Inefficiency of Collusion at English Auctions

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Abstract

In its attempts to deter and prosecute big rigging, U.S. antitrust authorities have focused on sealed-bid procurements, rather than on ascending-bid auctions. One possible justification for this focus is the idea, supported by the existing theoretical literature, that collusion creates inefficiency at sealed-bid auctions, but not at ascending-bid auctions. We show when there is no pre-auction communication and the collusive mechanism satisfies ex-post budget balance, collusion does affect efficiency. In particular, any collusive mechanism that increases cartel members’ expected payoffs relative to non-cooperative play results in inefficiency either in the allocation among cartel members or in the allocation between cartel and non-cartel bidders, or both.

Keywords: antitrust, bid rigging, bidding ring, cartel, ex-post budget balance

JEL Classification Codes: C72, D44, L41

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1 Introduction

Bid rigging is a violation of the Sherman Act for both auctions and procurements. Yet, the website posted by the Department of Justice (DoJ), “Price Fixing & Bid Rigging—They Happen: What They Are and What to Look For,”\(^1\) only provides guidance as to how to detect bid rigging at a sealed-bid procurement. The DoJ has prosecuted bid rigging at ascending-bid auctions, such as U.S. v. Seville Industrial Machinery, U.S. v. Ronald Pook (antiques), and District of Columbia v. George Basiliko, et al. (real estate); however, the emphasis of enforcement has been on sealed-bid procurements. In fact, according to the General Accounting Office (1990, p.4), from 1982 to 1988, over half of the criminal cases brought by DoJ’s Antitrust Division involved either price fixing or bid rigging in road construction or government procurement, which are typically sealed-bid procurements.

Despite the DoJ’s focus on sealed-bid procurements, a large amount of bidding in the United States occurs at auctions, especially ascending-bid auctions. For example, tobacco, timber, art, antiques, the assets of many bankrupt firms, and numerous other commodities are sold via ascending-bid auctions.\(^2\) Why would auctions like these warrant less attention from enforcement authorities than sealed-bid procurements? One possible answer relates to the legal issue of what evidence is required to establish a conspiracy. In a sealed-bid procurement, the bid of each bidder is known ex post, and if a player does not bid, then that is known. In contrast, at an ascending-bid auction, a bidder may appear inactive because at some point the bid exceeds his willingness to pay, even though he had every intention of participating competitively. In other words, the paper trail at a sealed-bid procurement is clearer than at an ascending-bid auction, potentially making prosecution easier at a sealed-bid than at an ascending-bid auction.

One can also present economic arguments for why antitrust authorities might not focus on ascending-bid auctions. For example, if collusion at an ascending-bid auction does not cause any inefficiency in the allocation of the object being sold, then an antitrust authority interested in promoting efficient outcomes might not concern itself with collusion.\(^3\) To see why collusion at an ascending-bid auction might not

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\(^{2}\) Our results for ascending-bid auctions also apply to procurements conducted via “reverse auctions,” where the bids decline until there is only one supplier willing to provide the commodity at the indicated price.

\(^{3}\) In this paper, we ignore the strategic behavior of the auctioneer or procurement agent that may
cause any inefficiency, note that a cartel can potentially secure a collusive gain if the colluding bidder with the highest value remains active up to his value (just as in his non-cooperative strategy) and all other cartel members bid zero. Since, in equilibrium, the non-colluding bidders remain active up to their values, the winner of the auction will be the bidder who values it most, which is efficient.

The idea that collusion does not create inefficiency at an ascending-bid auction is supported by the existing theoretical literature. For example, suppose bidders independently draw values (or costs) from some underlying distribution and these values are private information for each bidder. The collusive mechanisms described in the literature for an ascending-bid auction preserve efficiency (see, e.g., Robinson, 1985; Graham and Marshall, 1987; and Mailath and Zemsky, 1991). However, these collusive mechanisms assume that the colluding bidders communicate with one another, either directly or through a “center,” prior to the auction. Robinson (1985) assumes complete information among the cartel members, i.e., all cartel members credibly reveal their private information to one another prior to the auction, and shows that, given this type of pre-auction communication, it is incentive compatible for only the highest-valuing cartel member to bid at the auction. Thus, in this environment, an efficient collusive mechanism exists even in the absence of within-cartel transfer payments. Graham and Marshall (1987) relax the assumption of complete information among the cartel members. They show that if cartel members meet prior to the auction and make reports to a center that then recommends bids and assigns transfers, then there exists an efficient, ex-ante budget balanced collusive mechanism. Mailath and Zemsky (1991) consider a similar environment and show that there continues lead to increasingly inefficient outcomes as they attempt to thwart collusion. Specifically, optimal reserve prices that would be higher when bidders collude than when they act non-cooperatively are ignored in this paper. In addition, we ignore the long-run effect of depressed auction prices from collusion on the willingness of future sellers to bring items up for sale. Finally, for government auctions, such as timber sales, we ignore the effect of depressed prices from collusion in leading government authorities increasingly to use distortionary taxes to make up for the revenue shortfall.

In contrast, one might expect collusion at a sealed-bid procurement to generate inefficiency. For example, if bidders are symmetric, collusion can create asymmetries among the bidders, and thus can create inefficiency at the auction. In addition, if the distributions of the bidders are ordered, for example, by first-order stochastic dominance, then it is natural to think that the “strongest” bidders would collude, in which case collusion exacerbates the already existing asymmetry. For numerical calculations illustrating this point, see Marshall, et al. (1994).

In Graham and Marshall (1987), the transfer payments depend on the identity of the winner and price paid at the auction as well as the cartel members’ reports to the center; however, Marshall and Marx (2004) show the existence of an efficient mechanism satisfying ex-ante budget balance that relies only on the reports to the center.
to exist an efficient collusive mechanism under the stronger requirement of *ex-post*
budget balance when the cartel has the power to prevent all but one cartel member
from bidding at the auction.\(^6\)

In U.S. v. Seville Industrial Machinery, U.S. v. Ronald Pook, and District of
Columbia v. George Basiliko, et al., all of which involve collusion at ascending-bid
auctions, the collusive mechanism used by the cartel involved no communication prior
to the auction, except possibly to establish the identity of the cartel members or, in
the case of District of Columbia v. George Basiliko, et al., to designate a cartel
member who would then bid on behalf of the cartel. As stated in the 1988 decision
in U.S. v. Ronald Pook:\(^7\)

> When a dealer pool was in operation at a public auction of consigned
> antiques, those dealers who wished to participate in the pool would agree
> not to bid against the other members of the pool. If a pool member
> succeeded in purchasing an item at the public auction, pool members
> interested in that item could bid on it by secret ballot at a subsequent
> private auction (“knock out”) .... The pool member bidding the highest
> at the private auction claimed the item by paying each pool member
> bidding a share of the difference between the public auction price and
> the successful private bid. The amount paid to each pool member (“pool
> split”) was calculated according to the amount the pool member bid in
> the knock out.

A similar mechanism was used by the industrial machinery cartel of U.S. v. Seville
Industrial Machinery in the period after 1970,\(^8\) and by the real estate cartel of District
of Columbia v. George Basiliko, et al.\(^9\) Consistent with these examples, we assume
no pre-auction communication, except to establish the collusive mechanism.

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\(^6\) The transfer payments of Mailath and Zemsky (1991) depend only on the reports made by the
cartel members.


members of the industrial machinery cartel were given an opportunity to make vague indications of
interest prior to the auction, and then only the cartel organizer submitted bids at the auction based
on his educated guess about the likely high value for the object from among the ring members. This
mechanism was clearly inefficient.

\(^9\) See District of Columbia, ex rel. John Payton, Corporation Counsel v. George Basiliko, et al.,
that: “the defendants and the co-conspirators discussed and agreed ... not to compete with one
another to win the bid; selected a designated bidder to act for the conspirators ...; discussed and
In addition to having no pre-auction communication regarding cartel members’ values for the objects being sold, the collusive mechanisms used by the antiques, industrial machinery, and real estate cartels had the feature that they were ex-post budget balanced. Any collusive gain achieved was divided among the colluding bidders. Although collusive mechanisms such as that of Graham and Marshall (1987) assume the presence of an incentiveless agent who facilitates the actions of the cartel subject only to ex-ante budget balance, i.e., budget balance in expectation, in practice it would seem unlikely that there would exist such an agent ready to facilitate the felony actions of the conspirators. Thus, we assume ex-post budget balance.

In this paper, we consider collusive mechanisms for an ascending-bid auction that involve no pre-auction communication and that satisfy ex-post budget balance. We show that under these assumptions, contrary to the results of the previous literature, collusion at ascending-bid auctions does affect efficiency. In particular, any such collusive mechanism that increases cartel members’ expected payoffs relative to non-cooperative play results in inefficiency either in the allocation among cartel members or in the allocation between cartel and non-cartel bidders, or both.

We establish our result for the variant of the ascending-bid auction described by Milgrom and Weber (1982), which we refer to as a “button auction.” If we relax the assumption of no pre-auction communication, then the results of Mailath and Zemsky (1991) imply that an efficient, ex-post budget balanced mechanism exists if we allow the cartel to prevent all but one cartel member from bidding (allowing the cartel to have this power is not useful in the absence of pre-auction communication). If we relax the assumption of ex-post budget balance, then a modification of the collusive mechanism of Graham and Marshall (1987) produces efficient outcomes without pre-auction communication.10 Thus, both of our assumptions are crucial for our result.

agreed on specific payoffs that conspirators present would receive for not bidding, or discussed and agreed to hold a private, secret auction among themselves after the designated bidder won the public real estate auction ...; in many instances, held a secret auction in which the conspirators bid solely among themselves to acquire the property for a price higher than the price paid by the designated bidder at the public real estate auction and agreed to divide the difference between the public real estate auction price and the secret auction price by making payoffs among the conspirators; arranged by contract or other means for the secret auction winner to take title or ownership of the property; and made the payoffs that they had agreed to make.”

10 Replace pre-auction communication with the recommendation that cartel members should exit when the price reaches their value or when all non-cartel bidders have exited. After the auction, all ring members receive a fixed payment from the center (calculated to satisfy budget balance for the center in expectation). If a ring member wins the object for a price \( p \), then all ring members make reports to the center, the object is awarded to the ring member with the highest report, and that ring
To understand our result, first note that colluding bidders at a button auction can potentially achieve a collusive gain without creating inefficiency if each colluding bidder remains active at the auction until either the bid reaches his value or all non-colluding bidders exit. As soon as the last non-colluding bidder exits, all colluding bidders who are still active exit immediately. In this case, one of the colluding bidders wins the auction, but the colluding bidders still face the problem of how to allocate the object among themselves. We assume that the colluding bidders participate in a post-auction mechanism that determines this allocation.

As we show, setting up an efficient, ex-post budget balanced post-auction mechanism is not, itself, the problem. For example, a mechanism similar to that of Mailath and Zemsky (1991) can be used, or if bidders are symmetric, a mechanism similar to that of Cramton, Gibbons, and Klemperer (1987) can be used. The problem is that these mechanisms give positive expected payoff to some types of cartel members, and so they provide incentives for these cartel members to overbid at the auction in an attempt to acquire the object for the cartel and then benefit from the post-auction mechanism. This overbidding results in inefficiency in the allocation of the object between cartel bidders and non-cartel bidders.

The key to our non-existence result is that one cannot construct a post-auction mechanism that gives the cartel members higher expected payoff than non-cooperative play and that simultaneously gives colluding bidders the incentive to report truthfully to the post-auction mechanism and the incentive to bid at the auction in a way that guarantees that a cartel member wins the auction if and only if a cartel member has the highest value among all the bidders.

Myerson and Satterthwaite (1983) found the non-existence of an efficient, ex-post budget balanced, individually rational mechanism for the problem of bilateral exchange—one player owns an object and can sell it to another. We assume that a cartel member who wins the object purchases it for the cartel, and so a cartel member pays $p$ to the ring member who paid for the object at the auction and pays $\max\{0, r^{(2)} - p\}$ to the center, where $r^{(2)}$ is the second-highest report from among the ring members.

Cramton, Gibbons, and Klemperer (1987) show that when several partners jointly own an asset, with the ownership shares given exogenously, and when those partners have private values for sole ownership of the asset drawn from the same distribution, then there sometimes exists an incentive compatible, individually rational, ex-post budget balanced mechanism for allocating the asset to the highest-valuing partner. In our paper, when one of the ring members wins the object at the auction, one could think of the ring members as becoming joint owners in the object, with some ownership shares. Then the role of the post-auction mechanism would be the same as a mechanism to dissolve a partnership.
member who wins the object at the auction is not the “owner” of the object. In addition, whenever a cartel member wins the object, we require that all cartel members participate in the post-auction mechanism.\footnote{Even though we do not require that participation in the post-auction mechanism be individually rational, we are able to prove a non-existence result. Adding such a constraint would further limit the set of feasible mechanisms, and so our result would continue to hold.} Because we do not require individual rationality for participation in the post-auction mechanism, Myerson and Satterthwaite’s impossibility result does not apply—in the overall collusive mechanism, gains from collusion at the auction may provide a subsidy for the post-auction mechanism. Furthermore, as described in Section 3, it is, in fact, possible to construct a post-auction mechanism with the properties we desire. So the questions we are addressing are (1) whether a post-auction mechanism can be constructed that does not interfere with efficiency at the auction, and (2) whether the expected gains from collusion in terms of reducing the price paid at the auction more than offset any expected losses associated with the post-auction mechanism.

Perhaps one way to pose the conundrum addressed in this paper is as follows: If an item is sold by an auction that is known to be efficient, and the item is awarded to a colluding bidder who participates in a cartel that will use an efficient auction to determine ultimate ownership of the item, then it seems reasonable to conjecture that the allocation of the item will be efficient. We show that this is not the case.

Our result has an obvious implication for antitrust policy. The result implies that common collusive mechanisms for ascending-bid auctions result in inefficient outcomes. Since the promotion of efficiency is the fundamental premise of U.S. antitrust laws, ascending-bid auctions warrant attention from enforcement authorities.

The remainder of the paper is organized as follows: Section 2 describes the model. Section 3 states our main result. In Section 4, we explore whether the inefficiency of profitable collusive mechanisms can be attributed to behavior by ring members at the auction or to behavior by ring members at the post-auction mechanism, and we provide examples of profitable, but inefficient, collusive mechanisms. In Section 5 we conclude.
2 Definitions and Model

We consider a single-object auction within a heterogeneous IPV framework with a non-strategic seller. Bidders are risk neutral, and bidder $i$ independently draws a value $v_i$ from a distribution $F_i$ that has continuous density with support $[0, \bar{v}]$. We assume bidders 1 and 2 are members of a bidding ring. We use RM1 to denote ring member 1, and RM2 to denote ring member 2. Restricting attention to only two ring members simplifies some of the proofs by limiting the combinations of bidding strategies for the ring members that can achieve efficiency. Bidders not in the ring, which we refer to as the outside bidders, bid non-cooperatively. Since our results rely only on the behavior of the highest-valuing outside bidder, we can without loss of generality assume only one outside bidder, denoted bidder 3.

We consider the following “button auction.”\(^{13}\) The price of the object rises continuously from zero. Each bidder is “active” until he “exits.” A bidder can exit at any price. Once a bidder exits, he cannot reenter. The auctioneer and all bidders observe each bidder’s decision to exit. When only one active bidder remains, the price stops increasing, and the object is awarded to the last active bidder at the price at which the second-to-last bidder exited.

So that our analysis can proceed assuming continuous prices, we make two technical assumptions on how the button auction operates. First, if all three bidders are active and the outside bidder exits, then we assume the price stops momentarily and the ring members are given the opportunity to exit at the same price. This assumption eases the analysis because it rules out the case in which the outside bidder exits at price $p$ and then both ring members want to exit at a price infinitesimally greater than $p$. Second, we assume that if all three bidders are active and the outside bidder exits simultaneously with one or both of the ring members, then only the exit decision of the outside bidder is recognized, and, as before, the price stops momentarily and the ring members are given the opportunity to exit at the same price. Under this assumption, if RM1 observes that the outside bidder exits at $p$, it is still possible that RM2 attempted to exit simultaneously at price $p$. This means that the set of feasible reports for RM2 at the post-auction mechanism is a closed interval, which prevents problems with the existence of an optimal report for RM2.

We consider the following game: First, a collusive mechanism is announced to

\(^{13}\)See, e.g., Milgrom and Weber (1982) and Izmalkov (2002).
the ring members (there is credible commitment to the mechanism). The mechanism includes (i) a non-binding recommendation for bidding behavior by ring members at the auction, and (ii) a specification of a post-auction mechanism to be used in the event that one of the ring members is awarded the object. We refer to this type of mechanism as one without pre-auction communication because, even though there is communication about the mechanism itself, there is no communication regarding ring members’ private information about their values. Second, the button auction is held. We assume that the outside bidder bids according to his weakly dominant strategy of remaining active up to his value. Third, if a ring member wins the object at the auction, that ring member pays the auctioneer and brings the object to the post-auction mechanism. At the post-auction mechanism, both ring members report their values. Based both on these reports and on the outcome of the auction, the mechanism determines the allocation of the object and specifies transfers between the ring members. If the outside bidder wins the object, then the post-auction mechanism does not operate and there are no transfers between ring members.\textsuperscript{14}

We say that a post-auction mechanism is \textit{ex-post budget balanced} if for all reports made by ring members, the sum of transfers received by the ring members is equal to zero. We say that a collusive mechanism is \textit{efficient} if, when that collusive mechanism is in place, the object is awarded to the highest-valuing bidder (whether ring member or outside bidder) for all value realizations.

\section{Main Result}

We are interested in whether an efficient, ex-post budget balanced collusive mechanism exists for a button auction in the absence of pre-auction communication.

We begin with the observation that in any efficient mechanism, both ring members must participate in the auction. To see this, note that if the ring used a mechanism that involved one ring member bidding on behalf of the ring, then since there is no communication prior to the auction, the designated ring bidder would have no way to know the maximum value among the ring members, and so the mechanism would not be efficient.

\textsuperscript{14}This assumption is not required for our main result, but it simplifies the proofs in Section 4. The assumption is consistent with the collusive mechanisms used in U.S. v. Seville Industrial Machinery and U.S. v. Ronald Pook.
Let $\beta_i(v)$ be the price at which ring member $i$ with value $v$ exits the auction, conditional on the other ring member and the outside bidder being active at that price (different bid functions apply if the other ring member or the outside bidder has already exited). We assume that $\beta_i$ is a measurable function. Our first lemma shows that efficiency of the collusive mechanism requires that a ring member with value zero exit immediately and that ring members with values greater than zero not exit immediately.

**Lemma 1** In the absence of pre-auction communication, in any efficient collusive mechanism, for all $i \in \{1, 2\}$, $\beta_i(v_i) = 0$ if and only if $v_i = 0$.

*Proof.* See the Appendix.

Lemma 1 implies that, in the absence of pre-auction communication, in any efficient collusive mechanism for a button auction, if $v_1 = 0$, RM1 exits the button auction immediately and reveals that $v_1 = 0$. When this happens, and when $v_2 > 0$, there are two possible continuations. First, if $v_2 < v_3$, then an outside bidder wins the object. Second, if $v_3 \leq v_2$, then RM2 wins the object and is awarded the object by the post-auction mechanism. The next lemma establishes that in this second case, the post-auction mechanism involves no transfers between ring members.

**Lemma 2** In the absence of pre-auction communication, in any efficient, ex-post budget balanced collusive mechanism, if for $i \in \{1, 2\}$, $0 = v_{3-i} < v_3 < v_i$, then RM$i$ wins the object at the button auction and the post-auction mechanism leaves the object with RM$i$ and specifies no transfers between ring members.

*Proof.* See the Appendix.

To understand Lemma 2, suppose RM2 has value zero and note that by Lemma 1, RM2 must exit at a price of zero. Once RM2 exits, efficiency requires that RM1 remain active up to his value. If RM2 expects to make a payment to RM1 or receive a payment from RM1, then, using ex-post budget balance, there must exist value realizations for RM1 and the outside bidder such that RM1 wins the object and makes a non-zero transfer to RM2. But then there must also exist value realizations for RM1 and the outside bidder such that RM1 has an incentive to distort his bid, either exiting below his value to avoid having to make a payment to RM2 or staying
active above his value to capture a payment from RM2. This distortion of RM1’s bidding behavior potentially creates an inefficiency.

Using Lemma 2, we can prove the following result.

**Proposition 1** In the absence of pre-auction communication, there does not exist an efficient, ex-post budget balanced collusive mechanism that increases the expected payoff to the ring members relative to non-cooperative play.

*Proof.* Let $Q_i(v_i)$ and $M_i(v_i)$ denote RM$i$’s interim probability of getting the object and the interim expected payment, respectively, from participation in the collusive mechanism, and let $U_i(v_i) = v_i Q_i(v_i) - M_i(v_i)$. By standard arguments (see, e.g., McAfee and McMillan, 1992), incentive compatibility implies that for all $v_i \in [0, \bar{v}]$,

$$U_i(v_i) = U_i(0) + \int_0^{v_i} Q_i(x) dx. \quad (1)$$

Efficiency of the mechanism implies $Q_i(v_i) = Q_i^*(v_i) \equiv F_{3-i}(v_i) F_3(v_i)$. Note that under non-cooperative play, the interim probability function is also $Q_i^*$. Let $M_i^{nc}(v_i)$ denote RM$i$’s interim expected payment under non-cooperative play, and let $U_i^{nc}(v_i) = v_i Q_i^*(v_i) - M_i^{nc}(v_i)$. It follows that for all $v_i \in [0, \bar{v}]$, $U_i(v_i) - U_i^{nc}(v_i) = U_i(0) - U_i^{nc}(0) = U_i(0) = -M_i(0) = 0$, where the first equality uses (1), the second equality uses $M_i^{nc}(0) = 0$, the third equality uses the definition of $U_i$, and the last equality uses Lemma 2. Q.E.D.

Proposition 1 provides a general result on the effect of collusive mechanisms of the type we consider on allocative efficiency. If we let $q_i(v_1, v_2, v_3)$ be the probability with which bidder $i$ receives the object given the values of the three bidders, then by revenue equivalence $q_1$, $q_2$, and $q_3$ determine the bidders’ expected payoffs up to a constant. The proposition shows that if $q$ is the efficient allocation, $q^*$, then payoffs must be the same as under non-cooperative play. However, the proposition does not identify whether a collusive mechanism that increases ring members’ expected payoffs would necessarily have inefficiency for the outside bidder, i.e., $q_3 \neq q_3^*$, or perhaps only inefficiency among the ring members, i.e., $q_3 = q_3^*$ but $q_i \neq q_i^*$ for $i \in \{1, 2\}$.

In the next section, we consider whether the inefficiency of a profitable collusive mechanism is due to inefficiency in the allocation of the good between ring and non-ring bidders or due to inefficiency in the allocation determined by the post-auction
4 Source of Inefficiency

Proposition 1 implies that the presence of a bidding ring that uses a post-auction mechanism to reallocate the good among ring members affects efficiency. We now explore whether the inefficiency can be attributed to behavior by ring members at the auction or to behavior by ring members at the post-auction mechanism. To do this, it is useful to focus on collusive mechanisms in which both ring members use the same exit strategy \( \beta_1 = \beta_2 = \beta \), so we do this in Section 4.1 below. Then in Section 4.2, we consider the case of asymmetric bid functions. In Section 4.3, we provide an example showing that (necessarily inefficient) mechanisms exist that increase the ring members’ payoffs above non-cooperative play. And in Section 4.4, we characterize the mechanism that maximizes ring members’ expected payoffs conditional on there being no inefficiency in the post-auction mechanism.

4.1 Symmetric Bid Strategies

As shown in Lemma 3 below, when the ring members use symmetric bid functions, efficiency requires that if both ring members and the outside bidder are active, then a ring member exits when the price reaches his value. Without pre-auction communication, the ring members must use their bids at the auction to communicate their values, and the only symmetric way for them to do this that does not distort efficiency is for them to stay active up to their values as long as the outside bidder is active.

Lemma 3 In the absence of pre-auction communication, in any efficient collusive mechanism with \( \beta_1 = \beta_2 = \beta \), for all \( v \in [0, \bar{v}] \), \( \beta(v) = v \).

Proof. See the Appendix.

Consider what happens if both ring members are active when the last outside bidder exits at price \( p \) (a positive probability event given Lemma 3). In this case, if the ring members’ values are \( v_1 \) and \( v_2 \), we let \( x_i(v_1, v_2 \mid p) \) be the transfer received by ring member \( i \) in the post-auction mechanism, net of the payment of \( p \) by the ring member to whom the object is assigned to the ring member who paid for the object at
the auction. Note that ex-post budget balance implies that \( \sum_{i=1}^{2} x_i(v_1, v_2 \mid p) = 0 \). We assume that the expectation \( E_{v_{-i}}[x_i(v_i, v_{-i} \mid p) \mid v_{-i} \geq p] \) exists and is a measurable function of \( v_i \) for all \( v_i \in [0, \bar{v}] \).

Consider an example in which a ring member is awarded the object at a price \( p \). Suppose the post-auction mechanism is incentive compatible (and that it is not the “null” mechanism requiring no transfers and not reallocating the object), in which case a ring member’s expected transfer must be decreasing in his report (he must expect to pay more or receive less, the higher is his report). If the post-auction mechanism is such that both ring members’ expected transfers from making the lowest possible report, \( p_i \), are non-positive, then, since the expected transfers must be decreasing functions of the ring members’ reports, one can show that there must exist realizations of the ring members’ reports such that ex-post budget balance is violated. Thus, in order to maintain proper incentives at the post-auction mechanism, at least one ring member must receive a positive expected transfer at the post-auction mechanism when it has value \( p \). This intuition is formalized in Lemma 4 below.

**Lemma 4** In the absence of pre-auction communication, in any efficient, ex-post budget balanced collusive mechanism with \( \beta_1 = \beta_2 \), unless the post-auction mechanism never reallocates the object, for all \( p \in (0, \bar{v}) \), there exists \( i \in \{1, 2\} \) such that \( E_{v_{-i}}[x_i(p, v_{-i} \mid p) \mid v_{-i} \geq p] > 0 \).

**Proof.** See the Appendix.

Lemma 4 focuses on incentives for truthful reporting at the post-auction mechanism. As an illustration of Lemma 4, suppose both ring members were active when the last ring member exited at price \( p \), and suppose a ring member won the object at price \( p \). A possible post-auction mechanism is one based on Mailath and Zemsky (1991): Each ring member makes a report. The highest-reporting ring member gets the object and pays \( p \) to the ring member who paid for the object at the auction, and the transfer payment received by ring member \( i \) is \( \int_p^{r-i}(v-p)f_{-i}(v)dv - \int_p^{r-i}(v-p)f_i(v)dv \). This mechanism is incentive compatible,\(^{15}\) ex-post budget balanced, and efficient.\(^{16}\)

\(^{15}\)Ring member \( i \) chooses his report to solve \( \max_{r_i}(v_i - p)F_{-i}(r_i) + \int_p^{r-i}(v-p)f_i(v)dv - \int_p^{r-i}(v-p)f_{-i}(v)dv \), which has first-order condition \( (v_i - p)f_{-i}(r_i) - (r_i - p)f_i(r_i) = 0 \).

\(^{16}\)The mechanism is also individually rational in the following sense. Suppose that when a ring member wins an object at the auction, that ring member immediately hands the object over to a ring center, who reimburses the ring member for the price paid at the auction. Then ring members
However, the expected payoff to a ring member who reports $p$ is positive—he has zero probability of winning the object and gets a payment of $\int_p^{v_i}(v - p)f_i(v)dv > 0$. Thus, when we look at the incentives for truthful bidding at the auction, we may find that ring members have an incentive to overbid to make sure they get to participate in the post-auction mechanism.

Now move from Lemma 4, which focuses on incentives at the post-auction mechanism, to incentives for truthful bidding at the auction. Suppose that the bidding at the main auction has reached 100, and the outside bidder as well as both ring members are still active. If RM1 has value of 100 and exits from the bidding at 100, RM1’s expected transfer is zero (see Lemma 2). If RM1 stays active for a small increment beyond 100, say to 101, then there is some chance that the outside bidder will exit, and a ring member will win the object. If, in this case, RM1 earns a positive payoff by reporting $p$ at the post-auction mechanism, then such a deviation may be profitable for RM1. (To see this, note that if RM2 exits between 100 and 101, then RM1 can quickly exit from the bidding, limiting the probability that RM1 wins the object at a price greater than his value.) Thus, if a ring member can earn a strictly positive payoff simply by reporting $p$ at the post-auction mechanism, a ring member will sometimes have an incentive to bid more than his value at the auction, violating Lemma 3. This intuition is formalized in Lemma 5 below.

Note that Lemma 4 above tells us that incentive compatibility at the post-auction mechanism implies that a ring member reporting a value of $p$ must expect a positive transfer at any meaningful post-auction mechanism. However, Lemma 5 below implies that, to the contrary, incentive compatibility at the auction implies that a ring member reporting a value of $p$ must expect a non-positive transfer at the post-auction mechanism.

**Lemma 5** In the absence of pre-auction communication, in any efficient, ex-post budget balanced collusive mechanism with $\beta_1 = \beta_2$, for all $p \in (0, \bar{v})$ and $i \in \{1, 2\}$, $E_{v_{-i}}[x_i(p, v_{-i} | p) | v_{-i} \geq p] \leq 0$.

*Proof.* See the Appendix.

Lemmas 4 and 5 imply that the requirements for incentive compatibility at the auction and at the post-auction mechanism are in conflict. The only post-auction can decide whether to participate in the post-auction mechanism or not. Ring members will always choose to participate in the mechanism proposed.
mechanism satisfying both lemmas is one that does not reallocate the object among
the ring members, in which case efficiency requires that each ring member bid up to
his value at the auction, and so there is no collusive gain.

These results show that it is neither the auction nor the post-auction mecha-
nism that is necessarily inefficient, but that when the two are combined, incentive
compatibility for one prevents incentive compatibility at the other.

4.2 Asymmetric Bid Strategies

Lemmas 3–5 focus on the case of symmetric bid strategies for the ring members. In
this case, one reason collusion results in inefficiency is that there is positive probab-
ility that the outside bidder will exit prior to both ring members. In this case, Lemma
4 implies that a ring member with the lowest value must receive a positive expected
transfer, and this distorts the incentives for truthful bidding at the auction. If the
ring members use asymmetric bid strategies, one can essentially eliminate the case
addressed in Lemma 4 in which both ring members must report to the post-auction
mechanism. In particular, one ring member could exit essentially immediately, sig-
nalling his value to the other ring member through the price at which he exited. Then
the other ring member could remain active up to the maximum of his value and the
value of the ring member who exited.\textsuperscript{17}

We know from Proposition 1 that this type of signalling strategy does not eliminate
the inefficiency. In this section, we provide the intuition for why this is the case.

Consider the following bidding strategies: RM2 stays active until $\varepsilon v_2$, where $\varepsilon > 0$
is infinitesimally small. RM1 exits at zero if $v_1 = 0$, but otherwise RM1 remains active
until RM2 exits, and then remains active up to $\max\{v_1, v_2\}$. Given these strategies,
RM2 exits prior to RM1 and, with very high probability, prior to the outside bidder.
For the purposes of providing intuition, we assume that RM2 exits prior to the outside
bidder with probability one. Given these strategies and our assumption, RM1 and
the post-auction mechanism learn RM2’s value when RM2 exits. Then RM1 wins
the object whenever $\max\{v_1, v_2\}$ is greater than or equal to the value of the outside
bidder.

Consider the following post-auction mechanism. If RM1 wins the object at a price
$p$, he pays $p$ to the auctioneer and brings the object to the post-auction mechanism.

\textsuperscript{17}We thank an anonymous referee for this idea.
If \( p \geq v_2 \), RM1 receives the object and there are no transfers between ring members. If \( p < v_2 \), RM1 makes a report to the post-auction mechanism. If his report \( r_1 \) is less than \( v_2 \), then RM2 receives the object and reimburses RM1 for the cost \( p \) of the object and pays RM1 an amount \( g(v_2) \geq 0 \) that depends upon RM2’s revealed value. If RM1’s report is greater than or equal to \( v_2 \), then RM1 receives the object and pays \( v_2 - p \) to RM2 and RM2 pays \( g(v_2) \) to RM1.

Given this post-auction mechanism, it is incentive compatible for RM1 to remain active until RM2 exits and then bid up to \( \max \{v_1, v_2\} \) at the auction. And it is incentive compatible for RM1 to report truthfully to the post-auction mechanism—RM1’s payoff if he reports \( r_1 < v_2 \) is \( g(v_2) \), and his payoff if he reports \( r_1 \geq v_2 \) is \( v_1 - v_2 + g(v_2) \). One can show that if \( g(v_2) = \frac{1}{f_3(v_2)} \int_0^{v_2} (1 - F_1(x)) F_3(x)dx \), then it is also incentive compatible for RM2 to exit at \( \varepsilon v_2 \), truthfully revealing his value to RM1.\(^{18}\)

We have now constructed an incentive compatible, efficient, ex-post budget balanced collusive mechanism; however, RM2’s expected payoff from participation in this collusive mechanism is no greater than his expected payoff from non-cooperative play. To see this, note that RM2’s expected payoff under noncooperative play is \( \int_0^{v_2} F_1(x)F_3(x)dx \), but if we evaluate the expression for RM2’s expected payoff from footnote 18 at \( r_2 = v_2 \) and substitute in the expression for \( g(v_2) \), we find that RM2’s expected payoff under the collusive mechanism is also \( \int_0^{v_2} F_1(x)F_3(x)dx \). Thus, in an environment in which RM2’s value is revealed to RM1 prior to the post-auction mechanism, there is no way for RM2 to do better than under non-cooperative play. In effect, forcing RM2 to reveal his value allows RM1 to extract all the surplus generated through collusion.

### 4.3 Example

It remains to be shown whether there exists any ex-post budget balanced collusive mechanism for a button auction that increases the ring members’ expected payoffs

\(^{18}\)To see this, suppose RM2 exits at \( \varepsilon r_2 \) and RM1 wins the object at a price \( p < r_2 \). Then RM2 receives the object if \( v_1 < r_2 \) and has payoff \( v_2 - p - g(r_2) \), but if \( v_1 \geq r_2 \), RM2 does not receive the object and has payoff \( r_2 - p - g(r_2) \). So RM2’s expected payoff from exiting at \( \varepsilon r_2 \) when his value if \( v_2 \) is \( F_1(r_2) \int_0^{r_2} (v_2 - v_3) f_3(v_3)dv_3 + (1 - F_1(r_2)) \int_0^{v_2} (r_2 - v_3) f_3(v_3)dv_3 - g(r_2)F_3(r_2) \). Incentive compatibility requires that this expression be maximized with respect to \( r_2 \) when \( r_2 = v_2 \). The first-order condition defines a differential equation in the function \( g \), which we can solve. In particular, using \( g(v_2) \geq 0 \) to determine the constant of integration, we get \( g(v_2) = \frac{1}{f_3(v_2)} \int_0^{v_2} (1 - F_1(x)) F_3(x)dx \).
relative to non-cooperative play. By Proposition 1, such a mechanism must necessarily induce inefficient outcomes. As an illustration, the following example provides a simple environment in which one can easily construct an ex-post budget balanced collusive mechanism that allows ring members to increase their expected payoffs relative to non-cooperative play. But, of course, by Proposition 1 the mechanism is not efficient. In this particular example, there are inefficiencies both in the allocation of the object among ring members and in the allocation between ring members and outside bidders.

Suppose there is one outside bidder who has value $\frac{1}{4}$ with probability 1, and suppose that the two ring bidders draw their values from the uniform distribution on $[0, 1]$.

Assume, as above, that the outside bidder exits when the price reaches his value.

Consider a collusive mechanism that recommends that both ring members remain active at the auction up to a price of $\frac{1}{4}$ regardless of their values. More precisely, consider the following bidding strategies for the ring members: 1. If a ring member’s value is less than or equal to $\frac{1}{4}$, then that ring member exits when the outside bidder exits or when the price reaches $\frac{1}{4}$, whichever comes first. 2. If a ring member’s value is greater than $\frac{1}{4}$, then that ring member exits when the outside bidder exits or when the price reaches the ring member’s value, whichever comes first.

Consider the following post-auction mechanism: Each ring member submits a report $r_i \in \{0, \frac{1}{8}\}$. The ring member with the higher report wins the object and pays the amount of his report to the other ring member and pays the price paid at the auction ($\frac{1}{4}$ in equilibrium) to the ring member who paid for the item at the auction. If the ring members’ reports are the same, the object is allocated at random to one of the ring members, who then pays the amount of his report to the other ring member.

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19 Although our earlier results assume all bidders draw their values from distributions with the same support, the results extend to the case in which the outside bidder’s distribution does not have the same support as those of the ring members.

20 More precisely, consider the following bidding strategies for the ring members: 1. If a ring member’s value is less than or equal to $\frac{1}{4}$, then that ring member exits when the outside bidder exits or when the price reaches $\frac{1}{4}$, whichever comes first. 2. If a ring member’s value is greater than $\frac{1}{4}$, then that ring member exits when the outside bidder exits or when the price reaches the ring member’s value, whichever comes first.

21 Assuming that a deviation from equilibrium play at the auction does not affect ring members’ beliefs in the continuation game (we have the flexibility to specify off-equilibrium beliefs in this way), then a ring member cannot profitably deviate at the auction.

22 For completeness, assume that if the outside bidder exits at a price greater than $\frac{1}{4}$, then there is no post-auction mechanism.
and pays the price paid at the auction to the ring member who paid for the item at the auction.

In the equilibrium of the post-auction mechanism, each ring member reports zero if his value is less than or equal to $\frac{1}{2}$, and reports $\frac{1}{2}$ otherwise.\footnote{To see this, assume that RM2 uses this reporting strategy. Then RM1’s expected payoff if he reports zero to the post-auction mechanism is $\frac{1}{2} (v_1 - 1/4) \Pr (v_2 \leq 1/2) + \frac{1}{8} \Pr (v_2 > 1/2) = v_1/4$. And, RM1’s expected payoff if he reports $\frac{1}{2}$ to the post-auction mechanism is $(v_1 - 1/4 - 1/8) \Pr (v_2 \leq 1/2) + 1/8 ((v_1 - 1/4 - 1/8) \Pr (v_2 > 1/2) = 3v_1/4 - 1/4$. Thus, RM1 prefers to report zero if and only if $\frac{1}{2} v_1 \geq 3v_1/4 - 1/4$, i.e., if and only if $v_1 \leq 1/2$, and similarly for RM2.}

Given the equilibrium of the post-auction mechanism, ring member $i$ has expected payoff from participation in the cartel of $\frac{1}{2} v_i$ if $v_i \leq \frac{1}{2}$, and $\frac{3}{4} v_i - \frac{1}{4}$ otherwise. In contrast, ring member $i$’s expected payoff if he does not participate in the cartel is his expected payoff from non-cooperative play at the auction, which is zero if $v_i \leq \frac{1}{4}$ and $(v_i - \frac{1}{4}) \frac{1}{4} + \int_{\frac{1}{4}}^{v_i} (v_i - x) dx = \frac{v_i^2}{2} - \frac{1}{32}$ if $v_i > \frac{1}{4}$. A ring member with value $v_i \in [0, \frac{1}{4}]$ prefers participation if $\frac{1}{4} v_i > 0$, which holds for $v_i > 0$. A ring member with value $v_i \in (\frac{1}{4}, \frac{1}{2}]$ prefers participation if $\frac{1}{4} v_i > \frac{v_i^2}{2} - \frac{1}{32}$, which always holds for $v_i \in (\frac{1}{4}, \frac{1}{2}]$. And a ring member with value $v_i > \frac{1}{2}$ prefers participation if $\frac{3}{4} v_i - \frac{1}{4} > \frac{v_i^2}{2} - \frac{1}{32}$, which always holds for $v > \frac{1}{2}$. Thus, ring members with positive values strictly prefer participation in the ring over non-cooperative play.

Together, the two ring members have a joint expected payoff from non-cooperative play of $\frac{9}{32}$.\footnote{This is calculated as $\int_{\frac{1}{2}}^{\frac{1}{2}} \int_{0}^{\frac{1}{2}} (y - x) 2dxdy + \int_{\frac{1}{2}}^{\frac{1}{2}} \int_{0}^{\frac{1}{2}} (y - \frac{1}{2}) 2dxdy$.} But as members of a cartel with the collusive mechanism described above, the two ring members have a larger joint expected payoff of $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$.\footnote{This is calculated as $\int_{\frac{1}{2}}^{\frac{1}{2}} \int_{0}^{\frac{1}{2}} y^2dxdy + \frac{1}{2} (\int_{\frac{1}{2}}^{\frac{1}{2}} \int_{0}^{\frac{1}{2}} x2dxdy + \int_{\frac{1}{2}}^{\frac{1}{2}} \int_{0}^{\frac{1}{2}} y2dxdy) + \frac{1}{2} (\int_{0}^{\frac{1}{2}} \int_{0}^{\frac{1}{2}} x2dxdy + \int_{0}^{\frac{1}{2}} \int_{0}^{\frac{1}{2}} y2dxdy) - \frac{1}{4}$.} If the collusive mechanism always allocated the object to the highest valuing ring member, then the two ring members would have joint expected payoff of $\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$, which is even larger. The difference $\frac{2}{3} - \frac{5}{8} = \frac{1}{24}$ represents losses to the cartel associated with not allocating the object to the highest-valuing ring member.

In addition to the possibility of an inefficient allocation among the ring members, the collusive mechanism induces inefficiency in the overall allocation because the object is allocated to one of the ring members even when both ring members’ values are less than $\frac{1}{4}$, which is the value of the outside bidder. This occurs with probability $\frac{1}{16}$ in this example.

In the presence of this collusive mechanism, the auctioneer’s revenue is $\frac{1}{4}$. Under
non-cooperative play, the auctioneer’s revenue is \(\frac{37}{96}\), which is greater than \(\frac{1}{4}\). Thus, the auctioneer loses \(\frac{37}{96} - \frac{1}{4} \approx 0.14\) in expected revenue as a result of collusion.

### 4.4 Optimal Collusion

The collusive mechanism of Section 4.3 shows that collusive mechanisms exist that increase ring members’ expected payoffs above non-cooperative play. The mechanism in that example is inefficient both in its allocation between the two ring members and in its allocation between the ring and non-ring bidders. But, as discussed following Lemma 4, it is possible to construct ex-post budget balanced post-auction mechanisms that are efficient. In this section, we construct the optimal collusive mechanism, in the sense of maximizing the expected payoff of the ring members, conditional on there being no inefficiency in the post-auction mechanism.

For the purposes of this section, we assume the bidders are symmetric, drawing their values from distribution \(F\). In this case, we can restrict attention without loss of generality to symmetric mechanisms.\(^{27}\)

One can show that the optimal post-auction mechanism has the following form. If RM1 exits prior to RM2 and the outside bidder, then RM2 remains active up to his value; if RM2 wins the auction, then RM2 receives the object and there are no transfers between ring members, i.e., there is no post-auction mechanism, and similarly reversing the roles of RM1 and RM2. If the outside bidder exits prior to both ring members at price \(p\), then the ring members exit immediately and participate in a post-auction mechanism. We can assume that the ring member who pays the auctioneer receives half of the price paid from the other ring member as the initial part of the post-auction mechanism. Then ring members report their values, and the ring member with the higher report receives the object and makes a transfer to the losing ring member. Incentive compatibility is guaranteed if, when ring member \(i\) has the higher report, he pays

\[
\frac{1}{2} \int_{\beta^{-1}(p)}^{r_i} x \left( F(x) - F\left( \beta^{-1}(p) \right) \right) f(x) dx,
\]

where \(\beta^{-1}(p)\) is defined to be zero for \(p \geq \beta(0)\).\(^{28}\) This payment rule can be implemented by

\begin{align*}
\text{\textsuperscript{26}} & \text{This can be calculated as } \frac{1}{4} \int_{0}^{1} \int_{0}^{2} 2xdxdy + \int_{\frac{1}{3}}^{1} \int_{0}^{2} 2xdxdy + \int_{0}^{\frac{1}{3}} \int_{0}^{2y} 2ydydy = \frac{37}{96}. \\
\text{\textsuperscript{27}} & \text{A short proof for this is in footnote 11 of Maskin and Riley (1984).} \\
\text{\textsuperscript{28}} & \text{For example, if } F \text{ is the uniform distribution on } [0,1], \text{ if ring member } i \text{ has the higher report at the post-auction mechanism, he pays } \\
& \begin{cases} 
\frac{1}{4}(3p + 4r_i - 1), & \text{if } p \geq \frac{1}{3} \\
\frac{3p - 1}{2}, & \text{if } p < \frac{1}{3} 
\end{cases} \text{ to the other ring member,} \\
\text{where the lowest possible report is } \beta^{-1}(p) = \begin{cases} 
\frac{3p - 1}{2}, & \text{if } p \geq \frac{1}{3} \\
0, & \text{if } p < \frac{1}{3} 
\end{cases}
\end{align*}
having the ring members bid for the object, with the high bidder winning and paying the amount of his bid to the loser.

As shown in Lemma 4, the requirement that the post-auction mechanism be efficient implies that ring members will have an incentive to overbid at the auction. The following lemma confirms this, calculating the ring members’ bidding strategies for the case when both ring members and the outside bidder are active.

**Proposition 2** Assuming symmetry, in the optimal collusive mechanism conditional on the post-auction mechanism being efficient, \( \beta(v) = v + \frac{1}{\beta v} \left( \frac{1-F(t)}{1-F(v)} \right)^2 dt. \)

*Proof.* See the Appendix.

To be more concrete, assume \( F \) is the uniform distribution on \([0, 1]\). Then Lemma 2 implies that \( \beta(v) = \frac{1}{3} + \frac{2}{3} v \), which is greater than \( v \) for \( v \in [0, 1) \). Thus, a ring member remains active past his value as long as the outside bidder and other ring member are active. This implies that the ring members win the object when the outside bidder has the highest value, which is inefficient, whenever \( \max \{v_1, v_2\} < v_3 < \min \{\beta(v_1), \beta(v_2)\} \), which has probability \( \frac{1}{54}. \)

The seller’s expected revenue when facing a cartel using this mechanism is \( \frac{71}{162}. \) In contrast, the seller’s expected revenue with three bidders under non-cooperative play is \( \frac{1}{2} \). So collusion reduces the seller’s expected revenue by \( \frac{1}{2} - \frac{71}{162} = \frac{5}{84}. \)

The mechanism described here is the optimal collusive mechanism conditional on an efficient post-auction mechanism, but it is not necessarily the optimal collusive mechanism. For example, it might be possible to increase the ring members’ expected payoffs by using an inefficient post-auction mechanism that reduced overbidding by the ring members at the auction. At the extreme, a post-auction mechanism that randomly allocated the object could be used to eliminate overbidding completely, but such a mechanism would be highly inefficient and not optimal. We leave as an open question the characterization of the optimal collusive mechanism.

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29 This can be calculated as \( 2 \int_0^1 \int_{v_2}^{\beta(v_2)} \int_{v_2}^{v_3} dv_1 dv_2 dv_3. \)

30 Considering the cases in which \( v_2 < v_1 \), we have 1. \( v_2 < v_1 < \beta(v_2) < v_3, p = \beta(v_2) \); 2. \( v_3 < \beta(v_2) \) (and \( v_2 < v_1 \), \( p = v_3 \); 3. \( v_2 < \beta(v_2) < v_3 < v_1, p = v_3 \); 4. \( v_2 < \beta(v_2) < v_1 < v_3, p = v_1 \).

Calculating the seller’s expected revenue in these cases, we get \( \int_0^1 \int_{v_2}^{\beta(v_2)} \int_{v_2}^{v_3} dv_1 dv_2 dv_3 + \int_0^{v_1} \int_0^{\beta(v_2)} \int_{v_2}^{v_3} dv_1 dv_2 dv_3 + \int_0^{v_1} \int_{\beta(v_2)}^{v_1} \int_{v_2}^{v_3} dv_1 dv_2 dv_3 \) = \( \frac{1}{27} + \frac{1}{12} + \frac{4}{84} + \frac{4}{84} \). Multiplying this by two to account for the possibility that \( v_2 > v_1 \), we get \( \frac{71}{162}. \)
5 Conclusion

English auctions have the nice property that their outcomes are generally efficient. However, using Milgrom and Weber’s (1982) model of an English auction, we show that if a cartel at an English auction is restricted to use an ex-post budget balanced mechanism and if the cartel members do not communicate information regarding their values prior to the auction, then the presence of the cartel necessarily introduces inefficiency in the overall allocation. The inefficiency in the allocation introduced may be one of three types: inefficiency in the allocation among cartel members, inefficiency in the allocation between cartel members and non-cartel members, or some combination of the two.

Our result potentially has broader implications. For example, the literature on bilateral exchange typically takes the initial ownership of the object to be traded as fixed. But our result suggests that even when a mechanism exists that guarantees efficient exchange, that mechanism may create incentives at the stage when the initial ownership of the object is established that lead to inefficiency in the overall allocation. As another example, Cramton, Gibbons, and Klemperer (1987) provide a mechanism that can dissolve a partnership efficiently once the partnership shares are fixed, but our result suggests that in some environments this mechanism may introduce inefficiency at the stage when partnership shares are determined.

The fact that collusion at an English auction can result in inefficiency in the allocation between cartel members and non-cartel members is relevant from the perspective of antitrust enforcement since authorities might be less concerned if the only inefficiency introduced by collusion was in the allocation of the object among cartel members. Instead, we show that the inefficiency introduced by collusion may result in an “innocent” non-colluding bidder not receiving an object that he would have in the absence of collusion.

The auctioneer was not a player in our game. An increase in the reserve price by the auctioneer in response to collusion would lead to more inefficiency. But, in addition, auctioneers at ascending-bid auctions sometimes attempt to rein-in a cartel by using strategies such as a “quick knock,” perhaps with the help of a “protecting bidder.” The quick knock involves awarding the item to an outside bidder while cartel bidders are still willing to bid higher, and a protecting bidder is a shill for the auctioneer who can be awarded the item. The intent is to destabilize the cartel with
allocations that are intentionally adverse to the cartel. Of course, these strategies introduce their own inefficiencies.
A Appendix—Proofs

Proof of Lemma 1. We first show that \( \beta_i(0) = 0 \). Suppose that \( \beta_1(0) > 0 \) and \( \beta_2(0) > 0 \). If \( v_1 = v_2 = 0 < v_3 < \min\{\beta_1(0),\beta_2(0)\} \), then a ring member wins the object even though the outside bidder has the highest value, which is inefficient. Suppose \( \beta_2(0) = 0 < \beta_1(0) \). We must specify RM1’s exit strategy in the event that \( v_1 = 0 \) and RM1 observes that RM2 exits at price \( p \in [0,\beta_1(0)) \) while the outside bidder remains active. Let \( \hat{\beta}_1(p) \) denote RM1’s exit strategy in this case, and note that \( \hat{\beta}_1(p) > p \). If \( v_1 = v_2 = 0 < v_3 < \hat{\beta}_1(0) \), then RM1 wins the object, even though the outside bidder has the highest value, which is inefficient. A similar argument holds if \( \beta_1(0) = 0 < \beta_2(0) \). Thus, \( \beta_1(0) = \beta_2(0) = 0 \).

We now show that \( \beta_i(v_i) = 0 \) implies \( v_i = 0 \). Suppose there exists \( i \in \{1,2\} \) and \( v'_i > 0 \) such that \( \beta_i(v'_i) = 0 \). If \( v_{3-i} = 0 < v_3 < v_i = v'_i \), then the outside bidder wins the object even though RM\( i \) has the highest value, which is inefficient. Q.E.D.

Proof of Lemma 2. Suppose that \( 0 = v_2 < v_3 < v_1 \). By Lemma 1, RM2 exits at a price of zero, revealing his value to be zero. In the continuation of the button auction, efficiency requires that RM1 exit at a price equal to his value. If RM1 wins the object, he wins at a price of \( v_3 \). For \( i \in \{1,2\} \), let \( t_i(v_1,v_3) \) denote the transfer to bidder \( i \) when RM2 exits at a price of zero, RM1 has value \( v_1 \), and RM1 wins the object at price \( v_3 \). At the post-auction mechanism, efficiency requires that bidder 1 receive the object, and so incentive compatibility of the post-auction mechanism requires that \( t_1(v_1,v_3) \) be independent of \( v_1 \). By ex-post budget balance, \( \forall v_1 \in [v_3,\bar{v}] \), \( t_1(v_1,v_3) = -t_2(v_1,v_3) \equiv \tau(v_3) \). Thus, RM1’s payoff if it wins the object at price \( v_3 \) is \( v_1 + \tau(v_3) - v_3 \).

When RM2 exits at price zero, his expected transfer is \( \hat{t}_2 \equiv E_{v_3}[-\tau(v_3) \mid v_3 \leq v_1] \) \( \Pr[v_3 \leq v_1] \). Suppose that \( \hat{t}_2 > 0 \). Then there exists \([v'_3,\bar{v}'_3]\) such that \( \tau \) is continuous on \([v'_3,\bar{v}'_3]\) and \( \forall v_3 \in [v'_3,\bar{v}'_3] \), \( \tau(v_3) > 0 \). Let \( X \equiv \min_{v_3 \in [v'_3,\bar{v}'_3]} \tau(v_3) \) and note that \( X > 0 \). Suppose RM1 has value \( v_1 = v'_3 \) and observes that RM2 exits at zero, the current price is \( v'_3 \), and the outside bidder remains active at price \( v'_3 \). Efficiency requires that RM1 exit at price \( v'_3 \), in which case his payoff is zero. But RM1 can increase his expected payoff by remaining active until a price of \( \min\{v'_3 + X, v''_3\} \) since, if the outside bidder exits at a price \( p \in (v'_3,\min\{v'_3 + X, v''_3\}) \), which is a positive probability event, RM1’s payoff is \( v'_3 + \tau(p) - p \geq v'_3 + X - p > 0 \).
Similarly, if $\hat{t}_2 < 0$, there exists $[v'_3, v''_3]$ such that $\tau$ is continuous on $[v'_3, v''_3]$ and $\forall v_3 \in [v'_3, v''_3]$, $\tau(v_3) < 0$. In this case, let $X \equiv \min_{v_3 \in [v'_3, v''_3]} (-\tau(v_3))$ and note that $X > 0$. Suppose RM1 has value $v_1 = v''_3$ and observes that RM2 exits at zero, the current price is $v'_3$, and the outside bidder remains active at price $v'_3$. Efficiency requires that RM1 remain active until price $v''_3$. But if the outside bidder exits at a price $p \in (\max \{v'_3, v''_3 - X\}, v''_3)$, which is a positive probability event, then RM1 wins the object and has payoff $v''_3 + \tau(p) - p \leq v''_3 - X - p < 0$. Thus, RM1 can increase his expected payoff by exiting at $\max \{v'_3, v''_3 - X\}$.

Thus, we conclude that when a ring member exits at price zero, his expected transfer from the post-auction mechanism must be zero. Q.E.D.

**Proof of Lemma 3.** Recall that we assume that the outside bidder exits when, and only when, the price reaches his value. Suppose there exits $\hat{v}$ such that $\beta(\hat{v}) > \hat{v}$. If $v_1 = v_2 = \hat{v}$ and if the outside bidder has value $v_3 \in (\hat{v}, \beta(\hat{v}))$, then one of the ring members wins the object at a price $v_3$, which is greater than both ring members’ values and so is inefficient. Similarly, if $\beta(\hat{v}) < \hat{v}$, and if $v_1 = v_2 = \hat{v}$ and the outside bidder has value $v_3 \in (\beta(\hat{v}), \hat{v})$, then the outside bidder wins the object at a price $\beta(\hat{v})$, when efficiency requires that one of the ring members win the object. Thus, if $\beta_1(v) = \beta_2(v) = \beta(v)$, then $\beta(v) = v$. Q.E.D.

**Proof of Lemma 4.** To begin, note that when both ring members are active when the outside bidder exits, bidder $i$’s report to the post-auction mechanism must solve (assuming the post-auction mechanism allocates the object to the highest-reporting ring member):

$$\max_{r_i} E_{v_{-i}} [x_i(r_i, v_{-i} | p) | v_{-i} \geq p] + (v_i - p) \Pr [r_i \geq v_{-i} | v_{-i} \geq p] .$$

Suppose that $p < \bar{v}$ and that the first term in the maximand above is non-decreasing in $r_i$ for some $\hat{r}_i \geq p$. Then incentive compatibility of the post-auction mechanism requires that the second term be non-increasing in $r_i$ at $\hat{r}_i$, which fails to hold for $v_i > p$ if the post-auction mechanism does, in fact, allocate the object to the highest-reporting ring member. Thus, it must be that the first term in the maximand is strictly decreasing in $r_i$ for all $p < \bar{v}$ and $r_i \geq p$.

Suppose there exists $p \in (0, \bar{v})$ such that for all $i \in \{1, 2\}$, $E_{v_{-i}} [x_i(p, v_{-i} | p) |
\( v_{-i} \geq p \) \leq 0. Then for all \( v_i > p \), \( E_{v_{-i}} [ x_i(v_i, v_{-i} | p) | v_{-i} \geq p ] < 0 \). Thus,

\[
E_{v_1, v_2} [ x_1(v_1, v_2 | p) + x_2(v_1, v_2 | p) | v_1, v_2 \geq p ] < 0,
\]

which implies the existence of some \( \hat{v}_1, \hat{v}_2 \geq p \) such that \( x_1(\hat{v}_1, \hat{v}_2 | p) + x_2(\hat{v}_1, \hat{v}_2 | p) < 0 \), contradicting ex-post budget balance. Thus, we conclude that for all \( p \in (0, \hat{v}) \), there exists \( i \in \{1, 2\} \) such that \( E_{v_{-i}} [ x_i(p, v_{-i} | p) | v_{-i} \geq p ] > 0 \). Q.E.D.

**Proof of Lemma 5.** To begin, note that arguments as in the proof of Lemma 2 imply that if RM2 exits while RM1 and the outside bidder remain active, then RM2’s expected transfer is zero (and similarly reversing the roles of RM1 and RM2). If it were not, then there would exist values such that RM1 would either exit prior to his value to avoid having to make a payment to RM2 or exit after his value in order to receive a payment from RM2.

Assume an efficient, ex-post budget balanced, incentive compatible collusive mechanism. Suppose there exists \( \hat{p} \in (0, \hat{v}) \) such that \( E_{v_2} [ x_1(\hat{p}, v_2 | \hat{p}) | v_2 \geq \hat{p} ] > 0 \). We assume there exists \( \varepsilon > 0 \) sufficiently small such that \( \varepsilon < \hat{p} \) such that \( E_{v_2} [ x_1(p, v_2 | p) | v_2 \geq p ] \) is continuous in \( p \) for \( p \in [\hat{p} - \varepsilon, \hat{p}] \). Then there exists \( \hat{\varepsilon} > 0 \) such that \( \hat{\varepsilon} < \hat{p} \) and such that for all \( p \in [\hat{p} - \hat{\varepsilon}, \hat{p}] \), \( E_{v_2} [ x_1(p, v_2 | p) | v_2 \geq p ] > 0 \).

Letting

\[
X \equiv \min_{p \in [\hat{p} - \hat{\varepsilon}, \hat{p}]} E_{v_2} [ x_1(p, v_2 | p) | v_2 \geq p ],
\]

\( X \) is well defined and \( X > 0 \).

Suppose \( v_1 = \hat{p} - \hat{\varepsilon} \) and that both ring members and the outside bidder are active at price \( \hat{p} - \hat{\varepsilon} \). If RM1 exits at price \( \hat{p} - \hat{\varepsilon} \), his expected payoff is zero.

Consider the strategy for RM1: (i) if no bidders exit prior to price \( \hat{p} \), then exit at price \( \hat{p} \); (ii) if the outside bidder exits prior to (or simultaneously with) the other ring member at a price \( p' \in [\hat{p} - \hat{\varepsilon}, \hat{p}] \), which occurs with positive probability denoted \( \rho > 0 \), then exit at the same price and report a value of \( p' \) at the post-auction mechanism; (iii) if the other ring member exits prior to the outside bidder at price \( p' \in [\hat{p} - \hat{\varepsilon}, \hat{p}] \), then exit at price \( p' + \delta (\hat{p} - p') \), where \( \delta > 0 \) is small.

In case (i), the RM1’s expected payoff is zero. In case (ii) the object is allocated between the ring members by the post-auction mechanism. Using Lemma 3 and \( \hat{p} < \hat{v} \), RM1 believes that by reporting value \( p' \) at the post-auction mechanism, he will win the object with probability zero. Thus, his expected payoff from the post-
auction mechanism in this case is bounded below by $X > 0$. In case (iii), if RM1 exits prior to the outside bidder, i.e., $p' + \delta (\hat{p} - p') < v_3$, his payoff is zero, but if the outside bidder exits first, i.e., $p' + \delta (\hat{p} - p') > v_3$, RM1 wins the object at a price of $v_3$ and so RM1’s payoff is $v_1 - v_3 = \hat{p} - \hat{\epsilon} - v_3 > -\hat{\epsilon}$. Let $\rho'(\delta)$ denote the probability with which case (iii) occurs and the outside bidder exits prior to RM1. Note that $\lim_{\delta \downarrow 0} \rho'(\delta) = 0$. Thus, RM1’s expected payoff from the deviation (given his information when the price is $\hat{p} - \hat{\epsilon}$) is bounded below by $\rho X - \rho'(\delta) \hat{\epsilon}$, which is positive for $\delta$ sufficiently small. Thus, the deviation is profitable, a contradiction.

Now consider the case in which there does not exist $\epsilon > 0$ sufficiently small such that $E_{v_2} [x_1(p, v_2 \mid p) \mid v_2 \geq p]$ is continuous in $p$ for $p \in [\hat{p} - \epsilon, \hat{p}]$. In this case, $E_{v_2} [x_1(p, v_2 \mid p) \mid v_2 \geq p]$ is discontinuous from the left in $p$ at $p = \hat{p}$. If the discontinuity involves a jump down, then we can repeat the above argument for a point $\tilde{p}$ slightly less than $\hat{p}$ (our assumptions rule out the possibility that the expectation is everywhere discontinuous on $(0, \hat{p})$). If the discontinuity involves a jump up at $p = \hat{p}$, then a similar argument implies that a ring member with value slightly less than $\hat{p}$ has an incentive to stay active up to $\hat{p}$ at the auction, in contradiction to Lemma 3.

We conclude that for all $p \in (0, \bar{v})$, $E_{v_2} [x_1(p, v_2 \mid p) \mid v_2 \geq p] \leq 0$, and similarly, for all $p \in (0, \bar{v})$, $E_{v_1} [x_2(v_1, p \mid p) \mid v_1 \geq p] \leq 0$. Q.E.D.

**Proof of Proposition 2.** Conditional on bidder 3 being the first to exit at price $p$, a ring member $i$ with value $v_i$ receives the object with probability $\Pr(v_i > v_{3-i} \mid v_i, v_{3-i} \geq \beta^{-1}(p))$, which we write as $\Phi(v_i \mid p) \equiv \frac{F(v_i) - F(\beta^{-1}(p))}{1 - F(\beta^{-1}(p))}$. Let $V_i(v_i \mid p)$ denote RM$i$’s interim expected surplus in the post-auction mechanism conditional on bidder 3 exiting first at price $p$, not including the price $p$ paid at the auction. Since the post-auction mechanism must satisfy incentive compatibility, we have for any $v \in [\underline{v}(p), \bar{v}]$,

$$V_i(v \mid p) = V_i(\underline{v}(p) \mid p) + \int_{\underline{v}(p)}^{v} \Phi(z \mid p) \, dz,$$

(2)

where $\underline{v}(p)$ denotes the lowest type that participates in the mechanism. Equation (2) implies that $V_i(\cdot \mid p)$ is increasing; thus, the set of types who are not willing to exit at price $p$ is an interval $[\underline{v}(p), \bar{v}]$. Since we assume each ring member pays half of $p$, the lowest type $\underline{v}(p)$ is determined by the indifference condition $V_i(\underline{v}(p) \mid p) = \frac{1}{2}p$. 

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Using (2) and integrating by parts yields

\[
\sum_{i=1}^{2} \int_{\bar{v}(p)}^{\hat{v}} V_i (y \mid p) \, d\Phi_i (y \mid p) = \sum_{i=1}^{2} \int_{\bar{v}(p)}^{\hat{v}} \left( V_i (\bar{v}(p) \mid p) + \int_{\bar{v}(p)}^{y} \Phi (z \mid p) \, dz \right) \, d\Phi_i (y \mid p)
\]

\[
= \sum_{i=1}^{2} V_i (\bar{v}(p) \mid p) + \sum_{i=1}^{2} \int_{\bar{v}(p)}^{\hat{v}} [1 - \Phi (y \mid p)] \, \Phi_i (y \mid p) \, dy
\]

\[
= \sum_{i=1}^{2} V_i (\bar{v}(p) \mid p) + E \left[ \max \{v_1, v_2\} \mid v_1, v_2 > \bar{v}(p) \right] - E \left[ \min \{v_1, v_2\} \mid v_1, v_2 > \bar{v}(p) \right].
\]

Since the object is reallocated efficiently, the total surplus generated by the post-auction mechanism is

\[
\sum_{i=1}^{2} \int_{\bar{v}(p)}^{\hat{v}} V_i (y \mid p) \, d\Phi_i (y \mid p) = E \left[ \max \{v_1, v_2\} \mid v_1, v_2 > \bar{v}(p) \right].
\]

Combining the previous equalities, we obtain

\[
\sum_{i=1}^{2} V_i (\bar{v}(p) \mid p) = E \left[ \min \{v_1, v_2\} \mid v_1, v_2 > \bar{v}(p) \right].
\]

Since the mechanism is symmetric, we have \( V_1 (\bar{v}(p) \mid p) = V_2 (\bar{v}(p) \mid p) = \frac{1}{2} E \left[ \min \{v_1, v_2\} \mid v_1, v_2 > \bar{v}(p) \right] \). Therefore, the critical type \( \bar{v}(p) \equiv \beta^{-1} (p) \) is determined by the indifference condition \( \frac{1}{2} p = \frac{1}{2} E \left[ \min \{v_1, v_2\} \mid v_1, v_2 > \bar{v}(p) \right], \) i.e., \( \beta (v) = v + \int_{v}^{1} \frac{[1-F(t)]^2}{[1-F(v)]^2} \, dt \).

Q.E.D.
References


