Predatory accommodation: below-cost pricing without exclusion in intermediate goods markets

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We show that below-cost pricing can arise in intermediate goods markets when a monopolist retailer negotiates sequentially with two suppliers of substitute products. Below-cost pricing by one supplier allows the retailer to extract rents from the second supplier. Thus, the retailer and one supplier can increase their joint profit at the expense of the second supplier. We consider the welfare implications of below-cost pricing (welfare can increase or decrease as a result of below-cost pricing) and provide suggestions for when the courts should view below-cost pricing in intermediate goods markets as anticompetitive and when they should not.

1. Introduction

Predatory pricing is a violation of Section 2 of the Sherman Act and, if it occurs in an intermediate goods market, of Section 2(a) of the Robinson-Patman Act. Both acts require that there be substantial injury to competition, but neither statute specifies exactly what constitutes predatory pricing. This task has been left to the judicial system, which has recently ruled that a claim of predatory pricing under either act must establish that the alleged predator engaged in below-cost pricing and that it had a reasonable prospect of recouping its losses, e.g., after it had driven its rivals out of the market.

In this article we focus on pricing in intermediate goods markets when a retailer negotiates sequentially with two suppliers of imperfect substitutes. We show that below-cost pricing with recoupment can arise in a model of complete information, absent

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1 Our focus in the Robinson-Patman Act is on primary-line price discrimination, which occurs when one seller is injured by a discriminatory price that is offered by another seller at the same level.

2 A plaintiff must show two things: (a) that prices were below "an appropriate measure of its rival's costs," and (b) that its rival had "a reasonable prospect . . . of recouping its investment in below-cost prices." See Brooke Group Ltd. v. Brown & Williamson Tobacco Corp., 113 U.S. 2578 (1993).

3 We implicitly assume the existence of a second market in which the alleged predator does not engage
reputational or signalling concerns. The below-cost pricing we identify satisfies the two prerequisites for predatory pricing; yet although the rival supplier is harmed, it is not driven out of the market. We label such behavior predatory accommodation and show that welfare (the sum of profit and consumer surplus) can increase because of it.

Our work is related to that of Aghion and Bolton (1987), who first demonstrate the possibility of rent shifting in a sequential contracting environment. In Aghion and Bolton, the rent-shifting mechanism takes the form of an exclusive dealing arrangement with a penalty escape clause. In our model, by contrast, the rent-shifting mechanism takes the form of below-cost pricing. In both models, one retailer negotiates in sequence with two suppliers. Aghion and Bolton find that when there is no uncertainty, all surplus is extracted from the second supplier and there is no pricing distortion. In their model, contracts depend on the quantities the retailer purchases from both suppliers. In contrast, we find that equilibrium contracts exhibit below-cost pricing when we restrict attention to contracts that depend only on the quantity purchased from a single supplier. In our model, penalty escape clauses are not feasible, but we show that the retailer and the first supplier are still able to extract some surplus from the second supplier. As in Aghion and Bolton, our results are sensitive to assumptions about the possibility of renegotiation (see Section 5).

When the retailer negotiates with the second supplier, the retailer’s disagreement payoff is a decreasing function of the price at which it can buy additional units from the first supplier. The lower this price, the greater the retailer’s opportunity cost of buying from the second supplier (assuming the products of the two suppliers are substitutes), and therefore the more concessions the second supplier must make. Thus, by first contracting with one supplier and obtaining marginal units below cost, the retailer can extract more surplus from the second supplier. In equilibrium, we find that while the retailer’s optimal contract with the first supplier may have the appearance of predatory pricing, the intent is not to drive the second supplier from the market, but rather to extract jointly as much surplus from it as possible, conditional on its remaining viable. We say “jointly” because some of the surplus the retailer extracts from the second supplier must be shared with the first supplier—the contract negotiated between the retailer and the first supplier gives the first supplier higher inframarginal payments in exchange for its commitment to sell marginal units below cost. As a consequence, the sales and profits of the second supplier decrease.

Nevertheless, we show that predatory accommodation can be procompetitive. This result obtains because below-cost pricing causes consumer surplus to increase, and this increase can outweigh the reduction in overall joint profit associated with the pricing distortion. Since below-cost pricing in intermediate goods markets is not necessarily anticompetitive, the two prerequisites for identifying predatory pricing described above are not sufficient to imply anticompetitive behavior in intermediate goods markets. This suggests that claims under Section 2(a) of the Robinson-Patman Act, which addresses predatory pricing in intermediate goods markets, should not be treated the same as predatory pricing claims under Section 2 of the Sherman Act. Thus, our findings do not provide support for the recent Supreme Court ruling, in *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*, 509 U.S. 221 (1993), that “primary-line competitive injury under the Robinson-Patman Act is of the same general character as the injury inflicted by predatory pricing schemes” under the Sherman Act.

in below-cost pricing. Thus, when we find below-cost pricing in this article, we satisfy a requirement of the Robinson-Patman Act that the alleged predatory firm sell its product at different prices to different purchasers.

Dobson (1994) and Marshall and Merlo (1996) also consider rent shifting via sequential contracting. They do so, however, from the perspective of a wage-negotiating union (one supplier and two retailers, in contrast to our two suppliers and one retailer).

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Previous research on predatory pricing has split along two lines. One line aims to provide a framework to help policymakers distinguish anticompetitive behavior from procompetitive price cutting (e.g., Areeda and Turner, 1975; Williamson, 1977; Baumol, 1979; and Joskow and Klevorick, 1979). A second line of research uses game theory to rationalize the use of predatory pricing in certain settings. Predation has been found to be more likely, for instance, if (a) capital markets are imperfect and the predator has greater financial resources than its prey, e.g., Fudenberg and Tirole (1985), (b) there is an informational asymmetry about demand or cost between an established firm and new entrant, e.g., Fudenberg and Tirole (1986) and Saloner (1987), (c) there are multiple potential entrants, e.g., Rosenthal (1981) and Milgrom and Roberts (1982), or (d) technology is characterized by learning by doing, e.g., Cabral and Riordan (1994, 1997). Our work follows this second line of research.

The article is organized as follows. Section 2 describes the model. Section 3 characterizes the equilibrium and provides some comparative statics results. Section 4 presents welfare results. Section 5 discusses extensions and limitations of the model. Section 6 concludes.

2. Model

- We consider an intermediate goods market in which suppliers X and Y sell their products to a common retailer for subsequent distribution to final consumers. We assume that the retailer is a local monopolist in the final goods market, that supplier X is the sole supplier of product X, and that supplier Y is the sole supplier of product Y. We assume also that suppliers X and Y incur zero fixed costs and have constant-marginal-cost production technologies with marginal costs c_x and c_y, respectively. This assumption is made to avoid any ambiguity in identifying below-cost pricing as such when it occurs.5

A distinguishing feature of supply contracts in intermediate goods markets, as opposed to final goods markets, is that they are often negotiated. Consistent with this, supply contracts in our model are negotiated. To illustrate our idea that the retailer can use its contract with one supplier to gain a stronger bargaining position in negotiations with another supplier, we assume that the negotiations with the suppliers are sequential. We let supplier X be the first supplier to negotiate with the retailer. Thus, we consider a three-stage game. In stage one, the retailer negotiates a contract with supplier X for the purchase of product X. In stage two, the retailer negotiates a contract with supplier Y for the purchase of product Y. In stage three, the retailer purchases quantities \( x \) of product X and \( y \) of product Y and sells these quantities in the final goods market.

Another characteristic of supply contracts in intermediate goods markets is that they typically exhibit nonlinear pricing; the retailer and each supplier must reach an agreement both over the price of a marginal unit and over how the surplus is to be divided. Since the simplest contract form that captures these elements is a two-part tariff, we restrict attention to \( T_x \) and \( T_y \) of the form

\[
T_x(x | w_x, F_x) = \begin{cases} 
  w_x x + F_x, & x > 0 \\
  0, & x = 0
\end{cases} \quad \text{and} \quad T_y(y | w_y, F_y) = \begin{cases} 
  w_y y + F_y, & y > 0 \\
  0, & y = 0
\end{cases}
\]

5 The Supreme Court declined to specify in Brooke how below-cost pricing is to be identified. Most lower courts have adopted average variable cost as their measure, although some use marginal cost and others use average total cost. These measures are the same when marginal costs are constant and there are no fixed costs.

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where $w_t$ is the marginal price of an additional unit and $F_i$ divides the surplus. If the retailer buys quantity $x$ of product $X$, supplier $X$'s profit is $T_i(x|w_1, F_i) - c_i x$. If the retailer buys quantity $y$ of product $Y$, supplier $Y$'s profit is $T_i(y|w_1, F_i) - c_i y$. In Section 5, we discuss generalizations of the contract form.

We let $R(x, y)$ denote the retailer's maximum resale revenue if it purchases quantities $x$ and $y$. Then, the retailer's profit is $R(x, y) = T_i(x|w_1, F_i) - T_i(y|w_1, F_i)$. We make minimal assumptions about $R(x, y)$, assuming only that it is differentiable, strictly concave, bounded, has $R(0, 0) = 0$, and for all $b > a$, $\frac{\partial R(\cdot, a)}{\partial x} > \frac{\partial R(\cdot, b)}{\partial x}$, which implies that products $X$ and $Y$ are (imperfect) substitutes. One can also interpret the retailer as being the final consumer, in which case $R(x, y)$ is the consumer's utility function.

We also make minimal assumptions about the bargaining outcomes, assuming only that bargaining between the retailer and supplier $i$ results in maximization of the two firms' joint profit, and that the division of surplus is such that each firm receives its disagreement payoff plus a share of the incremental gains from trade (the joint profit of the retailer and supplier $i$ when the two firms trade minus their joint profit when they do not trade), with proportion $\lambda_i \in [0, 1]$ going to supplier $i$. These assumptions are consistent with the commonly used bargaining solutions, all of which require that players maximize bilateral joint profits and divide the incremental gains from trade. Our assumption of a fixed division of the incremental gains from trade admits several interpretations. For example, if the suppliers make take-it-or-leave-it offers to the retailer, then $\lambda_i = 1$. If the retailer makes take-it-or-leave-it offers to the suppliers, then $\lambda_i = 0$. In the bargaining game, the maximum payoffs for the players above their disagreement points are the same (and equal to the maximum joint profit minus the disagreement payoffs), so the Nash bargaining solution and Kalai-Smorodinsky bargaining solution both divide the gains from trade equally (see Nash, 1953; Binmore, Rubinstein, and Wolinsky, 1986; and Kalai and Smorodinsky, 1975), and, by definition, the egalitarian bargaining solution divides the gains equally (see Mas-Colell, Whinston, and Green, 1995). Thus, the Nash bargaining solution, the egalitarian bargaining solution, and the Kalai-Smorodinsky bargaining solution all imply $\lambda_i = \frac{1}{2}$.

3. Equilibrium

- We solve for the equilibrium strategies of the retailer and suppliers $X$ and $Y$ by working backward, taking our assumptions about the outcome of negotiations as given. Our solution concept corresponds to subgame perfection in that equilibrium behavior in early periods is not supported by noncredible threats in later periods.\(^8\)

\[
\max_{x, y} R(x, y) - T_i(x|w_1, F_i) - T_i(y|w_1, F_i). \tag{1}
\]

\(^a\) We assume that a supplier's profit is zero if it does not trade with the retailer.

\(^7\) In Rubinstein-alternating-offers bargaining with a common discount factor $\delta$, if the supplier makes the first offer, then $\lambda_i = [1 - \delta]/[1 - \delta^2]$ and if the retailer makes the first offer, then $\lambda_i = [\delta(1 - \delta)]/[1 - \delta^2]$ (see Rubinstein, 1982). In the limit as $\delta \to 1$, the surplus in the Rubinstein bargaining solution is divided equally. For additional discussion of bargaining models, see Roth (1985).

\(^8\) The equilibrium we identify corresponds to the subgame-perfect equilibrium of a related three-stage game in which the assumed bargaining solution is embedded in the players' payoff functions.
By the strict concavity and boundedness of $R$, if $w_i > 0$ for $i \in \{x, y\}$, the maximizers in (1) exist and are uniquely defined when they are both positive. We define $\hat{x}$ and $\hat{y}$ to be the positive maximizers of (1) when positive maximizers exist, and zero otherwise.

If bargaining with supplier $Y$ breaks down, the retailer chooses $x$ to solve

$$\max_{x \geq 0} R(x, 0) - T_y(x \mid w_x, F_y),$$

and if bargaining with supplier $X$ breaks down, the retailer chooses $y$ to solve

$$\max_{y \geq 0} R(0, y) - T_y(y \mid w_y, F_y).$$

Define $x^m$ and $y^m$ to be the positive maximizers of (2) and (3) when positive maximizers exist, and zero otherwise. (The superscript $m$ is mnemonic for monopoly.)

We now define six mutually exclusive and collectively exhaustive sets of contracts, $\Omega_{x, x}, \Omega_{x, 0}, \Omega_{y, x}, \Omega_{y, 0}, \Omega_{x, 0}$, and $\Omega_{y, 0}$, which will be useful in what follows. Let $\Omega_{x, x} = \{T_x, T_y \mid \hat{x} > 0, \hat{y} > 0\}$ be the set of contracts such that the retailer purchases a positive amount of both products, and let $\Omega_{x, 0} = \{T_x, T_y \mid \hat{x} = 0, \hat{y} = 0\}$ be the set of contracts such that the retailer does not purchase either product. Let

$$\Omega_{x, x} = \{T_x, T_y \mid \hat{x} > 0, \hat{y} = 0, x^m > 0\}$$

be the set of contracts such that the retailer purchases a positive amount of product $X$ only, and $y^m > 0$; and let $\Omega_{x, 0} = \{T_x, T_y \mid \hat{x} > 0, \hat{y} = 0, y^m = 0\}$ be the set of contracts such that the retailer purchases a positive amount of product $X$ only, and $y^m = 0$. Define $\Omega_{y, x}$ and $\Omega_{y, 0}$ analogously, i.e.,

$$\Omega_{y, x} = \{T_x, T_y \mid \hat{x} = 0, \hat{y} > 0, x^m > 0\} \quad \text{and} \quad \Omega_{y, 0} = \{T_x, T_y \mid \hat{x} = 0, \hat{y} > 0, x^m = 0\}.$$

It will sometimes be convenient to write $((w_x, F_x), (w_y, F_y)) \in \Omega_t$ to mean that a particular $T_x$ with parameters $(w_x, F_x)$ and $T_y$ with parameters $(w_y, F_y)$ satisfy $(T_x, T_y) \in \Omega_t$.

The retailer’s optimal choices in stage three can be written as

$$\hat{x} = \begin{cases} x^*(w_x), & \text{if } (T_x, T_y) \in \Omega_{x, x} \cup \Omega_{x, 0} \\ x^*(w_x, w_y), & \text{if } (T_x, T_y) \in \Omega_{x, x} \\ 0, & \text{if } (T_x, T_y) \in \Omega_{x, 0} \cup \Omega_{y, x} \cup \Omega_{y, 0} \end{cases}$$

$$\hat{y} = \begin{cases} y^*(w_y), & \text{if } (T_x, T_y) \in \Omega_{x, x} \cup \Omega_{x, 0} \\ y^*(w_x, w_y), & \text{if } (T_x, T_y) \in \Omega_{y, x} \\ 0, & \text{if } (T_x, T_y) \in \Omega_{x, 0} \cup \Omega_{y, x} \cup \Omega_{y, 0} \end{cases}$$

Our convention is that two asterisks in the superscript indicate that both $\hat{x}$ and $\hat{y}$ are positive, and a single asterisk in the superscript indicates that only one of $\hat{x}$ or $\hat{y}$ is positive. Note that $x^*$ is equal to $x^m$ when $x^m$ is positive, and $y^*$ is equal to $y^m$ when $y^m$ is positive. Note also that $x^*(w_x)$ satisfies the first-order condition

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\[
\frac{\partial R(x^*, 0)}{\partial x} = w^r, \tag{4}
\]

\(y^*(w^r)\) satisfies the first-order condition

\[\frac{\partial R(0, y^*)}{\partial y} = w^r, \tag{5}\]

and \(x^{**}(w^r, w^y)\) and \(y^{**}(w^r, w^r)\) satisfy the two first-order conditions

\[\frac{\partial R}{\partial x}(x^{**}, y^{**}) = w^r \quad \text{and} \quad \frac{\partial R}{\partial y}(x^{**}, y^{**}) = w^r. \tag{6}\]

That is, at any interior optimum involving supplier \(i\), the retailer equates marginal revenue from sales of product \(i\) to its marginal cost of purchasing product \(i\).

In what follows, it will be useful to note that our assumption that the products are imperfect substitutes implies that \(x^{**}(c^r, c^y) < x^*(c^r)\). To see this, note that \(x^{**}(c^r, c^y) \geq x^*(c^r)\), the concavity of \(R\), and the definition of substitute goods implies that \([\partial R(x^{**}, y^{**})] / \partial x \leq [\partial R(x^*, y^{**})] / \partial x < [\partial R(x^*, 0)] / \partial x\), which is a contradiction since (4) and (6) imply that \([\partial R(x^{**}, y^{**})] / \partial x = [\partial R(x^*, 0)] / \partial x\).

\(\square\) Stage two. In stage two, the retailer negotiates with supplier \(Y\) over contract \(T_y\). In the negotiations, the firms take \(T_y\) as given and choose \(T_y\) to maximize their joint profit,

\[
\max_{w^r, F^r} R(\hat{x}, \hat{y}) - T_y(\hat{x} \mid w^r, F^r) - c^y \hat{y}, \tag{7}
\]

and divide the incremental gains from trade so that each receives its disagreement payoff plus a share of the gains, with proportion \(\lambda\), going to supplier \(Y\). Given \(T_y\) and \(T_y\), the incremental gains from trade are

\[
g_y(T_y, T_y) \equiv R(\hat{x}, \hat{y}) - T_y(\hat{x} \mid w^r, F^r) - c^y \hat{y} - (R(x^m, 0) - T_y(x^m \mid w^r, F^r)). \tag{8}
\]

Thus, \(F^r\) is chosen so that supplier \(Y\)'s profit is equal to \(\lambda\) times \(g_y(T_y, T_y)\).

Proposition 1. If the gains from trade between the retailer and supplier \(Y\), given \(T_y\), are positive, the unique equilibrium \(T_y\) has \(\hat{w}_y = c^y\) and \(\hat{F}_y(w^r) = \lambda_y g_y(T_y, (c^y, 0)).^9\)

Proof. See Appendix A.

Proposition 1 shows that the joint profit of the retailer and supplier \(Y\) is maximized when supplier \(Y\)'s per-unit price is equal to its marginal cost. In effect, supplier \(Y\) and the retailer agree that the latter should be the residual claimant to their joint profits, with supplier \(Y\) receiving its profits in the form of a lump-sum payment \(\hat{F}_y(w^r)\). Intuitively, since supplier \(Y\) is the last one to negotiate with the retailer, there is no incentive to distort supplier \(Y\)'s per-unit price or the retailer's quantity choice for product \(Y\).

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\(^9\) If there are no gains from trade with supplier \(Y\), the optimal quantity of product \(Y\) is zero, so \(w^r = c^y\) and \(F^r = 0\) is an equilibrium but is not unique.

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Stage one. In stage one, the retailer negotiates with supplier \( X \) over contract \( T_i \). In the negotiations, the firms take as given the equilibrium strategies in stages two and three. We assume that \( T_i \) is chosen to maximize the joint profit of the retailer and supplier \( X \).

\[
\max_{w, \hat{w}} R(\hat{x}, \hat{y}) - c, \hat{x} - T_i(\hat{y} | c, \hat{F}(w)).
\]

We henceforth assume that the maximand in (9) is concave in \( w \), for all \( w \), such that \( \hat{x} > 0 \). We also assume that each firm receives its disagreement payoff plus a share of the incremental gains from trade, with proportion \( \lambda \), going to supplier \( X \). Given \( T_i \), if the retailer purchases a positive quantity of product \( X \), the incremental gains from trade are

\[
g_i(T_i) = R(\hat{x}, \hat{y}) - c, \hat{x} - T_i(\hat{y} | c, \hat{F}(w)) - (1 - \lambda)(R(0, y^m) - c, y^m).
\]

Thus, \( F_i \) is chosen so that supplier \( X \)'s profit is equal to \( \lambda \), times \( g_i(T_i) \).

We now consider two cases with different assumptions about the production costs of suppliers \( X \) and \( Y \) and the demand for products \( X \) and \( Y \). We define the production of product \( i \), \( i \in \{ X, Y \} \), to be "efficient" if and only if a fully integrated firm (horizontally and vertically) would strictly choose to produce it. We define the production of product \( i \) to be "inefficient" if it is optimal for a fully integrated firm not to produce product \( i \). We also distinguish between two kinds of inefficient production. We call the production of product \( i \) "weakly inefficient" if its production would be efficient if the fully integrated firm were constrained not to produce product \( j, j \neq i \), and "strongly inefficient" otherwise.

Case 1 (production of products \( X \) and \( Y \) is efficient). We begin with the case in which the production of products \( X \) and \( Y \) is efficient, i.e., \( (c, 0), (c, 0) \) \( \in \Omega_i \), and establish that both products are sold in equilibrium.

Proposition 2. If the production of products \( X \) and \( Y \) is efficient, then sales of products \( X \) and \( Y \) are positive in equilibrium.

Proof. See Appendix A.

Proposition 2 establishes that if the production of both products is efficient, the retailer and supplier \( X \) do not use a contract that excludes supplier \( Y \) from the market.\(^{10}\) Nevertheless, they do use below-cost pricing as shown in the following proposition.

Proposition 3. If the production of products \( X \) and \( Y \) is efficient and \( \lambda, > 0 \), the unique equilibrium \( T_i \) has \( w < c \) (if \( \lambda, = 0 \), then \( w = c \)).

Proof. Since Proposition 2 implies \( \hat{x} > 0 \) and \( \hat{y} > 0 \) in equilibrium, and since \( x^ > 0 \) implies \( x^m > 0 \), we can characterize the optimal contract \( T_i \), using first-order conditions. Differentiating the joint profit of the retailer and supplier \( X \), given in (9), with respect to \( w \), the optimal per-unit price \( \hat{w} \), satisfies

\[
\left( \frac{\partial R(x^*, y^*)}{\partial x} - c \right) \frac{\partial x^*}{\partial w} + \left( \frac{\partial R(x^*, y^*)}{\partial y} - c \right) \frac{\partial y^*}{\partial w} - \frac{\partial \hat{F}(w)}{\partial w} = 0,
\]

\(^{10}\) O'Brien and Shaffer (1997) show that if a monopolist retailer contracts with two suppliers simultaneously, there exist parameters of the model such that one of the suppliers is excluded in equilibrium even though the production of both goods is efficient.
where $x^{**}$ and $y^{**}$ are evaluated at $(\hat{w}_r, c_r)$. Differentiating $\hat{F}_r(w_r) = \lambda_x T_r, (c_r, 0))$ with respect to $w_r$, yields $[\partial \hat{F}_r(w_r)]/\partial w_r = -\lambda_x (x^{**} - x^*)$. Substituting this into (11), and using the first-order conditions for the case in which the retailer buys both $X$ and $Y$, given in (6), we get

$$\begin{align*}
(\hat{w}_r - c_r) \frac{\partial x^{**}(\hat{w}_r, c_r)}{\partial w_r} + \lambda_x (x^{**}(\hat{w}_r, c_r) - x^*(\hat{w}_r)) &= 0.
\end{align*}$$

(12)

Evaluating the left-hand side of (12) at $\hat{w}_r = c_r$, we get $\lambda_x (x^{**}(c_r, c_r) - x^*(c_r))$, which is negative if $\lambda_x > 0$ because the products are imperfect substitutes, i.e., $x^{**}(c_r, c_r) < x^*(c_r)$. Thus, since $[\partial x^{**}(c_r, c_r)]/\partial w_r < 0$ by the strict concavity of $R$, we have $\hat{w}_r < c_r$. If $\lambda_x = 0$, then (12) implies $\hat{w}_r = c_r$. Q.E.D.

Proposition 3 shows that if the production of products $X$ and $Y$ is efficient, the retailer and supplier $X$ have an incentive to agree to a per-unit price for product $X$ that is less than supplier $X$’s marginal cost. The intuition for the result has two parts. First, a lower per-unit price from supplier $X$ increases the retailer’s disagreement payoff, giving the retailer a stronger bargaining position in its subsequent negotiations with supplier $Y$. At the margin, a lower per-unit price increases the retailer’s disagreement payoff in proportion to $x^*$. This consideration provides the retailer with an incentive for below-cost pricing with supplier $X$. However, a lower per-unit price from supplier $X$ also increases the retailer’s joint profit with supplier $Y$, giving the retailer a weaker bargaining position in its subsequent negotiations with supplier $Y$. At the margin, a lower per-unit price increases the retailer’s joint profit with supplier $Y$ in proportion to $x^{**}$. This consideration provides the retailer with an incentive for above-cost pricing with supplier $X$. Proposition 3 implies that the first consideration dominates, provided that there is additional surplus to extract from supplier $Y$ ($\lambda_x > 0$). Thus, a lower per-unit price from supplier $X$ allows the retailer to extract more of the gains from trade with supplier $Y$, but the gains available to be extracted are smaller.

Since a lower per-unit price for product $X$ weakens supplier $Y$’s bargaining position and reduces the gains from trade between the retailer and supplier $Y$, below-cost pricing by supplier $X$ reduces supplier $Y$’s profits relative to marginal-cost pricing. This is the predatory aspect of the retailer and supplier $X$’s strategy. However, we know from Proposition 2 that the below-cost pricing does not drive supplier $Y$’s sales to zero. This is the accommodation aspect of the retailer and supplier $X$’s strategy. The strategy of predatory accommodation is optimal for the retailer and supplier $X$ because surplus is higher when both products $X$ and $Y$ are sold than when just product $X$ is sold, and the retailer and supplier $X$ extract from supplier $Y$ some of the surplus contributed by its product.

To conclude this case, note that supplier $X$’s fixed fee, $\hat{F}_r$, is chosen such that supplier $X$ earns profit equal to $\lambda_x$ times the incremental gains from trade with the

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11 To see this, note that

$$\frac{\partial \hat{F}_r(w_r)}{\partial w_r} = \lambda_x \left[ \left( \frac{\partial R(x^{**}, y^{**})}{\partial x} - w_r \right) \frac{\partial x^{**}}{\partial w_r} + \left( \frac{\partial R(x^{**}, y^{**})}{\partial y} - c_r \right) \frac{\partial y^{**}}{\partial w_r} - \left( \frac{\partial R(x^*, 0)}{\partial x} - w_r \right) \frac{\partial x^*}{\partial w_r} - x^{**} - x^* \right]$$

and use (4) and (6).

12 If the retailer has all of the bargaining power vis-à-vis supplier $Y$, then there is no additional surplus to extract from supplier $Y$, and below-cost pricing has no value to the retailer and supplier $X$. If products $X$ and $Y$ are complements, i.e., $[\partial R(a, b)]/\partial x > [\partial R(a, b)]/\partial x$ for all $a > b$, then $x^{**} > x^*$ and the optimal distortion calls for above-cost pricing.
retailer. In other words, \( \hat{F}_x \) satisfies \( (\hat{\check{\nu}}_x - c_0) x**(\hat{\check{\nu}}_x, c_0) + \hat{F}_x = \lambda_x g_x(\hat{\check{\nu}}_x, 0) \), which implies that

\[
\hat{F}_x = (c_x - \hat{\check{\nu}}_x) x**(\hat{\check{\nu}}_x, c_x) + \lambda_x g_x(\hat{\check{\nu}}_x, 0).
\]

(13)

Intuitively, the first term in (13) brings supplier \( X \)'s payoff up to its disagreement payoff of zero (supplier \( X \) is fully compensated for its direct loss from selling each unit at below marginal cost), and the second term gives supplier \( X \) a proportion \( \lambda_x \) of the incremental gains from trade with the retailer. Thus, the two prerequisites to establish predatory pricing are satisfied; supplier \( X \)'s prices below cost and recoupment is assured.

**Case 2 (production of one of the products is weakly inefficient).** We now consider the case in which the production of one of the products is weakly inefficient.\(^{13}\) The following lemma characterizes the equilibrium when the production of product \( Y \) is weakly inefficient, i.e., \( (c_y, 0), (c_x, 0) \) \( \in \Omega_{x,y} \).

**Lemma 1.** If the production of product \( Y \) is weakly inefficient, the unique equilibrium \( T_x \) has \( w_x = c_x \), and the retailer does not sell product \( Y \) in equilibrium.

**Proof.** See Appendix A.

Lemma 1 shows that if the production of product \( Y \) is weakly inefficient, then sales of product \( Y \) are zero. Moreover, supplier \( X \)'s per-unit price is equal to its marginal cost, which induces the retailer to purchase and then resell the monopoly quantity \( x^*(c_x) \).

We now consider the case in which the production of product \( X \) is weakly inefficient, i.e., \( (c_x, 0), (c_y, 0) \) \( \in \Omega_{x,y} \).

**Lemma 2.** If the production of product \( X \) is weakly inefficient and \( \lambda_x > 0 \), any equilibrium \( T_x \) has \( w_x < c_x \). There exist cases in which the retailer sells product \( X \) in equilibrium.

**Proof.** See Appendix A.

The retailer and supplier \( X \) negotiate a per-unit price that is less than marginal cost to extract surplus from the second supplier, just as in the case in which the production of both products is efficient. The retailer's incentives in stage three are then no longer the same as those of a fully integrated firm. It should not be surprising, therefore, that the retailer sometimes wants to sell a positive quantity of product \( X \) in equilibrium, even though positive sales of both \( X \) and \( Y \) yield lower overall joint profit than no sales of product \( Y \) alone. For example, if the retailer is just indifferent to selling positive amounts of product \( X \) at \( w_x = c_x \), then, by continuity, the retailer strictly prefers to sell \( x > 0 \) when \( w_x < c_x \).\(^{14}\)

The next lemma compares the retailer's equilibrium profit when it negotiates first with the supplier with the weakly inefficient production process versus when it negotiates second with that supplier.

---

\(^{13}\) If the production of a supplier's product is strongly inefficient, then it can be shown that the supplier's presence has no impact on the equilibrium contract with the other supplier, so there is no below-cost pricing and no distortion. Thus, we focus on the case in which the production of one of the products is weakly inefficient.

\(^{14}\) O'Brien and Shaffer (1997) show that a product whose production is inefficient is never sold in equilibrium when two suppliers simultaneously make take-it-or-leave-it offers to a retail monopolist. However, Lemma 2 and the example in Section 4 show that this result does not extend to the case of sequential contract offers with bargaining.
Lemma 3. If the production of product $i$, $i \in \{X, Y\}$, is weakly inefficient, and $\lambda_j > 0$, $j \neq i$, the retailer has higher profit if it negotiates first with supplier $i$ than if it negotiates second with supplier $i$.

Proof. See Appendix A.

Lemma 3 implies that the retailer prefers to negotiate first with the supplier of the product whose production is weakly inefficient. To see the intuition for Lemma 3, note that if a supplier with an inefficient production process negotiates second, then that supplier only affects the retailer’s profit insofar as it affects the retailer’s disagreement payoff should negotiations break down with the first supplier. Thus, a supplier with a weakly inefficient production process provides some value to the retailer as a threat if it negotiates second. However, if it negotiates first, then that supplier can provide an even stronger threat through the use of a contract that has below-cost pricing. The proof is by revealed preference; the retailer can never be worse off negotiating first with a supplier with an inefficient production process, since the retailer and this supplier can always choose a contract with marginal-cost pricing, in which case the supplier’s product is not purchased in equilibrium. By Lemma 2, the retailer and the supplier with the weakly inefficient production process can earn strictly higher profit by pricing below cost.

Using Lemmas 1, 2, and 3, we have the following result.

Proposition 4. If the production of one product is weakly inefficient and the retailer chooses the order of negotiations, then the unique equilibrium contract with the first supplier has a per-unit price that is less than the first supplier’s marginal cost.

Propositions 3 and 4 imply that equilibrium contracts exhibit below-cost pricing when the retailer contracts sequentially and can choose the order of negotiations, provided the production of each product is either efficient or weakly inefficient.

□ Comparative statics. In this subsection we assume that the production of both products is efficient. Then, the degree to which supplier $X$’s per-unit price is distorted below its marginal cost depends on a number of factors. First, consider how the distortion is affected by an increase in the bargaining power of supplier $Y$. Totally differentiating (12) yields

$$\frac{\partial \hat{w}_i}{\partial \lambda_i} = \frac{x^* - x^{**}}{(\hat{w}_i - e_i) \frac{\partial^2 x^{**}}{\partial w_i^2} + \frac{\partial x^{**}}{\partial w_i} + \lambda_i \left( \frac{\partial x^{**}}{\partial w_i} - \frac{\partial x^*}{\partial w_i} \right)}.$$ 

Since $x^* > x^{**}$, we know that $\partial \hat{w}_i/\partial \lambda_i$ is less than zero if and only if the denominator is negative, which it must be given our assumption that the joint profit of the retailer and supplier $X$ is concave in $w_i$. Hence, an increase in the bargaining power of supplier $Y$ increases the distortion. Second, consider how capacity constraints on supplier $X$’s output can affect the distortion. For example, suppose supplier $X$ has capacity constraint $\bar{x} \not\in [x^{**}(\hat{w}_i, e_i), x^*(\hat{w}_i)]$. Then, if negotiations between the retailer and supplier $Y$ break down, the retailer can only purchase min $\{\bar{x}, x^*(\hat{w}_i)\}$ from supplier $X$. In this case,

$$\frac{\partial F_i(c_i)}{\partial w_i} = \begin{cases} \lambda_i(x^*(c_i) - x^{**}(c_i, e_i)), & \text{if } x^*(c_i) \leq \bar{x} \\ \lambda_i(\bar{x} - x^{**}(c_i, e_i)), & \text{if } x^*(c_i) > \bar{x} \end{cases}.$$  

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so the value of $w_1$ that maximizes the retailer and supplier $X$'s joint profit is still less than $c_r$, but the distortion is reduced. The lower is $\bar{x}$, the less effective is a distortion in $w_1$ at strengthening the retailer's stage-two bargaining position, and so the smaller is the equilibrium distortion. These results are summarized in the following proposition.

**Proposition 5.** If $\lambda_x > 0$, then the equilibrium distortion in supplier $X$'s per-unit price is increasing in $\lambda_x$ and increasing in supplier $X$'s capacity in the range from $x^{**}(\hat{w}_r, c_r)$ to $x^*(\hat{w}_r)$.

Proposition 5 describes how supplier $Y$'s bargaining power and supplier $X$'s capacity affect the equilibrium distortion in $w_r$. This distortion is also affected by the degree to which products $X$ and $Y$ are substitutes. However, the effect of an increase in the substitutability between $X$ and $Y$ is, in general, ambiguous. If the products are unrelated, then the quantity of $X$ sold does not depend on whether product $Y$ is sold, i.e., $x^{**} = x^*$, and there is no distortion. As $X$ and $Y$ move from being unrelated toward being perfect substitutes, the presence of product $Y$ has a negative effect on sales of product $X$, so the difference $x^* - x^{**}$ increases. This, by itself, would cause the distortion to increase. But as the products become better substitutes, the quantity of product $X$ sold becomes more sensitive to its own price, so $\partial x^{**}/\partial w_r$ decreases. By itself, this would cause the distortion to decrease. The net effect of an increase in the products' substitutability on the distortion is thus ambiguous.

4. Welfare

- Is welfare (the sum of consumer surplus and overall joint profit) higher or lower in the below-cost pricing regime ($w_r = \hat{w}_r$) than in the marginal-cost pricing regime ($w_r = c_r$)? Although we know that overall joint profit is lower when $w_r = \hat{w}_r$, consumer surplus cannot readily be scaled in general (a lower $w_r$ implies larger quantities of $x$ sold, but lower quantities of $y$ sold), thus making welfare comparisons difficult. Nevertheless, we are able to obtain unambiguous welfare results in some circumstances, e.g., in the cases discussed below when consumer surplus is zero, and in the special case of symmetric linear demands.

Define $u(x, y)$ to be the utility of a representative consumer who purchases quantities $x$ and $y$. Then equilibrium consumer surplus is $CS = u(\bar{x}, \bar{y}) - R(\bar{x}, \bar{y})$, overall joint profit is $Π = R(\bar{x}, \bar{y}) - c_r\bar{x} - c_y\bar{y}$, and welfare is $W = Π + CS$.

Since the expression for the difference between welfare in the two regimes is difficult to sign, even with symmetric linear demands, we look for conditions under which we can sign the derivative of welfare with respect to $w_r$ over the relevant range of values for $w_r$. If the production of both products is efficient, then, for all $w_r < c_r$, $\partial Π/\partial w_r = (w_r - c_r)\partial x^{**}/\partial w_r$ is positive, so overall joint profit is lower in the below-cost pricing regime. However, consumers can be either better or worse off because

$$\frac{\partial CS}{\partial w_r} = \left(\frac{\partial u(x^{**}, y^{**})}{\partial x} - w_r\right)\frac{\partial x^{**}}{\partial w_r} + \left(\frac{\partial u(x^{**}, y^{**})}{\partial y} - c_r\right)\frac{\partial y^{**}}{\partial w_r},$$

(14)

where $x^{**}$ and $y^{**}$ are evaluated at $(w_r, c_r)$.

Given that the sign of $\partial CS/\partial w_r$ is ambiguous, it is not surprising that the sign of the derivative of welfare with respect to $w_r$ is also ambiguous:

15 The first pair of terms is nonpositive, since $[\partial u(x^{**}, y^{**})]/\partial x \geq w_r$, and $\partial x^{**}/\partial w_r < 0$, and the second pair of terms is nonnegative, since $[\partial u(x^{**}, y^{**})]/\partial y \geq c_r$, and $\partial y^{**}/\partial w_r > 0$. 

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\[
\frac{\partial W}{\partial w_s} = \left( \frac{\partial u(x^{**}, y^{**})}{\partial x} - c_s \right) \frac{\partial x^{**}}{\partial w_s} + \left( \frac{\partial u(x^{**}, y^{**})}{\partial y} - c_s \right) \frac{\partial y^{**}}{\partial w_s}.
\]

(15)

Unambiguous welfare results, however, can be obtained in some circumstances.

First, consider how the retailer's ability to price discriminate in the final goods market affects welfare. In particular, suppose that the retailer can perfectly price discriminate. In this case, \(\partial u/\partial x = w_s\) and \(\partial u/\partial y = c_s\), so \(\partial W/\partial w_s = \partial \Pi/\partial w_s\). Thus, our previous observation that \(\partial \Pi/\partial w_s > 0\) for all \(w_s < c_s\) implies that welfare is lower when \(w_s < c_s\) than when \(w_s = c_s\).

**Proposition 6.** If the retailer can perfectly price discriminate in the final goods market, then welfare is lower when \(w_s < c_s\) than when \(w_s = c_s\).

Second, consider the interpretation of our model in which the retailer is the final consumer. In this case, \(R(x, y)\) is the utility function, and welfare becomes \(R(\hat{x}, \hat{y}) - c_s\hat{x} - c_s\hat{y}\). Once again, welfare is lower when \(w_s < c_s\) than when \(w_s = c_s\).

**Proposition 7.** If suppliers \(X\) and \(Y\) negotiate directly with a final consumer, then welfare is lower when \(w_s < c_s\) than when \(w_s = c_s\).

In Propositions 6 and 7, the welfare comparison is unambiguous because \(CS\) is zero; either the representative consumer literally has zero surplus, or the consumer and retailer are indistinguishable. In general, neither of these circumstances holds. Moreover, welfare need not be lower in the below-cost pricing regime.

To verify that welfare can actually be higher in the below-cost pricing regime, and to obtain further welfare results, we now specialize to the case of symmetric linear demands, using the inverse demand functions

\[
p_s(x, y) = a - x - \gamma y \quad \text{and} \quad p_s(x, y) = a - y - \gamma x, \quad \text{where} \quad \gamma \in (0, 1).
\]

This allows us to establish necessary and sufficient conditions for below-cost pricing in intermediate goods markets to increase welfare.

**Proposition 8.** In the symmetric linear demand case with no retail price discrimination, necessary and sufficient conditions for welfare to be higher when \(w_s = \hat{w}_s\) than when \(w_s = c_s\) are, respectively, \(c_s < a(1 - \gamma) + \gamma c_s\) and

\[
c_s < \frac{(a(1 - \gamma) + c_s \gamma)(1 + \gamma^2 \lambda_s) - \gamma \lambda_s (a(1 - \gamma) - c_s)}{(1 + 2 \gamma^2 \lambda_s)}.
\]

(16)

**Proof.** See Appendix B.

For example, if \(a = 1\), \(\gamma = .5\), \(c_s = c_r = .25\), and \(\lambda_s = \lambda_r = .5\), then the sufficient condition in Proposition 8 is satisfied, and welfare increases as a result of below-cost pricing. Table 1 shows that for these parameter values, the joint profit of the retailer and supplier \(X\) increases slightly (1.59%) at the expense of supplier \(Y\), who suffers a large decrease in profit (−20.99%). There are increases in consumer surplus (12.35%) and welfare (3.29%). (See Appendix B for the details of the numerical examples.)

However, if we increase \(c_s\) from .25 to .7, the necessary condition in Proposition 8 is not satisfied, and below-cost pricing reduces welfare. Table 2 shows that welfare decreases by −1.65%. In this case, even though the change in consumer surplus is positive, the decrease in overall joint profit dominates.
TABLE 1  Example in Which Below-cost Pricing Increases Welfare

<table>
<thead>
<tr>
<th></th>
<th>Quantity of Product X</th>
<th>Quantity of Product Y</th>
<th>Overall and X's Joint Profit</th>
<th>Retailer and X's Joint Profit</th>
<th>Y's Profit</th>
<th>Consumer Surplus</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent change</td>
<td>22.22%</td>
<td>-11.11%</td>
<td>-1.23%</td>
<td>1.59%</td>
<td>-20.99%</td>
<td>12.35%</td>
<td>3.29%</td>
</tr>
</tbody>
</table>

Note: Percent change when below-cost pricing is allowed versus when marginal-cost pricing is used. Parameter values are \( a = 1 \), \( \gamma = .5 \), \( c_x = c_r = .25 \), and \( \lambda_x = \lambda_r = .5 \).

The following corollary is a consequence of the fact that the sufficient condition in Proposition 8 holds if \( c_x \leq c_r \).

**Corollary 1.** In the symmetric linear demand case with no retail price discrimination, if \( c_x \leq c_r \), then welfare is higher when \( w_x = \hat{w}_x \) than when \( w_x = c_r \).

Intuitively, since below-cost pricing by supplier X increases sales of product X and decreases sales of product Y, we would expect welfare to increase when supplier X has a lower-cost production process than supplier Y. To fully understand the corollary, however, recall from the previous section that the distortion in supplier X's per-unit price is increasing in the difference \( x^* - x^{**} \). Although both \( x^* \) and \( x^{**} \) are decreasing in supplier X's marginal cost, in the linear demand case, the difference \( x^* - x^{**} \) is increasing in supplier X's marginal cost. Thus, a high marginal cost for supplier X implies a high distortion. This distortion increases sales of product X relative to product Y, and if product X is sufficiently more costly to produce than product Y, the distortion in quantities causes welfare to decrease.

We conclude this section by considering the effect of product substitutability on the welfare comparison between the two pricing regimes. The derivative of the right-hand side of (16) with respect to \( \gamma \) is

\[
\frac{(a - c_r)(1 - \gamma^2 \lambda_x^2 + \gamma^2 \lambda_x (1 - \lambda_x) + \lambda_x + 2 \gamma^2 \lambda_x^2)}{(1 + 2 \gamma^2 \lambda_x)^2},
\]

which can be shown to be negative if the production of product Y is efficient (which implies \( a > c_r \)) and \( \gamma \) and \( \lambda \) are elements of \([0, 1]\). Thus, if products X and Y are weaker substitutes, then \( \gamma \) is smaller, and it is more likely that the sufficient condition (16) holds. For example, in the second symmetric linear demand example, which has \( c_y = .25 \), \( c_r = .7 \), and \( \gamma = .5 \), welfare is lower under below-cost pricing, but if the products are made less substitutable so that \( \gamma < .28 \), then welfare is higher under below-cost pricing.

TABLE 2  Example in Which Below-cost Pricing Decreases Welfare

<table>
<thead>
<tr>
<th></th>
<th>Quantity of Product X</th>
<th>Quantity of Product Y</th>
<th>Overall and X's Joint Profit</th>
<th>Retailer and X's Joint Profit</th>
<th>Y's Profit</th>
<th>Consumer Surplus</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent change</td>
<td>Increase from 0 to 0.04</td>
<td>-5.19%</td>
<td>-2.88%</td>
<td>17.42%</td>
<td>-26.71%</td>
<td>0.81%</td>
<td>-1.65%</td>
</tr>
</tbody>
</table>

Note: Percent change when below-cost pricing is allowed versus when marginal-cost pricing is used. Parameter values are \( a = 1 \), \( \gamma = .5 \), \( c_x = .7 \), \( c_r = .25 \), and \( \lambda_x = \lambda_r = .5 \).
5. Discussion

- In this section we discuss limitations of our model, as well as ways in which it can be generalized.

- **Limitations.** There are three key assumptions in our model: the restriction to two-part tariffs, the assumption that there is no renegotiation, and the assumption that contracts are observable. The assumption of two-part tariff contracts is important for some of our results, but our below-cost pricing result holds for all \( T_c: \mathbb{R}_+ \to \mathbb{R} \), as we discuss in the next subsection. In this subsection we discuss the assumptions of no renegotiation and observability. We also comment on our assumptions about the timing of negotiations and purchases.

**Renegotiation.** Spier and Whinston (1995) show that although equilibrium outcomes in Aghion and Bolton's (1987) model can be inefficient when there is uncertainty, adding renegotiation to the model eliminates the inefficiency. Our model is also subject to this same criticism. Once contracts are negotiated, all parties have an incentive to renegotiate so as to eliminate any inefficiencies. However, as we now show, merely allowing the retailer and supplier \( X \) to renegotiate if negotiations between the retailer and supplier \( Y \) break down actually strengthens our results. To see this, suppose that if the initial negotiations between the retailer and supplier \( X \) do not break down, then the retailer and supplier \( X \) can renegotiate if negotiations between the retailer and supplier \( Y \) break down. In this case, the retailer and supplier \( X \) would negotiate a new contract that had \( w_i = c_i \). As a result of the renegotiation, the retailer's profit would increase by a share \((1 - \lambda_i)\) of the increase in the joint profit of the retailer and supplier \( X \):

\[
(1 - \lambda_i)[R(x^*(c_i), 0) - c_i x^*(c_i) - (R(x^*(w_i), 0) - c_i x^*(w_i))].
\]

Thus, the fixed fee that would be negotiated between the retailer and supplier \( Y \) in anticipation of the outcome of renegotiation between the retailer and supplier \( X \) if negotiations with supplier \( Y \) break down, is \( \tilde{F}_i(w_i) \), where

\[
\tilde{F}_i(w_i) = \tilde{F}_i(w_i) - (1 - \lambda_i)[R(x^*(c_i), 0) - c_i x^*(c_i) - (R(x^*(w_i), 0) - c_i x^*(w_i))].
\]

Differentiating \( \tilde{F}_i(w_i) \) with respect to \( w_i \), we get

\[
\frac{\partial \tilde{F}_i(w_i)}{\partial w_i} + (1 - \lambda_i) \left( \frac{\partial R(x^*(w_i), 0)}{\partial x} - c_i \right) \frac{\partial x^*(w_i)}{\partial w_i} = -\lambda_i (x^*(w_i) - x^*(w_i)) + (1 - \lambda_i)(w_i - c_i) \frac{\partial x^*(w_i)}{\partial w_i}.
\]

Substituting this into (11), the optimal per-unit price in this case, \( \tilde{w}_i \), satisfies

\[
(\tilde{w}_i - c_i) \frac{\partial x^*(\tilde{w}_i, c_i)}{\partial w_i} - (1 - \lambda_i)(\tilde{w}_i - c_i) \frac{\partial x^*(\tilde{w}_i)}{\partial w_i} + \lambda_i (x^*(\tilde{w}_i, c_i) - x^*(\tilde{w}_i)) = 0.
\]

Since the left-hand side in this expression is negative at \( \tilde{w}_i = c_i \), we continue to get below-cost pricing in intermediate goods markets even when we allow renegotiation between the retailer and supplier \( X \) if negotiations with supplier \( Y \) break down. Indeed,
the left-hand side is also negative at \( \tilde{w} = \tilde{w} \), implying that the distortion in \( w \) is larger when renegotiation between the retailer and supplier \( X \) is allowed than when it is not allowed.

Allowing renegotiation between the retailer and supplier \( X \) if negotiations break down between the retailer and supplier \( Y \) does not remove the distortion in \( w \), because, although the renegotiation fixes the joint profit of the retailer and supplier \( X \) at \( R(x^*(c_\ast), 0) - c_\ast x^*(c_\ast) \), the disagreement payoff of the retailer is not independent of \( w \). To see this, note that if \( \tilde{F}_x \) is the fixed fee that the retailer would pay supplier \( X \) if negotiations with supplier \( Y \) did not break down, and \( F'_x \) is the fixed fee that would be renegotiated with supplier \( X \) if negotiations with supplier \( Y \) did break down, then the retailer’s disagreement payoff is given by

\[
R(x^*(c_\ast), 0) - c_\ast x^*(c_\ast) - F'_x \\
= R(x^*(w_\ast), 0) - w_\ast x^*(w_\ast) - \tilde{F}_x + (1 - \lambda_\ast) \\
\times [R(x^*(c_\ast), 0) - c_\ast x^*(c_\ast) - (R(x^*(w_\ast), 0) - c_\ast x^*(w_\ast))].
\]

Thus, allowing the retailer to renegotiate with supplier \( X \) after a breakdown of negotiations with supplier \( Y \) does not affect our results. However, allowing the retailer to renegotiate with supplier \( X \) after the conclusion of successful negotiations with supplier \( Y \) does eliminate the distortion, unless, for example, there is an exogenously fixed maximum number of rounds of renegotiation. In that case, similar to our model without renegotiation, the final contract renegotiated would have a per-unit price equal to the supplier’s marginal cost, but the second-to-last contract renegotiated would have a distorted per-unit price.

**Observability of contracts.** Our results rely on the assumption that supplier \( Y \) can observe \( T_\ast \) before it negotiates with the retailer and that the retailer and supplier \( X \) are fully committed to the contract. To understand the role of observability and commitment, we can view the model in terms of the strategic use of agents (see Fershtman, Judd, and Kalai, 1991). In our model, the first supplier commits to an observable contract with the retailer and then allows the retailer to act as an agent on behalf of both of them in negotiations with the second supplier. Through the observable contract, the retailer and the first supplier can commit to playing a cooperative strategy, enabling them to extract surplus from the second supplier.

Additional assumptions would be required to define the outcome of contract negotiations between the retailer and supplier \( Y \) if supplier \( Y \) did not know \( T_\ast \) at the time of its negotiations with the retailer (see, for example, the assumptions of Crémer and Riordan (1987) and O’Brien and Shaffer (1992) in defining a contract equilibrium). It may be that if \( T_\ast \) is not observed, there is no value to a commitment by supplier \( X \) to sell marginal units at below cost, so below-cost pricing would not obtain in equilibrium.

**Timing of negotiations and purchases.** When contracts are negotiated simultaneously, the retailer purchases the efficient quantity of any product that is sold (Bernheim and Whinston, 1998; O’Brien and Shaffer, 1997), so below-cost pricing does not arise. Thus, the assumption of sequential negotiations is important. However, fixing the bargaining power of the firms, a retailer, if given the choice, would choose sequential negotiations over simultaneous negotiations. To see this, note that with sequential negotiations, the retailer and supplier \( X \) have the option of choosing a contract that involves marginal-cost pricing. Such a contract results in the same choices of quantities and division of surplus as would arise from simultaneous negotiations. Since the retailer...
and supplier X do not choose such a contract, and since they divide the gains from trade, both must be better off under sequential negotiations than under simultaneous negotiations. Of course, if the bargaining power of the retailer differs in sequential versus simultaneous negotiations, then the retailer might prefer simultaneous negotiations.

We assume that the retailer makes its purchasing decisions after negotiations with both suppliers are completed. If the retailer made its purchasing decision with respect to product X before negotiating with supplier Y, then we would expect no distortion in equilibrium. Although, as above, if given the choice the retailer would prefer to postpone its purchasing decision with respect to product X until after negotiations with supplier Y were completed.

**Generalized contracts.** In this subsection we show that the equilibrium $T_i$ exhibits below-cost pricing even when the restriction to two-part tariff contracts is relaxed, provided that contracts with supplier X cannot be contingent on the quantity the retailer purchases from supplier Y.

If contracts with supplier X can be contingent on the quantity the retailer purchases from supplier Y, then the equilibrium $T_i$ does not exhibit below-cost pricing. For example, consider a contract between the retailer and supplier X that specifies a payment from supplier X to the retailer at the time the contract is signed and another payment from the retailer to supplier X of $R(x, y) = c_x y$ if the retailer purchases quantities $x$ and $y$. Then, if the production of both products is efficient, there exists an equilibrium in which the retailer chooses the efficient quantities, $x^*(c_x, c_y)$ and $y^*(c_x, c_y)$, and pays $c_x y^*(c_x, c_y)$ to supplier Y. Such a contract extracts all of the surplus from supplier Y and thus achieves the first best. Other contracts that can achieve the first best include contracts that impose a large penalty on the retailer if it chooses quantities other than the efficient quantities. Thus, if we allow the retailer’s contract with supplier X to depend on the quantities purchased from both suppliers, then equilibrium contracts do not exhibit below-cost pricing. However, such contracts would require that supplier X be able to verify the quantity that the retailer purchases from supplier Y, which may be difficult or costly in reality.

In the remainder of this subsection, we restrict attention to contracts that are contingent only on a supplier’s own quantity and relax our assumption of two-part tariffs. Thus, we consider contracts $T_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ and $T_i: \mathbb{R}_+ \rightarrow \mathbb{R}$. The next proposition implies that in any equilibrium, the contract between supplier Y and the retailer induces the retailer to choose the quantity of product Y that is efficient, given $T_i$.

**Proposition 9.** Let $\bar{r}(T_i, T_i)$ and $\bar{y}(T_i, T_i)$ be the retailer’s equilibrium choices of products X and Y, respectively, given contracts $T_i$ and $T_i$. If the gains from trade between the retailer and supplier Y, given $T_i$, are positive, then the equilibrium $T_i$ is such that

$$\bar{y}(T_i, T_i) \in \arg \max_{y \geq 0} R(\bar{r}(T_i, T_i), y) - c_x y - T_i(\bar{r}(T_i, T_i)).$$

**Proof.** See Appendix A.

We now prove that equilibrium contracts exhibit below-cost pricing in intermediate goods markets even when the restriction to two-part tariffs is relaxed. First, we show that the joint profit of the retailer and supplier X, if they negotiate a two-part tariff contract with $w_x = c_x$ and $F_x = 0$, is greater than or equal to the joint profit they would

\footnote{We thank Glenn Ellison for suggesting this contract.}
receive if they negotiated some other contract in which supplier $X$ promised to sell marginal units to the retailer at or above these units' marginal cost. Then, we prove that the joint profit of the retailer and supplier $X$ must be strictly higher when their contract involves below-cost pricing.

Define $T_s(x) = c_s x$ and let $\tilde{T}_s \in \Phi$, where $\Phi$ is the set of contracts such that marginal units of $X$ are everywhere sold at or above these units' marginal cost, i.e., the set of contracts for which there is no below-cost pricing:

$$\Phi = \{ T_s : \mathbb{R}_+ \rightarrow \mathbb{R} \mid T_s(x_1) - T_s(x_2) \geq c_s x_1 - c_s x_2, \ \forall x_1 \geq x_2 \}.$$ 

Using Proposition 9, we can prove the following lemma.

**Lemma 4.** The joint profit of the retailer and supplier $X$ is at least as large under the contract $T_s$ as under any contract $\tilde{T}_s \in \Phi$.

**Proof.** See Appendix A.

We now use Lemma 4 to prove the following proposition.

**Proposition 10.** If the production of products $X$ and $Y$ is efficient, or the production of product $X$ is weakly inefficient, and $\lambda > 0$, then, in equilibrium, $T_s$ exhibits below-cost pricing.

**Proof.** Suppose not. Then, since the equilibrium $T_s$ is an element of $\Phi$, we know from Lemma 4 that it yields joint profit for the retailer and supplier $X$ that is bounded above by their joint profit under the contract $T_s$. But since $T_s$ belongs to the class of two-part tariff contracts considered in Section 3, we know from Propositions 3 and 4 that there exists a different two-part tariff contract that has $w_i < c_s$ and yields strictly higher joint profit. Thus, $T_s \in \Phi$ cannot arise in equilibrium. Q.E.D.

6. Conclusion

- We identify a motivation for below-cost pricing in intermediate goods markets that is not currently described in the literature. We show that below-cost pricing by one supplier can facilitate the transfer of surplus from a second supplier to a retailer and the first supplier by strengthening the retailer's bargaining position vis-à-vis the second supplier. We also show that the pricing behavior stops short of driving the second supplier from the market—hence the label "predatory accommodation." Our results raise a number of legal issues having to do with Section 2 of the Sherman Act and Section 2(a) of the Robinson-Patman Act, and so we conclude with a discussion of these issues.

Some definitions of predation in the economics literature and academic legal writings encompass only behavior that is aimed at excluding rivals, e.g., "Predatory pricing is defined generally as conduct that is aimed at excluding business rivals on some basis other than efficiency." Other definitions of predation, e.g., Baker (1989), include as predatory pricing conduct that is aimed at deterring entry or disciplining rivals in order to soften subsequent competition. These definitions do not describe the conduct we have identified.

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17 ABA Antitrust Section, Monograph No. 22, (1996, p. 2).

18 Many scholars criticize the classic story of predatory pricing, which involves the exclusion of rivals and then the recoupment of losses, because it presumes a fortuitous chain of events for the predator, e.g., a breakdown in the capital markets and a lack of entry at later dates. Predatory accommodation is not subject to this criticism because it is the result of bargaining between the retailer and supplier $X$ and can be repeated each time contracts are negotiated.

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Nevertheless, the two elements that are necessary to establish liability for predatory pricing, as outlined by the Supreme Court in *Brooke*, are satisfied. In our model, supplier \( X \) allows the retailer to purchase additional units from it at below cost, and since this occurs as part of supplier \( X \)'s equilibrium strategy, recoupment is certain. Thus, although the predatory accommodation that occurs in our model would not be considered predatory conduct by those who view exclusion as a defining feature of "true" predatory pricing, it would have standing as predatory pricing in the courts under the criteria set forth in *Brooke*.

A court case involving predatory accommodation would have some interesting features. First, there would be no disputing that supplier \( Y \) had been injured. Below-cost pricing by supplier \( X \) not only reduces overall joint profit, it also allows the retailer and supplier \( X \) to capture a larger share. Thus, supplier \( Y \) is left with a smaller fraction of a smaller profit pie. Second, supplier \( X \)'s below-cost pricing would be evidence that competition was not on the merits, and thus that the injury to supplier \( Y \) did not necessarily occur because it had an inferior product or because its production costs were higher. Nevertheless, as we show in Section 4, below-cost pricing by supplier \( X \) need not have anticompetitive consequences.

Our results suggest that the two antitrust statutes under which predatory pricing claims can be made are not, as suggested by the Supreme Court in *Brooke*, of the same general character. Since sequential bargaining of the kind we consider is more likely to occur in intermediate goods markets than in final goods markets, we believe that predatory accommodation is more likely to be found in intermediate goods markets. This suggests that predatory pricing claims (primary-line claims) under the Robinson-Patman Act should not be treated the same as predatory pricing claims under the Sherman Act.

**Appendix A**

- The proofs of Propositions 1, 2, and 9 and of Lemmas 1, 2, 3, and 4 follow.

**Proof of Proposition 1.** Let \( T_i \) be such that \( g(T_i, (c_i, 0)) > 0 \). First, note that since \( \tilde{F}(w_i) \) is less than or equal to the incremental gains from trade between the retailer and supplier \( Y \), an increase in the fixed fee from zero to \( \tilde{F}(w_i) \) does not cause the retailer to stop purchasing product \( Y \), i.e., \( g(T_i, (c_i, \tilde{F}(w_i))) > 0 \). Define \( \tilde{x}(w) = \tilde{x}(w, w, F_i, \tilde{F}(w_i)) \) and \( \tilde{y}(w) = \tilde{y}(w, w, F_i, \tilde{F}(w_i)) \). We show that \( c_i \) maximizes the joint profit of the retailer and supplier \( Y \), given in (7). Suppose not. Then there exists \( \tilde{w}_i \neq c_i \) such that

\[
R(\tilde{x}(\tilde{w}_i), \tilde{y}(\tilde{w}_i)) - T(x(\tilde{w}_i), F_i) - c_i \tilde{y}(\tilde{w}_i) > R(\tilde{x}(c_i), \tilde{y}(c_i)) - T(x(c_i), F_i) - c_i \tilde{y}(c_i). \tag{A1}
\]

However, the definition of \( \tilde{x} \) and \( \tilde{y} \) implies that the first line in (A1) is less than or equal to the second line, a contradiction. Since \( g(T_i, (c_i, \tilde{F}(w_i))) > 0 \), then \( \tilde{y}(c_i) > 0 \), which implies that \( c_i \) is the unique maximizer in (7). Thus, \( \tilde{w}_i = c_i \). Since supplier \( Y \)'s disagreement payoff is zero, it is clear that when \( w_i = c_i, \tilde{F}(w_i) \) gives supplier \( Y \) its disagreement payoff plus proportion \( \lambda \) of the gains from trade. Q.E.D.

**Proof of Proposition 2.** Assume that the production of products \( X \) and \( Y \) is efficient. When sales of product \( Y \) are zero, the joint profit of the retailer and supplier \( X \) is bounded above by \( \pi_i = R(x^*(c_i), 0) - c_i x^*(c_i) \). When sales of product \( X \) are zero, the joint profit of the retailer and supplier \( X \) is \( \pi_i = (1 - \lambda) R(0, y^*(c_i)) - c_i y^*(c_i) \). Let \( \pi_o \) be the joint profit of the retailer and supplier \( X \) when sales of both products are positive,

\[
\pi_o(w_i) = R(x^{**}, y^{**}) - c_i x^{**} - c_i y^{**} - \tilde{F}(w_i), \tag{A2}
\]

where \( x^{**} \) and \( y^{**} \) are evaluated at \( (w_i, c_i) \). To show that \( \max_{\pi_o} \pi_o(w_i) \) is greater than \( \max \{ \pi_o, \pi_i \} \), it suffices to show that \( \max_{x, y} R(x, y) - c_i x - c_i y \) is positive. Define \( \pi_o = \max_{x, y} R(x, y) - c_i x - c_i y \) and note that

\[
\pi_o(c_i) = \pi_o - \tilde{F}(c_i) = (1 - \lambda) \pi_o + \lambda \pi_i \geq \max \{ \pi_o, \pi_i \}.
\]

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where the first equality uses (A2), the second equality uses $\hat{F}(c_e) = \lambda \left( \pi_n - \pi_r \right)$, and the inequality uses $\pi_n > \pi_r$ and $((1 - \lambda_1) \pi_n > \pi_r$, which hold because the production of products $X$ and $Y$ is efficient. Q.E.D.

Proof of Lemma 1. If the production of product $Y$ is weakly inefficient, the solution to the joint-profit-maximization problem of the retailer and supplier $X$, given in $(9)$, has $\delta = 0$ and $\delta > 0$, in which case the first-order condition, given in $(4)$, implies $w_r = c_r$. Given the equilibrium $T_r$, there does not exist a $T_s$ in stage two that both induces the retailer to purchase $y > 0$ and allows supplier $Y$ to earn nonnegative profit. Q.E.D.

Proof of Lemma 2. Suppose the production of product $X$ is weakly inefficient and $\lambda_1 > 0$ but that $T_s$ has $w_r \geq c_r$ in equilibrium. Then $\delta = 0$ and $\delta = y^*(w_r)$. Since the production of product $X$ is weakly inefficient, $x^*(c_e) > 0$. Differentiating the joint profit of the retailer and supplier $X$, given in $(9)$, yields $-(\partial \hat{F}(w_r))/\partial w_r = -\lambda_1 x^*(w_r)$, which implies that the retailer and supplier $X$ can increase their joint profit by lowering $w_r$ below $c_r$. Hence, $T_r$ has $w_r < c_r$ in equilibrium. In Section 4, we give an example with symmetric linear demands in which the production of product $X$ is weakly inefficient, but nevertheless a positive quantity is purchased in equilibrium due to below-cost pricing. Q.E.D.

Proof of Lemma 3. Using Lemma 1, if the production of product $Y$ is weakly inefficient, then the retailer’s profit is $R(x^*(c_e), 0) - c_x x^*(c_e) - \lambda_1 c_e (0) - c_x x^*(c_e))$, which can be written as

$$(1 - \lambda_1)(R(x^*(c_e), 0) - c_x x^*(c_e)) + \lambda_1 (1 - \lambda_1)(R(0, y^*(c_e)) - c_x x^*(c_e)). \quad (A3)$$

Suppose the production of product $X$ is weakly inefficient. We show the retailer prefers to negotiate first with supplier $X$. If the retailer negotiates first with supplier $Y$, then using Lemma 1, the retailer’s profit is given by (A3), with the roles of $X$ and $Y$ reversed:

$$\pi_r = (1 - \lambda_1)(R(0, y^*(c_e)) - c_x x^*(c_e)) + \lambda_1 (1 - \lambda_1)(R(x^*(c_e), 0) - c_x x^*(c_e)). \quad (A4)$$

If the retailer negotiates first with supplier $X$ and the negotiated contract has unit price $w_{x_r}$, then the retailer’s profit is given by $\pi(w_{x_r})$, where

$$\pi(w_{x_r}) = R(\delta, \delta) - w_{x_r} - c_x \delta - F_x - \hat{F}(w_{x_r}) = R(\delta, \delta) - c_x \delta - c_x \delta - \lambda_1 g_x(w_{x_r}, 0) - \hat{F}(w_{x_r}) = (1 - \lambda_1)(R(0, y^*(c_e)) - c_x x^*(c_e)) + (1 - \lambda_1)g_x(w_{x_r}, 0). \quad (A5)$$

where the first equality uses $F_x = (c_x - w_{x_r})x^* + \lambda_1 g_x(w_{x_r}, 0)$ and the second equality uses the definition of $g_x(w_{x_r}, 0)$ in (10). Since production of product $X$ is weakly inefficient, if $w_{x_r} = c_r$, then $\delta = 0$ and $\delta = y^*(c_e)$, so

$$\pi(c_e) = (1 - \lambda_1)(R(0, y^*(c_e)) - c_x x^*(c_e)) + (1 - \lambda_1)g_x(c_r, 0)$$

$$= (1 - \lambda_1)(R(0, y^*(c_e)) - c_x x^*(c_e)) + \lambda_1 (1 - \lambda_1)(R(x^*(c_e), 0) - c_x x^*(c_e)) = \pi_r. \quad (A6)$$

where the first equality uses (A5), the second inequality uses the definition of $g_x(w_{x_r}, 0)$ in (10) and the definitions of $\hat{F}$ and $g_x$ from Proposition 1 and (8), and the third equality uses (A4). Since we know from Lemma 2 that any equilibrium $T_s$ has $w_{x_r} < c_r$, it follows that $\pi(w_{x_r}) > \pi(c_e) = \pi_r$. Q.E.D.

Proof of Proposition 9. Let $T_s$ be such that $g_x(T_r, (c_r, 0)) > 0$. Define $T(x) = c_x y + F$, where

$$F = \lambda_1 g_x(T_s, (c_r, 0)).$$

Since $g_x(T_s, (c_r, 0)) > 0$, then $T(x, T_s) > 0$. We show that $T_s$ maximizes the joint profit of the retailer and supplier $Y$. Suppose not. Then there exists $T_s \neq T_s$ such that $T(x, T_s) > T(x, T_s)$ and

$$R(\tilde{X}(T_s, T_s), \tilde{Y}(T_s, T_s)) - c_x \tilde{Y}(T_s, T_s) > R(\tilde{X}(T_s, T_s), \tilde{Y}(T_s, T_s)) - c_x \tilde{Y}(T_s, T_s). \quad (A7)$$

However, since $(\tilde{X}(T_s, T_s), \tilde{Y}(T_s, T_s)) \in \arg \max_{x,y} R(x, y) - T(x) - T(y)$, the first line in (A7) is less than or equal to the second line, a contradiction. Q.E.D.

Proof of Lemma 4. Let $(x', y') \in \arg \max_{x,y} R(x, y) - c_x y - c_y$. If supplier $X$ and the retailer negotiate the contract $T'(x) = c_x x$ in the first stage, then, using Proposition 9, the retailer purchases $(x', y')$. In this

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case, the joint profit of the retailer and supplier \(X\) is \(R(x', y') - c.x' - c.y' - \lambda.g(T'_i, (c, _0))\), which can be written as

\[
(1 - \lambda_i) R(x', y') - c.x' - c.y') = -\lambda_i(R(x^*(c, _0), 0) - c.x^*(c, _0)). \tag{A8}
\]

Consider an arbitrary contract \(\tilde{T}_i \in \Phi\). Suppose \(\tilde{T}_i\) induces the retailer and supplier \(Y\) to negotiate \(\tilde{T}_i\) in stage 2. Let \((\tilde{x}, \tilde{y})\) denote the retailer's quantity choices given \((\tilde{T}_i, \tilde{T}_j)\). In this case, the joint profit of the retailer and supplier \(X\) is \(R(\tilde{x}, \tilde{y}) - c.\tilde{x} - \tilde{T}_i(\tilde{y})\), which can be written as

\[
R(\tilde{x}, \tilde{y}) - c.\tilde{x} - c.\tilde{y} - \lambda_i(R(\tilde{x}, \tilde{y}) - \tilde{T}_i(\tilde{x}) - c.\tilde{y}) + \lambda_i \text{max}_{x=0} R(x, 0) - \tilde{T}_i(x) \tag{A9}
\]

since supplier \(Y\) receives share \(\lambda_i\) of the gains from trade with the retailer in equilibrium. We now show that the expression in (A8) is greater than or equal to the expression in (A9). Since \(\tilde{T}_i(x) \geq c.x - c.\tilde{x}\) for \(x \geq \tilde{x}\), the profit in (A9) is bounded above by

\[
(1 - \lambda_i) R(\tilde{x}, \tilde{y}) - c.\tilde{x} - c.\tilde{y} + \lambda_i R(x^*(\tilde{T}_i), 0) - c.x^*(\tilde{T}_i)) \tag{A10}
\]

where we use \(x^*(\tilde{T}_i)\) to denote an element of \(\arg\max_{x \geq 0} R(x, 0) - \tilde{T}_i(x)\). Thus, it suffices to show that (A8) is greater than or equal to (A10), which holds by the definitions of \(x', y'\), and \(x^*(c, _0)\). \(Q.E.D.\)

**Appendix B**

- The details of the symmetric linear demand example and the proof of Proposition 8 follow.
- The quantity and two-part tariff for product \(Y\) when below-cost pricing is used are

\[
\hat{x} = \left[\alpha(1 - \gamma) - c_i + \gamma c_i\right]/[2(1 - \gamma^2)(1 + \gamma^2 \lambda_i)], \quad \hat{y}_i = c_i, \quad \text{and}
\]

\[
\hat{F}_i = \lambda_i(\alpha(1 - \gamma) - c_i + \gamma c_i)^2/[4(1 - \gamma^2)(1 + \gamma^2 \lambda_i)].
\]

To guarantee an interior solution for \(\hat{y}_i\), we assume \(c_i < \alpha(1 - \gamma)\). Supplier \(X\)'s per-unit price when below-cost pricing is used is

\[
\hat{w}_i = \frac{c_i - \gamma \lambda_i(\alpha(1 - \gamma) - c_i)}{1 + \gamma^2 \lambda_i},
\]

which is less than \(c_i\) since we assume \(c_i < \alpha(1 - \gamma)\). The quantity of product \(X\) is

\[
\hat{x}_i = \left[\alpha(1 - \gamma) - \hat{w}_i + \gamma c_i\right]/[2(1 - \gamma^2)],
\]

which, substituting in the expression for \(\hat{w}_i\), gives

\[
\hat{x}_i = \frac{\gamma \lambda_i(\alpha(1 - \gamma) - c_i) + (\alpha(1 - \gamma) + \gamma c_i)(1 + \gamma^2 \lambda_i) - c_i}{2(1 - \gamma^2)(1 + \gamma^2 \lambda_i)}. \tag{B1}
\]

To guarantee an interior solution for \(\hat{x}_i\), we assume that the numerator in (B1) is positive, i.e., that

\[
c_i < \gamma \lambda_i(\alpha(1 - \gamma) - c_i) + (\alpha(1 - \gamma) + \gamma c_i)(1 + \gamma^2 \lambda_i). \tag{B2}
\]

The expression for \(\hat{F}_i\) is tedious, but is constructed as

\[
\hat{F}_i = (c_i - \hat{w}_i)\hat{x}_i + \lambda_i(R(\hat{x}_i, \hat{y}_i) - c_i\hat{x}_i - c_i\hat{y}_i - \hat{F}_i) - (1 - \lambda_i)(R(0, y^*(c, _0) - c_i y^*(c, _0))). \tag{B3}
\]

where \(y^*(c, _0) = \left[\alpha - c_i\right]/2\).

For comparison, the profit-maximizing quantities for a fully integrated firm are

\[
x^* = \max \left\{0, \frac{\alpha(1 - \gamma) - c_i + \gamma c_i}{2(1 - \gamma^2)} \right\}, \quad y^* = \max \left\{0, \frac{\alpha(1 - \gamma) - c_i + \gamma c_i}{2(1 - \gamma^2)} \right\}.
\]

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Since we assume $c_i < a(1 - \gamma), \gamma_i = [a(1 - \gamma) - c_i + \gamma_c]/[2(1 - \gamma)],$ and since $\gamma_i > 0, \gamma_i$ is clearly less than $\gamma_i$. If $c_i < a(1 - \gamma) + \gamma_c,$ then $\gamma_i = 0,$ so clearly, $\gamma_i > x_i$; and if $c_i < a(1 - \gamma) + \gamma_c,$ then $\gamma_i - x_i = [\gamma_i, (a(1 - \gamma) - c_i + \gamma_c)]]/(2(1 - \gamma)(1 + \gamma_i)),$ which is positive given our assumptions. Thus, the quantity of product $X$ is higher and the quantity of product $Y$ is lower than the quantities that would be chosen by a fully integrated firm.

Proof of Proposition 8. Using $\partial u/\partial x = p_x$ and $\partial u/\partial y = p_y,$ then $u(\xi, \eta) = \int_{\xi}^{\eta} p_x(s, 0) ds + \int_{\xi}^{\eta} p_y(\xi, s) ds$ and we can write net consumer surplus as follows:

$$CS = (p_x(\xi, 0) - p_x(\xi, \eta))\xi + \frac{1}{2}(p_x(0, 0) - p_x(\xi, 0))\xi + \frac{1}{2}(p_x(\xi, 0) - p_x(\xi, \eta))\eta.$$  \hspace{1cm} (B4)

Welfare is then $W = p_x(\xi, \eta)\xi + p_x(\xi, \eta)\eta - c, \xi - c, \eta + CS,$ which equals

$$W = \frac{1}{4(1 - \gamma)^3}[(1 - \gamma)(3a(a - c_i) - aw) - \frac{1}{2}w_i^2 + \gamma w_i c_i + \frac{3}{2}c_i^2 - 2c_s(a - w_s - \gamma(a - c_s))].$$  \hspace{1cm} (B5)

Viewing $W$ as a function of $w,$ the derivative of welfare with respect to $w$ is

$$\frac{\partial W}{\partial w} = \frac{2c_s - w_s - a(1 - \gamma) - \gamma_c}{4(1 - \gamma^2)}.$$  \hspace{1cm} (B6)

The value of $(\partial W/\partial w)|_{w_s - c}$ can be negative or positive. In particular, since the denominator in (B6) is positive, $(\partial W/\partial w)|_{w_s - c} > 0$ if and only if $c_i < a(1 - \gamma) + \gamma_c,$ which, since $\partial W/\partial w,$ is decreasing in $w,$ is a necessary condition for welfare to increase as a result of below-cost pricing. (When $\gamma = 1,$ the goods are perfect substitutes and $(\partial W/\partial w)|_{w_s - c} < 0$ if and only if $c_i < c_s.$) Evaluating $\partial W/\partial w,$ at $w = w_s,$ we get

$$\frac{\partial W}{\partial w}|_{w = w_s} = \frac{c_s (1 + 2c_s \gamma_i) - a(1 - \gamma) + c_i\gamma_i(1 + c_i \gamma_i) + \gamma_i a(1 - \gamma) - c_i}{4(1 - \gamma^2)(1 + c_i \gamma_i)}.$$  \hspace{1cm} (B7)

Thus, $(\partial W/\partial w)|_{w_s - c} > 0$ if and only if

$$c_i < \frac{a(1 - \gamma) + c_i \gamma_i(1 + c_i \gamma_i) - \gamma_i a(1 - \gamma) - c_i}{(1 + 2c_i \gamma_i)},$$  \hspace{1cm} (B8)

which is a sufficient condition for welfare to be higher when $w = w_s.$ \hspace{1cm} Q.E.D.

References


