Coordinated Effects*

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Abstract

Coordination, modeled as bid suppression in a procurement, is individually rational only if each coordinating supplier is selected to bid with a probability exceeding its critical share. Thus, one minus the sum of the critical shares measures the risk of coordination. This risk is greater among larger suppliers, more symmetric suppliers, in second-price versus first-price auctions, and when buyers are not powerful. Mergers—including 3-to-2 mergers—need not increase the risk for coordination, and there is a trade-off between coordinated and unilateral effects. We also provide a definition of a maverick firm that permits rigorous maverick-based merger evaluation.

Keywords: merger review, buyer power, mavericks, unilateral effects, competitively neutral spread

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1 Introduction

Competition authorities regularly review proposed mergers and oppose those that they determine will have sufficiently detrimental effects.\(^1\) They recognize that one source of detrimental effects is that a merger can change “the nature of competition in such a way that firms that previously were not coordinating their behaviour, are now significantly more likely to coordinate and raise prices or otherwise harm effective competition” (EC Guidelines, para. 22(b)).\(^2\) Adverse competitive effects of mergers that arise in this way are referred to as “coordinated effects” and play a prominent role in antitrust thinking and practice.\(^3\) For example, Kolasky (2002) observes: “Concern over what we now call coordinated effects has long been at the core of U.S. merger policy.” Similarly, in evaluating a hospital merger, Judge Richard Posner wrote that “the ultimate issue is whether the challenged acquisition is likely to facilitate collusion” and, in particular, “whether the challenged acquisition is likely to hurt consumers, as by making it easier for the firms in a market to collude, expressly or tacitly, and thereby force price above or farther above the competitive level.”\(^4\) However, thus far, coordinated effects of mergers have proven elusive to define, let alone to analyze or test for.\(^5\) In light of the far-reaching implications of decisions that are based on coordinated effects, this state of affairs is, obviously, problematic.

Motivated by this, we provide a model of coordination that allows us to define what it means for a market to be at risk for coordination, to quantify the risk, and to analyze the effects of mergers on that risk. Our approach is novel in two fundamental ways. First, we model coordination by suppliers in a procurement setting as participation in a bidder selection scheme whereby each coordinating supplier is chosen to be the designated bidder with some probability. With private information about stochastic costs, bidder selection schemes are suboptimal coordination devices but are appealing because they do not require transfers or the communication of any private, production-relevant information, which

\(^1\)The U.S. Horizontal Merger Guidelines (U.S. Guidelines) guide courts in the United States on how to evaluate the potential anticompetitive effects of a merger. A similar role is played in Europe by the European Commission’s Guidelines on the Assessment of Horizontal Mergers (EC Guidelines) and in Australia by the Australian Competition and Consumer Commission’s Merger Guidelines.

\(^2\)Similarly, the U.S. Guidelines (p. 2) recognize that a merger “can enhance market power by increasing the risk of coordinated, accommodating, or interdependent behavior among rivals.”

\(^3\)Coordinated effects arguments have played a central role in U.S. merger cases such as Heinz/BeechNut, Anheuser-Busch InBev/Grupo Modelo, and H&R Block/TaxACT. For European cases, see Amelio et al. (2009) on ABF/GBI Business, Motta (2000) on Airtours/First Choice, and Aigner et al. (2006) on Sony/BMG and Impala and on the evolution of coordinated effects’ assessment in the EU.

\(^4\)Hospital Corp. Of America v. FTC, 807 F.2d 1381, 1386 (7th Cir. 1986), paras. 1 and 7.

\(^5\)As stated by a former U.S. DOJ official, “One particular problem is that neither the theoretical nor empirical literature tells us much at all about whether the disappearance of a single firm through merger will increase the likelihood of coordination, other than, perhaps, in the extreme case where a merger reduces the number of firms in a market from three to two” (Kolasky, 2002, p. 7).
likely would be deemed illegal. This allows us to ask whether, for a given set of candidate coordinators, there are selection probabilities that sum up to less than one that make each supplier better off coordinating than not. Second, the model based on bidder selection schemes allows us to focus on whether coordination is individually rational and to abstract, by and large, from the question of what makes coordination sustainable. This allows us to develop a test that can be implemented using data that are typically available before a merger takes place. Specifically, we test whether a market is at risk for coordination and quantify the extent to which it is at risk using the \textit{coordinated effects index (CEI)}, which is one minus the sum of the critical shares (or selection probabilities) necessary to make participation in a bidder selection scheme individually rational for all firms under consideration.

Because of its focus on individual rationality, the test we develop is biased in the direction of overestimating the gains from coordination. Thus, our test provides a screen that allows one to rule out concerns of coordinated effects in some cases. Whenever the index is negative, coordination as a bid suppression scheme without communication or transfers cannot happen. If the index is positive, it may. Importantly, under parametric assumptions, data on the pre-merger market shares of candidate coordinating suppliers and one supplier’s margin are sufficient to construct the CEI, making it operational for practical purposes.

The interpretation of a larger, positive value of the CEI as indicating an increase in risk may also prove useful for practitioners. For example, the \textit{U.S. Guidelines} (p. 26) state: “The Agencies regard coordinated interaction as more likely, the more the participants stand to gain from successful coordination.” A larger positive value of the CEI implies that there is more leeway for coordination to remain profitable in the face of additional costs of coordination, including potential penalties for being caught coordinating. In contrast, when the index is close to zero, small positive costs or frictions could deter coordination. A larger index also implies that the coordinating firms can use coarser, less sophisticated coordination devices because there is more leeway in choosing the selection probabilities, which likewise facilitates coordination. In the limit, as the index goes towards 1, uniform selection probabilities suffice.

In our framework, coordinated effects are a greater concern when coordinating suppliers are larger, more symmetric, and when they face less outside competition. Buyer power reduces the risk of coordination among symmetric suppliers, as does a first-price auction compared to a second-price auction. This is in line with the repeated games literature (notwithstanding different frameworks and alternative modes of coordination),\textsuperscript{6}

\textsuperscript{6}Compte et al. (2002), Vasconcelos (2005), and Ivaldi et al. (2007) find increased concerns from symmetry in repeated oligopoly setups. For the contrasting view that asymmetries can facilitate collusion in some settings, see Ganslandt et al. (2012).
the auctions literature,\(^7\) and the perceived wisdom and antitrust practice.\(^8\) In contrast to the perceived wisdom, we show that consolidation itself, including a 3-to-2 merger, does not necessarily put a market at (a greater) risk for coordination.\(^9\) In particular, a merger can either increase or decrease the CEI.

By having a clear-cut and operational definition of coordinated effects, we can also give precise meaning to another term that features prominently in antitrust thinking and practice but has proved elusive to define, that of a maverick firm. The particular concerns raised by mergers involving mavericks are discussed in, for example, the U.S., EC, and Australian merger guidelines, and they arise in many merger cases.\(^10\) Baker (2002, pp. 140–141) argues that “the identification of a maverick that constrains more effective coordination is the key to explaining ... which particular changes in market structure from merger or exclusion are troublesome, and why.”\(^11\) Current thinking is that an acquisition that eliminates a maverick is likely to cause adverse coordinated effects if the market is vulnerable to coordinated conduct (\textit{U.S. Guidelines}, para. 25),\(^12\) with some arguing that the elimination of a maverick may be necessary for a coordinated effects argument.\(^13\)

Notwithstanding its prominence, the definition of a maverick has remained vague.\(^14\)

\(^7\)See, for example, Ausubel and Milgrom (2006), Rothkopf (2007), and Marshall and Marx (2007).
\(^8\)For example, the \textit{EC Guidelines} (para. 48) state, “Firms may find it easier to reach a common understanding on the terms of coordination if they are relatively symmetric, especially in terms of cost structures, market shares, capacity levels and levels of vertical integration.” Amelio et al. (2009) explain that for the \textit{ABF/GBI Business} case, the EC identified the increase in competitors’ symmetry as one of the critical factors that would make tacit collusion easier to implement, monitor, and sustain. Related to the Hertz–Dollar Thrifty merger, Doane et al. (2019, p. 93) explain that, “Symmetry makes a market more conducive to coordination by making competitors prefer the same terms of coordination—the same prices, for example.” See footnote 17 for references to antitrust guidelines relating coordinated effects to buyer power.

\(^9\)The fact that the proposed acquisition of TaxACT by H&R Block merger would have reduced the number of major do-it-yourself tax software suppliers from 3 to 2 motivated concerns of coordinated effects. Similarly, the reduction in the number of yeast makers from 3 to 2 in Spain and Portugal in \textit{ABF/GBI Business} motivated calls for divestitures to avoid coordinated effects.

\(^10\)Examples include the proposed acquisition of maverick T-Mobile by AT&T, the acquisition of maverick Northwest Airlines by Delta Airlines, and the proposed acquisition of maverick baby food maker Beech-Nut by Heinz. On mavericks in EC merger decisions, see Bromfield and Olczak (2018).

\(^11\)Baker (2002, p. 197) goes on to argue for a “maverick-centered approach” to coordinated effects, saying: “In many settings, regulators reliably can identify an industry maverick that prevents or limits coordination” by either observing that the firm constrains industry pricing or identifying factors that would cause the firm to prefer low prices (e.g., excess capacity).

\(^12\)Ivaldi et al. (2007) identify market characteristics that may affect the sustainability of collusion.

\(^13\)Kolasky (2002, p. 8) states that the “loss of a firm that does not behave as a maverick is unlikely to lead to increased coordination.”

\(^14\)Antitrust officials have described a maverick as “a firm that declines to follow the industry consensus and thereby constrains effective coordination” (Kolasky, 2002, p. 7), while the \textit{U.S. Guidelines} (p. 4) describe a maverick as “a firm that has often resisted otherwise prevailing industry norms to cooperate on price setting or other terms of competition.” Ivaldi et al. (2007, pp. 224, 228) define a maverick as “a firm that has a drastically different cost structure, production capacity or product quality, or that is affected by different factors than the other market participants” and “is thus unwilling to participate to a
An operational definition of a maverick requires that a maverick firm be identifiable based only on pre-merger observables, which we are able to provide. For a fixed set of firms that contemplate coordinating, we say that an outside firm is a maverick if the CEI is negative when the maverick is present in the market, but is positive when the maverick is not present. A key insight is that, although the elimination of a maverick would, by definition, put the market at risk for coordination, the acquisition of a maverick does not necessarily put a market at risk because the acquisition of a firm does not eliminate the firm’s production capacity. The newly merged entity’s production capacity will be larger after the merger than the acquirer’s was before the merger. In general, this affects every firm’s critical share and, thereby, the scope for coordination. We provide conditions under which the answer to whether the acquisition of a maverick puts a market at risk for coordination depends critically on the size of the acquiring firm. Thus, in addition to providing a rigorous and disciplined approach to defining and analyzing mavericks and their effects, our framework and analysis suggest refinements to maverick-based merger evaluation.

Our framework also provides a new, perhaps surprising, result that a merger that generates moderate, merger-related cost synergies results in a larger post-merger CEI than if the merger generated no cost synergies at all. This result, in addition to others that we develop, points to a tension between coordinated and unilateral effects. For example, as we discuss, divestiture-based remedies for merger harm that reduce the CEI tend to increase concerns of unilateral effects.

Our result that, with ex ante symmetric suppliers, buyer power reduces the risk for coordination is intuitive because powerful buyers rely less on rivalry among bidders than buyers without power to obtain favorable prices. In addition, as mentioned above, it is in line with antitrust practice. Yet, to our knowledge, there has been no formalization of this intuitive idea in the literature.

As an overview of the model, we study a procurement model with independent private

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15The same observation—that a merger does not eliminate the production capacity but only a bid—explains the different effects of bargaining power in the merger context of Loertscher and Marx (2019b) versus the setup of Bulow and Klemperer (1996), where a bidder is eliminated.

16A similar tension arises in the repeated game setup of Compte et al. (2002), where divestitures of capacity that prevent the creation of a dominant firm can help tacit collusion.

17The U.S. Guidelines (p. 27) state that the agencies “consider the possibility that powerful buyers may constrain the ability of the merging parties to raise prices. This can occur, for example, ... if the conduct or presence of large buyers undermines coordinated effects.” The EC Guidelines discuss the possibility that “buyer power would act as a countervailing factor to an increase in market power resulting from the merger” (para. 11). The Australian Merger Guidelines view “countervailing power” as a competitive constraint that can limit merger harms (paras. 1.4, 5.3, 7.48).
types. This setup is tractable and explicitly models price formation in the presence of cost uncertainty. The setup neither presumes nor precludes efficiency and, in general, involves a tradeoff between profit and social surplus that is at the heart of industrial organization and antitrust economics. This procurement model is well-accepted in the theoretical literature and enables us to capture the insights of George Stigler regarding the importance of information costs (and incomplete information) in determining firm behavior. As noted by Stigler (1961, p. 213), “some important aspects of economic organization take on a new meaning when they are considered from the viewpoint of the search for information” and, in particular, “one important problem of information—the ascertainment of market price.” Stigler (1964, p. 44) emphasizes the particular role of information in policing collusive agreements, which he says “proves to be a problem in the theory of information.”

We model a merger by assuming that the two merging suppliers form a merged entity whose cost is the minimum of the merging suppliers’ costs. This implies that the merged entity draws its type from a “better” distribution than the pre-merger suppliers. We model coordination by assuming that only one of the set of coordinating suppliers competes for the buyer’s business, while the other coordinating suppliers suppress their bids. Because we assume that the coordination does not rely on the suppliers’ private information, it need not result in the most competitive supplier being selected. Thus, suppliers that coordinate may lose sales to outside suppliers, and coordination may reduce total surplus even when one of the coordinating suppliers wins if it is the “wrong” (not lowest cost) supplier that wins. But the coordinating suppliers stand to gain higher payments by suppressing bids. Thus, on net, the change in expected surplus from coordination can be positive or negative for the coordinating suppliers.

There is related legal and economics literature on coordinated effects. For an overview of the related legal literature, see Baker (2002, 2010) and Harrington (2013). Miller

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18 For auction-based models of unilateral effects, see Gagnepain and Martimort (2016), Waehrer (1999), Waehrer and Perry (2003), Miller (2014), and Froeb et al. (2017).

19 This is as in, e.g., Farrell and Shapiro (1990), Salant et al. (1983), and Perry and Porter (1985).

20 This mechanism resembles a bid rotation scheme (see, e.g., McAfee and McMillan (1992), Hendricks et al. (2015), and U.S. Department of Justice (2015)). An example rotation scheme is the electrical contractors conspiracy, sometimes referred to as the “phases of the moon” conspiracy (Richard A. Smith, “The Incredible Electrical Conspiracy,” Fortune, April 1961, 63, 132–80, and May 1961, 63, 161–224).

21 A number of authors consider collusion in the absence of communication and transfers in repeated Bertrand or repeated auctions setups, including Athey and Bagwell (2001) and Athey et al. (2004), where past prices are observed, Skrzypacz and Hoppenhain (2004) and Blume and Heidhues (2008), where identities of past winners are observed, and Hörner and Jamison (2007), where a firm observes whether or not it has won the unit mass of consumers. The literature shows that colluding bidders can improve upon a bid rotation scheme in a repeated game if they can condition rotations on the history of who won past auctions, where, in a sense, this conditioning plays the role of transfers. Imhof et al. (2018) provide empirical methods to screen for bid rotation schemes.
and Weinberg (2017) provide an empirical analysis of coordinated effects following the MillerCoors joint venture. The theoretical economics literature, including Compte et al. (2002), Vasconcelos (2005), Ivaldi et al. (2007), and Bos and Harrington (2010), has focused on repeated oligopoly models where the measure of coordinated effects is the critical discount factor required to support all-inclusive perfect collusion, based on grim trigger strategies and assuming any deviations are perfectly observed. A decrease in a firm’s critical discount factor is interpreted as an increase in its incentive to collude. In that setup, Compte et al. (2002) show that asymmetric capacities make collusion more difficult to sustain when the aggregate capacity is limited. Vasconcelos (2005) shows that mergers that increase asymmetries in asset holdings hinder collusion. Ivaldi et al. (2007) provide a detailed discussion of factors that enhance collusion, including symmetry in capacity constraints. Bos and Harrington (2010) find enhanced concerns for mergers of medium-sized firms. Overall, the focus has been on constraints imposed by incentive compatibility, in contrast to our focus on individual rationality.

For simulation-based results in the repeated oligopoly framework, see, e.g., Ivaldi and Lagos (2017) and Brito et al. (2018). These simulations suggest that mergers strengthen the incentives for the merged entity to collude but weaken incentives for the nonmerging firms, with the effect on the merged firm being stronger. Our results are strikingly different. We show that the overall effect of a merger can be to increase or decrease incentives for coordination. Further, in our model, when the buyer is not powerful and when there are no cost synergies, nonmerging suppliers are not affected by a merger (which is in contrast to Cournot and Bertrand setups), so a merger has no effect on their incentives to coordinate, and with cost synergies, a merger increases incentives for nonmerging suppliers to coordinate.

The paper is structured as follows. Section 2 defines the setup and describes our model of coordination. Section 3 introduces the coordinated effects index and its use in identifying whether a market is at risk for coordination. Section 4 shows that the effects of supplier characteristics on the risk for coordination within our framework are consistent with the existing literature. In Section 5, we analyze the effects of mergers, including mergers involving mavericks. Section 6 analyzes cost synergies and related tensions between mitigating unilateral and coordinated effects. In Section 7, we provide extensions that allow for buyer power and multi-unit demands and supplies, and applications. Section 8 concludes.

22 In alternative approaches, Harrington and Wei (2012) consider coordination in an infinitely repeated prisoner’s dilemma game, and Kovacic et al. (2007, 2009) and Gayle et al. (2011) view coordination as analogous to incremental mergers (i.e., perfect collusion) among post-merger firms and propose quantifying coordinated effects by using existing merger simulation tools to model these incremental mergers.
2 Framework

In this section, we introduce the procurement setup and the model of coordination.

2.1 Setup

We assume a procurement setup with one product and one buyer. In the pre-merger market, there are \( n \geq 2 \) suppliers, indexed by the elements of \( N \equiv \{1, \ldots, n\} \). In the market following a merger of suppliers \( k \) and \( \ell \), there are \( n - 1 \) suppliers, including the suppliers in \( N \) other than \( k \) and \( \ell \) plus the merged entity, which is denoted by \( \mu_{k,\ell} \). Thus, we index the post-merger set of suppliers by elements of \( \hat{N}_{k,\ell} \equiv \mu_{k,\ell} \cup N \setminus \{k, \ell\} \). We denote by \( M \) the set of active suppliers, where \( M = N \) for the pre-merger market and \( M = \hat{N}_{k,\ell} \) for the market following the merger of suppliers \( k \) and \( \ell \).

In either the pre-merger or post-merger market, each active supplier \( i \) draws a cost \( c_i \) independently from continuously differentiable distribution \( G_i \) with support \([c, \bar{c}]\) and density \( g_i \) that is positive on the interior of the support. We model a merger as allowing the merging suppliers to rationalize production by using the lower of their two costs, which implies that the merged entity \( \mu_{k,\ell} \) draws its cost from the distribution of the minimum of the pre-merger costs of suppliers \( k \) and \( \ell \), i.e., \( G_{\mu_{k,\ell}}(c) \equiv 1 - (1 - G_k(c))(1 - G_\ell(c)) \) with density \( g_{\mu_{k,\ell}} \).

The timing of the realization of private information in relation to procurements is as follows. Before bidding in a procurement, each bidder who participates in it learns its own cost. That means that at the procurement stage, each bidder is privately informed about its type and that the suppliers’ types are unknown to the buyer. The buyer has value \( v > c \) for one unit of the product. The buyer and suppliers are risk neutral. The buyer’s payoff is zero if it does not trade and equal to its value minus the price it pays if it does trade. Similarly, a supplier’s payoff is zero if it does not trade and equal to the payment that it receives minus its cost if it does trade. All of this is common knowledge.

With the exception of Section 7, where we assume that the buyer is powerful, we focus on the case without buyer power. Following Loertscher and Marx (2019b) and consistent with Bulow and Klemperer (1996), a buyer without power is assumed to use an efficient procurement. Specifically, we assume that the buyer uses a second-lowest-price auction (or, equivalently, a descending clock auction) with reserve equal to the minimum of the buyer’s value and the upper bound of the support of the suppliers’ cost distribution, \( r \equiv \min \{v, \bar{c}\} \). We assume that suppliers follow their weakly dominant strategies of reporting truthfully. As a result, the lowest-cost supplier wins whenever its cost is less than or equal to \( r \) and is paid the minimum of the second-lowest cost and \( r \). Otherwise, there is no trade.

In order to investigate the effects of changes in the size (i.e., market shares) of suppliers
on incentives for coordination, it is sometimes useful to consider a parameterized class of cost distributions. In the capacity-based parameterization, we assume that for all \( i \in N \),

\[
G_i(c) = 1 - (1 - c)^{\alpha_i},
\]

with support \([0, 1]\), where \( \alpha_i > 0 \) can be interpreted as supplier \( i \)'s capacity.\(^{23}\) In this parameterization, a supplier with a larger capacity has a better cost distribution insofar as it is first-order stochastically dominated by the cost distribution of a supplier with a smaller capacity, and a merged entity that combines suppliers \( k \) and \( \ell \) has capacity \( \alpha_k + \alpha_\ell \). For additional details on this parameterization, see Appendix A.

As shown by Loetscher and Marx (2019b), in this setting, a merger decreases expected buyer surplus. Thus, even in the absence of coordinated effects, a merger among suppliers is harmful for the buyer—termed “unilateral effects” in the argot of antitrust. Here, we study the possibility and quantification of incremental harm related to coordinated effects. We discuss the possibility of a combined measure of unilateral and coordinated effects in Section 6.

### 2.2 Coordination

We focus on coordination that does not involve the communication of firms’ private information and does not involve transfers. (Coordination involving either would presumably be considered illegal.) Without such communication, the coordinating suppliers are not able to identify which of them is the lowest-cost supplier, and, consequently, efficient coordination is not possible. Further, without transfers, coordination is individually rational only if it increases the individual expected surplus of each coordinating supplier.\(^{24}\) In contrast, with transfers, each supplier’s individual rationality constraint can be satisfied if coordination increases the joint expected surplus of the coordinating suppliers.

We model coordination among a subset of bidders \( K \) as a bidder selection scheme that randomly designates one supplier in \( K \) to be the only supplier from \( K \) to bid in the procurement. For given \( K \), the bidder selection scheme is defined by a vector \( s = (s_i)_{i \in K} \) of feasible selection probabilities, where feasibility requires that for all \( i \in K \), \( s_i \in [0, 1] \) and \( \sum_{i \in K} s_i = 1 \). With probability \( s_i \), supplier \( i \) is selected to participate in the procurement and the bids of suppliers in \( K \) other than \( i \) are suppressed. We assume that a coordinating supplier that is not designated by the bidder selection scheme has no ability to participate.

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\(^{23}\)See Froeb et al. (2017) on the usefulness of this class of distributions in merger evaluation and Hendricks et al. (2008) for an example of the use of power distributions in an auction context.

\(^{24}\)Baker (2002, p. 164) notes that: “coordinating firms may not be able to allocate the monopoly rents they achieve in a manner satisfactory to all the participants, because they may be unable to compensate each other directly.”
in the procurement, other than perhaps submitting a deliberately losing bid.\footnote{If \( v < \bar{c} \), deliberately losing bids (those with probability zero of being winning bids) include any bid in \([v, \bar{c}]\). If \( v \geq \bar{c} \), then nondesignated coordinating suppliers must submit a bid of \( \bar{c} \) or decline to bid in order to have zero probability of winning. In practice, there are examples in which a bidding ring’s designated bidder ensures that another ring member does not submit a competitive bid by turning in the bid form for that other ring member (\textit{U.S. v. W.F. Brinkley \& Son Construction Company, Inc.}, 783 F.2d 1157, 4th Cir. 1986).}

Given a set of suppliers \( K \) and selection probabilities \( s \), the timeline is as follows:

**Stage 1: Coordination stage.** All suppliers in \( K \) simultaneously and independently decide whether to coordinate by participating in the bidder selection scheme. If all suppliers in \( K \) choose to coordinate, then the bidder selection scheme operates, designating one and only one of the coordinating suppliers, according to probability vector \( s \), to participate in the procurement. If one or more supplier in \( K \) chooses not to coordinate, then the bidder selection scheme does not operate. (In Section 3.1, we refine this to account for the possibility that the failure by one or more suppliers in \( K \) to coordinate might result in a new coordination stage that involves the remaining members of \( K \).)

**Stage 2: Procurement stage.** All suppliers’ costs are realized, and the procurement is held: if the bidder selection scheme does not operate, then all active suppliers participate in the procurement; otherwise, active suppliers not in \( K \) plus the supplier in \( K \) designated by the bidder selection scheme participate in the procurement.

Regardless of whether the bidder selection scheme operates, by the usual second-price auction logic, truthful bidding is a weakly dominant strategy for all suppliers participating in the procurement. In particular, the incentives of the noncoordinating suppliers are not affected by the presence of coordination by rivals, and the incentives of the designated coordinating bidder are not affected by the coordination.

Suppliers contemplating coordination face tradeoffs. Coordinating suppliers may lose surplus in two ways. They may lose profitable sales to outside suppliers by suppressing winning bids, and they may make less profitable sales even when winning because the bidder selection scheme does not necessarily select the lowest-cost supplier. However, the coordinating suppliers stand to gain higher payments by suppressing all but one of the coordinating suppliers’ bids. Thus, on net, coordination can be positive or negative for the coordinating suppliers.

When a bidder decides whether to coordinate, the bidder contrasts its expected surplus from coordination with its expected surplus from no coordination, given truthful bidding in the procurement stage. We characterize when coordination by all suppliers in \( K \) is a Nash equilibrium.
Throughout the paper, we assume that the buyer’s procurement mechanism is the same with and without coordination. However, in Section 7, where we consider the optimal procurement of a powerful buyer, the buyer’s procurement mechanism discriminates among asymmetric suppliers and so treats the merged entity differently from the individual merging suppliers.

2.3 Coordination harms the buyer

We first show that a buyer is harmed by coordination among suppliers. In what follows, expectations are taken with respect to the vector of suppliers’ costs and we let \( c_X \) denote the vector of costs for suppliers in set \( X \), i.e., \( c_X \equiv \times_{i \in X} c_i \). Letting the operator \( 2^{nd} \) select the second-lowest element of a set, the change in the buyer’s expected payoff as a result of coordination by suppliers in \( K \subseteq M \) using selection probabilities \( s \) is

\[
- \sum_{i \in K} s_i \mathbb{E} \left[ \left( 2^{nd} \{ r, c_i, c_{M \setminus K} \} - 2^{nd} \{ r, c \} \right) \cdot 1_{\min\{ c_i, c_{M \setminus K} \} < r} \right] \\
- \sum_{i \in K} s_i \mathbb{E} \left[ (v - 2^{nd} \{ r, c \}) \cdot 1_{\min\{ c_{K \setminus \{i\}} \} < r < \min\{ c_i, c_{M \setminus K} \}} \right],
\]

where the first summation corresponds to the buyer having to pay more in expectation as a result of the suppression of some bids, and the second summation corresponds to the buyer no longer receiving any bids below the reserve as a result of the suppression of some bids. Because this expression is negative, it follows that a buyer is harmed by coordination. Further, because coordination results in the suppression of the bid of the lowest-cost supplier with positive probability, coordination decreases expected social surplus.

Thus, we have the following result:

**Proposition 1.** Coordination reduces expected buyer surplus and expected social surplus.

Proposition 1 underscores the relevance of understanding when a merger increases the risk of coordination by transforming a market from one that is not at risk for coordination into one that is.

3 Coordinated effects index

In this section, we introduce the notion of critical shares and define the coordinated effects index, which allows us to define whether a market is at risk for coordination.
3.1 Key concepts

Given active suppliers \( M \) and \( i \in M \), we let \( \Pi_i \) denote supplier \( i \)'s expected surplus in the absence of coordination. Thus,

\[
\Pi_i(M) = \mathbb{E}\left[\max\{0, \min\{r, c_{M \setminus \{i\}}\} - c_i\}\right].
\]

We use \( K \) to denote a set of two or more suppliers whose potential coordination we are considering. For \( K \subseteq M \) and \( i \in K \), we let \( \Pi^K_i \) denote supplier \( i \)'s expected surplus when it is the bidder in \( K \) designated by the bidder selection scheme to participate in the auction. It follows that

\[
\Pi^K_i(M) = \mathbb{E}\left[\max\{0, \min\{r, c_{M \setminus K}\} - c_i\}\right].
\]

Observe that \( \Pi^K_i(M) > \Pi_i(M) \). Given \( M, K \subseteq M \), and the selection probability \( s_i \), supplier \( i \)'s expected payoff if the suppliers in \( K \) coordinate is \( s_i \Pi^K_i(M) \). Because supplier \( i \)'s expected payoff is \( \Pi_i(M) \) without coordination, choosing to coordinate weakly dominates not doing so for supplier \( i \) if and only if

\[
s_i \Pi^K_i(M) > \Pi_i(M).
\]

Thus, when evaluating coordination by a subset \( K \) of suppliers in market \( M \), we define the critical share for supplier \( i \in K \) as

\[
s^K_i(M) \equiv \frac{\Pi_i(M)}{\Pi^K_i(M)}.
\]

Choosing to coordinate is weakly dominant for supplier \( i \in K \) if and only if supplier \( i \) is selected with probability greater than its critical share, which, of course, is only possible for all suppliers in \( K \) if their critical shares sum to less than 1. This leads to our definition of the coordinated effects index (CEI):

\[
CEI_K(M) \equiv 1 - \sum_{i \in K} s^K_i(M),
\]

where we drop the argument \( M \) when the market at issue is clear. In words, the CEI is 1 minus the sum of the critical shares, which is less than or equal to 1 and possibly negative because each critical share is a number between 0 and 1.

Letting \( L_X \) be the distribution of the minimum cost among suppliers in \( X \), i.e., \( L_X(c) = 1 - \times_{i \in X}(1 - G_i(c)) \), the analytic expressions for the critical shares are as follows:
Lemma 1. For all $M, K \subseteq M$, and $i \in K$,

$$s^K_i(M) = \frac{\Pi_i(M)}{\Pi^K_i(M)} = \frac{\int_{c_r}^{c_c} (1 - L_{M\setminus\{i\}}(c))G_i(c)dc}{\int_{c_r}^{c_c} (1 - L_{M\setminus K}(c))G_i(c)dc}.$$  \hspace{1cm} (2)

Proof. See Appendix B.

Markets at risk for coordination

We use the $CEI_K$ to measure the risk for coordination by suppliers in $K$, where a positive $CEI_K$ means that a market is at risk.

Definition 1. A market is “at risk for coordination by suppliers in $K$” if $CEI_K > 0$ and “not at risk for coordination by suppliers in $K$” if $CEI_K \leq 0$.

When a market is at risk for coordination by suppliers in $K$, by definition there exists a vector of selection probabilities $s^* = (s^*_i)_{i \in K}$ such that for all $i \in K$, $s^*_i \in (s^K_i, 1]$ and $\sum_{i \in K} s^*_i = 1$. This implies that a bidder selection scheme based on selection probabilities $s^*$ is feasible and makes coordination a weakly dominant strategy for each supplier in $K$. However, if $CEI_K$ is less than or equal to zero, then $\sum_{i \in K} s^K_i \geq 1$ and there is no feasible vector of selection probabilities such that coordination increases the expected payoff of all suppliers in $K$, relative to not coordinating. For any feasible vector of selection probabilities, at least one supplier would have weakly greater expected surplus from no coordination than from coordination.\footnote{One could incorporate a “cost of coordination” by requiring that the $CEI_K$ be greater than some positive threshold in order to view a market as at risk.}

As this suggests, a nonpositive $CEI_K$ is an indication that coordination by the suppliers in $K$ in the form of a bidder selection scheme is unlikely to be a reason for concern because it is not individually rational. In contrast, a positive $CEI_K$ tells us that coordination by suppliers in $K$ is feasible. Given a positive $CEI_K$, the question of whether coordination by suppliers in $K$ is “likely” requires further analysis. We offer a couple observations related to this question. First, for a market at risk for coordination, the larger the $CEI_K$, the more flexibility the suppliers in $K$ have to arrange mutually beneficial coordination, which suggests that coordination is more likely. This leads us to the following definition:

Definition 2. A market that is at risk for coordination by suppliers in $K$ is “at greater risk” the higher is the $CEI_K$.

This quantitative interpretation can be justified, for example, on the grounds that there are additional costs associated with coordination that are not captured in $\Pi^K_i(M)$,
such as possible penalties for being caught coordinating. Specifically, letting $P_i > 0$ be the expected penalty of bidder $i$ and denoting by $\hat{s}_i$ the critical share for $i$ that takes $P_i$ into account, we have $\hat{s}_i = (\Pi_i(M) + P_i)/\Pi_K^i(M) > s_i$. Thus, the larger is the $CEI_K$, the larger are the expected penalties required to deter coordination. Moreover, as the $CEI_K$ increases, coarser and less sophisticated coordination devices will suffice, making coordination easier. In the limit, as $CEI_K$ goes to 1, uniform randomization will do the trick.

Second, if the $CEI_K$ is positive, but for all subsets of $K$ containing two or more suppliers, the corresponding $CEI$ is nonpositive, then each member of $K$ is, in a sense, pivotal for the feasibility of coordination. This second consideration can be used to identify stable subsets of suppliers that are of greater concern for coordination. We formalize this using the following notion of stability:

**Definition 3.** Coordination among suppliers in $K$ is “stable” if (i) $CEI_K > 0$ and (ii) for every $\hat{K} \subsetneq K$ containing two or more suppliers, $CEI_{\hat{K}} \leq 0$.

Coordination can be viewed as more likely if it is stable because then no participant in the coordination could expect to benefit from the coordination of others in the group should it decline to participate.

### 3.2 CEI test has power

We begin with a characterization of when a market with symmetric suppliers is at risk for coordination. Under the assumption of symmetric suppliers, we let $c(i:j)$ denote the $i$-th lowest order statistic out of $j$ draws from the common cost distribution.

**Proposition 2.** A market with $n$ symmetric suppliers is at risk for coordination among $k \in \{2, \ldots, n\}$ suppliers if and only if

$$\frac{\mathbb{E}\left[\max\{0, \min\{r, c_{(2n-k+1)}\} - c_{(1n-k+1)}\}\right]}{\mathbb{E}\left[\max\{0, \min\{r, c_{(2n)}\} - c_{(1n)}\}\right]} > \frac{k}{n} (n - k + 1).$$  \hspace{1cm} (3)

**Proof.** See Appendix B.

Proposition 2 implies that when $v \geq \overline{v}$, which means that $r = \overline{v}$, a market with $n$ symmetric suppliers is at risk for coordination among all $n$ suppliers if and only if

$$\overline{v} - \mathbb{E}\left[c_i\right] > \mathbb{E}\left[c_{(2n)} - c_{(1n)}\right].$$  \hspace{1cm} (4)

Inequality (4) says that the distance between the upper bound of support of the cost distribution and the expected cost is greater than the expected distance between the
first and second order statistics. Inequality (4) is satisfied, for example, for the uniform
distribution on \([0, 1]\) and \(n = 2\).\(^{27}\) It is not satisfied, for example, when \(n = 2\) and the
density \(g(c)\) is such that \(g(c) = 0.05\) for \(c \in [0, 0.9]\) and \(g(c) = 9.55\) for \(c \in (.9, 1]\),\(^{28}\)
which has a long left tail and high probability close to the upper bound of support.\(^{29}\) In
this case, even with competition, the price paid to the winning supplier is likely to be
close to the reserve, so the incremental expected payment under cooperation is small and
outweighed by the incremental cost associated with possibly having the “wrong” supplier
trade. As this example shows, a “3-to-2” merger that results in a duopoly characterized
by the long left tail distribution does not raise concerns of coordinated effects.

To explore this further, note that when there are two suppliers drawing their costs
from the uniform distribution on \([0, 1]\), coordination increases the expected payment to
the winner from \(2/3\) to \(1\), that is, by \(1/3\). Coordination also increases the expected cost
of the winner, from \(1/3\) to \(1/2\), that is, by \(1/6\). Because the increase in expected payment
exceeds the increase in expected cost, coordination is profitable. However, if the suppliers
draw their costs from the long left tail distribution described above, coordination only
increases the expected payment to the winner from 0.965 to 1, that is, by 0.035, while
it increases the expected cost of the winner from 0.891 to 0.928, which is an increase of
0.037. Thus, for the long left tail distribution, coordination increases the expected cost
to the winner by more than it increases the expected payment to the winner, making
coordination unprofitable.

This establishes the following result.

**Corollary 1.** Some, but not all, markets are at risk for coordination, including coordina-
tion by all suppliers in the market.

Corollary 1 shows that our test for markets at risk for coordination is powerful in that
some, but not all, markets are at risk. This is in contrast to notions of a market being at
risk for perfect collusion in which only the lowest-cost supplier trades. Coordination of
the type we consider introduces an inefficiency because it is not necessarily the lowest-cost
agent who trades, and if the buyer’s reserve is binding, coordination can cause there to
be no trade when there would be trade in the absence of coordination.

## 4 Consistency

In this section, we briefly review the broad lessons learned from the literature on collusion
and show that our framework, despite substantial differences, generates results that are
consistent with the extant literature and some of the perceived wisdom.

\(^{27}\)In this case, \(\bar{c} - \mathbb{E}[c_i] = 1/2\) and \(\mathbb{E} \left[ c_{(2:n)} - c_{(1:n)} \right] = 1/3.\)

\(^{28}\)The corresponding cdf has \(G(c) = 0.05x\) for \(c \in [0, 0.9]\) and \(G(c) = 9.55c - 8.55\) for \(c \in (0.9, 1]\).

\(^{29}\)Specifically, \(1 - \mathbb{E}[c] = 0.0725\) and \(\mathbb{E} \left[ c_{(2:n)} - c_{(1:n)} \right] = 0.0740.\)
4.1 Markets with sufficient outside competition are not at risk

As one would expect, coordination is more challenging when coordinating suppliers face more outside competition. An increase in the number of (symmetric) outside suppliers decreases the payoffs of the suppliers in $K$ both with and without coordination. However, as we show in Proposition 3, the decrease in their payoffs under coordination is larger, and so critical shares increase and the market becomes less at risk for coordination. Intuitively, increased outside competition has a greater effect on the payoffs of suppliers in $K$ under coordination because, under coordination, the price paid to the selected bidder only depends on the bids of outsiders and possibly the reserve. In contrast, without coordination, prices also depend on the bids of insiders.

As we show in Proposition 3 for a symmetric setup, the risk for coordination vanishes as the number of outside suppliers increases.

**Proposition 3.** With symmetric suppliers, holding fixed the number of suppliers in $K$, as the number of outside suppliers increases, eventually the market is not at risk, i.e., there exists $\hat{n}$ such that for all $n > \hat{n}$, $CEI_K < 0$.

**Proof.** See Appendix B.

As an illustration, when $m$ symmetric suppliers draw their costs from the uniform distribution on $[0, 1]$, then $CEI_{\{1, 2\}}(\{1, \ldots, m\}) = (3 - m)/(m + 1)$, which is decreasing in $m$ and is less than or equal to 0 for $m \geq 3$. Thus, increasing the number of outside suppliers decreases the CEI until eventually the market is not at risk for coordination.

4.2 Markets are more at risk with large coordinators

To analyze the risk for coordination by large versus small suppliers, we consider the capacity-based parameterization, which allows us to rank suppliers by their capacities. Proposition 4 shows that if there is a set of suppliers that are sufficiently large relative to the other suppliers, then the market is at risk for coordination by those large suppliers. Conversely, if there is a set of suppliers that are sufficiently small relative to the other suppliers, then the market is not as risk for coordination only by those suppliers. Furthermore, the CEI for a set of $k$ suppliers is greatest when the set contains the $k$ largest suppliers.

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30In the case of costs drawn from the uniform distribution on $[0, 1]$, $\Pi_i = \frac{1}{m(m+1)}$, and for $i \in \{1, \ldots, k\}$, $\Pi_{\{1, \ldots, k\}} = \frac{1}{m-k+1} \frac{1}{m-k+2}$, so $s_{i}^{\{1, \ldots, k\}} = (m - k + 1)(m - k + 2)/(m(1 + m))$ and $CEI_{\{1, \ldots, k\}}(\{1, \ldots, m\}) = 1 - ks_{i}^{\{1, \ldots, k\}}$. Given $k$, as $m$ increases, $CEI_{\{1, \ldots, k\}}(\{1, \ldots, m\})$ is eventually decreasing in $m$ and less than or equal to 0 for $m$ sufficiently large.

---
Proposition 4. In the capacity-based parameterization with \( v \geq c \), if suppliers in \( K \) all have capacity \( \alpha \), then \( \lim_{\alpha \to \infty} CEI_K > 0 \) and \( \lim_{\alpha \to 0} CEI_K < 0 \). Furthermore, with asymmetric suppliers, the \( CEI_K \) is largest, conditional on \( |K| = k \) if \( K \) includes the \( k \) highest-capacity suppliers.

Proof. See Appendix B.

It follows from Proposition 4 that a market with both large and small suppliers is most at risk for the type of coordination that is most problematic for buyers, namely coordination among the large suppliers. In addition, a market is more at risk for coordination among a group of the largest suppliers in the market, relative to an equal number of other suppliers. This is consistent with the prevailing view that a competition authority should be most concerned about coordination among the largest suppliers in a market.

4.3 Greater symmetry increases the CEI

Competition authorities have expressed the view that a merger that increases symmetry among potentially coordinating suppliers is more likely to raise concerns of coordinated effects. As we show, our framework provides support for this view.

Let \( K \) be a set of three or more suppliers, including suppliers 1 and 2, and suppose that suppliers 1 and 2 are symmetric with one another (although not necessarily with other suppliers) with cost distribution \( G \). From this scenario, we construct a second scenario by keeping the distributions for suppliers other than 1 and 2 the same and by making supplier 1 stronger and supplier 2 weaker, while preserving the distribution of the minimum cost of suppliers 1 and 2. That is, the second scenario has distributions for suppliers 1 and 2 of \( F_1 \) and \( F_2 \) such that, for all \( c \in [\underline{c}, \overline{c}] \),

\[
F_1(c) \geq G(c) \quad \text{and} \quad F_2(c) \leq G(c),
\]

with a strict inequality for an open subset of \( [\underline{c}, \overline{c}] \), and for all \( c \in [\underline{c}, \overline{c}] \),

\[
(1 - F_1(c))(1 - F_2(c)) = (1 - G(c))^2.
\]

We refer to a shift in the distributions of two symmetric suppliers that satisfies (5) and (6) as a competitively neutral spread of \( G \). Moreover, we say that \((H_1, H_2)\) is a larger competitively neutral spread of \( G \) than \((F_1, F_2)\) if for all \( c \in [\underline{c}, \overline{c}] \), \( H_1(c) \geq F_1(c) \geq G(c) \) and \( H_2(c) \leq F_2(c) \leq G(c) \), with a strict inequality between \( H_i \) and \( F_i \) for an open subset of \([\underline{c}, \overline{c}]\), and \((1 - H_1(c))(1 - H_2(c)) = (1 - G(c))^2 \). For example, in the capacity-based parameterization, a larger competitively neutral spread of the distribution \( G \) with capacity \((\alpha_1 + \alpha_2)/2\) is created if the capacities \( \alpha_1 \) and \( \alpha_2 \) of firms 1 and 2 satisfying \( \alpha_1 < \alpha_2 \) are
redistributed to $\hat{\alpha}_1$ and $\hat{\alpha}_2$ satisfying $\hat{\alpha}_1 + \hat{\alpha}_2 = \alpha_1 + \alpha_2$ and $\hat{\alpha}_1 < \alpha_1 < \alpha_2 < \hat{\alpha}_2$.

It follows that a competitively neutral spread applied to two symmetric suppliers in $K$ does not affect the critical shares of any other suppliers in $K$. As we show in the proof of Proposition 5, a competitively neutral spread causes the sum of the critical shares of the affected suppliers to increase, thereby decreasing the CEI. In addition, Proposition 5 shows that for the capacity-based parameterization, a larger competitively neutral spread results in a larger decrease in the CEI.

**Proposition 5.** A competitively neutral spread applied to two symmetric suppliers in $K$ does not affect the outsiders’ expected payoffs, reduces the $CEI_K$, and increases the buyer’s expected price. The larger is the spread, the larger is the price increase and, for the capacity-based parameterization with $v \geq \bar{v}$, the larger is the decrease in the $CEI_K$.

*Proof.* See Appendix B.

The result in Proposition 5 regarding the negative effects of a competitively neutral spread on the buyer resonates with the idea that the presence of a firm with a “dominant position” harms buyers and that greater dominance results in greater harm.\(^{31}\) It is noteworthy that the buyer harm in Proposition 5 arises irrespective of the sizes of the firms under consideration relative to their rival. Any competitively neutral spread increases the expected price. This gives rise to a tension between coordinated and unilateral effects that we explore in more detail in Section 6.

In passing, we note as a corollary to Proposition 5 that a competitively neutral spread decreases the incentives to merge. To make the statement precise, denote by $\Delta\pi_G$, $\Delta\pi_F$, and $\Delta\pi_H$, respectively, the differences in the merging suppliers’ joint profits after and before merger when firms 1 and 2 draw their costs independently from $G$, from $F_1$ and $F_2$, and from $H_1$ and $H_2$, respectively, where $(H_1, H_2)$ and $(F_1, F_2)$ are two competitively neutral spreads of $G$.

**Corollary 2.** The larger is a competitively neutral spread, the smaller are the incentives to merge. That is, assuming $H_1(c) \geq F_1(c) \geq G(c)$ and $H_2(c) \leq F_2(c) \leq G(c)$, we have

$$\Delta\pi_H \leq \Delta\pi_F \leq \Delta\pi_G,$$

with strict inequalities if the inequalities for the corresponding distributions are strict for an open subset of $[\underline{c}, \min\{v, \bar{v}\}]$.

\(^{31}\)Regarding the possibility of a “significant impediment to effective competition,” the EC Guidelines (para. 2) states, “The creation or the strengthening of a dominant position is a primary form of such competitive harm.” See also Compte et al. (2002).
Proof. See Appendix B.

As observed by Loertscher and Marx (2019b), in the absence of buyer power and cost synergies, a merger of two firms is equivalent to perfect collusion between them, with perfect collusion understood as requiring communication of production-relevant information about costs. Viewed in this light, Corollary 2 also says that more asymmetry, in the sense of a larger competitively neutral spread, decreases incentives for perfect collusion.

4.4 The CEI is lower when the buyer uses a first-price auction

First-price auctions are commonly perceived as being less vulnerable to collusion than second-price auctions. Consistent with this, the CEI is weakly lower when the buyer uses a first-price auction rather than a second-price auction (or a strategically equivalent descending clock auction). As is well known, in general, first-price auctions are tractable and efficient if and only if all bidders draw their types from the same distribution. This is why in the following proposition we assume ex ante symmetric suppliers, that is, $G_i = G$ for all $i \in N$. Let $CEI_{FPA}^K$ denote the coordinated effects index when the buyer uses a first-price auction and the suppliers in the set $K$ coordinate. As before, $CEI_K$ denotes the index when the same bidders coordinate and the buyer uses a second-price or descending clock auction.

Proposition 6. With symmetric suppliers, coordination is less of a concern with a first-price auction, that is, for any $K \subset N$, $CEI_{FPA}^K \leq CEI_K$, with equality if and only if, under the first-price auction, the bidders $j \in N \setminus K$ know that the bidders $i \in K$ coordinate.

Proposition 6 uses the CEI to formalize the popular notion that first-price auctions are less vulnerable to collusion than second-price auctions. It also demonstrates that the dominant-strategy aspect of our procurement setting is what makes the framework tractable and why our framework permits clean comparisons. By definition, a dominant strategy is a best response that does not vary with other players’ actions. In other words, in dominant strategy mechanisms, actions are neither strategic substitutes nor strategic complements. This means that outsiders’ best responses do not depend on whether or not they know that there is coordination. This permits clean comparisons because the analyst can remain agnostic regarding the question of outsiders’ knowledge. It also means that there is no ambiguity regarding the appropriate notion of equilibrium—it is dominant strategy equilibrium with and without coordination, irrespective of what the outsiders know or anticipate.
5 Mergers and mavericks

In contrast to standard arguments that increased concentration increases the risk of coordination, we now show that in our framework a merger can increase or decrease the risk of coordination. Thus, we can distinguish mergers with coordinated effects concerns from those without. In addition, in our framework there is a natural definition of a maverick, which allows us to analyze the effects of a merger that involves the acquisition of a maverick. As we show, in some settings, resonating with perceived wisdom, the acquisition of a maverick increases the risk of coordination; however, in other settings it does not.

5.1 A merger need not increase the risk of coordination

In order to consider the effects of a merger among suppliers in $K$ on incentives for coordination, we take as given a set $K$ of three or more suppliers. Recall from the discussion above that $s^K_i$ depends only on the distribution of the minimum cost of suppliers other than $i$ in $K$ and on the distribution of the minimum cost of suppliers outside $K$. Because a merger of two suppliers does not affect the distribution of the minimum cost of the two merging suppliers, it follows that a merger of two suppliers in $K$ does not affect the critical shares of the nonmerging firms in $K$. Thus, the change in $CEI_K$ as a result of a merger depends on how the critical share of the merged entity compares to the sum of the critical shares of two merging suppliers in the pre-merger market. We state this formally in the following lemma, where $\hat{K}_{k,\ell} \equiv \mu_{k,\ell} \cup K \setminus \{k, \ell\}$ denotes the suppliers in $K$ following the merger of $k$ and $\ell$ in $K$.

**Lemma 2.** Given $K \subseteq N$ and merging suppliers $k, \ell \in K$ and

$$CEI_{K_{k,\ell}}(\hat{N}_{k,\ell}) - CEI_K(N) = s^K_K(N) + s^K_{\ell}(N) - s_{\hat{K}_{k,\ell}}(\hat{N}_{k,\ell}).$$

(7)

The next proposition follows immediately.

**Proposition 7.** A merger of suppliers in $K$ increases the $CEI_K$ if and only if the critical share of the merged entity is less than the sum of the pre-merger critical shares of the merging suppliers.

A merger of suppliers $k$ and $\ell$ is always profitable for those suppliers, i.e., $\Pi_k + \Pi_\ell < \Pi_{\mu_{k,\ell}}$ (Loertscher and Marx, 2019b, Prop. 6). At the same time, a merger increases the merging suppliers’ payoff from coordination by improving the efficiency of coordination. To see this, note that the merged entity has cost $\min\{c_k, c_\ell\}$, so the merger eliminates the possibility that the supplier with the higher cost $\max\{c_k, c_\ell\}$ is designated to be the sole bidder to represent the coordinating bidders, i.e., $\max\{\Pi^K_K, \Pi^K_\ell\} < \Pi^K_{\mu_{k,\ell}}$. Because either effect can dominate, the sum of the pre-merger critical shares of the merging suppliers
can be greater than or less than the critical share of the merged entity, implying that the CEI can increase or decrease.

In contrast to the case of a merger of suppliers in $K$, a merger of suppliers outside $K$ does not affect the $CEI_K$ as it only depends on suppliers outside $K$ through the distribution of the minimum of their costs, which is not affected by a merger. Thus, we have the following result:

**Proposition 8.** A merger of suppliers in $K$ can, but need not, cause a market not at risk for coordination among suppliers in $K$ to become at risk for coordination among the corresponding post-merger suppliers, and a merger of suppliers outside $K$ does not affect the risk for coordination among suppliers in $K$.

### 5.2 Maverick firms: definition and implications

As discussed in the introduction, the term “maverick” fares prominently in current antitrust economics and practice, but lacks a clear-cut and operational definition. If, as has been argued, regulators can reliably identify an industry maverick that prevents or limits coordination, but are unable to define what a maverick is, then it will be difficult, if not impossible, for merging parties to provide evidence and arguments that could invalidate the authorities’ concerns.

An operational definition of a maverick requires that a maverick firm be identifiable based only on pre-merger observables. In what follows, we provide such a definition and then analyze mergers involving a maverick in our setup. According to our definition, a maverick with respect to a set of suppliers $K$ is a supplier whose presence prevents a pre-merger market from being at risk for coordination among suppliers in $K$, i.e., the market is not at risk for coordination by suppliers in $K$ when the maverick is in the market, but is at risk when the maverick is not in the market.\(^{32}\)

**Definition 4.** Given a pre-merger market with set of suppliers $N$ and subset $K \subset N$, supplier $m \in N \setminus K$ is a maverick with respect to $K$ if $CEI_K(N) \leq 0$ and $CEI_K(N \setminus \{m\}) > 0$.

This definition allows the possibility that more than one supplier in a market could be a maverick with respect to a particular set $K$ of suppliers.\(^{33}\) However, because merger approval is typically required for each pairwise merger, we focus on transactions that

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\(^{32}\)For example, when the market contains three firms drawing their cost types from the uniform distribution on $[0, 1]$, then supplier 3 is a maverick with respect to coordination by suppliers 1 and 2, i.e., $CEI_{\{1, 2\}}(\{1, 2, 3\}) \leq 0$ and $CEI_{\{1, 2\}}(\{1, 2\}) > 0$.

\(^{33}\)Indeed, in the U.S. mobile communications market, prior to T-Mobile’s acquisition of MetroPCS, both were considered “mavericks’ with a history of disrupting the industry” (”Sprint CEO Sees ‘Enormous’ Synergies in T-Mobile Merger,” *Kansas City Star*, June 12, 2017).
involve the acquisition of only one maverick.\textsuperscript{34}

As a case in point, consider the milk processing market. After the acquisition of Foremost Farms by Dean Foods (see Baye et al., 2019), there were four major suppliers: 1. Dean, 2. Foremost, 3. Kemps, and 4. Prairie. In that matter, the DOJ argued that “without Foremost, the remaining producers would find it easier to achieve coordinated effects,” and Dean viewed Foremost as “unpredictable due to its excess capacity” (Baye et al., 2019, p. 160). Of particular concern was coordination by Dean and one of the other (unnamed) firms who had, it seems, coordinated in the past. Calibrating the capacity-based parameterization to the suppliers’ market shares and an assumed operating margin for Dean,\textsuperscript{35} Foremost satisfies the definition for being a maverick with respect to coordination by Dean, Kemps, and Prairie, with $CEI_{\{Dean, Kemps, Prairie\}}(N) = -10\% < 0$ and $CEI_{\{Dean, Kemps, Prairie\}}(N\{\{Foremost\}) = 5\% > 0$, where $N$ denotes the set of all pre-merger suppliers. Further, the corresponding CEI for the post-merger market is positive 12\%, indicating that the acquisition of Foremost by Dean puts the market at risk for coordination by the merged entity, Kemps, and Prairie Farms.

Following the merger between a supplier in $K$ and a maverick, the critical share of a nonmerging supplier $i \in K$ decreases relative to the case in which the maverick is not present, i.e., relative to $s^K_i(N\{m\})$, because their coordinated expected payoff remains the same, but their noncoordinated payoff decreases. The merged entity’s noncoordinated and coordinated expected payoffs increase, and in the capacity-based parameterization, the critical share of the merged entity increases relative to the critical share of the acquiring supplier in the market with suppliers $N\{m\}$.

The following proposition provides conditions under which the decrease in the critical share of the nonmerging supplier dominates if and only if the maverick is acquired by the smaller of two coordinating suppliers.

\textbf{Proposition 9.} In the capacity-based parameterization with $v \geq \tau$, if the market contains a maverick with respect to $K = \{1, 2\}$, then the acquisition of the maverick by the weakly smaller supplier in $K$ puts the market at risk for coordination, whereas the acquisition of the maverick by the larger supplier in $K$ does not if the weaker supplier in $K$ is sufficiently small.

\textsuperscript{34} If two mavericks with respect to $K$ are themselves the merging parties, then Proposition 8 implies that in the absence of buyer power, the merger does not affect the $CEI_K$, so the post-merger market is not at risk for coordination. With buyer power, a merger of two mavericks increases the payoffs of suppliers in $K$ both with coordination and without, with an ambiguous effect on the critical shares and hence on the $CEI_K$.

\textsuperscript{35} Market shares were Dean 45\%, Foremost 12\%, Kemps 17\%, and Prairie 15\%, with no other firm having more than 5\%. For identification, we assume that Dean’s operating margin (Lerner Index) is 51\%. See Appendix A for the details on using market shares and one firm’s margin to calibrate parameterized cost distributions. The resulting capacity parameters are $\alpha_{Dean} = 1.10$, $\alpha_{Foremost} = 0.29$, $\alpha_{Kemps} = 0.41$, and $\alpha_{Prairie} = 0.37$, with a total for the parameters of the other firms of 0.27.
Proof. See Appendix B.

Proposition 9 provides a setting in which the acquisition of the maverick by the weakly smaller of two suppliers in $K$ puts the post-merger market at risk for coordination. This is consistent with the view that a merger that eliminates a maverick raises concerns of coordinated effects. However, Proposition 9 also shows that the acquisition of a maverick by the larger of two suppliers in $K$ does not put the market at risk when the other supplier in $K$ is sufficiently small. Thus, Proposition 9 puts into question the view that any merger that involves the acquisition of a maverick raises coordinated effects concerns. Whether the acquisition of a maverick puts a market at risk for coordination depends on the size of the supplier that acquires the maverick.

Some other aspects of the perceived wisdom regarding mavericks are similarly not supported in our framework. In particular, one can easily construct examples in which the acquisition of a supplier that is not a maverick puts a market at risk for coordination. Thus, it is not the case that the acquisition of a maverick is necessary for coordinated effects. It is also not the case, as has been suggested, that the presence of a maverick prevents coordinated effects from mergers not involving the maverick.\footnote{To see that this argument (see, e.g., Baker, 2002, p. 180) does not hold in our setup, consider a market with five firms drawing their cost types from the uniform distribution on $[0, 1]$. Then supplier 5 is a maverick with respect to coordination by suppliers in $\{1, 2, 3\}$, i.e., $CEI_{\{1, 2, 3\}}(\{1, 2, 3, 4, 5\}) \leq 0$ and $CEI_{\{1, 2, 3\}}(\{1, 2, 3, 4\}) > 0$, and following the merger of suppliers 1 and 2, the market is at risk for coordination by the merged entity and supplier 3, i.e., $CEI_{\{\mu_{1, 2, 3}\}}(\{\mu_{1, 2, 3}, 4, 5\}) > 0$.}

6 Mitigating harms tradeoffs

In this section, we discuss merger-related cost synergies and divestitures, both of which are commonly viewed as mitigating harms from a merger. As we show, although they have the potential to mitigate unilateral effects, they also have the potential to exacerbate coordinated effects. Indeed, in the case of divestitures, changes to symmetry that mitigate unilateral effects are exactly those that exacerbate coordinated effects.

6.1 Cost synergies and merger harm

We model cost synergies by assuming that if suppliers $k$ and $\ell$ merge, then the cost distribution of the merged entity $\mu_{k, \ell}$ is

$$G_{\mu_{k, \ell}; z}(c) \equiv 1 - ((1 - G_k)(1 - G_\ell))^z,$$

where $z = 1$ corresponds to no cost synergies (that is, $G_{\mu_{k, \ell}; 1}(c) = G_{\mu_k, \ell}(c)$) and $z > 1$ corresponds to merger-related cost synergies.
Cost synergies increase the critical share of the merged entity for any group of coordinating suppliers that it might be a part of. But cost synergies for the merged entity decrease the noncoordinated payoffs for other suppliers, without affecting their coordinated payoffs, and so decrease the critical shares of those suppliers. As we show, the decrease in critical shares for the nonmerging suppliers dominates for a range of cost synergies. Specifically, when the post-merger set of coordinating suppliers $\hat{K}$ includes the merged entity and another supplier of comparable or greater size (in the sense of FOSD), then cost synergies increase the $CEI_{\hat{K}}$ as long as they are not too large.

**Proposition 10.** If $\hat{K}$ consists of the merged entity $\mu$ and one other supplier $i$ with $G_i \leq G_{\mu}$, then cost synergies increase the $CEI_{\hat{K}}$ as long as they are not too large.

**Proof.** See Appendix B.

Proposition 10 shows that coordinated effects between two post-merger suppliers are a greater concern when a merger generates (moderate) cost synergies than when it does not. To our knowledge, this relation between cost synergies and coordinated effects is novel. It raises the interesting possibility that two merging firms might choose not to fully implement available cost synergies in order to preserve the possibility of post-merger coordination.

This analysis highlights a tension between unilateral and coordinated effects. Proposition 10 shows that moderate merger-based cost synergies increase the CEI and so indicate heightened concerns for coordinated effects. However, Loertscher and Marx (2019b, Proposition 9) show that in the absence of coordination, such cost synergies decrease the expected buyer harm from a merger, reducing concerns of unilateral effects.

### 6.2 Divestiture-based remedies for merger harm

An additional source of tension between unilateral and coordinated effects arises when considering divestiture-based remedies for merger harm. Specifically, a divestiture that breaks the merged entity into two symmetric suppliers would, using Proposition 5, place the market more at risk than a divestiture that breaks the merged entity into asymmetric suppliers satisfying the conditions of a competitively neutral spread. For the purposes of the result below, we use the term “merger-plus-divestiture” to mean the case in which the assets of two suppliers are merged and then divided to create two new suppliers, in which case, we refer to the two new suppliers as the post-divestiture suppliers.

**Corollary 3.** A market affected by a merger-plus-divestiture has a greater CEI for coordination by the post-divestiture suppliers if they are symmetric (with distribution $G$) than if they differ by a competitively neutral spread (of $G$).

24
Corollary 3 offers some guidance to competition authorities focused on coordinated effects that are contemplating requiring a divestiture in order to remedy the anticompetitive effects of a merger—divestitures that result in asymmetric post-divestiture suppliers are preferred. However, as shown in Proposition 5, concerns regarding unilateral effects suggest the opposite approach. A competitively neutral spread replaces two symmetric firms with, in effect, one dominant and one non-dominant firm, with the larger firm’s dominant position being stronger the larger is the spread. This highlights a tension between divestitures that address concerns of unilateral effects versus those that address concerns of coordinated effects.\footnote{This tension is also noted by Compte et al. (2002) in the context of a repeated game model with capacity constrained firms, in which case tacit collusion is aided by symmetry.}

This tension suggests that a combined measure of unilateral and coordinated effects might be useful. Given estimates of suppliers’ pre-merger cost distributions, one can use the reduction in expected buyer surplus as a measure of unilateral effects. If the $CEI_K$ is positive, one can then calculate bounds on the incremental reduction in expected buyer surplus as a result of coordination by considering the possible selection probabilities for suppliers in $K$ that both sum to one and are greater than or equal to each supplier’s critical share. The total effect of the merger and coordination on expected buyer surplus would then give a measure of the combined effect of unilateral and coordinated effects.

7 Extensions and applications

We now extend our setup and results to allow for powerful buyers and for multi-unit demand and supply. Then we provide two applications.

7.1 Powerful buyers

As mentioned in the introduction, there is a notion in the U.S. Guidelines (p. 27) that “the conduct or presence of large buyers” could undermine coordinated effects.\footnote{The U.S. Guidelines (p. 27) also state: “In some cases, a large buyer may be able to strategically undermine coordinated conduct, at least as it pertains to that buyer’s needs, by choosing to put up for bid a few large contracts rather than many smaller ones, and by making its procurement decisions opaque to suppliers.”}

As we are unaware of theoretical foundations for this view, we will provide such a foundation in what follows.

Of course, in our setup with single-unit demand, there is no natural notion of buyer size. We therefore adhere to the view that large buyers are powerful, meaning that, as in Loertscher and Marx (2019b), they use—the dominant strategy implementation of—the optimal mechanism, that is, the mechanism that maximizes the buyer’s expected profit subject to suppliers’ incentive compatibility and individual rationality constraints. The optimal mechanism contrasts with the efficient mechanism used by buyers without power,
which we have analyzed thus far.\textsuperscript{39} We assume that buyer power itself is not affected by a merger among suppliers, which seems natural if buyer power derives from the size and/or sophistication of the buyer, as suggested by the \textit{EC Guidelines} (para. 65), or from the ability to vertically integrate upstream or sponsor entry, as suggested by the \textit{U.S. Guidelines} (p. 27).\textsuperscript{40}

In the optimal procurement, the buyer ranks suppliers according to their virtual costs, defined as

$$\Gamma_i(c) \equiv c + \frac{G_i(c)}{g_i(c)},$$  \hspace{1cm} (8)

and applies a supplier-specific reserve price to the supplier with the lowest virtual cost. We impose the standard regularity assumption that $\Gamma_i$ is increasing.\textsuperscript{41} Because we allow the possibility that the densities are zero at $c$ (and also possibly at $c$), define $\Gamma_i(c) = \lim_{c \to c} \Gamma_i(c) = c$. For $x > \Gamma_i(c)$, we define $\Gamma_i^{-1}(x) \equiv c$. The reserve price that a powerful buyer applies to supplier $i$ is $\hat{r}_i \equiv \Gamma_i^{-1}(v)$. (In the case of symmetric suppliers, we drop the subscript $i$ on $\hat{r}$.) Because $\Gamma_i(c) > c$ for $c > c$, it follows that $\hat{r}_i < v$.

As noted by Loertscher and Marx (2019b), buyer power consists of two components: the ability to discriminate between suppliers and the commitment to cancel a procurement even though it would be profitable, which may be called monopsony power. Whether the buyer optimally exerts one or both of these powers depends on the problem at hand. When all suppliers are ex ante symmetric, that is, when $G_i = G$ for all $i \in M$, there is no point for the buyer to discriminate because ranking the suppliers according to their virtual costs is the same as ranking them according to their costs. Without ex ante symmetry, the buyer will optimally use his power to discriminate some of the time. The buyer will optimally refrain from ever using his monopsony power if and only if $v > \max_{i \in M} \{\Gamma_i(c)\}$.

The assumption that suppliers follow their weakly dominant strategies of reporting truthfully implies that if supplier $i$ has the lowest virtual cost, then supplier $i$ wins if $c_i \leq \hat{r}_i$ and is paid $\min_{j \neq i} \{\hat{r}_i, \Gamma_j^{-1}(\Gamma_i(c_j))\}$. Otherwise, there is no trade.

\textsuperscript{39}Loertscher and Marx (2019b) build on Myerson (1981) and apply his mechanism design approach to merger analysis. Their notion of designers with and without power is the same as that of Bulow and Klemperer (1996).

\textsuperscript{40}The \textit{EC Guidelines} also raise the possibility that a merger could reduce buyer power “because a merger of two suppliers may reduce buyer power if it thereby removes a credible alternative” (\textit{EC Guidelines}, para. 67). A nuance on the view that mergers decrease buyer power is provided by Loertscher and Marx (2019b), who observe that, with symmetric suppliers, a merger \textit{increases} the buyer’s incentive to become powerful.

\textsuperscript{41}An intuitive interpretation of the virtual cost function and an understanding of the role of its monotonicity can be developed using standard monopsony pricing. Consider a buyer with value $v \leq \bar{c}$ who faces a single supplier $i$ who draws his cost from the distribution $G_i$. The buyer’s pricing problem is $\max_p (v - p) G_i(p)$, the first-order condition for which is $0 = g_i(p)(v - \Gamma_i(p))$. If $\Gamma_i$ is increasing, the second-order condition is satisfied if the first-order condition is, i.e., the problem is quasi-concave.
When the buyer is powerful,

\[ \Pi_i(M) = \mathbb{E} \left[ \max \{0, \min_{j \in M \setminus \{i\}} \{ \hat{r}_i, \Gamma_i \Gamma_i^{-1}(\Gamma_j(c_j)) \} - c_i \} \cdot 1_{\Gamma_i(c_i) \leq \min_{j \in M \setminus \{i\}} \Gamma_j(c_j)} \right] \]

and

\[ \Pi^K_i(M) = \mathbb{E} \left[ \max \{0, \min_{j \in K \setminus \{i\}} \{ \hat{r}_i, \Gamma_i \Gamma_i^{-1}(\Gamma_j(c_j)) \} - c_i \} \cdot 1_{\Gamma_i(c_i) \leq \min_{j \in K} \Gamma_j(c_j)} \right]. \]

The definitions of critical shares and of the CEI are then the same as in the case without buyer power. With buyer power, the expression for the critical shares is the same as in Lemma 1, with \( r \) replaced by \( \hat{r} \) and \( L_X \) replaced by \( \hat{L}_X \), where \( \hat{L}_X \) is the distribution of \( \min_{j \in X} \Gamma_{i}^{-1}(\Gamma_j(c_j)) \).

As in the case without buyer power, with buyer power, the buyer is harmed by coordination, and some but not all markets are at risk for coordination. In particular, with buyer power, Proposition 2 continues to hold when one replaces \( r \) with \( \hat{r} \) in (3). Interestingly, with buyer power, coordination can increase social surplus for some type realizations. This occurs with ex ante heterogeneous suppliers when, absent coordination, the buyer does not purchase from the lowest-cost supplier because he discriminates between suppliers on the basis of their virtual costs and purchases from a supplier with a lower cost when there is coordination because the bid of the supplier he buys from absent coordination is suppressed.

With buyer power, we continue to have results that (i) markets with sufficient outside competition are not at risk for coordination (Proposition 3 holds with buyer power by replacing \( r \) with \( \hat{r} \) in the proof); and (ii) in the absence of binding reserve prices, a competitively neutral spread applied to two symmetric suppliers in \( K \) decreases the \( CEI_K \) (the relevant portion of the proof of Proposition 5 holds with \( L_X \) replaced by \( \hat{L}_X \)). Assuming ex ante symmetric suppliers pre merger, a decrease in the critical share of the merged entity relative to the sum of the pre-merger critical shares of the merging suppliers is sufficient, but no longer necessary (as in Proposition 7) for the merger to increase the CEI.\(^{42}\) Nevertheless, it remains the case that, as in the first part of Proposition 8, a merger can, but need not, cause a market not at risk to become so.

To examine how the CEI is affected by buyer power in our framework, we first abstract from mergers and focus on ex ante symmetric suppliers. In this case, a powerful buyer never uses its power to discriminate, so the sole effect of buyer power is to reduce the reserve price from \( r \) to \( \hat{r} \), which allows us to focus on the effects of a change in the reserve.

\(^{42}\text{With buyer power, the equation in Lemma 2 holds as a weak inequality (with the left side of (7) weakly greater). This occurs because with buyer power, a merger of suppliers in } K \text{ increases the critical shares of the nonmerging suppliers in } K \text{ because it increases their expected payoffs without coordination but does not affect their expected payoffs if they are designated by the coordination mechanism.}\)
With buyer power and symmetric suppliers, analogous to (2), for \( i \in K \),

\[
s^K_i(M) = \frac{\int_{\hat{r}}^{\bar{r}} (1 - L_{M\setminus\{i\}}(c)) G(c) dc}{\int_{\hat{r}}^{\bar{r}} (1 - L_{M\setminus K}(c)) G(c) dc}.
\]

(9)

If \( v \geq \Gamma(\bar{r}) \), then \( \hat{r} = \bar{r} \), and so the critical shares are not affected by buyer power.

Focusing on the case with \( \hat{r} < \bar{r} \) and differentiating the expression in (9) with respect to \( \hat{r} \) and letting \( k \equiv |K| \) and \( m \equiv |M| \), we get an expression with sign equal to the sign of

\[
(1 - L_{M\setminus\{i\}}(\hat{r})) \int_{\hat{r}}^{\bar{r}} \left(1 - L_{M\setminus K}(c)\right) G(c) c - (1 - L_{M\setminus K}(\hat{r})) \int_{\hat{r}}^{\bar{r}} (1 - L_{M\setminus\{i\}}(c)) G(c) dc
\]

\[
= (1 - G(\hat{r}))^{m-1} \int_{\hat{r}}^{\bar{r}} (1 - G(c))^{m-k} G(c) dc - (1 - G(\hat{r}))^{m-k} \int_{\hat{r}}^{\bar{r}} (1 - G(c))^{m-1} G(c) dc
\]

\[
= (1 - G(\hat{r}))^{2m-1-k} \int_{\hat{r}}^{\bar{r}} \left\{ \left(\frac{1 - G(c)}{1 - G(\hat{r})}\right)^{m-k} - \left(\frac{1 - G(c)}{1 - G(\hat{r})}\right)^{m-1} \right\} G(c) dc.
\]

Because \( \frac{1-G(\hat{r})}{1-G(c)} > 1 \) for \( c < \hat{r} \) and because \( k \geq 2 \), which implies \( m-k < m-1 \), it follows that the expression above is negative. Thus, critical shares are weakly decreasing in buyer power, and strictly so for \( v < \Gamma(\bar{r}) \), which gives us the following result:

**Proposition 11.** Assuming symmetric suppliers, the CEI_K is increasing in the buyer’s reserve price, and thus decreasing with buyer power.

To the best of our knowledge, Proposition 11 is the first formal demonstration that, consistent with perceived wisdom, buyer power reduces concerns of coordination.

Let us now turn to a merger. With buyer power, a merger of two suppliers in \( K \) increases the noncoordinated payoff of the other suppliers in \( K \) as a result of the buyer’s more aggressive discrimination against the merged entity. However, a merger does not affect the payoff of a nonmerging supplier under coordination. Thus, we have the following result:

**Proposition 12.** With buyer power, a merger of two suppliers in \( K \) increases the critical shares of the non-merging suppliers in \( K \).

Proposition 12 implies that with buyer power, a merger of two suppliers in \( K \) reduces the incentive for the other suppliers in \( K \) to participate in coordination. In the face of buyer power, a merger of suppliers in \( K \) constrains the ability of those suppliers to coordinate by increasing the critical shares of the nonmerging suppliers.

Propositions 11 and 12 provide a foundation for the view that coordinated effects from a merger are less of a concern in the face of powerful buyers.
7.2 Multi-unit suppliers and buyer

Our definition of and test for coordinated effects generalize straightforwardly to the case in which the buyer has multi-unit demand and suppliers have multi-unit capacities if we assume no buyer power. (Without multi-unit demand, multi-unit capacities play no substantial role.) To be specific, we can allow the buyer to be characterized by a commonly known marginal value vector \( v = (v_1,...,v_Q) \), where \( Q \) is the buyer’s maximal demand, with \( v_i \geq v_{i+1} \) for \( i \in \{1,...,Q-1\} \) and for each supplier \( j \) to be characterized by a capacity \( \kappa_j \) and a vector of marginal costs \( c^j = (c^j_1,...,c^j_{\kappa_j}) \) satisfying \( c^j_i \leq c^j_{i+1} \) for \( i \in \{1,...,\kappa_j-1\} \), where \( \kappa_j \) is (now) an integer. Assume that each supplier \( j \)’s capacity \( \kappa_j \) is common knowledge, but that each supplier’s marginal costs are its own private information. Assume also that for all \( j \), \( c^j \) is distributed according to the commonly known, continuous distribution \( G_j(c^j) \) with support \([c,\bar{c}]^{\kappa_j}\).

A simple and particularly convenient specification for the multivariate distribution \( G_j(c^j) \) is to assume that \( j \)’s cost draw is the realization of \( \kappa_j \) independent, univariate random variables \( c \) drawn from the distribution \( G(c) \). This implies that \( G_j(c^j) \) is given by the distribution if the \( \kappa_j \)-th order statistic from \( G_j \). For example, the distribution of \( c^j_1 \) is \( G_j(\kappa_j)(c^j_1) = 1 - (1 - G(c))^\kappa_j \). Consequently, we refer to this as the order statistics model. This model also makes clear the sense in which the “capacity-based parameterization” captures a supplier’s capacity.

Following a merger between suppliers \( h \) and \( j \), the merged entity’s capacity is \( \kappa_h + \kappa_j \). In the order statistics model, assuming pre-merger symmetry between \( j \) and \( h \), so that \( G_j = G_h = G \), the distribution of the minimum cost \( c^h_{1j} \) of the merged firm is \( G_{hj,1}(c) = 1 - (1 - G(c))^{\kappa_h+\kappa_j} \).

No buyer power

In the presence of multi-unit demand and multi-unit supply, a buyer without buyer power continues to use an efficient procurement mechanism. The payoff (or revenue) equivalence theorems for multi-dimensional type spaces of Williams (1999) and Krishna and Maenner (2001) imply that the generalized second-price auction with reserve prices for the \( m \)-th unit given by \( \min\{v_m,\bar{c}\} \) is without loss of generality insofar as this is the profit-maximizing mechanism for the buyer subject to efficiency and individual rationality and incentive compatibility constraints for the suppliers. Consequently, the profit of every supplier \( j \) is pinned down by \( v \) and the distributions \( (G_i(c^i))_{i \in N} \) when all suppliers play their dominant strategies of reporting their types \( c^i \) truthfully.

Likewise, the expected profit \( \Pi^K_i \) when the suppliers \( j \in K \) participate in a bidder selection scheme when \( i \in K \) is the designated bidder is pinned down by \( G_i(c^i) \) for \( i \in K \) and \( (G_h(c^h))_{h \in N \setminus K} \). Consequently, \( s^K_i = \Pi_i/\Pi^K_i \) as in the single-unit case, and the CEI\(_K\).
can be defined in the same way and with the same interpretation as before.

Buyer power

With buyer power, the main obstacle to the generalization to multi-unit supply and demand is that the optimal mechanism is not known when agents have multi-dimensional types. Even if one assumed single-unit suppliers in the pre-merger market, a merger would naturally lead to a multi-unit supplier.

However, all is not lost because there are circumstances in which even multi-unit buyers restrict themselves to buying at most one unit from each individual supplier. This may be due to (non-modelled) preferences for diversification, protection against further hold-up, or imposed by law (as in one of the applications in Section 7.3). Under these circumstances, all that matters for the buyer’s optimal mechanism are the distributions of each seller $j$’s lowest cost $c^*_j$, that is, $G_j(1)(c^*_j)$, which is a one-dimensional variable. Hence, the standard mechanism design tools and results apply.

Let us briefly elaborate. The profit-maximizing mechanism for the buyer subject to incentive compatibility and individual rationality constraints given $n > Q$ is characterized as follows: For notational simplicity, let $c_j \equiv c^*_j$ and $G_j(c_j) \equiv G_j(1)(c^*_j)$ with support $[c, \bar{c}]$ and density $g_j(c_j)$ for all $j \in N$. Moreover, to simplify the analysis, assume as before that, for all $j \in N$, the virtual cost function $\Gamma_j(c_j)$, as defined in (8), is increasing in $c_j$. Then, for a given realization $c = (c_1, \ldots, c_n)$ and for given $v$, the profit-maximizing mechanism for the buyer has the allocation rule of purchasing $m \in \{0, \ldots, Q\}$ units from the $m$ suppliers with the lowest virtual costs, where, if $m < Q$, $m$ is such that the $m$-th lowest virtual cost is less than $v_m$ and the $m + 1$-st lowest virtual cost exceeds $v_{m+1}$.

In the dominant strategy implementation of this mechanism, suppliers who do not produce receive (and make) no payments. Each supplier who trades is paid a threshold payment, that is, the highest cost that it could have reported without changing the fact that it trades. This pins down $\Pi_i$ and $\Pi^K_i$, and thereby $s^K_i$ and $CEI^K_i$, just as in the single-unit case. For example, in the special case in which all suppliers are ex ante symmetric with $G_j = G$ for all $j$ and thus $\Gamma_j = \Gamma$ for all $j$, the optimal mechanism can be implemented as via a second-price auction, in which the reserve price for the $l$-th unit is $\Gamma^{-1}(v_l)$. If the quantity traded is $m$, the $m$ successful suppliers are paid $\min\{\Gamma^{-1}(v_m), c_{[m+1]}\}$, where $c_{[m+1]}$ denotes the $m + 1$-st lowest cost.

7.3 Applications

We now provide two applications. The first application examines the French audit industry and illustrates the case of multi-unit demand. The second examines the oilfield services industry and highlights the effects of buyer power.
**French audit industry**

We first consider the French audit industry studied by Ivaldi et al. (2012). Those authors raise the questions of whether the French audit industry is at risk for coordination among the Big 4 firms and whether the fifth-largest firm, Mazars, should be viewed as a maverick, by which they mean “a firm with a drastically different cost structure, which is thus unwilling to participate to a collusive action” (Ivaldi et al., 2012, p. 40). Ultimately, they conclude that Mazars is not a maverick based on their qualitative and quantitative analysis, including econometric results indicating that Mazars is not properly viewed as a competitor with comparable capabilities to a Big 4 firm. Given that, they then conclude that the market is at risk for coordination by the Big 4 based on the *Airtours* criteria of sufficient transparency, the possibility of retaliation, and the absence of either a disruptive rival (i.e., a maverick) or powerful buyers.\(^{43}\)

We calibrate cost distributions to the data and then examine both whether the market is at risk for coordination by the Big 4 firms according to our framework and whether Mazars is a maverick according to our definition.

Ivaldi et al. (2012) note that a key feature of the French audit industry is that large companies must each hire two independent auditors. In addition, they note that buyer power does not seem to be a strong force on the audit market (Ivaldi et al., 2012, p. 81). To account for these features within our framework, we model the buyer as having value \(v \geq c\) for two-units, one from each of two different suppliers (and zero value for anything else), and as not having buyer power.

To calibrate cost distributions, we use the data on 2006 revenue-based market shares for the top eight firms given in Ivaldi et al. (2012, Table 4.2) and assume parameterized cost distributions \(G_i(c) = 1 - (1 - c)^{\alpha_i}\) defined on \([0, 1]\). It is then straightforward to write each supplier \(i\)'s revenue share, \(R_i / \sum_{j=1}^n R_j\), in terms of the parameters of the cost distributions (for details, see Appendix C). For identification, we assume that Ivaldi et al.’s “other” category consists of six symmetric suppliers (which ensures that they are among the smallest suppliers) and that the average cost parameter is equal to one, which pins down the level of the parameters.

Using the calibrated cost distributions, we can then calculate the critical shares and CEI\(_{\text{Big 4}}\), which are shown in Table 1. We find that the CEI\(_{\text{Big 4}}\) is positive, which implies that, consistent with the findings of Ivaldi et al. (2012), the market is at risk for coordination among the Big 4 and Mazars is not a maverick (because Mazars’ presence in the market does not prevent the market from being at risk for coordination).

To analyze the stability of coordination, we calculate the CEIs for various groups of coordinating suppliers, as shown in Table 2. As the table shows, coordination by any

Table 1: Calibration and analysis of coordination in the French audit industry

<table>
<thead>
<tr>
<th>Revenue-based shares</th>
<th>$\alpha_i$</th>
<th>$s_i^{Big 4}$</th>
<th>CEI_{Big 4}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ernst &amp; Young</td>
<td>29.8%</td>
<td>3.8668</td>
<td>0.1289</td>
</tr>
<tr>
<td>Deloitte</td>
<td>21.4%</td>
<td>2.9934</td>
<td>0.1052</td>
</tr>
<tr>
<td>KPMG</td>
<td>22.2%</td>
<td>3.0820</td>
<td>0.1077</td>
</tr>
<tr>
<td>PWC</td>
<td>17.2%</td>
<td>2.5061</td>
<td>0.0913</td>
</tr>
<tr>
<td>Mazars</td>
<td>7.3%</td>
<td>1.1819</td>
<td></td>
</tr>
<tr>
<td>Grant Thornton</td>
<td>0.4%</td>
<td>0.0703</td>
<td></td>
</tr>
<tr>
<td>BDO</td>
<td>0.2%</td>
<td>0.0352</td>
<td></td>
</tr>
<tr>
<td>Constantin</td>
<td>0.3%</td>
<td>0.0528</td>
<td></td>
</tr>
<tr>
<td>6 others</td>
<td>0.2% each</td>
<td>0.0352</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

Three of the Big 4 is stable (the CEI for any three of the Big 4 is positive, but for any two is negative). The table also shows that the market is not at risk for pairwise coordination among any of the Big 4 firms (and the market is not at risk for pairwise coordination between Mazars and any one of the Big 4).

Table 2: CEI$_K$ for various sets $K$ of coordinating suppliers. Firms are: 1. Ernst & Young, 2. Deloitte, 3. KPMG, 4. PWC, 5. Mazars.

<table>
<thead>
<tr>
<th>K</th>
<th>CEI$_K$</th>
</tr>
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<tbody>
<tr>
<td>{1,2,3,4}</td>
<td>0.5670</td>
</tr>
<tr>
<td>{1,2,3}</td>
<td>0.2905</td>
</tr>
<tr>
<td>{1,2,4}</td>
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</tr>
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<td>0.2852</td>
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<tr>
<td>{2,3,4}</td>
<td>0.2917</td>
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</table>

<table>
<thead>
<tr>
<th>K</th>
<th>CEI$_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,2}</td>
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</tr>
<tr>
<td>{1,3}</td>
<td>-0.0640</td>
</tr>
<tr>
<td>{1,4}</td>
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</tr>
<tr>
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<td>-0.1044</td>
</tr>
<tr>
<td>{2,4}</td>
<td>-0.1710</td>
</tr>
<tr>
<td>{3,4}</td>
<td>-0.1617</td>
</tr>
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<table>
<thead>
<tr>
<th>K</th>
<th>CEI$_K$</th>
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<tbody>
<tr>
<td>{1,2,3,4,5}</td>
<td>0.6115</td>
</tr>
<tr>
<td>{1,2,5}</td>
<td>0.0103</td>
</tr>
<tr>
<td>{1,3,5}</td>
<td>0.0172</td>
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<td>{1,4,5}</td>
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<td>{2,3,5}</td>
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</tr>
<tr>
<td>{3,4,5}</td>
<td>-0.1271</td>
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Overall our results are consistent with those of Ivaldi et al. (2012)—we find that the market is at risk for coordination by the Big 4 and that Mazars is not a maverick—but our analysis also points to the perhaps greater concern of coordination among subsets of three of the Big 4. Although the market is at risk for coordination by the Big 4, each individual member of the Big 4 would prefer not to coordinate if the alternative were for the remaining three firms to coordinate. But for any subset of three Big 4 firms, each of
the firms is pivotal to the feasibility of the coordination.

**Oilfield services**

The U.S. DOJ’s analysis of the proposed merger of oilfield services providers Halliburton and Baker Hughes identifies the $400 million market of offshore cementing services as a relevant antitrust market.\(^{44}\) According to the DOJ Complaint, three suppliers, Halliburton, Baker Hughes, and Schlumberger, had combined market share of 99%.\(^{45}\) Further, the information in the DOJ’s complaint indicates that Halliburton and Baker Hughes had pre-merger market shares of 32% and 24% and that Schlumberger had a pre-merger market share of 43%.\(^{46}\)

For this application of our framework, it seems reasonable to assume (as the merging parties argued) that the buyers, which include BP, Shell, and Exxon-Mobil, have buyer power. Thus, we calibrate the suppliers’ distributions and calculate the CEI under the assumption of buyer power, but we also contrast the results with the case of no buyer power.\(^{47}\)

As Table 3 shows, the market is at risk for coordination, both pre-merger and post-merger, despite the presence of powerful buyers. Furthermore, the post-merger risk of coordination between Schlumberger and the merged entity, as measured by the CEI, is greater than the pre-merger risk of coordination between any two of the top three suppliers. The CEIs would be even higher in the absence of buyer power. Holding fixed the distributions, in the absence of buyer power, we would have instead $CEI_{\{S,1,2\}} = 0.9510$ and $CEI_{\{S,\mu_1,2\}} = 0.8960$, which are larger than their corresponding values with buyer power.

---

\(^{44}\)U.S. v. Halliburton Co. and Baker Hughes Inc., Complaint, Case 1:16-cv-00233-UNA, filed 6 April 2016 (DOJ Complaint).

\(^{45}\)“In a strategic planning session, Halliburton’s cementing executives recognized that this market is already a ‘pure oligopoly’ among the Big Three” (DOJ Complaint, p. 18).

\(^{46}\)This can be deduced from the information provided in the DOJ Complaint that Schlumberger’s market share was approximately 43%, the combined market share of Halliburton and Baker Hughes was approximately 56%, the pre-merger HHI was approximately 3500, and the post-merger HHI was approximately 5000. Although we can identify the shares of Halliburton and Baker Hughes as approximately 32% and 24%, it is not clear which supplier has the 32% share and which has the 24% share.

\(^{47}\)To facilitate the analysis of the case with buyer power, we use the parameterization $G_i(c) = c^{\beta_i}$ (which implies linear virtual cost functions, i.e., we have $\Gamma_i(c) = (1 + \beta_i)c/\beta_i$) and assume that $v$ is sufficiently large that $v \geq \Gamma_i(\bar{c})$ for all $i$. As an identifying assumption, we assume that $\sum_{i=1}^{4} \beta_i = 4$. Letting supplier 1 be Schlumberger and letting supplier 2 have market share 34% and supplier 3 have market share 24%, our calibration delivers $\beta_1 = 0.0760$, $\beta_2 = 0.0999$, $\beta_3 = 0.1274$, and $\beta_4 = 3.6967$. Notice that, in contrast to the capacity-based parameterization, increasing a supplier’s cost distribution parameter means that supplier $i$ becomes less efficient.
Table 3: Results for the oilfield services market of offshore cementing assuming powerful buyers. Supplier $S$ is Schlumberger, with pre-merger share 43%. Suppliers 2 and 3 are Halliburton and Baker Hughes (in unknown order), with pre-merger shares 34% and 24%. We denote the supplier that would result from the merger of Halliburton and Baker Hughes by $\mu_{2,3}$.

<table>
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<tr>
<th>Pre-merger</th>
<th>Post-merger</th>
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<td>$CEI_K$</td>
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<tr>
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</tr>
<tr>
<td>${S, \mu_{2,3}}$</td>
<td>0.6338</td>
</tr>
</tbody>
</table>

8 Conclusion

We provide a framework for analyzing the potential coordinated effects of mergers. We model coordination as a bidder selection scheme among coordinating suppliers, where only one of the coordinating suppliers is designated to participate in a buyer’s procurement. This approach allows the possibility of identifying markets that are not at risk for coordination prior to a merger but that are at risk for coordination after a merger and for quantifying the extent of that risk.

Our analysis confirms existing results that coordinated effects of a merger are more of a concern when the coordinating suppliers are larger, more symmetric, and face little outside competition. We formalize the views that the use of first-price auctions and the presence of powerful buyers reduce coordinated effects. In contrast to existing approaches, in our framework, the increased consolidation from a merger need not increase a market’s risk for coordinated effects. As we show, coordinated effects are a greater concern when the merger generates moderate cost synergies. We provide a definition of a maverick firm and find some support for the view that a merger that eliminates a maverick puts a market at risk for coordination, but only in some settings, such as when the acquiring supplier has low capacity relative to the supplier with which it would coordinate. We examine the tension between unilateral and coordinated effects concerns, for example as it relates to divestiture-based merger remedies.

Although the typical view of causality is that a merger potentially induces coordinated effects, the opposite can be true in our setup with buyer power. With buyer power, a merger is not necessarily profitable for the merging suppliers because after the merger a powerful buyer more aggressively asserts its power against the merged entity. Thus, in an environment with buyer power, the prospect of post-merger coordination can make the difference between a merger being profitable and not. In that sense, the possibility of post-
merger coordination can make a merger profitable that would not have been otherwise, implying that coordinated effects can induce a merger, rather than the other way around.

Much remains for future research. For example, in a dynamic setup, the type of bidder selection scheme that we analyze may form the basis for the division of surplus in subsequent explicit collusion, which may suitably be analyzed along the lines of Cramton et al. (1987). Auction-based models such as the one presented here have broad application in markets involving business-to-business transactions, where purchasing predominantly entails the simultaneous consideration of offers. The question remains to what extent the ideas put forth in this paper extend to standard oligopoly models with price-taking consumers. Our procurement setting, and for that matter, any auction of a single unit that endows agents with dominant strategies, has the convenient feature that coordination among some bidders does not change the optimal strategies of the others, regardless of the coordination scheme. This allows us to focus on the individual rationality constraints of the insiders, without departing from equilibrium for outsiders. In first-price auctions, bids are strategic complements, and the designated bidder is better off if the outsiders are aware of the bid suppression. While creating such awareness is in the designated bidder’s interest, it also raises the question of how that awareness can be achieved without raising the suspicions of the buyer or authorities. However, if the outsiders are not aware that there is coordinated bidding in a first-price auction, they are “duped” because they would bid differently if they were aware, raising the question of what is the appropriate notion of equilibrium when there is no such awareness. In an oligopoly setting, one would need to develop a notion of suboptimal coordination and understand how outsiders would react.
Appendix: Capacity-based parameterization

In the capacity-based parameterization defined in (1), assuming no buyer power and \( v \geq c \), it is straightforward to show, letting \( A_x \equiv \sum_{j \in x} \alpha_j \) and \( A \equiv A_M \), that for \( i \in K \),

\[
\Pi_i(M) = \frac{\alpha_i}{(1 + A_{M \setminus \{i\}})(1 + A)},
\]

\[
\Pi_i^K(M) = \frac{\alpha_i}{(1 + \alpha_i + A_{M \setminus K})(1 + A_{M \setminus K})},
\]

and, thus,

\[
s_i^K(M) = \frac{(1 + \alpha_i + A_{M \setminus K})(1 + A_{M \setminus K})}{(1 + A_{M \setminus \{i\}})(1 + A)}.
\]

Differentiating with respect to \( \alpha_i \) reveals that \( s_i^K(M) \) is increasing in \( \alpha_i \).

More generally, using \( r = \min\{v, c\} \), we have

\[
s_i^K(M) = \frac{(1 + \alpha_i + A_{M \setminus K})(1 + A_{M \setminus K})}{(1 + A_{M \setminus \{i\}})(1 + A)} \cdot \frac{\alpha_i + (1 - r)^{A_{M \setminus \{i\}}}[1 - r^{A_{M \setminus \{i\}}}(1 + A_{M \setminus \{i\}}) - (1 + A)]}{\alpha_i + (1 - r)^{1 + A_{M \setminus K}}[1 - r^{A_{M \setminus K}}(1 + A_{M \setminus K}) - (1 + \alpha_i + A_{M \setminus K})]},
\]

where the expression on the second line is equal to one if \( r = c = 1 \).

Noting that the capacity-based market share for supplier \( i \) is

\[
\sigma_i = \frac{\alpha_i(1 - (1 - r)^A)}{A},
\]

we can write the critical shares and CEI for market \( M \) as a function of the suppliers’ market shares and the total capacity. Specifically, if \( v \geq c \), then

\[
\sigma_i = \alpha_i/A,
\]

\[
s_i^K(M) = \frac{(1 + A(\sigma_i + \sum_{j \in M \setminus K} \sigma_j)(1 + A \sum_{j \in M \setminus K} \sigma_j))}{(1 + A(1 - \sigma_i))(1 + A)}, \quad (10)
\]

and

\[
CEI_K(M) = 1 - \frac{1 + A \sum_{j \in M \setminus K} \sigma_j}{1 + A} \sum_{i \in K} \frac{1 + A \sigma_i + A \sigma_j}{1 + A(1 - \sigma_i)}. \quad (11)
\]

In addition, if \( v \geq c \), the total capacity \( A \) can be related to supplier \( i \)'s operating
margin (Lerner Index), \( \omega_i \equiv \Pi_i / \text{revenue}_i \), as follows:

\[
A = \frac{1}{1/\omega_i - 2 + \sigma_i},
\]
as long as supplier \( i \)'s operating margin is less than \( 1/(2 - \sigma_i) \).\(^{48}\) (A sufficient condition is that the operating margin is less than 50\%.) Or, noting that \( \omega_i = 1/(1/A + 2 - \sigma_i) \), one could derive \( A \) from the (share weighted) average margin for the industry or for any subset of suppliers.

In the absence of margin data or other identifying information, an alternative is to impose a restriction on the suppliers’ aggregate capacity. For example, assuming that \( A \) is equal to the number of suppliers would be equivalent to assuming that the suppliers’ cost distributions are uniform on average.

**B  Appendix: Proofs**

*Proof of Lemma 1.* Given \( M \) and \( i \in M \), \( \Pi_i(M) \) is given as

\[
\Pi_i(M) = \int_r^c \int_c^r (1 - L_{M\setminus\{i\}}(x)) \, dx \, G_i(c) = \int_r^c (1 - L_{M\setminus\{i\}}(c)) \, G_i(c) \, dc,
\]
which follows from the payoff (or revenue) equivalence theorem (see, e.g., Myerson (1981), Krishna (2002), or Börgers (2015)) and then by integration by parts. Similarly, given \( K \subseteq M \) and \( i \in K \), one can write \( \Pi^K_i(M) \) as

\[
\Pi^K_i(M) = \int_r^c (1 - L_{M\setminus K}(c)) \, G_i(c) \, dc.
\]

\[\blacksquare\]

*Proof of Proposition 2.* Under symmetry, for all \( i \in K = \{1, ..., k\} \), the definition of \( \Pi^K_i \) implies that

\[
\Pi^K_i = \mathbb{E} \left[ \max\{0, \min\{r, c_{N\setminus K}\} - c_i\} \right] = \frac{1}{n - k + 1} \mathbb{E} \left[ \max\{0, \min\{r, c_{(2:n-k+1)}\} - c_{(1:n-k+1)}\} \right],
\]

\(^{48}\)For the case with \( r = \min\{v, \bar{v}\} \), we have \( \Pi_i = \frac{1-(1-r)^{1+A-\alpha_i}}{1+A-\alpha_i} - \frac{1-(1-r)^A}{A} \) and revenue \( i = \frac{(1-(1-r)^{1+A-\alpha_i}) - 1-(1-r)^A}{A(1+A)} + (1 + \alpha_i) \frac{1-(1-r)^A}{A(1+A)} \).
and the definition of $\Pi_i$ implies that

$$
\Pi_i = \frac{1}{n} \mathbb{E} \left[ \max\{0, \min\{r, c(2:n)\} - c(1:n)\} \right].
$$

Using symmetry, the market is at risk for coordination among $k = |K|$ suppliers if and only if coordination based on symmetric selection probabilities increases the expected surplus for all suppliers, i.e., for all $i \in K$, $\frac{1}{k} \Pi_i^K > \Pi_i$, which holds if and only if

$$
\frac{1}{n - k + 1} \mathbb{E} \left[ \max\{0, \min\{r, c(2:n-k+1)\} - c(1:n-k+1)\} \right]
$$

$$
> \frac{k}{n} \mathbb{E} \left[ \max\{0, \min\{r, c(2:n)\} - c(1:n)\} \right],
$$

which completes the proof. ■

Proof of Proposition 3. In the limit as $n$ grows large, the conditioning events in (3) are irrelevant because with probability 1 both the lowest and second-lowest order statistics are less than the reserve $r$. Thus, it is sufficient to show that there exists $\hat{n}$ such that for all $n > \hat{n}$,

$$
\lim_{n \to \infty} \left( \mathbb{E} \left[ c(2:n-k+1) - c(1:n-k+1) \right] - \frac{k}{n} (n - k + 1) \mathbb{E} \left[ c(2:n) - c(1:n) \right] \right) < 0. \quad (12)
$$

As shown by Loertscher and Marx (2019a),

$$
j \mathbb{E}[c(j+1:n) - c(j:n)] = \mathbb{E} \left[ \frac{G(c(j:n))}{g(c(j:n))} \right],
$$

which allows us to write the left side of (12) as

$$
\mathbb{E} \left[ \frac{G(c(1:n-k+1))}{g(c(1:n-k+1))} \right] - \frac{k}{n} (n - k + 1) \mathbb{E} \left[ \frac{G(c(1:n))}{g(c(1:n))} \right].
$$

Then using $\mathbb{E} \left[ \frac{G(c(j+1))}{g(c(j+1))} \right] = \int_\xi^\tau jG(x)(1 - G(x))^{j-1}dx$, we can rewrite it again as

$$
\int_\xi^\tau (n - k + 1)G(x)(1 - G(x))^{n-k}dx - \frac{k(n - k + 1)}{n} \int_\xi^\tau nG(x)(1 - G(x))^{n-1}dx
$$

$$
= (n - k + 1) \int_\xi^\tau G(x)(1 - G(x))^{n-k} (1 - k(1 - G(x))^{k-1}) dx,
$$

38
which has the sign of
\[
\int_{\xi}^{c^*} G(x)(1 - G(x))^{n-k} (1 - k(1 - G(x))^{k-1}) \, dx.
\]

Let \( H(x) \equiv G(x)(1 - G(x)) \left(1 - k(1 - G(x))^{k-1}\right) \) and let \( c^* \equiv G^{-1}(1 - \left(\frac{1}{k}\right)^{k-1}) \). Note that \( H(\xi) = H(c^*) = H(\bar{\xi}) = 0 \) and that for \( x \in (\xi, c^*) \) \( H(x) < 0 \) and that for \( x \in (c^*), H(x) > 0 \). It follows that

\[
\int_{\xi}^{c^*} G(x)(1 - G(x))^{n-k}(1 - k(1 - G(x))^{k-1}) \, dx \\
= \int_{\xi}^{c^*} H(x)(1 - G(x))^{n-k-1} \, dx \\
= \int_{\xi}^{c^*} H(x)(1 - G(x))^{n-k-1} \, dx + \int_{c^*}^{\bar{\xi}} H(x)(1 - G(x))^{n-k-1} \, dx \\
= \int_{\xi}^{c^*} H(x)(1 - G(x))^{n-k-1} \, dx \\
+ \int_{\xi}^{c^*} H \left( \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi} \right) \left(1 - G \left( \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi} \right) \right)^{n-k-1} \frac{\bar{\xi} - c^*}{c^* - \xi} \, dy \\
= \int_{\xi}^{c^*} \left[ H(x)(1 - G(x))^{n-k-1} \\
+ H \left( \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi} \right) \left(1 - G \left( \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi} \right) \right)^{n-k-1} \frac{\bar{\xi} - c^*}{c^* - \xi} \right] \, dx,
\]

where the change of variables uses

\[
y = \frac{c^* - \xi}{\bar{\xi} - c^*} x + \frac{\xi c^* - c^{*2}}{\bar{\xi} - c^*} \quad \text{or equivalently} \quad x = y \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi},
\]

so that \( x = c^* \) corresponds to \( y = \xi \) and \( x = \bar{\xi} \) corresponds to \( y = c^* \) and \( dx = \frac{\bar{\xi} - c^*}{c^* - \xi} \, dy \).

Factoring, we can then rewrite the expression as

\[
\int_{\xi}^{c^*} H(x)(1 - G(x))^{n-k-1} \left[ 1 + \left( \frac{1 - G(x) \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi}}{1 - G(x)} \right) \right]^{n-k-1} \frac{H(x) \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi}}{H(x) \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi}} \, dx.
\]

Because \( \frac{1 - G(x) \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi}}{1 - G(x)} \in (0, 1) \) for \( x \in (\xi, c^*) \) and \( \frac{H(x) \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi}}{H(x) \frac{\bar{\xi} - c^*}{c^* - \xi} - \frac{\xi c^* - c^{*2}}{c^* - \xi}} \) is negative and
bounded for $x \in (\underline{c}, c^*)$, there exists $\hat{n}$ such that for all $n > \hat{n}$ and $x \in (\underline{c}, c^*)$,
\[
\left(1 - G\left(x \frac{x^*-c^*}{c^*-\underline{c}} - \frac{x^*-c^*}{c^*-\underline{c}}\right)\right) \frac{H(x) - \frac{x^*-c^*}{c^*-\underline{c}}}{1 - G(x)} H(x) \frac{x^*-c^*}{c^*-\underline{c}} c^*-\underline{c} \in (-1, 0),
\]
which implies that for all $n > \hat{n}$ and $x \in (\underline{c}, c^*)$,
\[
\frac{H(x)(1 - G(x))^{n-k-1}}{1 + \left(1 - G\left(x \frac{x^*-c^*}{c^*-\underline{c}} - \frac{x^*-c^*}{c^*-\underline{c}}\right)\right) \frac{H(x) - \frac{x^*-c^*}{c^*-\underline{c}}}{1 - G(x)} H(x) \frac{x^*-c^*}{c^*-\underline{c}} c^*-\underline{c}} < 0.
\]
This proves that for all $n > \hat{n}$,
\[
\int_{\underline{c}}^{c^*} G(x)(1 - G(x))^{n-k}(1 - k(1 - G(x))^{k-1})dx < 0
\]
and thus completes the proof. ■

**Proof of Proposition 4.** Using the expression for $s^K_i(M)$ given in Appendix A and given that $K$ contains two or more suppliers, it is straightforward to show that for all $i \in K$, $\lim_{\alpha \to 0} s^K_i(M) = 1$ and $\lim_{\alpha \to \infty} s^K_i(M) = 0$. Thus, $\lim_{\alpha \to 0} CEI_K = 1 - |K| < 0$ and $\lim_{\alpha \to \infty} CEI_K = 1$.

Turning to the second part of the proposition, let $K \equiv \{1, \ldots, k\}$ and $M \equiv \{1, \ldots, m\}$. Then
\[
\sum_{i \in K} s^K_i = \sum_{i \in K} \frac{(1 + \alpha_i + A_{M\setminus K})(1 + A_{M\setminus K})}{(1 + A - \alpha_i)(1 + A)}.
\]
Because we assume that $1 \in K$, we can rewrite this as
\[
\sum_{i \in K} s^K_i = \sum_{i \in K \setminus \{1\}} \frac{(1 + \alpha_i + A_{M\setminus K})(1 + A_{M\setminus K})}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{M\setminus K})(1 + A_{M\setminus K})}{(1 + A - \alpha_1)(1 + A)}.
\]
Suppose we add $\varepsilon > 0$ to the capacity $\alpha_j$ for some $j \in M \setminus K$ and deduct $\varepsilon$ from the capacity $\alpha_1$. Then the CEI is
\[
\sum_{i \in K \setminus \{1\}} \frac{(1 + \alpha_i + A_{M\setminus K} + \varepsilon)(1 + A_{M\setminus K} + \varepsilon)}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{M\setminus K})(1 + A_{M\setminus K} + \varepsilon)}{(1 + A - \alpha_1 + \varepsilon)(1 + A)}.
\]
Differentiating with respect to $\varepsilon$, we have

$$
\sum_{i \in K \setminus \{1\}} \frac{(1 + A_{M \setminus K} + \varepsilon) + (1 + \alpha_i + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + A - \alpha_1 + \varepsilon) - (1 + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_1 + \varepsilon)^2}
$$

$$
= \sum_{i \in K \setminus \{1\}} \frac{(1 + A_{M \setminus K} + \varepsilon) + (1 + \alpha_i + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{M \setminus K})(A - \alpha_1 - A_{M \setminus K})}{(1 + A)(1 + A - \alpha_1 + \varepsilon)^2}
$$

$$
> 0.
$$

Thus, the CEI decreases when a member of $K$ is replaced with a member of $M \setminus K$ with a lower capacity. It follows that the CEI is maximized when its members are the set $K$ of suppliers with the largest capacities. ■

**Proof of Proposition 5.** For symmetric suppliers 1 and 2, we have critical shares of (dropping the argument $x$ to conserve of notation and integrating from $c$ to $r$)

$$
s_1 = s_2 = \frac{\int (1 - L_{M \setminus \{1,2\}})(1 - G)G \, dx}{\int (1 - L_{M \setminus K})G \, dx}
$$

and following a competitively neutral spread of

$$
\hat{s}_1 = \frac{\int (1 - L_{M \setminus \{1,2\}})(1 - F_2)F_1 \, dx}{\int (1 - L_{M \setminus K})F_1 \, dx} \quad \text{and} \quad \hat{s}_2 = \frac{\int (1 - L_{M \setminus \{1,2\}})(1 - F_1)F_2 \, dx}{\int (1 - L_{M \setminus K})F_2 \, dx},
$$

The competitively neutral spread causes the CEI to decrease if and only if $\hat{s}_1 + \hat{s}_2 - s_1 - s_2 > 0$, which holds as we now show.
Using the definitions of $s_1$, $s_2$, $\hat{s}_1$, and $\hat{s}_2$ in (13) and (14) we have

\[
\begin{align*}
\hat{s}_1 + \hat{s}_2 - s_1 - s_2 &= \frac{\int(1 - L_{M_\{1,2\}})(1 - F_2)F_1dx}{\int(1 - L_{M\{K\}})F_1dx} + \frac{\int(1 - L_{M\{1,2\}})(1 - F_1)F_2dx}{\int(1 - L_{M\{K\}})F_2dx} \\
&- 2\frac{\int(1 - L_{M\{1,2\}})(1 - G)Gdx}{\int(1 - L_{M\{K\}})Gdx} \\
&> \frac{\int(1 - L_{M\{1,2\}})(1 - F_2)F_1dx}{\int(1 - L_{M\{K\}})F_1dx} + \frac{\int(1 - L_{M\{1,2\}})(1 - F_1)F_2dx}{\int(1 - L_{M\{K\}})Gdx} \\
&- 2\frac{\int(1 - L_{M\{1,2\}})(1 - G)Gdx}{\int(1 - L_{M\{K\}})Gdx} \\
&= \frac{\int(1 - L_{M\{1,2\}})(1 - F_2)F_1dx}{\int(1 - L_{M\{K\}})F_1dx} + \frac{\int(1 - L_{M\{1,2\}})(G^2 - F_1)dx}{\int(1 - L_{M\{K\}})Gdx} \\
&> \frac{\int(1 - L_{M\{1,2\}})(1 - F_2)F_1dx}{\int(1 - L_{M\{K\}})F_1dx} + \frac{\int(1 - L_{M\{1,2\}})(G^2 - F_1)dx}{\int(1 - L_{M\{K\}})F_1dx} \\
&= \frac{\int(1 - L_{M\{1,2\}})^2(G - F_1)^2dx}{\int(1 - L_{M\{K\}})F_1dx} \\
&> 0,
\end{align*}
\]

where the first inequality uses $G \geq F_2$ (with a strict inequality for a positive measure subset of $[c, \tilde{c}]$), the second equality collects terms and uses the implication of (6) that $(1 - F_1)F_2 - 2(1 - G)G = G^2 - F_1$, the second inequality uses $F_1 \geq G$ (with a strict inequality for a positive measure subset of $[c, \tilde{c}]$), the third equality collects terms and uses the implication of (6) that $(1 - F_2)F_1 + G^2 - F_1 = \frac{(F_1 - G)^2}{1 - F_1}$ (and also equal to $\frac{(G - F_2)^2}{1 - F_2}$), and the final inequality rearranges. This completes the proof of the first part of the proposition.

Turning to the second part, having two different competitively neutral spreads in the capacity-based parameterization implies the existence of $a$ such that (we abuse notation by using $h_i$ and $f_i$ for the parameters of the distributions $H_i$ and $F_i$ rather than for their pdfs):

\[
h_1 > f_1 > a > f_2 > h_2
\]

and

\[
h_1 + h_2 = 2a = f_1 + f_2.
\]

Thus, $h_2 = 2a - h_1$ and $f_2 = 2a - f_1$. Let $s^K_i$ be the critical share for supplier $i \in K$ under $(H_1, H_2)$ and $s^K_i$ be critical share of supplier $i \in K$ under $(F_1, F_2)$. The shift from $(F_1, F_2)$ to $(H_1, H_2)$ decreases the CEI$_K$ if and only if it increases the sum of the critical shares of suppliers 1 and 2. In what follows, we show that $\hat{s}^K_1 + \hat{s}^K_2 - s^K_1 - s^K_2 > 0$. In the
capacity-based parameterization,

\[ s^K_i = \frac{(1 + f_i + A_{M \setminus K})(1 + A_{M \setminus K})}{(1 + A - f_i)(1 + A)} \quad \text{and} \quad \hat{s}^K_i = \frac{(1 + h_i + A_{M \setminus K})(1 + A_{M \setminus K})}{(1 + A - h_i)(1 + A)}. \]

Consequently, the sign of \( \hat{s}^K_i + \hat{s}^K_i - s^K_i - s^K_i \) is equal to the sign of

\[
\frac{(1 + h_1 + A_{M \setminus K})}{(1 + A - h_1)} + \frac{(1 + h_2 + A_{M \setminus K})}{(1 + A - h_2)} - \frac{(1 + f_1 + A_{M \setminus K})}{(1 + A - f_1)} - \frac{(1 + f_2 + A_{M \setminus K})}{(1 + A - f_2)}.\]

Substituting \( 2a - h_1 \) for \( h_2 \) and \( 2a - f_1 \) for \( f_2 \) and collecting terms, we get

\[
\frac{2(f_1 - h_1 - 2a)(h_1 - f_1)(1 + a + X)(2 + 2a + X)}{(1 + 2a - f_1 + X)(1 + f_1 + X)(1 + 2a - h_1 + X)(1 + h_1 + X)},
\]

where \( X \equiv A_{M \setminus \{1,2\}} + A_{M \setminus K} \), which is positive.

Last, we prove the result related to buyer harm. Observe first that whether firms 1 or 2 produce depends only on their lowest cost draw whose distribution is, by the definition of a competitively neutral spread, the same with or without the spread. Second, the price that a successful rival of the two firms pays in a second-price auction similarly only depends on the lowest cost draw of firms 1 and 2 (if it is affected by their costs at all). Thus, a competitively neutral spread can affect the price the buyer pays only in the event that one of the two firms has the lowest cost and the other one the second lowest cost. Since the distribution of the lowest cost draw is by construction the same, all that is left to do is to compare the distribution of their second-lowest draw, which is \( G(c)^2 \) without the spread and \( G_1(c)G_2(c) \) after the spread. We are now going to show that for all \( c \in [c_0, \bar{c}] \), \( H_1(c)H_2(c) \leq G_1(c)G_2(c) \leq G(c)^2 \).

For any given \( c \), the conditions for a competitively neutral spread can be written as (i) \( AB = C^2 \) and (ii) \( 1 - A \geq 1 - C \geq 1 - B \) with \( A, B, C \in (0,1) \). Consider now the problem

\[
\max_{(A,B)\in[0,1]^2} (1 - A)(1 - B) \quad \text{s.t.} \quad AB = C^2.
\]

Substituting the constraint using \( A = C^2/B \) yields the univariate maximization problem

\[
\max_{B\in[0,1]} \frac{B - C^2}{B}(1 - B),
\]

whose first and second derivatives are \(-1 + \frac{C^2}{B^2}\) and \(-\frac{2C^2}{B^3} < 0\), respectively. Thus, the problem is strictly concave and maximized at \( B = C \). This implies that for any \( A \) and \( B \) satisfying (i) and (ii) with \( B \neq C \), we have \((1 - A)(1 - B) < (1 - C)^2 \). Setting \( A = 1 - G_1 \), \( B = 1 - G_2 \), and \( C = 1 - G \), this establishes stochastic dominance of the distributions of
the second-lowest cost, that is, $G_1 G_2 \leq G^2$. Because the second-lowest cost determines the buyer’s price, this implies that the buyer’s expected price is higher under $(G_1, G_2)$ than when both suppliers draw their costs independently from $G$. Moreover, strict concavity implies that $H_1 H_2 \leq G_1 G_2$, which in turn implies that the buyer’s expected price is higher under $(H_1, H_2)$ than under $(G_1, G_2)$. ■

Proof of Corollary 2. Denote by $L_{-1,-2}(c)$ the distribution of the lowest cost of the rivals of firms 1 and 2. We have

$$
\Delta \pi_G = \int_{L}^{\min\{v, \pi\}} (1 - L_{-1,-2}(c)) G(c)^2 dc
$$

$$
\Delta \pi_F = \int_{L}^{\min\{v, \pi\}} (1 - L_{-1,-2}(c)) F_1(c) F_2(c) dc
$$

$$
\Delta \pi_H = \int_{L}^{\min\{v, \pi\}} (1 - L_{-1,-2}(c)) H_1(c) H_2(c) dc.
$$

Because $1 - L_{-1,-2}(c)$ is nonincreasing, the result follows from the stochastic dominance established in the proof of Proposition 5. ■

Proof of Proposition 6. Let $\pi_{i,a}(c)$ be the expected payoff of supplier $i$ when its cost is $c$, there is no coordination, and the auction format is $a \in \{FPA, SPA\}$. Likewise, let $\pi^K_{i,a}(c)$ be the expected payoff or profit of supplier $i$ when its cost is $c$, it is the selected bidder in the set of coordinators $K$, the auction format is $a$, and the outsiders—that is, the suppliers $j \in N \setminus K$—know that the suppliers in $K$ coordinate. Denote by $\hat{\pi}^K_{i,a}(c)$ the profit of supplier $i$ when its cost is $c$, it is the selected bidder in the set of coordinators $K$, the auction format is $a$, and the outsiders do not know that the suppliers in $K$ coordinate. We use upper case letters to denote the expectation of these profits, e.g., $\Pi_{i,a} = \mathbb{E}[\pi_{i,a}(c)]$.

Let $s^K_{i,a}$ and $\hat{s}^K_{i,a}$ be $i$’s critical share under auction format $a$ when the outsiders know, respectively do not know, that the suppliers in $K$ coordinate. Lastly, under auction format $a$ let $CEI_{K,a}$ and $\hat{CEI}_{K,a}$ denote the coordinated effects index when the outsider know, respectively do not know, that the suppliers in $K$ coordinate.

Because of dominant strategies, we have $\pi^K_{i,SPA}(c) = \hat{\pi}^K_{i,SPA}(c)$, implying $\Pi^K_{i,SPA} = \hat{\Pi}^K_{i,SPA}$, $s^K_{i,SPA} = \hat{s}^K_{i,SPA}$, and thus $\hat{CEI}_{K,SPA} = CEI_{K,SPA}$.

Next, notice that in both the dominant strategy equilibrium of the second-price auction and the symmetric Bayes Nash equilibrium of the first-price auction, when all bidders know that there is coordination or that there is no coordination, the allocation is efficient insofar as the lowest cost supplier among the active bidders wins. Therefore, by the revenue equivalence theorem, we have $\pi^K_{i,FPA}(c) = \pi^K_{i,SPA}(c)$ and $\pi_{i,SPA}(c)$, 

implying

$$\Pi^K_{i,FPA} = \Pi^K_{i,SPA} \quad \text{and} \quad \Pi_{i,FPA} = \Pi_{i,SPA}.$$  

Consequently, $$s^K_{i,FPA} = s^K_{i,SPA}$$ and $$CEI^K_{i,FPA} = CEI^K_{i,SPA} = \hat{CEI}^{K,SPA}_{K,SPA}.$$  

We are thus left to consider the effect of outsiders’ (not) knowing that there is coordination in the first-price auction. Suppose the outsiders do not know that the suppliers in $$K$$ coordinate. In a first-price auction, this means that all outsiders bid as if they face all $$n - 1$$ rivals. In contrast, when they know that the suppliers in $$K$$ coordinate, the outsiders bid less aggressively, namely under the hypothesis that they face $$n - 1 - |K - 1| < n - 1$$ rivals. Let $$\hat{b}(c)$$ be the optimal bid that the designated bidder uses when its cost is $$c$$ and the outsiders do not know that there is coordination. Because the designated bidder could always use $$\hat{b}(c)$$, but optimally chooses not to when the outsiders know that there is coordination, and because the outsiders bid less aggressively when they know that there is coordination, it follows that

$$\hat{\pi}_{i,FPA}(c) \leq \pi_{i,FPA}(c),$$

with strict inequality for $$c$$ sufficiently small (and with equality only when $$c$$ is larger than $$v$$, in which case $$\hat{\pi}_{i,FPA}(c) = 0 = \pi_{i,FPA}(c)$$). This implies that $$\hat{\Pi}_{i,FPA} < \Pi_{i,FPA}$$ and thus $$\hat{CEI}^{K,FPA} < CEI^{K,FPA} = CEI^{K,SPA} = \hat{CEI}^{K,SPA}_{K,SPA},$$ which completes the proof.  

**Proof of Proposition 9.** As shown in Appendix A, in the capacity-based parameterization,

$$s^K_i(N) = \frac{(1 + \alpha_i + A_{N\setminus K})(1 + A_{N\setminus K})}{(1 + A_{N\setminus \{i\}})(1 + A_{N})}.$$  

Let $$N = \{1, ..., n\}$$ for some $$n \in \{3, 4, ..., \}$$. Let $$K = \{1, 2\}$$ and assume that supplier $$m \in \{3, ..., n\}$$ is a maverick with respect to $$K$$. Define $$X \equiv \sum_{i \in N \setminus \{1, 2, m\}} \alpha_i$$. By the definition of a maverick, $$1 - \sum_{i \in K} s^K_i(N \setminus \{m\}) > 0$$, which we can be written as

$$\sum_{i \in \{1, 2\}} \frac{(1 + \alpha_i + X)(1 + X)}{(1 + \alpha_1 + \alpha_2 - \alpha_i + X)(1 + \alpha_1 + \alpha_2 + X)} < 1. \tag{15}$$

Following the merger of suppliers 1 and $$m$$, the CEI for suppliers in $$\hat{K} \equiv \{\mu_1, m, 2\}$$ is

$$CEI_{\hat{K}} = 1 - \frac{(1 + \alpha_1 + X + \alpha_m)(1 + X)}{(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)} = \frac{(1 + \alpha_2 + X)(1 + X)}{(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)}.$$  

It follows that

$$\lim_{\alpha_2 \to 0} CEI_{\hat{K}} = 1 - 1 - \frac{(1 + X)^2}{(1 + \alpha_1 + X + \alpha_m)^2} < 0,$$
which proves the second part of the proposition.

Using the above expression for $CEI_K$ and (15), we have

$$CEI_K > 1 - \frac{(1 + \alpha_1 + X + \alpha_m)(1 + X)}{(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)} - \frac{(1 + \alpha_2 + X)(1 + X)}{(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)}$$

$$- 1 + \frac{(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X)}{(1 + \alpha_1 + X)(1 + \alpha_1 + \alpha_2 + X)}$$

$$\equiv (1 + X)f(\alpha_1, \alpha_2, \alpha_m, X).$$

Thus, $CEI_K > 0$ if $f(\alpha_1, \alpha_2, \alpha_m, X) > 0$. Collecting the terms in $f(\alpha_1, \alpha_2, \alpha_m, X)$ over the common denominator of

$$(1 + \alpha_1 + X)(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X),$$

it follows that $CEI_K > 0$ if the associated numerator, denoted by $\hat{f}(\alpha_1, \alpha_2, X)$ (we drop the argument $\alpha_m$ to conserve on notation), is positive, i.e., if $\hat{f}(\alpha_1, \alpha_2, X) > 0$, where

$$\hat{f}(\alpha_1, \alpha_2, X) = -(1 + \alpha_1 + X)(1 + \alpha_1 + X + \alpha_m)^2(1 + \alpha_1 + \alpha_2 + X)$$

$$+ (1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_1 + X)^2$$

$$- (1 + \alpha_2 + X)^2(1 + \alpha_1 + X)(1 + \alpha_1 + \alpha_2 + X)$$

$$+ (1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_2 + X)^2(1 + \alpha_1 + X + \alpha_m).$$

Differentiating with respect to $X$, we have

$$\frac{\partial \hat{f}(\alpha_1, \alpha_2, X)}{\partial X} \bigg|_{X=0} = \alpha_m(4\alpha_2^2 + 2\alpha_1(2 + \alpha_2) + 2(3 + \alpha_m) + \alpha_2(8 + \alpha_m)) > 0$$

and

$$\frac{\partial^2 \hat{f}(\alpha_1, \alpha_2, X)}{\partial X^2} = 2\alpha_m(6 + 2\alpha_1 + 4\alpha_2 + \alpha_m + 6X) > 0.$$
and
\[
\frac{\partial^2 \hat{f}(\alpha_1, \alpha_2, 0)}{\partial \alpha_2^2} = 2\alpha_m(4 + 2\alpha_1 + 3\alpha_2 + \alpha_m) > 0.
\]

Thus, \(\hat{f}(\alpha_1, \alpha_2, 0)\) is convex in \(\alpha_2\) and positive and increasing in \(\alpha_2\) at \(\alpha_2 = \alpha_1 = \alpha\), which implies that \(\hat{f}(\alpha_1, \alpha_2, 0) > 0\) for all \(\alpha_2 \geq \alpha_1\), i.e., as long as the acquiring supplier has the weakly smaller capacity, which completes the proposition. \(\blacksquare\)

**Proof of Proposition 10.** Let \(\mu\) be the index of the merged entity and \(o\) be the index of the other supplier in \(\hat{K}\), so that \(\hat{K} = \{\mu, o\}\). Then, dropping the argument \(x\), we have

\[
\sum_{i \in \hat{K}} s_i \hat{K} = \frac{\int_{\mu}^r (1 - L_{M\setminus\{\mu\}})G_{\mu}dx}{\int_{\mu}^r (1 - L_{M\setminus\{K\}})G_{\mu}dx} + \frac{\int_{o}^r (1 - L_{M\setminus\{o\}})G_{o}dx}{\int_{o}^r (1 - L_{M\setminus\{K\}})G_{o}dx}
\]

\[
= \frac{\int_{\mu}^r (1 - G_o)\prod_{j \in M\setminus K}(1 - G_j)G_{\mu}dx}{\int_{\mu}^r \prod_{j \in M\setminus K}(1 - G_j)G_{\mu}dx} + \frac{\int_{o}^r (1 - G_{\mu})\prod_{j \in M\setminus K}(1 - G_j)G_{o}dx}{\int_{o}^r \prod_{j \in M\setminus K}(1 - G_j)G_{o}dx}
\]

\[
= \frac{\int_{\mu}^r (1 - G_o)BG_{\mu}dx}{\int_{\mu}^r BG_{\mu}dx} + \frac{\int_{o}^r (1 - G_{\mu})BG_{o}dx}{\int_{o}^r BG_{o}dx},
\]

where \(B \equiv \times_{j \in M\setminus K}(1 - G_j)\). Differentiating with respect to \(z\), we have

\[
\int_{\mu}^r (1 - G_o)B \frac{dG_{\mu}}{dz}dx \int_{\mu}^r BG_{\mu}dx - \int_{\mu}^r (1 - G_o)BG_{\mu}dx \int_{\mu}^r B \frac{dG_{\mu}}{dz}dx
\]

\[
\left(\int_{\mu}^r BG_{\mu}dx\right)^2 - \int_{\mu}^r B \frac{dG_{\mu}}{dz}BG_{o}dx.
\]

Letting \(z = 1\) and noting that \(\frac{dG_{\mu}}{dz} = -z((1 - G_{\mu_1})(1 - G_{\mu_2}))z^{-1}\) so that \(\left.\frac{dG_{\mu}}{dz}\right|_{z=1} = -1\), we have

\[
- \int_{\mu}^r (1 - G_o)B dx \int_{\mu}^r BG_{\mu}dx + \int_{o}^r (1 - G_{\mu})BG_{\mu}dx \int_{o}^r B dx
\]

\[
\left(\int_{\mu}^r BG_{\mu}dx\right)^2 - 1,
\]

which is negative if and only if

\[
\int_{\mu}^r (1 - G_o)BG_{\mu}dx \int_{\mu}^r B dx - \int_{\mu}^r (1 - G_o)B dx \int_{\mu}^r BG_{\mu}dx < \left(\int_{\mu}^r BG_{\mu}dx\right)^2,
\]

which can be rewritten as

\[
\int_{\mu}^r BG_{\mu}dx \int_{\mu}^r BG_{\mu}(x)dx - \int_{\mu}^r BG_{o}G_{\mu}dx \int_{\mu}^r B dx < \left(\int_{\mu}^r BG_{\mu}dx\right)^2. \quad (16)
\]

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If $G_o \leq G_\mu$, then

$$\int_\xi^r B G_o dx \int_\xi^r B G_\mu dx - \int_\xi^r B G_o G_\mu dx \int_\xi^r B dx \leq \left( \int_\xi^r B G_\mu dx \right)^2 - \int_\xi^r B G_o G_\mu dx \int_\xi^r B dx < \left( \int_\xi^r B G_\mu dx \right)^2,$$

and so (16) holds. ■

C Appendix: Two-unit demand

With two-unit demand, $v \geq \bar{v}$, and no buyer power, supplier $i$’s expected revenue is the expected value of the second-lowest cost among the other $n - 1$ suppliers, conditional on that cost being greater than supplier $i$’s cost, and multiplied by the probability that supplier $i$’s cost is one of the two lowest, which we can write as:

$$R_i = \int_\xi^\bar{v} \int_\xi^v y dH_i(y) dG_i(c),$$

where $H_i$ is the distribution of the second-lowest cost among suppliers other than $i$. Similarly, supplier $i$’s expected profit is

$$\Pi_i = \int_\xi^\bar{v} \int_\xi^v (y - c) dH_i(y) dG_i(c).$$

Letting $H_K$ be the distribution of the second-lowest cost among suppliers not in set $K$, then for $i \in K$, supplier $i$’s expected profit when suppliers in $K$ coordinate and supplier $i$ is selected to be the only member of $K$ to bid is

$$\Pi_i^K = \int_\xi^\bar{v} \int_\xi^v (y - c) dH_K(y) dG_i(c).$$

Now consider the capacity-based parameterization. Letting $A \equiv \sum_{k \in N} \alpha_k$, $A_{-i} \equiv \sum_{k \in N \setminus \{i\}} \alpha_k$, and $A_{-i,j} \equiv \sum_{k \in N \setminus \{i,j\}} \alpha_k$, we can write $R_i$, $\Pi_i$, and $\Pi_i^K$ in terms of the parameters of the cost distributions as shown in the following lemma.

Lemma 3. Assuming $G_i(c) = 1 - (1 - c)^{\alpha_i}$ and $n \geq 3$, if the buyer has two-unit demand with $v \geq 1$ and no buyer power, then supplier $i$ trades with probability $q_i =$
\[ \alpha_i \sum_{\ell \neq i} \left( \frac{1}{A_{-\ell}} - \frac{A_{-i,\ell}}{A_{-i}A} \right), \] has expected revenue

\[ R_i = \alpha_i \sum_{\ell \neq i} \left( \frac{\alpha_i + A^2_{-\ell}}{(1 + A_{-i,\ell}) A_{-\ell}(1 + A_{-\ell})} - \frac{A_{-i,\ell} (1 + \alpha_i)}{A_{-i}(1 + A_{-i}) A (1 + A)} \right) \]

\[ = \frac{q_i (1 + \alpha_i)}{(1 + A_{-i})(1 + A)} + \alpha_i \sum_{\ell \neq i} \frac{1}{A_{-\ell}} \left( \frac{\alpha_i + A^2_{-\ell}}{(1 + A_{-i,\ell})(1 + A_{-\ell})} - \frac{1 + \alpha_i}{(1 + A_{-i})(1 + A)} \right), \]

and has expected profit \( \Pi_i = R_i - C_i, \) where \( C_i \) is supplier \( i \)'s expected cost, given by

\[ C_i = \alpha_i \sum_{\ell \neq i} \left( \frac{1}{A_{-\ell}(1 + A_{-\ell})} - \frac{A_{-i,\ell}}{A_{-i}A(1 + A)} \right) \]

\[ = \frac{q_i}{1 + A} + \alpha_i \sum_{\ell \neq i} \frac{1}{A_{-\ell}} \left( \frac{\alpha_{\ell}}{(1 + A_{-i})(1 + A)} \right). \]

In addition, for \( i \in K, \) \( q_i, R_i \) and \( C_i \) can be adjusted for the case of coordination by suppliers in \( K \) by summing over \( \ell \in N \setminus K \) and letting \( \alpha_j \) be zero for all \( j \in K \setminus \{i\} \).

**Proof of Lemma 3.** Using the parameterization \( G_i(c) = 1 - (1 - c)^{\alpha_i}, \) for \( n \geq 3, \) the cdf of the second-lowest among the \( n - 1 \) suppliers other than supplier \( i \) is

\[ H_i(c) \equiv 1 - \left( \times_{j \neq i} (1 - G_j(c)) + \sum_{\ell \neq i} G_\ell(c) \times_{j \in N \setminus \{i, \ell\}} (1 - G_j(c)) \right) \]

\[ = 1 - \left( (1 - c)^{A_{-i}} + \sum_{\ell \neq i} (1 - (1 - c)^{\alpha_i})(1 - c)^{A_{-i,\ell}} \right), \]

so the pdf is

\[ h_i(c) = A_{-i}(1 - c)^{A_{-i}-1} + \sum_{\ell \neq i} (1 - (1 - c)^{\alpha_i}) A_{-i,\ell}(1 - c)^{A_{-i,\ell}-1} - \sum_{\ell \neq i} \alpha_{\ell}(1 - c)^{\alpha_{\ell}-1}(1 - c)^{A_{-i,\ell}} \]

\[ = A_{-i}(1 - c)^{A_{-i}-1} + \sum_{\ell \neq i} (1 - (1 - c)^{\alpha_i}) A_{-i,\ell}(1 - c)^{A_{-i,\ell}-1} - \sum_{\ell \neq i} \alpha_{\ell}(1 - c)^{A_{-i}-1} \]

\[ = \sum_{\ell \neq i} (1 - (1 - c)^{\alpha_{\ell}}) A_{-i,\ell}(1 - c)^{A_{-i,\ell}-1}. \]

For example, if all the capacity parameters are equal to 1, these are

\[ H_i(c) = 1 - \left( (1 - c)^{(n-1)} + (n - 1) c(1 - c)^{n-2} \right) \]
and
\[
h_i(c) = (n-1)(n-2)c(1-c)^{n-3}.
\]

The probability of trade for supplier \(i\) is
\[
q_i = \int_0^1 \int_c^1 \left( \sum_{\ell \neq i} (1 - (1 - y)^{\alpha_i}) A_{-i,\ell}(1 - y)^{A_{-i,\ell}-1} \right) \alpha_i (1-c)^{\alpha_i-1} dy dc
\]
\[
= \int_0^1 \left( \sum_{\ell \neq i} A_{-i,\ell} \int_c^1 ((1 - y)^{A_{-i,\ell}-1} - (1 - y)^{A_{-i}-1}) dy \right) \alpha_i (1-c)^{\alpha_i-1} dc
\]
\[
= \int_0^1 \left( \sum_{\ell \neq i} A_{-i,\ell} \left( \frac{(1 - c)^{A_{-i,\ell}}}{A_{-i,\ell}} - \frac{(1 - c)^{A_{-i}}}{A_{-i}} \right) \right) \alpha_i (1-c)^{\alpha_i-1} dc
\]
\[
= \alpha_i \int_0^1 \left( \sum_{\ell \neq i} A_{-i,\ell} \left( \frac{1}{A_{-i,\ell} A_{-\ell}} - \frac{1}{A_{-i} A} \right) \right) dc
\]
\[
= \alpha_i \sum_{\ell \neq i} A_{-i,\ell} \left( \frac{1}{A_{-i,\ell} A_{-\ell}} - \frac{1}{A_{-i} A} \right)
\]
\[
= \alpha_i \sum_{\ell \neq i} \left( \frac{1}{A_{-\ell}} - \frac{A_{-i,\ell}}{A_{-i} A} \right).
\]

So the market share of supplier \(i\) is \(q_i/2\) (because the sum of all suppliers’ probabilities of trade is 2 in the case of two-unit demand and no buyer power and \(v \geq 1\)).

For the symmetric uniform case, \(q_i = \frac{2}{n}\), so all market shares are \(1/n\).
Supplier $i$’s expected revenue is

$$R_i = \int_0^1 \int_c^1 y \left( \sum_{\ell \neq i} (1 - (1 - y)^{\alpha_\ell})A_{-i,\ell} (1 - y)^{A_{-i,\ell} - 1} \right) \alpha_i (1 - c)^{\alpha_i - 1} dy dc$$

$$= \int_0^1 \int_c^1 \left( \sum_{\ell \neq i} A_{-i,\ell} \int_c^1 (y(1 - y)^{A_{-i,\ell} - 1} - y(1 - y)^{A_{-i,\ell} - 1}) dy \right) \alpha_i (1 - c)^{\alpha_i - 1} dc$$

$$= \int_0^1 \int_c^1 \left( \sum_{\ell \neq i} A_{-i,\ell} \left( \frac{(1 - c)^{A_{-i,\ell}} (1 + cA_{-i,\ell}) - (1 - c)^{A_{-i,\ell}} (1 + cA_{-i,\ell})}{A_{-i}(1 + A_{-i})} \right) \right) \alpha_i (1 - c)^{\alpha_i - 1} dc$$

$$= \alpha_i \int_0^1 \sum_{\ell \neq i} \left( \frac{1}{1 + A_{-i,\ell}} - \frac{A_{-i,\ell}}{A_{-i}(1 + A_{-i})} + \frac{A_{-i,\ell}}{A_{-i}(1 + A_{-i})} \right) \alpha_i (1 - c)^{\alpha_i - 1} dc$$

Supplier $i$’s expected cost is

$$C_i = \int_0^1 \int_c^1 c \left( \sum_{\ell \neq i} (1 - (1 - y)^{\alpha_\ell})A_{-i,\ell} (1 - y)^{A_{-i,\ell} - 1} \right) \alpha_i (1 - c)^{\alpha_i - 1} dy dc$$

$$= \int_0^1 \int_c^1 c \left( \sum_{\ell \neq i} A_{-i,\ell} \int_c^1 ((1 - y)^{A_{-i,\ell} - 1} - (1 - y)^{A_{-i,\ell} - 1}) dy \right) \alpha_i (1 - c)^{\alpha_i - 1} dc$$

$$= \int_0^1 \int_c^1 c \left( \sum_{\ell \neq i} A_{-i,\ell} \left( \frac{(1 - c)^{A_{-i,\ell}}}{A_{-i,\ell}} - \frac{(1 - c)^{A_{-i,\ell}}}{A_{-i}} \right) \right) \alpha_i (1 - c)^{\alpha_i - 1} dc$$

$$= \alpha_i \int_0^1 \sum_{\ell \neq i} \left( \frac{c(1 - c)^{A_{-i,\ell} - 1} - c(1 - c)^{A_{-i,\ell} - 1}}{A_{-i,\ell}} \right) dc$$

$$= \alpha_i \int_0^1 \sum_{\ell \neq i} \left( \frac{1}{A_{-i,\ell} A_{-i}(1 + A_{-i})} - \frac{1}{A_{-i} A_{-i}(1 + A)} \right) dc$$

The remaining results follow by substitution and some rearranging. ■
References


view, 82, 579–599.


