B Online Appendix: Formal statements of results and proofs for Section 3.4

This appendix contains the formal statements of propositions and the associated proofs for the material in Section 3.4.

Outside competition

Proposition B.1. *Assuming symmetric suppliers and a fixed set of coordinators K, (i) the market is not at risk if n is sufficiently large, i.e., there exists $\underline{n} > k$ such that for all $n > \underline{n}$, $\mathcal{I}^S(K) < 0$, and (ii) for the power-based distribution, $\mathcal{I}^S(K)$ decreases in n.*

Proof of Proposition B.1. In the limit as n grows large, with probability 1, both the lowest and second-lowest order statistics are less than the reserve r. Thus, we can focus on the case in which the reserve does not bind, in which case $\Pi_i = \frac{1}{n}\mathbb{E}\left[c_{(2:n)} - c_{(1:n)}\right]$ and $\Pi_i(K) = \frac{1}{n-k+1}\mathbb{E}\left[c_{(2:n-k+1)} - c_{(1:n-k+1)}\right]$. It is then sufficient to show that there exists $\underline{n}$ such that for all $n > \underline{n}$,

$$\frac{n-k+1}{n}\frac{\mathbb{E}\left[c_{(2:n)} - c_{(1:n)}\right]}{\mathbb{E}\left[c_{(2:n-k+1)} - c_{(1:n-k+1)}\right]} > \frac{1}{k}. \tag{11}$$

As shown by Loertscher and Marx (2020, Lemma 1),

$$j\mathbb{E}[c_{(j+1:n)} - c_{(j:n)}] = \mathbb{E}\left[\frac{G(c_{(j:n)})}{g(c_{(j:n)})}\right],$$

which, using $\mathbb{E}\left[\frac{G(c_{(1:\ell)})}{g(c_{(1:\ell)})}\right] = \int_{\underline{c}}^{\overline{c}} \ell G(x)(1-G(x))^{\ell-1}dx$, allows us to write (11) as

$$\frac{\int_{\underline{c}}^{\overline{c}} G(x)(1-G(x))^{n-1}dx}{\int_{\underline{c}}^{\overline{c}} G(x)(1-G(x))^{n-k}dx} - \frac{1}{k} > 0,$$

which we can write as

$$\frac{\int_{\underline{c}}^{\overline{c}}(1-G(x))^{n-k}H(x)dx}{k\int_{\underline{c}}^{\overline{c}} G(x)(1-G(x))^{n-k}dx} > 0, \tag{12}$$

where

$$H(x) \equiv kG(x)(1-G(x))^{k-1} - G(x).$$

Note that $H(\underline{c}) = 0$, $H'(\underline{c}) > 0$, and $H(x) < 1$ for all $x \in [\underline{c}, \overline{c}]$. Thus, there exists $c^* \in (\underline{c}, \overline{c})$ such that for all $x \in (\underline{c}, c^*)$,

$$H(x) \in (0,1). \tag{13}$$

In addition, note that for all $x \in [\underline{c}, \overline{c}]$,

$$H(x) \geq -1, \tag{14}$$

with equality only at $x = \overline{c}$.

Because the denominator on the left side of (12) is positive for all n, it is sufficient to show that there exists $\underline{n}$ such that the numerator is positive for all $n > \underline{n}$. Letting $\hat{c} \in (\underline{c}, c^*)$, we have

$$\begin{aligned}
& \int_{\underline{c}}^{\overline{c}} (1 - G(x))^{n-k} H(x) dx \\
= & \int_{\underline{c}}^{c^*} (1 - G(x))^{n-k} H(x) dx + \int_{c^*}^{\overline{c}} (1 - G(x))^{n-k} H(x) dx \\
> & \int_{\underline{c}}^{\hat{c}} (1 - G(x))^{n-k} H(x) dx - \int_{c^*}^{\overline{c}} (1 - G(x))^{n-k} dx \\
> & (1 - G(\hat{c}))^{n-k} \int_{\underline{c}}^{\hat{c}} H(x) dx - (1 - G(c^*))^{n-k} (\overline{c} - c^*) \\
= & (1 - G(\hat{c}))^{n-k} \left[\int_{\underline{c}}^{\hat{c}} H(x) dx - \left(\frac{1 - G(c^*)}{1 - G(\hat{c})} \right)^{n-k} (\overline{c} - c^*) \right],
\end{aligned}$$

where the first inequality, which replaces the upper bound of integration in the first integral with $\hat{c} \in (\underline{c}, c^*)$ and replaces $H(x)$ in the second integral with -1, uses (13) and (14); the second inequality uses the properties of G as a cdf; and the final equality rearranges. Setting the expression in square brackets in the last line equal to zero and solving for n, we define

$$\underline{n} \equiv \ln \left(\frac{\int_{\underline{c}}^{\hat{c}} H(x) dx}{\overline{c} - c^*} \right) / \ln \left(\frac{1 - G(c^*)}{1 - G(\hat{c})} \right) + k > k,$$

where, using (13) and the properties of G as a cdf, the two logarithms are both negative, which gives the result that $\underline{n} > k$. It follows that for all $n > \underline{n}$, $\int_{\underline{c}}^{\overline{c}} (1 - G(x))^{n-k} H(x) dx > 0$, and so (12) holds for all $n > \underline{n}$, which completes the proof of the first part of the proposition.

Turning to the second part of the proposition, using Lemma 1, under symmetry and the power-based distribution,

$$I^S(K) = 1 - k \frac{(1 + (n - k + 1)\alpha)(1 + (n - k)\alpha)}{(1 + (n - 1)\alpha)(1 + n\alpha)}.$$

Differentiating with respect to n, we have

$$\frac{\partial I^S(K)}{\partial n} = -\frac{a^2 (k - 1) k \left[2 - 2a(k - 2n) + a^2 (k - 2kn + 2n^2) \right]}{(1 + a(n - 1))^2 (1 + an)^2},$$

which has sign equal to the sign of

$$-2 - 2a(2n - k) - a^2(k - 2kn + 2n^2)$$

which is increasing in k and at $k = n$ is equal to $-2 - 2an - a^2 n$, which is negative. Thus, $I^S(K)$ is decreasing in n for all $k \in \{2, ..., n\}$. ■

Supplier strength

Proposition B.2. *For the power-based parameterization with $v \geq \bar{c}$ and symmetric suppliers in K, as the strength of suppliers in K decreases, eventually the market is not at risk, that is,* $\lim_{\alpha \to 0} \mathcal{I}^S(K) < 0$, *and as the strength of suppliers in K increases, eventually the market is at risk, that is,* $\lim_{\alpha \to \infty} \mathcal{I}^S(K) > 0$.

Proof. The proof follows straightforwardly from Lemma 1. ■

Proposition B.3. *For the power-based parameterization with $v \geq \bar{c}$, $\mathcal{I}^S(K)$ is largest, conditional on $|K| = k$ if K includes the k strongest suppliers. Further, increasing the strength of the suppliers in K, while holding fixed the distribution of the lowest cost draw in the market, causes $\mathcal{I}^S(K)$ to increase. That is, given parameters $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$ satisfying $\sum_{i \in N} \alpha_i = \sum_{i \in N} \beta_i$ and $\beta_i \geq \alpha_i$ for all $i \in K$ with a strict inequality for at least one i, $\mathcal{I}^S(K; \boldsymbol{\beta}) > \mathcal{I}^S(K; \boldsymbol{\alpha})$.*

Proof of Proposition B.3. We begin by proving the first statement of the proposition. If $n = k$, then the result holds trivially, so assume that $n > k$. Without loss of generality, assume that $K = \{1, ..., k\}$. Using Lemma 1, we have

$$\sum_{i \in K} s_i(K) = \sum_{i \in K} \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_i)(1 + A)}.$$

Because we assume that $1 \in K$, we can rewrite this as

$$\sum_{i \in K} s_i(K) = \sum_{i \in K \setminus \{1\}} \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_1)(1 + A)}.$$

Suppose we deduct $\varepsilon \in (0, \alpha_n)$ from α_n and add ε to α_1. Then the sum of all suppliers' strength parameters, A, remains unchanged, and we have

$$\begin{aligned} &\mathcal{I}^S(K; \alpha_1 + \varepsilon, \alpha_2, ..., \alpha_{n-1}, \alpha_n - \varepsilon) \\ = \; &1 - \left[\sum_{i \in K \setminus \{1\}} \frac{(1+\alpha_i+A_{-K}-\varepsilon)(1+A_{-K}-\varepsilon)}{(1+A-\alpha_i)(1+A)} + \frac{(1+\alpha_1+A_{-K})(1+A_{-K}-\varepsilon)}{(1+A-\alpha_1-\varepsilon)(1+A)} \right]. \end{aligned}$$

Differentiating with respect to ε, we have

$$-\frac{1}{1+A}\left[\sum\nolimits_{i\in K\backslash\{1\}}\frac{-(1+A_{-K}-\varepsilon)-(1+\alpha_i+A_{-K}-\varepsilon)}{1+A-\alpha_i}-\frac{(1+\alpha_1+A_{-K})(A-\alpha_1-A_{-K})}{(1+A-\alpha_1-\varepsilon)^2}\right],$$

which is positive. Thus, for all $\varepsilon\in(0,\alpha_n)$,

$$\mathcal{I}^S(K;\alpha_1+\varepsilon,\alpha_2,...,\alpha_{n-1},\alpha_n-\varepsilon)>\mathcal{I}^S(K;\alpha_1,\alpha_2,...,\alpha_{n-1},\alpha_n). \tag{15}$$

If follows that $\mathcal{I}^S(K)$ increases when a member of K is replaced with a member of $N\backslash K$ with a larger distributional parameter and that $\mathcal{I}^S(K)$ is maximized when its members are the set K of suppliers with the largest distributional parameters. This completes the proof of the first sentence of the proposition.

Turning to the second sentence of the proposition, let $\boldsymbol{\beta}$ be as given in the statement of the proposition. Because $\mathcal{I}^S(K)$ relies on the strength parameters of suppliers in $N\backslash K$ only through the sum of those parameters, we have

$$\mathcal{I}^S(K;\alpha_1,...,\alpha_k,\alpha_{k+1},...,\alpha_n)=\mathcal{I}^S(K;\alpha_1,...,\alpha_k,\beta_{k+1},...\beta_{n-1},A_{-K}-\sum_{i=k+1}^{n-1}\beta_i),$$

where $A_{-K}-\sum_{i=k+1}^{n-1}\beta_i>0$ by the assumptions on $\boldsymbol{\beta}$. But then, letting $\varepsilon_i\equiv\beta_i-\alpha_i\geq 0$ for $i\in K$, we have

$$\begin{aligned}&\mathcal{I}^S(K;\alpha_1,...,\alpha_k,\beta_{k+1},...\beta_{n-1},A_{-K}-\sum_{i=k+1}^{n-1}\beta_i)\\ <\ &\mathcal{I}^S(K;\alpha_1+\varepsilon_1,...,\alpha_k+\varepsilon_k,\beta_{k+1},...\beta_{n-1},A_{-K}-\sum_{i=k+1}^{n-1}\beta_i-\sum_{i=1}^{k}\varepsilon_i)\\ =\ &\mathcal{I}^S(K;\beta_1,...,\beta_k,\beta_{k+1},...\beta_{n-1},\beta_n),\end{aligned}$$

where the inequality uses the repeated application of (15) and the assumption that ε_i is strictly positive for at least one $i\in K$, and where the final inequality uses the definition of ε_i and the assumption that $\sum_{i\in N}\beta_i=A$. This completes the proof. ■

Symmetry

Proposition B.4. *For the power-based parameterization, an NR spread applied to two suppliers in K reduces $\mathcal{I}^S(K)$.*

Proof of Proposition B.4. We show that for the power-based parameterization, an NR spread applied to two suppliers in K reduces $\mathcal{I}^S(K)$. We denote the two suppliers experiencing the

NR spread as 1 and 2, with $\{1, 2\} \subseteq K$. Let F_1 and F_2 be their distributions prior to the spread, with parameters f_1 and f_2, and let H_1 and H_2 be their distributions after the spread, with parameters h_1 and h_2. Note that we abuse notation by using, for $i \in \{1, 2\}$, f_i and h_i to denote the parameters of the distributions F_i and H_i, rather than their pdfs. Denote the parameters of suppliers other than 1 and 2 by α_j for $j \in \{3, ..., n\}$.

An NR spread in the power-based parameterization implies the existence of $a > 0$ such that

$$h_1 > f_1 \geq a \geq f_2 > h_2$$

and

$$h_1 + h_2 = 2a = f_1 + f_2.$$

Thus, $h_2 = 2a - h_1$ and $f_2 = 2a - f_1$. Let $s_i(K)$ be critical share of supplier $i \in K$ under (F_1, F_2), and let $\hat{s}_i(K)$ be the critical share for supplier $i \in K$ under (H_1, H_2). The shift from (F_1, F_2) to (H_1, H_2) decreases the $\mathcal{I}^S(K)$ if and only if it increases the sum of the critical shares of suppliers 1 and 2. In what follows, we show that $\hat{s}_1(K) + \hat{s}_2(K) - s_1(K) - s_2(K) > 0$.

In the power-based parameterization, letting $A \equiv 2a + \sum_{i \in \{3,...,n\}} \alpha_i$,

$$s_i(K) = \frac{(1 + f_i + A_{-K})(1 + A_{-K})}{(1 + A - f_i)(1 + A)} \quad \text{and} \quad \hat{s}_i(K) = \frac{(1 + h_i + A_{-K})(1 + A_{-K})}{(1 + A - h_i)(1 + A)}.$$

Consequently, the sign of $\hat{s}_1(K) + \hat{s}_2(K) - s_1(K) - s_2(K)$ is equal to the sign of

$$\frac{(1 + h_1 + A_{-K})}{(1 + A - h_1)} + \frac{(1 + h_2 + A_{-K})}{(1 + A - h_2)} - \frac{(1 + f_1 + A_{-K})}{(1 + A - f_1)} - \frac{(1 + f_2 + A_{-K})}{(1 + A - f_2)}.$$

Substituting $2a - h_1$ for h_2 and $2a - f_1$ for f_2 and collecting terms, we get

$$\frac{2(f_1 - h_1 - 2a)(h_1 - f_1)(1 + a + X)(2 + 2a + X)}{(1 + 2a - f_1 + X)(1 + f_1 + X)(1 + 2a - h_1 + X)(1 + h_1 + X)},$$

where $X \equiv A_{-\{1,2\}} + A_{-K}$, which is positive. ■

C Online Appendix: Longer proofs

Proof of Lemma 4. Here we show that $\mathcal{I}^C(N) > 0$. (See Appendix A for the proof of (6).) The Cournot and monopoly quantities are given by:

$$q_i^C(\mathbf{c}) = \frac{1-(n+1)c_i + C_N}{n+1} \quad \text{and} \quad q_i^M(c_i) = \frac{1-c_i}{2}.$$

Because supplier i's Cournot and monopoly profits are the squares of the corresponding quantities,

$$\mathcal{I}^C(N) = 1 - \sum_{i \in N} \left(\frac{q_i^C(\mathbf{c})}{q_i^M(c_i)} \right)^2.$$

Consider the problem of minimizing $\mathcal{I}^C(N)$ with respect to $\mathbf{c}$, subject to $q_i^C(\mathbf{c}) \geq 0$ for all $i \in N$, i.e., temporarily, we allow quantities to be zero. Using (6), given $\mathbf{c}_{-i} \in [0,1)^{n-1}$, $\lim_{c_i \to 1} \mathcal{I}^C(N) = -\infty$. It follows that when $\mathcal{I}^C(N)$ is minimized with respect to $\mathbf{c}$, subject to $q_i^C(\mathbf{c}) \geq 0$ for all $i \in N$, at least one constraint must be binding, i.e., must be satisfied with equality.

Let $\mathbf{c}$ be such that $\mathcal{I}^C(N)$ is minimized subject to $q_i^C(\mathbf{c}) \geq 0$ for all $i \in N$. Let $Z \subset N$ be the set of suppliers such that $q_i^C(\mathbf{c}) > 0$ and let $z \equiv |Z|$. By the arguments above $z \in \{1, ..., n\}$.

Case 1: $z = 1$. Then $\mathcal{I}^C(N) = 0$.

Case 2: $z \geq 2$ and for all $i \in Z$, $c_i = 0$. Then for all $j \in N \backslash Z$, $q_j^C(\mathbf{c}) = 0$, which implies that $c_j = \frac{1}{z+1}$, so $C_N = \frac{n-z}{z+1}$. Using (6),

$$\mathcal{I}^C(N) = 1 - \sum_{i \in Z} \left(1 + \frac{n-z}{z+1}\right)^2 \frac{4}{(n+1)^2} = 1 - \frac{4z}{(z+1)^2} = \frac{(z-1)^2}{(z+1)^2} > 0,$$

which contradicts the supposition of being at the minimum.

Case 3: $z \geq 2$ and there exists $\ell \in Z$ such that $c_\ell > 0$ and for all $i \in Z \backslash \{\ell\}$, $c_i = 0$. Then for all $j \in N \backslash Z$, $q_j^C(\mathbf{c}) = 0$, which implies that $c_j = \frac{1+c_\ell}{1+z}$ and $C_N = \frac{(n-z)(1+c_\ell)}{1+z} + c_\ell = \frac{n-z+(n+1)c_\ell}{1+z}$. Further, in order to have $q_\ell(\mathbf{c}) > 0$, it must be that $1-(n+1)c_\ell + C_N > 0$, which we can write as $c_\ell < \frac{1}{z}$. Using (6),

$$\begin{aligned}\mathcal{I}^C(N) &= 1-(z-1)\left(1+\frac{n-z+(n+1)c_\ell}{1+z}\right)^2\frac{4}{(n+1)^2}\\ &\quad -\frac{\left(1-(n+1)c_\ell+\frac{n-z+(n+1)c_\ell}{1+z}\right)^2}{(1-c_\ell)^2}\frac{4}{(n+1)^2}\\ &= \frac{-(1+c_\ell)(z-1)(1-z+3(z-1)c_\ell-4c_\ell^2+4c_\ell^3)}{(1-c_\ell)^2(1+z)^2},\end{aligned}$$

which is positive for all $c_\ell < \frac{1}{z}$ and $z \in \{2, 3, ..., n\}$,[44] contradicting the supposition of being at the minimum.

Case 4: $z \geq 2$ and there exist $j, \ell \in Z$ with $j \neq \ell$ such that $c_j, c_\ell > 0$. Without loss of generality, suppose that $c_j \leq c_\ell$. Let $\Delta \in (0, \min\{c_j, q_\ell^C(\mathbf{c})\})$ and consider a change in costs that decreases c_j by Δ and increases c_ℓ by Δ. (As we discuss in Section 3.4, this is a neutral-for-rivals spread.) Let $\hat{\mathbf{c}}$ be the new vector of costs, i.e., $\hat{c}_j = c_j - \Delta$, $\hat{c}_\ell = c_\ell + \Delta$, and for all $i \in N \backslash \{j, \ell\}$, $\hat{c}_i = c_i$. Given our choice of Δ, $q_j^C(\hat{\mathbf{c}}) > 0$ and $q_\ell^C(\hat{\mathbf{c}}) > 0$. The change in supplier j's critical share is

$$\begin{aligned}\frac{q_j^C(\mathbf{c})}{q_j^M(c_j)} - \frac{q_j^C(\hat{\mathbf{c}})}{q_j^M(\hat{c}_j)} &= \frac{2}{n+1}\left(\frac{1-(n+1)c_j+C_N}{1-c_j} - \frac{1-(n+1)(c_j-\Delta)+C_N}{1-(c_j-\Delta)}\right)\\ &= \frac{2\Delta}{n+1}\left(\frac{C_N-n}{(1-c_j)(1-c_j+\Delta)}\right)\\ &< 0,\end{aligned}$$

and, analogously, the change in supplier ℓ's critical share is

$$\frac{q_\ell^C(\mathbf{c})}{q_\ell^M(c_\ell)} - \frac{q_\ell^C(\hat{\mathbf{c}})}{q_\ell^M(\hat{c}_\ell)} = \frac{-2\Delta}{n+1}\left(\frac{C_N-n}{(1-c_\ell)(1-c_\ell-\Delta)}\right) > 0.$$

[44]To see this, note that the expression has sign equal to the sign of $1+z-3(z-1)c_\ell+4c_\ell^2-4c_\ell^3$, which at $c_\ell = \frac{1}{z}$ is equal to

$$-4+4z+3z^2-4z^3+z^4,$$

which is zero for $z=2$ and positive for $z>2$. In addition, the expression has derivative with respect to c_ℓ of

$$\frac{8c_\ell(z-1)(2-z-2c_\ell^2+c_\ell^3)}{(1-c_\ell)^3(1+z)^2},$$

which has sign equal to the sign of $2-z-c_\ell^2(2-c_\ell)$, which is negative for $z \geq 2$ and $c_\ell \in (0,1)$. Thus, the expression is positive for all $z \geq 2$ and $c_\ell \in (0, 1/z)$.

Because $\Delta > 0$ and $c_j \leq c_\ell$, it follows that

$$\left| \frac{q_1^C(\mathbf{c})}{q_1^M(c_1)} - \frac{q_1^C(\hat{\mathbf{c}})}{q_1^M(\hat{c}_1)} \right| < \left| \frac{q_2^C(\mathbf{c})}{q_2^M(c_2)} - \frac{q_2^C(\hat{\mathbf{c}})}{q_2^M(\hat{c}_2)} \right|,$$

which implies that the decrease in supplier j's critical share is less than the increase in supplier ℓ's critical share, and so overall $\mathcal{I}^C(N)$ decreases as a result of the change. Because all constraints continue to be satisfied, this contradicts the supposition of being at the minimum.

Thus, we conclude that when $\mathcal{I}^C(N)$ is minimized subject to $q_i^C(\mathbf{c}) \geq 0$ for all $i \in N$, the quantity constraint is slack for one and only one supplier and $\mathcal{I}^C(N) = 0$. Because the minimum is not achieved when all constraints are satisfied strictly, we conclude that under our assumption that all suppliers in N have positive Cournot quantities, $\mathcal{I}^C(N) > 0$. ■

D Online Appendix: Extensions

D.1 Procurement setup with multi-unit suppliers and buyer power

Our definition of and test for coordinated effects generalize straightforwardly to an efficient procurement setup in which the buyer has multi-unit demand and suppliers have multi-unit capacities. (Without multi-unit demand, multi-unit capacities play no substantial role.) To be specific, we can allow the buyer to be characterized by a commonly known marginal value vector $\mathbf{v} = (v_1, ..., v_Q)$, where Q is the buyer's maximal demand, with $v_i \geq v_{i+1}$ for $i \in \{1, \ldots, Q-1\}$ and for each supplier j to be characterized by a capacity κ_j and a vector of marginal costs $\mathbf{c}^j = (c_1^j, \ldots, c_{\kappa_j}^j)$ satisfying $c_i^j \leq c_{i+1}^j$ for $i \in \{1, \ldots, \kappa_j - 1\}$, where κ_j is (now) an integer. Assume that each supplier j's capacity κ_j is common knowledge, but that each supplier's marginal cost is its own private information. Assume also that for all j, $\mathbf{c}^j$ is distributed according to the commonly known, continuous distribution $G_j(\mathbf{c}^j)$ with support $[\underline{c}, \overline{c}]^{\kappa_j}$.

A simple and particularly convenient specification for the multivariate distribution $G_j(\mathbf{c}^j)$ is to assume that j's cost draw is the realization of κ_j independent, univariate random variables c drawn from the distribution $G_j(c)$. This implies that $G_j(\mathbf{c}^j)$ is given by the distribution if the κ_j-th order statistic from G_j. For example, the distribution of c_1^j is $G_{j,[1]}(c) = 1 - (1 - G_j(c))^{\kappa_j}$. Consequently, we refer to this as the *order statistics model.* This model also makes clear the sense in which the power-based parameterization captures a supplier's strength.

Following a merger between suppliers h and j, the merged entity's capacity is $\kappa_h + \kappa_j$. In the order statistics model, assuming pre-merger symmetry between j and h, so that

$G_j = G_h = G$, the distribution of the minimum cost c_1^{hj} of the merged firm is $G_{hj,[1]}(c) = 1 - (1 - G(c))^{\kappa_h + \kappa_j}$.

The payoff (or revenue) equivalence theorems for multi-dimensional type spaces of Williams (1999) and Krishna and Maenner (2001) imply that the generalized second-price auction with reserve prices for the m-th unit given by $\min\{v_m, \bar{c}\}$ is without loss of generality insofar as this is the profit-maximizing mechanism for the buyer subject to efficiency and individual rationality and incentive compatibility constraints for the suppliers. Consequently, the profit of every supplier j is pinned down by $\mathbf{v}$ and the distributions $(G_i(\mathbf{c}^i))_{i \in N}$ when all suppliers play their dominant strategies of reporting their types $\mathbf{c}^i$ truthfully.

Likewise, the expected profit $\Pi_i(K)$ when the suppliers $j \in K$ participate in a bidder selection scheme when $i \in K$ is the designated bidder is pinned down by $G_i(\mathbf{c}^i)$ for $i \in K$ and $(G_h(\mathbf{c}^h))_{h \in N \setminus K}$. Consequently, $s_i(K) = \Pi_i / \Pi_i(K)$ as in the single-unit case, and the $\mathcal{I}^S(K)$ can be defined in the same way and with the same interpretation as before.

Two-unit demand

With two-unit demand, $v \geq \bar{c}$, supplier i's expected revenue is the expected value of the second-lowest cost among the other $n-1$ suppliers, conditional on that cost being greater than supplier i's cost, and multiplied by the probability that supplier i's cost is one of the two lowest, which we can write as:

$$R_i = \int_{\underline{c}}^{\bar{c}} \int_{c}^{\bar{c}} y \, dH_i(y) dG_i(c),$$

where H_i is the distribution of the second-lowest cost among suppliers other than i. Similarly, supplier i's expected profit is

$$\Pi_i = \int_{\underline{c}}^{\bar{c}} \int_{c}^{\bar{c}} (y - c) dH_i(y) dG_i(c).$$

Letting H_K be the distribution of the second-lowest cost among suppliers not in set K, then for $i \in K$, supplier i's expected profit when suppliers in K coordinate and supplier i is selected to be the only member of K to bid is

$$\Pi_i(K) = \int_{\underline{c}}^{\bar{c}} \int_{c}^{\bar{c}} (y - c) dH_K(y) dG_i(c).$$

Now consider the power-based parameterization. Letting $A \equiv \sum_{k \in N} \alpha_k$ and $A_{-X} \equiv \sum_{k \in N \setminus X} \alpha_k$, we can write R_i, Π_i, and $\Pi_i(K)$ in terms of the parameters of the cost distributions as shown in the following lemma.

Lemma D.1. *Assuming $G_i(c) = 1-(1-c)^{\alpha_i}$ and $n \geq 3$, if the buyer has two-unit demand with $v \geq 1$, then supplier i trades with probability $q_i = \alpha_i \sum_{\ell \neq i}\left(\frac{1}{A_{-\{\ell\}}} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}A}\right)$, has expected revenue*

$$R_i = \alpha_i \sum_{\ell \neq i}\left(\frac{\alpha_i + A^2_{-\{\ell\}}}{\left(1+A_{-\{i,\ell\}}\right)A_{-\{\ell\}}(1+A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}\left(1+\alpha_i\right)}{A_{-\{i\}}(1+A_{-\{i\}})A\left(1+A\right)}\right),$$

and has expected profit $\Pi_i = R_i - C_i$, where C_i is supplier i's expected cost, given by

$$C_i = \alpha_i \sum_{\ell \neq i}\left(\frac{1}{A_{-\{\ell\}}(1+A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}A(1+A)}\right).$$

In addition, for $i \in K$, q_i, R_i and C_i can be adjusted for the case of coordination by suppliers in K by summing over $\ell \in N \backslash K$ and letting α_j be zero for all $j \in K \backslash \{i\}$.

Proof. Using the parameterization $G_i(c) = 1-(1-c)^{\alpha_i}$, for $n \geq 3$, the cdf of the second-lowest among the $n-1$ suppliers other than supplier i is

$$\begin{aligned} H_i(c) &\equiv 1 - \left(\prod_{j \neq i}(1-G_j(c)) + \sum_{\ell \neq i} G_\ell(c) \times_{j \in N \backslash \{i,\ell\}} (1-G_j(c))\right) \\ &= 1 - \left((1-c)^{A_{-\{i\}}} + \sum_{\ell \neq i}(1-(1-c)^{\alpha_\ell})(1-c)^{A_{-\{i,\ell\}}}\right), \end{aligned}$$

and the associated pdf is

$$h_i(c) = \sum_{\ell \neq i}(1-(1-c)^{\alpha_\ell})A_{-\{i,\ell\}}(1-c)^{A_{-\{i,\ell\}}-1}.$$

The probability of trade for supplier i is

$$\begin{aligned} q_i &= \int_0^1 \int_c^1 \left(\sum_{\ell \neq i}(1-(1-y)^{\alpha_\ell})A_{-\{i,\ell\}}(1-y)^{A_{-\{i,\ell\}}-1}\right)\alpha_i(1-c)^{\alpha_i - 1}dydc \\ &= \int_0^1 \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left((1-y)^{A_{-\{i,\ell\}}-1} - (1-y)^{A_{-\{i\}}-1}\right)dy\right)\alpha_i(1-c)^{\alpha_i-1}dc \\ &= \alpha_i \sum_{\ell \neq i}\left(\frac{1}{A_{-\{\ell\}}} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}A}\right). \end{aligned}$$

So the market share of supplier i is $q_i/2$ (because the sum of all suppliers' probabilities of trade is 2 in the case of two-unit demand and no buyer power and $v \geq 1$).

Supplier i's expected revenue is

$$\begin{aligned} R_i &= \int_0^1 \int_c^1 y \left(\sum_{\ell \neq i} (1-(1-y)^{\alpha_\ell}) A_{-\{i,\ell\}} (1-y)^{A_{-\{i,\ell\}}-1} \right) \alpha_i (1-c)^{\alpha_i - 1} dy dc \\ &= \int_0^1 \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left(y(1-y)^{A_{-\{i,\ell\}}-1} - y(1-y)^{A_{-\{i\}}-1} \right) dy \right) \alpha_i (1-c)^{\alpha_i-1} dc \\ &= \alpha_i \sum_{\ell \neq i} \left(\frac{1}{1+A_{-\{i,\ell\}}} \left(1 - \frac{A_{-\{i,\ell\}}}{A_{-\{\ell\}}(1+A_{-\{\ell\}})} \right) - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}(1+A_{-\{i\}})A} \left(1 - \frac{A_{-\{i\}}}{1+A} \right) \right). \end{aligned}$$

Supplier i's expected cost is

$$\begin{aligned} C_i &= \int_0^1 \int_c^1 c \left(\sum_{\ell \neq i} (1-(1-y)^{\alpha_\ell}) A_{-\{i,\ell\}} (1-y)^{A_{-\{i,\ell\}}-1} \right) \alpha_i (1-c)^{\alpha_i - 1} dy dc \\ &= \int_0^1 c \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left((1-y)^{A_{-\{i,\ell\}}-1} - (1-y)^{A_{-\{i\}}-1} \right) dy \right) \alpha_i (1-c)^{\alpha_i-1} dc \\ &= \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}(1+A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}} A (1+A)} \right). \end{aligned}$$

The remaining results follow by substitution and rearrangement. ■

D.2 Applications with buyer power

In the U.S. DOJ's analysis of the proposed merger of oilfield services providers Halliburton and Baker Hughes, the agency identified the $400 million market of offshore cementing services as a relevant antitrust market.[45] According to the DOJ Complaint, the pre-merger market had essentially three suppliers: Halliburton, Baker Hughes, and Schlumberger.[46] Further, the information in the DOJ's complaint indicates that Halliburton and Baker Hughes had pre-merger market shares of 32% and 24% and that Schlumberger had a pre-merger market share of 43%, with the three suppliers accounting for 99% of the market.[47]

In this application, it seems reasonable to assume (as the merging parties argued) that the buyers, which include BP, Shell, and Exxon-Mobil, have buyer power. These buyers

[45] U.S. v. Halliburton Co. and Baker Hughes Inc., Complaint, Case 1:16-cv-00233-UNA, filed 6 April 2016 (DOJ Complaint).

[46] "In a strategic planning session, Halliburton's cementing executives recognized that this market is already a 'pure oligopoly' among the Big Three" (DOJ Complaint, p. 18).

[47] This can be deduced from the information provided in the DOJ Complaint that Schlumberger's market share was approximately 43%, the combined market share of Halliburton and Baker Hughes was approximately 56%, the pre-merger HHI was approximately 3500, and the post-merger HHI was approximately 5000. Although we can identify the shares of Halliburton and Baker Hughes as approximately 32% and 24%, it is not clear which supplier has the 32% share and which has the 24% share.

are large, sophisticated firms that purchased through competitive procurements. Thus, we calibrate distributions and calculate the coordinated effects index under the assumption of buyer power, but we also contrast the results with the case of no buyer power.

To facilitate the analysis of the case with buyer power, we use the parameterization $G_i(c) = c^{\alpha_i}$ (which implies linear virtual type functions), and we assume that v is sufficiently large that $v \geq \Gamma_i(\overline{c})$ for all i. As an identifying assumption, we assume that $\sum_{i=1}^{4} \alpha_i = 4$. Letting supplier 1 be Schlumberger and letting supplier 2 have market share 34% and supplier 3 have market share 24%, our calibration delivers $\alpha_1 = 0.0760$, $\alpha_2 = 0.0999$, $\alpha_3 = 0.1274$, and $\alpha_4 = 3.6967$. The calculation of the associated coordinated effects index for different sets of coordinators is shown in Table D.1.

Pre-merger		Post-merger	
K	$\mathcal{I}^S(K)$	K	$\mathcal{I}^S(K)$
$\{1,2,3\}$	0.7617	$\{1,\mu_{2,3}\}$	0.6338
$\{1,2\}$	0.4625		
$\{1,3\}$	0.4198		
$\{2,3\}$	0.3780		

Table D.1: Results for the oilfield services market of offshore cementing. Supplier 1 is Schlumberger, with pre-merger share 43%. Suppliers 2 and 3 are Halliburton and Baker Hughes (in unknown order), with pre-merger shares 34% and 24%. We denote the merger of Halliburton and Baker Hughes by $\mu_{2,3}$.

Holding fixed the distributions, without buyer power, we would have instead $\mathcal{I}^S(\{1,2,3\}) = 0.9510$ and $\mathcal{I}^S(\{1,\mu_{2,3}\}) = 0.8960$, which are larger than their corresponding values with buyer power, illustrating that the market would be at even greater risk, before and after the merger, if the buyers were not powerful.

As this shows, the market is at risk despite the presence of powerful buyers. And, holding fixed cost distributions, the market would be at greater risk if buyers were not powerful.

The market is also at risk for coordination by any pair of the suppliers in the Big 3.

D.3 Generalization to multi-unit demand with buyer power

With buyer power, the main obstacle to the generalization to multi-unit supply and demand is that the optimal mechanism is not known when agents have multi-dimensional types. Even if one assumed single-unit suppliers in the pre-merger market, a merger would naturally lead to a multi-unit supplier.

However, all is not lost because there are circumstances in which even multi-unit buyers restrict themselves to buying at most one unit from each individual supplier. This may be due to (non-modelled) preferences for diversification, protection against further hold-up, or imposed by law (as in one of the applications in Section 3.5). Under these circumstances, all that matters for the buyer's optimal mechanism are the distributions of each seller j's lowest cost c_1^j, that is, $G_{j,[1]}(c_1^j)$, which is a one-dimensional variable. Hence, the standard mechanism design tools and results apply.

Let us briefly elaborate. The profit-maximizing mechanism for the buyer subject to incentive compatibility and individual rationality constraints given $n > Q$ is characterized as follows: For notational simplicity, let $c_j \equiv c_1^j$ and $G_j(c_j) \equiv G_{j,[1]}(c_1^j)$ with support $[\underline{c}, \overline{c}]$ and density $g_j(c_j)$ for all $j \in N$. Moreover, to simplify the analysis, assume as before that, for all $j \in N$, the virtual cost function $\Gamma_j(c_j)$ defined by

$$\Gamma_j(c_j) \equiv c_j + \frac{G_j(c_j)}{g_j(c_j)}$$

is increasing in c_j. Then, for a given realization $\mathbf{c} = (c_1, \ldots, c_n)$ and for given $\mathbf{v}$, the profit-maximizing mechanism for the buyer has the allocation rule of purchasing $m \in \{0, \ldots, Q\}$ units from the m suppliers with the lowest *virtual* costs, where, if $m < Q$, m is such that the m-th lowest virtual cost is less than v_m and the $m+1$-st lowest virtual cost exceeds v_{m+1}.

In the dominant strategy implementation of this mechanism, suppliers who do not produce receive (and make) no payments. Each supplier who trades is paid a threshold payment, that is, the highest cost that it could have reported without changing the fact that it trades. This pins down Π_i and $\Pi_i(K)$, and thereby $s_i(K)$ and $\mathcal{I}^S(K)$, just as in the single-unit case. For example, in the special case in which all suppliers are ex ante symmetric with $G_j = G$ for all j and thus $\Gamma_j = \Gamma$ for all j, the optimal mechanism can be implemented as via a second-price auction, in which the reserve price for the l-th unit is $\Gamma^{-1}(v_l)$. If the quantity traded is m, the m successful suppliers are paid $\min\{\Gamma^{-1}(v_m), c_{[m+1]}\}$, where $c_{[m+1]}$ denotes the $m+1$-st lowest cost.

References for Online Appendix

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