Cournot Competition with a Common Input Supplier\textsuperscript{*}

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July 10, 2016

Abstract

In a bilateral-contracting game in which competing Cournot firms purchase inputs from a common input supplier, we endogenize the cost functions of the competing firms by allowing them to make offers to the common supplier. Equilibria in which all firms trade may not exist and are sensitive to whether the competing firms observe which of their rivals’ contracts have been accepted. Relatively more profitable firms may be excluded so that the ‘wrong firms’ trade. Our findings have implications for the evaluation of exclusionary conduct, buyer power both generally and in mergers, and export subsidies in international trade.

JEL Classification Codes: D43, D45, L14, L42
Keywords: Cournot equilibrium, exclusion, buyer power, trade policy

\textsuperscript{*}We thank Zhiqi Chen, Howard Marvel, Yaron Yehezkel, and seminar participants at Carleton University, Duke University, University of British Columbia, and Vanderbilt University for helpful comments.
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1 Introduction

Augustin Cournot in 1838 developed a model of strategic interaction among players whose actions exert externalities on one another. In the years since, Cournot’s model, in which competing firms simultaneously choose quantities to maximize their profits given the quantity choices of their rivals, has become a workhorse model of oligopoly. It has been used in a vast array of applications,¹ and it has been the subject of a large literature on its foundations.²

The focus of this literature has mostly been on the market at the level of the competing firms, taking as given the input costs of those firms. Surprisingly, there has been little discussion of where these costs actually come from, or how the firms obtain their inputs, leaving one to consider in each instance whether the firms are implicitly assumed to be producing the inputs themselves, purchasing the inputs on the world market, or relying on third-party suppliers. This raises concerns about the robustness of the insights obtained.

The concerns arise because, unlike when firms purchase inputs on the world market or produce the inputs themselves, when firms purchase from third-party suppliers, strategic considerations can come into play and firms may not be able to obtain their inputs at cost even when they have all the bargaining power. In this paper, we focus on the strategic considerations that arise when competing Cournot firms purchase their inputs from a common input supplier based on contracts, with the competing firms having the bargaining power. The firms’ contract offers are made simultaneously and consist of menus of price-quantity pairs.

We show that when one endogenizes the cost functions of Cournot competitors through the addition of a contracting stage with a common upstream supplier, the outcomes in the literature obtain only under restrictions on the cost function of the upstream supplier and are sensitive to whether the downstream firms observe which of their rivals have supply contracts with the upstream firm. In particular, for there to be the possibility that all downstream firms trade, the supplier’s costs must be linear over the relevant range (or additively separable) and the downstream firms must not observe which of their rivals have contracts.

When these conditions are not met, equilibrium outcomes necessarily involve the exclusion of one or more downstream firms. In addition, conditional on the number of firms that trade, we show that equilibrium outcomes may involve the ‘wrong firms’ competing in the

¹Applications include export subsidies (Brander and Spencer, 1985), dumping practices (Brander and Krugman, 1983), mergers (Salant et al., 1983; Salinger, 1988; Farrell and Shapiro, 1990), research joint ventures (Salant and Shaffer, 1999), and financial markets (Brander and Lewis, 1986; Allaz and Vila, 1993).

²One line of work is concerned with characterizing necessary and sufficient conditions for the existence and uniqueness of equilibria (Novshek, 1985; Gaudet and Salant, 1991). Another line of work considers the consistency of Cournot conjectures (Bresnahan, 1981; Daughety, 1985). Establishing when Cournot outcomes arise is also an area of research (Kreps and Scheinkman, 1983; Klemperer and Meyer, 1989).
sense that less profitable firms may be active while more profitable firms may be excluded. A distinguishing feature of the model is that the competing firms choose both the quantities in the downstream market and the terms they are willing to pay for their inputs (subject to acceptance by the supplier). Although the former has been well studied, the latter in conjunction with the former has not. We show that there are two types of outcomes that can arise when the firms compete on both dimensions. One is relatively benign and occurs when each firm uses its bargaining power to procure its inputs at the lowest possible cost. The other, less benign outcome occurs when firms use their bargaining power to foreclose their rivals. It is this possibility that calls into question the robustness of earlier insights.

When firms can observe the supplier’s accept-reject decisions, they can adjust their quantities accordingly. This can lead to exclusion because it allows the firms to design their contracts in a way that punishes the supplier for accepting other firms’ contracts. The design implicitly takes the form of “if you accept my rivals’ offers—or if you accept more offers than I am expecting—I will drastically cut back on the quantity that I purchase from you.” When this is combined with a payment that is below cost at the cutback quantity, the supplier faces a choice: accept the other firms’ offers or accept this firm’s offer. This invariably leads to exclusion, although surprisingly, it is not necessarily the weakest firms that are excluded.

Exclusion can also arise even when firms cannot observe the supplier’s accept-reject decisions, but other factors must then be present. In particular, there must be externalities in the supplier’s cost of serving each firm. When this is the case, for example, if the supplier’s costs are strictly convex over the relevant range, it can be profitable for a firm to increase its quantity demanded and induce the supplier to reject one or more of its rivals’ contracts.

Implications for policy
Our findings have implications for policy in several areas. These areas include the engagement of firms in exclusionary conduct, the exercise of buyer power, the role of buyer power in horizontal merger policy, and the role of export subsidies in international trade.

Consider first the implications of our findings for policy on exclusionary conduct. When exclusion is alleged to have occurred via contracts between large buyers and sellers, competition authorities typically focus on exclusive-dealing arrangements of the form “you agree not to buy from anyone else but me” (e.g., Aghion and Bolton, 1987; Rasmusen et al., 1991; Segal and Whinston, 2000) or other types of contracts that explicitly reference rivals, such as market-share requirements contracts (e.g., Calzolari and Denicolo, 2013; Chen and Shaffer, 2014). To determine culpability, authorities consider whether the firm that was alleged to

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3 The contract must of course be designed in a way that makes this threat credible.
4 See the 2014 joint workshop of the U.S. Department of Justice and U.S. Federal Trade Commission on
have engaged in exclusionary conduct was pricing below cost (making it difficult for others to match), had a first-mover advantage (meaning there was not equal opportunity), was less efficient than the excluded firm (meaning that it would not have competed well on the merits), offered lump-sum payments to the buyers (meaning that the buyers needed to be compensated for the exclusion), and so on. They also consider whether the excluded firms had economies of scale in production (making it easier for the excluding firm to exclude).

In contrast, in our model, contracts consist of menus of own price-quantity pairs, which, because the contracts do not explicitly reference rivals, are normally looked upon favorably by competition authorities. The contract offers are made simultaneously, and the transfer prices in any exclusionary equilibrium are always above cost. Neither the excluded firms nor the upstream supplier need have economies of scale in production. Moreover, the excluding firm may be more or less efficient than the excluded firm, and lump-sum payments are often given to the excluding firm, not paid by the excluding firm. What makes the analysis unusual is that the fulcrum of exclusion in our model is upside down. Instead of the upstream firms needing the acquiescence of downstream buyers to achieve exclusion, it is the downstream firms that need the acquiescence of the upstream supplier to exclude.

With respect to the exercise of buyer power, the policy debate typically revolves around whether the allegedly lower input prices that buyers with more bargaining power can obtain will be enough to offset any adverse effects on the upstream firms’ incentives to invest (given that their profit margins will be squeezed).\(^5\) In contrast, our findings suggest that buyers with bargaining power may not be focused on obtaining the lowest possible input prices. They may instead prefer to structure their contract terms in a way that harms competition.

Our findings also have implications for merger policy because they suggest that a consolidation of upstream firms might affect competition in unexpected ways. Modern merger analysis typically focuses attention on the level of the merging firms. In the case of an upstream merger, for example, it would consider whether the proposed consolidation would be expected to lead to higher prices and/or reduced choices in the upstream market.\(^6\) Our findings suggest, however, that if as a result of the proposed merger the downstream firms would be forced to buy their inputs from a common input supplier, then the harm from the upstream consolidation could in some cases manifest itself indirectly—not in the upstream market—but in potentially reduced choices and/or higher prices in the downstream market.

Modern merger analysis also recognizes that the presence of large buyers can be a miti-
gating factor that lessens the potential for harm because they can serve as a countervailing force to any proposed price increase. Our analysis confirms this for settings in which all downstream firms trade. In other settings, however, large buyers can exacerbate the potential for harm. In these settings, our analysis suggests that pronouncements of large buyers need not be informative for policy. A large buyer may, for example, be unopposed to an upstream merger that creates a common supplier not because it expects to keep price increases in check, but because it expects to be favored in the exclusionary equilibrium that follows.

Lastly, our findings have implications for international trade policy, where it has been widely recognized (e.g., Brander and Spencer, 1985) that countries may be able to gain a competitive advantage in the world market by subsidizing their firms’ exports when competition occurs a la Cournot. Similarly, calls for the creation of a domestic “national champion” can be viewed in the same way.\footnote{Arguments in favor of creating national champions, in which governments promote the R&D investments of some domestic firms at the expense of others, have long been advocated (Servan and Schreiber, 1968).} By subsidizing the R&D of the champion, or by pushing the champion farther down its learning curve, a country allegedly can ensure that it will be better positioned in the international marketplace. Although total output in the world market will increase as a result of the subsidies (this is so whether the subsidies lower the domestic firm’s marginal costs directly, as in an export subsidy, or indirectly via inducing the firm to increase its R&D expenditures), the favored firms and the country sponsoring them can gain because of the strategically induced cutback in the rival firms’ quantities.

However, our findings suggest that subsidizing exports or creating national champions can backfire when the downstream firms purchase their inputs from a common input supplier. Countries that engage in such policies can potentially be worse off than under laissez faire. This is so even when competition occurs a la Cournot, quantities are strategic substitutes, and conditions are conducive for all players to trade. The reason is that the supplier’s costs may no longer be linear over the relevant range after the increase in the quantity demanded of its inputs. When this is the case, our results suggest that at least one of the downstream firms will be excluded. If the domestic firm or firms happen to be among those excluded, then the firms and the country sponsoring them will be worse off. Even if the country’s subsidies cause its firms to have the lowest marginal costs in the industry, it may still be the case that the country and its firms can be worse off because of our finding that the ‘wrong firms’ may be excluded in equilibrium. There is simply no guarantee that the country can make itself better off by subsidizing its firms, and it could make itself worse off.

Related literature
Our model belongs to the class of bilateral-contracting games that have both common and
competing agents, with one side making contract offers to the other, and one side making quantity (or other payoff-relevant) choices given the contracts. This class includes the literatures on common agency and vertical control. However, the models in these literatures differ from ours by having one side make the offers and the other side make the choices.

In the literature on common-agency, “upstream” competing players make the offers and a “downstream” common agent makes the choices. The focus in this literature is on characterizing when the payoff of the overall coalition will be maximized. It is found that this will generally be the case when there is complete information on the agent’s preferences because then the competing players can do no better than to make the agent the residual claimant of the coalition’s payoff (e.g., Bernheim and Whinston, 1986a,b; Segal and Whinston’s bidding game, 2003). This is so even when exclusive contracts are feasible (e.g., O’Brien and Shaffer, 1997; Bernheim and Whinston, 1998). Inefficient outcomes can arise and exclusion of upstream rivals is possible, however, when exclusive contracts are feasible and the agent has asymmetric information because in addition to aligning incentives, contracts then also serve as screening devices (e.g., Martimort and Stole, 2010; Calzolari and Denicolo, 2013, 2014).\(^8\)

In the literature on vertical control, an upstream monopolist makes the offers and downstream competing firms make the choices. The focus in this literature is on determining whether and when the upstream firm will be able to induce the downstream firms to make the choices that maximize industry profits. It is found that when the competing firms can observe their rivals’ contract terms before making their choices (e.g., Mathewson and Winter, 1984; Winter, 1993; Krishnan and Winter, 2007), the efficient outcome will often be achievable as long as contracts are sufficiently flexible (e.g., two-part tariffs may suffice when the retail price is the only choice variable, whereas two-part tariffs and resale price maintenance may be needed when both retail prices and service levels are choice variables).\(^9\) However, when contract terms are unobservable (e.g., Hart and Tirole, 1990; O’Brien and Shaffer, 1992; Rey and Vergé, 2004), outcomes will depend on the competing firms’ out-of-equilibrium beliefs. It has been shown that with passive beliefs (McAfee and Schwartz, 1994), for example, quantity choices will be bilaterally optimal (i.e., pairwise stable), implying that the outcome will be inefficient when there are negative externalities in the payoffs of the competing firms. It has also been shown that the upstream firm may be better off committing to serve fewer downstream firms as a way of increasing overall profits. For example, it may be better off

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\(^8\)Martimort and Stole (2010) identify two sources of inefficiency: inefficient contracting given the set of active competing players, and inefficient participation (insufficient activity) by the competing players. The problem is that competing players must trade off bilateral efficiencies with information-rent considerations.

\(^9\)Ensuring that contracts are publicly observable, however, is not generally enough to obtain the efficient outcome when the competing firms have private information (e.g., Dequiedt and Martimort, 2015).
serving just one firm and obtaining monopoly profit from that one firm than it would be serving multiple firms and obtaining only the industry-wide Cournot profits from those firms.

The models in these literatures differ in important ways from our model. Whereas they tend to focus on characterizing conditions under which the payoff of the overall coalition will be maximized, our focus is on characterizing conditions under which quantity choices will be bilaterally optimal (which is known to be inefficient). In this sense, our model is similar to the vertical-control literature that assumes that contract terms are unobservable. We differ from this literature, however, in that we do not need to place restrictions on out-of-equilibrium beliefs to obtain our results. Moreover, the settings in which exclusion arises in our model have no counterpart in these other models. In the common-agency literature, for example, exclusion arises only when there is asymmetric information on the agent’s preferences. And in the vertical-control literature with unobservable contracts, the upstream firm commits to exclusion to increase overall joint profits. Here, in our setting, there is complete information on the agent’s preferences, and exclusion, when it arises, is destructive to overall joint profits.

Segal and Whinston (2003) look for robust predictions in a class of bilateral-contracting games between a common player and competing players when the common player makes the choices. As special cases, they consider a “bidding game” in which the competing players make the offers and the common player makes the choices, and an “offer game” in which the common player makes the offers, the competing players accept or reject the offers they receive, and the common player makes the choices. This allows them, for example, to sharpen the predictions of the vertical-control literature on unobservable contracts by focusing in the offer game on outcomes that can be supported regardless of the competing players’ beliefs. As Segal and Whinston (2003, p. 762) note, “A key idea of this paper is that the manufacturer may be able to avoid the problem of negative inferences by retailers by using contracts that specify a menu of possible trades from which she herself later chooses.” Thus, in Segal and Whinston’s offer game, the common player makes both the offers and the choices. This difference between our model and theirs as to who makes the offers and the choices gives rise to the possibility of exclusion in our model, whereas exclusion does not come up in theirs. It also leads to differences as to when we would expect pairwise-stable outcomes to arise.

This brings us to the literature that considers, as we do, the case of competing players making both the offers and the quantity choices (Marx and Shaffer, 2007; Miklós-Thal et al., 2011; and Rey and Whinston, 2013). Of these, Marx and Shaffer (2007) and Miklós-Thal et al. (2011) restrict attention to the case of two competing players and assume that

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10Our model also differs from Segal and Whinston’s (2003) model in that they assume throughout that the competing players’ accept-reject decisions are not observable, whereas we compare and contrast the case in which the common player’s accept-reject decisions are observable with the case in which they are not.
all contracts and accept-reject decisions are observable. Marx and Shaffer (2007) show that when contracts consist of three-part tariffs, all equilibria involve the exclusion of a competing player.\textsuperscript{11} Miklós-Thal et al. (2011) show that if the competing players can make offers that are contingent on whether they have exclusivity (i.e., if they can offer the common player one set of terms if he accepts the other player’s offer, and a different set of terms if he rejects the other player’s offer), there is always an equilibrium that maximizes industry profits and does not involve exclusion.

Similarly, Rey and Whinston (2013) assume that all contracts and accept-reject decisions are observable, but they allow for $n \geq 2$ competing players. They show that an equilibrium in which industry profits are maximized can be supported when each competing player can offer the common player a menu of contracts. Although only one contract per player is accepted in equilibrium, the $n-1$ contracts per player that are not accepted in equilibrium nevertheless play an important role in supporting the equilibrium outcome in the sense that they are needed to ensure that rival competing players do not have profitable deviations.

We contribute to this literature by considering the case in which contracts are unobservable. As noted above, assuming that contracts are unobservable can have dramatic effects on the set of outcomes that can be supported in equilibrium, and the same is true here. Moreover, in doing so, we are able to distinguish observable accept-reject decisions from observable contracts, and in particular, to isolate the importance of observable accept-reject decisions separately from the more restrictive assumption that competing players know both the common player’s accept-reject decisions and contract terms. This distinction has not previously been made in the literature, but it turns out to be of vital importance here. Lastly, by allowing for an arbitrary number of competing players, we are able to show that exclusionary equilibria may exist in which, conditional on the number of players trading, the wrong players may trade. This outcome is not possible with only two competing players.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 considers the case of unobservable accept-reject decisions. Section 4 considers the case of observable accept-reject decisions. Section 5 considers in turn cases involving additively separable costs, contracts that depend on rivals’ quantities, and menus of contracts. Section 6 concludes. The proofs of the lemmas and propositions for Section 3 are in the appendix, and the proofs for Sections 4 and 5 are in the online appendix.

\textsuperscript{11}A three-part tariff consists of an upfront payment that is paid whether or not the competing player chooses a positive quantity, and a two-part tariff that goes into effect if a positive quantity is chosen.
2 Model

We consider a bilateral contracting game between an input supplier (the “common player”) and \( n \geq 2 \) downstream buyers (the “competing players”). We assume that the buyers use the inputs to produce substitute products for resale to final consumers.

The game proceeds in three stages. In the first stage, the competing players simultaneously make contract offers to buy one or more units of the common player’s inputs. In the second stage, the common player accepts or rejects each offer. In the third stage, the competing players whose offers are accepted simultaneously make their quantity choices. Payoffs are realized and payments to the common player are made according to the accepted contracts. The players whose offers are not accepted do not trade and make no payments.

The competing players in our model thus make the contract offers in addition to the quantity choices. This distinguishes it from Segal and Whinston’s (2003) offer game, where the common player makes the offers and the quantity choices, and their bidding game, where the competing players make the offers but the common player makes the quantity choices. It also distinguishes it from the games in Hart and Tirole (1990) and McAfee and Schwartz (1994), where the common player makes the offers and the competing players make the choices.

Throughout the paper we assume that contract terms are observed only by the players who are party to the contract. Specifically, each competing player knows the terms at which she can purchase the common player’s inputs but does not know—and never learns—the terms at which her rivals can purchase their inputs. However, we consider different assumptions on the observability of the common player’s accept-reject decisions. In Section 3, we assume that a player can only observe whether her own contract was accepted or rejected. In Section 4, we assume that all accept-reject decisions by the common player are observable.

Notation and assumptions

The set of possible trades between the common player and competing player \( i \) is represented by \( \mathcal{X}_i \subseteq \mathbb{R}_+ \), where \( 0 \in \mathcal{X}_i \), and a typical trade is represented by \( x_i \in \mathcal{X}_i \). We assume their contract, which defines a monetary transfer \( t_i \) from the competing player to the common player as a function of how much the competing player purchases, consists of a finite menu of price-quantity pairs.\(^{12}\) That is, we assume the contract takes the form \( t_i : \mathcal{X}_i \to \mathbb{R} \cup \{\infty\} \), where \( t_i(x_i) = \infty \) for all but a finite subset of \( \mathcal{X}_i \).\(^{13}\) It is useful to let \( N \equiv \{1, \ldots, n\} \)

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\(^{12}\)This ensures that each competing player \( i \) has an optimal quantity choice in stage three for any possible contract that player may have offered in stage one and any possible set of acceptances in stage two.

\(^{13}\)For now, we restrict attention to contracts that specify player \( i \)'s payment as a function only of her own quantity. In Section 5, we allow player \( i \)'s payment to depend on the quantities purchased by all players.
denote the set of competing players. It is also useful to define the following additional sets:

\[ X \equiv X_1 \times \ldots \times X_n, \quad T_i \equiv \{ t_i : X_i \to \mathbb{R} \cup \{\infty\} \}, \quad \text{and} \quad T \equiv T_1 \times \ldots \times T_n. \]

A competing player’s payoff is given by the difference between the utility she receives from her quantity choice (given the other players’ choices) and the payment she must make to the common player. To capture the presence of (negative) externalities in the competing players’ payoffs, we let \( g_i : X \to \mathbb{R} \) denote competing player \( i \)'s utility and assume that for all trade profiles \( x = (x_1, \ldots, x_n) \in X \) in which \( x_i > 0 \), \( g_i(x) \) is lower when others also trade:

**Assumption 1** For all \( x \in X \) and all \( i \neq j \), if \( x_i, x_j > 0 \), then \( g_i(x) < g_i(0, x_{-j}) \).

Assumption 1 captures the notion that a positive quantity sold by one player in the downstream market will adversely affect the prices at which the other players can sell their products. It holds when competing player \( i \) trades. If player \( i \) does not trade, she receives her default utility. We assume this utility does not depend on the other players’ trades. That is, for all \( x_{-i} \in X_{-i} \), we assume \( g_i(0, x_{-i}) = 0 \). This is also a natural assumption to make (e.g., although the price at which a competing player can sell her product depends on the other players’ trades, her revenue will be zero if she herself does not trade), and it implies, using the terminology of Segal (1999), that there are no externalities on non-traders. When it is combined with our assumption that the default outcome is for no trade and no payment to the common player, it ensures that player \( i \)'s payoff will be zero if her contract is rejected.

The common player’s payoff is given by the difference between the sum of the payments he receives from the competing players and his cost of supplying the inputs. To capture the possible presence of externalities in the common player’s payoff, we let \( f : \mathbb{R} \to \mathbb{R} \) denote the common player’s cost of supplying inputs and assume that it depends on the total quantity of inputs supplied. Thus, given the trade vector \( x \), if \( X \) denotes the sum \( \sum_{i=1}^n x_i \), and \( X_{-j} \) denotes the sum \( \sum_{i \neq j} x_i \), then \( f(X) \) is the common player’s cost of supplying \( x \) and \( f(X_{-j}) \) is the common player’s cost of supplying \( (0, x_{-j}) \). We make the following assumptions on \( f \):

**Assumption 2** \( f(X) \) is non-decreasing and for \( X > 0 \) weakly convex.

Assumption 2 is more restrictive than assuming that \( f \) is lower semi-continuous, as in Segal and Whinston’s (2003) offer game, but not as restrictive as assuming that \( f \) is non-decreasing and strictly convex, as in Segal and Whinston’s bidding game. It is also not as restrictive as the corresponding assumptions in the models of Hart and Tirole (1990), McAfee and Schwartz (1994) and Rey and Verge (2004), which require \( f \) to be linear. An implicit assumption here and in these models is that the common player’s inputs are homogeneous.\(^{14}\)

\(^{14}\)In Section 5, we extend the analysis to allow for the possibility that the inputs are customized.
It follows that when the competing players have accepted contracts \( t = (t_1, ..., t_n) \), and the trade profile is \( x \), the common player’s payoff is \( \sum_{i \in N} t_i(x_i) - f(X) \), each competing player \( i \)'s payoff is \( g_i(x) - t_i(x_i) \), and the overall joint payoff is \( \Pi(x) \equiv \sum_{i \in N} g_i(x) - f(X) \).

**Solution concept**

We solve for a perfect Bayesian equilibrium of the game and restrict attention to pure-strategy equilibria, although we do not require the equilibrium to be unique. An equilibrium must specify: a) the contracts offered by the competing players; b) the set of contracts accepted by the common player for any possible vector of contracts offered; c) the quantity choices of each competing player for any possible contract that player might have offered and any possible set of acceptances; and d) beliefs. Thus, an equilibrium must specify: \((t^*, I^*, x^*)\), where \( t^* \in T \) is a vector of contract offers, \( I^* : T \rightarrow 2^N \) is a mapping from contract offers to the subset of competing players whose offers are accepted, and \( x_i^* : T_i \times 2^N \rightarrow X_i \) is player \( i \)'s quantity choice given her contract offer and the set of competing players whose offers are accepted by the common player. When the meaning is clear, we use \( x_i^* \) in place of \( x_i^*(t_i^*, I^*(t^*)) \) to denote player \( i \)'s quantity choice on the equilibrium path. Similarly, when the meaning is clear, we use \( I^* \) in place of \( I^*(t^*) \) to denote the equilibrium set of acceptances.

In the case of unobservable accept-reject decisions, we restrict \( x_i^*(t_i, I) \) to be equal to \( x_i^*(t_i^*, I^*) \) for all \( I \subset N \), so that quantity choices do not depend on the accept-reject decisions.

We discuss the specification of off-equilibrium beliefs as needed below.

### 3 Unobservable accept-reject decisions

We begin by assuming that the competing players cannot observe whether the common player has accepted or rejected their rivals’ contracts. This implies that the competing players will not know with certainty which of their rivals they will be competing against when they make their quantity choices, although in equilibrium their beliefs will be correct.

#### 3.1 Pairwise stability and at-cost payments

In our first result, we show that in any equilibrium in which all players trade and rival accept-reject decisions are unobserved, the bilateral surplus of the common player and each competing player \( i \) is maximized, and each competing player \( i \)'s payment is equal to the common player’s incremental cost of serving her. In this case, we say that the trade profile is “pairwise stable” and payments are “at cost.” Players must also have non-negative payoffs.
Lemma 1 Assuming rival accept-reject decisions are not observed, in any equilibrium in which all players trade, the trade profile $x^*$ is pairwise stable, $t^*_i$ is such that player $i$’s payment to the common player is at cost, and all players have non-negative payoff: $\forall i \in N,$

$$x^*_i \in \arg\max_{x_i \in X_i} g_i(x_i, X^*_{-i}) - f(x_i + X^*_i),$$

$$t^*_i(x^*_i) = f(X^*) - f(X^*_{-i}),$$

$$g_i(x^*) - t^*_i(x^*_i) \geq 0, \quad \sum_{i \in N} t^*_i(x^*_i) - f(X^*) \geq 0.$$

Payments are at cost because player $i$’s rivals cannot respond to deviations that they cannot observe, nor can they respond to the common player’s response to those deviations if accept-reject decisions are not observed. Hence, the common player knows that the quantity choices the other players would make if he accepts their offers, and hence the payments he would receive from them, do not depend on whether he accepts or rejects player $i$’s offer. It follows that in deciding whether to accept player $i$’s offer, the common player only needs to consider whether the payment he expects to receive from player $i$ will cover his cost of supplying her.

As for the requirement that trades be pairwise stable, the proof of Lemma 1 exploits the fact that gains from trade between the common player and competing player $i$ exist if their bilateral surplus holding all other players’ quantities fixed is not maximized. By offering to share these gains with the common player, player $i$ can make both herself and the common player strictly better off. This is so whether or not the common player accepts the other players’ offers; if he does, then there is no change in the other players’ quantities and player $i$ is better off, and if he does not, then the other players’ quantities either stay the same or default to zero, and the deviation is even more profitable for player $i$ (from Assumption 1).

Pairwise stability has also been found in other vertical settings when contract terms are unobservable. However, there are some differences between our results and those in the related literature. Among them, Lemma 1 establishes only that trade profiles are pairwise stable when all players trade. One can show that pairwise stability need not hold otherwise. The reason is that in the event of a deviation by player $i$ when only some players trade, the common player might respond by accepting a contract offer from a player that he would otherwise have rejected, and this decision can make the deviation unprofitable for player $i$.

By comparison, in Segal and Whinston’s (2003, p. 781) bidding game, pairwise stability holds only for upward deviations, even when all players trade. That is, it is shown only that there cannot exist $i$ and $\hat{x}_i > x^*_i$ such that the bilateral surplus of player $i$ and the common player is higher at quantity $\hat{x}_i$. Intuitively, for upward deviations by one competing
player, the common player will not increase her purchases from the other competing players in the case of substitutes (will not decrease in the case of complements), so the response by the common player to the deviation and the change in the deviating player’s quantity only makes the deviation more profitable. This need not be the case with downward deviations. In contrast, in our model, where the competing players make both the offers and the choices, pairwise stability holds for both upward and downward deviations when all players trade.

Pairwise stability also holds when all players trade in the models of Hart and Tirole (1990) and McAfee and Schwartz (1994), but only under certain restrictions on beliefs. In these models, where the common player makes the offers but the competing players make the choices, restrictions are needed on the buyers’ beliefs because the common player can make multiple deviation offers at a time.\(^{15}\) In contrast, in our model, pairwise stability always holds when all players trade without any restrictions on beliefs. The reason is that when any one competing player makes a deviation offer, she believes that she is the only one deviating.

Lastly, pairwise stability can arise in equilibrium when all players trade in Segal and Whinston’s (2003, p. 781) offer game. However, absent restrictions on the competing players’ beliefs, other equilibrium outcomes, including the competitive outcome, are also possible.

**Implications of pairwise stability**

Notice that pairwise-stable quantities correspond to Cournot-equilibrium quantities in a simultaneous-move model of quantity choice in which competing player \(i\)’s revenues are \(g_i(x_i, x_{-i})\) and costs are \(f(x_i + X_{-i}) - f(X_{-i})\). In this Cournot equilibrium, player \(i\) chooses quantity \(x_i^*\) and earns a net payoff \(g_i(x_i^*, x_{-i}^*) - (f(X^*) - f(X_{-i}^*))\), which is precisely what she would choose and earn in equilibrium in our model after paying the common player \(f(X^*) - f(X_{-i}^*)\) to obtain her inputs. It follows that when rival accept-reject decisions are not observed and all players trade, equilibria in our model correspond to static Cournot outcomes in which competing player \(i\)’s cost of producing her equilibrium quantity can be thought of as the at-cost payment she must make to the common player to obtain her inputs.

In what follows, to focus on the interesting case, we assume that there exists a strictly positive pairwise-stable trade vector. This corresponds to assumptions on fundamentals as described by e.g., Friedman, 1983; Novshek, 1985; Shapiro, 1989; Gaudet and Salant, 1991.

**Assumption 3** \(X^* \equiv \{x^* \in X \mid x^* \text{ is strictly positive and pairwise stable}\} \text{ is nonempty.}\)

It is well known that Cournot outcomes are generally inefficient. This can be seen by rewriting the bilateral surplus of competing player \(i\) and the common player as the difference

\(^{15}\)For example, pairwise stability holds in these models if there is “market-by-market” bargaining, as in Hart and Tirole (1990) or if the competing players have “passive beliefs,” as in McAfee and Schwartz (1994). In contrast, trade profiles are not pairwise stable if competing players have “wary” or “symmetric” beliefs.
between the overall payoff and the sum of the utilities of the rival competing players. Thus, for all $i \in N$, we can rewrite the condition for pairwise stability, which is given in (1), as:

$$x_i^* \in \arg \max_{x_i \in X_i} \Pi(x_i, x_{-i}^*) - \sum_{j \neq i} g_j(x_i, x_{-i}^*).$$

Rewriting the condition for pairwise stability in this way helps to clarify the source of the inefficiency. In maximizing the bilateral surplus of herself and the common player, player $i$ does not take into account the externality that her choice of $x_i$ imposes on the payoffs of the rival competing players. As a result, too much quantity is produced. Although it may be possible for efficient outcomes to arise if there are kinks in the competing players’ utilities or the common player’s costs, in the absence of these features, the outcome will be inefficient.

**Implications of at-cost payments**

Whether it is possible to satisfy the conditions in Lemma 1 depends in part on whether the common player’s payoff is non-negative when the competing players’ payments are at cost. Notice that a consequence of at-cost payments is that in the absence of non-convexities in the common player’s costs (for example, if the common player’s costs exhibit constant or decreasing returns to scale everywhere), the sum of the competing players’ payments always covers the common player’s costs. However, when there are non-convexities, this is no longer true. Condition (3) would fail to hold, for example, if there were increasing returns to scale everywhere: $f(0) = 0$ and $f(X) = k + cX$, for all $X > 0$, where $k > 0$ denotes the common player’s fixed costs and $c \geq 0$ denotes his marginal cost of supplying each unit. In these cases, there is no equilibrium in which all players trade because the common player would be better off rejecting the candidate equilibrium contracts and earning zero payoff.

It follows that the existence of equilibria in which all players trade depends on the nature of the common player’s costs. We further explore this relationship in the next subsection.

### 3.2 Cournot outcomes

We begin with the simple case in which the common player’s costs have constant returns to scale everywhere: $f(X) = cX$ for all $X \geq 0$. Using Lemma 1, it then follows that in any

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16This follows because then $f(X^*) - f(X_{-i}^*) \geq f(x_1)$, $f(X^*) - f(X_{-2}^*) \geq f(x_1 + x_2) - f(x_1)$, $f(X^*) - f(X_{-3}^*) \geq f(x_1 + x_2 + x_3) - f(x_1 + x_2)$, and so on. Summing both sides implies that $\sum_{i=1}^n t_i^*(x_i^*) \geq f(X^*)$.

17In this case, each player $i$’s payment to the common player in any equilibrium in which all players trade must be $t_i^*(x_i^*) = cx_i^*$. Adding these payments and comparing the sum to $f(X^*)$, it follows that the common player’s payoff if he were to accept the proposed contracts would be negative: $\sum_{i=1}^n t_i^*(x_i^*) - f(X^*) = -k$. 

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13
equilibrium in which all players trade and rival accept-reject decisions are not observed,

\[ x^*_i \in \arg \max_{x_i \in \mathcal{X}_i} g_i(x_i, x^*_{-i}) - cx_i \]  

and

\[ t^*_i(x^*_i) = cx^*_i, \]  

implying that competing player \( i \) earns \( g_i(x^*_i, x^*_{-i}) - cx^*_i \) and the common player earns zero.

As for existence, it is easy to see that an equilibrium can be supported with contract offers \( t_i(x_i) = cx_i \) (appropriately discretized) for all \( i \in N \).\(^{18}\) Because \( \mathcal{X}^* \) is non-empty by Assumption 3, it follows that there is an equilibrium in which all players trade and rival accept-reject decisions are not observed if the common player’s costs exhibit constant returns to scale.

An immediate implication of this is that having a common supplier need not imply higher prices downstream. When the common player’s costs are given by \( f(X) = cX \) for all \( X \geq 0 \), for example, the competing players pay the same per-unit prices (and earn the same payoff) buying from the common player as they would pay (and earn) if each bought instead from an independent supplier that had the same costs. The competing players thus help to keep in check the market power that the common player might otherwise have been able to exploit.\(^{19}\)

To analyze cases in which the common player does not have constant returns to scale, it is helpful to extend our results to an environment in which payoffs are continuous. To make this more precise, we define a **continuous environment** to be one in which for all \( i \in N \), \( \mathcal{X} = \mathbb{R}^n_+ \) and \( g_i \) is continuous in \( x_i \). Specializing to this environment allows us to assess, for example, whether equilibria in which all players trade can be supported when the common player’s costs have decreasing returns to scale, whether and to what extent equilibria in which all players trade can be supported when there are non-convexities in the common player’s costs, and whether and when the common player can earn positive payoff in equilibrium.

\(^{18}\)For the case of unobserved accept-reject decisions and an equilibrium in which all trade, off-equilibrium beliefs are of no consequence. To see that each player’s contract can be discretized and written as a finite menu, note that even when accept-reject decisions are observed, a finite number of quantity choices occur on and off the equilibrium path, corresponding to the different combinations of contracts that the common player might accept or reject.

\(^{19}\)This has implications for merger policy. If formerly independent suppliers with constant returns to scale technology everywhere merge to form a common supplier with the same constant returns to scale technology, we would expect each competing player to obtain her inputs post merger at the same per-unit price that she could get pre merger (absent any change in her ability to make offers). And if the same merger leads to efficiencies that reduce the merged firm’s marginal costs, we would expect the competing players’ per-unit prices to decrease post merger. In neither case would we expect the merger to be harmful to final consumers. These findings are consistent with the treatment of large buyers in the U.S. merger guidelines, which view large buyers as offering a countervailing force to mitigate what might otherwise be an unpleasant merger.
Proposition 1  Assuming rival accept-reject decisions are not observed and a continuous environment, there is an equilibrium in which all players trade

- if $\exists x^* \in X^*, c > 0, \text{ and } b \geq 0 \text{ such that for } X \geq \min_{i \in N} X^*_{-i}, f(X) = cX - b$;
- only if $\exists x^* \in X^*, c > 0, \text{ and } b \geq 0 \text{ such that for } X \in [\min_{i \in N} X^*_{-i}, X^*], f(X) = cX - b$.

Proposition 1 establishes necessary and sufficient conditions for an equilibrium to exist in which all players trade in a continuous environment. It shows that the common player’s cost function must be linear in the relevant range (it is necessary for it to be linear in the range from $\min_{i \in N} X^*_{-i}$ to $X^*$; it is sufficient for it to be linear for all $X \geq \min_{i \in N} X^*_{-i}$), and be such that the average cost of supplying inputs is less than the marginal cost in this range.

When the sufficient conditions for existence hold, at-cost payments imply that player $i$ earns a payoff of $g_i(x^*_i, x^*_{-i}) - cx^*_i \geq 0$ and that the common player earns a payoff of $b \geq 0$. Proposition 1 thus establishes that the common player will earn zero payoff when his costs exhibit constant returns to scale in the relevant range (i.e., when $b = 0$) and a positive payoff when his costs exhibit decreasing returns to scale in the relevant range (i.e., when $b > 0$).

These findings are illustrated in Figures 1(a) and 1(b). As drawn in these figures, $f(X)$ is linear for all $X \geq \alpha$ and has the property that the average cost of supplying inputs in this range is weakly less than the marginal cost of these inputs. Given this, it follows from Proposition 1 that if $\min_{i \in N} X^*_{-i} \geq \alpha$, there is an equilibrium in which all players trade in a continuous environment. If the average cost of supplying $X^*$ units is equal to the common player’s marginal cost, as illustrated in Figure 1(a), the common player’s payoff will be zero. If it is less than this, as illustrated in Figure 1(b), the common player’s payoff will be positive.

![Diagram](image-url)

(a) Zero profit for the common player  
(b) Positive profit for the common player

Figure 1: Illustration of possible cost functions for the common player

Notice that, in both figures, the supplier incurs fixed costs of production and $f(X)$ exhibits (weakly) increasing returns to scale for all $X \leq \alpha$. Because, as we have seen, the
sufficient conditions in Proposition 1 can nevertheless be satisfied, it follows that the mere presence of fixed costs (i.e., non-convexities) or increasing returns to scale in the common player’s cost function does not rule out the possibility of an equilibrium in which all players trade (nor does it preclude the common player from earning strictly positive payoff in equilibrium).

It is also true, however, that the absence of fixed costs does not imply that such an equilibrium will exist. As we now show, equilibria in which all players trade in a continuous environment may fail to exist in many familiar cases. Equilibria in which all players trade in a continuous environment may fail to exist, for example, even in the absence of non-convexities. Moreover, because the necessary conditions in Proposition 1 imply that there cannot be a player whose quantity is sufficiently large that there is some strict convexity in \( f \) between the total cost of supplying all players and the cost of supplying all but that one player, it follows that equilibria in which all players trade need not exist even if the competing players’ at-cost payments would be more than enough to cover the common player’s costs.

The necessary conditions in Proposition 1 also have surprising implications for what the competing players’ costs must be in equilibrium. The conditions require that the common player’s costs must be such that \( f(X) = cX - b \) for some \( c > 0 \) and \( b \geq 0 \), for \( X \in [\min_{i \in N} X_{-i}^*, X^*] \). At-cost payments require that \( t_i^*(x_i^*) = f(X^*) - f(X_{-i}^*) \). Combining the two, we see that the bs cancel, and therefore it must be that \( t_i^*(x_i^*) = cx_i^* \).

**Proposition 2** Assuming rival accept-reject decisions are not observed and a continuous environment, in any equilibrium in which all players trade, the equilibrium quantities correspond to the Cournot equilibrium quantities of competing firms in a simultaneous-move model of quantity choice in which each competing player i’s revenues are \( g_i(x_i, x_{-i}) \) and costs are \( cx_i \), where \( c \) satisfies the conditions of Proposition 1.

Proposition 2 has implications relating to a number of areas of economic research. These areas include: (1) the foundations for cost functions in the literature on Cournot models; (2) the ability of larger downstream firms to extract more favorable terms from upstream suppliers in the literature on buyer power; and (3) the interpretation of empirical studies of buyer power. We briefly discuss each of these in turn.

There is a large literature on the existence and uniqueness of Cournot equilibrium as well as a vast literature employing this model. It is common in this literature to assume that firms have exogenously given cost functions. In some cases, it is natural to think that these cost functions derive from the firms’ input purchases. If we take seriously the notion that firms purchase inputs from upstream suppliers, then a new avenue for strategic interaction opens up among the firms that may have implications for their costs. Proposition 2 shows that if firms
contract with a common upstream supplier as in our model with rival accept/reject decisions not observed, then one cannot have, for example, downstream firms with different constant marginal costs, unless there are intrinsic differences in the downstream firms’ production costs that are unrelated to their costs of procurement. Common, constant marginal costs, however, are consistent with firms obtaining their inputs from a common input supplier.

Proposition 2 has implications for the literature on buyer power that attempts to explain why larger buyers might be expected to pay less than smaller buyers. Prominent papers in this literature (e.g., Chipty and Snyder, 1999; Inderst and Wey, 2007) suggest that the source of the buyer power may be attributable to a common supplier having strictly increasing marginal costs (i.e., a strictly convex cost of supplying inputs). In these models, larger buyers can procure their inputs at better terms than smaller buyers because the average cost of supplying a larger buyer is lower, given the quantity demanded by the other buyers. This literature assumes that buyers operate independently in local downstream markets. Our results, however, suggest that these findings do not extend to the case in which buyers compete, which is the case in many real-world settings. Although our model can accommodate the presence of both large and small downstream firms (through different $g_i$ functions), Proposition 2 implies that large downstream firms in competition with smaller ones will not be able to obtain lower prices (i.e., more favorable terms of trade) than their smaller rivals.

Thus, the results in Proposition 2 may help to explain why empirical studies of buyer power sometimes have difficulty establishing that larger buyers receive better terms than smaller buyers. They may not always receive better terms. By aggregating across different suppliers and products, these studies may not be sensitive to settings in which buyers effectively make the offers (as they do in our model) and settings in which they do not.

### 3.3 Exclusionary outcomes

Proposition 1 implies that at least one player will be excluded if $f(X)$ is not linear for all $X \in \left[\min_{i \in N} X^*_{-i}, X^*\right]$. The reason for this is that in any candidate equilibrium in which all players trade, a competing player would be able to profitably deviate in such a way that the common player would strictly prefer to reject the contracts of another player or subset of players. More generally, there is an equilibrium in which all players trade in a continuous environment only if $\exists \delta > 0$ such that for any non-empty strict subset $J$ of $N$, $\forall \delta \in (0, \delta)$,

\[
f(X^* + \delta) - f\left(\sum_{j \in N \setminus J} x_j^* + \delta\right) \leq \sum_{j \in J} (f(X^*) - f(X^*_j)).
\]  

(6)

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20See the UK Competition Commission’s (2000) and (2008) reports on the UK supermarket industry.
To see this, suppose that condition (6) is violated for some subset of players and \( \hat{\delta} \), and consider a deviation offer by player \( i \) of

\[
\begin{align*}
t_i(x_i) \equiv & \begin{cases} 
  f(X^* + \hat{\delta}) - f(X^* - x_i) + \epsilon, & \text{if } x_i = x_i^* + \hat{\delta} \\
  \infty, & \text{if otherwise,}
\end{cases}
\end{align*}
\]

where \( \epsilon > 0 \) is small. By accepting player \( i \)'s deviation offer along with all other offers, the common player can earn \( \epsilon \) more than if he accepts all other offers but rejects player \( i \)'s offer. Hence, the common player will accept player \( i \)'s offer in any equilibrium of the continuation game. But notice that the common player can earn an even higher payoff by accepting player \( i \)'s offer and rejecting one or more of the other offers. This follows because the left side of (6) corresponds to what the common player would save on costs if he were to reject the contracts of the players in \( J \), and the right side of (6) corresponds to what the common player would lose in payments. When (6) is violated, the left side is greater than the right side, and the common player would earn higher payoff by rejecting these contracts. Because player \( i \) benefits when one or more of her rivals' contracts are rejected, her gain from deviating can exceed what she must offer the common player to accept her contract.

A sufficient condition for an exclusionary outcome to arise (i.e., for (6) to be violated) is that the common player’s costs be nonlinear in the relevant range. This would be the case, for example, if the common player’s costs are such that \( f(X) \) is strictly convex for all \( X > 0 \).

**Corollary 1** Assuming rival accept-reject decisions are not observed and a continuous environment, there does not exist an equilibrium in which all players trade if the common player’s costs are strictly convex (or if they exhibit decreasing returns to scale in the relevant range).

One might think that equilibria in which all players trade would exist in this case because when the common player’s costs are strictly convex, the common player can earn strictly positive payoff when payments are at cost. However, this inference is incorrect because when the common player’s costs are strictly convex, player \( i \)'s quantity imposes a negative externality on her rivals by increasing the common player’s costs of serving them. This allows player \( i \) to use her contract to induce the common player to exclude one or more rivals.\(^{21}\)

To see this, suppose the common player’s costs are strictly convex. Consider a candidate equilibrium in which there are three symmetric players, with each one choosing quantity \( x^* \) and making an at-cost payment of \( t^*(x^*) = f(3x^*) - f(2x^*) \) to the common player. All players earn strictly positive payoff in this case, and the conditions in Lemma 1 are satisfied.

\(^{21}\)When the common player’s costs exhibit constant returns to scale, there is no scope for a player to use her contract to exclude a rival because there are no externalities in the common player’s costs.
Nevertheless, a profitable deviation exists. Consider, for example, a deviation by player 1 to

\[
t^d(x) = \begin{cases} 
  f(3x^* + \delta) - f(2x^*) + \varepsilon, & \text{if } x = x^* + \delta \\
  \infty, & \text{otherwise}
\end{cases}
\]

where \(\delta\) and \(\varepsilon\) are suitably small positive numbers. When facing offers by two players of \(t^*\) and one player of \(t^d\), the common player could accept all three offers or alternatively accept only two of them. Relative to accepting all three offers, if the common player rejects one of the non-deviating offers, he loses revenue of \(t^*(x^*)\) but saves on costs in the amount \(f(3x^* + \delta) - f(2x^* + \delta)\). As depicted in Figure 2, the cost savings exceed the revenue loss. This suggests that the common player prefers to reject at least one offer rather than accept all three. Having some \(\varepsilon > 0\) in the deviator’s contract ensures that the deviator’s contract is not rejected. By continuity, there exists a positive \(\delta\) and \(\varepsilon\) sufficiently small that the deviator is better off producing slightly more, paying slightly more, and having one fewer rival.

![Figure 2: Illustration of the effects of strict convexity.](image)

To return to the international trade example mentioned in the introduction, suppose there are \(n \geq 2\) firms that compete in a global market, all purchasing inputs from a common supplier and producing strictly positive amounts \((x_1^*, ..., x_n^*)\). Suppose that the supplier’s cost function is as depicted in Figure 1(a) or 1(b), with \(\min_{i \in N} X_{-i}^* \geq \alpha\). Now consider the impact of a subsidy provided to firm 1 that increases firm 1’s output to \(x'_1 > x_1^*\), partially at the expense of the other firms so that the new quantities satisfy \(X'_{-1} < X_{-1}^*\) and \(\min_{i \in N} X'_{-i} < \alpha\). Then, by Proposition 1, there will be exclusion in the new equilibrium, perhaps of the subsidized firm. As we show in Section 4.5, even subsidies that guarantee that a firm is the most efficient among its rivals cannot guarantee that it will not be excluded.
Existence of exclusionary equilibria

We now consider the existence of equilibria in which not all players trade. We focus on the case of only one player trading (sufficient conditions for an equilibrium with \( k \in \{1, ..., n - 1\} \) players trading are given in Proposition A.1 in the Online Appendix). To this end, let \( x_i^m \) denote the quantity that maximizes the joint payoff of the common player and competing player \( i \) conditional on only player \( i \) trading, and let \( \Pi_i^m \equiv g_i(x_i^m, 0) - f(x_i^m) \) denote their maximized “monopoly” payoff.

**Proposition 3** Assuming rival accept-reject decisions are not observed, there exists an equilibrium in which only competing player \( i \) trades if \( \Pi_i = \max_j \Pi_j \), for all \( j \neq i \),

\[
\arg\max_{x_j \geq 0} g_j(x_j, x_i^m, 0) - (f(x_i^m + x_j) - f(x_i^m)) \leq 0
\]

and for some \( k \in \arg\max_{j \neq i} \Pi_j \),

\[
\arg\max_{x_k \geq 0} g_k(x_k, 0) - (f(x_i^m + x_k) - f(x_i^m)) \leq 0.
\]

Proposition 3 establishes that in any equilibrium in which only player \( i \) trades, the monopoly payoff of the common player and player \( i \) must be weakly higher than the monopoly payoff of the common player and any other player \( j \). Intuitively, if another player could generate a higher monopoly payoff than player \( i \), that player could deviate in a way that would make herself and the common player strictly better off if her deviation offer were accepted.

Proposition 3 also establishes that there exists an equilibrium in which only the strongest player trades if (i) there is no possibility for profitable entry by any other player when player \( i \) chooses her monopoly quantity \( x_i^m \) (this corresponds to condition (7)), and (ii) the common player cannot gain by accepting the contract offered in equilibrium by the second strongest player in addition to the contract offered by player \( i \) (this corresponds to condition (8)). Condition (7) ensures that no competing player \( j \neq i \) has a profitable deviation given player \( i \)'s quantity choice \( x_i^m \). Condition (8) ensures that the common player also does not have a profitable deviation.

However, Proposition 3 does not establish that conditions (7) and (8) are necessary for there to be an equilibrium in which only player \( i \) trades. For example, it could be that equilibria exist in which player \( i \) deters all other competing players from trading by distorting her quantity choice above \( x_i^m \), thereby engaging in a form of limit pricing.
4 Observable accept-reject decisions

We now assume that the competing players can observe the common player’s accept-reject decisions and thus know whom they will be competing against when they make their quantity choices (although we continue to assume the contract terms themselves are unobservable).

This seemingly innocuous change has many consequences. One is that the common player will be able to earn strictly positive payoff even when there are constant returns to scale. To see this, note that previously we showed that equilibria in this case could be supported with contract offers $t_i(x_i) = cx_i$ (appropriately discretized) for all $i \in N$. These contracts do not support an equilibrium in which all players trade when accept-reject decisions are observable, however, because if all players are offering to pay the common player $c$ per unit, any one player can profitably deviate by offering to pay the common player more than $c$ per unit. The common player would reject the other contract offers, knowing that because these rejections would be observed by the deviating player, the deviating player would be induced to choose a larger quantity than she otherwise would choose. Observable accept-reject decisions work to the advantage of the common player in this case because he knows that if he rejects contracts in which players are offering to pay him only for his costs, a player who is offering to pay him more can respond by increasing the quantity that she purchases.

As we now show, the consequences of observable accept-reject decisions are not limited to the common player’s payoff, but also affect our results on pairwise stability, the existence of equilibria when all players trade, and the existence of equilibria when not all players trade.

4.1 Pairwise stability

Recall that when accept-reject decisions are unobservable, pairwise stability need not hold if only some players trade. The reason is that if a player that trades were to deviate to her pairwise-stable quantity in this case, the common player might respond by accepting a contract offer from a player that he would otherwise have rejected, which could make the deviation unprofitable. With observable accept-reject decisions, however, a deviating player knows that she will be able to observe the common player’s response to her offer prior to making her downstream quantity choice, and this enables her to structure her deviation offer in a way that insures her payoff against the possibility that the common player’s acceptances might differ from what she was expecting. As a result, quantities are always pairwise stable.

Lemma 2 Assuming rival accept-reject decisions are observed, in any equilibrium in which
player $i$ trades, player $i$’s quantity is pairwise stable: for all $i \in N$ such that $x_i^* > 0$,

$$x_i^* \in \arg \max_{x_i} g_i(x_i, x_{-i}^*) - f(x_i + X_{-i}^*).$$

(9)

Lemma 2 implies that pairwise stability holds for all players that trade a positive quantity, regardless of how many players trade, when accept-reject decisions are observable. As with unobservable accept-reject decisions, a consequence of this is that when more than one player trades, outcomes will generally be inefficient for a given number of players that trade. The restriction to more than one player trading arises because the pairwise-stable quantity when only one player trades is the monopoly quantity, which is efficient given that only one player is trading.

4.2 Equilibria with exclusion

Players can also use their contracts to make implied threats when they can observe the common player’s accept-reject decisions. For example, the contract in (27), which we use to show that player $i$’s quantity must be pairwise stable in any equilibrium in which player $i$ trades, has this feature. In it, player $i$ essentially threatens to purchase zero (or a sufficiently low quantity) from the common player if his acceptance set differs from what she was expecting. Not only does this provide insurance for player $i$ if the common player’s accept-reject decisions are unexpected, it also serves to dissuade the common player from making such unexpected decisions. This has important implications for our results on exclusion.

In our next result, we show that if rival accept-reject decisions are observable, then an equilibrium exists in which only one competing player trades. Assume players are numbered such that player 1 is the ‘strongest’ player and player $n$ is the ‘weakest’ in the sense that

$$\Pi_1^m \geq ... \geq \Pi_n^m.$$  

(10)

Then exclusion can arise as follows: player 1 offers the contract (appropriately discretized)

$$t_1(x_1) = \begin{cases} \Pi_2^m - \Pi_1^m, & \text{if } x_1 = 0 \\ f(x_1) + \Pi_2^m, & \text{if } x_1 > 0, \end{cases}$$

and for all $i \in \{2, ..., n\}$, competing player $i$ offers the contract (also discretized)

$$t_i(x_i) = \begin{cases} 0, & \text{if } x_i = 0 \\ f(x_i) + \Pi_i^m, & \text{if } x_i > 0. \end{cases}$$
In this case, if only player \( i \)’s contract is accepted, there is an equilibrium of the continuation game in which \( x_i = x_i^m \). And if multiple contracts are accepted, there is an equilibrium of the continuation game in which \( x_1 = \ldots = x_n = 0 \) (note that each player \( i \) is indifferent between \( x_i = 0 \) and \( x_i = x_i^m \) given \( x_{-i} = 0 \)). Given these strategies, it is a best reply for the common player to accept only player 1’s contract, giving player 1 a payoff of \( \Pi_1^m - \Pi_2^m \) and the common player a payoff of \( \Pi_2^m \). Following a deviation by one competing player, it is still the case that no more than one competing player trades in the continuation game, and the common player can still obtain a payoff of \( \Pi_2^m \) by trading with either player 1 or player 2. Thus, no player has a profitable deviation that would be accepted by the common player.

These contracts have the dual feature described above. They provide “insurance” for the competing players and they “punish” the common player if he accepts more than one offer. In the case of unobserved accept-reject decisions, competing player 1, for example, can only induce exclusion under limited circumstances (see Proposition 3) because she must worry about the common player accepting multiple offers, which would make her worse off. In the case of observed accept-reject decisions, however, player 1 would be indifferent if the common player accepted multiple offers. Moreover, if the common player were to accept multiple offers in this case, player 1 would simply choose \( x_1 = 0 \), leaving the common player worse off. Hence the common player will not accept more than one offer. Both features of the equilibrium contracts are important. Without the indifference, player 1’s implied threat to purchase zero from the common player if more than one offer is accepted might not be credible, and unless the common player were strictly worse off, equilibria in the continuation game might exist in which the common player accepts more than one offer.

Lemma 2 implies that in any equilibrium in which the common player trades only with player \( i \), the quantity traded must be \( x_i^m \). And if there is an equilibrium in which he trades only with player \( j \neq 1 \), then it must be that (10) holds with equality for players 1, 2, \ldots, \( j \) (because otherwise player 1 can profitably deviate). Thus, conditional on there being an equilibrium in which only one competing player trades, overall joint payoff is maximized.

**Proposition 4** Assuming rival accept-reject decisions are observed, there exists an equilibrium in which all but one competing players are excluded. In any such equilibrium, the competing player that trades has payoff \( \Pi_1^m - \Pi_2^m \) and the common player has payoff \( \Pi_2^m \).

The equilibria identified in Proposition 4 are inefficient because they involve exclusion, but the proposition shows that there are no further inefficiencies. This statement has two elements to it. First, conditional on trade between the common player and only one competing player, the quantity traded maximizes their bilateral payoff. Second, only the ‘strongest’ of the competing players trades with the common player. If another player could generate a
larger payoff for herself and the common player, then there would exist a profitable deviation that would induce the common player to accept her contract and not her rival’s contract.

### 4.3 Exclusion in all equilibria

Proposition 4 leaves open the possibility that equilibria may exist in which all players trade. However, as we now show, with a restriction on off-equilibrium-path beliefs, this is not possible. When rival accept-reject decisions are observed, all equilibria involve exclusion.

Before introducing the refinement on beliefs, it is useful to begin with a characterization of the common player’s payoff in any equilibrium $E=(t^*, I^*, x^*)$ in which all players trade.

**Lemma 3** Assuming rival accept-reject decisions are observed, in any equilibrium $E$ in which all players trade, the common player’s equilibrium payoff is given by $D^E_i = \ldots = D^E_n$, where

$$D^E_i \equiv \max_{I \subset N \setminus i} \Pi(x^*_i(t^*_1, I), \ldots, x^*_n(t^*_n, I)) - \sum_{j \in I} (g_j(x^*_i(t^*_1, I), \ldots, x^*_n(t^*_n, I)) - t_j(x^*_j(t^*_j, I)))$$

is the maximum payoff the common player can earn if he rejects competing player $i$’s offer.

Lemma 3 implies that in any equilibrium in which all players trade and rival accept-reject decisions are observed, the common player’s payoff is equal to his disagreement payoff with each competing player. To see this, note that if the common player’s payoff were higher than his disagreement payoff with competing player $i$, player $i$ could deviate so as to earn a higher payoff without affecting the common player’s decision whether to accept her offer. And if the common player’s payoff were lower than his disagreement payoff with one of the competing players, then the common player could profitably deviate by rejecting that player’s offer.

We now proceed to our refinement on beliefs. On the equilibrium path, each player $i$’s quantity maximizes her payoff given the quantities of all other competing players:

$$x^*_i \in \arg \max_{x_i} g_i(x_i, x^*_{-i}) - t^*_i(x_i).$$

However, if players observe that the common player has accepted a different set of contracts than what is specified by the equilibrium, they might believe the equilibrium contracts were offered but that the common player deviated, or they might believe there was a deviation in the initial contract offers. We abstract away from this distinction and focus only on the expected effect of an acceptance set different from the one defined by the equilibrium path.

Let $\hat{g}_E^i(x_i; I)$ denote player $i$’s expected utility from quantity $x_i$ if she observes that the common player has accepted the contracts of the players in $I$. Because beliefs are correct in equilibrium, it follows that $\hat{g}_E^i(x_i; I^*) = g_i(x_i, x^*_{-i})$. However, off the equilibrium path, there
is greater flexibility in player $i$’s beliefs, and so for $I \neq I^*$ there is less structure on $\hat{g}_i^E(x_i; I)$. Thus, in what follows, it is useful to restrict the beliefs of a non-deviating competing player in the off-equilibrium case in which that player observes that the common player has accepted a strict subset of the contracts that he would have accepted on the equilibrium path.

**Definition 1** We say that players have substitutes beliefs if for all $i \in N$ and $I \subset I^*$,

$$\hat{g}_i^E(x_i^*; I) \geq \hat{g}_i^E(x_i^*; I^*)$$

where the inequality is strict if $x_i^* > 0$ and $I^* \setminus I$ includes at least one player $j$ such that $x_j^* > 0$.

With substitutes beliefs, each competing player that trades expects to be better off if one or more of her rivals’ offers is unexpectedly rejected, even if she does not re-optimize and continues to choose her equilibrium-path quantity. In the case of $n = 2$, the assumption of negative externalities in the players’ payoffs (Assumption 1) is sufficient for players to have substitutes beliefs. However, Assumption 1 is not sufficient if $n > 2$ because then all players switch to a new equilibrium of the continuation game when a player’s contract is rejected.

**Proposition 5** Assuming rival accept-reject decisions are observed, there is no equilibrium with substitutes beliefs in which all competing players trade with the common player.

The combination of Lemma 3 and substitute beliefs implies that the expected bilateral joint payoff of the common player and each player $i$ that trades increases when a rival competing player is excluded. The common player’s payoff does not change if it rejects one of the competing players’ contracts, but each competing player that trades expects to benefit. It follows that competing players have an incentive to offer contracts that induce exclusion.

To see how this works, suppose there is an equilibrium in which all competing players trade with the common player. Suppose also that conditional on rejecting player $j$’s contract, the common player would maximize his payoff by accepting player $i$’s contract, and that player $i$’s beliefs about the off-equilibrium behavior of the remaining competing players are such that player $i$ has higher expected payoff in this off-equilibrium case (substitute beliefs). Let $\pi_i^* = g_i(x^*) - t_i^*(x_i^*)$ denote player $i$’s equilibrium payoff and let $\pi_i^{\text{not } j}$ be player $i$’s expected payoff in the continuation game following the common player’s rejection of player $j$’s contract (and possibly other contracts), achieved with $x_i = x_i^{\text{not } j}$. Then, for
\( \varepsilon \in (0, \frac{2}{3} (\pi_i^{\text{not}, j} - \pi_i^*) ) \), player \( i \) can profitably deviate by offering the following contract:

\[
t_i(x_i) \equiv \begin{cases} 
-\pi_i^* - \frac{\varepsilon}{2}, & \text{if } x_i = 0 \\
t_i^*(x_i) + \varepsilon, & \text{if } x_i = x_i^{\text{not}, j} \\
\infty, & \text{otherwise}
\end{cases}
\] (11)

If the common player rejects player \( i \)'s offer, and possibly others as well, then Lemma 3 implies that the common player's payoff is bounded above by \( D_i^\varepsilon \). If the common player maximizes his payoff conditional on rejecting player \( j \)'s contract, then his payoff is \( D_i^\varepsilon + \varepsilon \). (To see that in this case player \( i \) would choose \( x_i = x_i^{\text{not}, j} \), note that her incremental payoff from choosing \( x_i^{\text{not}, j} \) over zero is \( \pi_i^{\text{not}, j} - \varepsilon - \pi_i^* - \frac{\varepsilon}{2} > 0 \).) Thus, the common player prefers this over rejecting player \( i \)'s contract. It follows that player \( i \)'s contract is accepted in the continuation game after her deviation to the contract in (11). This deviation is profitable for player \( i \), regardless of which other contracts are accepted following her deviation because she can always choose \( x_i = 0 \) and guarantee herself a strictly higher payoff than in equilibrium.

### 4.4 Equilibria with multiple trading players

Proposition 5 shows that under the refinement of substitute beliefs, all equilibria involve exclusion when accept-reject decisions are observed. However, it does not indicate whether all equilibria involve only one competing player trading, as in the example in Section 4.2, or whether, with \( n > 2 \), there might be equilibria in which multiple competing players trade.

In our next result, we give conditions for equilibria to exist in which \( k \) players trade. For this result, it is useful to introduce additional notation. Given \( J \subset N \setminus \{i\} \), let \((x_i, \hat{x}_J, 0)\) be a vector with \( i \)th component \( x_i \), \( j \)th component \( \hat{x}_j \) for all \( j \in J \), and other components zero.

**Proposition 6** Assuming rival accept-reject decisions are observed, given \( k \in \{1, ..., n-1\} \) and pairwise-stable trade profile \((\hat{x}_1, ..., \hat{x}_k, 0)\), there exists an equilibrium satisfying substitutes beliefs with the trade profile \((\hat{x}_1, ..., \hat{x}_k, 0)\) if there exists \( \hat{t}_1, ..., \hat{t}_k \) such that \( \forall i \in \{1, ..., k\} \) and \( \forall J \subset \{1, ..., k\} \),

\[
0 \leq g_i(\hat{x}_1, ..., \hat{x}_k, 0) - \hat{t}_i,
\] (12)

\[
\sum_{j=1}^k \hat{t}_j - f(\sum_{j=1}^k \hat{x}_j) = \max_{j \in \{k+1, ..., n\}} \Pi_j^m,
\] (13)

and

\[
\max_{x_i, \hat{x}_i, \hat{x}_{J \setminus \{i\}}, 0} g_i(x_i, \hat{x}_{J \setminus \{i\}}, 0) - f(x_i + \sum_{j \in J \setminus \{i\}} \hat{x}_j) + \sum_{j \in J \setminus \{i\}} \hat{t}_j \\
\leq g_i(\hat{x}_1, ..., \hat{x}_k, 0) - \hat{t}_i + \max_{j \in \{k+1, ..., n\}} \Pi_j^m.
\] (14)

26
The proof of Proposition 6 constructs an equilibrium in which \( k \) competing players trade. Three conditions must hold for this equilibrium to exist. First, the competing players must have non-negative payoff. This is condition (12). Second, the common player’s equilibrium payoff must equal the maximum of the monopoly payoffs of the common player and each excluded competing player, \( \max_{j \in \{k+1, \ldots, n\}} \Pi^m_j \). This is condition (13). Third, a competing player must not be able to profit by changing her quantity and having the common player accept only a subset of the \( k \) trading players’ contracts. That is, the equilibrium joint payoff of player \( i \in \{1, \ldots, k\} \) and the common player (the right side of (14)) must be weakly greater than the maximized joint payoff of player \( i \) and the common player when the common player accepts the contracts of \( i \) and of a subset of players \( \{1, \ldots, k\}\{i\} \) (the left side of (14)).

As an example, we specialize to the case of \( n = 3 \) and assume that the common player has constant returns to scale everywhere, \( f(X) = cX \) for all \( X \geq 0 \). Assume also that player 1 is the strongest competing player (i.e., has the highest monopoly payoff), and that players 2 and 3 are symmetric. Then, as we now show, there is an equilibrium in which player 1 and either player 2 or player 3 trade if the duopoly payoff of each player that trades is more than one-half of the monopoly payoff of the common player and the player who does not trade.

Let \( (x^d_{12}, x^d_{21}) \) be the duopoly quantities of players 1 and 2, respectively, such that \( x^d_{12} \in \arg \max_{x_{12}} g_1(x_{12}, x_{21}, 0) - cx \) and \( x^d_{21} \in \arg \max_{x_{21}} g_2(x^d_{12}, x_{21}, 0) - cx \), and define \( \pi^d_{12} \equiv g_1(x^d_{12}, x^d_{21}, 0) - cx^d_{12} \) and \( \pi^d_{21} \equiv g_2(x^d_{12}, x^d_{21}, 0) - cx^d_{21} \). Then, if \( \pi^d_{12} > \frac{\pi^m_3}{2} \) and \( \pi^d_{21} > \frac{\pi^m_3}{2} \), an equilibrium in which players 1 and 2 trade can be supported with the following contracts:

\[
\begin{align*}
\text{for } i, j \in \{1, 2\} \text{ with } i \neq j, \quad t_i(x_i) &\equiv \begin{cases} 
- (\pi^d_{ij} - \frac{\pi^m_3}{2}), & \text{if } x_i = 0 \\
px + \frac{\pi^m_3}{2}, & \text{if } x_i = x^d_{ij} \\
\infty, & \text{otherwise}
\end{cases} \\
t_3(x_3) &\equiv \begin{cases} 
0, & \text{if } x_3 = 0 \\
px + \pi^m_3, & \text{if } x_3 = x^m_3 \\
\infty, & \text{otherwise}.
\end{cases}
\end{align*}
\]

One can view the contract between the common player and player 1 as specifying an upfront payment of \( \pi^d_{12} - \frac{\pi^m_3}{2} \) from the common player to competing player 1 and a payment of \( \pi^d_{12} + cx^d_{12} \) from competing player 1 to the common player if competing player 1 chooses her duopoly quantity. The contract between the common player and player 2 can be viewed similarly. In contrast, the contract in (16) involves no upfront payment and specifies a payment of \( cx^m_3 + \pi^m_3 \) from competing player 3 to the common player if competing player 3 chooses her monopoly quantity. In equilibrium, trade occurs with players 1 and 2 only. The common player has payoff \( \pi^m_3 \), player 1 has payoff \( \pi^d_{12} - \frac{\pi^m_3}{2} \), and player 2 has payoff \( \pi^d_{21} - \frac{\pi^m_3}{2} \).
It is straightforward to show that no player has a profitable deviation. The common player can obtain a payoff of $\pi^m_3$ by rejecting the contracts of players 1 and 2 and accepting player 3’s offer, so players 1 and 2 cannot extract more surplus. And, given player 3’s contract offer, player 3 does not trade unless she is a monopolist in the downstream market, so the common player cannot profitably deviate by accepting a different set of contract offers.

4.5 Wrong players may trade

One might think that the strongest player would necessarily be among the players that trade in any equilibrium with $k$ competing players. However, this is not implied by Proposition 6, and indeed it is easy to come up with settings in which the ‘wrong’ players may trade. To see this in the context of our example with $n = 3$, let $(x^d_{23}, x^d_{32})$ be the duopoly quantities of players 2 and 3, respectively, such that $x^d_{23} \in \arg \max_{x_2} g_2(0, x_2, x^d_{32}) - cx_2$ and $x^d_{32} \in \arg \max_{x_3} g_3(0, x^d_{23}, x_3) - cx_3$, and define $\pi^d_{23} \equiv g_2(0, x^d_{23}, x^d_{32}) - cx^d_{23}$ and $\pi^d_{32} \equiv g_3(0, x^d_{23}, x^d_{32}) - cx^d_{32}$.

Under the assumption that $2\pi^d_{23} > \pi^m_1$, there is an equilibrium in which only players 2 and 3 trade. One can support it with contracts that are analogous to those in (15) and (16). Specifically, player $i$, $i \in \{2, 3\}$, specifies $t_i(x_i) = -\left(\pi^m_{23} - \frac{\pi^m_1}{2}\right)$ if $x_i = 0$, $t_i(x_i) = cx^d_{23} + \frac{\pi^m_1}{2}$ if $x_i = x^d_{23}$, and $\infty$ otherwise, and player 1 specifies $t_1(x_1) = 0$ if $x_1 = 0$, $t_1(x_1) = cx^m_1 + \pi^m_1$ if $x_1 = x^m_1$, and $\infty$ otherwise. The common player earns $\pi^m_1$, and players 2 and 3 get $\pi^d_{23} - \frac{\pi^m_1}{2}$.

To see that the common player cannot profitably deviate, note that if he accepts player 1’s contract, then in the continuation game either player 1 chooses zero, and so the common player’s payoff is the same as if he had rejected player 1’s contract, or only player 1 chooses a positive quantity, in which case the common player’s payoff is $\pi^m_1$ minus payments of $\pi^d_{23} - \frac{\pi^m_1}{2}$ to players 2 and 3 if their contracts are accepted. To see that the competing players have no profitable deviation, note that the incremental payoff to player 3 from being a monopolist is $\pi^m_3 - \left(\pi^d_{23} - \frac{\pi^m_1}{2}\right)$, but the common player must be compensated an amount $\pi^m_1$ to induce him to reject player 1’s contract. Because $\pi^m_3 - \left(\pi^d_{23} - \frac{\pi^m_1}{2}\right) < \pi^m_1$, there is no profitable deviation.

We have seen that pairwise stability implies that the quantities traded in equilibrium are not efficient conditional on the number of players $k > 2$ that trade. This inefficiency has two possible sources. It may be that the ‘correct’ players trade, conditional on the number trading, but that they choose inefficient quantities. Or, it may be that the ‘wrong’ players trade, conditional on the number trading. In particular, there may be equilibria in which weaker, less efficient players trade while stronger, more efficient players are excluded.

To continue with our example, if the sum of the duopoly profits when two competing players trade is higher when players 1 and 2 trade than when players 2 and 3 trade (i.e., if $\pi^d_{12} + \pi^d_{21} > 2\pi^d_{23}$), then the efficient outcome, conditional on two competing players trading,
is for players 1 and 2 to trade. In this case, the wrong players are trading in the equilibrium in this subsection. If instead $\pi_{12}^d + \pi_{21}^d < 2\pi_{23}^d$, then the opposite is true, and the efficient outcome, conditional on two competing players trading, is for players 2 and 3 to trade. In this case, the correct players trade in the equilibrium in this subsection, but the wrong players trade in the equilibrium that is supported by the contracts in (15) and (16).

Because both types of equilibria can coexist, one might wonder whether a simple selection criterion can be used to rule out equilibria in which the wrong players trade. To continue with our example, it is clear that all players prefer an outcome in which they are not excluded. Thus, for example, competing player 3 prefers the equilibrium in this subsection, whereas competing player 1 prefers the equilibrium in the previous subsection. Moreover, we know from Proposition 6 that the common player is better off when the strongest player is among the players who are excluded. Thus, for example, if $\pi_{12}^d + \pi_{21}^d > 2\pi_{23}^d$, then we know that the common player would prefer the equilibrium in which the wrong players trade to the equilibrium in which the correct players trade. However, player 2’s preferences are ambiguous in general. Her payoff in the equilibrium in which she trades with player 1 is $\pi_{21}^d - \frac{\pi_m^d}{2}$, whereas her payoff in the equilibrium in which she trades with player 3 is $\pi_{23}^d - \frac{\pi_m^d}{2}$. If $\pi_{23}^d > \pi_{21}^d$ then either payoff can dominate. It thus follows, surprisingly, that conditions exist in which three of the four players would strictly prefer the equilibrium in which the wrong players trade.

5 Extensions

In this section, we consider three extensions. First, we extend the model to allow for cost functions that do not exhibit constant returns to scale but nevertheless are additively separable. Second, we consider the possibility that each player’s offer can be made contingent on all players’ quantities (assuming rival accept-reject decisions are observable). Third, we discuss an environment in which each competing player can offer a menu of contracts.

5.1 Additively separable costs

In this subsection, we assume that the common player’s costs are additively separable. In other words, we suppose that $f$ depends on the vector of quantities purchased, such that $f(x) = \sum_{i=1}^{n} f_i(x_i)$, where each individual $f_i$ is non-decreasing and weakly convex. Proofs for the results of this subsection are contained in the Online Appendix.

When costs are additively separable, as they might be, for example, if the common player’s inputs are customized for each competing player, there are no externalities in the common player’s cost function. This has implications for the results in Section 3 as follows:
Proposition 7 Assuming rival accept-reject decisions are not observed, if the costs of the common player are additively separable, there exists an equilibrium in which all players trade. Moreover, in all such equilibria, the common player earns zero payoff.

Proposition 7 establishes that when the common player’s costs are additively separable, and accept-reject decisions are unobserved, equilibria exist in which all players trade (it is not possible to induce exclusion even when costs are strictly convex), and the common player necessarily earns zero payoff. These results contrast with those in Section 3 and are analogous to our findings when the common players’ costs exhibit constant returns to scale.

When rival accept-reject decisions are observed, however, the fact that costs may be additively separable is of no significance. There exist equilibria in which only one competing player trades, and there is no equilibrium in which all players trade with substitute beliefs.

Proposition 8 Assuming rival accept-reject decisions are observed, if the common player’s costs are additively separable, there exists an equilibrium in which only one competing player trades, there does not exist an equilibrium satisfying substitute beliefs in which all players trade, and conditions exist with substitute beliefs in which \( k \in \{1, ..., n - 1\} \) players trade.

The reason why the results differ sharply in the case of unobserved rival accept-reject decisions, but do not change in the case of observed rival accept-reject decisions is simple. In the former case, the externalities that can be exploited are all on the cost side. When these externalities are absent, equilibria in which all players trade necessarily exist and the common player necessarily earns zero payoff. In contrast, in the latter case, the externalities that can be exploited are in the competing players’ utilities, and the externalities that arise there are unaffected by different assumptions about the common player’s costs.

5.2 Contingent contracts

We now extend the model to allow competing player \( i \)’s payment to the common player to depend not only on the quantity that player \( i \) trades, but also on the quantities that the other competing players trade with the common player. We show that having the ability to condition on the full vector of quantity choices does not eliminate the existence of an exclusionary equilibrium, but it does allow equilibria in which all competing players trade.

We begin by showing that when contracts can depend on all competing players’ quantities, there exist equilibria in which all competing players trade with the common player. In fact, we show a stronger result: there exists an equilibrium in which all players trade even if contracts can only condition on whether the other players’ quantities are zero or positive.
To state this result, we continue to let \( \Pi_i^m \) denote the maximized joint payoff of the common player and player \( i \) when only player \( i \) trades with the common player, and we let \( \Pi^m \) denote the maximized value of overall joint payoff, with maximizing trade vector \( x^m \).

**Proposition 9** Assuming rival accept-reject decisions are observed, if contracts can depend on a player’s own quantity and whether rival quantities are zero or positive, there exists an equilibrium in which all players trade. More specifically, given nonnegative \( \pi_0, \pi_1, ..., \pi_n \) such that \( \pi_0 \geq \sum_{j=1}^{n} (\Pi_j^m - \Pi^m) \), \( \forall i \in N \), \( \pi_0 \geq \Pi_i^m - \pi_1 \), and \( \sum_{i=0}^{n} \pi_i = \Pi^m \), there exists an equilibrium with trade vector \( x^m \) and payoff \( \pi_0 \) for the common player and \( \pi_i \) for player \( i \).

To see the intuition for this result, note that because the joint payoff of the common player and competing player \( i \) is at least \( \Pi_i^m \), no deviation that results in the exclusion of the other competing players can increase their joint payoff. And, because overall joint payoff is maximized at \( \Pi^m \), there is also no possible deviation that can increase overall joint payoff.

Although Proposition 9 shows that, with more general contracts, overall joint payoff can be maximized in equilibrium, there also continue to be equilibria in which exclusion occurs. (The proof follows from the proof of Proposition 4.)

**Proposition 10** Assuming rival accept-reject decisions are observed, if contracts can depend on a player’s own quantity and whether rival quantities are zero or positive, there exists an equilibrium in which all but one competing players are excluded.

Proposition 10 implies that moving to a contracting environment in which competing players’ contracts can depend on rival players’ quantity choices does not guarantee that overall joint payoff will be maximized in equilibrium. To the contrary, there continue to be equilibria in which some competing players are excluded from trade with the common player.

### 5.3 Menus of contracts

Rey and Whinston (2013) show in an environment with observable accept-reject decisions and observable contracts that equilibria in bilateral-contracting games may be affected by the ability of competing players to offer menus of contracts from which the common player can choose. They show that, in contrast to Marx and Shaffer (2007), who do not allow menus and get inefficiency (overall joint payoff below its maximum) and exclusion in all equilibria, when menus are permitted, equilibria exist that are efficient and do not involve exclusion. The introduction of menus means that the common player can be made indifferent in equilibrium between accepting the equilibrium path contracts from each menu and rejecting any one menu while accepting off-equilibrium contracts from the other \( n - 1 \) players.
In the environment of the current paper in which rivals’ contract terms are not observed, a straightforward extension of Lemmas 1 and 2 shows that in any equilibrium in which all players trade, pairwise stability must be satisfied. Thus, outcomes will generally be inefficient. Furthermore, when contract terms are not observed, exclusion can arise in equilibrium even when menus are permitted, depending on off-equilibrium beliefs. For example, starting from a candidate equilibrium in which all trade, if the necessary conditions of Proposition 1 are not satisfied, a profitable deviation exists for some player as long as the other players view the observation that the common player has selected an off-equilibrium contract from their menu as sufficiently bad news regarding their future payoff in the downstream market.

6 Conclusion

We have considered a bilateral-contracting game in which competing Cournot firms purchase inputs from a common input supplier based on contracts, with the competing firms having the bargaining power. We show that when one endogenizes the cost functions of the competing firms through the addition of a contracting stage with the common supplier, equilibria in which all firms trade obtain only under restrictions on the cost function of the supplier and are sensitive to whether the competing firms can observe which of their rivals have contracts with the upstream firm. Our findings thus contribute to an understanding of when the Cournot model and the implications that have been derived from it are applicable.

In modeling the strategic interaction between Cournot downstream firms and a common input supplier, we found that Cournot models in which the downstream firms have common, constant marginal costs are consistent with firms obtaining their inputs from a common input supplier, whereas Cournot models in which this is not the case are not consistent with firms obtaining their inputs from a common input supplier. We also found that when the supplier’s costs are not linear in the relevant range or the competing firms can observe the common supplier’s accept-reject decisions, equilibrium outcomes necessarily involve the exclusion of one or more downstream firms and may involve the ‘wrong firms’ competing in the sense that less profitable firms are active while more profitable firms are excluded.

As we show, these findings have implications for policy ranging from the role of buyer power in horizontal merger policy and the engagement of firms in exclusionary conduct on the one hand, to the role of trade subsidies and the promotion of national champions on the other. With respect to the role of buyer power in merger policy, we identify conditions under which the buyers are able to keep in check the market power that a common supplier might otherwise have, and we identify conditions under which an upstream merger can have deleterious effects in the downstream market. With respect to the engagement of firms in
exclusionary conduct, we show that the usual precursors for exclusion need not be present. For example, there need not be economies of scale in production, the excluding firm may be more or less efficient than the excluded firm, and lump-sum payments may often be given to the excluding firm rather than paid by the excluding firm. Lastly, with respect to international trade policy, we show that subsidizing exports and/or creating national champions can backfire when the downstream firms purchase their inputs from a common input supplier, making countries that engage in such policies potentially worse off than under laissez faire.

More generally, our findings shed light on any bilateral-contracting environment in which one player trades with multiple competing players, but in which the competing players have both the bargaining power and make the payoff-relevant choices. While these environments have been the subject of increasing scrutiny in the policy arena, particularly with respect to the practice of powerful retailers demanding slotting allowances from small manufacturers, the theoretical literature has mostly focused on situations in which the player or the players making the offers differ from the player or the players making the payoff-relevant choices.

Given our results on exclusion and the result that quantities are pairwise stable in all equilibria in which all players trade, we found that, unlike in the typical models of common agency with contracts that depend only on the quantity transacted between the two parties, overall joint payoff is never maximized in equilibrium. The inefficiencies that we identify differ from Segal’s (1999) “direct externalities,” which stem from the fact that the quantity choices of one downstream firm impose externalities on the other downstream firms, and differ from Prat and Rustichini’s (2003) “strategic externalities,” which arise in an environment with multiple upstream and multiple downstream firms but no direct externalities among firms. The inefficiencies we identify do not arise at all in the standard common agency environment, where Bernheim and Whinston (1998, p. 74) show that efficient equilibria exist even if “forcing contracts” are allowed. Thus, we have identified a new source of inefficiency.

A Appendix: Proofs

Proof of Lemma 1. Assume an equilibrium with contracts $t^*$ and strictly positive trade vector $x^*$. Suppose (1) does not hold. Then there exists $i$ and $\hat{x}_i \in X_i$ such that $g_i(\hat{x}_i, x^*_i) - f(\hat{x}_i + X^*_i) > g_i(x^*) - f(X^*)$. Let $\varepsilon \in (f(\hat{x}_i + X^*_i) - f(X^*), g_i(\hat{x}_i, x^*_i) - g_i(x^*))$. Consider
a deviation offer by player $i$ of

$$t_i(x_i) \equiv \begin{cases} 
t^*_i(x^*_i) + \varepsilon, & \text{if } x_i = \hat{x}_i \\
\infty, & \text{otherwise}. \end{cases} \quad (17)$$

If the common player accepts all offers, then the trade vector is $(\hat{x}_i, x^*_{-i})$ (players other than player $i$ do not observe the deviation and so continue to choose their equilibrium quantities). In this case both the common player and player $i$ have payoffs greater than their equilibrium payoffs. If the common player rejects player $i$’s offer, his payoff is bounded above by his equilibrium payoff. Thus, the common player accepts player $i$’s offer in any equilibrium of the continuation game. If the common player accepts player $i$’s offer but rejects other offers, then by Assumption 1 player $i$ is even better off. Thus, player $i$’s deviation is profitable, contradicting the assumption of an equilibrium. Thus, (1) holds.

Suppose (2) does not hold. Then either $t^*_i(x^*_i) > f(X^*) - f(X^*_{-i})$ or $t^*_i(x^*_i) < f(X^*) - f(X^*_{-i})$. If the former holds, player $i$ could profitably deviate by offering the common player a forcing contract at quantity $x^*_i$ with a payment less than $t^*_i(x^*_i)$ but greater than $f(X^*) - f(X^*_{-i})$, which the common player strictly prefers to accept. If the latter holds, the common player could profitably deviate by rejecting $i$’s offer. Thus, (2) holds.

To see that (3) holds, note that competing players can earn a payoff of zero by offering null contracts and the common player can earn a payoff of zero by rejecting all offers. Q.E.D.

**Proof of Proposition 1.** Assume $x^* \in X^*$ and that for some $c > 0$ and $b \geq 0$, for all $X \geq \min_{i \in N} X^*_{-i}$, $f(X) = cX - b$. We show that there exists an equilibrium in with trade vector $x^*$ and contracts

$$t_i(x) = \begin{cases} cx_i, & \text{if } x_i = x^*_i \\
\infty, & \text{otherwise}. \end{cases}$$

Because $x^*$ is pairwise stable, it follows that for all $i \in N$, $g_i(x^*) - f(X^*) \geq -f(X^*_{-i})$, which implies that for all $i \in N$, $g_i(x^*) - cx^*_i \geq 0$. Thus, if all contracts are accepted, all competing players have nonnegative payoff. The common player’s payoff from accepting any nonempty subset of offers is less than or equal to his equilibrium payoff of $b$, so the common player cannot profitably deviate from accepting all contracts. As part of the common player’s strategy in the face of a deviation by a single competing player, assume that the common player continues to accept all non-deviating contracts as long as he is at least indifferent between accepting and rejecting these.

Consider a deviation by player $i$ to contract $t^d_i$ that results in a deviation quantity for player $i$ of $x^d_i$ if her deviation contract is accepted. The common player’s payoff if he accepts just the contract of the deviating player is $t^d_i(x^d_i) - f(x^d_i)$. If the common player accepts
the deviation contract plus the contract of any number of other players, his payoff is greater than or equal to \( t_i'(x_i^d) - f(x_i^d) \) because, using the weak convexity of \( f \) and the assumption that for all \( X \geq \min_{i \in N} X_{-i}^* \), \( f(X) = cX - b \), the incremental cost to the common player of the additional units of output is less than or equal to the incremental payments given by the contracts \( t_{-i} \) of the other players. Thus, if the common player accepts the deviating player’s contract, he also accepts the contracts of all other players. Thus, by pairwise stability, player \( i \) does not have a profitable deviation. This completes the proof of the first part of the proposition.

To prove the second part of the proposition, assume an equilibrium with contracts \( t^* \) and strictly positive trade vector \( x^* \). Let \( \pi_0 \) be the common player’s equilibrium payoff. By Lemma 1, \( x^* \in X^* \) and \( t_i'(x_i^*) = f(X^*) - f(X_{-i}^*) \). Suppose that it is not the case that there exists \( c > 0 \), and \( b \geq 0 \) such that for all \( X \in [\min_{k \in N} X_{-k}^*, X^*] \), \( f(X) = cX - b \). It then follows using the weak convexity of \( f \) that for all \( \delta > 0 \),

\[
      f(X^* + \delta) - f(\min_{k \in N} X_{-k}^* + \delta) > f(X^*) - f(\min_{k \in N} X_{-k}^*). \tag{18}
\]

For all \( j \neq i \), let \((x_i^*, x_{i,j}^*, 0)\) denote the vector \( x^* \) with the \( j \)th element replaced by zero. By Assumption 1, for all \( j \neq i \), \( g_i(x_i^*, x_{i,j}^*, 0) > g_i(x^*) \). Given the assumption of a continuous environment, \( g_i \) is continuous in \( x_i^* \), so using \( x^* \) strictly positive, there exists \( \delta' > 0 \) such that \( \forall j \neq i, \forall \delta \in (0, \delta') \),

\[
      \frac{1}{2}(g_i(x_i^*, x_{i,j}^*, 0) + g_i(x^*)) < g_i(x_i^* + \delta, x_{i,j}^*, 0),
\]

which we can rewrite as

\[
      \frac{1}{2}(g_i(x_i^*, x_{i,j}^*, 0) - g_i(x^*)) < g_i(x_i^* + \delta, x_{i,j}^*, 0) - g_i(x^*). \tag{19}
\]

Furthermore, given the assumption of a continuous environment, \( f \) is continuous, so there exists \( \delta'' > 0 \) such that \( \forall j \neq i, \forall \delta \in (0, \delta'') \),

\[
      f(X^* + \delta) - f(X^*) < \frac{1}{2}(g_i(x_i^*, x_{i,j}^*, 0) - g_i(x^*)).
\]

It then follows, using (19), that \( \forall \delta \in (0, \min \{\delta', \delta''\}) \),

\[
      f(X^* + \delta) - f(X^*) < \min_{j \neq i} \left( g_i(x_i^* + \delta, x_{i,j}^*, 0) - g_i(x^*) \right). \tag{20}
\]

Let \( \hat{\delta} \in (0, \min \{\delta', \delta''\}) \). Let player \( i \) be such that \( X_{-i}^* = \min_{k \in N} X_{-k}^* \). We show that player
\( i \neq i \) can profitably deviate by offering

\[
\tilde{t}_i(x_i) \equiv \begin{cases} 
  f(X^* + \hat{\delta}) - f(X^*_{-i}) + \varepsilon, & x_i = x^*_i + \hat{\delta} \\
  \infty, & \text{otherwise},
\end{cases}
\]

where \( \varepsilon \in \left(0, \min_{j \neq i} \left(g_i(x^*_i + \hat{\delta}, x^*_{-i,j}, 0) - g_i(x^*)\right) - f(X^* + \hat{\delta}) + f(X^*)\right) \). (Inequality (20) establishes that \( \varepsilon \) is well defined.) The common player’s payoff if he accepts all contracts is \( \pi_0 + \varepsilon \). If the common player rejects player \( i \)'s offer, then his payoff is bounded above by \( \pi_0 \). Thus, the common player accepts player \( i \)'s offer in the continuation game following the deviation. If the common player accepts player \( i \)'s offer, rejects the offer of player \( \hat{i} \), and accepts all other offers, then the common player’s payoff is

\[
\pi_0 - t^*_i(x^*_i) - t^*_i(x^*_i) + \tilde{t}_i(x^*_i) + \hat{\delta} + f(X^*) - f(X^* - x^*_i + \hat{\delta})
\]

\[
= \pi_0 - (f(X^*) - f(X^*_{-i})) + f(X^* + \hat{\delta}) - f(X^*_i) + \varepsilon + f(X^*) - f(X^* - x^*_i + \hat{\delta})
\]

\[
= \pi_0 + f(X^* + \hat{\delta}) - f(\min_{k \in N} X^*_{-k} + \hat{\delta}) - f(X^*) - f(\min_{k \in N} X^*_{-k}) + \varepsilon
\]

\[> \pi_0 + \varepsilon,\]

where the inequality uses (18). Thus, the common player accepts player \( i \)'s offer in the continuation game and rejects at least one rival player’s offer. To see that the deviation is profitable for player \( i \) if the common player rejects only one other player’s offer, note that for all \( j \neq i \),

\[
g_i(x^*_i + \hat{\delta}, x^*_{-i,j}, 0) - \tilde{t}_i(x^*_i) = g_i(x^*_i + \hat{\delta}, x^*_{-i,j}, 0) - f(X^* + \hat{\delta}) + f(X^*_i) - \varepsilon
\]

\[> g_i(x^*) - f(X^*) + f(X^*_i)
\]

\[= g_i(x^*) - t^*(x^*_i),\]

where the inequality uses the definition of \( \varepsilon \). If additional rival offers are also rejected, then player \( i \)'s payoff increases further. Thus, the deviation is profitable, a contradiction. Q.E.D.

**Proof of Proposition 3.** Let \( i \) be such that \( \Pi_i = \max_j \Pi_j \) and assume that for all \( j \neq i \), condition (7) holds. Let \( k \) be such that \( \Pi_k \geq \max_{j \neq i} \Pi_j \) and such that (8) holds. We show there is an equilibrium in which player \( i \) offers

\[
t_i(x_i) = \begin{cases} 
  \Pi_k + f(x^m_i), & \text{if } x_i = x^m_i \\
  \infty, & \text{otherwise},
\end{cases}
\]

\[36\]
player $k$ offers
\[ t_k(x_k) = \begin{cases} 
    g_k(x_k^m, 0), & \text{if } x_k = x_k^m \\
    \infty, & \text{otherwise},
\end{cases} \]
and players $j$ different from $i$ and $k$ offer
\[ t_j(x_j) = \begin{cases} 
    0, & \text{if } x_j = 0 \\
    \infty, & \text{otherwise}.
\end{cases} \]
In the equilibrium, the common player accepts only $i$'s contract, player $i$ chooses $x_i = x_i^m$, and has payoff $\Pi_i - \Pi_k$, which is non-negative by the assumption that $\Pi_i = \max_j \Pi_j$.

If the common player accepts only $i$'s contract or only $k$'s contract, his payoff is $\Pi_k$. If the common player accepts both contracts, his payoff is weakly less because
\[
    t_i(x_i^m) + t_k(x_k^m) - f(x_i^m + x_k^m) = g_k(x_k^m, 0) - f(x_i^m + x_k^m) + f(x_i^m) + \Pi_k \\
    \leq \max_{x_k} (g_k(x_k, 0) - f(x_i^m + x_k) + f(x_i^m)) + \Pi_k \\
    \leq \Pi_k,
\]
where the equality uses the definitions of $t_i$ and $t_k$, the first inequality uses the definition of the maximum, and the second inequality uses (8). Thus, it is a best reply for the common player to accept only $i$'s contract. The fact that $k$'s contract offers payoff $\Pi_k$ to the common player and Assumption 1 guarantee that player $i$ has no profitable deviation. The fact that $i$'s contract offers payoff $\Pi_k$ to the common player and condition (7) imply that players other than $i$ have no profitable deviation. Q.E.D.

**References**


