Chapter Four

Optimal Asset Allocation in Asset Liability Management

Jules H. van Binsbergen
Stanford GSB

Michael W. Brandt
Fuqua School of Business, Duke University

4.1 Introduction 2
4.2 Yield Smoothing 8
4.3 ALM problem 10
  4.3.1 Return and Yield Dynamics 10
  4.3.2 Preferences 13
  4.3.3 Constraints 15
    4.3.3.1 Short sale constraints 15
    4.3.3.2 VaR constraints 15
    4.3.3.3 Additional Financial Contributions (AFCs) 16
  4.3.4 Data description and estimation 16
4.4 Method 17
4.5 Single Period Portfolio Choice 19
  4.5.1 ALM with a VaR constraint 19
  4.5.2 ALM with AFCs 22
4.6 Dynamic Portfolio Choice 27
  4.6.1 Welfare and portfolio implications of yield smoothing 27
  4.6.2 Hedging demands and regulatory constraints 28
4.1 Introduction

Asset Liability Management (abbreviated ALM) refers to the portfolio choice problem of an investor who uses the principal and investment returns on assets to satisfy future liabilities. One leading example is a defined benefits pension plan that must pay promised benefit payments using pension contributions and the investment returns accumulated on those contributions. Because liabilities are typically modeled as coupon payments of more or less known timing and magnitude, ALM is at the heart a fixed income problem.

The traditional and most conservative approach to ALM is cash flow matching, where assets are invested in fixed income securities for which the coupon and principal payments match as closely as possible the liabilities both in terms of timing and magnitude, thereby eliminating most if not all risk. In larger liability portfolios, such as for an insurance company, however, liability cash flow matching is difficult so instead of matching individual payments, traditional ALM matches the risk profile, specifically interest rate risk and liquidity risk, of the liabilities. Cash flow or risk matching is commonly referred to as Immunization.7

Both liability cash flow or risk matching rely critically on the liabilities being fully funded. ALM in this case is simply a fixed income risk-management problem. Unfortunately, a growing number of defined benefits pension plans in particular are materially underfunded. For example, in 2013 the largest 100 pension plans in the United States reported 3.77 trillion USD of liabilities guaranteed with only 2.58 trillion USD of asset, which represents and underfunding of more than 30%.2 With such degree of underfunding, the plan sponsor cannot rely simply on interest to make up the shortfall. In-

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1 Investment professionals have in recent years relabeled ALM as Liability Driven Investing (LDI). For our purposes the two are synonymous.
2 See Milliman 2013 Public Pension Funding Study, www.milliman.com
stead, the assets must be invested in more sophisticated investment portfolio that earns risk premia in order to generate higher expected returns. A recent study by investment consultant Tower Watson shows the average asset allocation of US pension plans to be 57% equities, 23% bonds, and 20% others (typically real assets and alternatives). Therefore, ALM is no longer just a risk management problem. It has become a portfolio choice problem, and a rather complex and very dynamic one at that.

ALM (or its recent popularized version LDI) has become increasingly the standard for pension management. A 2013 survey by SEI, a pension consultant, found that the use of modern portfolio choice techniques for pension management has increased from 20 percent in 2007 to almost 60% in 2013 with the primary objective being control of the level and volatility of the funding status of the plan. Although pension plans are the primary focus of ALM in the academic and practitioner literature, because of their vastly underfunded nature, they are not the only financial institutions that face ALM problems. Other users of ALM are insurance companies, banks with liabilities on their balance sheet, and even central banks. Any institution that approaches asset management from a balance sheet perspective, meaning takes both assets and liabilities into account when making decisions, is solving an ALM problem and potentially has a different objective than simply maximizing the Sharpe ratio of return on assets. As a consequence, the total amount of assets that are involved in ALM problems is many times larger than the amounts discussed above.

In this paper we use an easy-to-implement dynamic programming algorithm to solve for the optimal asset allocation of a financial intermediary that faces an Asset Liability Management (ALM) problem under regulatory constraints. As an application, we focus on the ALM problem of a defined benefits pension plan. The defined plan faces a dynamic investment opportunity set with time-varying expected bond and stock returns, as well as two types of constraints: ex ante and ex post risk constraints, and we study examples of both. For the ex ante constraint, we consider a Value-at-Risk (VaR) constraint. The ex post constraint we study is the legal requirement for plan sponsors to make mandatory additional financial contributions (AFCs) once the plan is underfunded relative to the accounting definition of liabilities. This accounting definition can (and does) deviate from the market value of the liabilities, which we explicitly account for in our method.

Using this framework, we examine the impact of regulations on pension investment decisions. Because many plans invested heavily in stocks and only had limited positions in long-term bonds, the recent decrease in interest rates combined with the poor performance of global stock markets has further lowered the funding status of many plans. This raises the important question

1See Tower Watson, Global Pension Assets Study 2014, www.towerswatson.com
why pension funds hedge interest rate risk so poorly by investing in asset classes, such as stocks, that have a low correlation with interest rate changes. We argue in this paper that this investment behavior is (at least partially) due to the regulatory environment. In response to the underfunded status of many plans, congress has passed a law as part of the Transportation Bill (June 2012), that allows corporate defined benefit plans to discount their liabilities using a rolling average of yields over the past 25 years. Not only does this interest smoothing rule lead to a large deviation of the reported value of liabilities from their market value (by a factor 2 in 2012), but we show that it further discourages the hedging of interest rate risk. We study in this paper the optimal asset allocation decisions of a pension manager as a function of the plan’s funding ratio (defined as the ratio of its assets to liabilities), interest rates, and the equity risk premium. We compare the optimal investment decisions under several policy alternatives to understand better the real effects of financial reporting and risk management rules.

Arguably the most important aspect of the asset liability management (ALM) problem faced by defined benefits retirement plans is the discount rate used for computing the present value of the plan’s liabilities. Historically, two methodologies have been implemented, namely, the use of current market yields and the use of a weighted average of yields over a longer horizon (yield smoothing). On the one hand, discounting by current yields guarantees an accurate description of the fund’s financial situation. On the other hand, proponents of yield smoothing argue that discounting by smoothed yields gives a more long-term view of the fund’s financial position, as the liabilities are not subject to “short-term” interest rate fluctuations. The 2012 transportation bill, which established a 25-year smoothing period to compute the discount rate for liabilities, is the third time in a decade that discounting rules have been changed. The pension Protection Act of 2006 introduced a two-year smoothing period, replacing the four-year smoothing period of corporate bond rates established under the Pension Funding Equity Act of 2004.

In this context, we make the following three points. First, regardless of whether or not yield smoothing leads to a long-term view of a pension plan’s financial position, it can (and arguably has) led to an upward biased view. This upward bias is induced by a regulator who, in response to political pressure from the private sector and pension sector, changes the length of the smoothing rule depending on the path of interest rates. Figure 4.1 shows that under the 25-year smoothing rule, reported liabilities are twice as small as they would be under market-based discounting in 2012. However, the figure also shows that when interest rates were high, 25-year smoothing would have led to higher reported liabilities than what market-based accounting implied. Not surprisingly, 25-year smoothing was not the rule at the time. Surely, when in the next decades interest rates are high once again, the 25-year smoothing rule

See also Novy-Marx and Rauh (2010).
will be abandoned, and market-based accounting methods will be applied, lowering reported liabilities.

**FIGURE 4.1** Smoothing the 15-year government bond yield and the value of liabilities. We plot the present value of 1 Billion in future liabilities with an assumed duration of 15 years under four discounting regimes. Actual discounting (no smoothing), 4-year smoothing, 25-year smoothing and constant discounting (infinite smoothing) between 1956 and 2012.

Second, yield smoothing distorts incentives and leads to poor hedging against interest rate risk. We show that even when the long-term objective of a plan manager is to maximize the ratio of assets and *market-based* liabilities, her short-term objective (and/or requirement) of satisfying risk constraints and/or avoiding additional financial contributions (AFCs) from the plan sponsor (which are based on the reported liabilities), can induce poor interest rate hedging and inadvertently increase risk taking behavior. We investigate portfolio choice under different discounting regimes, taking a given policy as fixed. This provides a lower bound on how distorting yield smoothing can be. If we add to the model the realistic feature that smoothing rules can be adjusted by a regulator to minimize reported liabilities, for example as a consequence of political pressure, the effects become even stronger. Adjusting the smoothing rule forms a put option on funds’ reported financial positions, encouraging risk taking and further lowering incentives to hedge interest rate risk.

Third, we investigate the influence of ex ante versus ex post risk constraints on the investment decisions of pension managers. At first glance, ex ante and ex post risk constraints may seem similar as both aim to decrease the
risk-taking behavior of the manager. However, we show that they can have different implications for the gains to dynamic, as opposed to myopic, decision making. We hypothesize that ex ante (preventive) constraints, such as a Value-at-Risk (VaR) constraint, can decrease the gains to dynamic investment, as these constraints restrict the choice set of the manager and hence do not allow the manager to respond to the time-varying investment opportunity set. Ex post (punitive) constraints, such as ACFs, in contrast, can increase the gains from solving the dynamic program. In other words, under ex ante constraints, the myopic solution may provide a good approximation for the optimal solution whereas under ex post constraints it requires dynamic optimization to make the optimal investment decision. As such, ex post constraints induce the manager to behave strategically.

ALM problems are inherently long-horizon problems with potentially important strategic aspects. They differ from standard portfolio choice problems (Markowitz (1952), Merton (1969,1971), Samuelson (1969) and Fama (1970)), not only because of the short position in the pension liabilities, but also because of the regulatory risk constraints and mandatory ACFs discussed above. We assume that the investment manager dislikes drawing ACFs from the plan sponsor and directly model this dislike as a “utility” cost. We interpret this utility cost as a reduced form for the loss of compensation or reputation of the investment manager. In other words, drawing mandatory ACFs serves as an ex post (punitive) risk constraint. The associated utility cost introduces a kink in the value function of the investment manager’s dynamic optimization problem that causes the manager to become first-order risk averse whenever the (reported) funding ratio approaches the critical threshold that triggers ACFs. We show that this kink in the value function leads to substantial hedging demands and large certainty equivalent utility gains from dynamic investment.

The investment behavior of corporate pension plans has been studied by Sundaresan and Zapatero (1997) and by Boulier, Trussant and Florens (2005). Sundaresan and Zapatero (1997) model the marginal productivity of the workers of a firm and solve the investment problem of its pension plan assuming a constant investment opportunity set consisting of a risky and a riskless asset. We instead allow for a time-varying investment opportunity set including cash, bonds, and stocks. More importantly, we consider the ALM problem from the perspective of the investment manager as a decision maker and investigate how regulatory rules influence the optimal investment decisions. In order to focus attention on the asset allocation side of the ALM problem,

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7 First-order risk aversion implies that even over very small lotteries investors are very risk averse. This is different for second-order risk aversion where, as the lottery becomes smaller and smaller, the investor essentially becomes risk neutral.
we model the liabilities of the pension plan in reduced form by assuming a constant duration of 15 years.

Boulier, Trussant, and Florens (2005) also assume a constant investment opportunity set with a risky and a risk-free asset. In their problem, the investment manager chooses his portfolio weights to minimize the expected discounted value of the contributions over a fixed time horizon, with the constraint that the value of the assets cannot fall below that of the liabilities at the terminal date. This problem setup implicitly assumes that the pension plan terminates at some known future date and that the investment manager’s horizon is equal to this terminal date. By taking the investment manager’s preferences and horizon as the primitive, our perspective is different. The manager has a motive to minimize (the disutility from) the sponsor’s contributions, captured by the AFCs in our case. However, the manager also wants to maximize the funding ratio at the end of his investment horizon, for example due to career concerns. The end of the manager’s investment horizon may be long before the pension plan terminates, which is why we hold the duration of the liabilities fixed.

Our contributions to the portfolio choice literature are the following. First, we attempt to bridge further the gap between the dynamic portfolio choice literature and the ALM literature.\(^8\) We pose the ALM problem as a standard dynamic portfolio choice problem by defining terminal utility over the ratio of assets and liabilities, as opposed to over assets only. This approach allows a parsimonious representation of the ALM problem under a time-varying investment opportunity set. Solving this dynamic program is relatively straightforward compared to the usual, more complicated, stochastic programming techniques. We then assess the interplay between dynamic hedging demands, risk constraints, and first-order risk aversion. We show that the solution to the ALM problem under ex post (punitive) constraints involves economically significant hedging demands, whereas ex ante (preventive) constraints decrease the gains from dynamic investment. Finally, we explicitly model the trade-off between the long-term objective of maximizing terminal utility and the short-term objective of satisfying VaR constraints and avoiding AFCs from the plan sponsor.\(^9\) We show that if these short-term objectives are based on reported liabilities that are different from actual liabilities, they can lead to large utility losses with respect to the long-term objective.

\(^8\)Campbell and Viceira (2002) and Brandt (2005) survey the dynamic portfolio choice literature.

\(^9\)We could easily incorporate other short-term objectives, such as beating a benchmark portfolio over the course of the year (see also Basak, Shapiro, and Teplá (2006) and Basak, Pavlova, and Shapiro (2007)). Whenever this short-term objective is defined with respect to reported liabilities that are different from actual liabilities, this leads to a similar misalignment of incentives as the one we explore in this paper. It is interesting to note that in practice pension fund managers are often assessed relative to an assets-only benchmark, which is a benchmark that implicitly assumes constant liabilities (see van Binsbergen, Brandt and Koijen (2009)).
For ease of exposition, there are at least three important aspects of the ALM problem that we do not address explicitly. First, there is a literature, starting with Sharpe (1976), that explores the value of the so-called “pension put” arising from the fact that U.S. defined-benefit pension plans are insured through the Pension Benefit Guarantee Corporation. Sharpe (1976) shows that if insurance premiums are not set correctly, the optimal investment policy of the pension plan may be to maximize the difference between the value of the insurance and its cost. This obviously induces perverse incentives. Second, we do not incorporate inflation. Besides affecting the allocation to real versus nominal assets (Hoevenaars et al. (2004)), inflation drives another wedge between the long-term objective of maximizing the real funding ratio, computed with liabilities that are usually pegged to real wage levels, and the short-term objective of satisfying risk controls and avoiding AFCs based on nominal valuations. Third, we ignore the taxation issues described by Black (1980) and Tepper (1981).

The paper proceeds as follows. Section 4.2 describes the different smoothing rules that have been proposed to compute the discount factor and then assesses their impact on discount rates and reported liabilities. Section 4.3 describes the return dynamics, the preferences of the investment manager, and the constraints under which the manager operates. Section 4.4 describes our numerical solution method for the dynamic optimization problem. Section 5 and Section 4.6 present our results, and Section 7 concludes.

### 4.2 Yield Smoothing

As mentioned earlier, the 2012 transportation bill, which established a 25-year smoothing period to compute the discount rate for liabilities, is the third time in a decade that discounting rules have been changed. The pension Protection Act of 2006 introduced a two-year smoothing period, replacing the four-year smoothing period of corporate bond rates established under the Pension Funding Equity Act of 2004. To illustrate the impact of yield smoothing, Figure 4.1 plots the net present value of $1 billion in future 15-year liabilities (nominal) under four different discounting regimes: (1) market-based accounting using the 15-year (nominal) government bond yield, (2) 4-year smoothing of the 15-year government bond yield, (3) 25-year smoothing of the 15-year bond yield (4) constant discounting using the average 15-year yield between 1956 and 2012. The graph shows that as of July 2012, the present value of $1 Billion in liabilities using market-based accounting methods is $760 million, whereas under the newly approved 25-year smoothing rule, this present value

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The PBGC pays pension benefits for failed defined benefit plans up to the maximum guaranteed benefit of $54,000 (2011) to participants who retire at age 65.
4.2 Yield Smoothing

is reduced by almost half to $420 million. The assumption that all liabilities are due at 15 years is obviously a simplification, but using the usual duration arithmetic this is accurate up to a first-order approximation.

The underlying yields used to compute the liabilities in Figure 4.1 are plotted in Figure 4.2. The graph shows that the unconditional variance of the 15-year bond yield is close to the unconditional variance of the 4-year smoothed 15-year bond yield. In other words, the 15-year yield is so persistent that a 4-year smoothing rule is not long enough to decrease its unconditional variance. To the extent that the purpose of yield smoothing is to create “stability” (as defined by its proponents) in the pension system by decreasing the unconditional variance of the discount factor, we have to conclude that this goal was not reached by previous regulations that allowed for 2-year or 4-year smoothing, but is achieved by 25-year smoothing.

![Figure 4.2: Smoothing the 15-year government bond yield.](image)

There is however another important implication of yield smoothing that can have large (potentially unintended) consequences even with 2-year or 4-

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11In the rest of this paper we use the 15-year government bond yield as the discount rate as for many plans, 15 years is their average duration. As the dynamics of the 15-year and the 30-year yield are very similar, our conclusions do not change when using the 30-year bond yield.

12As noted before, similar results hold for the 30-year bond yield.
year smoothing. Denote by \( y_{15,t} \) the yield to maturity of a 15-year government zero coupon bond at time \( t \). The conditional variance of the \( N \)-year smoothed series (conditional on time \( t \) information) is given by:

\[
\text{var}_t \left[ \frac{1}{N} \sum_{i=0}^{N-1} y_{15,t+1-i} \right] = \frac{1}{N^2} \text{var}_t [y_{15,t+1}] . \tag{4.1}
\]

where \( N \) is the smoothing period. This conditional variance of the \( N \)-year smoothed series is a factor \( N^2 \) smaller than the conditional variance of the actual (market-based) yield series. For example, for 2-year smoothing, the formula simplifies to:

\[
\text{var}_t \left[ \frac{y_{15,t+1} + y_{15,t}}{2} \right] = \frac{1}{4} \text{var}_t [y_{15,t+1}] . \tag{4.2}
\]

which shows that the variance is reduced by a factor 4. For 25-year smoothing, the conditional variance is reduced by a factor 625, essentially eliminating interest rate risk in reported liabilities.

### 4.3 ALM problem

The ALM problem requires that we specify the investment opportunity set (or return dynamics), the preferences of the investment manager, and the risk constraints the investment manager faces. The next three subsections describe these three items in turn.

#### 4.3.1 RETURN AND YIELD DYNAMICS

We consider a pension plan that can invest in three asset classes: stocks, bonds, and the risk-free asset. Stocks are represented by the Standard and Poors (S&P) 500 index, bonds by a 15-year constant maturity Treasury bond, and the risk-free asset by a one-year Treasury bill. We consider an annual rebalancing frequency. We reduce the investment opportunity set to three asset classes driven by two state variables, to keep the dimensionality of the problem low. Considering only three asset classes may seem restrictive, however, these asset classes can also be interpreted as broader categories where long-term bonds represent assets that are highly correlated with the liabilities; stocks and one-year Treasury bills represent assets that have a low correlation with liabilities and have respectively a high risk/high return and low risk/low return profile. We assume that the one-year and 15-year yield levels follow a first-order VAR process. We model stock returns with a time-varying risk premium that depends on the level and slope of the yield curve (e.g., Ang and Bekaert (2005)).
4.3 ALM problem

We model the return dynamics as follows:

\[
\begin{bmatrix}
    r_{s,t} \\
    \ln (y_{1,t}) \\
    \ln (y_{15,t})
\end{bmatrix}
= A + B \begin{bmatrix}
    \ln (y_{1,t-1}) \\
    \ln (y_{15,t-1})
\end{bmatrix} + \varepsilon_t \text{ with } \varepsilon_t \sim MVN (0, \Sigma),
\]

where \( r_{s,t} \) is the annual log return on the S&P 500 index (including distributions), \( y_{1,t} \) and \( y_{15,t} \) are the annualized continuously compounded zero coupon yields for the one-year Treasury bill and the 15-year Treasury bond, \( \varepsilon_t \) is a 3-by-1 vector of innovations and \( \Sigma \) is a 3-by-3 covariance matrix of the innovations. We model the dynamics of the log yields in the spirit of Black and Karasinski (1991) to ensure that nominal yields are positive. The return dynamics we propose allow for both a time-varying risk free rate, time-varying expected bond returns, and a time-varying equity risk premium, all as a function of two state variables, the short-term and the long-term yield. The estimation results are presented in Appendix A.

We assume that the pension plan has liabilities with a fixed duration of 15 years. We measure the value of these liabilities in three ways. First, we compute the actual present value of the liabilities by discounting by the actual 15-year government bond yield:

\[ L_t = \exp (-15y_{15,t}) \] (4.3)

Our second measure is based on recent regulations prescribing that the appropriate discount factor is the four-year average bond yield:

\[ \hat{L}_t = \exp (-15\hat{y}_{15,t}) \] (4.4)

where

\[ \hat{y}_{15,t} = \frac{y_{15,t} + y_{15,t-1} + y_{15,t-2} + y_{15,t-3}}{4}. \] (4.5)

Finally, we compute the value of the liabilities using a constant yield equal to the steady state value of the long-term bond yield \( \bar{y}_{15} \) implied by the VAR (see Appendix A):

\[ \bar{L}_t = \exp(-15\bar{y}_{15}). \] (4.6)

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13 We assume here that asset returns are homoskedastic. Evidence by Chacko and Viceira (2005) suggests that the volatility of stock returns is not variable enough to create sizeable hedging demands.

14 For recent work on return predictability see Binsbergen and Kooijen (2010), Ang and Bekaert (2005), Lewellen (2004), Campbell and Yogo (2005), and Torous, Valkanov, and Yan (2005) for stock returns, as well as Dai and Singleton (2002) and Cochrane and Piazzesi (2005) for bond returns.

15 To maintain a parsimonious representation, we use the 15-year bond yield to determine the discount factor instead of the 30-year bond yield. Since the dynamics of both yields are very similar, this simplification does not alter our results.
With all three measures the liabilities follow a stationary stochastic process. The model could easily be extended to include a deterministic time trend representing demographic factors. However, to maintain a parsimonious representation, we focus on the detrended series. Our specification also abstracts from inflows (premium payments) and outflows (pension payments) to the fund. We assume that in each year the inflows equal the outflows, which allows us to focus purely on the investment management part of the fund. The only inflows we consider are cash injections by the plan sponsor required to meet the regulator's minimal funding level. Note further that the three liability measures above are driven by only one risk factor, the 15-year government bond yield. This could suggest that a one-factor model for the term structure would suffice in our model. However, we assume a two-factor model to allow for a time-varying riskfree rate.

We compute the simple gross returns on the three asset classes as follows:

\[
\begin{align*}
R_{f,t} &= \exp(y_{1,t-1}) \\
R_t &= \begin{bmatrix} R_{s,t} \\ R_{b,t} \end{bmatrix} = \begin{bmatrix} \exp(r_{s,t}) \\ \exp(-14y_{15,t})/\exp(-15y_{15,t-1}) \end{bmatrix},
\end{align*}
\]

where \( R_{s,t} \) is the simple gross return on stocks, \( R_{f,t} \) is the return on the one-year T-bill (riskfree), and \( R_{b,t} \) is the simple gross return on long-term bonds. Our expression for the bond return assumes that the yield curve is flat between 14 and 15 years to maturity.\(^{16}\)

The funding ratio of the pension plan is defined as the ratio of its assets to liabilities:

\[
S_t = \frac{A_t}{L_t},
\]

where assets evolve from one period to the next according to:

\[
A_t = A_{t-1} (R_{f,t} + \alpha_{t-1} \cdot (R_t - R_{f,t})) + c_t \exp(-15y_{15,t}) \quad \text{for } t \geq 1
\]

and \( \alpha_t \equiv [\alpha_{s,t}, \alpha_{b,t}] \)' denotes the portfolio weights in stocks and bonds. We let \( c_t \) denote the contributions of the plan sponsor at time \( t \) as a percentage of the liabilities which, under actual discounting, are equal to \( \exp(-15y_{15,t}) \). Note that defining the contributions as a percentage of the liabilities is equivalent to expressing contributions in future \((t + 15)\) dollars. When liabilities are determined through constant discounting or four-year average discounting, we define the contributions as a percentage of those liability measures and the last term in expression (4.9) is adapted accordingly. We use \( \hat{S}_t \) and \( \bar{S}_t \)

\(^{16}\)To check that this implicit assumption is innocuous we also considered a specification in which the 14-year yield is included in the VAR. The results from this slightly expanded specification are identical. Recall that short positions are not allowed.
to denote the funding ratios computed using the liability measures $\hat{L}_t$ and $\bar{L}_t$, respectively.

Finally, we define $A^*_t$ as the assets in period $t$ before the contributions are received, and $S^*_t$ as the ratio of $A^*_t$ and the liabilities:

$$A^*_t = A_{t-1} (R_{f,t} + \alpha_{t-1} \cdot (R_t - R_{f,t})) \quad \text{for } t \geq 1 \quad (4.10)$$

$$S^*_t = \frac{A^*_t}{L_t} \quad (4.11)$$

4.3.2 PREFERENCES

We take the perspective of an investment manager facing a realistic regulatory environment. We assume that the manager’s utility is an additively separable function of the funding ratio at the end of the investment horizon, $S_T$, and the requested extra contributions from its sponsor as a percentage of the liabilities, $c_t$. We assume that the manager suffers disutility in the form of unmodeled reputation loss or loss in personal compensation for requesting these contributions. The utility function of the manager is given by:

$$U \left( S_T, \{c_t\}_{t=1}^{T-1} \right) = E_0 \left[ u \left( S_T \right) - \sum_{t=1}^{T} v \left( c_t, t \right) \right]$$

$$= E_0 \left[ \beta^T \frac{S^1}{1 - \gamma} - \lambda \sum_{t=1}^{T} \beta^T c_t \right], \quad \text{where } \gamma \geq 0 \text{ and } \lambda \geq 0. \quad (4.12)$$

The first term in the utility function is the standard power utility specification with respect to the funding ratio at the end of the investment horizon. We call this wealth utility. We assume that this wealth utility always depends on the actual funding ratio. That is, we use the actual yields to compute the liabilities in the denominator, regardless of government regulations, as opposed to using a smoothed or constant yield. The motivation for this assumption is that ultimately the manager is interested in maximizing the actual financial position of the fund, which is also the position the pension holders care about.\textsuperscript{17} If we assumed that wealth utility was also based on the smoothed

\textsuperscript{17}It is interesting to note that even when both wealth utility and the risk constraints/AFCs are determined through four-year average discounting, there is still a misalignment of incentives for a multi-period investment problem. The risk constraints (which apply in every period) still induce the use of the risk-free asset. This is a consequence of the large reduction of the conditional variance that yield smoothing induces: \( \text{var}_t(\hat{y}_{15,t+1}) = \text{var}_t(\{\hat{y}_{15,t+1} + y_{15,t} + y_{15,t-1} + y_{15,t-2}\}) = \frac{1}{4} \text{var}_t(y_{15,t+1}) \). Wealth utility, on the other hand, depends on the funding ratio in year $T$. The conditional variance of $y_{15,T}$ is given by: \( \text{var}_t(\hat{y}_{15,T}) = \text{var}_t(\{\hat{y}_{15,T} + y_{15,T-1} + y_{15,T-2} + y_{15,T-3}\}) \). Note that for a 10-year investment problem, $T = 10$, the yields in year 10, nine, eight and seven (which jointly determine the liabilities in year 10) are all unknown before year seven. Therefore, in periods one through six, long-term bonds are still the preferred instrument to hedge against liability risk when maximizing wealth utility, but in years seven through ten they are not.
liability measure, this would further strengthen our results, as the incentive
to hedge interest rate risk would be even further reduced.

The term $\sum_{t=1}^{T} \beta^t v(c_t)$ represents the investment manager’s disutility
(penalty) for requesting and receiving extra contributions $c_t$ from the plan
sponsor. This penalty can be interpreted as loss in reputation or compens-
ation. The linear function just reflects the first-order effect of these penalties
and higher order terms could be included in our analysis. Furthermore,
when contributions from the sponsor are set equal to the funding ratio short-
fall, linearity of the function $v(\cdot)$ implies that the utility penalties are scaled
versions of the expected loss, which, next to a VaR constraint, is often used
as a risk measure. As noted by Campbell and Viceira (2005), the weakness
of a VaR constraint is that it treats all shortfalls greater than the VaR as equiva-
 lent, whereas it seems likely that the cost of a shortfall is increasing in the
size of the shortfall. They, therefore, propose to incorporate the expected
loss directly in the utility function, which in our framework is achieved by
the linearity of the function $v(\cdot)$. Finally, the investment manager discounts
next period’s utility and disutility by the subjective discount factor $\beta$.

Another appealing interpretation of our utility specification is the fol-
lowing. In the context of private pension plans, the investment manager acts
in the best interest of two stakeholders of the plan, (i) the pension holders
who are generally risk averse and (ii) the sponsoring firm which we assume
to be risk neutral. The parameter $\lambda$ then measures the investment manager’s
tradeoff between these two stakeholders. If one believes that the investment
manager merely acts in the best interest of the firm, the value of $\lambda$ is high.
Conversely, if one believes that the investment manager acts mainly in the
interest of the beneficiaries, $\lambda$ is low.

Finally, we can interpret the proposed utility specification in yet two
other interesting ways. First we can interpret it as a portfolio choice problem
with intermediate consumption and bequest. In the literature on life-time
savings and consumption, it is common to assume that utility from consump-
tion is additively separable from bequest utility. The only difference is that,
in our case, consumption is strictly negative and not strictly positive. In other
words, the investment manager can increase his wealth by suffering negative
consumption which leads to a tradeoff between maximizing (the utility from)
the funding ratio at the end of the investment horizon and minimizing (the
disutility from) the contributions along the way. The second interpretation
is that similar utility specifications have been used in the general equilibrium
literature with endogenous default, where agents may choose to default on
their promises, even if their endowments are sufficient to meet the required
payments (e.g., Geanakoplos, Dubey, Shubik (2005)). Agents incur utility
penalties which are linearly increasing in the amount of real default. The idea
of including default penalties in the utility specification was first introduced by Shubik and Wilson (1977).

The tradeoff between the disutility from contributions and wealth utility is captured by the coefficient $\lambda$. When we impose that in each period the sponsor contributions are equal to the funding ratio shortfall, and this shortfall is determined through actual discounting, a value of $\lambda = 0$ implies that the investment manager owns a put option on the funding ratio with exercise level $S^* = 1$. This gives the manager an incentive to take riskier investment positions. When $\lambda \to \infty$, the disutility from contributions is so high that the investment manager will invest conservatively to avoid a funding ratio shortfall when the current funding level is high. Depending on how liabilities are computed, investing conservatively either implies investing fully in the risk-free asset or investing fully in bonds (to immunize the liabilities) or a mixture of the two.\(^{19}\)

Increasing the funding ratio at time zero affects the expected utility in three ways. First, it increases current wealth and therefore, keeping the investment strategy constant, also increases expected wealth utility. Second, if there is a period-by-period risk constraint, a higher funding ratio will make the risk constraint less binding in the current period and also decreases its expected impact on future decisions. Third, keeping the investment strategy constant, the probability of incurring contribution penalties in future periods decreases.

### 4.3.3 Constraints

#### 4.3.3.1 Short sale constraints

We assume that the investment manager faces short sales constraints on all three assets:

\[ \alpha_t \geq 0 \quad \text{and} \quad \alpha_t' \leq 1. \]  

#### 4.3.3.2 VaR constraints

Pension funds often operate under Value-at-Risk (VaR) constraints. A VaR constraint is an ex ante (preventive) risk constraint. It is a risk measure based on the probability of loss over a specific time horizon. For pension plans, regulators typically require that over a specific time horizon the probability of underperforming a benchmark is smaller than some specified probability.

\(^{19}\)When $\lambda \geq 1$, concavity of the utility function is guaranteed under actual discounting. For $\lambda = 1$, the utility is smooth, but for $\lambda > 1$, it is kinked at $S^* = 1$. The right derivative of the function $\frac{1}{1-\gamma} \left( \max\left( S^*, 1 \right) \right)^{1-\gamma} - \lambda \max(1 - S^*, 0)$ is 1 whereas its left derivative equals $\lambda$. The risk neutrality over losses combined with the kinked utility function at $S^* = 1$ resembles elements of prospect theory (Kahneman and Tversky (1979)).
The most natural candidate for this benchmark is the fund’s reported liabilities. In this case, the VaR constraint requires that in each period the probability of being underfunded (i.e., reported liabilities exceeding assets) in the next period is smaller than probability $\delta$. We set $\delta$ equal to 0.025. Depending on prevailing regulations, the relevant benchmark can be the actual liabilities ($L_t$), constant liabilities ($\bar{L}_t$) or, as under current regulations, $\hat{L}_t$.

We also compute the optimal portfolio weights and certainty equivalents when there are no additional contributions from the sponsor. In that case, there is no external source of funding that guarantees the lower bound equal to 1 on the funding ratio. It may therefore be that in some periods the fund is underfunded to begin with. In those cases, the VaR constraint described above can not be applied and requires adaptation. When at the beginning of the period the fund has less assets than liabilities, we impose that the probability of a decrease in the funding ratio is less than 0.025. In other words, if the fund is underfunded to begin with, the manager faces a VaR constraint as if the funding ratio equals one.

4.3.3.3 Additional Financial Contributions (AFCs)

Under current regulations, a pension plan is required to receive AFCs from its sponsor whenever it is underfunded. As the manager dislikes drawing AFCs from the plan sponsor, this requirement serves as an ex post (punitive) risk constraint. The government regulation around these mandatory AFCs is not at all trivial and has changed over time. The Bush era reforms have substantially shortened the amortization period over which shortfalls can be amortized. This amortization period is generally equal to 7 years but extensions can be applied in certain cases. Previous regulation has also allowed for building up so-called credits, in which excess contributions in the past could be used to lower current contributions, regardless of the current financial position of the fund.

In our setup, we set the contributions of the sponsor equal to the funding ratio shortfall in each period. The measurement of this shortfall depends on the way liabilities are computed, which is what we study in this paper. We do not allow for credits nor do we allow for amortization of the shortfall. Since the latter can easily be mimicked by a bond that amortizes over time in combination with a different value for $\lambda$, we do not consider this to be a severe restriction in our model.

4.3.4 DATA DESCRIPTION AND ESTIMATION

To obtain a reasonably representative data generating process we use the following annual data from June 1954 through June 2008. For stock returns we take the natural logarithm of the return on the S&P 500 composite index including distributions. For bond yields we use the continuously compounded
constant maturity yields as published by the Federal Reserve Bank. Whenever data on 15-year government bonds is missing, we take an average of the 10 and 20-year bond yields. We estimate the model by OLS and we include dummy variables for the period 1978-1983 in our estimation to correct for this exceptional period with high inflation. The estimation results are given in Appendix A.

### 4.4 Method

The ALM investment problem, even in stylized form, is a complicated and path-dependent dynamic optimization program. We use the simulation-based method developed by Brandt, Goyal, Santa-Clara, and Stroud (2005) to solve this program. The main idea of their method is to parameterize the conditional expectations used in the backward recursion of the dynamic problem by regressing the stochastic variables of interest across simulated sample paths on a polynomial basis of the state variables. More specifically, we generate $N = 25,000$ paths of length $T$ from the estimated return dynamics. We then solve the dynamic problem recursively backward, starting with the optimization problem at time $T-1$:

$$\max_{\alpha_{T-1}} U(S_T) = \max_{\alpha_{T-1}} E_{T-1} \left[ \frac{\beta}{1-\gamma} S_{T-1}^{1-\gamma} - \lambda \beta c_T \right], \quad (4.14)$$

subject to equations (4.7), (4.8) and (4.9) as well as the short sale constraints and the definition of the required contributions.

The solution of this problem depends on $S_{T-1}$. To recover this dependence, we solve a range of problems for $S_{T-1}$ varying between 0.4 and three. For each value of $S_{T-1}$, we optimize over the portfolio weights $\alpha_{T-1}$ by a grid search over the space $[0, 1] \times [0, 1]$. This grid search over the portfolio weights avoids a number of numerical problems that can occur when taking first order conditions and iterating to a solution. We then evaluate the conditional expectation $E_{T-1} \left( S_{T-1}^{1-\gamma} \right)$ by regressing for each value of $S_{T-1}$ and each grid point of $\alpha_{T-1}$ the realizations of $S_{15,T-1}$ and $y_{1,T-1}$. Define:

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_{1,T-1} \\ y_{15,T-1} \end{bmatrix}, \quad (4.15)$$

The dummies correct for the average level of interest rates over this period, and also lower the persistence estimate of yields. We also run a version of the VAR where we exclude these dummy variables. This does not change any of our conclusions and leads to highly similar quantitative results.

This approach is inspired by Longstaff and Schwartz (2001) who first proposed this method to price American-style options by simulation. See Chapter XX of this handbook for a review of this methodology.
then
\[
X = \begin{bmatrix}
1 & z_{1,1} & (z_{1,1})^2 & (z_{1,1})^2 & (z_{1,1}) (z_{2,1}) & \cdots \\
1 & z_{1,2} & (z_{1,2})^2 & (z_{1,2})^2 & (z_{1,2}) (z_{2,2}) & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\
1 & z_{1,N} & (z_{1,N})^2 & (z_{1,N})^2 & (z_{1,N}) (z_{2,N}) & \cdots
\end{bmatrix}
\] (4.16)

where each row of \(X\) corresponds to a different simulation. The conditional expectation can then be computed as:
\[
E_{T-1} (S_T^{1-\gamma}) = X' \hat{\beta},
\] (4.17)

where
\[
\hat{\beta} = (X'X)^{-1} X' (S_T^{1-\gamma}).
\] (4.18)

When liabilities are discounted by the rolling four-year average yield, we have to include polynomial expansions of all four lags of both state variables in our solution method. To evaluate the conditional expectation of the contributions in period \(T\), \(E_{T-1} (c_T)\), we first regress \(1 - S_T^*\) on \(X\):
\[
\hat{\zeta} = (X'X)^{-1} X' (1 - S_T^*).
\] (4.19)

Assuming normality for the error term in the regression and letting \(\hat{\sigma}\) denote its standard deviation, we find:
\[
E_{T-1} (c_T) = E_{T-1} [\max (1 - S_T^*, 0)] = \Phi \left( \frac{X' \hat{\zeta}}{\hat{\sigma}} \right) X' \hat{\zeta} + \hat{\sigma} \phi \left( \frac{X' \hat{\zeta}}{\hat{\sigma}} \right),
\] (4.20)

where \(\Phi (\cdot)\) and \(\phi (\cdot)\) denote respectively the cumulative and probability density functions of the standard normal distribution. Note that this conditional expectation also represents the expected loss of the fund over the next period.

In the exposition above, contributions are determined by actual discounting. When we use constant or four-year average discounting we simply replace \(S_T^*\) by \(\bar{S}_T^*\) or \(\hat{S}_T^*\) and we replace \(S_T^*\) by \(\bar{S}_T^*\) or \(\hat{S}_T^*\).

Given the conditional expectations in (17) and (20), we can finally solve problem (14), obtaining the optimal weights \(\alpha_{T-1}\). These weights depend on \(S_{T-1}\). Given the solution at time \(T - 1\), meaning the mapping from \(S_{T-1}\) to the optimal \(\alpha_{T-1}\), we iterate backwards through time. The iterative steps are as described above with just a few additions. For ease of exposition we now describe these additions for period \(T - 2\), but they equally apply for periods \(T - 3, T - 4, \ldots, 1\). At time \(T - 2\) we determine for each grid point of \(\alpha_{T-2}\) the return on the portfolio in path \(i \in N\) from \(T - 2\) to \(T - 1\). Using this return to compute \(S_{T-1,i}\), we can then compute the return in path \(i\) from \(T - 1\) to \(T\) by interpolating over the mapping from \(S_{T-1}\) to \(\alpha_{T-1}\) derived in the previous step. Similarly, we interpolate in each path the expected penalty payments.
In the case of a VaR constraint, we impose the constraint in each period and along each path. For given values of $S_t$, we determine for each $\alpha_t$ the conditional mean and conditional variance of the funding ratio in period $t+1$ through regressions on the polynomial basis of the state variables. By assuming log normality, we then evaluate whether the probability of a funding ratio shortfall (i.e., a funding ratio smaller than one) in period $t+1$ is less than $\delta$. If this requirement is not met, those particular portfolio weights are excluded from the investment manager’s choice set. As described above, the VaR can be imposed with respect to $S^*_t$ (discounting at actual yields), $\bar{S}^*_t$ (discounting at constant yields), or $\hat{S}^*_t$ (discounting at the four-year average yield).

For ease of exposition, we will use first order approximations throughout our simulations, but higher order terms in the matrix $X$ can easily be accommodated.

### 4.5 Single Period Portfolio Choice

In this section we investigate the investment manager’s optimal portfolio choice when he is faced with risk constraints that are based on the smoothed liability measure. We consider a VaR constraint as the ex ante (preventive) constraint. For the ex post (punitive) constraint, we consider the requirement to draw AFCs whenever the plan becomes underfunded. Note again that both the VaR constraint and the AFCs are short-term considerations based on the smoothed liability measure whereas the long-term objective of wealth utility is defined with respect to the actual liability measure. We quantify in this section the losses that result from the wedge that yield smoothing drives between these short- and long-term considerations. We first solve a one-period problem to explain the main intuition in a parsimonious setting. We then explain how the results change in a multi-period setup.

#### 4.5.1 ALM WITH A VAR CONSTRAINT

First we investigate optimal portfolio decisions and corresponding certainty equivalents in a one-period context ($T=1$) under a VaR constraint. We set the state variables at time zero equal to their long-run averages. We set the VaR probability $\delta = 0.025$ and we do not include contributions from the sponsor (i.e., $c_T = 0$). We compare a VaR constraint imposed on $S^*_T$ (discounting at actual yields) with one imposed on $\bar{S}^*_T$ (discounting at a constant yield) and one on $\hat{S}^*_T$ (discounting at the four-year average yield).

Figures 4.3 and 4.4 present the optimal portfolio weights and scaled certainty equivalents for a VaR based on actual (market-based) discounting, four-year average discounting, and constant discounting, for two different levels of risk aversion, $\gamma = 2$ and $\gamma = 5$. Note that the VaR constraint is more binding...
when the funding ratio at time zero, denoted by $S_0$, is lower. Therefore, as $S_0$ decreases, the manager has to substitute away from stocks to satisfy the constraint. The key insight of these results is that under actual discounting the manager substitutes into the long-term bond, whereas under constant discounting he moves into the riskfree asset. Because the utility from wealth depends on the actual (market-based) funding ratio, which is computed using current yields, investing in the riskless asset leads to large utility losses. The riskless asset does not hedge against liability risk and has a low expected return. In other words, when the VaR is imposed with constant discounting, the manager is torn between the objective of maximizing utility from wealth and satisfying the VaR constraint. When the VaR constraint is based on actual yields these two objectives are more aligned.
FIGURE 4.3 Portfolio weights: one-period CRRA ALM problem under a VaR constraint ($\delta = 0.025$). The VaR constraint is determined under actual liability discounting (liability-adjusted VaR), four-year average discounting (part-liability-adjusted VaR) and constant discounting (non-liability-adjusted VaR). We solve the problem for relative degrees of risk aversion equal to two and five.
CHAPTER 4  Asset Allocation in Asset Liability Management

FIGURE 4.4 Scaled certainty equivalents for a one-period CRRA ALM problem under a Value-at-Risk constraint (δ = 0.025). The Value-at-Risk constraint is determined under actual liability discounting (liability-adjusted VaR) four-year average discounting (part-liability-adjusted VaR) and constant discounting (non-liability-adjusted VaR). We solve the problem for relative degrees of risk aversion equal to two and five. We report certainty equivalents that are scaled by the initial funding ratio $S_0$, comparable to an annual gross risk free return (so 1.06 means a 6% return).

The utility loss from constant discounting can be large and up to four percent of wealth. This loss is increasing in the degree of risk aversion. Substituting away from bonds into the riskless asset and stocks leaves a larger exposure to liability risk, leading to larger utility losses when the degree of risk aversion is higher. As risk aversion increases, the manager’s preferred position in stocks decreases and she prefers to invest more in bonds to hedge against liability risk. As a consequence, the VaR constraint under actual discounting, which requires a substantial weight in bonds, does not affect the manager much. The VaR constraint under constant discounting, on the other hand, forces the manager into the riskfree asset and stocks leading to large welfare losses.

4.5.2 ALM WITH AFCS

We now assess the impact of smoothing yields by comparing optimal portfolio decisions and corresponding certainty equivalents when the investment manager has to request AFCs whenever the fund is underfunded. The notion of being underfunded depends on the liability measure used. We set the contributions at time 1 equal to the realized funding ratio shortfall, which implies
\( c_T = \max(1 - S_T^+, 0) \) under constant discounting and \( c_T = \max(1 - S_T^+, 0) \) under actual discounting. As before, we consider a one-period setup (\( T = 1 \)). We set \( \lambda > 1 \) to ensure concavity of the utility function and, for ease of exposition, we do not impose the VaR constraint. Finally, we set the state variables equal to their long-run averages at time zero.

Figures 4.5 and 4.6 present the optimal portfolio weights and certainty equivalents for \( \lambda = 2 \) for degrees of risk aversion \( \gamma \) equal to 2 and 5 as a function of the funding ratio \( S_0 \geq 1 \). Figures 4.7 and 4.8 show the results for \( \lambda = 5 \). The graphs show that when sponsor contributions and their consequent reputation loss are determined under smoothed yield measures, the investment manager does not substitute into bonds but hedges against the utility penalties through a higher weight in the risk free asset combined with a higher weight in stocks. This goes against the investment manager’s desire to maximize wealth utility, leading to large welfare losses.
FIGURE 4.5 Portfolio weights: one-period CRRA ALM problem with sponsor contributions ($\lambda = 2$). The sponsor contributions (AFCs) are determined under actual liability discounting (liability-adjusted sponsor contributions), four-year average discounting (part-liability-adjusted sponsor contributions) and constant discounting (non-liability-adjusted sponsor contributions). We solve the problem for relative degrees of risk aversion equal to two and five.
FIGURE 4.6 Certainty equivalents: one-period CRRA ALM problem with AFCs ($\lambda = 2$). The sponsor contributions (AFCs) are determined under actual liability discounting (liability-adjusted sponsor contributions) four-year average discounting (part-liability-adjusted sponsor contributions) and constant discounting (non-liability-adjusted sponsor contributions). We solve the problem for relative degrees of risk aversion equal to two and five. We report certainty equivalents that are scaled by the initial funding ratio $S_0$, comparable to a gross annual riskfree return (so 1.06 means a 6% return)
FIGURE 4.7 Portfolio weights: one-period CRRA ALM problem with AFCs ($\lambda = 1$). The sponsor contributions are determined under actual liability discounting (liability-adjusted sponsor contributions), four-year average discounting (part-liability-adjusted sponsor contributions) and constant discounting (non-liability-adjusted sponsor contributions). We solve the problem for relative degrees of risk aversion equal to two and five.
4.6 Dynamic Portfolio Choice

4.6.1 Welfare and Portfolio Implications of Yield Smoothing

In the previous section we have assessed the welfare and portfolio choice impact of smoothing yields in a parsimonious one-period framework. When we extend our analysis to a multi-period framework, we find results that are highly comparable to the one-period case, with welfare losses of up to 2-4% per year in certainty equivalent terms and portfolio choices that hedge poorly...
CHAPTER 4  Asset Allocation in Asset Liability Management

against interest rate risk. That said, there are a few subtle but important differences with the one-period framework that are worth discussing, particularly when considering the VaR constraint.

Because the VaR constraint only binds for levels of the funding ratio close to (and less than) one and the funding ratio has a positive drift, the VaR becomes on average less binding over time. Therefore, for fully funded pension plans, the impact of the VaR and the difference between constant and actual discounting decreases over time as the plan’s funding ratio increases. However, highly underfunded plans without AFCs can be confronted with the VaR constraint over a very long time span. Therefore, the welfare loss of constant discounting for such underfunded plans, is very large and in the same order of magnitude as in the one-period model, i.e., between two to four percent per year.

We would expect that, as in the one-period model, the impact of four-year average discounting takes an average of the impacts of constant and actual discounting. However, another important implication of smoothing yields now emerges: in around 10-20 percent of the cases, not a single portfolio weight in the choice space satisfies the VaR constraint. The reason is as follows. Consider the following example of a plan with a funding ratio equal to one. Suppose that in the last four periods ($t - 3$, $t - 2$, $t - 1$ and $t$) the 15-year yield took the path 0.060, 0.055, 0.045 and 0.040, leading to a four-year rolling average of 0.050. Assume further that the short term yield is currently very low at 0.020. The investment manager knows that next year the yield at time $t - 3$ (i.e., 0.060) will be dropped from the average and the yield at $t + 1$ will be added. The 15-year bond yield is expected to rise, due to mean reversion, but it is unlikely to rise back to 0.060 in one period. Therefore, the four-year average yield will most probably decrease, leading to a deterioration of the fund’s position. Investing fully in the riskless asset is not allowed because the expected return is too low to compensate for the deterioration of the fund’s reported position. As a result, investing fully in the riskless asset leads to an almost certain shortfall, whereas the maximum allowed probability of a shortfall under the VaR constraint is only 0.025. Investing in bonds is not too attractive either, as the long-term yield is expected to rise, leading to low expected returns on long-term bonds. The investment manager needs to offset a drop in the four-year rolling average yield, whereas the long-term yield is expected to increase. Therefore, the probability of a shortfall will be larger than 0.025 for all available portfolio weights. Surprisingly, it turns out that in cases like this, the portfolio composition that leads to the lowest probability of a shortfall, is investing 100 percent in stocks, which is undesirable from a wealth utility perspective. We conclude once again that smoothing yields can lead to excessive risk taking and large welfare losses.\textsuperscript{22}

\textsuperscript{22}When long-term yields have been rising consistently, and the short term yield is high, the opposite argument holds, and the investment environment is very favorable to the manager.
4.6.2 HEDGING DEMANDS AND REGULATORY CONSTRAINTS

In the previous sections we have discussed the welfare implications of yield smoothing in a dynamic framework. In this section we address the impact of regulatory constraints on hedging demands. That is, the differences between the dynamic (also called strategic) and myopic portfolio weights. They hedge against future changes in the investment opportunity set. It is well-known that the value function of standard CRRA utility function is relatively flat at the maximum, implying that moderate deviations from the optimal portfolio policy only lead to small utility losses (e.g., Cochrane (1989) and Brandt (2005)). As a consequence, the economic gains to dynamic (strategic) as opposed to myopic (tactical) investment are usually small even when the hedging demands are large in magnitude. Intuitively, ex ante (preventive) risk constraints could enhance these gains as it could be profitable to strategically avoid the constraints in future periods. Specifically when the investment opportunity set is time-varying, the investment manager might want to avoid being constrained in the future when expected returns are high. We show that for the return generating process that we consider, the exact opposite result seems to hold: ex ante risk constraints further decrease the gains to dynamic investment. However, we also show that under ex post (punitive) constraints such as the requirement to draw AFCs when the plan becomes underfunded, strategically (dynamically) avoiding these contributions can lead to economically large utility gains.

4.6.2.1 Dynamic ALM Benchmark

As a benchmark, we first present the optimal portfolio weights for an ALM problem without sponsor contributions, AFCs, or VaR constraint. Table 4.1 shows the optimal portfolio weights and certainty equivalents for the dynamic and the myopic investor for different values of $\gamma$. Given the short position in the liabilities, it is no longer optimal to invest in the riskless asset, so the manager spreads his wealth between stocks and bonds. The dynamic investor now substitutes away from long-term bonds into stocks. When investing myopically, the uncertainty caused by the liabilities induces the manager to invest more in long-term bonds, which are a good hedge against liability risk. However, when investing dynamically, liability risk is not as important due to the mean-reverting nature of yields. In other words, future bond returns are negatively correlated with current bond returns. Suppose that the 15-year yield is at its long-run average and is hit by a negative shock. Current liabilities will increase, which leads to a deterioration of the funds financial position. However, yields are consequently expected to increase which will

In this case he is less restricted by the VaR constraint under four-year average discounting than under actual discounting.

\[ 23 \] An even more basic benchmark is to study an asset-only (no liabilities) problem given the return dynamics, which we do in Appendix B.
CHAPTER 4 Asset Allocation in Asset Liability Management

TABLE 4.1 Portfolio weights and standardized certainty equivalents for a 10-period CRRA ALM portfolio optimization without VaR constraint or sponsor contributions. Due to the power utility specification over the funding ratio, the weights are independent of the funding ratio at time zero. The portfolio weights are rebalanced annually. We run 50 simulations and report averages and standard deviations (between brackets). In the last column (gain in basis points per year) we take the ratio of the certainty equivalents to the power 1/10, subtract one and multiply by 10,000 to get the gains to dynamic (strategic) investment compared to myopic (tactical) investment in basis points per year.

<table>
<thead>
<tr>
<th>γ</th>
<th>Stocks</th>
<th>Riskfree</th>
<th>Bonds</th>
<th>CE scaled</th>
<th>Stocks</th>
<th>Riskfree</th>
<th>Bonds</th>
<th>CE scaled</th>
<th>Gains (bp/a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.96</td>
<td>0.00</td>
<td>0.04</td>
<td>2.951</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>2.956</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.000)</td>
<td>(0.028)</td>
<td>(0.000)</td>
<td>(0.012)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.012)</td>
<td>(0.1)</td>
</tr>
<tr>
<td>5</td>
<td>0.37</td>
<td>0.00</td>
<td>0.63</td>
<td>2.415</td>
<td>0.43</td>
<td>0.00</td>
<td>0.57</td>
<td>2.426</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.000)</td>
<td>(0.008)</td>
<td>(0.000)</td>
<td>(0.028)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.028)</td>
<td>(1.0)</td>
</tr>
</tbody>
</table>

ameliiorate the fund’s position. In other words, in our setup bonds are a good hedge against changes in the investment opportunities for bonds. Apparently, this effect even dominates the use of bonds as a hedge against changes in the investment opportunities for stocks. The gains to dynamic investment are again small, not exceeding 10 basis points per year.

4.6.2.2 Dynamic ALM with a VaR constraint

We now investigate the impact of ex ante (preventive) risk constraints on hedging demands. In particular, we focus on a VaR constraint. We set the VaR probability \( \delta \) equal to 0.025 and maintain the assumption of no sponsor contributions, so \( c_t = 0 \) for all times \( t \).

Intuitively we would expect the VaR constraint to increase the value of solving the dynamic program. Strategically avoiding the VaR constraint should lead to utility increases. Our results indicate exactly the opposite: the VaR constraint further reduces the already small gains to dynamic investing. Figures 4.9 and 4.10 show the optimal portfolio weights for \( \gamma = 5 \) as a function of the funding ratio at time zero for both the dynamic and myopic investor. The graphs also plot the certainty equivalent gains of dynamic investing relative to the myopic solution. In Figure 4.9, the VaR is imposed with respect to \( S_t \) (discounting at current yields) and in Figure 4.10 it is imposed with respect to \( \bar{S}_t \) (discounting at a constant yield). Recall that the VaR constraint applies period-by-period for both the dynamic and the myopic investor. For ease of exposition, let us consider low and intermediate levels of the funding ratio at time zero. In this case the VaR constraint binds and it reduces the weight in stocks. That is, the VaR constraint leads the investor away from the preferred portfolio weight in stocks, leading to a utility loss. The upper bound on the weight in stocks as required by the VaR depends
on the funding ratio at time zero and is the same for the dynamic and the myopic investor. From the unconstrained ALM problem presented in the previous subsection, we know that to hedge against changes in the investment opportunity set, the dynamic investor wants to invest more in stocks than the myopic one. However, the VaR constraint prevents him from doing so, thereby eliminating the gains to dynamic investment.

![Diagram showing dynamic vs myopic portfolio choice](image)

**Figure 4.9** Dynamic vs Myopic Portfolio Choice under a VaR. Dynamic and myopic portfolio weights and certainty equivalent gains for a 10-period CRRA ($\gamma = 5$) ALM problem (no sponsor contributions) with a VaR constraint imposed on the actual (actual discounting) funding ratio ($\delta = 0.025$). The top panel summarizes the dynamic (strategic) portfolio weights and the middle panel summarizes the myopic (tactical) ones. The bottom panel shows the certainty equivalent gains, which are computed in basis points per year by taking the ratio of the certainty equivalents of the ten year problem, raising this to the power $1/10$ subtract one and multiply by $10,000$.

Even though the dynamic investor cannot invest more in stocks than the myopic one, he could choose to invest less in stocks in the current period, thereby strategically lowering the probability of being constrained by the VaR in the future. However, the current VaR already decreases his weight in stocks, which already lowers the probability of being constrained by the VaR in the future. The current period’s portfolio loss of decreasing the weight in stocks even further outweighs the potential future gains. As a result, both investors make the same portfolio choices, leading to almost equal certainty equivalents. The VaR constraint in the current period is a strong remedy in trying to avoid the VaR constraint in the future. Furthermore, it is interesting to note that the expected returns on stocks are high when the 15-year
yield is high. This means that the liabilities are low when stock returns are high. This ameliorates the negative impact of the VaR in future periods and allows the manager to invest in stocks when it is most profitable for him to do so. However, this last argument only holds when the VaR is imposed on the actual funding ratio $S_t$ and may not hold when liabilities are smoothed. This is yet another unattractive feature of yield smoothing.

We conclude that the dynamic investor faces a tradeoff between forming an optimal portfolio in the current period given the current VaR and lowering the probability and impact of hitting the VaR in the future. Our results suggest that the current loss from decreasing the weight in stocks by more than is prescribed by the current VaR outweighs the gain of a lower probability of hitting the VaR in the future, at least for the return generating process (VAR) that we consider. When the 15-year yield at time zero is no longer at its unconditional mean but below it, investing in stocks in the current period becomes less appealing compared to investing in stocks in the future. However it also implies that current liabilities are high and are expected to decrease.
4.6 Dynamic Portfolio Choice

This downward trend in liabilities decreases the impact of the VaR constraint in the future under actual discounting but not under constant discounting.

4.6.2.3 Dynamic ALM with AFCs

Finally, we consider ex post (punitive) risk constraints. In particular we focus on the investment manager’s requirement to request additional financial contributions (AFCs) from the plan sponsor whenever the plan becomes underfunded. In the previous section we showed that lowering the weight in stocks today to strategically avoid the VaR constraint in the future does not pay off. Now, however, lowering the weight in stocks to lower the probability of being underfunded in the future can lead to very large utility gains. Including sponsor contributions in the utility function leads to a kinked utility function. The induced first order risk aversion enhances the gains to dynamic investment. Furthermore, contributions have a direct utility impact and apply each period as opposed to utility from wealth, which only depends on the funding ratio in time \( T \). We set the subjective discount factor \( \beta = 0.99 \), and set \( \delta = 1 \) (no VaR constraint).\(^{24}\) Recall that \( \lambda \) is the parameter that describes the tradeoff between the sponsor contributions and wealth utility. When \( \lambda \) is set sufficiently high, the gains of lowering the probability of being underfunded in the future will outweigh the portfolio loss of lowering the weight in stocks today. In this case the gains to dynamic investment are very large. Figure 4.10 shows the portfolio weights and certainty equivalent gains for \( \lambda = 2 \) when in each period the contributions are set equal to the realized funding ratio shortfall under actual discounting (\( \epsilon_t = \max(1 - S_t^*, 0) \)). The certainty equivalent gains are in terms of wealth, which assumes that the utility penalties associated with AFCs can be converted into monetary amounts according to the tradeoff in the utility function.

The figure shows that highly underfunded plans invest heavily in stocks, which is a consequence of the linearity of the utility penalties, which effectively make the managers risk neutral for low levels of the funding ratio. This gambling for resurrection in our model also illustrates that including the pension put (i.e. the fact that U.S. defined-benefit pension plans are insured through the Pension Benefit Guarantee Corporation) in our framework would not change the portfolio weights very much.\(^{25}\) The manager already invests 100% in stocks when the plan is highly underfunded. The figure also shows that the gains from dynamic investment are very large. By lowering the weight in stocks today, the investment manager can avoid costly contributions from the sponsor in the future, thereby realizing large utility gains.

\(^{24}\) Setting the subjective discount factor \( \beta \) to 0.95 or 0.9 does not influence our results.

\(^{25}\) For a similar V-shaped policy function of portfolio weights, see Berkelaar and Kouwenberg (2003).
4.7 Conclusion

We address in this paper the investment problem of the investment manager of a defined benefits pension plan and show that financial reporting and risk control rules have real effects on investment behavior. The requirement to discount liabilities at a rolling average yield can induce grossly suboptimal investment decisions, both myopically and dynamically. Both a VaR constraint and mandatory AFCs by the plan sponsor should decrease the manager’s risky holdings as the funding ratio approaches the critical threshold of one. We show that when these short-term objectives are defined with respect to the smoothed liability measure, they can inadvertently induce the manager to increase the riskiness of the portfolio. We therefore conclude that smoothing yields may lead to highly perverse investment behavior and large welfare losses. Furthermore, if the regulator yields to political pressure by allowing additional smoothing when interest rates are low, and less smoothing when interest rates are high, this encourages additional risk taking behavior, thereby increasing the probability of underfunded pension plans relative to the non-smoothed (true market value) of liabilities. We therefore argue for

We compared the influence of preventive and punitive constraints on the gains to dynamic decision making. We conclude that ex ante (preventive) constraints such as VaR constraints, short sale constraints and an upper bound on the share of stocks in the portfolio, decrease the size of the choice set (i.e. the space of admissible portfolio weights) and thereby substantially decrease the gains to dynamic investment. However, ex post (punitive) constraints, such as mandatory AFCs from the plan sponsor, make the investment manager first-order risk averse at the critical threshold that triggers the constraint, leading to large utility gains to dynamic investment. In other words, if the investment manager is concerned about being underfunded and dislikes the resulting AFCs, a dynamic investment strategy leads to large expected utility gains by strategically avoiding to be underfunded in the future.

4.8 Appendix: Return model parameter estimates

We estimate the Vector Auto Regression (VAR) of order one that describes the return dynamics by OLS, equation by equation. The estimates are given below with their respective standard errors between brackets.

\[
A = \begin{bmatrix}
0.252 & (0.1677) \\
-0.518 & (0.4189) \\
-0.309 & (0.1562)
\end{bmatrix}
\] (4.21)
As the starting values, we take the unconditional average of the yields in the data, which differs slightly from the average yields implied by the VAR.

### 4.9 Appendix: Benchmark Without Liabilities

As a second benchmark, we solve a standard dynamic portfolio optimization problem without liabilities, AFCs, utility penalties, or a VaR constraint. We compare the certainty equivalent achieved under the solution of the dynamic 10-year investment problem with that of a myopic setup. The myopic problem involves solving 10 sequential one-year optimizations. Hence the only difference between the dynamic and the myopic problems is the utility function that the manager maximizes. In the myopic problem, the manager optimizes the one period utility function 10 times and in the dynamic problem he optimizes the 10-period utility function. We then use the optimal weights for both problems to compute certainty equivalents with respect to the 10-period utility function. In other words, we use the myopic and dynamic policy functions to compute the certainty equivalent when the investment manager has a 10-year utility function. In this case the myopic policy function is suboptimal. The important question is how suboptimal it is. We define the gains to dynamic investment as the ratio of the dynamic and myopic certainty equivalents.

Table 4.2 shows for different values of $\gamma$ the optimal portfolio weights and certainty equivalents for the dynamic and the myopic problem. In the myopic case, the manager spreads his funds between stocks and the riskless asset and hardly invests in long-term bonds. In the dynamic case, however, it is optimal to invest part of the funds in long-term bonds as a hedge against changes in the investment opportunity set for stocks. When there is a drop in the 15-year yield, the return on bonds in the current period are high, which forms a hedge against the lower future risk premium on stocks. Even though the hedging demands can be large, the utility gains, as expressed by the net ratio of the dynamic and myopic certainty equivalents, are relatively small. They vary between 3 and 23 basis points per year depending on the degree of risk aversion. These low gains to dynamic investment are caused by the relatively flat peak of the 10-period value function.
CHAPTER 4 Asset Allocation in Asset Liability Management

Table 4.2: Portfolio weights and standardized per-period certainty equivalents for a 10-year CRRA portfolio optimization problem without liabilities, VaR constraint or sponsor contributions. Due to the power utility specification over wealth, the weights are independent of the wealth level at time zero. The portfolio weights are rebalanced annually. We run 10 simulations and report averages and standard deviations (between brackets). In the last column (gain in basis points per year) we take the ratio of the certainty equivalents to the power 1/10, subtract one and multiply by 10,000 to get the gains to dynamic (strategic) investment compared to myopic (tactical) investment in basis points per year.

<table>
<thead>
<tr>
<th>γ</th>
<th>Stocks</th>
<th>Riskfree</th>
<th>Bonds</th>
<th>CE scaled</th>
<th>Stocks</th>
<th>Riskfree</th>
<th>Bonds</th>
<th>CE scaled</th>
<th>Gains (bp/a)</th>
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<td>2</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>2.881</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>2.7</td>
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<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.008)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.012)</td>
<td>(0.1)</td>
</tr>
<tr>
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<td>0.50</td>
<td>0.00</td>
<td>2.384</td>
<td>0.57</td>
<td>0.07</td>
<td>0.16</td>
<td>2.439</td>
<td>23.08</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.007)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.009)</td>
<td>(1.14)</td>
</tr>
</tbody>
</table>

REFERENCES


FIGURE 4.11 Dynamic vs Myopic Portfolio Choice under AFCs. Dynamic and myopic portfolio weights and certainty equivalent gains for a 10-period CRRA ($\gamma = 5$) ALM problem with sponsor contributions (AFCs), where AFCs are computed on the actual funding ratio ($\lambda = 2$). The top panel summarizes the dynamic (strategic) portfolio weights and the middle panel summarizes the myopic (tactical) ones. The bottom panel shows the certainty equivalent gains, which are computed in basis points per year by taking the ratio of the certainty equivalents of the ten year problem, raising this to the power $1/10$ subtract one and multiply by $10,000$.

CHAPTER 4 Asset Allocation in Asset Liability Management


