Coordinated Effects in Merger Review*

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Abstract

Coordinated effects are merger-related harms that arise because a subset of post-merger firms modify their conduct to limit competition among themselves, particularly in ways other than explicit collusion, that is, without transfers or communication of private information. We provide a measure of the risk of such conduct by examining the individual rationality of participation by subsets of firms in market allocation schemes. This measure of risk for coordinated effects allows one to distinguish markets that are at risk from those that are not, and to distinguish mergers that increase risk from those that do not. We show that a market’s risk for market allocation by a subset of firms varies with the degree of outside competition, the symmetry and strength of the subset of firms, buyer power, and vertical integration of buyers. We are able to make precise the widely used, but rarely rigorously defined, notion of a maverick firm, and we provide foundations for a maverick-based approach to coordinated effects. In addition, we identify previously unrecognized tradeoffs between unilateral and coordinated effects.

Keywords: merger review, buyer power, mavericks, unilateral effects, neutral for rivals spread

JEL Classification: D44, D82, L41
1. Introduction

Competition authorities regularly review proposed mergers and oppose those deemed likely
to have sufficiently detrimental effects, recognizing that one source of detrimental effects is
that a merger can change “the nature of competition in such a way that firms that previously
were not coordinating their behaviour, are now significantly more likely to coordinate and
raise prices or otherwise harm effective competition.”\(^1\) Adverse competitive effects of mergers
that arise in this way are referred to as *coordinated effects* and play a central role in antitrust
thinking and practice.\(^2\) Despite their prominence and in contrast to theories of harm based
on unilateral effects, which are adverse competitive effects resulting from the elimination of
competition between the merging parties (see, for example, *U.S. Guidelines*), and which are
supported by a range of well-accepted tools for the quantification of harm (see, for example,
Davis and Garcés (2010)), theories of harm based on coordinated effects have proved difficult
to formulate rigorously and a methodology to quantify these harms has proved elusive.

Of course, a major obstacle to quantifying the effect of a merger on the risk of collusive
conduct is that perfect collusion is always profitable, both before and after a merger. Conse-
quently, any theory of harm based on coordinated effects must rely on a form of imperfectly
collusive conduct. Of particular appeal to real-world agents and concern to competition
authorities are market allocation schemes, whereby firms take turns in serving a given mar-
ket.\(^3\) The “phases of the moon” conspiracy that involved 29 suppliers of industrial electrical
generators in the 1950s is a classic and colourful example of this notorious but popular
(mal)practice (Asker, 2010), and additional examples of allocation schemes are plentiful.\(^4\)

In this paper, we provide a theory of collusive behaviour based on market allocation
schemes that permits us to quantify the extent to which a market is at risk for such conduct.
The basic idea is that for participation in an allocation scheme to pay off, each participant
has to be selected to be the active supplier with sufficiently high probability. Thus, any
allocation scheme can be defined by a set of *critical shares*—the shares of the market that
leave participants indifferent between participating in the allocation scheme and not. Of
course, each supplier’s critical share is strictly less than one because the allocation scheme suppresses competition and thereby increases the profits of the active supplier. However, for a market to be at risk for allocation by a given set of suppliers, the critical shares of those suppliers need to sum up to less than one. Because the scheme can be inefficient (for example, because the suppliers have different marginal costs or because the reaction of outsiders erodes the benefits accruing to insiders), there is no a priori reason for this sum of critical shares to be less than one. Thus, for a given set of candidate participants, the sum of their critical shares provides a natural way to define whether a market is at risk for allocation by those firms—if the sum is less than one, then it is at risk. This is the definition for being at risk for the suppression of rivalry by market allocation that we use in this paper. Notably, this framework for evaluating the risk for coordinated effects can be combined with whatever model of a market’s price-formation process is most appropriate for the problem at hand, be that a model of oligopoly, procurement, Nash-in-Nash bargaining, or other bargaining model. Moreover, the theory provides the basis for a test of the extent to which a market is at risk and the extent to which this risk increases with a merger. Further, the test is operational based on data that are commonly available during merger review.

Our approach puts the participation constraints for collusion front and center. This contrasts with the folk theorem based literature on collusion in repeated games, whose focus is on the incentive compatibility and sustainability of collusion and the calculation of critical discount factors. As emphasized by Farrell and Baker (2020), the applicability of the traditional, repeated-game approach to coordinated effects is limited because subgame perfect equilibria abound; collusive subgame perfect equilibria exist both pre and post merger; and it fails to satisfy the “quantification-hungry” nature of the policy world. In this sense, our paper is in line with the conclusion drawn by Farrell and Baker that approaches to coordinated effects should depart from repeated-game models in the direction of quantifiable models and measures.

Closely tied to the notion of coordinated effects, both in the literature and in practice, is
the notion of a *maverick* firm. Broadly and vaguely, a maverick is a firm whose acquisition will put a market at risk for allocation when the market is not at risk for allocation with the maverick firm present. Baker (2002, pp. 140–141, 197) argues for a “maverick-centered approach” to coordinated effects, stating that “the identification of a maverick that constrains more effective coordination is the key to explaining ... which particular changes in market structure from merger or exclusion are troublesome, and why” and that “In many settings, regulators reliably can identify an industry maverick that prevents or limits coordination.” Kolasky (2002) argues that the elimination of a maverick may be necessary for coordinated effects.

Because our framework allows us to quantify whether and the extent to which a market is at risk for the suppression of rivalry by various subsets of suppliers, we are able to offer a precise definition of a maverick firm. Relative to a given set of suppliers potentially engaged in suppression of rivalry, we say that some other firm $m$ outside this set is a *maverick* if the market is not at risk for allocation by these firms with $m$ present and is at risk for allocation without $m$. As we show, an approach to merger review that focuses on blocking mergers that involve mavericks makes sense for markets characterized by the Cournot model with constant marginal costs. However, in other instances, such as procurement markets, a merger involving a maverick need not put the market at risk because the acquisition of a supplier is not the same as eliminating its productive assets.

Consistent with agencies' concerns related to coordinated effects, we find that market allocation schemes reduce expected buyer/consumer surplus and social surplus. However, for a variety of widely used models of the price-formation process, we also show that by our measure of being at risk for coordinated effects, some, but not all, markets are at risk and some, but not all, mergers put markets at risk. While a maverick-based approach has a foundation in the Cournot model, in general a more nuanced approach to mavericks is required. We show that a market’s risk varies with the degree of outside competition, symmetry and strength of participating firms, buyer power, and vertical integration of buyers.
We identify tradeoffs between unilateral and coordinated effects, including that structural remedies based on divestitures may not be able to simultaneously address concerns related to unilateral and coordinated effects.

There is a large legal and economics literature on coordinated effects and the intertwined notion of maverick firms. Baker (2002, 2010), Kaplow (2011), and Harrington (2013) provide overviews of the legal literature, with Kaplow (2013) providing an in-depth discussion; Ivaldi et al. (2007), Porter (2020), and Farrell and Baker (2020) provide overviews of the economics literature. With the notable exceptions of Kwoka (1989) and Miller and Weinberg (2017), who analyze coordinated effects in static models by incorporating conjectural variation and a behavioral common ownership parameter, respectively, most of the coordinated effects literature has taken a repeated-game approach. Theoretical contributions along these lines include Compte et al. (2002), Vasconcelos (2005), and Bos and Harrington (2010). The first two papers analyze all-inclusive collusion with price-setting and quantity-setting firms, respectively. The last paper analyzes non-all-inclusive collusion with price-setting firms.\textsuperscript{9} Rotemberg and Saloner (1990) provide a model in which price leadership facilitates collusion, with the leader earning higher profits, which raises the possibility that coordinated effects could arise as a result of a subset of large firms allocating the right to act as the leader among themselves. Empirical work includes Igami and Sugaya (2019) related to the vitamins industry and Miller et al. (2019) related to the beer industry, finding that Grupo Modelo acted as a maverick that constrained interdependent pricing between ABI and MillerCoors. Ivaldi and Lagos (2017) provide simulation-based results.

Our paper shares with the repeated-game approach the quantitative interpretation of a market being more at risk when, in our setup, the coordinated effects index is larger and, in the repeated-game framework, the critical discount factor is smaller.\textsuperscript{10} A key distinguishing feature of our approach is that it naturally gives rise to a threshold that distinguishes markets that are at risk from those that are not and therefore allows one to hone in on mergers that would transform a market from not being at risk to being at risk. It is this threshold that
also allows us to define mavericks, to test for whether a firm is a maverick, and clarify the value of maverick-based merger policies.

The remainder of the paper is organized as follows. In Section 2, we present our approach, first without imposing specific assumptions about the price-formation process, and then using a model of differentiated products price competition to fix ideas. In Section 3, we examine procurement markets. Section 4 provides a discussion of policy implications, including a micro-foundation for a maverick-based approach to merger review and an analysis of tradeoffs between unilateral and coordinated effects. Section 5 concludes the paper. The Online Appendix provides an application and extensions.

2. Risk for Market Allocation

We denote by $N \equiv \{1, \ldots, n\}$, with $n \geq 2$, the set of all suppliers, such as the firms in an oligopoly model, and we consider the possibility that the suppliers in a subset $K \subseteq N$ engage in market allocation, where $K$ contains $k \geq 2$ suppliers. An allocation scheme among suppliers in $K$ is an arrangement in which each supplier $i \in K$ is designated to be the active member of $K$ with some probability $s_i$, in which case all other members of $K$ are inactive. In a merger-review context, competition authorities may have reason to focus on specific subsets of suppliers based on historical conduct or other evidence; as we illustrate in the Online Appendix, one can use the framework developed here to identify which subsets of suppliers, if any, pose a concern.

Let $\Pi_i$ denote supplier $i$’s payoff when there is no market allocation. Given a market allocation among suppliers in $K$, we denote by $\Pi_i(K)$ the payoff of supplier $i \in K$ when it is the only supplier from $K$ that is active in the market, which occurs when the other suppliers in $K$ are not active in that market.

A supplier’s decision to participate in a market allocation depends on its expected payoff when it participates as well as what happens if the supplier declines to participate. Because a market allocation among a subset $K$ of suppliers provides a public good to suppliers outside
the conservative approach is to assume that the failure of any one of the suppliers in $K$ to participate in the allocation scheme results in there being no market allocation by suppliers in $K$. If a market is not at risk for allocation by suppliers in $K$ under this assumption, then it is also not at risk for allocation by suppliers in $K$ under alternative assumptions on continued market allocation by subsets of $K$ when a supplier in $K$ declines to participate. This is the approach that we take.

In this setup, participation by supplier $i \in K$ in a market allocation with the other suppliers in $K$ is profitable for supplier $i$ if and only if supplier $i$’s expected payoff under the market allocation is greater than its payoff when there is no market allocation. This occurs if and only if the market is allocated to supplier $i$ with a sufficiently high probability (or supplier $i$ is allocated a sufficiently large share of the geographic areas, products, or customers). Specifically, participation by supplier $i \in K$ in a market allocation scheme among suppliers in $K$ is individually rational for supplier $i$ if and only if the market is allocated to supplier $i$ with probability greater than supplier $i$’s critical share $s_i(K)$ defined by

$$s_i(K) \equiv \frac{\Pi_i}{\Pi_i(K)}.$$

Of course, the market allocation is only feasible if it is profitable for all of the suppliers in $K$. This means that each supplier in $K$ needs to have the market allocated to it with probability greater than its critical share. Because the probabilities that define the allocation scheme must sum to one, the participation constraints can be satisfied for all of the suppliers in $K$ only if those suppliers’ critical shares sum to less than one. This naturally leads to the coordinated effects index:

$$\mathcal{I}(K) \equiv 1 - \sum_{i \in K} s_i(K).$$

If $\mathcal{I}(K) > 0$, then shares exist for the suppliers in $K$ such that each of those suppliers finds it profitable to participate in the market allocation. In that case, we say that the market is at risk for allocation by the suppliers in $K$. In contrast, if $\mathcal{I}(K) \leq 0$, then no such shares
exist, and we say that the market is not at risk for allocation by the suppliers in $K$. Given a positive index, a further increase in the index allows greater scope for an allocation scheme to operate in the sense that some inefficiencies or imperfections can then be accommodated. In the extreme, as the index approaches 1, a market allocation can be sustained by selecting each participating supplier with an equal probability (or dividing geographic areas, products, or customers evenly among the participating suppliers).

Because the coordinated effects index focuses on a necessary condition for market allocation, it is biased in the direction of overestimating the gains from market allocation. It thus provides a screen that allows one to dismiss concerns of coordinated effects as unlikely whenever the index is nonpositive. In this case, a market allocation without communication or transfers is not profitable for the suppliers in $K$. If the index is positive, an allocation scheme may be profitable depending on the challenges of implementation, including the issue of inducing compliance from suppliers that are designated to be inactive.\textsuperscript{12} Importantly, the coordinated effects index is operational for practical purposes—indeed, the model of the pre-merger price-formation process that is required to calculate $\Pi_i$ and $\Pi_i(K)$ is something that is currently typically constructed using pre-merger data for the purpose of unilateral effects analysis.\textsuperscript{13}

As discussed in the introduction, the notion of a maverick firm is prominent in coordinated effects analyses, but lacks a precise definition. Our approach also allows us to make headway in this direction. We define a maverick with respect to a set of suppliers $K$ to be a supplier whose presence prevents the pre-merger market from being at risk for a market allocation by suppliers in $K$, that is, the market is not at risk for allocation by suppliers in $K$ when the maverick is in the market, but is at risk when the maverick is not in the market. More formally, writing $I(K; N)$ to denote the coordinated effects index for the subset $K$ of $N$ suppliers, supplier $m \in N \setminus K$ is a maverick if

$$I(K; N) \leq 0 \text{ and } I(K; N \setminus \{m\}) > 0.$$
This definition of a maverick captures the view that a maverick firm stands separate from its rivals and interferes with its rivals’ ability to enhance their profits by dividing the market (or geographic areas, products, or customers) among themselves. The definition allows the possibility that there is no maverick, as well as the possibility that more than one supplier in a market could be a maverick with respect to a particular set $K$ of suppliers.\textsuperscript{14}

The conditions for a market to be at risk or for structural changes, such as the acquisition of a maverick, to put a market at risk naturally depend on the specifics of the price-formation process. Thus, the implementation of this approach requires a model of the price-formation process.

To illustrate, suppose that we have a market with $n$ suppliers engaged in differentiated-products price competition, where supplier $i$’s cost function is $C_i$ and demand for supplier $i$’s product given price vector $\mathbf{p}$ is $D_i(\mathbf{p})$. Then supplier $i$’s payoff given price vector $\mathbf{p}$ is

$$\pi_i(\mathbf{p}) \equiv p_i D_i(\mathbf{p}) - C_i(D_i(\mathbf{p})).$$

Letting $\mathbf{p}^*(X)$ denote the vector of Nash equilibrium prices for the game in which suppliers in $X \subseteq N$ choose their prices to maximize their profits and the suppliers in $N \setminus X$ choose their prices so that their equilibrium quantities are zero, we have

$$\Pi_i = \pi_i(\mathbf{p}^*(N)) \quad \text{and} \quad \Pi_i(K) = \pi_i(\mathbf{p}^*((N \setminus K) \cup \{i\})).$$

For example, in the symmetric differentiated Bertrand model of Singh and Vives (1984) with inverse demand $P_i(\mathbf{q}) = 1 - q_i - s \sum_{j \neq i} q_j$ and marginal costs of zero, where $s \in (0, 1)$ is a substitution parameter, the coordinated effects indices for subsets of $k = 2$ out of $n$ suppliers are as shown in Figure 1(a) as a function of the substitution parameter.

As Figure 1(a) illustrates, whether a market is at risk for market allocation by pairs of suppliers depends on the total number of suppliers in the market and the substitutability between the suppliers’ products. As shown, a market with more suppliers is less at risk
because $I(K)$ decreases with $n$, and greater substitutability among products increases the risk because $I(K)$ increases with $s$. These comparative statics align well with traditional thinking on which markets pose the greatest risk for coordinated effects.

Figure 1(b) shows the effects of a merger on the risk of market allocation by $k = 2$ suppliers, one of which being the merged entity, when there are $n = 5$ symmetric firms before the merger, with a merger by two firms modelled as creating a new firm that chooses prices to maximize the joint profit from the sale of both of its products. As the figure shows, the merger increases the risk of allocation and, for products that are sufficiently strong substitutes, the merger causes a market that was not at risk for pairwise market allocation to become at risk.

3. Procurement Markets

The importance of business-to-business (or business-to-government) transactions, where prices are typically determined through competitive procurements, and the regularity of antitrust concerns related to collusion and supplier mergers in this sphere mean that a key application of coordinated effects analysis is to procurement markets.

3.1. Procurement Market Setup

We consider a standard procurement model, in which a buyer with value $v$ for a single unit of a good can potentially purchase the good from any of $n$ suppliers. Each supplier $i$ independently draws its cost of producing a unit, which is its private information, from a distribution $G_i$ with support $[c, \bar{c}]$ and bounded density $g_i$ that is positive on $(c, \bar{c})$. To focus on the case where gains from trade are possible, we assume that $c < v$. Symmetry among a set of suppliers means that the suppliers in that set draw their costs from the same distribution: $G_i = G$ for all $i$ in the set.

To model mergers, we assume as do Farrell and Shapiro (1990) that merging suppliers rationalize production by producing using the lower of their two costs. Thus, a merged entity that combines suppliers $i$ and $j$ has a cost distribution that is the distribution of
the minimum cost of the two pre-merger suppliers, in which case the merged entity’s cost
distribution is \( \hat{G}(c) \equiv 1 - (1 - G_i(c))(1 - G_j(c)) \). It follows that the productivity of the
merged entity is enhanced relative to that of either of the individual merging suppliers.\(^{18}\)

At the time when the suppliers in \( K \) must decide whether to participate in a market
allocation, all distributions, which characterize the firms’ production technologies, are com-
monly known, but the suppliers’ costs—their opportunity costs of using these technologies
to serve the buyer—have not been realized. These costs are realized after the participation
decisions are made but prior to bidding in the procurement.\(^{19}\) Thus, the relevant payoffs as-
sociated with joining an allocation scheme are the suppliers’ ex ante expected payoffs, with
the expectation taken over all cost realizations and the allocation probabilities. Assuming
that the suppliers in \( K \) participate in an allocation scheme based on allocation probabilities
\((s_i)_{i \in K}\), the timing is as follows: First, a procurement arrives and is allocated to one of the
suppliers in \( K \) according to the probabilities \((s_i)_{i \in K}\). Second, costs are realized for the sup-
plier in \( K \) to whom the procurement is allocated and for the suppliers outside \( K \), with those
costs remaining the private information of the individual suppliers. Third, the procurement
occurs, with the designated supplier from \( K \) competing against suppliers outside \( K \). Finally,
the winner is determined according to the procurement rules and payoffs are realized. A
key assumption is that the suppliers in \( K \) other than the one to whom the procurement is
allocated do not submit bids.

For some results, it is useful to parameterize the suppliers’ cost distributions as \( G_i(c) = 1 - (1 - c)^{\alpha_i} \) with \( \alpha_i > 0 \) and support \([c, \bar{c}] = [0, 1]\), which we refer to as the power-based
parameterization.\(^{20}\) We refer to \( \alpha_i \) as supplier \( i \)'s strength parameter because under efficient
procurement, in a market with set \( N \) of competing suppliers, supplier \( i \)'s market share is
\( \alpha_i / \sum_{j \in N} \alpha_j \). This parameterization allows us to talk about suppliers with larger values of
\( \alpha_i \) as being “larger” or “stronger.”
3.2. Efficient Procurement Markets

To model an efficient procurement market, we assume that the buyer uses a second-price procurement with reserve \( r \equiv \min\{v, \bar{c}\} \), which, together with the assumption that suppliers follow their weakly dominant strategies of bidding truthfully, ensures that the procurement is efficient. We let \( I^S(K) \), with the superscript \( S \) for second-price, denote the coordinated effects index for efficient procurement markets.

A market allocation suppresses competition from one or more suppliers, and with positive probability, the participation of the lowest-cost supplier is suppressed. Thus, a market allocation increases the expected cost conditional on trade and also reduces the probability of trade if the buyer’s reserve binds with positive probability. As a result, a market allocation reduces expected social surplus. In addition, the elimination of bids as a result of a market allocation results in a higher expected price, which harms the buyer. Thus, we have the following result:

**Proposition 1.** In an efficient procurement market, market allocation reduces expected buyer surplus and expected social surplus.

Proposition 1 provides a foundation for competition authorities’ concerns about coordinated effects in procurement markets. Further, if we view the buyer’s purchase as an input to production for downstream consumers, and if a failure to purchase translates into a higher marginal cost or lower quality, then market allocation harms downstream consumers when \( v < \bar{c} \) because it reduces the probability of trade.

Using the notation \( c_{(j:n)} \) to denote the \( j \)-th lowest order statistic out of \( n \) independent draws from a common cost distribution, we have the following result:

**Proposition 2.** In an efficient procurement market with symmetric suppliers with cost distribution \( G \) and \( v \geq \bar{c} \), the market is at risk for an all-inclusive market allocation, \( I^S(N) > 0 \), if and only if

\[
\mathbb{E}_c[c_{(2:n)}] - c_{(1:n)} < \bar{c} - \mathbb{E}_c[c].
\]
Furthermore, this condition holds if \( G(c)/g(c) \) is increasing in \( c \) for all \( c \in [c, \overline{c}] \).

Proof. See Appendix A.

Proposition 2 provides conditions under which an efficient procurement market with symmetric suppliers is at risk for an all-inclusive market allocation scheme and conditions under which it is not. In particular, (1) fails to hold—implying that the market is not at risk for all-inclusive allocation—for cost distributions with a “long left tail,” where the price paid to the winning supplier is likely to be close to the reserve under competition, and the expected incremental payment under cooperation is small and outweighed by the loss associated with the possibility that the supplier is not selected to participate.\(^{21}\)

It follows that the coordinated effects index test has power: based on the index, some, but not all, markets are at risk for allocation. Using “model specification” to mean the buyer’s value and suppliers’ cost distributions, we have:

**Corollary 1.** For an efficient procurement market, there exist \( N, K \subseteq N \), and model specifications such that \( \mathcal{I}^S(K) > 0 \), and other \( N, K \subseteq N \), and model specifications such that \( \mathcal{I}^S(K) \leq 0 \).

3.2.1. Mergers and Mavericks

A merger raises concerns of coordinated effects if the merger enables or encourages post-merger coordinated interaction (U.S. Guidelines, p. 24). A key scenario of concern is, therefore, a situation in which, based on pre-merger data, the pre-merger market is characterized by \( \mathcal{I}^S(K) \leq 0 \) and, based on the same data, the post-merger market would be characterized by \( \mathcal{I}^S(K) > 0 \). In this case, the data would be consistent with pre-merger competition and with the risk of post-merger market allocation. More generally, accounting for the inevitable uncertainty and imprecision, if a merger is expected to increases the coordinated effects index, regardless of the sign of the index, the merger can be viewed as increasing the likelihood of a market allocation by the set of suppliers in \( K \).
As another scenario, if one obtains $I^S(K) > 0$ for the pre-merger market, then concerns that the merger could exacerbate coordinated effects are justified when the post-merger market has an even larger value of $I^S(K)$ because this implies that a larger set of possibilities for allocation become available following the merger. Either scenario suggests potential for merger-specific harm based on coordinated effects.

An implication of Proposition 2 is that even a “3-to-2” merger does not raise concerns of coordinated effects if the resulting duopoly is characterized by symmetric suppliers that draw their costs from a distribution that does not satisfy (1). Thus, while it is possible to have a 3-to-2 merger that does not raise concerns of coordinated effects, such circumstances are limited, which is consistent with the view in practice that 3-to-2 defines a significant, but not insurmountable, hurdle for antitrust approval.\textsuperscript{22}

Next consider the effects of a merger among three or more suppliers in $K$ on the profitability of participation in a market allocation among those suppliers. In an efficient procurement market, the critical share for a supplier $i \in K$, $s_i(K)$, depends only on the distribution of the minimum cost of suppliers other than $i$ in $K$ and on the distribution of the minimum cost of suppliers outside $K$. Because a merger of two suppliers does not affect the distribution of the minimum cost of the two merging suppliers, it follows that in an efficient procurement market, a merger of two suppliers in $K$ does not affect the critical shares of the nonmerging firms in $K$. Thus, the change in $I^S(K)$ as a result of the merger depends on how the critical share of the merged entity compares to the sum of the critical shares of two merging suppliers in the pre-merger market. Letting $\hat{N}$ denote the set of post-merger suppliers and $\hat{K}$ denote the post-merger suppliers in $K$ following the merger of suppliers $k$ and $\ell$ in $K$, and letting $\mu$ denote the merged entity, we have

$$I^S(\hat{K}; \hat{N}) - I^S(K; N) = s_k(K; N) + s_\ell(K; N) - s_\mu(\hat{K}; \hat{N}).$$ (2)

The next lemma follows immediately.
Lemma 1. For an efficient procurement market, a merger of suppliers in $K$ increases the coordinated effects index if and only if the critical share of the merged entity is less than the sum of the pre-merger critical shares of the merging suppliers.

To see the forces at work, consider a merger of suppliers $k$ and $\ell$, both of which are members of $K$, and assume for purposes of illustration that $\Pi_k(K; N) = \Pi_{\ell}(K; N)$. Under this assumption, we have

$$s_k(K; N) + s_{\ell}(K; N) = \frac{\Pi_k + \Pi_{\ell}}{\Pi_k(K; N)}. \quad (3)$$

A merger of suppliers $k$ and $\ell$ is always profitable for those suppliers, that is, $\Pi_k + \Pi_{\ell} < \Pi_{\mu}$ (Loertscher and Marx, 2019a, Prop. 6), so the numerator in the merged entity’s critical share, $\Pi_{\mu}$, is larger than the numerator in (3). At the same time, the denominator in the merged entity’s critical share, $\Pi_{\mu}(\hat{K}; \hat{N})$, is greater than the denominator in (3) because the merged entity draws its cost from a better distribution than does either supplier $k$ or $\ell$. Because either effect can dominate, the critical share of the merged entity can be greater than or less than the sum of the pre-merger critical shares of the merging suppliers, implying that the coordinated effects index can increase or decrease as a result of the merger.

In contrast to the case of a merger of suppliers in $K$, a merger of suppliers outside $K$ does not affect $\mathcal{I}(K)$ because $\mathcal{I}(K)$ only depends on suppliers outside $K$ through the distribution of the minimum of their costs, which is not affected by a merger. Thus, we have the following result:

Proposition 3. In an efficient procurement market, a merger of suppliers in $K$ can, but need not, cause a market not at risk for a market allocation by suppliers in $K$ to become at risk for a market allocation by the corresponding post-merger suppliers, and a merger of suppliers outside $K$ does not affect the risk for market allocation by suppliers in $K$.

An implication of Proposition 3’s result that a merger of outsiders does not affect the coordinated effects index is that merging parties cannot reduce the perceived risk of a market
allocation as a result of their merger by strategically varying the timing of their merger so that it falls either before or after a merger of outsiders.

As we now show, the acquisition of a maverick can, but need not, put the market at risk. In a procurement market, the acquisition of a maverick is not the same as the elimination of the maverick’s productive capability. Rather, it eliminates a bid in the procurement. Thus, if supplier $i \in K$ is the acquirer, then both $\Pi_i$ and $\Pi_i(K)$ increase after the acquisition, so that the critical share $s_i(K)$ may well be larger after the acquisition than before it. For all other suppliers $j \in K\{i\}$, $\Pi_j(K)$ increases because the maverick has been eliminated as an outside supplier. In an efficient procurement, $\Pi_j$ is not affected, so $s_j(K)$ decreases.

Consequently, in an efficient procurement market, the overall effect of the acquisition of a maverick with respect to $K$ on the coordinated effects index depends on the details and can, as we show, go either way.

The following proposition provides conditions under which the decrease in the critical share of the nonmerging supplier dominates if and only if the maverick is acquired by the smaller of two participating suppliers.

**Proposition 4.** For an efficient procurement market characterized by the power-based parameterization and $v \geq \overline{v}$, if $k = 2$, then the acquisition of a maverick by the weakly smaller supplier in $K$ puts the market at risk, whereas the acquisition by the larger supplier in $K$ does not if the weaker supplier is sufficiently small.

**Proof.** See Appendix A.

Proposition 4 implies that maverick-based merger review that focuses on blocking mergers that involve the acquisition of a maverick can be misleading for efficient procurement markets. In addition, some other aspects of the perceived wisdom regarding mavericks are also not supported in our framework. In particular, one can easily construct examples in which the acquisition of a supplier that is not a maverick puts a market at risk. Thus, it is not the case that the acquisition of a maverick is necessary for coordinated effects. Contrary to what has
be suggested, it is also not the case that the presence of a maverick prevents coordinated
effects from mergers not involving the maverick.\textsuperscript{23}

Similar to the neutrality result of Nocke and Whinston (2010) showing that the order in
which a competition authority addresses mergers does not matter for unilateral effects, we
have a form of neutrality for coordinated effects because outside mergers do not affect the
participation constraint for a market allocation. If $I^S(K) > 0$, a merger of outsiders cannot
stop the market from being at risk nor create a maverick with respect to $K$ because mergers
among firms outside $K$ do not affect $I^S(K)$, which depends on outsiders only through the
distribution of the minimum of their costs. Thus, a competition authority cannot reduce
the risk of coordinated effects among one set of firms by first approving “balancing” mergers
among outsiders. However, if $I^S(K) < 0$, then a merger of outsiders could create a maverick,
which would then be a potential acquisition target for suppliers in $K$. For example, suppose
that there are two suppliers outside $K$ and that $I^S(K) < 0$. If neither of the outsiders
is a maverick, but if they are jointly a maverick in the sense that if both outsiders were
eliminated, then the market would be at risk for a market allocation by $K$, then a merger
of those outsiders creates a maverick.

The merger of U.S. telecom firms Sprint and T-Mobile raised the question whether the
merger of two mavericks might create a “super maverick”, with former FCC Commissioner
Robert McDowell stating, “Verizon and AT&T have nearly 70 percent market share and 93
percent of the industry cash flow. Combining T-Mobile and Sprint will create a supercharged
maverick, which will still be only Number Three. This newly invigorated third carrier will
be better able to compete against the larger two.”\textsuperscript{24} In our model, a merger involving a
maverick with respect to $K$ and another supplier outside $K$ does not affect $I^S(K; N)$ and
increases $I^S(K; N\backslash \{\hat{m}\})$, where $\hat{m}$ denotes the merged entity formed by the maverick and
the other outside supplier. As a result of such a merger, there continues to be a maverick
with respect to $K$, and the difference between the coordinated effects index with and without
the maverick is increased.\textsuperscript{25} Thus, there is a sense in which the merger of two mavericks could
indeed be described as creating a “super maverick.”

3.2.2. Market and Supplier Characteristics

We now show that the effects of market and supplier characteristics on the risk of market allocation in our model are broadly consistent with the perceived wisdom and the prior literature. In particular, we discuss the effects of outside competition, supplier strength, and symmetry among the participating suppliers on the coordinated effects index.

A natural conjecture is that a market allocation by a given set of suppliers is more challenging when those suppliers face more outside competition. Although an increase in the number of (symmetric) outside suppliers decreases the payoffs of the suppliers in $K$ both with and without a market allocation, one can show that, for symmetric suppliers, the decrease in payoffs under market allocation is larger, and so critical shares increase and the market becomes less at risk for market allocation.

**Proposition 5.** *In an efficient procurement market with symmetric suppliers, given $K$ with $k \geq 2$ members, the market is not at risk if $n$ is sufficiently large: $\lim_{n \to \infty} I(K) = 1 - k < 0$.*

*Proof.* See Appendix A.

The intuition is simple. As mentioned, a market allocation provides a public good whose costs are borne by the insiders and whose benefits accrue to all active suppliers, which include the designated supplier among the insiders and all outsiders. For a market allocation to pay off, it must be the case that the insiders internalize enough of the benefits that their conduct generates, which requires that the outside competition is limited.

Interestingly, the effect of the degree of outside competition on the incentives for a subset of suppliers to participate in a market allocation may make such conduct “contagious” in the following sense: Suppose that, if the suppliers in some set $K_1$ do not coordinate, then $\mathcal{I}^S(K_2)$ is negative for some disjoint set of suppliers $K_2$. If the suppliers in $K_1$ coordinate, implying that the lowest-cost supplier in $K_1$ is not active with positive probability, this is as if the outside competition for the suppliers in $K_2$ has weakened, making it possible
that once the suppliers in $K_1$ coordinate, $I^S(K_2)$ becomes positive. Of course, because the same logic applies to the suppliers in $K_1$, it is possible that each set of suppliers only finds it beneficial to coordinate if the other set coordinates as well.

Next, we consider the effect of increases in the strength of the participating suppliers. We focus on the power-based parameterization, where an increase in strength corresponds to an increase in the distributional parameter and to a first-order stochastically dominated shift in the cost distribution.

**Proposition 6.** For the power-based parameterization and $v \geq \bar{c}$,

(a) if suppliers in $K$ have strength $\alpha$, then $\lim_{\alpha \to 0} I^S(K) < 0$ and $\lim_{\alpha \to \infty} I^S(K) > 0$;

(b) $I^S(K)$ is largest, conditional on $|K| = k$ if $K$ includes the $k$ strongest suppliers;

(c) given $\alpha = (\alpha_i)_{i \in N}$ and $\beta = (\beta_i)_{i \in N}$ satisfying $\sum_{i \in N} \alpha_i = \sum_{i \in N} \beta_i$, and adding the distributional parameters as an argument to $I^S$, we have

$$I^S(K; \alpha) < I^S(K; \beta)$$

if $\alpha_i \leq \beta_i$ for all $i \in K$ with a strict inequality for at least one $i \in K$.

**Proof.** See Appendix A.

For the power-based parameterization, by Proposition 6(a), efficient procurement markets are not at risk when the participants in the allocation scheme are sufficiently weak and are at risk when those participants are sufficiently strong. Intuitively, participating in a market allocation with a stronger supplier is more profitable because the bid suppression by that supplier is more likely to affect the outcome of the procurement. Further, by Proposition 6(b), market allocations in efficient procurement markets are characterized by “positive assortative matching” insofar as the coordinated effects index for a set of $k$ suppliers is greatest when the set contains the $k$ strongest suppliers. Moreover, by Proposition 6(c), the coordinated effects for a given set of suppliers $K$ increases as these suppliers become stronger while keeping
fixed the distribution of the lowest cost draw in the market. Proposition 6(b) implies that an efficient procurement market is most at risk for the allocation that is most problematic for buyers, namely a market allocation involving the large suppliers. This is consistent with the prevailing view that a competition authority should be most concerned about a market allocation among the largest suppliers in a market.

We now turn our attention to the effects of asymmetries among participants in an allocation scheme. The existing literature has argued that asymmetries make it more difficult for firms to agree to a common pricing policy and that incentive compatibility may be difficult to satisfy for low-cost firms that face relatively large gains from deviations and small costs from punishment.\textsuperscript{27} We now show that a similar effects arise in our context.

To consider the effects of asymmetries among suppliers in a market, we begin by defining a \textit{neutral for rivals spread} (NR spread). We say that a change in the cost distributions for suppliers $i$ and $j$ from $(G_i, G_j)$ to $(H_i, H_j)$ is an NR spread if for all $c \in [\underline{c}; \overline{c}]$,\textsuperscript{28}

\begin{equation}
H_i(c) \leq G_i(c), G_j(c) \leq H_j(c),
\end{equation}

with strict inequalities for costs in an open subset of $[\underline{c}; \min\{\nu, \overline{c}\}]$, and

\begin{equation}
(1 - G_i(c))(1 - G_j(c)) = (1 - H_i(c))(1 - H_j(c)).
\end{equation}

The inequalities in (4) represent the spread, while (5) captures neutrality for rivals because it means that the distribution of the minimum cost of suppliers $i$ and $j$ is the same under $(H_i, H_j)$ as under $(G_i, G_j)$.

As we now show, consistent with the existing literature showing that collusion is harder to sustain among more asymmetric suppliers, we provide conditions under which an NR spread applied to two suppliers in $K$ reduces the coordinated effects index:

\textbf{Proposition 7.} For the power-based parameterization and $\nu \geq \tau$, an NR spread applied to two suppliers in $K$ reduces $I^S(K)$.
Proof. See Appendix A.

Proposition 7 highlights the impact of asymmetries among suppliers on the risk for coordinated effects. Intuitively, when two suppliers are made less symmetric, their expected second-lowest cost increases, which increases the expected prices and profits in the absence of a market allocation and so decreases $I^S(K)$. At the same time, expected profits in the presence of a market allocation decrease for the supplier made weaker and increase for the supplier made stronger, with an ambiguous effect on $I^S(K)$. Combining these, in the case of the power-based parameterization, the overall effect is to reduce $I^S(K)$.

3.3. First-price Procurement Markets

We now provide conditions under which buyers that employ first-price rather than second-price procurements are less vulnerable to coordinated effects.

In first-price procurement auctions, the bidder with the lowest bid that is less than the reserve wins and is paid the amount of its bid. We further restrict attention to ex ante symmetry among bidders, namely $G_i = G$ for all $i \in N$. In a first-price auction, a market allocation by suppliers in $K \subset N$ harms the buyer and reduces social surplus just like it does in a second-price auction. It does so because, first, the active supplier in $K$ increases its bid for any cost realization $c \in [c_{\text{min}}, c]$ because it faces less competition, which results in a higher expected price for the buyer and inefficient production. Moreover, if an outsider—that is, a supplier that does not participate in the market allocation—is aware of the market allocation, then it also bids higher according to the unique Bayes Nash equilibrium bidding strategy (Lebrun, 1999) of the procurement game in which $n + 1 - k$ symmetric suppliers simultaneously submit bids. If the outsiders are not aware of the market allocation, then they bid according to the Bayes Nash equilibrium bidding strategy with $n$ suppliers, and the active supplier in $K$ best responds to these strategies.

To distinguish between the scenarios in which outsiders are and are not aware of the market allocation, we denote the coordinated effects index by $I^F_1(K)$ when outsiders are
aware of the market allocation and by $I^F_0(K)$ when they are not. Because of strategic complementarity, we have $I^F_1(K) \geq I^F_0(K)$. It will be useful in what follows to note that the payoff equivalence theorem (see, for example, Myerson, 1981; Krishna, 2002; Börgers, 2015) implies that under symmetry, suppliers’ competitive payoffs are the same under second-price and first-price procurements. If, in addition, the market allocation is observable, then the resulting payoffs are also the same under second-price and first-price procurements because then, letting $k$ be the number of participating suppliers, we simply have competition among $n - k + 1$ symmetric suppliers. Thus, for symmetric suppliers,

$$I^S(K) = I^F_1(K) \geq I^F_0(K),$$

with a strict inequality if and only if $K \neq N$. This formalizes the commonly held notion that first-price procurements are less susceptible to collusive conduct than second-price procurements (See, for example, (Marshall and Marx, 2009) and Kovacic et al. (2006).)

**Proposition 8.** Given a procurement market with $n$ symmetric suppliers, if allocation schemes are not observable, then the market is at lower risk for allocation by a subset of $k < n$ suppliers when the procurement is first price rather than second price; but the risk is not affected by the auction format if the allocation scheme is observable or if the allocation scheme is all inclusive, namely $k = n$.

While Proposition 8 confirms the usual thinking that first-price procurements tend to be more resistant to anticompetitive conduct by bidders than second-price procurements, it also provides conditions under which a first-price procurement provides no additional protection from a market allocation relative to a second-price procurement. Thus, while a buyer’s use of first-price procurements may mitigate concerns related to coordinated effects, it does not eliminate them.
3.4. **Procurement Markets with Powerful Buyers**

An entire section of the *U.S. Guidelines* is devoted to “Powerful Buyers,” where it is argued that “the conduct or presence of large buyers” could undermine coordinated effects (p. 27). Indeed, for procurement markets, it is natural to consider the possibility that powerful buyers design their procurement mechanisms to maximize their expected payoffs. (In contrast, it is not clear how one would incorporate buyer power into standard oligopoly models, which assume price-taking buyers.) Intuitively, one expects buyer power to reduce the profitability of market allocation schemes because powerful buyers rely less on rivalry among bidders to police prices than do buyers without power. Yet, to our knowledge, there has been no formalization of this intuitive idea in the literature.

We follow Bulow and Klemperer (1996) and Loertscher and Marx (2019a) in modeling a powerful buyer as one that has the sophistication and commitment power to employ the optimal procurement mechanism, that is, the mechanism that maximizes the buyer’s expected profit subject to suppliers’ dominant strategy incentive compatibility and individual rationality constraints. We assume that buyer power itself is not affected by mergers among suppliers, which is natural if buyer power derives from the size and/or sophistication of the buyer, as suggested by the *EC Guidelines* (para. 65), or from the ability to vertically integrate upstream or sponsor entry, as suggested by the *U.S. Guidelines* (p. 27), and if the merger does not give suppliers the power to influence the procurement mechanism that is employed by the buyer.

As noted by Loertscher and Marx (2019a), buyer power consists of two components: the ability to discriminate between suppliers and the commitment to cancel a procurement even though it would be profitable, which may be called *monopsony power*. Whether the buyer optimally exerts one or both of these powers depends on the problem at hand. When all suppliers are ex ante symmetric, that is, when \( G_i = G \) for all \( i \in N \), there is no discrimination because ranking the suppliers according to their virtual costs is the same as ranking them according to their costs; however, in the absence of ex ante symmetry, the buyer optimally
uses its power to discriminate some of the time. The buyer optimally refrains from ever using its monopsony power if and only if \( v \geq \min_{i \in N} \Gamma_i(\bar{c}) \), where \( \Gamma_i(c) \equiv c + \frac{G_i(c)}{g_i(c)} \) is supplier \( i \)’s virtual cost, which we assume is increasing in \( c \).

In procurement markets with buyer power, as in the case of efficient procurement markets, the buyer is harmed by a market allocation, and some but not all markets are at risk for market allocation. Interestingly, with buyer power, a market allocation can increase social surplus for some type realizations. This occurs with ex ante heterogeneous suppliers when, absent a market allocation, the buyer does not purchase from the lowest-cost supplier because it discriminates between suppliers on the basis of their virtual costs and purchases from a supplier with a lower cost when there is a market allocation because the bid of the supplier it buys from in the absence of a market allocation is suppressed.

With buyer power, the result that markets with sufficient outside competition are not at risk for market allocation continues to hold. Assuming ex ante symmetric pre-merger suppliers, a decrease in the critical share of the merged entity relative to the sum of the pre-merger critical shares of the merging suppliers is sufficient, but no longer necessary for the merger to increase the coordinated effects index. Nevertheless, it remains the case that a merger can, but need not, cause a market not at risk to become so.

To examine how the coordinated effects index is affected by buyer power in our framework, we focus on ex ante symmetric suppliers. In this case, a powerful buyer never uses its power to discriminate, so the sole effect of buyer power is to reduce the reserve price, which allows us to focus on the effects of a change in the reserve in a second-price procurement.

**Proposition 9.** In a procurement market with symmetric suppliers, \( I^S(K) \) is increasing in the buyer’s reserve price, and thus decreases with buyer power.

*Proof.* See Appendix A.

To the best of our knowledge, Proposition 9 is the first formal demonstration that, consistent with perceived wisdom, buyer power reduces concerns of coordinated effects.
Let us now turn to a merger. With buyer power, and regardless of whether suppliers are ex ante symmetric, a merger of two suppliers in $K$ increases the payoff of the other suppliers in $K$ when there is no market allocation as a result of the buyer’s more aggressive discrimination against the merged entity. However, a merger between two suppliers in $K$ does not affect the payoff of a nonmerging supplier in $K$ under a market allocation. Thus, we have the following result:

**Proposition 10.** In a procurement market with buyer power, a merger of two suppliers in $K$ increases the critical shares of the non-merging suppliers in $K$.

Proposition 10 implies that with buyer power, a merger of two suppliers in $K$ constrains the profitability of a market allocation by increasing the critical shares of the nonmerging suppliers in $K$. Propositions 9 and 10 provide a foundation for the view that coordinated effects from a merger are less of a concern in the face of powerful buyers.\(^{34}\)

### 3.5. Vertically Integrated Buyer

Another question of concurrent interest concerns the effects of vertical integration on market outcomes.\(^{35}\) To shed light on this question, we return to a second-price procurement without buyer power and stipulate that there are $n \geq 3$ ex ante symmetric suppliers and a buyer with willingness to pay $v \geq \overline{c}$ in the absence of integration. This implies that prior to integration, the reserve price in the second-price auction is $\overline{c}$. The exposition simplifies by assuming that $G$ is such that for any $a \in [0, 1]$, both $\Gamma(c; a) \equiv c + a \frac{G(c)}{g(c)}$ and $\Phi(c; a) \equiv c - a\frac{1 - G(c)}{g(c)}$ are increasing in $c$, a sufficient condition for which is that $\Gamma(c; 1)$ and $\Phi(c; 1)$ are increasing in $c$.

Without loss of generality, assume that the buyer integrates with supplier 1. This implies that after integration, the buyer’s willingness to pay for the service or product of the independent suppliers is $c_1$, implying that the post-integration market has two-sided private information. By a straightforward extension of the impossibility theorem of Myerson and Satterthwaite (1983) (see, for example, Gresik and Satterthwaite, 1989; Delacrétaz et al., 2019), this implies that post-integration ex post efficient trade is impossible without running
a deficit. For the post-integration market, it is then natural to focus on the second-best mechanism that maximizes ex ante expected social surplus subject to incentive compatibility, individual rationality, and budget constraints. As is well known (see, again, for example Gresik and Satterthwaite, 1989), this mechanism is characterized by an allocation rule that induces trade from the lowest-cost independent supplier \( i \) to the integrated firm if and only if
\[
\Phi(c_1; a^*) \geq \Gamma(c_i; a^*),
\]
where \( a^* \in (0, 1) \). For any \( K \subset N \setminus \{1\} \), denote by \( \mathcal{I}^S(K) \) and \( \mathcal{I}^S_{VI}(K) \) the coordinated effects index before and after integration, respectively.

Intuitively, vertical integration has no effect on the efficient quantities under the assumptions that we stipulate in the sense that under ex post efficiency, for any given cost realization, it is the same supplier that produces. Hence, if the quantities were still efficient following integration, then vertical integration would have no impact on the coordinated effects index. However, because vertical integration induces a Myerson-Satterthwaite problem, the allocation following integration is no longer efficient. Qualitatively, the second-best mechanism has the same effects as endowing the buyer with buyer power. To see this, notice that any allocation rule according to which trade occurs between the integrated firm and the independent supplier with the lowest cost if and only if
\[
\Phi(c_1; a) \geq \Gamma(c_i; a),
\]
is implementable via a second-price auction in which the integrated buyer sets the reserve
\[
p(c_1; a) \equiv \Gamma^{-1}(\Phi(c_1; a); a),
\]
which is decreasing in \( a \) and satisfies \( p(c_1; 0) = c_1 \). Thus, for a given realization of \( c_1 \), vertical integration has the same effect as having the buyer set a more aggressive reserve. As we know from Proposition 9, buyer power reduces the risk for allocation among symmetric suppliers. Hence, from Proposition 9 follows Corollary 2:

**Corollary 2.** Vertical integration decreases the risk for allocation by any subset of independent, symmetric suppliers in the sense that: for symmetric suppliers and \( v \geq \tau \), if \( \Phi(c; 1) \) and \( \Gamma(c; 1) \) are increasing in \( c \) and \( K \subset N \setminus \{1\} \), then \( \mathcal{I}^S(K) > \mathcal{I}^S_{VI}(K) \).

To our knowledge, Corollary 2 is the first instance of a formal connection between buyer power and vertical integration. Merger guidelines customarily refer to vertical integration as a source of buyer power. Although in the independent private values framework that underlies
our setup there is no connection between the designer’s type (that is, its realized willingness
to pay) and its ability to use the profit-maximizing mechanism, vertical integration in this
setting has qualitatively the same effect as endowing the buyer with buyer power, as the
above shows.

4. Discussion

In this section, we discuss micro-foundations for a maverick-based approach to coordinated
effects and tradeoffs between coordinated effects and unilateral effects.

4.1. Micro-foundation for a Maverick-based Approach

We now show that the Cournot model provides a theoretical foundation for a maverick-based
approach to coordinated effects.

For this purpose, consider a Cournot model in which the inverse demand function for
aggregate quantity $Q \in [0, 1]$ is $P(Q) = 1 - Q$ and suppliers have constant marginal costs
$(c_1, \ldots, c_n)$ that are common knowledge. 39 We assume that costs are such that all suppliers are
active in equilibrium in the absence of a market allocation. 40 To model a market allocation, we
assume that $\Pi_i(K)$ is equal to supplier $i$’s Cournot payoff when only supplier $i$ and suppliers
in $N \setminus K$ are present in the market. An immediate result is that a market allocation, which
suppresses competition from one or more suppliers, reduces total output, and so reduces
consumer surplus and social surplus. Thus, the concerns of competition authorities related
to coordinated effects apply to Cournot markets.

In the Cournot model, we model mergers as resulting in the elimination of the merging
supplier with the higher cost, or simply one of the suppliers if their costs are the same.
This means that the effect of a merger in the Cournot model is the elimination of the
productive assets of the weakly less efficient merging supplier, which leaves the productivity
of the merged entity the same as that of the weakly more efficient merging supplier. 41 This
immediately implies that a merger between a maverick and a lower-cost supplier eliminates
the maverick and so, by the definition of a maverick, puts the market at risk.
Further, as we now show, in the Cournot model, the only way a merger can cause a market that is not at risk to become at risk for allocation by a subset of symmetric suppliers is if the merger involves a maverick. For the result below, we assume that when a supplier \( i \in K \) merges with supplier outside \( K \), the merged entity takes the place of supplier \( i \) as a member of \( K \) in the post-merger market. Because the proof is relatively short and straightforward, we include it here.

**Proposition 11.** In the Cournot model, a merger between a maverick with respect to \( K \) and a weakly lower cost supplier puts the market at risk for market allocation by suppliers in \( K \); moreover, the only merger that can cause a market that is not at risk for allocation by a subset \( K \) of symmetric suppliers to become at risk for allocation by those suppliers is a merger involving a maverick with respect to \( K \).

**Proof.** The first part follows by the definition of a maverick and the assumption that a merger in the Cournot model eliminates the higher-cost supplier. Turning to the second part, letting \( q_i^C(X) \) denote the Cournot quantity of supplier \( i \) when suppliers in \( X \) participate in the market, we have \( q_i^C(X) = \frac{1+C_X-|X|c_i}{|X|+1} \), where \( C_X \equiv \sum_{i \in X} c_i \). Because supplier \( i \)'s Cournot profit is the square of its quantity, the index for the Cournot model is \( \mathcal{I}^C(K) = 1 - \sum_{i \in K} \left( \frac{q_i^C(N)}{q_i^C(N \setminus K \cup \{i\})} \right)^2 \), which for symmetric suppliers in \( K \) becomes

\[
\mathcal{I}^C(K) = 1 - k \left( \frac{n-k+2}{n+1} \right)^2.
\]  

(6)

Given symmetric suppliers in \( K \), if for all \( i \in K \), \( \mathcal{I}^C(K; N) \leq 0 \) implies \( \mathcal{I}^C(K \setminus \{i\}; N \setminus \{i\}) < 0 \), then a merger between firms in \( K \) does not put the market at risk. Thus, the only mergers that could put the market at risk involve either (i) two firms outside \( K \), where the weakly higher cost is a maverick with respect to \( K \), or (ii) one supplier inside \( K \) and one outside supplier that is a maverick with respect to \( K \). ■

As Proposition 11 shows, the mergers that raise concerns of coordinated effects in the Cournot model correspond closely to those that involve a maverick, providing a foundation
for maverick-based merger review. Moreover, consistent with the results of (Salant et al., 1983; Perry and Porter, 1985), a market allocation by two symmetric firms in the Cournot model only pays off if the allocation scheme is all inclusive, that is, if there are only two firms. However, $K$ contains more than two firms, a non-all-inclusive allocation scheme can be profitable.

4.2. Tradeoffs between Unilateral Effects and Coordinated Effects

Our definition and measure of a market being at risk for coordinated effects brings to light tradeoffs between unilateral effects and coordinated effects. To see this, reconsider the efficient procurement market. Because an NR spread results in a spread of the affected suppliers’ market shares, without changing their total market share, an NR spread increases the Herfindahl-Hirschman Index (HHI), defined as the sum of the squared market shares, which is a widely used measure of merger effects.

As shown in Proposition 7, for the power-based parameterization, an NR spread applied to two firms within $K$ decreases $I_S(K)$. In this sense, an NR spread is good medicine because it decreases the risk of allocation. At the same time, an NR spread implies that $H_iH_j < G_iG_j$, that is, the distribution of the higher of the two cost draws after the spread first-order stochastically dominates the distribution of the higher of the two cost draws before the spread. This is illustrated in Figure 2, which shows a symmetry-reducing NR change from $(G_1, G_2)$ to $(H_1, H_2)$. The shaded rectangles below the NR curve show that for any symmetry-reducing NR change, $H_1H_2 < G_1G_2$. This means that the buyer’s expected price increases, and the more so the larger is the spread, where we say that $(\hat{H}_1, \hat{H}_2)$ is a larger NR spread $(G_1, G_2)$ than $(H_1, H_2)$ if $(\hat{H}_1, \hat{H}_2)$ is a NR spread of $(H_1, H_2)$. Thus, an NR spread is bad medicine if the firms bid competitively because it harms the buyer.

We summarize these results in the following proposition:

**Proposition 12.** An NR spread has the unilateral effect of increasing the HHI and the buyer’s expected price, with a larger NR spread resulting in both a larger increase in the HHI and a larger increase in the expected price.
Proof. See Appendix A.

Proposition 12 resonates with the idea that the presence of a supplier with a “dominant position” harms buyers and that greater dominance results in greater harm.\textsuperscript{43} It is noteworthy that the buyer harm from a decrease in symmetry identified in Proposition 12 arises irrespective of the sizes of the suppliers under consideration relative to their rivals—any NR spread increases the expected price.

Interestingly, while Proposition 12 shows that an NR spread increases the HHI, Proposition 7 shows that for the power-based parameterization, an NR spread applied to suppliers in $K$ reduces $I^S(K)$. In contrast to the positive and reaffirming news from Proposition 12 for the use of the HHI as in indicator of unilateral effects, the fact that a NR spread applied to two suppliers can decrease the coordinated effects index implies that a larger change in the HHI need not be indicative of an increase in risk for allocation as measured by the coordinated effects index.\textsuperscript{44} In other words, the HHI is not a reliable indicator of coordinated effects, except inversely so for some efficient procurement markets. This raises concerns given competition authorities’ historical reliance on the HHI as an indicator of coordinated effects.\textsuperscript{45}

The contrast between unilateral effects and the risk for coordinated effects extends to dynamic considerations. For example, Nocke and Whinston (2013) raise the question whether in a dynamic setting, a competition authority might have an incentive to block a merger with a large supplier in order to induce a merger with smaller supplier that is less harmful in terms of unilateral effects. Our results show that when considering the risk of coordinated effects, a competition authority might have an incentive to block a large firm from acquiring a smaller firm in order to induce it to instead acquire a larger firm, with a resulting lower coordinated effects index.

The inherent conflict between unilateral and coordinated effects is particularly salient in the context of merger remedies. For example, a competition authority considering requiring a divestiture as a condition for merger approval faces the dilemma that a divestiture that results
in relatively more symmetric post-merger suppliers reduces concerns of unilateral effects, but increases concerns of coordinated effects, and conversely for a divestiture that results in relatively less symmetric post-merger suppliers. Letting a “merger plus divestiture” be a transaction that takes two suppliers and reorganizes them to create two different suppliers:

**Corollary 3.** *A merger plus divestiture involving suppliers in K that results in an NR spread increases the HHI, but in an efficient procurement market characterized by the power-based parameterization and υ ≥ τ, it decreases I^S(K).*

This raises the interesting point that, although suppliers contemplating a merger might have an incentive to manipulate their market shares (for example by shifting sales from one merging party to the other) in hopes of improving their chances of surviving a merger review, a manipulation that reduces concerns of unilateral effects based on the HHI could increase concerns of coordinated effects based on the coordinated effects index.

5. Conclusion

Modeling coordinated effects as a market allocation scheme among a subset of suppliers, where only one of the suppliers in the subset is designated to be active in the market, we provide a framework that allows one to define and measure when a market is at risk for market allocation and to define mavericks. This analysis provides guidance for competition authorities regarding whether a merger raises concerns of coordinated effects. Mergers that create a symmetric set of large suppliers in a market with limited outside competition and buyers that lack significant buyer power and are not vertically integrated are of particular concern for putting markets at risk for allocation schemes. We show that the Cournot model provides a foundation for a maverick-based approach to mergers. This contrasts with procurement markets, where a merger involving a maverick need not put a market at risk because the acquisition of a supplier does not correspond to eliminating its productive assets.

The framework of this paper allows one to see where prior thinking related to coordinated effects was sound and where it was muddled, in some instances because it was, without
being explicit about it, switching between different models of the price-formation process. For example, merger guidelines promote the idea that buyer power lessens concerns of coordinated effects, and our framework concurs, but with the clarification that this is grounded in procurement-based thinking. At the same time, there is a view that mavericks play a critical role, but, as we show, that is grounded in Cournot-based thinking. Thus, another key contribution of our paper is to separate the overall framework for analyzing coordinated effects, which is general, from the model of the price-formation process, which is specific to a market.
Notes

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1. European Commission’s Guidelines on the Assessment of Horizontal Mergers (EC Guidelines), para. 22(b). Similarly, the U.S. Horizontal Merger Guidelines (U.S. Guidelines) recognize that a merger “can enhance market power by increasing the risk of coordinated, accommodating, or interdependent behavior among rivals” (p. 2). Similar guidance is provided by the Australian Competition and Consumer Commission’s Merger Guidelines.

2. Coordinated effects arguments have played a central role in U.S. merger cases such as Heinz/Beech-Nut, Anheuser-Busch InBev/Grupo Modelo, and H&R Block/TaxACT. For European cases, see Amelio et al. (2009) on ABF/GBI Business, Motta (2000) on Airtours/First Choice, and Aigner et al. (2006) on Sony/BMG and Impala and on the evolution of coordinated effects’ assessment in the EU.

3. The U.S. Department of Justice states that bid rigging is one of the most common violations that it prosecutes and that four basic schemes are involved in most bid-rigging conspiracies, all of which involve one bidder being designated to represent the participating firms: bid suppression, complementary bidding, bid rotation,
and customer or market allocation (U.S. DOJ, 2015).

4. As described in the European Commission’s decisions, allocation schemes were used by cartels in Choline Chloride, Copper Plumbing Tubes, Electrical and Mechanical Carbon and Graphite Products, Food Flavour Enhancers, Industrial and Medical Gases, Industrial Bags, Industrial Tubes, Methylglucamine, Monochloroacetic Acid, and Zinc Phosphate (Marshall and Marx, 2012, Table 6.1).

5. Stigler’s (1964) seminal paper on collusion takes as its starting point that oligopolists wish to collude to maximize profits, but that collusion is much more effective in some circumstances than in others, even to the point that it may be impossible. Stigler (1964, p. 47) notes that “the conditions appropriate to the assignment of customers will exist in certain industries, and in particular the geographical division of the market has often been employed.” Although Stigler’s focus is on the issue of secret price cutting, that is, incentive compatibility constraints, his point that conditions supporting an assignment of customers hold in some cases and not others continues to be true when one focuses, as we do, on the profitability of market allocations.

6. “One particular problem is that neither the theoretical nor empirical literature tells us much at all about whether the disappearance of a single firm through merger will increase the likelihood of coordination, other than, perhaps, in the extreme case where a merger reduces the number of firms in a market from three to two” (Kolasky, 2002, p. 7).

7. The particular concerns raised by mergers involving mavericks are discussed in, for example, the U.S., EC, and Australian merger guidelines, and they arise in many merger cases. Examples include the proposed acquisition of maverick T-Mobile by AT&T, the acquisition of maverick Northwest Airlines by Delta Airlines, and the proposed acquisition of maverick baby food maker Beech-Nut by Heinz. On mavericks in EC merger decisions, see Bromfield and Olczak (2018).

8. Antitrust officials have described a maverick as “a firm that declines to follow the industry consensus and thereby constrains effective coordination” (Kolasky, 2002, p. 7), while the U.S. Guidelines (p. 4) describe a maverick as “a firm that has often resisted otherwise prevailing industry norms to cooperate on price setting or other terms of competition.” Ivaldi et al. (2007, pp. 224, 228) define a maverick as “a firm that has a drastically different cost structure, production capacity or product quality, or that is affected by different factors than the other market participants” and “is thus unwilling to participate to a collusive action.” Kwoka (1989, p. 410) identifies a maverick as the relatively “more rivalrous” firm, and Ivaldi and Lagos (2017) take the view that a maverick is a small firm, while in de Roos and Smirnov (2019) a maverick is a fringe firm...
that disrupts coordination.

9. In an alternative approach, Kovacic et al. (2007a, 2009) and Gayle et al. (2011) view coordinated effects as analogous to incremental mergers among post-merger firms and propose quantifying coordinated effects by using existing merger simulation tools to model coordinated effects as incremental mergers.

10. Details of the repeated-game approach can be found in, for example, Ivaldi et al. (2007). In this context, the effects on collusion of having multiple markets, which could potentially be allocated along similar lines to the allocation scheme that we consider, are explored by, for example, Bernheim and Whinston (1990), Belleflamme and Bloch (2008), and Byford and Gans (2014).

11. Under the interpretation that an allocation scheme allocates one market to each supplier in $K$ with some probability, it is the expected payoff from participation that is relevant. Under the alternative interpretation that a supplier is allocated a share of a large number of individual geographic areas, products, or customers, then one need not take expectations for complete information models.

12. One might expect that repeated interaction could resolve compliance concerns, although that is outside the model that we consider.


14. We provide additional discussion and examples in Section 3.2.1. The notion of multiple mavericks arises in practice, for example, in the U.S. mobile communications market, prior to T-Mobile’s acquisition of MetroPCS, both were considered “‘mavericks’ with a history of disrupting the industry” (“Sprint CEO Sees ‘Enormous’ Synergies in T-Mobile Merger,” *Kansas City Star*, June 12, 2017).


16. The U.S. Department of Justice recently increased by more than 60% the size of its Procurement Collusion Strike Force, which is committed “combatting collusion, antitrust crimes and related fraudulent schemes, which undermine competition in government procurement, grant and program funding” (U.S. DOJ, Press Release 20-1230, November 12, 2020).
17. This is also the approach taken by, for example, Salant et al. (1983), Perry and Porter (1985), Waehrer (1999) Dalkir et al. (2000), and Loertscher and Marx (2019a).

18. For future research, one could incorporate additional merger-related cost synergies into the merged entity’s distribution along the lines laid out in Loertscher and Marx (2019a).

19. Whether the costs are realized before or after the designated bidder is selected in a market allocation is immaterial as long as this selection is independent of the realized costs.

20. In this case, $G_i$ is the Beta distribution with parameters $(1, \alpha_i)$.

21. If $G$ is the Beta distribution with parameters $(1, a)$, then $G(c)/g(c)$ is increasing in $c$ for all $c \in [\underline{c}, \overline{c}]$ if $a \geq 1$ and not if $a \in (0, 1)$. The market is not at risk, for example, when $n = 2$ and $G$ is the Beta distribution with parameters $(1, 0.3)$, in which case the distribution $G_{(1)}$ of $c_{(1:n)}$ is the Beta distribution with parameters $(1, 0.6)$, and we have $E_{c \sim G} \left[ c_{(2:n)} - c_{(1:n)} \right] = E_{c \sim G_{(1)}} \left[ G(c) / g(c) \right] \geq 0.2885$ and $1 - E_c[c] = \frac{0.3}{n+1} \geq 0.2308$. Hence, (1) is not satisfied.

22. This is reflected in headlines such as “Is 4-3 the New 3-2? FTC Continues to Target Pharmaceutical Mergers” (Bruce Sokler and Helen Kim, Mintz Levin Antitrust Alert, 2014, https://www.mintz.com/newsletter/2014/Advisories/3893-0414-NAT-AFR/index.html) and analysis of antitrust enforcement trends (see, for example, Kovacic et al., 2007b; Hawkins and King-Kafsack, 2014)).

23. To see that this argument (for example, by Baker, 2002, p. 180) does not apply for efficient procurement markets, consider a market with five firms drawing their cost types from the uniform distribution on $[0, 1]$. Then supplier 5 is a maverick with respect to participation in a market allocation by suppliers in $\{1, 2, 3\}$, that is, $\mathcal{T}^S(\{1, 2, 3\}; \{1, 2, 3, 4, 5\}) \leq 0$ and $\mathcal{T}^S(\{1, 2, 3\}; \{1, 2, 3, 4\}) > 0$, and following the merger of suppliers 1 and 2, the market is at risk for market allocation by the merged entity and supplier 3, that is, $\mathcal{T}^S(\{\mu, 3\}; \{\mu, 3, 4, 5\}) > 0$, where $\mu$ is the merged entity.

25. This suggests that one could quantify the extent of the maverickness by the difference in the coordinated effects index with and without the maverick, that is, one could measure the strength of a maverick \( m \) by 
\[ I_S(K; N\backslash\{m\}) - I_S(K; N). \]
As an example, \( n = 4, a_1 = 2, a_2 = 2, a_3 = 1, a_4 = 1 \): 2 is a maverick for \( K = \{1, 3\} \) and “more of a maverick” after 2 merges with 4.

26. The contagion effect would not be present if the conduct were efficient, as might be the case with explicit collusion involving the communication of private information and transfers.

27. See, for example, Compte et al. (2002), Kühn (2004), Vasconcelos (2005), and Ivaldi et al. (2007). For the contrasting view that asymmetries can facilitate collusion in some settings, see Ganslandt et al. (2012). Miklós-Thal (2008, 2011) show that cost asymmetries can facilitate collusion if transfer payments are feasible.

28. An NR spread applied to two suppliers in a market produces a market that is “more concentrated” according to Waehrer (2019). For example, if 
\[ G(c; \alpha) = 1 - (1-c)^\alpha, \]
we can construct an NR spread of \( (G(c; \alpha_1), G(c; \alpha_2)) \), where \( \alpha_1, \alpha_2 > 0 \), using 
\[ H_1(c) = G(c; \alpha_1 - \Delta) \text{ and } H_2(c) = G(c; \alpha_2 + \Delta) \text{ for } \Delta \in (0, \min\{\alpha_1, \alpha_2\}]. \]

29. The U.S. Guidelines (p. 27) state that the agencies “consider the possibility that powerful buyers may constrain the ability of the merging parties to raise prices.” This can occur, for example, ... if the conduct or presence of large buyers undermines coordinated effects.” U.S. Guidelines (p. 27) also state: “In some cases, a large buyer may be able to strategically undermine coordinated conduct, at least as it pertains to that buyer’s needs, by choosing to put up for bid a few large contracts rather than many smaller ones, and by making its procurement decisions opaque to suppliers.”

30. In contrast, the EC Guidelines raise the possibility that a merger could reduce buyer power “because a merger of two suppliers may reduce buyer power if it thereby removes a credible alternative” (EC Guidelines, para. 67).

31. Loertscher and Marx (2020b) provide a formalization of the “countervailing power” argument that a merger between suppliers might shift power away from the buyer and towards suppliers (see, for example, EC Guidelines (para. 11). and the Australian Merger Guidelines (paras. 1.4, 5.3, 7.48). A nuance on the view that mergers decrease buyer power is provided by Loertscher and Marx (2019a), who observe that, with symmetric suppliers, a merger increases the buyer’s incentive to become powerful.

32. Asker et al. (2019) quantify the contribution of market power to the misallocation induced by OPEC.

33. Because we allow the possibility that the densities are zero at \( c \) (and also possibly at \( \tau \)), define \( \Gamma_i(c) = \)
\( \lim_{c \to \tau} \Gamma_i(c) = \tau \). For \( x > \Gamma_i(\tau) \), we define \( \Gamma_i^{-1}(x) \equiv \tau \). An intuitive interpretation of the virtual cost function and an understanding of the role of its monotonicity can be developed using standard monopsony pricing. Consider a buyer with value \( v \leq \tau \) who faces a single supplier \( i \) who draws his cost from the distribution \( G_i \). The buyer’s pricing problem is \( \max_p (v - p) G_i(p) \), the first-order condition for which is \( 0 = g_i(p)(v - \Gamma_i(p)) \). If \( \Gamma_i \) is increasing, the second-order condition is satisfied if the first-order condition is satisfied (that is, the problem is quasi-concave).

34. In the Online Appendix, we demonstrate the tractability of the analysis of markets with buyer power through an application to the oilfield services market.

35. See the recently released “Vertical Merger Guidelines” of the U.S. Department of Justice and Federal Trade Commission.

36. See Loertscher and Marx (2020b) for a more detailed analysis of vertical integration and the efficiency of the price-formation process.

37. For our purposes, the derivation of \( a^* \) and its precise size, above and beyond \( a^* > 0 \), does not matter. It is the smallest number \( a \in [0, 1] \) such that an incentive compatible and individually rational mechanism that is based on the allocation rule that induces trade between the lowest-cost independent supplier \( i \) and the integrated firm if and only if \( \Phi_a(c_1) \geq \Gamma_a(c_i) \) does not run a deficit.

38. For example, the U.S. Guidelines (p. 27) describe the “ability and incentive to vertically integrate upstream” as a possible source of buyer power.

39. For the relevance of the linear-demand, constant-marginal-cost Cournot setup in an empirical context, see, for example, Igami and Sugaya (2019).

40. Farrell and Shapiro (1990) consider unilateral effects of mergers in a Cournot setup and provide conditions, which are satisfied in our Cournot setup with linear demand and constant marginal cost, under which a merger that involves no synergies must increase the price.

41. The role the assumption of constant marginal costs plays for this results is worth mentioning. As noted in McAfee and Williams (1992), in the Cournot model, a merger would no longer amount to the elimination of a supplier if, for example, supplier \( i \)’s cost function for producing quantity \( q_i \) were \( q_i^2 / \kappa_i \), where \( \kappa_i \) is \( i \)'s capacity. It that case, a merger of suppliers \( i \) and \( j \) would result in a supplier whose cost was only \( q_i^2 / (\kappa_i + \kappa_j) \). Related to this model of costs in a Cournot setup, see also Perry and Porter (1985); Farrell
and Shapiro (1990); Whinston (2006).

42. Indeed, the result that $H_i H_j < G_i G_j$ is implied by any symmetry reducing change, where a change in distributions for suppliers 1 and 2 from $(F_1, F_2)$ to $(H_1, H_2)$ is *symmetry reducing* if, for all $c \in [c, \bar{c}]$, we have $\max_i \{H_i(c)\} \geq \max_i \{F_i(c)\}$ and $\min_i \{H_i(c)\} \leq \min_i \{F_i(c)\}$, with strict inequalities for costs in an open subset of $[c, \min\{v, \bar{c}\}]$.

43. Regarding the possibility of a “significant impediment to effective competition,” the *EC Guidelines* (para. 2) states, “The creation or the strengthening of a dominant position is a primary form of such competitive harm.” See also Compte et al. (2002).

44. Igami and Sugaya (2019) show that when one measures coordinated effects based on the incentive compatibility constraint, it is again possible for the HHI and the incentive to collude to go in opposite directions.


46. For further analysis of mergers plus divestitures, see Cabral (2003), Vergé (2010), Vasconcelos (2010), and Loertscher and Marx (2019b).

47. See Grout and Sonderegger (2005) on the implications of buyer power for the ability of suppliers to sustain collusion. See Green et al. (2015) illustrating how explicit collusion can defeat the buyer power of even a small number of large strategic buyers.
A. Appendix: Proofs

Proof of Proposition 2. Under symmetry and \( v \geq \bar{v} \), we have \( r = \bar{v} \) and the definition of \( \Pi_i(N) \) implies that
\[
\Pi_i(N) = E_c [\bar{v} - c],
\]
and the definition of \( \Pi_i \) (and the payoff equivalence theorem) implies that
\[
\Pi_i = \frac{1}{n} E_c [c_{(2:n)} - c_{(1:n)}].
\]

Using symmetry, the market is at risk for all-inclusive allocation if and only if allocation based on symmetric selection probabilities increases the expected surplus for all suppliers, that is, for all \( i \in N \), \( \frac{1}{n} \Pi_i(N) > \Pi_i \), which holds if and only if
\[
E_c [\bar{v} - c] > E_c [c_{(2:n)} - c_{(1:n)}],
\]
which establishes (1).

Turning to the claim that (1) is satisfied if \( G(c)/g(c) \) is increasing, note that
\[
E_c [\bar{v} - c] = \int_0^1 (1 - c)g(c)dc = \int_0^1 G(c)dc = \int_0^1 \frac{G(c)}{g(c)}dG(c) = E_{c:G} \left[ \frac{G(c)}{g(c)} \right],
\]
and using \( G_{(1)}(c) \equiv 1 - (1 - G(c))^n \) and \( G_{(2)}(c) \equiv 1 - (1 - G(y))^n - nG(y)(1 - G(y))^{n-1} \),
\[
E_c [c_{(2:n)} - c_{(1:n)}] = \int_0^1 c \frac{dG_{(2)}(c)}{dc} dc - \int_0^1 c \frac{dG_{(1)}(c)}{dc} dc
= \int_0^1 (G_{(2)}(c) - G_{(1)}(c)) dc
= \int_0^1 nG(c)(1 - G(c))^{n-1} dc
= \int_0^1 \frac{G(c)}{g(c)} dG_{(1)}(c)
= E_{c:G_{(1:n)}} \left[ \frac{G(c)}{g(c)} \right].
\]
Because \( G_{(1:n)} \) is first-order stochastically dominated by \( G \), it follows that if \( \frac{G(c)}{g(c)} \) is increasing, then

\[
E_{c \sim G_{(1:n)}} \left[ \frac{G(c)}{g(c)} \right] < E_{c \sim G} \left[ \frac{G(c)}{g(c)} \right],
\]

and so (1) holds.

**Proof of Proposition 4.** We begin by stating and proving the following lemma, where we let

\[ A \equiv \sum_{j \in N} \alpha_j \quad \text{and} \quad A_X \equiv \sum_{j \in N \setminus X} \alpha_j. \]

**Lemma A.1.** For an efficient procurement market characterized by the power-based parameterization and \( v \geq \bar{v} \),

\[
\mathcal{I}^S(K) = 1 - \sum_{i \in K} \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A_{-(i)})(1 + A)}. 
\]

**Proof of Lemma A.1.** It is straightforward to show that for \( i \in K \),

\[
\Pi_i = \frac{\alpha_i}{(1 + A_{-(i)})(1 + A)} \quad \text{and} \quad \Pi_i(K) = \frac{\alpha_i}{(1 + \alpha_i + A_{-K})(1 + A_{-K})},
\]

and, thus,

\[
s_i(K) = \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A_{-(i)})(1 + A)}. 
\]

The expression for \( \mathcal{I}^S(K) \) then follows.

**Continuation of the proof of Proposition 4.** Using Lemma A.1, in the power-based parameterization,

\[
s_i(K; N) = \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A_{-(i)})(1 + A)},
\]

where we augment the arguments of the critical share to include the set of all suppliers. Let \( N = \{1, \ldots, n\} \) for some \( n \in \{3, 4, \ldots\} \). Let \( K = \{1, 2\} \) and assume that supplier \( m \in \{3, \ldots, n\} \) is a maverick with respect to \( K \). Define \( X \equiv \sum_{i \in N \setminus \{1,2,m\}} \alpha_i \). By the definition of a maverick,
\[ 1 - \sum_{i \in K} s_i(K; N \setminus \{m\}) > 0, \] which we can be written as
\[
\sum_{i \in \{1, 2\}} \frac{(1 + \alpha_i + X)(1 + X)}{(1 + \alpha_1 + \alpha_2 - \alpha_i + X)(1 + \alpha_1 + \alpha_2 + X)} - 1 < 0. \tag{7}
\]

Following the merger of suppliers 1 and \( m \), the coordinated effects index for suppliers in \( \hat{K} \equiv \{\mu_{1,m}, 2\} \), where \( \mu_{1,m} \) denotes the merged entity, is
\[
I^S(\hat{K}) = 1 - \frac{(1 + \alpha_1 + X + \alpha_m)(1 + X)}{(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)} - \frac{(1 + \alpha_2 + X)(1 + X)}{(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)}.
\]

It follows that
\[
\lim_{\alpha_2 \to 0} I^S(\hat{K}) = 1 - 1 - \frac{(1 + X)^2}{(1 + \alpha_1 + X + \alpha_m)^2} < 0,
\]
which proves the second part of the proposition.

Using the above expression for \( I^S(\hat{K}) \) and adding the expression on the left side of (7), which is negative, we have
\[
I^S(\hat{K}) > -\frac{(1 + \alpha_1 + X + \alpha_m)(1 + X)}{(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)} - \frac{(1 + \alpha_2 + X)(1 + X)}{(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)} + \frac{(1 + \alpha_1 + X)(1 + X)}{(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X)} + \frac{(1 + \alpha_2 + X)(1 + X)}{(1 + \alpha_1 + X)(1 + \alpha_1 + \alpha_2 + X)}
\equiv (1 + X)f(\alpha_1, \alpha_2, \alpha_m, X),
\]
where we factor out \((1 + X)\) and define \( f(\alpha_1, \alpha_2, \alpha_m, X) \) to be equal to the remainder. Thus, \( I^S(\hat{K}) > 0 \) if \( f(\alpha_1, \alpha_2, \alpha_m, X) > 0 \). Collecting the terms in \( f(\alpha_1, \alpha_2, \alpha_m, X) \) over the common denominator of
\[
(1 + \alpha_1 + X)(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X),
\]

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it follows that $I^S(\hat{K}) > 0$ if the associated numerator,

$$
\hat{f}(\alpha_1, \alpha_2, \alpha_m, X) \equiv -(1 + \alpha_1 + X)(1 + \alpha_1 + X + \alpha_m)^2(1 + \alpha_1 + \alpha_2 + X) \\
+(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_1 + X)^2 \\
-(1 + \alpha_2 + X)^2(1 + \alpha_1 + X)(1 + \alpha_1 + \alpha_2 + X) \\
+(1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_2 + X)^2(1 + \alpha_1 + X + \alpha_m),
$$

is positive. Differentiating with respect to $X$, it is straightforward to show that $\hat{f}(\alpha_1, \alpha_2, \alpha_m, X)$ is convex in $X$ and increasing at $X = 0$, which implies that for all $X \geq 0$, $\hat{f}(\alpha_1, \alpha_2, \alpha_m, X) \geq \hat{f}(\alpha_1, \alpha_2, \alpha_m, 0)$. Thus, it is sufficient to show that $\hat{f}(\alpha_1, \alpha_2, \alpha_m, 0) > 0$. Straightforward calculations show that $\hat{f}(\alpha_1, \alpha_2, \alpha_m, 0)$ is convex in $\alpha_2$ and is positive and increasing in $\alpha_2$ at $\alpha_2 = \alpha_1$, which implies that $\hat{f}(\alpha_1, \alpha_2, \alpha_m, 0) > 0$ for all $\alpha_2 \geq \alpha_1$, that is, as long as the acquiring supplier has the weakly smaller distributional parameter, completing the proposition.

\[\blacksquare\]

**Proof of Proposition 5.** In the limit as $n$ grows large, with probability 1, both the lowest and second-lowest order statistics are less than the reserve $r$. Thus, we can focus on the case in which the reserve does not bind, in which case $\Pi_i = \frac{1}{n}E[c(2:n) - c(1:n)]$ and $\Pi_i(K) = \frac{1}{n-k+1}E[c(2:n-k+1) - c(1:n-k+1)]$. Then we have

$$
I(K) = 1 - k \frac{\Pi_i}{\Pi_i(K)} = 1 - kA_n,
$$

where $A_n \equiv \frac{n-k+1}{n} \frac{E[c(2:n)-c(1:n)]}{E[c(2:n-k+1)-c(1:n-k+1)]}$. To complete the proof, we show that $\lim_{n \to \infty} A_n = 1$.

First, note that as shown by Loertscher and Marx (2020a, Lemma 1),

$$
\hat{f}E[c(1:n) - c(1:n)] = E \left[ \frac{G(c(1:n))}{g(c(1:n))} \right],
$$

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which, using $E \left[ \frac{G(c_{1,0})}{g(c_{1,0})} \right] = \int_\xi^\tau \ell G(x)(1 - G(x))^{\ell-1} dx$, implies that

$$A_n = \frac{\int_\xi^\tau G(x)(1 - G(x))^{n-1} dx}{\int_\xi^\tau G(x)(1 - G(x))^{n-k} dx} \leq 1. \quad (8)$$

The denominator in the expression for $A_n$ in (8) satisfies

$$\int_\xi^\tau G(x)(1 - G(x))^{n-k} dx = \int_\xi^\tau \left[ G(x)(1 - G(x))^{n-1} \right]^{\frac{n-k}{n-1}} G(x)^{\frac{k-1}{n-1}} dx \quad (9)$$

where the inequality uses Hölder’s inequality. Using (8) and (9), we have

$$A_n \geq \left( \frac{\int_\xi^\tau G(x)(1 - G(x))^{n-1} dx}{\int_\xi^\tau G(x) dx} \right)^{\frac{k-1}{n-1}}. \quad (10)$$

Letting $C \equiv 1/\max_{x \in [\xi, \tau]} g(x) > 0$, where the inequality uses the assumption that $g$ is bounded, we then have

$$\int_\xi^\tau G(x)(1 - G(x))^{n-1} dx = \int_0^1 u(1 - u)^{n-1} \frac{1}{g(G^{-1}(u))} du \geq \int_0^{1 - \frac{1}{n-1}} u(1 - u)^{n-1} \frac{1}{g(G^{-1}(u))} du \geq \int_0^{1 - \frac{1}{n-1}} u(1 - (n-1)u) \frac{1}{g(G^{-1}(u))} du \geq \int_0^{1 - \frac{1}{n-1}} u(1 - (n-1)u)C du = \frac{C}{6(n-1)^2},$$

where the first equality uses the change of variables $u = G(x)$, the first inequality reduces the upper bound of integration, the second inequality uses $(1 - u)^{n-1} \geq 1 - (n-1)u$ for all $n \geq 1$ and $u \in [0,1]$, the third inequality uses the definition of $C$, and the final equality
integrates. Putting these together, and using \( \lim_{n \to \infty} (n - 1)^{1/(n-1)} = 1 \), we have

\[
1 \geq A_n \geq \left( \frac{\int_{c_x}^{c} G(x) (1 - G(x))^{n-1} dx}{\int_{c_x}^{c} G(x) dx} \right)^{\frac{n-1}{n}} \geq \left( \frac{1}{n} \right)^{\frac{1}{n-1}} = 1,
\]

and so by the sandwich theorem, \( \lim_{n \to \infty} A_n = 1 \), which completes the proof. ■

**Proof of Proposition 6.** The proof of (a) follows straightforwardly from Lemma A.1. Considering (b), if \( n = k \), then the result holds trivially, so assume that \( n > k \). Without loss of generality, assume that \( K = \{1, \ldots, k\} \). Using Lemma A.1, we have

\[
\sum_{i \in K} s_i(K) = \sum_{i \in K} \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_i)(1 + A)}.
\]

Because we assume that \( 1 \in K \), we can rewrite this as

\[
\sum_{i \in K} s_i(K) = \sum_{i \in K \setminus \{1\}} \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_1)(1 + A)}.
\]

Suppose we deduct \( \varepsilon \in (0, \alpha_n) \) from \( \alpha_n \) and add \( \varepsilon \) to \( \alpha_1 \). Then the sum of all suppliers’ strength parameters, \( A \), remains unchanged, and we have

\[
\mathcal{T}^S(K; \alpha_1 + \varepsilon, \alpha_2, \ldots, \alpha_{n-1}, \alpha_n - \varepsilon)
= 1 - \left[ \sum_{i \in K \setminus \{1\}} \frac{(1 + \alpha_1 + A_{-K} - \varepsilon)(1 + A_{-K} - \varepsilon)}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{-K})(1 + A_{-K} - \varepsilon)}{(1 + A - \alpha_1 - \varepsilon)(1 + A)} \right].
\]

Differentiating with respect to \( \varepsilon \), we have

\[
-\frac{1}{1 + A} \left[ \sum_{i \in K \setminus \{1\}} \frac{-(1 + A_{-K} - \varepsilon)(1 + \alpha_i + A_{-K} - \varepsilon)}{1 + A - \alpha_i} + \frac{(1 + \alpha_1 + A_{-K})(A - \alpha_1 - A_{-K})}{(1 + A - \alpha_1 - \varepsilon)^2} \right],
\]

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which is positive. Thus, for all \( \varepsilon \in (0, \alpha_n) \),

\[
I_S(K; \alpha_1 + \varepsilon, \alpha_2, \ldots, \alpha_{n-1}, \alpha_n - \varepsilon) > I_S(K; \alpha_1, \alpha_2, \ldots, \alpha_{n-1}, \alpha_n).
\]  

(10)

If follows that \( I_S(K) \) increases when a member of \( K \) is replaced with a member of \( N \setminus K \) with a larger distributional parameter and that \( I_S(K) \) is maximized when its members are the set \( K \) of suppliers with the largest distributional parameters. This completes the proof of the first sentence of the proposition.

Turning to (c), let \( \beta \) be as given in the statement of the proposition. Because \( I_S(K) \) relies on the strength parameters of suppliers in \( N \setminus K \) only through the sum of those parameters, we have

\[
I_S(K; \alpha_1, \ldots, \alpha_k, \alpha_{k+1}, \ldots, \alpha_n) = I_S(K; \alpha_1, \ldots, \alpha_k, \beta_{k+1}, \ldots, \beta_{n-1}, A_{-K} - \sum_{i=k+1}^{n-1} \beta_i),
\]

where \( A_{-K} - \sum_{i=k+1}^{n-1} \beta_i > 0 \) by the assumptions on \( \beta \). But then, letting \( \varepsilon_i \equiv \beta_i - \alpha_i \geq 0 \) for \( i \in K \), we have

\[
I_S(K; \alpha_1, \ldots, \alpha_k, \beta_{k+1}, \ldots, \beta_{n-1}, A_{-K} - \sum_{i=k+1}^{n-1} \beta_i)
\]

\[
< I_S(K; \alpha_1 + \varepsilon_1, \ldots, \alpha_k + \varepsilon_k, \beta_{k+1}, \ldots, \beta_{n-1}, A_{-K} - \sum_{i=k+1}^{n-1} \beta_i - \sum_{i=1}^{k} \varepsilon_i)
\]

\[
= I_S(K; \beta_1, \ldots, \beta_k, \beta_{k+1}, \ldots, \beta_{n-1}, \beta_n),
\]

where the inequality uses the repeated application of (10) and the assumption that \( \varepsilon_i \) is strictly positive for at least one \( i \in K \), and where the final inequality uses the definition of \( \varepsilon_i \) and the assumption that \( \sum_{i \in N} \beta_i = A \). This completes the proof. ■

Proof of Proposition 7. Denote the two suppliers experiencing the NR spread as 1 and 2, with \( \{1, 2\} \subseteq K \). Let \( F_1 \) and \( F_2 \) be their distributions prior to the spread, with parameters
$f_1$ and $f_2$, and let $H_1$ and $H_2$ be their distributions after the spread, with parameters $h_1$ and $h_2$. Note that we abuse notation by using, for $i \in \{1, 2\}$, $f_i$ and $h_i$ to denote the parameters of the distributions $F_i$ and $H_i$, rather than their pdfs. Denote the parameters of suppliers other than 1 and 2 by $\alpha_j$ for $j \in \{3, ..., n\}$.

An NR spread in the power-based parameterization implies the existence of $a > 0$ such that

$$h_1 > f_1 \geq a \geq f_2 > h_2$$

and

$$h_1 + h_2 = 2a = f_1 + f_2.$$ 

Thus, $h_2 = 2a - h_1$ and $f_2 = 2a - f_1$. Let $s_i(K)$ be critical share of supplier $i \in K$ under $(F_1, F_2)$, and let $\hat{s}_i(K)$ be the critical share for supplier $i \in K$ under $(H_1, H_2)$. By Lemma 1, the shift from $(F_1, F_2)$ to $(H_1, H_2)$ decreases $I^S(K)$ if and only if it increases the sum of the critical shares of suppliers 1 and 2. In what follows, we show that $\hat{s}_1(K) + \hat{s}_2(K) - s_1(K) - s_2(K) > 0$.

Using Lemma A.1 and letting $A \equiv 2a + \sum_{i \in \{3, ..., n\}} \alpha_i$,

$$s_i(K) = \frac{(1 + f_i + A_{-K})(1 + A_{-K})}{(1 + A - f_i)(1 + A)} \quad \text{and} \quad \hat{s}_i(K) = \frac{(1 + h_i + A_{-K})(1 + A_{-K})}{(1 + A - h_i)(1 + A)}.$$

Consequently, the sign of $\hat{s}_1(K) + \hat{s}_2(K) - s_1(K) - s_2(K)$ is equal to the sign of

$$\frac{(1 + h_1 + A_{-K})}{(1 + A - h_1)} + \frac{(1 + h_2 + A_{-K})}{(1 + A - h_2)} - \frac{(1 + f_1 + A_{-K})}{(1 + A - f_1)} - \frac{(1 + f_2 + A_{-K})}{(1 + A - f_2)}.$$

Substituting $2a - h_1$ for $h_2$ and $2a - f_1$ for $f_2$ and collecting terms, we get

$$\frac{2(f_1 - h_1 - 2a)(h_1 - f_1)(1 + a + X)(2 + 2a + X)}{(1 + 2a - f_1 + X)(1 + f_1 + X)(1 + 2a - h_1 + X)(1 + h_1 + X)},$$

where $X \equiv A_{-(1,2)} + A_{-K}$, which is positive.
Proof of Proposition 9. From Myerson (1981), it is well known that in the optimal procurement, the buyer applies a supplier-specific reserve price to the supplier with the lowest virtual cost, where the reserve price applied to supplier \( i \) is \( \hat{r}_i \equiv \Gamma_i^{-1}(v) \). Because \( \Gamma_i(c) > c \) for \( c > \underline{c} \), it follows that \( \hat{r}_i < v \). The assumption that suppliers follow their weakly dominant strategies of reporting truthfully implies that if supplier \( i \) has the lowest virtual cost, then supplier \( i \) wins if \( c_i \leq \hat{r}_i \) and is paid (in the dominant strategy implementation) its threshold type of \( \min_{j \in N \setminus \{i\}} \{ \hat{r}_i, \Gamma_i^{-1}(\Gamma_j(c_j)) \} \). Otherwise, there is no trade. Consequently, when the buyer is powerful,

\[
\Pi_i = \mathbb{E}_c \left[ \max \left\{ 0, \min_{j \in N \setminus \{i\}} \{ \hat{r}_i, \Gamma_i^{-1}(\Gamma_j(c_j)) \} - c_i \right\} \cdot 1_{\Gamma_i(c_i) \leq \min_{j \in N \setminus \{i\}} \Gamma_j(c_j)} \right]
\]

and

\[
\Pi_i(K) = \mathbb{E}_c \left[ \max \left\{ 0, \min_{j \in N \setminus K} \{ \hat{r}_i, \Gamma_i^{-1}(\Gamma_j(c_j)) \} - c_i \right\} \cdot 1_{\Gamma_i(c_i) \leq \min_{j \in N \setminus K} \Gamma_j(c_j)} \right].
\]

The definitions of critical shares and of the coordinated effects index are then the same as in the case without buyer power.

With buyer power and symmetric suppliers, letting \( L_X(c) \) denote the distribution of the lowest cost among suppliers in \( X \), that is, \( L_X(c) = 1 - (1 - G(c))^{|X|} \), for all \( i \in K \),

\[
s_i(K) = \frac{\int_{\underline{c}}^{\hat{r}} (1 - L_{N \setminus \{i\}}(c)) G(c) dc}{\int_{\underline{c}}^{\hat{r}} (1 - L_{N \setminus K}(c)) G(c) dc}, \tag{11}
\]

If \( v \geq \Gamma(\bar{c}) \), then \( \hat{r} = \bar{c} \), and so the critical shares are not affected by buyer power. Focusing on the case with \( \hat{r} < \bar{c} \) and differentiating the expression in (11) with respect to \( \hat{r} \), we get an
expression with sign equal to the sign of

\[
(1 - L_{N\setminus\{i\}}(\hat{r})) \int_{c}^{\hat{r}} (1 - L_{N\setminus K}(c)) G(c) c - (1 - L_{N\setminus K}(\hat{r})) \int_{c}^{\hat{r}} (1 - L_{N\setminus\{i\}}(c)) G(c) dc
\]

\[
= (1 - G(\hat{r}))^{n-1} \int_{c}^{\hat{r}} (1 - G(c))^{n-k} G(c) dc - (1 - G(\hat{r}))^{n-k} \int_{c}^{\hat{r}} (1 - G(c))^{n-1} G(c) dc
\]

\[
= (1 - G(\hat{r}))^{2n-1-k} \int_{c}^{\hat{r}} \left\{ \left( \frac{1 - G(c)}{1 - G(\hat{r})} \right)^{n-k} - \left( \frac{1 - G(c)}{1 - G(\hat{r})} \right)^{n-1} \right\} G(c) dc.
\]

Because \( \frac{1 - G(c)}{1 - G(\hat{r})} > 1 \) for \( c < \hat{r} \) and because \( k \geq 2 \), which implies \( n - k < n - 1 \), it follows that the expression above is negative. Thus, critical shares are weakly decreasing in buyer power, and strictly so for \( v < \Gamma(\bar{c}) \), which completes the proof. ■

**Proof of Proposition 12.** Suppose we have an NR spread for suppliers 1 and 2, causing their distributions to change from \((G_1, G_2)\) to \((H_1, H_2)\) satisfying (4) and (5) for \( i = 1, j = 2 \), and all \( c \in [\underline{c}, \bar{c}] \), with (4) satisfied with strict inequalities for costs in an open subset of \([\underline{c}, \min\{v, \bar{c}\}]\). Letting \( p_1 \) and \( p_2 \) be the probabilities of trade for suppliers 1 and 2, respectively, prior to the NR spread and \( \hat{p}_1 \) and \( \hat{p}_2 \) be their probabilities after the NR spread, then \( \hat{p}_1 < \min\{p_1, p_2\} \leq \max\{p_1, p_2\} < \hat{p}_2 \). Because an NR spread affects neither the overall probability of trade nor the probability of trade of suppliers other than 1 and 2, we have \( p_1 + p_2 = \hat{p}_1 + \hat{p}_2 \). Letting \( \Delta \equiv \min\{p_1, p_2\} - \hat{p}_1 > 0 \), the change in HHI as a result of the NR spread is

\[
\hat{p}_1^2 + \hat{p}_2^2 - p_1^2 - p_2^2 = (\min\{p_1, p_2\} - \Delta)^2 + (p_1 + p_2 - (\min\{p_1, p_2\} - \Delta))^2 - p_1^2 - p_2^2
\]

\[
= 2\Delta (\max\{p_1, p_2\} - \min\{p_1, p_2\}),
\]

which is positive and increasing in \( \Delta \).

An NR spread affects the price that the buyer pays only in the event that one of suppliers 1 and 2 has the lowest cost and the other one has the second-lowest cost. Because the
distribution of their lowest cost is by construction not affected by the NR spread, all that is left to do is to compare the distribution of their second-lowest draw, which is \(G_1(c)G_2(c)\) without the spread and \(H_1(c)H_2(c)\) after the spread.

Take as given a \(c \in (c, \bar{c})\) in the open subset of \([c, \min\{v, \bar{c}\}]\) such that (4) is satisfied with strict inequalities. Let \(A = 1 - H_1(c), B = 1 - H_2(c), C = 1 - F_1(c), \) and \(D = 1 - F_2(c)\). Then we have (i) \(AB = CD\) and (ii) \(1 - A \geq 1 - C, 1 - D \geq 1 - B,\) with \(A, B, C, D \in (0, 1)\).

Consider the problem

\[
\max_{(A,B) \in [0,1]^2} (1 - A)(1 - B) \quad \text{s.t.} \quad AB = CD. \tag{12}
\]

Noting that \((1 - A)(1 - B) = (1 - \sqrt{AB})^2 - (\sqrt{A} - \sqrt{B})^2\), we can rewrite the problem as

\[
\max_{(A,B) \in [0,1]^2} (1 - \sqrt{CD}) - (\sqrt{A} - \sqrt{B})^2 \quad \text{s.t.} \quad AB = CD.
\]

Because the maximand is less than or equal to \(1 - \sqrt{CD}\) and equal to \(1 - \sqrt{CD}\) if and only if \(A = B\), we conclude that the unique solution to (12) is \(A = B = \sqrt{CD}\). Thus, for any \(A\) and \(B\) satisfying (i) and (ii) with \(B \neq \sqrt{CD}\), we have \((1 - A)(1 - B) < (1 - \sqrt{CD})^2\), which we can write as (dropping the argument \(c\)):

\[
H_1H_2 < \left(1 - \sqrt{(1 - G_1)(1 - G_2)}\right)^2 = G_1G_2 - G_1 - G_2 - 2\sqrt{(1 - G_1)(1 - G_2)} < G_1G_2.
\]

This establishes that the distribution of the second-lowest cost after the NR spread first-order stochastically dominates the distribution of the second-lowest cost prior to the spread, for all \(c \in [c, \bar{c}]\), \(H_1H_2 \leq G_1G_2\), with a strict inequality for costs in an open subset of \([c, \min\{v, \bar{c}\}]\).

Because the second-lowest cost determines the buyer’s price, this implies that the buyer’s expected price is higher under \((H_1, H_2)\) than under \((G_1, G_2)\).

Further, a larger NR spread, that is, a change from \((G_1, G_2)\) to \((\hat{H}_1, \hat{H}_2)\), where \((\hat{H}_1, \hat{H}_2)\) is an NR spread of \((H_1, H_2)\), implies a larger \(\Delta\), and so a larger increase in the HHI, and
also implies an increase in the buyer’s expected price relative to \((H_1, H_2)\). ■
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Figure Legends

Figure 1: $\mathcal{I}(K)$ for the Singh-Vives model of symmetric differentiated Bertrand competition

Figure 2: Illustration of a symmetry-reducing NR change from $(G_1, G_2)$ to $(H_1, H_2)$
(a) $\mathcal{I}(K)$ for $K$ containing 2 firms

(b) $\mathcal{I}(K)$ before and after merger

Figure 1: $\mathcal{I}(K)$ for the Singh-Vives model of symmetric differentiated Bertrand competition
Figure 2: Illustration of a symmetry-reducing NR change from \((G_1, G_2)\) to \((H_1, H_2)\)