Rev. Econ. Design 3, 371–387 (1998)

Review
of Economic
Design

Efficient venture capital financing combining debt and equity

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Received: 9 September 1997 / Accepted: 3 April 1998

Abstract. I present a model of venture capital contracting in which contracts that involve a mixture of both debt and equity are efficient and dominate pure-equity and pure-debt financing. The optimal contract balances the venture capitalist's incentive to intervene in the project and the entrepreneur's desire for control.

JEL classification: G24, G32

Key words: Venture capital financing, convertible preferred equity

1 Introduction

This paper examines the contracting problem faced by entrepreneurs and venture capitalists. Because of the long time horizons, riskiness, and sizable capital requirements of venture capital projects, contracts are an essential part of the relationship between entrepreneurs and venture capitalists. Venture capital projects are typically financed with a mixture of debt and equity, where the equity is typically a convertible security.[1] This paper presents a model of venture capital financing and shows that a financing arrangement that has both debt and equity components can achieve the first best, whereas pure equity and pure debt financing cannot.

This paper is a revision of the working paper titled "Negotiation and Renegotiation of Venture Capital Contracts." I thank Bernard Black, Steven Matthews, Andrew Winton, two anonymous referees, and an associate editor for their advice and comments. I thank seminar participants at Rice, Stanford, Texas A&M, University of Arizona, University of Chicago, University of Pennsylviana, and University of Rochester. The Support of a National Sience Foundation Graduate Fellowship and a Sloan Doctoral Dissertation Fellowship during work on this paper is gratefully acknowledged. I also thank the American Compensation Association Emerging Scholar Program for financial support. The views expressed are sololy the author's and do not reflect the views or opinions of the NSF, Sloan Foundation, or American Compensation Association.

[1] See Gompers (1997), Pozdena (1990), Sahlman (1990), Silver (1987), Testa (1988), and Tyebjee and Bruno (1984).

In the model of this paper, a wealth-constrained entrepreneur has a project that generates an uncertain but ex-post verifiable return. The entrepreneur must raise financing for the project from a venture capitalist. In exchange for investment capital, the entrepreneur trades rights to the stochastic return. After the project begins, information arrives about the project's potential for success. At this point the venture capitalist can intervene in the project (at a cost), but doing so eliminates the entrepreneur's benefits of control. I restrict the sharing rule for the project's return *a priori* to the class of sharing rules that have only debt and equity components, and then show that the first best can be achieved with a sharing rule in this class.[2] Thus, an efficient sharing arrangement for the project's return is generated by selling both debt and equity to the venture capitalist.[3] The intuition for this result is that while pure debt gives the venture capitalist too great an incentive to intervene, and pure equity too little, a mixed debt-equity sharing rule enables the optimal level of intervention to be achieved.

While the contracting problem in this paper is discussed in terms of a venture capital project, the optimality of combining a fixed payment with a proportional share of any remainder may apply to other environments as well. For example, if an agent needs funding for an activity with an uncertain return and values his autonomy, and if the available funding sources cannot commit not to intervene in order to prevent the worst outcomes, then the relationship fits loosely into the framework of the model. We would expect a fixed repayment amount to result in too much intervention and a proportional repayment to result in too little intervention, while an arrangement that involves some fixed repayments and some proportional sharing could implement the optimal level of intervention.

Section 2 provides a brief description of the main characteristics of the venture capital industry and reviews the related literature. In Sect. 3, a model of venture capital contracting is described. Analysis of the model and results on optimal financing arrangements are presented in Sect. 4. Section 5 concludes and proposes extensions. The more tedious proofs are left to the Appendix.

2 Venture capital financing

One might question why venture capitalists are needed, given the existence of commercial banks. However, the ability of banks in the United States to invest in commercial enterprises is limited. Banks are allowed to extend loans to industrial firms, but they cannot hold controlling amounts of equity in such firms or simultaneously lend to a commercial firm and hold its equity (Pozdena 1990). Thus, equity financing is not an option for banks. To control the risks associated with pure debt financing, U.S. banks generally make loans only to well-capitalized, established firms selling established products or to firms with ample collateral (Pozdena 1990). At the other end of the spectrum of potential loan clients are

[2] I leave the characterization of the set of optimal efficient contracts when no *a priori* restrictions are placed on the form of the contract for future research.

[3] This can be interpreted as the entrepreneur's selling to the venture capitalist shares of participating preferred equity or convertible preferred equity.

entrepreneurial firms. They are, by definition, firms with little or no current cash flow to support debt obligations. In addition, venture capitalists are largely unrestricted in the financial relationship they can establish with entrepreneurs (Pozdena 1990). Because venture capitalists can have rights of control over their investments and can utilize financial arrangements that involve both debt and equity, they have a comparative advantage in venture capital investing.[4]

The equity component of the financing arrangement between a venture capitalist and an entrepreneur generally takes the form of convertible preferred stock. Preferred stock gives the venture capitalist some debt-like priority over common stock holders, while the requirement that the preferred stock be convertible to common equity provides some of the upside potential of common stock. Testa (1988), Silver (1987), and Sahlman (1990) argue that the possibility of the venture capitalist receiving both fixed payments and a share of the project's remaining return is a key characteristic of convertible preferred stock. Preferred stock offers a fixed dividend like debt, but typically the decision whether to pay dividends or allow them to accrue is at the discretion of the directors. Conversion typically occurs if and when the firm holds an initial public offering or when the firm consistently generates more than a target level of earnings.[5]

In addition to receiving equity, venture capitalists usually obtain inside management rights, for example, the right to appoint one or more directors or to serve as an officer of the company.[6] Gompers (1997) shows that, in a sample of fifty convertible preferred equity venture investments, contracts usually explicitly allocate control rights to the venture capitalist, including giving them enough board seats to control the board of directors. In addition, venture capitalists provide a wide range of support for the ventures they finance and sometimes take over day-to-day operations (Sahlman 1990, p.508). The issue of control is often difficult since the entrepreneur values the independence of his company.[7]

Most research on the venture capital industry is descriptive in nature, e.g., Tyebjee and Bruno (1984), Sahlman (1988, 1990), and Barry et al. (1990), but some papers attempt to model venture capital contracting. Authors such as Amit, et al. (1990) study a wealthy entrepreneur's decision whether to involve an outside investor. In contrast, in this paper I do not model the decision to involve an outside investor, but rather assume that it is necessary. Authors such as Hansen (1991), Neher (1992), and Admati and Pfleiderer (1994) examine the conditions that determine when a project that requires multiple rounds of investment should be continued and when it should be terminated.

Trester (1993), Berglöf (1994), and Cornelli and Yosha (1997) compare payoffs and control outcomes under different types of contracts, but do not characterize an efficient contract, as is done in this paper. The model of Berglöf

[4] Amit et al. (1997) argue that venture capitalists' advantage lies in their ability to reduce information-based asymmetries, particularly through their selection and monitoring of entrepreneurial projects.

[5] See Testa (1988), Sahlman (1990), Brealey and Myers (1991), and Golder (1991).

[6] See Perez (1986), Pozdena (1990), and Brealey and Myers (1991).

[7] See Perez (1986, pp.8-9), Gorman and Sahlman (1989, p.241), and Golder (1991).

(1994) is somewhat similar to the model of this paper and also shows that debt and equity are complementary in an environment in which control issues are important. Cornelli and Yosha (1997) show that, in an environment with staged financing, convertible debt is better than a mixture of debt and equity because it reduces the entrepreneur's incentives to focus his effort on the short-term success of the project. Although the model in this paper allows only one round of financing, it allows a first-best contract to be characterized rather than just providing comparative results.

The model of Hellmann (1998) shows that an entrepreneur may voluntarily relinquish control rights over his project to the venture capitalist when the venture capitalist must be given incentives to engage in costly search for a new CEO for the project. Other papers that describe how venture capitalists retain control rights over projects include Rosenstein (1988), Sahlman (1990), Gompers (1995), and Lerner (1995).

The structure of the model of this paper has much in common with that of Aghion and Bolton (1992). In both models, a wealth constrained entrepreneur makes a take-it-or-leave-it offer, which may later be renegotiated, to an investor to raise money to finance a project. The entrepreneur, who envisioned and initiated the project, cares not only about monetary returns but also about other less tangible things such as reputation, specific human capital, etc. The utility from these things is modeled as a non-monetary element in the entrepreneur's payoff and is referred to as private benefits of control and is neither observable nor verifiable by third parties. The investor is only interested in her monetary return. In contrast to Aghion and Bolton (1992), this paper presents a model in which, after the state is revealed, the investor must decide whether or not to proceed with costly intervention, which deprives the entrepreneur of his private benefits of control.

3 Model

The model has three periods, $t = 0, 1, 2$. There is a venture capitalist and an entrepreneur. The entrepreneur "owns" the rights to the return of a project. The entrepreneur can trade rights to the return to the venture capitalist in exchange for investment capital. In order for the project to be executed, at $t = 0$ the entrepreneur and venture capitalist must sign a contract, and the venture capitalist must provide investment I^0. The entrepreneur proposes a contract and the venture capitalist accepts if she has nonnegative expected payoff. The venture capitalist can, at a cost, affect the distribution of the project's return by intervening in the project, but then the entrepreneur loses his nonpecuniary benefits of control. The need to balance intervention by the venture capitalist with control for the entrepreneur forms the primary contracting problem in this model. The venture capitalist does not internalize the full cost of intervention because she does not take into account the entrepreneur's loss of benefits of control. I assume that intervention by the venture capitalist is a sufficiently complex action that contracts cannot be contingent on intervention. Thus, it is not possible to enforce contracts that make the venture capitalist internalize the full cost of intervention by requiring that she pay an amount equal to the entrepreneur's benefits of control when she intervenes. In the model, there is no effort choice by the entrepreneur. Although providing incentives for the entrepreneur to work hard may be important, I wish to focus on other issues. If the entrepreneur is sufficiently motivated by the private benefits of control or other self-motivation, then the venture capitalist need not offer additional incentives for high effort.

Figure 1 shows the timeline for the model. At $t = 1$, the state ω is revealed. The state ω is observable to both the venture capitalist and the entrepreneur, but it is not verifiable, so contracts cannot be contingent on the observed value of ω. The state ω is an element of the set Ω, which is a compact interval of the real line, $[\underline{\omega}, \overline{\omega}]$. I let $\omega > \omega'$ indicate that ω is a "better" state than ω' in the sense that returns in state ω first-order stochastically dominate returns in state ω' (see Assumption 3). Ex ante, the distribution of ω is given by the differentiable function h, which is common knowledge. After the state is revealed, the contract can be renegotiated, with the entrepreneur making offers that the venture capitalist must accept or reject, and then the venture capitalist chooses whether or not to intervene. Intervention reduces the venture capitalist's payoff by $c > 0$, the cost of intervening, and deprives the entrepreneur of his benefits of control, $b > 0$. The variable $\mu \in \{0, 1\}$ represents the intervention decision, with $\mu = 1$ if the venture capitalist intervenes and $\mu = 0$ if she does not. For monetary return r and intervention decision μ, the entrepreneur's utility is $r + (1 - \mu)b$. For monetary return R and intervention decision μ, the venture capitalist's payoff is $R - \mu c$.

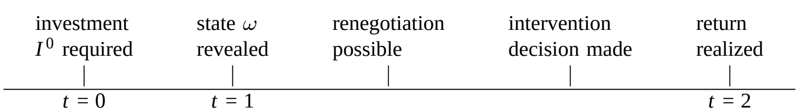

Fig. 1.

At $t = 2$, the project's payoff $y \in [0, \infty)$ is realized. The distribution of the project's payoff depends on the state and the intervention decision μ. Given the state and the intervention decision, the cumulative distribution function for the project's return is denoted by $F(\cdot \mid \omega, \mu)$. The following assumptions are made regarding the distribution of the project's return and the distribution of the state. They are maintained throughout the paper. An additional assumption that c is large relative to b requires additional notation and is stated in Sect. 4.

Assumption 1. *For all $\omega \in \Omega$, $F(\cdot \mid \omega, 0)$ is absolutely continuous and has continuous density function $f(\cdot \mid \omega, 0)$.*

Assumption 2. *For all $\omega \in \Omega$, the support of $f(\cdot \mid \omega, 0)$ is an interval of the form $[0, x^{\omega}]$, where $x^{\omega} > I^0$.*

Assumption 3. *For all $\omega, \omega' \in \Omega$, if $\omega > \omega'$ then the distribution $F(\cdot \mid \omega, 0)$ first-order stochastically dominates $F(\cdot \mid \omega', 0)$. Furthermore, given ω and $y \in (0, x^{\omega})$, $F(y \mid \omega, 0)$ is differentiable with respect to ω.*

Assumption 4. $\int_{\omega \in \Omega} \int_0^{\infty} yf(y \mid \omega, 0)h(\omega)dyd\omega > I^0$.

Assumption 1 is a technical assumption. Assumption 2 has the important implication that there is always positive probability of low returns if the venture capitalist does not intervene. In the absence of intervention, failure is always possible. Further, the assumption implies that, regardless of the state, recovery of the initial investment is also possible. Assumption 3 says that the distributions $F(\cdot \mid \omega, 0)$ are ordered according to first-order stochastic dominance, with distributions for larger, "better" states first-order stochastically dominating distributions for smaller, "worse" states. The assumption of differentiability is for convenience. Assumption 4 is a feasibility assumption and ensures that the project has positive expected return when the venture capitalist does not intervene.

Intervention

Intervention by the venture capitalist improves the project's prospects for success, but I assume that only the entrepreneur has the creative or project-specific abilities necessary to achieve high returns, so intervention by the venture capitalist can prevent some bad returns but cannot affect the probabilities of high returns. The venture capitalist brings managerial and organizational skills to the project, not new creative ideas or initiatives. The venture capitalist may be able to salvage a struggling venture with good administration and marketing or by using her contacts with suppliers and other businesses; however, she cannot affect the probability of high returns because these depend on the quality of the entrepreneur's idea and other factors beyond her control.

Specifically, assume intervention does not affect the probabilities of returns above some amount L, where $L \leq I^0$. Although the basic results of this paper hold as long as intervention results in a first-order stochastic dominance improvement for returns below L,[8] for simplicity I assume that whenever the project would have produced a return less than L without intervention, the project returns exactly L with intervention. So for all $\omega \in \Omega$, $F(y \mid \omega, 1) = 0$ for $y < L$ and $F(y \mid \omega, 1) = F(y \mid \omega, 0)$ for $y \geq L$. Under these assumptions, one can take two possible views of intervention. First, one can view intervention as bringing additional skills to the project so that, the project will return at least L. Second, one can view intervention as putting the venture capitalist in a position to assess whether the project's return will be less than L, in which case the venture capitalist liquidates the project's assets at liquidation value L rather than allowing the project proceed.

[8] A proof of this result is available from the author. It requires the assumption that, for returns in $(0, L)$, a change in the state has a larger effect on the distribution with no intervention than it does on the distribution with intervention, and it requires that Assumption 3 be extended to cover $F(\cdot \mid \omega, 1)$.

Contracts

A contract requires payment of I^0 by the venture capitalist, and it specifies a sharing rule for the final return. Sharing rules are written as functions $s(\cdot)$, where if the project's return is y, the venture capitalist receives $s(y)$ and the entrepreneur receives $y - s(y)$. In order to incorporate wealth constraints on the entrepreneur, sharing rules are restricted to satisfy $s(y) \leq y$ for all y.

I define three types of sharing rules: debt, equity, and mixed debt-equity. A *debt* sharing rule with debt level $\hat{d} \geq 0$ is defined by $s^d(y \mid \hat{d}) \equiv \min\{y, \hat{d}\}$. An *equity* sharing rule with share $\beta \in [0,1]$ is defined by $s^e(y \mid \beta) \equiv \beta y$.

A *mixed debt-equity* sharing rule with dividend $v \geq 0$ and share $\gamma \in [0,1]$ is defined by $s^m(y \mid v, \gamma) \equiv \min\{y, v + \gamma(y - v)\}$. Note that the mixed debt-equity sharing rule can be written as

$$s^m(y \mid v, \gamma) = \begin{cases} y, & y < v \\ v + \gamma(y - v), & y \geq v, \end{cases}$$

so it can be viewed as a fixed dividend payment v in conjunction with a proportional sharing rule for returns in excess of the dividend. If $\gamma = 0$, then the mixed debt-equity sharing rule is actually a debt sharing rule, and if $v = 0$, then it is an equity sharing rule.

The mixed debt-equity sharing rule can be interpreted as convertible preferred equity or participating preferred equity, whose holders receive a preferred dividend and then share equally with the common shareholders in the remaining dividends. To see this, note that, in a model in which returns are available in only one period, a mixed debt-equity sharing rule captures the basic characteristics of convertible preferred equity financing. With returns possible only at $t = 2$, the decision to allow dividends to accrue and decisions on the timing of conversion can be ignored. One can view v as the level of accrued dividends at $t = 2$. When the project's return at $t = 2$ is less than the level of accrued dividends, the preferred share holder has priority and receives all of the project's returns. When the project's return is greater than the accrued dividends, the investor receives the accrued dividends and converts her preferred stock into the fraction γ of the common stock in the venture, which has value $\gamma(y - v)$.

4 Analysis of the model

The venture capitalist's break-even constraint under sharing rule s is

$$\begin{aligned} I^0 \leq \; & \int_{\omega \notin I(s)} \int_0^\infty s(y) f(y \mid \omega, 0) h(\omega) dy d\omega \\ & + \int_{\omega \in I(s)} \left[s(L) F(L \mid \omega, 0) + \int_L^\infty s(y) f(y \mid \omega, 0) dy - c \right] h(\omega) d\omega, \end{aligned} \tag{1}$$

where $I(s)$ is the set of states such that the venture capitalist prefers to intervene at $t = 1$ when the sharing rule is $s(\cdot)$. I assume that the venture capitalist does not intervene if she is indifferent between intervening and not.

Given Assumption 4, for all $v \in [0, I^0]$, there exists $\gamma^*(v) \in [0,1]$ such that $s^m(\cdot \mid v, \gamma^*(v))$ satisfies the venture capitalist's break-even constraint, (1), with equality. Moreover, the function $\gamma^* : [0, I^0] \to [0,1]$ defined in this way is continuous on $[0, I^0]$. This is stated formally as Lemma 1, which is proven in the Appendix.

Lemma 1. *For all* $v \in [0, I^0]$, *there exists a continuous function* $\gamma^* : [0, I^0] \to [0,1]$ *such that* $s^m(\cdot \mid v, \gamma^*(v))$ *satisfies the venture capitalist's break-even constraint,* (1), *with equality.*

There also exist equity share β^* and debt level d^* such that $s^e(\cdot \mid \beta^*)$ and $s^d(\cdot \mid d^*)$ satisfy the break-even constraint (1) with equality. To see this, for β^*, let $\beta^* \equiv \gamma^*(0)$ and use Lemma 1. For d^*, first note that, under a debt sharing rule with debt level d, the venture capitalist's revenue can never be greater than d. So, in order for the venture capitalist to have expected revenue greater than or equal to I^0 under a debt sharing rule, it must be that the debt level is greater than or equal to I^0. Thus, if d^* exists, then $d^* \geq I^0$ (and the inequality is strict if $\gamma^*(I^0) > 0$). Second, note that, even when the venture capitalist is not permitted to intervene, by Assumption 4 there exists d sufficiently large such that $s^d(\cdot \mid d)$ satisfies the venture capitalist's break-even constraint.[9] Since intervention by the venture capitalist increases the venture capitalist's expected revenue, when the venture capitalist can intervene, there also exists d sufficiently large such that the venture capitalist's break-even constraint is satisfied, i.e., for d sufficiently large,

$$\begin{aligned} I^0 \quad \leq \quad & \int_{\omega \notin I(s^d(\cdot|d))} \left(\int_0^d y f(y \mid \omega, 0) dy + \int_d^\infty d f(y \mid \omega, 0) dy \right) h(\omega) d\omega \\ & + \int_{\omega \in I(s^d(\cdot|d))} \Bigg(L \cdot F(L \mid \omega, 0) + \int_L^d y f(y \mid \omega, 0) dy \\ & \qquad\qquad + \int_d^\infty d f(y \mid \omega, 0) dy - c \Bigg) h(\omega) d\omega. \end{aligned}$$

The right-hand side in the above inequality is less than or equal to I^0 when $d = I^0$, and it increases continuously as d increases (since intervention only affects returns below L, the sets $I(s^d(\cdot \mid d))$ do not change as d increases above L). Thus, there exists $d^* \geq I^0$ such that the venture capitalist's break-even constraint is satisfied with equality under debt sharing rule $s^d(\cdot \mid d^*)$.

In state ω, without intervention the total benefit of the project is

$$b + \int_0^\infty y f(y \mid \omega, 0) dy,$$

[9] Without intervention, the venture capitalist's break even constraint is $I^0 \leq \int_{\omega \in \Omega} \left(\int_0^d y f(y \mid \omega, 0) dy + \int_d^\infty d f(y \mid \omega, 0) dy \right) h(\omega) d\omega$, and as d approaches infinity, the right-hand side in this inequality approaches $\int_{\omega \in \Omega} \left(\int_0^d y f(y \mid \omega, 0) dy \right) h(\omega) d\omega$, which, by Assumption 4, is strictly greater than I^0.

and with intervention the total benefit of the project is

$$L \cdot F(L \mid \omega, 0) + \int_L^\infty yf(y \mid \omega, 0)dy - c.$$

Intervention is efficient in state ω when the total benefit of the project is larger with intervention than without (to be consistent with the assumption about when the venture capitalist intervenes, I call intervention efficient if and only if the total benefit is *strictly* larger with intervention than without). Thus, intervention is efficient if

$$b + \int_0^\infty yf(y \mid \omega, 0)dy < L \cdot F(L \mid \omega, 0) + \int_L^\infty yf(y \mid \omega, 0)dy - c,$$

which, integrating by parts, is

$$\int_0^L F(y \mid \omega, 0)dy > b + c. \tag{2}$$

Using the assumption on first-order stochastic dominance, Assumption 3, if intervention is not efficient in state $\hat{\omega}$, then intervention is also not efficient in any state $\omega \in \Omega$ such that $\omega \geq \hat{\omega}$.

Define state $\tilde{\omega}$ to be the smallest state such that intervention is not efficient, or $\overline{\omega}$ if intervention is always efficient (we can use "min" in the definition of $\tilde{\omega}$ instead of "inf" since F is continuous in ω by Assumption 3),

$$\tilde{\omega} \equiv \begin{cases} \min\{\omega \in [\underline{\omega}, \overline{\omega}] \mid \int_0^L F(y \mid \omega, 0)dy \leq b + c\}, & \text{if } \int_0^L F(y \mid \overline{\omega}, 0)dy \leq b + c \\ \overline{\omega}, & \text{otherwise.} \end{cases}$$

Consider the intervention decision faced by the venture capitalist at $t = 1$ after the state has been revealed and after any renegotiation is complete. To begin, assume a debt sharing rule $s^d(y \mid \hat{d})$ is in place. Assume that $\hat{d} \geq I^0$, since otherwise, as discussed after Lemma 1, the sharing rule could not satisfy the venture capitalist's ex-ante break-even constraint. If the venture capitalist does not intervene, her expected payoff (from the perspective of $t = 1$) is

$$\int_0^\infty \min\{y, \hat{d}\} f(y \mid \omega, 0)dy.$$

If the venture capitalist does intervene, her expected payoff is

$$L \cdot F(L \mid \omega, 0) + \int_L^\infty \min\{y, \hat{d}\} f(y \mid \omega, 0)dy - c.$$

Thus, integrating by parts, the expected increase in the venture capitalist's utility from intervening is

$$\int_0^L F(y \mid \omega, 0)dy - c. \tag{3}$$

Under a debt sharing rule with debt level greater than or equal to I^0, the venture capitalist intervenes if and only if (3) is greater than zero. Letting ω^d be the smallest state such that the venture capitalist does not intervene under a debt sharing rule, or $\overline{\omega}$ if the venture capitalist always intervenes under debt (as before, we can use "min" instead of "inf" by Assumption 3),

$$\omega^d \equiv \begin{cases} \min\{\omega \in [\underline{\omega}, \overline{\omega}] \mid \int_0^L F(y \mid \omega, 0)dy \leq c\}, & \text{if } \int_0^L F(y \mid \overline{\omega}, 0)dy \leq c \\ \overline{\omega}, & \text{otherwise,} \end{cases}$$

then for all $\omega \geq \omega^d$, the venture capitalist does not intervene when a debt sharing rule is used.

If intervention is efficient in state ω, then by (2), $\int_0^L F(y \mid \omega, 0)dy > b + c$. But then (3) is greater than zero, and so the venture capitalist intervenes. Thus, when a debt sharing rule is used, the venture capitalist always intervenes when it is efficient and may sometimes intervene when it is not efficient. Thus, there is at least as much intervention when a debt sharing rule is used (i.e. $\omega^d \geq \tilde{\omega}$).

Now consider an equity sharing rule $s^e(y \mid \beta)$. Under the equity sharing rule, the venture capitalist's expected utility, from the perspective of $t = 1$, if she does not intervene is

$$\int_0^\infty \beta y f(y \mid \omega, 0)dy.$$

If the venture capitalist does intervene, her expected utility is

$$\beta L \cdot F(L \mid \omega, 0) + \int_L^\infty \beta y f(y \mid \omega, 0)dy - c.$$

So, integrating by parts, the expected increase in the venture capitalist's utility from intervening under an equity sharing rule is

$$\beta \int_0^L F(y \mid \omega, 0)dy - c. \qquad (4)$$

Under an equity sharing rule, the venture capitalist intervenes if and only if (4) is greater than zero. Define state ω^e to be the smallest state such that the venture capitalist does not intervene under an equity sharing rule with share β^*, or $\overline{\omega}$ if the venture capitalist always intervenes under equity (again, we can use "min" instead of "inf" by Assumption 3),

$$\omega^e \equiv \begin{cases} \min\{\omega \in [\underline{\omega}, \overline{\omega}] \mid \beta^* \int_0^L F(y \mid \omega, 0)dy \leq c\}, & \text{if } \beta^* \int_0^L F(y \mid \overline{\omega}, 0)dy \leq c \\ \overline{\omega}, & \text{otherwise.} \end{cases}$$

Then for all $\omega \geq \omega^e$, the venture capitalist does not intervene under an equity sharing rule with share β^*.

Comparing expressions (3) and (4), one can see that the gain to the venture capitalist from intervening is at least as great when the initial sharing rule is debt than when it is equity. So the venture capitalist intervenes at least as often (for at least as many states) under debt contracts than equity contracts, i.e. $\omega^e \leq \omega^d$.

Under a debt sharing rule, if the return is below L, the venture capitalist receives one dollar for every dollar increase in the project's return. But under an equity sharing rule, the venture capitalist receives only a fraction of a dollar for every dollar increase in the project's return. Thus, when the project's return is low, the marginal benefit to the venture capitalist of intervening is at least as great under a debt sharing rule than under an equity sharing rule.

I now state one additional assumption, which is maintained in the remainder of the paper.

Assumption 5. *The cost of intervention, c, is sufficiently large relative to the benefits of control, b, such that* $(1-\beta^*)c \geq \beta^* b$.

Since β^* does not depend on b and is strictly less than 1 by Assumption 4, there exists c sufficiently large such that Assumption 5 is satisfied. For example, if $F(\cdot \mid \omega, 0)$ is the uniform distribution on $[0, \omega]$ and $\beta^* = 1/2$, then Assumption 5 is satisfied as long as $c \geq b$. The assumption is reasonable in the context of venture capital projects since its failure means that the entrepreneur has large benefits of control (perhaps he is essential to the project or is himself a defining characteristic of the project) but that the venture capitalist is still able to improve the project's returns at low cost, something that is unlikely.

Using Assumption 3 and the definitions of $\tilde{\omega}$ and ω^e, Assumption 5 implies that $\omega^e \leq \tilde{\omega}$, which implies that intervention is efficient in state ω^e. To see this, suppose that $\omega \leq \omega^e$. Then by Assumption 3 and the definition of ω^e, $\beta^* \int_0^L F(y \mid \omega, 0)dy \geq c$. This inequality and Assumption 5 implies that $\int_0^L F(y \mid \omega, 0)dy \geq b + c$, which, by Assumption 3 and the definition of $\tilde{\omega}$, implies that $\omega \leq \tilde{\omega}$, thus establishing that $\omega^e \leq \tilde{\omega}$.

When Assumption 5 does not hold, there is a state in which intervention is suboptimal and in which the venture capitalist intervenes under sharing rule $s^e(\cdot \mid \beta^*)$. So, without the assumption, there is too much intervention using an equity sharing rule as well as too much intervention under debt, and the strategy of using a mixture of debt and equity to obtain efficient intervention fails. Thus, Assumption 5 is necessary for the results on the optimality of mixed debt-equity.

Define v^* to be the largest dividend in $[0, L]$ for which there is no intervention in state $\tilde{\omega}$ under the sharing rule $s^m(\cdot \mid v^*, \gamma^*(v^*))$,

$$v^* \equiv \max\left\{ v \in [0, L] \mid \int_0^v F(y \mid \tilde{\omega}, 0)dy + \gamma^*(v) \int_v^L F(y \mid \tilde{\omega}, 0)dy \leq c \right\}. \quad (5)$$

Under the sharing rule $s^m(\cdot \mid v^*, \gamma^*(v^*))$, the state $\tilde{\omega}$ is the pivotal state for the venture capitalist's intervention decision. Lemma 2 confirms that the dividend v^* is well defined.

Lemma 2. *Dividend v^* is well defined.*

Proof. Intervention is not efficient in state $\tilde{\omega}$, so, since Assumptions 3 and 5 imply that $\omega^e \leq \tilde{\omega}$, the venture capitalist does not intervene in state $\tilde{\omega}$ under sharing rule $s^e(\cdot \mid \beta^*)$. Thus, $\beta^* \int_0^L F(y \mid \tilde{\omega}, 0)dy \leq c$. Since $\gamma^*(0) = \beta^*$, this implies $\int_0^0 F(y \mid \tilde{\omega}, 0)dy + \gamma^*(0) \int_0^L F(y \mid \tilde{\omega}, 0)dy \leq c$, so the expression in the definition of v^* is satisfied at $v = 0$. The proof is completed by noting that the left-hand side of the inequality in (5) is continuous in v, which follows from Lemma 1's result that $\gamma^*(v)$ is continuous. □

I now state the main result of the paper.

Proposition 1. *The sharing rule $s^m(\cdot \mid v^*, \gamma^*(v^*))$ is efficient, and maximizes the entrepreneur's ex-ante expected utility subject to the venture capitalist's break-even constraint* (1)*.*

The proof of Proposition 1 uses Lemma 3 below, which states that a convertible preferred sharing rule with dividend v^* and share $\gamma^*(v^*)$ results in intervention if and only if the realized state is less than $\tilde{\omega}$. The proof of Lemma 3 is in the Appendix.

Lemma 3. *Under sharing rule $s^m(\cdot \mid v^*, \gamma^*(v^*))$, the venture capitalist intervenes in state ω if and only if $\omega < \tilde{\omega}$.*

Proof of Proposition 1. By the definition of $\tilde{\omega}$ and Assumption 3, intervention is efficient if and only if $\omega < \tilde{\omega}$. By Lemma 3, under sharing rule $s^m(\cdot \mid v^*, \gamma^*(v^*))$, the venture capitalist intervenes if and only if it is efficient. By the definition of $\gamma^*(\cdot)$, $s^m(\cdot \mid v^*, \gamma^*(v^*))$ satisfies (1) with equality and so is acceptable to the venture capitalist. This, together with the optimality of intervention resulting from the sharing rule, implies that the sharing rule maximizes the entrepreneur's expected utility subject to (1). □

Note that the sharing rule of Proposition 1 is optimal for the entrepreneur in the class of mixed-debt equity contracts, and since the sharing rule achieves the first best, there is no sharing rule that makes both the entrepreneur and the venture capitalist better off, and thus there is no scope for renegotiation.

While the results of this section show that the contract with sharing rule $s^m(\cdot \mid v^*, \gamma^*(v^*))$ is optimal, it has not been shown that equity and debt contracts are not also optimal. Debt and equity are also optimal if, after the state is revealed and before the intervention decision is made, they are renegotiated to new contracts that give the venture capitalist proper incentives for intervention. If the initial sharing rule is debt, the venture capitalist has the incentive to intervene in states in which intervention is inefficient, and if the initial sharing rule is equity, the venture capitalist has the incentive not to intervene in states in which intervention is efficient. If, in these situations, the entrepreneur can propose a new contract under which intervention occurs if and only if it is efficient, then debt and equity can be optimal initial contracts. However, transfers are a necessary part of the renegotiation – from the entrepreneur to the venture capitalist when renegotiating debt and from the venture capitalist to the entrepreneur when renegotiating equity. Wealth constraints may prevent the first type of transfer.[10] Thus, even considering renegotiation, a debt sharing rule may not be efficient. Furthermore, if renegotiation is costly, both parties prefer that the initial sharing rule be mixed debt-equity.

The result on the optimality of mixed debt-equity requires that the debt and equity be held by the same investor and that the debt have priority. The inseparability of the fixed component and the proportional component of mixed debt-equity is crucial for the optimality result. Once the two features are separated, the holders of the separate instruments have different incentives for intervention than a single investor holding mixed debt-equity.[11] Even if debt and equity are issued to the same investor, if the two securities are separable, she might have the incentive to sell one or the other. In addition, the implications of defaulting on debt may be quite different from those of deferring or cancelling dividend payments.

An important additional point is that the venture capital project of this model is distinct from the investment project of a pre-existing firm, and thus the results of this section should not be interpreted as suggesting that all firms should finance their projects with mixed debt-equity. First, if a firm already has outstanding debt or equity, mixed debt-equity may not be optimal. Second, if the value of a pre-existing firm is in its continuing existence and profitability, then any income from a new project might be best used by reinvesting it in the company. In the case of a venture capital project, the $t = 2$ return is typically the result of an initial public offering, and thus the venture capitalist can cash out her investment without adversely affecting the future of the firm.

5 Conclusion

This paper shows that, in the optimal contract between a wealth constrained entrepreneur and a risk neutral venture capitalist, the project is financed through a mixed debt-equity sharing rule. Since intervention by the venture capitalist is sometimes efficient and contracts can provide proper incentives for intervention to occur when and only when it is efficient, it is optimal to give the venture capitalist the right to intervene.

A possible extension involves allowing the venture capitalist to separate the debt and equity components of the project's financing. Then one would need to consider the incentives for the venture capitalist to see either the debt or the equity component and how that affects her incentives to intervene in the project.

[10] If the venture capitalist made additional payments to the entrepreneur at $t = 0$, then renegotiation to an optimal contract might be possible since the entrepreneur could compensate the venture capitalist for accepting the new sharing rule.

[11] Incentives for intervention will also be different if there are multiple investors. However, Barry et al. (1990, p.462) finds that when more than one venture capitalist invests in a venture-backed firm, one venture-capital investor typically takes the lead role in coordinating the investors and working with the firm. Brander et al. (1997) view having multiple venture capitalists as potentially valuable since then the entrepreneur benefits from the expertise of more people.

Other extensions include relaxing the all-or-nothing nature of intervention to allow degrees of intervention with varying costs to the venture capitalist or allowing for more than one infusion of capital. In a model that allows staged financing, or if the project has returns in more than one period, we must consider when payments to the venture capitalist should be made and how the venture capitalist's incentive to intervene changes over time. If signals of the project's performance are available in multiple periods, then, as in Cornelli and Yosha (1997), the sharing rule may need to provide incentives that prevent the entrepreneur from being overly focused on short-term results.

Another interesting extension involves allowing the venture capitalist's right to intervene to be contingent on some signal. For instance, if dividends were scheduled to be paid in an interim period, then the venture capitalist might have the right to intervene if and only if the dividend obligation were not met. This corresponds well with provisions in some convertible preferred equity sales contracts that give the venture capitalist control of the board of directors if dividends payments are missed.

6 Appendix

Proof of Lemma 1

Note that, using integration by parts, the venture capitalist does not intervene under a mixed debt-equity sharing rule with dividend v and share γ if and only if

$$\int_0^{\min\{v,L\}} F(y \mid \omega, 0)dy + \gamma \int_{\min\{v,L\}}^{L} F(y \mid \omega, 0)dy \leq c.$$

Define the function $\omega^* : [0, I^0] \times [0,1] \to \Omega$ such that $\omega^*(v,\gamma)$ is the smallest state such that the venture capitalist does not intervene under a mixed debt-equity sharing rule with dividend v and share γ. To do this, for $v > 0$, define

$$\omega^*(v,\gamma) \equiv \begin{cases} \min\{\omega \in \Omega \mid \int_0^{\min\{v,L\}} F(y \mid \omega, 0)dy + \gamma \int_{\min\{v,L\}}^{L} F(y \mid \omega, 0)dy \leq c\}, \\ \qquad \text{if } \int_0^{\min\{v,L\}} F(y \mid \bar{\omega}, 0)dy + \gamma \int_{\min\{v,L\}}^{L} F(y \mid \bar{\omega}, 0)dy \leq c \\ \bar{\omega}, \qquad \text{otherwise.} \end{cases}$$

Since

$$\int_0^{\min\{v,L\}} F(y \mid \omega, 0)dy + \gamma \int_{\min\{v,L\}}^{L} F(y \mid \omega, 0)dy \tag{6}$$

is strictly decreasing in ω for $v > 0$ (by Assumptions 2 and 3) and continuous in v and γ, $\omega^*(v,\gamma)$ is continuous and well defined for $v > 0$. For $v = 0$, define $\omega^*(0,\gamma)$ to be $\lim_{v \downarrow 0} \omega^*(v,\gamma)$, so that ω^* is continuous for all $(v,\gamma) \in [0, I^0] \times [0,1]$. Note that ω^* is nondecreasing in v and γ. If we subtract c from (6), then we have the difference between the expected returns to the venture capitalist in state ω with and without intervention.

The venture capitalist's expected return from sharing rule $s^m(\cdot \mid v, \gamma)$ is given below as the function M^V,

$$M^V(v,\gamma) \equiv \int_{\omega^*(v,\gamma)}^{\bar{\omega}} \int_0^\infty s^m(y \mid v,\gamma) f(y \mid \omega, 0) h(\omega) dy d\omega + \int_{\underline{\omega}}^{\omega^*(v,\gamma)} \big[s^m(L \mid v,\gamma) F(L \mid \omega, 0) + \int_L^\infty s^m(y \mid v,\gamma) f(y \mid \omega,0) dy - c \big] h(\omega) d\omega. \tag{7}$$

Substituting in the functional form for $s^m(\cdot \mid v, \gamma)$ and integrating by parts, we get

$$M^V(v,\gamma) = \int_{\underline{\omega}}^{\bar{\omega}} \int_0^\infty s^m(y \mid v,\gamma) f(y \mid \omega, 0) h(\omega) dy d\omega + \int_{\underline{\omega}}^{\omega^*(v,\gamma)} \left(\int_0^{\min\{v,L\}} F(y \mid \omega, 0) dy + \gamma \int_{\min\{v,L\}}^{L} F(y \mid \omega, 0) dy - c \right) h(\omega) d\omega.$$

It is clear from this expression that M^V is continuous in (v, γ) and nondecreasing in γ. Since the supremum of the support of f is greater than I^0 by Assumption 2, for $v \in [0, I^0]$, the first term in the above expression for M^V is strictly increasing in γ. Thus, M^V is also strictly increasing in γ. Furthermore, the correspondence $\varphi : [0, I^0] \rightrightarrows [0,1]$ defined by $\varphi(v) = \{\gamma \in [0,1] \mid M^V(v,\gamma) \geq I^0\}$ is continuous.

We need to show that for $v \in [0, I^0]$, there exists $\gamma \in [0,1]$ such that $M^V(v,\gamma) = I^0$. Letting $\gamma = 0$, then for all $y \geq 0$, $s^m(y \mid v,\gamma) \leq v$. Thus, it is clear from (7) that $M^V(v,0) \leq v$, which implies that, for all $v \in [0, I^0]$, $M^V(v,0) \leq I^0$. Letting $\gamma = 1$, for all $v \in [0, I^0]$, $M^V(v,1) = M^V(0,1)$, which is greater than I^0 by Assumption 4. Therefore, given $v \in [0, I^0]$, $M^V(v,\gamma) = I^0$ for one and only one $\gamma \in [0,1]$. This completes the proof that $\gamma^* : [0, I^0] \to [0,1]$ is a well-defined function.

To prove that γ^* is continuous, note that continuity of M^V in (v,γ) implies that the graph of $\varphi(v)$ is closed in $[0, I^0] \times [0,1]$. Since M^V is strictly increasing in γ, the lower boundary of the graph of $\varphi(v)$ is the graph of $\gamma^*(v)$. If $\gamma^*(v)$ were not continuous, then the graph of $\varphi(v)$ would not be closed. □

Proof of Lemma 3

Define $g(v) : [0,L] \to \mathbb{R}$ by $g(v) \equiv \int_0^v F(y \mid \tilde{\omega}, 0) dy + \gamma^*(v) \int_v^L F(y \mid \tilde{\omega}, 0) dy$. Then g is continuous in v (since γ^* is continuous) for all $v \in [0,L]$. To begin the proof, I claim that, if intervention occurs in state $\tilde{\omega}$ under a sharing rule $s^d(\cdot \mid d^*)$, then $g(v^*) = c$. To prove the claim, suppose that intervention occurs in state $\tilde{\omega}$ under debt sharing rule $s^d(\cdot \mid d^*)$. Then, it must be that the venture capitalist has a higher payoff if she intervenes than if she does not intervene, i.e., from (3),

$$\int_0^L F(y \mid \tilde{\omega}, 0) dy > c. \tag{8}$$

Inequality (8) implies that $g(L) > c$. Since $\omega^e \leq \tilde{\omega}$ by Assumptions 3 and 5, the venture capitalist does not intervene in state $\tilde{\omega}$ under sharing rule $s^e(\cdot \mid \beta^*)$, i.e. $\beta^* \int_0^L F(y \mid \tilde{\omega}, 0) dy \leq c$. Since $\beta^* = \gamma^*(0)$, this implies that

$$\gamma^*(0) \int_0^L F(y \mid \tilde{\omega}, 0) dy \leq c. \tag{9}$$

Inequality (9) implies that $g(0) \leq c$. Since $g(L) > c$ and $g(0) \leq c$, by the Intermediate Value Theorem, there exists $v^* \in [0, L]$ such that $g(v^*) = c$. Define $\overline{v} \equiv \sup\{v \in [0, L] \mid g(v) \leq c\}$, and note that $g(\overline{v}) = c$. Then, using the definition of v^* in equation (5), $v^* = \overline{v}$, so $g(v^*) = c$, completing the proof of the claim.

Continuing, let $\omega \in \Omega$. Since $g(v^*) = c$, the venture capitalist does not intervene when the state is $\tilde{\omega}$. By first-order stochastic dominance, if $\omega \geq \tilde{\omega}$ the venture capitalist also does not intervene in state ω, completing the proof of the "only if" part of the lemma.

Suppose $\omega < \tilde{\omega}$ and that the venture capitalist prefers not to intervene in state ω. Then

$$\int_0^{v^*} F(y \mid \omega, 0) dy + \gamma^*(v^*) \int_{v^*}^L F(y \mid \omega, 0) dy \leq c.$$

But this and first-order stochastic dominance (Assumption 3) implies that $g(v^*) < c$, which contradicts the earlier claim that $g(v^*) = c$, completing the "if" part of the proof. □

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