes deviations from the collusive agreement no longer an issue. A common theme of all these papers is that a leniency policy makes it harder for firms to support collusion, although they recognise the potential reduction in deterrence due to the reduced fines firms expect to pay under a leniency policy. Among this literature, we are not the first to raise the concern that cartels may exploit a leniency policy. Others have studied how a cartel may do so to obtain reduced fines.

In addition to the theoretical literature, there is an economic literature devoted to the empirical analysis of the impact of leniency\footnote{14} and to economic experiments related to leniency.\footnote{15} A leniency policy generates a co-ordination game among members of a cartel. If a cartel member applies for leniency, it is then in the best interests of all other members of the cartel also to apply; indeed, to do so first. Conversely, if no other firm applies, and the probability of being convicted without a leniency application is sufficiently low, it is best not to apply for leniency, so as to avoid penalties altogether. Co-ordination games of this kind typically have multiple equilibria. As we have done elsewhere,\footnote{16} we use the economic theory of global games to model the co-ordination game among firms that is generated by a leniency policy. The theory of global games selects a unique equilibrium based on the game theoretic notion of ‘risk dominance’, which incorporates both the level and the riskiness of pay-offs.\footnote{17}

In a paper co-authored with Robert C Marshall, we showed that profiling may be useful to counter some undesirable incentives created by a leniency policy (such as ‘Penalty Plus’) that limits future leniency applications by firms colluding in relation to multiple products if they choose not to apply for leniency at the time they are found to be colluding in relation


\footnote{16} See Marx, Mezzetti and Marshall (n 5); LM Marx and C Mezzetti, ‘Effects of Antitrust Leniency on Concealment Effort by Colluding Firms’ (2014) 2 Journal of Antitrust Enforcement (forthcoming).

to any one of their products.\textsuperscript{18} We showed that Penalty Plus increases detection in relation to the first product but decreases it in relation to the second; firms have a reduced incentive to apply for leniency in relation to the second product in the Penalty Plus environment because there is uncertainty about whether that second product offering would even have come under investigation in the absence of the leniency application. As we showed, profiling can serve to balance this effect by decreasing leniency applications in relation to the first product but increasing detection in relation to the second product.

IV. SUMMARY OF THE MODEL

Appendix A contains a formal statement of the model used in this chapter. In this section, we provide a description of only the key components of the model.

We consider two multi-product firms engaged in collusion. These firms, denoted firm $A$ and firm $B$, are the only producers of product 1 and are engaged in collusion in relation to that product. We also assume that firm $A$ produces product 2 and is colluding in relation to that product with firm $C_A$, and that firm $B$ produces product 2 and is colluding in relation to that product with firm $C_B$. Because firms $A$ and $B$ interact only in relation to product 1, they cannot impose externalities on their product 1 co-conspirator through some other product. Our results extend to other environments, but it is useful to consider this simple environment in order to illustrate the key results.

We assume that the firms’ pay-offs differ by product and by whether they are found to be colluding, with reduced penalties for any firm that applies for and receives leniency. Firms $A$ and $B$ each have a collusive pay-off of $\pi$ in relation to product 1, firms $A$ and $C_A$ each have a pay-off of $\pi$ in relation to product 2, and firms $B$ and $C_B$ each have a pay-off of $\pi$ in relation to product 2. (The assumption that firms have the same pay-offs in different product markets is not important; its purpose is merely to reduce notation.)

We assume that if a firm receives leniency in relation to a product, its pay-off in relation to that product is zero. Thus, we implicitly assume that the firm loses the collusive gain of $\pi$ but is not fined. A firm that is found to be colluding and does not receive leniency has a pay-off of $-f\pi$, where $f > 0$, implying that the firm not only loses the collusive gain, but also pays a fine. We also assume that $f < 2$, so that the fine cannot exceed twice the collusive gain.\textsuperscript{19}

We consider the case in which a competition authority has initiated an investigation of anti-competitive conduct by firms $A$ and $B$ in relation to product 1. At the time that the investigation begins, but before the firms have chosen whether to apply for leniency, the competition authority commits to the probability with which it will investigate the firms’ other products, conditional on the outcome in relation to product 1. In particular, the competition authority commits to investigate products 2 and 2 with probability $h_N$ if the firms are not found to be colluding in relation to product 1 and with probability $h_p$ if they are found to be colluding. If $h_p > h_N$, we say that the competition authority engages in profiling, and if $h_p < h_N$, we say that the competition authority engages in reverse profiling.

\textsuperscript{18} See Marx, Mezzetti and Marshall (n 5).

\textsuperscript{19} This assumption corresponds to Assumption A1 in Marx, Mezzetti and Marshall (n 5).
Profiling can be viewed as a policy by which firms that are found to be colluding in relation to one of their products are viewed with greater suspicion, with the result that their other products are targeted for investigation with a higher probability. Reverse profiling can be viewed as a policy by which firms that are investigated for collusion but not ultimately found to have colluded (perhaps because no firm ‘confessed’ by applying for leniency) are viewed with suspicion, with the result that their other products are targeted for investigation with a higher probability.

The timeline in relation to product 1 is as follows:

1. **Feasibility of leniency and the threat of detection:** having learned that they are under investigation in relation to product 1, firms \( A \) and \( B \) conduct internal investigations that reveal whether or not there is sufficient evidence to support a leniency application in relation to product 1. To simplify notation, we assume that, with probability \( \frac{1}{2} \), sufficient evidence is revealed and that leniency is therefore a feasible option for firm \( A \), and similarly for firm \( B \). In addition, we assume that each firm obtains information, with probability \( \tau \), with which the competition authority will be able to detect the cartel in product 1 in the event that no participant applies for leniency.

2. **Leniency:** firms that have uncovered evidence sufficient to support a leniency application simultaneously and independently make a choice about whether to apply for leniency. If both firms apply for leniency, we assume that each firm has a 50–50 chance of being selected as the leniency recipient.\(^{20}\)

3. **Detection:** if at least one firm applies for leniency, we assume that the cartel is detected. If no firm applies for leniency, we assume that with probability \( \tau \) the cartel is detected, and with the complementary probability it is not.

Once the outcome is determined for product 1, the competition authority turns its attention to the firms’ other products. If the alleged cartel in product 1 was not detected, we assume that with probability \( h_N \) the competition authority launches an investigation of the second products \( 2A \) and \( 2B \) and the game above is played with respect to those products. And if the alleged cartel in product 1 was detected, we assume that with probability \( h_P \) the competition authority investigates products \( 2A \) and \( 2B \) and the game above is played with respect to those products.

V. IMPLICATIONS FOR PROFILING

In the model, firms must consider the effects of applying for leniency. If a firm applies for leniency in relation to product 1, it will be detected and lose its collusive pay-off in relation to that product, but it will not pay a fine unless another firm also applies and is designated as the leniency applicant. If a firm does not apply for leniency, the cartel might or might not be detected in relation to product 1. If the cartel is not detected, the cartel firms will continue to receive the collusive pay-off in relation to product 1, but if it is detected, the firms will lose the collusive pay-off and will also have to pay fines.

\(^{20}\) An equivalent, alternative, interpretation is that each firm pays half the fine.
In addition to considerations relating directly to product 1, firms considering applying for leniency in relation to that product must also consider the effect on product 2. If there is profiling, ie \( h_p > h_N \), applying for leniency in relation to product 1 will increase the probability that the firm’s collusive conduct in relation to product 2 will be investigated and detected. This reduces a firm’s incentive to apply for leniency in relation to product 1. Conversely, if there is reverse profiling, ie \( h_p < h_N \), applying for leniency in relation to product 1 will reduce the probability of investigation and detection in relation to product 2 and so increase a firm’s incentive to apply for leniency in relation to product 1.

In the model, the decision to apply for leniency hinges on the firm’s estimate of the probability that it will be detected when no firm applies for leniency and on the competition authority’s policy regarding profiling. A firm will apply for leniency if it believes that it is sufficiently likely that the competition authority will be able to detect collusion in the absence of a leniency applicant and if profiling is not too severe, but not otherwise.\(^{21}\)

We now consider the effects on detection of changes in the competition authority’s policy on profiling, taking as the starting point the case in which there is neither profiling nor reverse profiling, ie \( h_p = h_N = \bar{h} \) where \( \bar{h} < 1 \) is the base level of investigation. We assume that the competition authority can make marginal changes to the probability of an investigation of a second product either by profiling—that is, increasing the probability of investigating a cartel involving a firm that has previously been found to have colluded in relation to a different product—or by reverse profiling—that is, increasing the probability of investigating a cartel involving a firm that has previously been investigated but ultimately not found to have been colluding in relation to a different product. Changes in the probability of a future investigation of the second product affect the probability that firms apply for leniency in relation to the first product. These marginal changes can be substantial, rather than infinitesimal, but are assumed to be such that the probability that a firm applies for leniency in relation to the first product remains interior; that is, it is neither one nor zero.

In our model, the goal of the competition authority in making these changes from the base level \( \bar{h} \)—ie, choosing its profiling policy—is to maximise the sum of the probabilities of detecting collusion in relation to product 1 and the two product 2s. The competition authority is subject to a ‘budget constraint’ that the total probability of a product 2 investigation is bounded above by a given total investigation level \( \bar{h} \), but it can move the probabilities \( h_N \) and \( h_p \) either by profiling or by reverse profiling up to a given maximum amount \( \bar{h} + \Delta \). We assume \( \bar{h} + \Delta < 1 \) and that for all feasible levels of \( h_p \) and \( h_N \), the threshold level for the probability of detection without a leniency applicant remains strictly between zero and one, implying that the equilibrium of the model is well specified.\(^{22}\)

Thus, the competition authority chooses \( h_N \) and \( h_p \) to maximise the sum of the probabilities of detection in relation to product 1, firm A’s product 2, and firm B’s product 2, subject to the probability of a product 2 investigation being less than or equal to \( \bar{h} \) and subject to \( h_N \) and \( h_p \) each being less than or equal to \( \bar{h} + \Delta \). As stated in Proposition 1, reverse profiling is optimal in this environment.

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21 This follows from Proposition A1 in Appendix A.

22 Formally, and using the notation of Appendix A, we assume that for all feasible levels of \( h_p \) and \( h_N \), \( \tau_1 \in (0, 1) \).
**Proposition 1** The optimal solution of the competition authority involves reverse profiling.

The proof of this proposition can be found in Appendix B.

Although it may initially seem counterintuitive that it would be optimal to investigate the firms’ second products with higher probability if the firms are not found to be colluding in relation to product 1, one must consider the effects of profiling and reverse profiling on the firms’ incentives to apply for leniency in relation to the first product. If a finding of collusion in relation to product 1 reduces the probability of a subsequent investigation, firms will have a greater incentive to apply for leniency in relation to product 1. Applying for leniency in relation to product 1 guarantees the detection of the cartel in that product, but doing so also garners the firms the benefit of a reduced probability of investigation, and potentially detection, in relation to product 2.

To understand this result, note that in our model the detection of the product 1 cartel is not informative about the probability of collusion and detection in relation to product 2. A key effect of profiling and reverse profiling is, rather, its influence on the probability of detecting collusion in relation to product 1. The way to increase detection in relation to product 1 is through reverse profiling, which gives firms an added incentive to apply for leniency in relation to that product. If, on the contrary, detection of the product 1 cartel provides an informative signal to the competition authority that cartel firms are more likely to have colluded in relation to other products, it may be optimal to profile firms, increasing the monitoring and likelihood of investigation of their other products.

Whether profiling or reverse profiling is optimal therefore rests partially on whether there is correlated collusion in the various products of multi-product firms. If there is, it seems reasonable to increase investigation of a cartel participant’s other products after it has been found to have colluded in relation to one product. If collusion is not correlated across products, reverse profiling could be optimal.

**VI. CONCLUSION**

Our results suggest that a competition authority’s use of profiling or reverse profiling can affect the incentives provided by leniency policies and the effectiveness of those policies in facilitating the detection of cartels. Although the value of reverse profiling is potentially counterintuitive, we provide an economic model of leniency that shows that reverse profiling, rather than profiling, may be valuable in some settings. The value of reverse profiling is counterintuitive because it seems natural that competition authorities should more aggressively monitor and investigate the other products offered by multi-product firms when those firms have been found to have colluded, rather than when they have not been found to have colluded.

Reverse profiling incentivises leniency applications in relation to the first product because it provides an additional benefit to colluding firms that apply for leniency, namely a reduced probability that their other products will come under investigation. This can be valuable in inducing firms to apply for leniency in relation to an initial product. In contrast, a policy of profiling provides a disincentive for firms to apply for leniency in relation to the first product because they will recognise that doing so increases the probability that their other products will come under investigation.
This chapter highlights the fact that the trade-offs involved in providing incentives for firms to apply for leniency in relation to one product versus detecting potential collusion in relation to a second product can go in the direction of reverse profiling. However, it should be recognised that a policy of reverse profiling has the drawback of increasing the probability that collusion in the second product will go unpunished. This could invite gaming by firms that may decide to apply for leniency in relation to a less profitable product in order to reduce the probability of an investigation of the more profitable product by the competition authority. More generally, when detection of collusion in relation to one product is a strong indicator that the firms involved may have colluded in relation to additional products, the benefits of profiling in terms of enhanced detection of collusion will be greater and profiling, rather than reverse profiling, is likely to be desirable.

Profiling can be valuable in other modelling environments. For example, under the United States Penalty Plus policy, firms already found to have colluded in relation to one product that do not report collusion in relation to additional products face increased penalties should they later be found to have colluded in relation to those products. Together with Robert C Marshall, we have pointed out that Penalty Plus decreases detection in those additional products, and that profiling may be an effective counter-measure.23

In the model that we consider in this chapter, whether or not a firm is found to have colluded in relation to one product is not informative as to whether the firm is colluding in relation to a second product.24 Thus, in our model, the dominant effect of profiling or reverse profiling is its effect on firms’ incentives to apply for leniency in relation to the first product.

The general lesson of this research is that when considering enhancements to leniency policies, competition authorities should recognise that firms will respond in a strategic way that may affect the optimality of various policies or combinations of policies.

APPENDIX A: FORMAL STATEMENT OF THE MODEL

The model is derived from the two-product model that we developed in a paper co-authored with Robert C Marshall.25 The text below is largely drawn from that paper.

Assuming an investigation has begun in relation to product 1, the timeline is as follows:

1. Each firm \( i \in \{A, B\} \) conducts an internal investigation, which uncovers evidence sufficient to support a leniency application with probability \( \frac{1}{2} \), in which case firm \( i \) observes a conditionally independent random variable \( \theta_i \), uniformly distributed in the interval \([\tau - \varepsilon, \tau + \varepsilon]\) centred on the realised value of the random variable \( \tau \), defined below. We will think of \( \varepsilon > 0 \) as ‘small’, so that \( \tau \) is ‘almost’ perfectly observed by each firm, and focus on the limit as \( \varepsilon \downarrow 0 \).26

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23 See Marx, Mezzetti and Marshall (n 5).
24 The conditions under which collusion is facilitated when firms operate in multiple markets have been studied by BD Bernheim and MD Whinston, ‘Multimarket Contact and Collusive Behavior’ (1990) 21 RAND Journal of Economics 1. See also Choi and Gerlach (n 11).
25 See Marx, Mezzetti and Marshall (n 5).
26 As described in Carlsson and van Damme, ‘Global Games and Equilibrium Selection’ (n 17), iterated dominance selects the risk-dominant equilibrium of the game provided that \( \varepsilon \) is sufficiently small.
2. If the internal investigation uncovered evidence sufficient to support a leniency application, the firm decides whether to apply for leniency. If only one firm applies for leniency, it receives leniency. If both firms apply, one (and only one) is randomly designated as receiving leniency.

3. If there is a leniency application, the cartel is detected. If not, the cartel is detected with probability $\tau$. For the firms, $\tau$ is a random variable uniformly distributed on $[0, 1]$ with expected value $E[\tau] = \frac{1}{2}$.

4. If the cartel in relation to product $1$ is detected, with probability $h_p$ the same game is played for product $2$ and with probability $1 - h_p$ firms are not investigated in relation to product $2$. If the cartel in relation to product $1$ is not detected, with probability $h_N$ the same game is played for product $2$ and with probability $1 - h_N$ firms are not investigated in relation to product $2$.

To describe the equilibrium, it is useful to begin with product $2i$.

Assuming there is an investigation in relation to product $2i$, if a firm $j \in \{i, C_i\}$ uncovers evidence sufficient to support a leniency application, it faces a strategic game. The firm (the row player) must decide whether to apply for leniency ($L$) or not ($N$), and its pay-off depends on whether the other firm (the column player) applies for leniency in the case where it has also uncovered evidence sufficient to support an application.

In relation to product $2i$, the pay-off of the row player $j$ is given by the entries in the table below.

<table>
<thead>
<tr>
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<th>$L$</th>
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<tr>
<td>$L$</td>
<td>$-\frac{1}{4} f \pi$</td>
<td>$0$</td>
</tr>
<tr>
<td>$N$</td>
<td>$\frac{1}{2} (1 - \theta_j)(1 + f) \pi - f \pi$</td>
<td>$(1 - \theta_j)(1 + f) \pi - f \pi$</td>
</tr>
</tbody>
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If the row player applies for leniency, it receives leniency and a pay-off of $0$ if the other firm does not apply after uncovering evidence (upper right cell). If the other firm does apply after uncovering evidence (upper left cell), the row player is fined when the other firm receives leniency, which occurs with probability $\frac{1}{4}$. When the row player $j$ does not apply for leniency, it is not detected and receives a pay-off of $\pi(1 + f)$ above the baseline $-f \pi$ with probability $1 - \theta_j$ if the other firm does not apply after uncovering evidence (lower right cell) and with probability $\frac{1}{2} (1 - \theta_j)$ if the other firm applies for leniency after uncovering evidence (lower left cell).

As we and Robert C Marshall showed, when $\theta_j$ is sufficiently large, applying for leniency is a strictly dominant strategy and $(L, L)$ is the unique Nash equilibrium. As $\theta_j$ decreases, there is a range of values for $\theta_j$ such that there are two pure strategy Nash equilibria $(L, L)$ and $(N, N)$ and equilibrium $(L, L)$ is risk dominant. As $\theta_j$ decreases further, there

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27 Because the basic leniency game is symmetric, $(L, L)$ is risk dominant if $L$ is the best reply to the opponent’s strategy of randomising with equal probability between $L$ and $N$. 
is a range of values for $\theta_j$ such that there continue to be two pure strategy Nash equilibria $(L, L)$ and $(N, N)$ but $(N, N)$ is risk dominant. Finally, when $\theta_j$ is sufficiently small (and making the assumption that $f < 2$), no leniency is a dominant strategy and $(N, N)$ is the unique Nash equilibrium.

Because we are interested in the case of a small error in the observation by firm $j$ of the probability of detection, $\theta_j$ is approximately equal to $\tau$. Define the cut-off value for the probability of detection below which $(N, N)$ is risk dominant and above which $(L, L)$ is risk dominant in the leniency game for product $2i$ as:

$$
\tau_2' = \frac{6 - f}{6 + 6f} > 0.
$$

We are now in a position to state Proposition A1, which is analogous to Proposition 1 in our paper with Robert C Marshall and which exploits the fact that $\tau$ is a random variable that is imperfectly observed by the firms.

**Proposition A1** If $f < 2$ then for $\epsilon$ sufficiently small, the product $2i$ subgame taking place after an investigation by the competition authority has started has a unique Bayesian equilibrium that survives the iterated elimination of strictly dominated strategies. In such an equilibrium, when firm $j$ uncovers evidence, it applies for leniency if it receives a signal $\theta_j > \tau_2'$, and does not apply if it receives a signal $\theta_j < \tau_2'$.

As Proposition A1 shows, firms for which leniency is feasible may or may not choose to apply for leniency depending on the signals they receive. Henceforth, when computing payoffs and probabilities of detection, we take the limit as $\epsilon \downarrow 0$, with the implication that the firms co-ordinate; either both firms apply for leniency when that is feasible or both do not apply.

Conditional on an investigation of product 2, the probability that the cartel in relation to product 2 will be detected is:

$$
\Psi_2 = 1 - \frac{1}{4} \left(1 - \tau^\epsilon\right) - \frac{3}{4} \int_0^{\tau_2'} \left(1 - \tau\right) d\tau = \frac{1}{2} + \frac{3}{8} \left(1 - \tau_2'\right)^2
$$

To understand (3), note that the cartel is not detected if neither firm finds sufficient evidence to apply for leniency and the competition authority is unable to detect collusion, which occurs with probability $\frac{1}{4} \left(1 - \tau^\epsilon\right)$, or if at least one firm finds sufficient evidence (probability $\frac{3}{4}$) but $\tau$ is less than $\tau_2'$ and the authority is unable to detect collusion.

The expected pay-off of a cartel firm that is investigated with probability $h$ is $V_2(h)\pi$, where:

$$
V_2(h) = 1 - h \Psi_2 (1 + f) + h^3 \frac{3}{8} f \left(1 - \tau_2'\right),
$$

where $\Psi_2$ is defined as above.
and so the change in the pay-off to a firm from avoiding being profiled and being investigated in relation to product 2 with probability $h_N$ rather than $h_P$ is $\Delta V_2 \pi$, where:

$$\Delta V_2 \equiv (h_P - h_N) \Psi_2 (1 + f) - (h_P - h_N) \frac{3}{8} f \left( 1 - \tau_i^* \right)$$

$$= (h_P - h_N) \left( \frac{1 + f}{2} + \frac{7f^2}{96(1 + f)} \right).$$

Incorporating expectations about product 2 into the leniency game for product 1, the pay-off of the row player $i$ in the product 1 game is given by adding the baseline pay-off $V_2(h_P) \pi$ to the entries in the table below:

<table>
<thead>
<tr>
<th></th>
<th>$L$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>$-\frac{1}{4} f \pi$</td>
<td>0</td>
</tr>
<tr>
<td>$N$</td>
<td>$\frac{1 - \theta_i}{2} \left[ (1 + f) \pi + \Delta V_2 \pi \right] - f \pi$</td>
<td>$(1 - \theta_i) \left[ (1 + f) \pi + \Delta V_2 \pi \right] - f \pi$</td>
</tr>
</tbody>
</table>

Then, relying on Proposition 3 from our paper with Robert C Marshall, we have a result analogous to Proposition A1 for product 1, but with the signal threshold of

$$\tau_i^* \equiv 1 - \frac{7f}{6(1 + f + \Delta V_2)}.$$

This allows us to define the probability of detection in relation to product 1.

$$\Psi_i \equiv 1 - \frac{1}{4} (1 - \tau^e) - \frac{3}{4} \int \left( 1 - \tau \right) d\tau = \frac{1}{2} + \frac{3}{8} \left( 1 - \tau_i^* \right)^2$$

$$= \frac{1}{2} + \frac{49f^2}{96(1 + f + \Delta V_2)^2}$$

$$= \frac{1}{2} + \frac{49f^2}{96 \left( 1 + f + (h_P - h_N) \left( \frac{1 + f}{2} + \frac{7f^2}{96(1 + f)} \right) \right)^2}.$$
Proof of Proposition 1

As described in the text, the competition authority chooses $h_N$ and $h_P$ to solve

$$\max_{h_N, h_P \in [0, 1]} \Psi_1 + 2\Psi_2 (\Psi_1 (h_P - h_N) + h_N)$$

s.t. $\Psi_1 (h_P - h_N) + h_N - \bar{h} \leq 0$, $h_P \leq \bar{h} + \Delta$, and $h_N \leq \bar{h} + \Delta$,

where we ignore the non-negativity constraints on $h_N$ and $h_P$, which one can show do not bind. The first constraint is the total probability constraint. It says that the total probability of investigating the second product is bounded above by $\bar{h}$. We can write the Lagrangian as:

$$L = \Psi_1 + 2\Psi_2 (\Psi_1 (h_P - h_N) + h_N) - \lambda (\Psi_1 (h_P - h_N) + h_N - \bar{h}) - \mu_p (h_P - \bar{h} - \Delta) - \mu_N (h_N - \bar{h} - \Delta).$$

Note that $\Psi_1$ is a decreasing function of $(h_P - h_N)$, while $\Psi_2$ is independent of $h_p$ and $h_N$. It is convenient to define

$$\Psi'_1 = \frac{\partial \Psi_1}{\partial h_P} = -\frac{\partial \Psi_1}{\partial h_N}.$$

The Kuhn-Tucker conditions of the competition authority maximisation policy are:

$$\Psi'_1 \left[ 1 + (2\Psi_2 - \lambda)(h_P - h_N) \right] + (2\Psi_2 - \lambda)\Psi'_1 - \mu_p = 0$$

$$-\Psi'_1 \left[ 1 + (2\Psi_2 - \lambda)(h_P - h_N) \right] + (2\Psi_2 - \lambda)(1 - \Psi_1) - \mu_N = 0$$

$$\lambda \left( \Psi_1 (h_P - h_N) + h_N - \bar{h} \right) = 0 \quad \text{and} \quad \left( \Psi_1 (h_P - h_N) + h_N - \bar{h} \right) \leq 0$$

$$\mu_p (h_P - \bar{h} - \Delta) = 0 \quad \text{and} \quad h_P \leq \bar{h} + \Delta$$

$$\mu_N (h_N - \bar{h} - \Delta) = 0 \quad \text{and} \quad h_N \leq \bar{h} + \Delta$$

$$\lambda \geq 0, \quad \mu_N \geq 0, \quad \mu_p \geq 0$$

Observe that it cannot be $h_P = h_N = \bar{h} + \Delta$, otherwise the total probability constraint would be violated. This implies that at least one of $\mu_p$ and $\mu_N$ must be zero. In turn, this implies that the total probability constraint must bind and that $\lambda > 0$, otherwise the competition authority would profit from raising the $h_i$ corresponding to $\mu_i = 0$. 
Adding the first two conditions yields:
\[ 2\Psi'_2 - \lambda - \mu_p - \mu_N = 0. \]  
(8)

Using (8), we can show that it cannot be that \( \mu_p = \mu_N = 0 \). Suppose, to the contrary, that the two equalities hold. Then, by (8), \( \lambda = 2\Psi'_2 \) and the two first order conditions require \( \Psi'_1 = 0 \), which is a contradiction because \( \Psi'_1 < 0 \). It follows that there are only two possible cases:
\[ \mu_p > \mu_N = 0 \text{ or } \mu_N > \mu_p = 0. \]

We now prove that it cannot be that \( \mu_p > \mu_N = 0 \). Suppose, to the contrary, that \( \mu_p > \mu_N = 0 \). This implies that \( h_p = \tilde{h} + \Delta > h_N \). It also implies that \( 2\Psi'_2 - \lambda > 0 \), \( (2\Psi'_2 - \lambda)(h_p - h_N) > 0 \) and \( (2\Psi'_2 - \lambda)(1 - \Psi'_1) > 0 \), where the latter follows from \( \Psi'_1 < 1 \). As a result, it must be that
\[ -\Psi'_1 \left[ 1 + (2\Psi'_2 - \lambda)(h_p - h_N) \right] + (2\Psi'_2 - \lambda)(1 - \Psi'_1) - \mu_N > 0, \]
which violates the second Kuhn-Tucker condition, which is a contradiction.

It follows that in equilibrium it must be:
\[ h_N = \tilde{h} + \Delta, \]
\[ h_p = \tilde{h} - \frac{1 - \Psi'_1}{\Psi'_1} \Delta, \]
\[ \lambda = 2\Psi'_2 + \frac{\Psi'_1}{\Psi'_1 - \frac{\Delta}{\Psi'_1} \Psi'_1}, \]
\[ \mu_p = 0, \]
\[ \mu_N = -\frac{\Psi'_1}{\Psi'_1 - \frac{\Delta}{\Psi'_1} \Psi'_1}. \]

Thus, \( h_N > h_p \) and reverse profiling is optimal. QED.