Contract Renegotiation
for Venture Capital Projects

Leslie M. Marx*
University of Rochester
March 1999

Abstract
This paper models the strategic behavior of venture capitalists and entrepreneurs under financing arrangements that involve a mixture of debt and equity. It is shown that a range of behavior can be expected, including the forgiveness of dividends and the combined reduction of the debt obligation of the entrepreneur and increase in the equity share of the venture capitalist. Although intervention by the venture capitalist does not occur on the equilibrium path, the possibility of intervention plays an important role in contract renegotiation.

*This paper is a revision of an earlier paper titled “Negotiation of Venture Capital Contracts.” I thank Rajshree Agarwal, Thomas Hellmann, Lynn Hunicutt, Gretchen Kalsow, Kala Krishna, Steven Matthews, Irfan Saif, and Andrew Winton for their advice and comments. I also thank seminar participants at the Toronto-Rochester Conference. I thank Ian Turvill for valuable research assistance and the American Compensation Association Emerging Scholar Program for financial support. The views expressed are solely the author’s and do not reflect the views or opinions of the American Compensation Association.
1 Introduction

This paper examines the contracting problem faced by entrepreneurs and venture capitalists when negotiating the financing arrangements for venture capital projects. Venture capital projects are typically financed with a mixture of debt and equity. This paper examines the strategic behavior of venture capitalists and entrepreneurs when venture capital projects are financed in this way.

The most common security used in venture capital financing is convertible preferred stock.\(^1\) Although convertible preferred equity typically specifies a fixed conversion rate and dividend, these terms may be, and often are, renegotiated once the project is under way. Some venture capital financing does not involve explicit dividend requirements, but requires fixed payments through contracts that require interest payments or a repayment of the initial investment. Any contract that requires a fixed payment to the venture capitalist before allowing the remaining payoffs to be shared proportionally between the venture capitalist and the entrepreneur falls within the scope of this study.

I present a principal-agent model in which a venture capitalist and entrepreneur enter a contractual relationship, which can (and in some states will) be renegotiated after information is revealed about the initial progress of the project. I show that, in equilibrium, when renegotiation occurs, it results either in the forgiveness of part of the entrepreneur’s debt obligation or a reduction in the debt obligation together with an increase in the equity share of the venture capitalist. The results of the model fit well with the stylized facts of venture capital financing.

It is typical for the behavior of venture capitalists and entrepreneurs to vary based

\(^1\)Convertible preferred stock gives the venture capitalist some debt-like priority over common stock holders and can be converted to common equity. Preferred stock often offers a fixed dividend like debt, but typically the decision whether to pay dividends or allow them to accrue is at the discretion of the directors. Conversion typically occurs if and when the firm holds an initial public offering or when the firm consistently generates more than a target level of earnings. See Tyebjee and Bruno (1984), Silver (1987), Testa (1988), Pozzena (1990), Sahlman (1990), Brealey and Myers (1991), Golder (1991), and Gompers (1997).
on the level of success or failure of their projects. When a project proceeds as hoped, the venture capitalist remains distant and allows the entrepreneur to make decisions for the project on his own. If a project is not quite on target, there may be some renegotiation of the initial contract, typically in the direction of increasing the venture capitalist’s proportional share in the project and decreasing required fixed payments. Although cash-flow constraints are a major reason for this type of renegotiation, the model developed in this paper shows that this type of renegotiation can be expected for strategic reasons even in the absence of interim cash-flow constraints. Finally, when a project is going badly, the venture capitalist may step in and take over control of the project. The venture capitalist may do this herself or hire someone else to take over the decision making for the project.

As part of the financing arrangement, venture capitalists usually obtain inside management rights. These may include the right to appoint one or more directors or to serve as an officer of the company.² Gompers (1997) shows that, in a sample of fifty convertible preferred equity venture investments, contracts usually explicitly allocate control rights to the venture capitalist, including giving them enough board seats to control the board of directors. Sahlman (1990, p.508) states that,

Venture capitalists sit on boards of directors, help recruit and compensate key individuals, work with suppliers and customers, help establish tactics and strategy, play a major role in raising capital, and help structure transactions such as mergers and acquisitions. They often assume more direct control by changing management and are sometimes willing to take over day-to-day operations themselves.

Perez (1986, pp.8-9) comments on the relationship saying, “The matter of control has become one of the crucial ‘sticking points’ between entrepreneurs and venture capitalists. The entrepreneur wants to maintain the independence of his company

²See Perez (1986), Pozdena (1990), and Brealey and Myers (1991).
free of outside control. But to get the vital initial capital to finance the venture, they usually must compromise their independence somewhat.” When things are proceeding reasonably well in the underlying business, arrangements between venture capitalists and entrepreneurs allow each to advance the interest of the other merely by following their own self interests. But when things go badly, the interests of the two parties diverge.

As an illustration of the kinds of contracts that are used and the frequency and types of renegotiation, consider a sample of fifteen venture-backed companies that held IPOs during June-September 1997.3 These firms all achieved some measure of success since they all reached the point of holding a public offering, but the public nature of the prospectuses allows us to obtain information on the firms’ financing arrangements. All fifteen firms were financed with a mixture of debt and equity.4 Seven of the firms had, prior to their IPO, renegotiated some feature of their financing, including: the conversion of payables to preferred stock or convertible preferred stock, the forgiveness of debt, and the waiver of debt compliance provisions. Four of the firms issued common or preferred stock or stock options in exchange for services or equipment. There were no instances revealed in the prospectuses of the conversion of equity to debt or the reduction of the venture capitalists’ equity shares in the projects.

Similar renegotiations (conversions of notes to equity and forgiveness of debt) are found in the investments of a large Texas venture capital fund that agreed to share its investment history from 1993-1996. In addition to successful investments, the investment history for the Texas fund also contains information on failed investments.

3The Venture Capital Journal (December 1997 issue) reports that a total of 50 venture-backed companies held IPOs during June-September 1997. I was able to obtain prospectuses for 15 of these firms: AutoCyte; Best Software, Inc.; Boron, Lepore and Associates, Inc.; Cardima, Inc.; Compass Plastics and Technologies, Inc.; EduTrek International, Inc.; Genesys Telecommunication Laboratories, Inc.; II Fornaio; Megabios; New Era of Networks, Inc.; Omtool, Inc.; Peapod LR; Pegasus Systems, Inc.; Waterlink, Inc.; and 3DFX Interactive, Inc.

4One firm is a limited partnership, so is financed with a mixture of debt and limited partnership units.
During 1993–1996, the company wrote off three of its thirty-eight investments as complete failures and disposed of its interest in two investments at a loss. Again, although there were conversions of debt to equity and the forgiveness of debt, there were no instances of the conversion of equity to debt or the reduction of the venture capitalists’ equity shares in the projects.

In Section 2, I describe some related literature. In Section 3, I describe a model of venture capital contracting. In Section 4, I present the analysis of the model and a characterization of the equilibrium behavior of the venture capitalist and entrepreneur. Section 5 concludes.

2 Related literature

A number of authors develop models of venture capital contracting. Authors such as Amit, Glosten, and Muller (1990) study a wealthy entrepreneur’s decision whether to involve an outside investor. In Amit, et al. (1990), the entrepreneur can finance his project with his own funds (or a fully collateralized loan) or sell out completely to an outside investor. In this paper, in contrast, the decision to involve an outside investor is not modeled, but rather is assumed to be necessary, and a range of financing arrangements are considered, including risky debt, equity, and mixtures of the two.

Another general category of papers examines the circumstances that determine when a project that requires multiple rounds of investment should be continued and when it should be terminated. Examples of research on this topic include Admati and Pfleiderer (1991), Hansen (1991), and Neher (1994). Admati and Pfleiderer (1991) conclude that pure equity financing should be used, while Hansen (1991) concludes that the optimal contract is a complex financial arrangement similar to convertible preferred equity but has no conversion in equilibrium.

For more details of venture capital financing see the references in footnote 1 and Sahlman (1988), Gorman and Sahlman (1989), Barry, et al. (1990), and Morris (1991).
The model of Hellmann (1998) shows that an entrepreneur may voluntarily relinquish control rights over his project to the venture capitalist when the venture capitalist must be given incentives to engage in costly search for a new CEO for the project. Other papers that describe how venture capitalists retain control rights over projects include Rosenstein (1988), Sahlman (1990), Gompers (1995), and Lerner (1995).

Another category of papers compare payoffs and control outcomes under different types of contracts, but do not attempt to characterize the optimal contract, e.g., Trester (1993), Berglöf (1994), and Cornelli and Yosha (1997). The model of Berglöf (1994) shows that debt and equity are complementary in an environment in which control issues are important. Both Trester (1993) and Berglöf (1994) show that contracts more complicated that simple debt or equity are preferred. Cornelli and Yosha (1997) show that, in an environment with staged financing, convertible debt is better than a mixture of debt and equity because it reduces the entrepreneur’s incentives to focus his effort on the short-term success of the project. In this paper, I consider only one round of financing. In Marx (1998), a contract involving both debt and equity components, is shown to obtain the first best in a model with one round of financing since the debt level can be chosen to balance the venture capitalist’s ability to intervene in the project against the entrepreneur’s desire for control over the project. In contrast, this paper takes as given the use of financing that is a mixture of debt and equity, and examines the implications of this type of contract for efficiency and for the incentives and behavior of the venture capitalist and entrepreneur.

3 Model

In this section, I present a model of venture capital contracting. The model has three periods, $t = 0, 1, 2$ as shown in the timeline below. The entrepreneur, who is either a good type or a bad type, contracts with the venture capitalist in order to obtain
investment capital for his project. During the initial contracting period, the venture capitalist cannot observe the entrepreneur’s type. The entrepreneur and venture capitalist contract over three elements: (i) the initial investment by the venture capitalist, (ii) the sharing rule for the final return of the project, and (iii) the venture capitalist’s control over the project. An initial investment of $I$ is required. The sharing rule must respect the entrepreneur’s wealth constraint, i.e. must not specify a payment to the venture capitalist that is greater than the project’s return. And the venture capitalist’s right to exert control over the project must be all or nothing, although, when the venture capitalist is granted the right to intervene, she need not exercise that right.

### Timeline

<table>
<thead>
<tr>
<th>contract offer and investment $I$</th>
<th>state $\omega$ revealed</th>
<th>intervention or renegotiation</th>
<th>choice of standard or risky strategy</th>
<th>return realized</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>$t = 1$</td>
<td></td>
<td></td>
<td>$t = 2$</td>
</tr>
</tbody>
</table>

At $t = 0$, the entrepreneur makes a contract offer to the venture capitalist, which the venture capitalist must accept or reject. The contract specifies the venture capitalist’s control rights and the share of the profits that the venture capitalist receives in exchange for her initial investment in the project of $I$.

At $t = 1$, information about the potential success of the project becomes available. This information, referred to as the state, is observed by the entrepreneur and the venture capitalist but is nonverifiable. The state is denoted by $\omega$ and is assumed to be an element of the real interval $\Omega$, which has finite lower bound $\omega$. If $\omega > \omega'$, then $\omega$ is a “better” state than $\omega'$ in the sense that returns in state $\omega$ first-order stochastically dominate returns in state $\omega'$. For a good entrepreneur, I assume a probability distribution for $\omega$ with full support, finite first moment, and no mass points. For a bad entrepreneur, I assume the state is $\omega$ with probability one. Other than the different distributions over states, good and bad entrepreneurs are identical.
After the state is revealed, the contract can be renegotiated. In the renegotiation phase, the venture capitalist makes a take-it-or-leave-it offer to the entrepreneur. If the venture capitalist’s offer is rejected, she may still choose unilaterally to make changes in the terms of the contract that “favor” the entrepreneur, such as forgiving part or all of the entrepreneur’s debt obligation or giving up some of her equity share in the project.

After renegotiation is concluded, a venture capitalist with control rights can choose to intervene in the project at cost $c \geq 0$ to herself. A venture capitalist without control rights may not intervene. If the venture capitalist intervenes, then the venture capitalist chooses the strategy herself. I assume that the entrepreneur, or the venture capitalist if she is in control, can choose a standard strategy, which we can think of as the one that maximizes the expected payoff of the project, or a risky strategy, which has lower expected payoff and increases the probability of both the worst and the best outcomes for the project. The entrepreneur’s choice of strategy is not observable.

At $t = 2$, the project’s payoff is realized. If the standard strategy was used, the project’s payoff is drawn from the cumulative distribution function $F(\cdot \mid \omega)$. If the risky strategy was used, the project’s payoff is drawn from the cumulative distribution function $G(\cdot \mid \omega)$. Distributions $F(\cdot \mid \omega)$ and $G(\cdot \mid \omega)$ are assumed to be absolutely continuous and to have continuous density functions $f(\cdot \mid \omega)$ and $g(\cdot \mid \omega)$, respectively. The support of both $f(\cdot \mid \omega)$ and $g(\cdot \mid \omega)$ is $[0, \bar{g}(\omega)]$, where $\bar{g}(\omega)$ is increasing in $\omega$. This assumption implies that there is always positive probability of low returns and that, given the state, the risky and standard strategies produce returns in the same range.

The assumption that the risky strategy is always suboptimal, in that it generates lower expected return in every state, is formalized in Assumption 1.

**Assumption 1** For all $\omega \in \Omega$, $\int_0^{\bar{g}(\omega)} yg(y \mid \omega)dy < \int_0^{\bar{g}(\omega)} yf(y \mid \omega)dy$. 

7
The assumption that the risky strategy places higher weight on the worst and best outcomes relative to the standard strategy is captured by Assumption 2 and illustrated in Figure 1.

![Figure 1: Graph of $F(y \mid \omega)$ and $G(y \mid \omega)$ as functions of $y$.](image)

**Assumption 2** For all $\omega \in \Omega$, there exists $y^* \in (0, \bar{y}(\omega))$ such that $G(y \mid \omega) > F(y \mid \omega)$ if and only if $0 < y < y^*$.

Finally, in Assumption 3, I assume that the project’s expected payoff under the standard strategy exceeds the required initial investment. And, so that it is not efficient for venture capitalists to finance bad entrepreneurs, I assume that the expected return for bad entrepreneurs, even under the standard strategy, is less than the required initial investment.

**Assumption 3** $E_{\omega} \left[ \int_{0}^{g(\omega)} y f(y \mid \omega) dy \right] \geq I$ and $\int_{0}^{\bar{y}(\omega)} y f(y \mid \omega) dy < I$.

As mentioned in the introduction, I take the contract form as fixed. I focus on contracts that can be characterized as a mixture of debt and equity financing, which
is typical of venture capital financing. I define a mixed debt-equity sharing rule with dividend \( v \geq 0 \) and share \( \gamma \in [0, 1] \) by \( s(y \mid v, \gamma) \equiv \min\{y, v + \gamma(y - v)\} \). A mixed debt-equity sharing rule, shown in Figure 2, can also be written as

\[
s(y \mid v, \gamma) = \begin{cases} 
y, & y < v \\
v + \gamma(y - v), & y \geq v,
\end{cases}
\]

so it can be viewed as a fixed dividend payment \( v \) in conjunction with a proportional sharing rule for returns in excess of the dividend. If \( \gamma = 0 \), then the mixed debt-equity sharing rule is actually a debt sharing rule, and if \( v = 0 \) and \( \gamma \in [0, 1] \), then it is an equity sharing rule.

\[
\begin{array}{c}
s(y \mid v, \gamma)
\end{array}
\]

\[
\begin{array}{c}
slope = \gamma
\end{array}
\]

\[
\begin{array}{c}
slope = 1
\end{array}
\]

\[
\begin{array}{c}
v
\end{array}
\]

\[
\begin{array}{c}
y
\end{array}
\]

Figure 2: Graph of mixed debt-equity sharing rule \( s(y \mid v, \gamma) \) as a function of \( y \).

Since a mixed debt-equity sharing rule is concave, a venture capitalist who receives payment \( s(y \mid v, \gamma) \) when a project’s profit is \( y \) is risk averse. Similarly, an entrepreneur who receives payment \( y - s(y \mid v, \gamma) \) when a project’s profit is \( y \) is risk loving.

In a model such as this one, in which returns are available in only one period, a mixed debt-equity sharing rule captures the basic characteristics of convertible
preferred equity financing. With returns possible only at $t = 2$, the decision to allow dividends to accrue and decisions on the timing of conversion can be ignored. One can view $v$ as the level of accrued dividends at $t = 2$. When the project’s return at $t = 2$ is less than the level of accrued dividends, the holder of preferred shares has priority and receives all of the project’s returns. When the project’s return is greater than the accrued dividends, the investor receives the accrued dividends and converts her preferred stock into the fraction $\gamma$ of the common stock in the venture, which has value $\gamma(y - v)$.

4 Results

In this section, I first examine the players’ choices of the standard or risky strategy, then the renegotiation and intervention stage, and finally the entrepreneur’s initial contract offer. The propositions characterize a perfect Bayesian equilibrium of the game. I focus on the equilibrium that involves the least amount of renegotiation, i.e., I focus on the equilibrium that involves forgiveness of contract terms by the venture capitalist only when that makes her strictly better off and that, when renegotiation occurs, involves the smallest possible changes to the terms of the contract.

Standard and Risky Strategies

If, after the state is revealed, the venture capitalist intervenes, then the venture capitalist chooses between the standard and risky strategies. The venture capitalist is assumed to maximize her expected payoff. The venture capitalist’s payoff under a mixed debt-equity contract with dividend $v$ and share $\gamma$, conditional on the standard strategy, is

$$\pi(\omega, v, \gamma \mid s) \equiv \begin{cases} \int_0^{\gamma(\omega)} y f(y \mid \omega) dy, & \text{if } v \geq \bar{y}(\omega) \\ \int_0^{v} y f(y \mid \omega) dy + \int_{v}^{\gamma(\omega)} (v + \gamma(y - v)) f(y \mid \omega) dy, & \text{otherwise,} \end{cases}$$
and the venture capitalist’s payoff conditional on the risky strategy is

\[
\pi(\omega, v, \gamma \mid r) \equiv \begin{cases} 
\int_0^{g(\omega)} yg(y \mid \omega)dy, & \text{if } v \geq \bar{y}(\omega) \\
\int_0^v yg(y \mid \omega)dy + \int_v^{g(\omega)} (v + \gamma(y - v))g(y \mid \omega)dy, & \text{otherwise}.
\end{cases}
\]

Given these payoff functions, the venture capitalist is risk averse, and so always prefers the standard strategy. More formally, by Assumption 1, the standard strategy has higher expected payoff, and by Assumption 2, the risky strategy is riskier in the sense of second-order stochastic dominance. Then the concavity of the venture capitalist’s payoff function implies that the venture capitalist has higher expected payoff under the standard strategy.

If, after the state is revealed, the venture capitalist chooses not to intervene, then the entrepreneur decides whether to pursue the risky strategy or the standard strategy. The entrepreneur chooses the strategy that gives him the higher expected return. I assume the entrepreneur chooses the standard strategy if he is indifferent between the two strategies. The entrepreneur’s payoff conditional on the standard strategy, \(s\), is

\[
u(\omega, v, \gamma \mid s) \equiv \begin{cases} 
0, & \text{if } v \geq \bar{y}(\omega) \\
(1 - \gamma) \int_v^{g(\omega)} (y - v)f(y \mid \omega)dy, & \text{otherwise},
\end{cases}
\]

and the entrepreneur’s payoff conditional on the risky strategy, \(r\), is

\[
u(\omega, v, \gamma \mid r) \equiv \begin{cases} 
0, & \text{if } v \geq \bar{y}(\omega) \\
(1 - \gamma) \int_v^{g(\omega)} (y - v)g(y \mid \omega)dy, & \text{otherwise}.
\end{cases}
\]

The entrepreneur chooses the risky strategy if \(u(\omega, v, \gamma \mid r) > u(\omega, v, \gamma \mid s)\). Whether this inequality holds is independent of \(\gamma\), so the entrepreneur’s decision does not depend on the share \(\gamma\). The entrepreneur’s choice of risky or standard strategy depends only on the fixed component of the sharing rule. By Assumption 1, when the dividend is zero, the standard strategy gives the entrepreneur greater expected payoff than the risky strategy, so the entrepreneur always prefers the standard strategy.
when the project is financed entirely with equity. But if the contract has some debt component, then the entrepreneur may prefer the risky strategy.

Define the dividend \( v^s(\omega) \) to be the largest dividend such that the entrepreneur still prefers the standard strategy in state \( \omega \).\(^6\)

\[
v^s(\omega) \equiv \max\{v \in [0, \bar{y}(\omega)] \mid u(\omega, v, \gamma | r) \leq u(\omega, v, \gamma | s)\}.
\] (1)

The \( s \) in \( v^s \) is mnemonic for standard strategy since \( v^s \) is the maximal dividend such that the entrepreneur chooses the standard strategy. Figure 3 shows the entrepreneur’s expected payoffs for the standard and risky strategies as a function of the dividend \( v \), holding fixed the state \( \omega \) and the share \( \gamma \). The dividend \( v^s \) is the dividend such that for smaller dividends, the entrepreneur prefers the standard strategy and for larger dividends the entrepreneur prefers the risky strategy.

![Figure 3: Entrepreneur’s expected payoffs for the standard and risky strategies as a function of the dividend \( v \), holding fixed the state \( \omega \) and the share \( \gamma \).](image)

\(^6\)Since \( u(\omega, 0, \gamma | r) < u(\omega, 0, \gamma | s) \) and \( u \) is continuous in \( v \), the dividend \( v^s(\omega) \) is well defined.
As mentioned before, the entrepreneur’s choice does not depend on \( \gamma \), so \( v^s \) also does not depend on \( \gamma \). And, when the dividend is zero, the entrepreneur is risk neutral with respect to the project’s returns, and so the entrepreneur strictly prefers the standard strategy, implying that \( v^s(\omega) > 0 \). For all dividends less than \( v^s \), the entrepreneur chooses the standard strategy, and for dividends greater than \( v^s \), the entrepreneur chooses the risky strategy.

Lemma 1 states a property of \( v^s \), that for all \( \omega \in \Omega \), \( v^s(\omega) \) is less than the point at which the distributions \( F \) and \( G \) intersect, \( y^\omega \), which is assumed to be strictly less than \( \bar{y}(\omega) \) (see Figure 1). To see the intuition for this result, note that the entrepreneur’s payoff depends only on project returns that are above the dividend, and, conditional on profits being greater than \( y^\omega \), the risky strategy results in higher expected profit (the cdf for \( G \) is below the cdf for \( F \) for \( y > y^\omega \)). Thus, for any dividend greater than \( y^\omega \), the entrepreneur prefers the risky strategy, and so \( v^s(\omega) \) cannot be greater than \( y^\omega \).

**Lemma 1** For all \( \omega \in \Omega \), \( v^s(\omega) \leq y^\omega < \bar{y}(\omega) \).

**Proof.** Suppose \( v^s(\omega) > y^\omega \). Then by Assumption 2,
\[
\int_{v^s(\omega)}^{\bar{y}(\omega)} [G(y | \omega) - F(y | \omega)] \, dy < 0,
\]
but, using the definition of \( u \) and integrating by parts, this implies that
\[
u(\omega, v^s(\omega), \gamma | r) > u(\omega, v^s(\omega), \gamma | s),\]
which violates the definition of \( v^s(\omega) \). This contradiction implies that \( v^s(\omega) \leq y^\omega \), and Assumption 2 implies \( y^\omega < \bar{y}(\omega) \).

*Renegotiation and intervention*

In state \( \omega \), the venture capitalist is willing to reduce the fixed payment to \( v^s(\omega) \) to avoid having the entrepreneur choose the risky strategy if and only if that change
increases the venture capitalist’s expected utility, i.e., if and only if \( \pi(\omega, v^s(\omega), \gamma \mid s) \) is greater than the payoff the venture capitalist would get otherwise. If the venture capitalist has control rights, then the venture capitalist is willing to reduce the fixed payment to \( v^s(\omega) \) if and only if

\[
\pi(\omega, v^s(\omega), \gamma \mid s) > \max \{ \pi(\omega, v, \gamma \mid r), \pi(\omega, v, \gamma \mid s) - c \}.
\]

If the venture capitalist does not have control rights, then the venture capitalist is willing to reduce the fixed payment to \( v^s(\omega) \) if and only if

\[
\pi(\omega, v^s(\omega), \gamma \mid s) > \pi(\omega, v, \gamma \mid r).
\]

The venture capitalist will not always be willing to forgive dividends in order to give the entrepreneur the incentive to choose the standard strategy. If the state is such that \( v^s(\omega) \) is much less than \( v \), then the venture capitalist may not be willing to reduce the dividend from \( v \) to \( v^s(\omega) \) just to give the entrepreneur the incentive to choose the standard strategy.

When the venture capitalist has control rights, let \( v^f_c(\omega, \gamma) \) be the maximum dividend such that the venture capitalist is willing to forgive dividends down to \( v^s(\omega) \) to avoid having the entrepreneur choose the risky strategy when the state is \( \omega \) and the share is \( \gamma \). When the venture capitalist does not have control rights, let \( v^f_n(\omega, \gamma) \) be similarly defined.\footnote{Since the inequalities in (2) and (3) are satisfied at \( v = v^s(\omega) \), and since \( \pi \) is continuous in \( v \), the dividends \( v^f_c(\omega, \gamma) \) and \( v^f_n(\omega, \gamma) \) are well defined.}

\[
v^f_c(\omega, \gamma) \equiv \max \{ v \in [v^s(\omega), \bar{v}(\omega)] \mid \pi(\omega, v^s(\omega), \gamma \mid s) \geq \max \{ \pi(\omega, v, \gamma \mid r), \pi(\omega, v, \gamma \mid s) - c \} \}, \quad (2)
\]

and

\[
v^f_n(\omega, \gamma) \equiv \max \{ v \in [v^s(\omega), \bar{v}(\omega)] \mid \pi(\omega, v^s(\omega), \gamma \mid s) \geq \pi(\omega, v, \gamma \mid r) \}. \quad (3)
\]

The \( f \) in \( v^f_c \) is mnemonic for forgive since \( v^f \) is the maximum dividend such that the venture capitalist is willing to unilaterally forgive part of the entrepreneur’s dividend obligation.
By construction, given state $\omega$ and share $\gamma$, the venture capitalist prefers to forgive dividends down to $v^* (\omega)$ rather than continue with the original contract if and only if $v \leq v^f_c (\omega, \gamma)$ or $v \leq v^f_n (\omega, \gamma)$, depending on whether the venture capitalist has control rights or not. If $v > v^f_c (\omega, \gamma)$, then the venture capitalist prefers to continue with the original contract and either let the entrepreneur choose the risky strategy or intervene and implement the standard strategy. If the venture capitalist does not have control rights and $v > v^f_n (\omega, \gamma)$, then the venture capitalist prefers to continue with the original contract and let the entrepreneur choose the risky strategy.

Figure 4 shows the venture capitalist’s payoff as a function of $v$, holding fixed the state $\omega$ and share $\gamma$. The figure shows that for dividends between $v^* (\omega)$ and $v^f_c (\omega, \gamma)$, the venture capitalist prefers to forgive dividends down to $v^* (\omega)$ rather than continue with the original contract since $\pi (\omega, v, \gamma \mid r) < \pi (\omega, v^* (\omega), \gamma \mid s)$. In Figure 4, the cost of intervention, $c$, is sufficiently large that the venture capitalist prefers to let the entrepreneur choose the risky strategy rather than intervene, so the dividend $v^f_c (\omega, \gamma)$ as well as the dividend $v^f_n (\omega, \gamma)$ are determined by the intersection of $\pi (\omega, v, \gamma \mid r)$ and $\pi (\omega, v^* (\omega), \gamma \mid s)$. If the cost of intervention were sufficiently small, the venture capitalist would prefer to intervene rather than let the entrepreneur choose the risky strategy, and so $v^f_c (\omega, \gamma)$ would be determined by the intersection of $\pi (\omega, v, \gamma \mid s) - c$ and $\pi (\omega, v^* (\omega), \gamma \mid s)$.

If $v > v^f_c (\omega, \gamma)$, the venture capitalist is not willing to forgive dividends so as to induce the entrepreneur to choose the standard strategy because the amount that would have to be forgiven is so large that doing so would decrease the venture capitalist’s expected payoff. However, renegotiation of the contract is possible. In renegotiation, the venture capitalist offers a new contract that has dividend $v^* (\omega)$, which is less than the dividend in the original contract, and a share that is greater than the share in the original contract.

When the venture capitalist has control rights, there are two cases to consider. First, if $\pi (\omega, v, \gamma \mid r) \geq \max \{\pi (\omega, v, \gamma \mid s) - c, \pi (\omega, v^* (\omega), \gamma \mid s)\}$, then the en-
Figure 4: Venture capitalist’s payoff as a function of \( v \), holding fixed the state \( \omega \) and share \( \gamma \).

The entrepreneur accepts any renegotiation offer that gives him payoff at least \( u(\omega, v, \gamma | r) \), since this is the payoff the entrepreneur gets if he rejects the renegotiation offer. We define \( \gamma^r \) to be the share that leaves the entrepreneur indifferent between accepting and rejecting the renegotiation offer, i.e., \( \gamma^r(\omega, v, \gamma) \) satisfies

\[
u(\omega, v^s(\omega), \gamma^r(\omega, v, \gamma) | s) = u(\omega, v, \gamma | r).
\]

Second, if \( \pi(\omega, v, \gamma | s) - c > \max \{ \pi(\omega, v, \gamma | r), \pi(\omega, v^s(\omega), \gamma | s) \} \), then the entrepreneur accepts any renegotiation offer that gives him payoff at least \( u(\omega, v, \gamma | s) \), since this is the payoff the entrepreneur gets if he rejects the renegotiation offer. (If the entrepreneur rejected the offer, the venture capitalist would intervene and choose the standard strategy.) So \( \gamma^i(\omega, v, \gamma) \) satisfies

\[
u(\omega, v^s(\omega), \gamma^i(\omega, v, \gamma) | s) = u(\omega, v, \gamma | s).
\]

The \( r \) in \( \gamma^r \) is mnemonic for the share that is acceptable rather than choosing the
“risky strategy, and the $i$ in $\gamma^i$ is mnemonic for the share the is acceptable rather than facing "i"ntervention. Note that I assume that the venture capitalist does not intervene when she is indifferent between intervening and not.

When the venture capitalist does not have control rights, the entrepreneur accepts any renegotiation offer that gives him payoff at least $u(\omega, v, \gamma \mid r)$, since this is the payoff the entrepreneur would get if he rejected the renegotiation offer. So the same share $\gamma^r$ from above leaves the entrepreneur indifferent between accepting and rejecting the renegotiation offer in this case.

As shown in Lemma 2, the shares $\gamma^r$ and $\gamma^i$ are well defined, leave the entrepreneur indifferent between accepting the renegotiation offer and not, and leave the venture capitalist at least as well off as if she did not make the renegotiation offer. The proof of Lemma 2 is given in the Appendix.

**Lemma 2** There exist functions $\gamma^r, \gamma^i : \Omega \times \mathbb{R}_+ \times [0, 1] \to [0, 1]$ such that for all $\omega \in \Omega$, $\gamma \in [0, 1]$,

(i) If $v \geq v^r_\omega(\omega, \gamma)$ and $\pi(\omega, v, \gamma \mid r) \geq \max \{\pi(\omega, v, \gamma \mid s) - c, \pi(\omega, v^s(\omega), \gamma \mid s)\}$ or if $v \geq v^i_\omega(\omega, \gamma)$, then $u(\omega, v^s(\omega), \gamma^r(\omega, v, \gamma) \mid s) = u(\omega, v, \gamma \mid r)$ and

$$\pi(\omega, v^s(\omega), \gamma^r(\omega, v, \gamma) \mid s) \geq u(\omega, v, \gamma \mid r);$$

and

(ii) if $v \geq v^i_\omega(\omega, \gamma)$ and $\pi(\omega, v, \gamma \mid s) - c > \max \{\pi(\omega, v, \gamma \mid r), \pi(\omega, v^s(\omega), \gamma \mid s)\}$, then $u(\omega, v^s(\omega), \gamma^i(\omega, v, \gamma) \mid s) = u(\omega, v, \gamma \mid s)$ and

$$\pi(\omega, v^s(\omega), \gamma^i(\omega, v, \gamma) \mid s) \geq \pi(\omega, v, \gamma \mid s).$$

Lemma 2 implies that it is always possible for the venture capitalist and entrepreneur to renegotiate out of a situation in which the venture capitalist would intervene or the entrepreneur would choose the risky strategy. If $v \geq v^r_\omega(\omega, \gamma)$ and

$$\pi(\omega, v, \gamma \mid s) - c > \max \{\pi(\omega, v, \gamma \mid r), \pi(\omega, v^s(\omega), \gamma \mid s)\},$$


then the venture capitalist would intervene in the absence of renegotiation. In this case, the entrepreneur must be offered a contract that gives him payoff at least 
\( u(\omega, v, \gamma | s) \). This leaves a payoff of \( \pi(\omega, v, \gamma | s) \) for the venture capitalist, and this is at least as large as the payoff she would get if she intervened in the project, 
\( \pi(\omega, v, \gamma | s) - c \). If either \( v \geq v^f_\epsilon(\omega, \gamma) \) and

\[
\pi(\omega, v, \gamma | r) \geq \max \{ \pi(\omega, v, \gamma | s) - c, \, \pi(\omega, v^s(\omega), \gamma | s) \}
\]

or \( v \geq v^f_\eta(\omega, \gamma) \), then the entrepreneur must be offered a contract that gives him payoff at least \( u(\omega, v, \gamma | r) \). Assuming the standard strategy is chosen, this leaves a payoff of \( \pi(\omega, v, \gamma | s) + u(\omega, v, \gamma | s) - u(\omega, v, \gamma | r) \) for the venture capitalist, and this is at least as large as the payoff she would get if she intervened in the project, \( \pi(\omega, v, \gamma | s) - c \), or allowed the entrepreneur to choose the risky strategy, \( \pi(\omega, v, \gamma | r) \).

We can now characterize the subgame perfect equilibrium starting from the renegotiation stage. Before doing that, it will be useful to define two partitions of the state space, one for the case in which the venture capitalist has control rights, and one for the case in which she does not. For the case in which the venture capitalist has control rights, a partition is

\[
\Omega^1(v) = \{ \omega \in \Omega | v \leq v^s(\omega) \},
\]

\[
\Omega^2_\epsilon(v, \gamma) = \{ \omega \in \Omega | v^s(\omega) < v < v^f_\epsilon(\omega, \gamma) \},
\]

\[
\Omega^3_\epsilon(v, \gamma) = \left\{ \omega \in \Omega \middle| v^f_\epsilon(\omega, \gamma) \leq v \text{ and } \pi(\omega, v, \gamma | r) \geq \max \{ \pi(\omega, v, \gamma | s) - c, \, \pi(\omega, v^s(\omega), \gamma | s) \} \right\},
\]

and

\[
\Omega^4_\epsilon(v, \gamma) = \left\{ \omega \in \Omega \middle| v^f_\epsilon(\omega, \gamma) \leq v \text{ and } \pi(\omega, v, \gamma | s) - c > \max \{ \pi(\omega, v, \gamma | r), \, \pi(\omega, v^s(\omega), \gamma | s) \} \right\}.
\]

For the case in which the venture capitalist does not have control rights, a partition
is \( \Omega^1(v) \) together with

\[
\Omega^2_{\Omega}(v, \gamma) \equiv \{ \omega \in \Omega \mid v^*(\omega) < v < v^g_1(\omega, \gamma) \}, \text{ and } \\
\Omega^2_{\Omega}(v, \gamma) \equiv \{ \omega \in \Omega \mid v^g_1(\omega, \gamma) \leq v \}.
\]

Given the earlier assumptions that \( \Omega \) is a real interval and that \( F(\cdot \mid \omega) \) and \( G(\cdot \mid \omega) \) first-order stochastically dominate \( F(\cdot \mid \omega') \) and \( G(\cdot \mid \omega') \), respectively, when \( \omega > \omega' \), then the sets \( \Omega^2_{\Omega} \) are real intervals or the union of real intervals.

We can now define the subgame perfect strategies starting from period \( t = 1 \), given the initial sharing rule and allocation of control rights. Proposition 1 implies that, when the venture capitalist has control rights, an equilibrium outcome starting from \( t = 1 \) is that there is no renegotiation if \( v \leq v^s(\omega) \), dividends are forgiven down to \( v^s(\omega) \) if \( v^s(\omega) < v < v^g_1(\omega, \gamma) \), and the sharing rule is changed to \( s(\cdot \mid v^s(\omega), \gamma^*(\omega, v, \gamma)) \) or \( s(\cdot \mid v^g_1(\omega, \gamma), \gamma^*(\omega, v, \gamma)) \) if \( v^g_1(\omega, \gamma) \leq v \). When the venture capitalist does not have control rights, there is no renegotiation if \( v \leq v^s(\omega) \), dividends are forgiven down to \( v^s(\omega) \) if \( v^s(\omega) < v < v^g_1(\omega, \gamma) \), and the sharing rule is changed to \( s(\cdot \mid v^s(\omega), \gamma^*(\omega, v, \gamma)) \) if \( v^g_1(\omega, \gamma) \leq v \). These new sharing rules are preferred (at least weakly) by both the venture capitalist and the entrepreneur to the old sharing rule, and there is no sharing rule that gives higher expected payoff to both.

**Proposition 1** Let \( \omega \in \Omega, v \geq 0, \text{ and } \gamma \in [0,1] \) be given. The following strategies are subgame perfect for the subgame starting at \( t = 1 \):

**Renegotiation with control rights:** If \( \omega \in \Omega^1(v) \cup \Omega^2_{\Omega}(v, \gamma) \), the venture capitalist makes no renegotiation offer; if \( \omega \in \Omega^2_{\Omega}(v, \gamma) \), the venture capitalist proposes a new sharing rule with dividend \( v^s(\omega) \) and share \( \gamma^*(\omega, v, \gamma) \); and if \( \omega \in \Omega^1(v, \gamma) \), the venture capitalist proposes a new sharing rule with dividend \( v^s(\omega) \) and share \( \gamma^*(\omega, v, \gamma) \). If \( \omega \in \Omega^1(v) \cup \Omega^2_{\Omega}(v, \gamma), \omega \in \Omega^2_{\Omega}(v, \gamma), \text{ or } \omega \in \Omega^2_{\Omega}(v, \gamma) \), the entrepreneur accepts a renegotiation offer if and only if it gives him expected payoff at least \( u(\omega, v, \gamma \mid s) \), \( u(\omega, v^s(\omega), \gamma \mid s) \), or \( u(\omega, v, \gamma \mid r) \), respectively.
**Renegotiation without control rights:** If \( \omega \in \Omega^1(v) \cup \Omega^2_{n1}(v, \gamma) \), the venture capitalist makes no renegotiation offer; and if \( \omega \in \Omega^2_{n1}(v, \gamma) \), the venture capitalist proposes a new sharing rule with dividend \( v^*(\omega) \) and share \( \gamma^r(\omega, v, \gamma) \). If \( \omega \in \Omega^1(v) \), \( \omega \in \Omega^2_{n1}(v, \gamma) \), or \( \omega \in \Omega^3_{n1}(v, \gamma) \), the entrepreneur accepts a renegotiation offer if and only if it gives him expected payoff at least \( u(\omega, v, \gamma \mid s) \), \( u(\omega, v^*(\omega), \gamma \mid s) \), or \( u(\omega, v, \gamma \mid r) \), respectively.

**Forgiveness and intervention:** Suppose that after renegotiation, the contract has dividend \( \hat{v} \) and share \( \hat{\gamma} \). If the venture capitalist has control rights, she forgives dividends to \( v^*(\omega) \) if and only if \( \omega \in \Omega^2_{c}(\hat{v}, \hat{\gamma}) \) and intervenes if and only if \( \omega \in \Omega^4_{c}(\hat{v}, \hat{\gamma}) \). If the venture capitalist does not have control rights, she forgives dividends to \( v^*(\omega) \) if and only if \( \omega \in \Omega^2_{n}(\hat{v}, \hat{\gamma}) \).

**Strategy choice:** Suppose that after any forgiveness or intervention, the contract in place has dividend \( v' \). If the venture capitalist intervened, she chooses the standard strategy. If the venture capitalist did not intervene, the entrepreneur chooses the standard strategy if and only if \( \omega \in \Omega^1(v') \).

**Proof of Proposition 1.** By earlier arguments using Assumptions 1 and 2, the venture capitalist’s choice of the standard strategy is subgame perfect. By the definition of \( v^*(\omega) \), the entrepreneur’s strategy choice is subgame perfect. By the definitions of \( v^*(\omega) \), \( \Omega^1 \), \( \Omega^j_n \), for \( j \in \{2, 3, 4\} \), and \( \Omega^j_c \), for \( j \in \{2, 3\} \), the forgiveness and intervention strategies are subgame perfect. Using those definitions again, the definitions of \( \gamma^r \), \( \gamma^i \), and Lemma 2, the venture capitalist’s strategy for renegotiation offers and the entrepreneur’s strategy for accepting renegotiation offers are subgame perfect. 

**Intervention**

By intervening, the venture capitalist prevents the entrepreneur from using the risky strategy, and thus precludes negotiation, but intervening also imposes costs on the venture capitalist. Since I assume the entrepreneur is indifferent between the
original and the renegotiated contract, all gains to the change in strategy achieved through renegotiation are captured by the venture capitalist. Thus, the venture capitalist always prefers the renegotiated contract over intervention. However, in reality we do observe instances in which venture capitalists intervene in venture capital projects, install new management, and even remove the founding entrepreneur. If the model of this paper were modified to allow the entrepreneur to have some bargaining power in renegotiation, then in some states the venture capitalist would prefer to intervene rather than renegotiate. This suggests that we should expect increases in the entrepreneur’s bargaining power to increase the likelihood of intervention by the venture capitalist. The addition to the model of frictions such as costly renegotiation would also result in intervention by the venture capitalist in some states.

The addition of costly renegotiation or bargaining power for the entrepreneur in renegotiations also implies that some efficient projects will not receive financing, since either the choice of the risky strategy by the entrepreneur or costly intervention by the venture capitalist will be unavoidable in some states, reducing the ex-ante expected payoff of some projects below the required initial investment.

Initial contract choice

To determine the equilibrium initial contract, I define the venture capitalist’s ex-ante break-even constraint, taking into account the subgame perfect equilibrium strategies starting in period $t = 1$ given in Proposition 1, but assuming a good entrepreneur. With control rights, the constraint is

$$I \leq \Pr [\omega \in \Omega^1(v)] E_{\omega} [\pi(\omega, v, \gamma | s) | \Omega^1(v)]$$

$$+ \Pr [\omega \in \Omega^2(v, \gamma)] E_{\omega} [\pi(\omega, v^s(\omega), \gamma | s) | \Omega^2(v, \gamma)]$$

$$+ \Pr [\omega \in \Omega^3(v, \gamma)] E_{\omega} [\pi(\omega, \gamma^s(\omega), \gamma(\omega, v, \gamma) | s) | \Omega^3(v, \gamma)]$$

$$+ \Pr [\omega \in \Omega^4(v, \gamma)] E_{\omega} [\pi(\omega, v^i(\omega), \gamma^i(\omega, v, \gamma) | s) | \Omega^4(v, \gamma)],$$
and without control rights it is

\[ I \leq \Pr [\omega \in \Omega^1(v)] E_\omega [\pi(\omega, v, \gamma | s) | \Omega^1(v)] \\
+ \Pr [\omega \in \Omega^2_n(v, \gamma)] E_\omega [\pi(\omega, v^*(\omega), \gamma | s) | \Omega^2_n(v, \gamma)] \\
+ \Pr [\omega \in \Omega^2_n(v, \gamma)] E_\omega [\pi(\omega, v^*(\omega), \gamma^*(\omega, v, \gamma) | s) | \Omega^2_n(v, \gamma)]. \tag{5} \]

Let \( \bar{v}_c \) and \( \bar{v}_n \) be the smallest dividend levels such that the venture capitalist’s break-even constraints (4) and (5), respectively, hold with equality when \( \gamma = 0 \).

For \( v \in [0, \bar{v}_c] \), we can define \( \gamma^*_c(v) \) to be the share such that the sharing rule \( s(\cdot | v, \gamma^*_c(v)) \) satisfies the venture capitalist’s ex-ante break-even constraint (4) with equality, and for \( v \in [0, \bar{v}_n] \), we can define \( \gamma^*_n(v) \) to be the share such that the sharing rule \( s(\cdot | v, \gamma^*_n(v)) \) satisfies the venture capitalist’s ex-ante break-even constraint (5) with equality. By the following lemma, the shares \( \gamma^*_c(v) \) and \( \gamma^*_n(v) \) are well defined (asterisks denote equilibrium values). The proof is in the Appendix.

**Lemma 3** For \( j \in \{c, n\} \), there exists \( \gamma^*_j : [0, \bar{v}_j] \rightarrow [0, 1] \), such that for all \( v \in [0, \bar{v}_j] \), the sharing rule with dividend \( v \) and share \( \gamma^*_j(v) \) is feasible and satisfies the venture capitalist’s ex-ante break-even constraint (4) if \( j = c \) or (5) if \( j = n \) with equality.

I now complete the statement of the equilibrium.

**Proposition 2** A perfect Bayesian equilibrium consists of the strategies in Proposition 1 together with (i) beliefs for the venture capitalist that an offer comes from a good entrepreneur if and only if it offers control rights to the venture capitalist and has dividend at least \( \bar{y}(\omega) \), and (ii) the following strategies: A good entrepreneur gives control rights to the venture capitalist and offers a sharing rule with dividend \( \bar{y}(\omega) \) and share equal to \( \gamma^*_c(\bar{y}(\omega)) \) if \( \bar{y}(\omega) \leq \bar{v}_c \) and equal to zero otherwise. A bad entrepreneur does not give control rights to the venture capitalist and offers a sharing rule with dividend zero and share \( \gamma^*_n(0) \). The venture capitalist accepts an initial offer if and only if it includes control rights, has dividend at least \( \bar{y}(\omega) \), and satisfies the break-even constraint (4).

---

\(^8\)Given the continuity assumed in the model and Assumption 3, it is clear that \( \bar{v}_c \) and \( \bar{v}_n \) exist.
Proof. Proposition 1 establishes subgame perfection starting in period $t = 1$ for good and bad entrepreneurs. In equilibrium, the standard strategy is always chosen and the venture capitalist never intervenes, so a sharing rule that satisfies the venture capitalist’s break-even constraint with equality gives the entrepreneur the maximum possible payoff. If $\bar{y}(\omega) \leq \bar{\nu}_c$, then, by the definition of $\gamma^*_c$, the equilibrium contract satisfies the venture capitalist’s break-even constraint (4) with equality. By the first part of assumption 3, the entrepreneur has nonnegative expected payoff in this case. If $\bar{y}(\omega) > \bar{\nu}_c$, then, by the definition of $\bar{\nu}_c$, (4) is satisfied when $(v, \gamma) = (\bar{y}(\omega), 0)$. Since $\bar{y}(\omega)$ is increasing in $\omega$, a good entrepreneur has nonnegative expected payoff under such a sharing rule. Thus, given the venture capitalist’s acceptance strategy, the strategy for a good entrepreneur maximizes his expected payoff. Suppose a bad entrepreneur offers a sharing rule that satisfies the venture capitalist’s acceptance criteria. Then the bad entrepreneur must offer a dividend greater than or equal to $\bar{y}(\omega)$. By the definition of $v^*_\omega$, for all $\gamma \in [0, 1]$, $v^*_\omega(\omega, \gamma) \leq \bar{y}(\omega)$, so for all $\gamma \in [0, 1]$ and $v \geq \bar{y}(\omega), \omega \in \Omega^1(\omega, \gamma) \cup \Omega^1(\omega, \gamma)$. This implies that the bad entrepreneur’s contract is always renegotiated. Since the entrepreneur has zero expected payoff in the absence of renegotiation, his disagreement payoff is zero and so he gets a payoff of zero in the renegotiated contract. Thus, it is payoff maximizing for a bad entrepreneur to offer a contract that is, in equilibrium, not accepted by the venture capitalist. Finally, given the venture capitalist’s beliefs, her acceptance strategy maximizes her expected payoff, and given the strategies for the two types of entrepreneur, the venture capitalist’s beliefs are consistent. ■

As shown in Proposition 2, a good entrepreneur can signal his type by offering control rights to the venture capitalist and proposing a sharing rule that has a positive dividend. The equilibrium dividend is sufficiently large that a bad entrepreneur has zero expected payoff under such a contract.\textsuperscript{9} In this model there is no reason other

\textsuperscript{9}The addition of an effort cost borne by the entrepreneur or a requirement that the entrepreneur
than signalling for an entrepreneur to offer control rights to the venture capitalist since intervention does not occur in equilibrium. However, whether the venture capitalist has control rights affects the entrepreneur’s disagreement payoff in renegotiation and so does affect the renegotiation offer that is made in equilibrium. In some cases, i.e., when \( \bar{y}(\omega) \leq \bar{y}_c \),\(^{10}\) the entrepreneur captures the differences in renegotiation outcomes in the initial sharing rule, so that the venture capitalist’s ex ante break-even constraint is satisfied with equality.

The equilibrium contract described in Proposition 2 is efficient. In equilibrium, good entrepreneurs receive financing and bad entrepreneurs do not. In equilibrium, the venture capitalist never chooses to undertake costly intervention, and the entrepreneur never chooses the inefficient risky strategy. Although I restrict the sharing rule proposed by the entrepreneur to be a mixed debt-equity sharing rule, Proposition 2 shows that contracts of that form are sufficiently complex so as to achieve efficiency.

In summary, in the equilibrium of Proposition 2, contracts are sometimes renegotiated (either one-sided forgiveness or two-sided renegotiation) so that the entrepreneur has the incentive to choose the standard strategy. Renegotiation always prevents a situation in which the entrepreneur has the incentive to choose the risky strategy. Equilibrium renegotiation or forgiveness involves either no change, the reduction of the entrepreneur’s debt obligation, or the reduction of debt together with an increase in the venture capitalist’s equity share. Although the renegotiation outcome described is not unique, we can think of it as the least-cost renegotiation in that it involves the minimal adjustments to the contract terms.

\(^{10}\)A sufficient condition for this is that \( E_\omega \left[ \int_0^{\bar{y}(\omega)} y \, f(y \mid \omega) \, dy \right] < I \).
5 Conclusions

The analysis from the previous section allows us to characterize the behavior of entrepreneurs and venture capitalists when projects are financed by mixtures of debt and equity. Some notable features of the equilibrium are as follows: (i) renegotiation occurs if the state is revealed to be low, (ii) renegotiation never leads to an increase in the dividend, (iii) renegotiation never leads to a decrease in the venture capitalist’s share, and (iv) renegotiation may affect only the dividend but never affects only the venture capitalist’s share. These results are consistent with observed behavior between venture capitalists and entrepreneurs. Furthermore, the results show that mixed debt-equity contracts can achieve efficiency and that positive dividend levels can be used by an entrepreneur to effectively signal the expected profitability of his project.

It is interesting that, while intervention by the venture capitalist does not occur on the equilibrium path, the threat of intervention is important in determining the entrepreneur’s disagreement payoff in renegotiation, and thus the outcome of contract renegotiation. The addition of frictions to the model, such as costly renegotiation, would result in intervention by the venture capitalist in some states. Also, increases in the entrepreneur’s bargaining power in renegotiation would result in intervention by the venture capitalist in some states. Either of these changes to the model would also result in some efficient projects not being financed.

In the model, good entrepreneurs use a combination of positive dividends and the granting of control rights to distinguish themselves from bad entrepreneurs. An extension of the model would allow a greater role for control rights, so that giving control rights to the venture capitalist would serve as a stronger signal of the entrepreneur’s quality.
6 Appendix

Proof of Lemma 2

(i) Suppose \( \omega \in \Omega, v \geq 0, \gamma \in [0, 1] \), and either \( v \geq v^*_c(\omega, \gamma) \) and \( \pi(\omega, v, \gamma \mid r) \geq \max \{ \pi(\omega, v, \gamma \mid s) - c, \pi(\omega, v^*(\omega), \gamma \mid s) \} \) or \( v \geq v^*_d(\omega, \gamma) \). I claim \( \gamma^r \) is uniquely defined by

\[
u(\omega, v^*(\omega), \gamma^r(\omega, v, \gamma) \mid s) = \nu(\omega, v, \gamma \mid r).
\]

Using the definition of \( u \) and integrating by parts, this equality implies (suppressing the arguments of \( F, G, v^s, \) and \( \bar{y} \)):

\[
(1 - \gamma^r(\omega, v, \gamma)) \left( \bar{y} - v^s - \int^0_{v^s} Fdy \right) = (1 - \gamma) \left( \bar{y} - \min\{v, \bar{y}\} - \int^0_{\min\{v, \bar{y}\}} Gdy \right).
\]

Since \( v^s(\omega) < \bar{y}(\omega) \) by Lemma 1, (6) uniquely defines \( \gamma^r(\omega, v, \gamma) \). One can easily show that \( \gamma^r(\omega, v, \gamma) > \gamma \), i.e., the decrease in dividend requires an increase in the share to keep the entrepreneur indifferent. To see that \( \gamma^r(\omega, v, \gamma) < 1 \), so that the share \( \gamma^r \) is feasible, note that

\[
\gamma^r(\omega, v, \gamma) = \frac{\left( \bar{y} - v^s - \int^0_{v^s} Fdy \right) - (1 - \gamma) \left( \bar{y} - \min\{v, \bar{y}\} - \int^0_{\min\{v, \bar{y}\}} Gdy \right)}{\left( \bar{y} - v^s - \int^0_{v^s} Fdy \right)} < 1.
\]

To show that \( \pi(\omega, v^s(\omega), \gamma^r(\omega, v, \gamma) \mid s) \geq \pi(\omega, v, \gamma \mid r) \), note that

\[
\pi(\omega, v^s(\omega), \gamma^r(\omega, v, \gamma) \mid s) - \pi(\omega, v, \gamma \mid r)
= v^s - \int_{v^s}^{\bar{y}} Fdy + \gamma^r(v) (\bar{y} - v^s - \int_{v^s}^{\bar{y}} Fdy)
- v + \int_{\min\{v, \bar{y}\}}^{\bar{y}} Gdy - \gamma (\bar{y} - \min\{v, \bar{y}\} - \int_{\min\{v, \bar{y}\}}^{\bar{y}} Gdy)
= \bar{y} - \min\{v, \bar{y}\} - \int_{\min\{v, \bar{y}\}}^{\bar{y}} Gdy + \int_{0}^{\bar{y}} [G - F] dy
\geq 0,
\]

where the first inequality uses the definition of \( \pi \) and integration by parts, the second equality uses (7), and the inequality uses Assumption 1.

(ii) Suppose \( \omega \in \Omega, v \geq 0, \gamma \in [0, 1] \), \( v \geq v^*_c(\omega, \gamma) \), and \( \pi(\omega, v, \gamma \mid r) - c > \max \{ \pi(\omega, v, \gamma \mid s), \pi(\omega, v^*(\omega), \gamma \mid s) \} \). I claim \( \gamma^i \) is uniquely defined by

\[
u(\omega, v^*(\omega), \gamma^i(\omega, v, \gamma) \mid s) = \nu(\omega, v, \gamma \mid s).
\]
Using the definition of \( u \) and integrating by parts, this equality implies (suppressing the arguments of \( F, G, v^s, \) and \( \bar{y} \)):

\[
(1 - \gamma^i(\omega, v, \gamma)) \left( \bar{y} - v^s - \int_{v^s}^\omega Fdy \right) = (1 - \gamma) \left( \bar{y} - \min \{v, \bar{y} \} - \int_{\min \{v, \bar{y} \}}^\omega Fdy \right).
\]

(8)

Since \( v^s(\omega) < \bar{y}(\omega) \) by Lemma 1, (8) uniquely defines \( \gamma^i(\omega, v, \gamma) \). One can easily show that \( \gamma^i(\omega, v, \gamma) > \gamma \), i.e., the decrease in dividend requires an increase in the share to keep the entrepreneur indifferent. To see that \( \gamma^i(\omega, v, \gamma) < 1 \), so that the share \( \gamma^i \) is feasible, note that

\[
\gamma^i(\omega, v, \gamma) = \frac{\left( \bar{y} - v^s - \int_{v^s}^\omega Fdy \right) - (1 - \gamma) \left( \bar{y} - \min \{v, \bar{y} \} - \int_{\min \{v, \bar{y} \}}^\omega Fdy \right)}{\left( \bar{y} - v^s - \int_{v^s}^\omega Fdy \right)} < 1.
\]

(9)

To show that \( \pi(\omega, v^s(\omega), \gamma^i(\omega, v, \gamma) \mid s) \geq \pi(\omega, v, \gamma \mid s) - c \), note that

\[
\pi(\omega, v^s(\omega), \gamma^i(\omega, v, \gamma) \mid s) - \pi(\omega, v, \gamma \mid s) + c
= v^s - \int_0^{v^s} Fdy + \gamma^i(\omega, v, \gamma) \left( \bar{y} - v^s - \int_{v^s}^\omega Fdy \right)
- \min \{v, \bar{y} \} + \int_0^{\bar{y}} Fdy - \gamma \left( \bar{y} - \min \{v, \bar{y} \} - \int_{\min \{v, \bar{y} \}}^{\bar{y}} Fdy \right) + c
= c
\geq 0,
\]

where the first inequality uses the definition of \( \pi \) and integration by parts, the second equality uses (9), and the inequality uses the assumption that \( c \geq 0 \). ■

**Proof of Lemma 3.** Let dividend \( v \in [0, \bar{v}_j] \) and \( \gamma \in [0, 1] \) be given. When the break-even constraint is satisfied with equality, the following equation holds (suppressing the arguments of the \( \Omega^k_j \)s and defining \( \Omega^4_n \equiv \emptyset \)):

\[
\gamma \left( \Pr [\omega \in \Omega^1] E_\omega \left[ \int_0^{\bar{v}(\omega)} (y - v)f(y \mid \omega)dy \mid \Omega^1 \right] + \Pr [\omega \in \Omega^2_1] E_\omega \left[ \int_0^{\bar{v}(\omega)} (y - v^s(\omega))f(y \mid \omega)dy \mid \Omega^2_1 \right] \right)
= I - \Pr [\omega \in \Omega^1] E_\omega \left[ \int_0^{\bar{v}(\omega)} yf(y \mid \omega)dy + \int_0^{\bar{v}(\omega)} v^s(\omega)f(y \mid \omega)dy \mid \Omega^1 \right]
- \Pr [\omega \in \Omega^2_1] E_\omega \left[ \int_0^{\bar{v}(\omega)} yf(y \mid \omega)dy + \int_0^{\bar{v}(\omega)} v^s(\omega)f(y \mid \omega)dy \mid \Omega^2_1 \right]
- \Pr [\omega \in \Omega^3_1] E_\omega \left[ \int_0^{\bar{v}(\omega)} yf(y \mid \omega, \gamma^i(\omega, \gamma, \omega, \gamma)f(y \mid \omega)dy \mid \Omega^3_1 \right]
- \Pr [\omega \in \Omega^4_1] E_\omega \left[ \int_0^{\bar{v}(\omega)} yf(y \mid \omega, \gamma^i(\omega, \gamma, \omega, \gamma)f(y \mid \omega)dy \mid \Omega^4_1 \right].
\]

(10)
Given our assumptions about the probability distribution for $\omega$, both sides of (10) are continuous in $\gamma$. When $\gamma = 0$, the left-hand side is zero. When $\gamma = 0$, the right-hand side is zero if $v = \bar{v}_j$ and strictly positive if $v < \bar{v}_j$ (by the definition of $\bar{v}_j$ and using Assumption 3). When $\gamma = 1$, $\gamma^*(\omega, v, \gamma) = \gamma^*(\omega, v, \gamma) = 1$, so the left-hand side of (10) is
\[
\text{Pr} [\omega \in \Omega^1] E_\omega \left[ \int_0^{\gamma(\omega)} (y - v)f(y \mid \omega)dy \mid \Omega^1 \right] \\
+ \text{Pr} [\omega \in \Omega^2_j] E_\omega \left[ \int_{\gamma(\omega)}^{\gamma(\omega)} (y - \gamma^*(\omega))f(y \mid \omega)dy \mid \Omega^2_j \right].
\]
(11)
and the right-hand side is
\[
I - \text{Pr} [\omega \in \Omega^1] E_\omega \left[ \int_0^{\gamma(\omega)} yf(y \mid \omega)dy + \int_0^{\gamma(\omega)} vf(y \mid \omega)dy \mid \Omega^1 \right] \\
- \text{Pr} [\omega \in \Omega^2_j] E_\omega \left[ \int_{\gamma(\omega)}^{\gamma(\omega)} yf(y \mid \omega)dy + \int_{\gamma(\omega)}^{\gamma(\omega)} \gamma^*(\omega)f(y \mid \omega)dy \mid \Omega^2_j \right] \\
- \text{Pr} [\omega \in \Omega^3_j] E_\omega \left[ \int_{\gamma(\omega)}^{\gamma(\omega)} yf(y \mid \omega)dy \mid \Omega^3_j \right] \\
- \text{Pr} [\omega \in \Omega^4_j] E_\omega \left[ \int_{\gamma(\omega)}^{\gamma(\omega)} yf(y \mid \omega)dy \mid \Omega^4_j \right].
\] (12a)

Taking the difference of expression (11) minus expression (12a), we get $E_\omega \int_0^{\gamma(\omega)} yf(y \mid \omega)dy - I$ which is nonnegative by Assumption 3. Therefore, for all $v \in [0, \bar{v}_j]$, there exists $\gamma^*_j(v) \in [0, 1]$. ■
7 References


Marx, Leslie M. “Efficient Venture Capital Financing Combining Debt and Equity.” 


