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Executive effort and selection of risky projects

Richard A. Lambert*

This article examines the incentives of executives to adopt risky projects. The agency problem considered is that of motivating the executive to expend effort to generate information about the profitability of projects and to select the “best” project conditional upon the information that his effort generates. We show that the executive and the principal will not always agree regarding which project is best. We provide conditions under which this conflict of interest leads (from the principal’s perspective) to either an underinvestment or an overinvestment in risky projects.

1. Introduction

One of the most frequently cited problems with executives concerns their incentives to invest in new projects (Rappaport, 1978; Kaplan, 1982). One factor that is thought to contribute to this problem is the difference in attitudes toward risk of executives and shareholders. In particular, it is usually suggested that executives are excessively risk averse in their strategy selection. As a result, they are alleged to turn down projects that would be profitable from the point of view of shareholders because the executives perceive the projects to be overly risky from the point of view of their own welfare.

This article uses optimal contracting theory to examine the incentives of risk-averse executives to invest in risky projects. Conventional models of the principal-agent relationship are inadequate to address this issue because they typically do not model the agent as choosing among alternative projects. Instead, most of the agency literature (Harris and Raviv, 1978, 1979; Holmström, 1979) models the agent as selecting a single-dimensional action, which is interpreted as effort. The agency problem arises because the agent dislikes effort. Moreover, in conventional models increased effort is assumed to increase cash flow in the sense of first-order stochastic dominance. This implies that the actions available to the organization can be ranked according to their financial returns independently of the decisionmaker’s risk preferences. Therefore, the issue of selecting among projects that differ in riskiness does not arise in a conventional agency setting.

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1 A second factor that is also thought to contribute to underinvestment is a difference in the decisionmaking horizon of managers versus shareholders. Managers are often alleged to have a shorter time horizon than shareholders, and therefore reject projects that do not return an immediate profit. This issue is discussed further in Section 7. Holmström and Ricart i Costa (1984) provide additional analysis of the time-horizon issue when the manager is concerned about his reputation in future periods.
This article models the executive as deciding among alternative projects solely on the basis of the financial returns (to him) of the projects. A key feature of the model is that these projects cannot be ranked according to stochastic dominance; specifically, the executive must choose between a risky project and a safe one. A conflict of interest exists in the model because the executive must expend effort to produce additional information about the profitability of the available projects. The principal would like to motivate the executive to work to produce information and to select the “best” project, given the information signal generated. We show that even if the principal can observe which project the executive has selected, an agency problem exists because the principal cannot determine why the project was selected. For example, if the executive selects the safe project, the principal cannot determine whether this was done because the executive worked and his effort indicated that the risky project would not be very profitable, or because the executive worked, and although the risky project would be profitable to the principal, the executive’s contract imposed so much risk on him that he decided to “play it safe,” or because the executive did not work and decided to select the safe project all of the time.

We show that the optimal incentive contract imposes risk upon the executive to motivate him to work to produce information. Unlike in conventional agency settings, however, imposing excessive risk can result in the executive’s supplying less effort. The risk imposed by the contract will also lead the principal and the executive sometimes to disagree about which project should be selected on the basis of the information signal received by the executive. As a result, the executive does not always select the project that maximizes the principal’s utility. Interestingly, we show that this conflict of interest can result in an over-investment in the risky project.

We also analyze the investment problem when communication between the executive and the principal is permitted. We incorporate communication by allowing the executive to select from a menu of contracts on the basis of the information signal he observes. We find that underinvestment problems disappear when the executive is permitted to communicate his information in this fashion. This suggests that factors that contribute to the inability of executives to communicate their information regarding the profitability of their projects may contribute significantly to problems of underinvestment.

Section 2 develops the model of risky project selection. We discuss the first-best solution to this problem in Section 3. Section 4 begins the analysis of the situation in which the executive’s actions are unobservable, and compares the model with conventional agency models of effort. In Section 5 we characterize the optimal contract and analyze the executive’s project selection strategy in the second-best solution. Section 6 analyzes the role that communication plays in the investment problem. We discuss multiperiod considerations in Section 7.

2. Model

We model the executive as choosing between a “safe” and a “risky” project. The safe project will return a certain cash flow, $x_O$. A special case is the situation in which $x_O = 0$, which can be interpreted as not investing. The “risky” project will return either a high cash flow, $x_H > x_O$, or a low cash flow, $x_L < x_O$. If the executive selects the risky project without acquiring additional information, the probability of $x_H$’s occurring is $.5$. Alternatively, he can expend effort to produce additional information about the profitability of the risky project. He can then select a project on the basis of the information signal that his effort generates. For convenience, we express the information signal in terms of the posterior probability of $x_H$’s occurring if he selects the risky project. There exists a continuum of

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2 At least three cash flows are required in the model to allow the projects to be noncomparable according to first-order stochastic dominance, and to allow issues related to the riskiness of the projects to be examined.
possible risky projects, each characterized by \( r \), the probability of the high cash flow’s occurring. If the risky project is selected after the signal \( r \) is received, the conditional probability of \( x_H \)'s occurring is \( r \). We assume that the signal \( r \) is generated from the uniform distribution over the interval zero to one.\(^3\)

This formulation of asymmetric information differs from conventional agency models.\(^4\) In conventional agency settings the agent is exogenously endowed with superior information. On the basis of the information he receives, he then decides how much effort to supply. In our model the executive must work to acquire the superior information. His effort provides him with superior information about the nature of the risky project he faces. He then chooses either the risky or the safe project solely on the basis of the projects’ financial returns.

We assume that the principal is risk neutral and that the executive is effort and risk averse. We also assume that the executive’s utility function is additively separable into an income and an effort component. We normalize his utility function so that the disutility of not working is zero and the disutility of working is \( V \). Let \( U(z) \) be his utility function for income and let \( S(U) \) be the inverse of the executive’s utility function for income; i.e., \( S(U(z)) = z \).

### 3. First-best solution

- In the absence of incentive problems, it is optimal for the principal to absorb all of the risk; the executive will be paid a constant, \( s \). It is easy to see that the optimal strategy for the executive in selecting projects is to choose a cutoff point, \( \hat{r} \), such that he selects the safe project if \( r < \hat{r} \) and the risky project if \( r > \hat{r} \); he selects the risky project only if the probability of obtaining \( x_H \) is sufficiently high. The benefits of having the executive work therefore arise because the information that his effort generates allows him to select the project on the basis of superior information.\(^5\)

If the executive works and the cutoff point \( \hat{r} \) is chosen, the probability that cash flow \( x_r \) occurs, denoted \( p_j(\hat{r}) \), is:

\[
\begin{align*}
  p_D(\hat{r}) &= \hat{r} \\
  p_H(\hat{r}) &= .5(1 - \hat{r}^2) \\
  p_L(\hat{r}) &= .5(1 - \hat{r})^2.
\end{align*}
\]

(1)

For notational convenience, we shall hereafter drop the dependence of \( p_j \) on \( \hat{r} \).

In the absence of incentive problems, and under the assumption that the principal wishes to have the executive work, we can write the principal’s problem as

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\(^3\) The analysis holds for any \textit{a priori} probability \( p \) of \( x_H \)'s occurring and any continuous distribution for the information signal \( r \). Of course, we require the expected value of \( r \) to equal \( p \).

\(^4\) Our analysis is an application of Demski and Sappington’s (1984) model of “delegated expertise.” In other agency-based models of project selection (Holmström and Ricart i Costa, 1984; Antle and Eppen, 1985), the agent is exogenously endowed with private information regarding the profitability of investment projects.

\(^5\) It is only optimal to have the executive work if the increase in the gross cash flow that is achieved by his obtaining the information exceeds the increase in the expected compensation that must be provided to him if he works. If the cash flows are such that the first-best cutoff point approaches zero (one), the expected gross cash flow that is obtained by having the executive work will approach the gross cash flow that can be obtained by his not working and always selecting the risky (safe) project. Under these conditions it will not be optimal to have the executive work to obtain the information.

\(^6\) More generally, we have

\[
\begin{align*}
  p_D &= F(\hat{r}), \\
  p_H &= \int_{\hat{r}}^1 r f(r) dr, \\
  p_L &= \int_{\hat{r}}^1 (1 - r) f(r) dr,
\end{align*}
\]

where \( f(r) \) is the probability distribution of \( r \) and \( F(r) \) is the cumulative distribution of \( r \).
\[
\max_{x} p_0 x_0 + p_H x_H + p_L x_L - s
\]

subject to \( U(s) - V \geq \theta \), where \( \theta \) is the utility value of the executive's next best employment opportunity.

From equation (1) the first-order condition on \( \hat{\theta} \) is \( x_0 - \hat{\theta} x_H + (1 - \hat{\theta}) x_L \). When we assume an interior solution for \( \hat{\theta} \), we can denote the first-best cutoff point, \( r_f \), for determining which project to select by

\[
r_f = \frac{x_0 - x_L}{x_H - x_L}.
\]

Note that the second-order condition on \( \hat{\theta} \) is \( - (x_H - x_L) \), which is negative. This implies that the objective function is strictly concave, and that the \( r_f \) given by equation (3) is necessary and sufficient for an interior maximum.

Intuitively, \( r_f \) is the probability of \( x_H \)'s occurring such that the risky project has the same expected profitability as the safe project. When the information signal indicates that the probability of \( x_H \) exceeds (is less than) \( r_f \), it is optimal to select the risky (safe) project.

4. **Effort unobservable**

- In this section we assume that the principal can observe neither the executive's effort nor the information signal that his effort generates. Moreover, we assume that the cost of the executive's communicating the information signal to the principal is excessive. The principal and the executive jointly observe only the cash flow that arises at the end of the period. Therefore, the executive's compensation can depend only on this cash flow.

The agency problem occurs because all three cash flows can arise in the optimal project selection strategy (assuming the principal wants the executive to work), but the principal cannot observe the executive's strategy for choosing projects.\(^7\) Although the principal can determine ex post which project the executive has selected, the principal cannot determine why the project was selected.\(^8\)

To analyze the executive's strategy for selecting a project, suppose he is offered the contract, \((U_0, U_H, U_L)\), where \( U_j \) is the utility of the executive's payment if the cash flow is \( x = x_j \). If the executive works and receives the information signal \( r \), he must decide whether to invest in the safe project (and receive \( U_0 \) for certain) or to invest in the risky project (and face the lottery \( r U_H + (1 - r) U_L \)). As in the first-best solution, the executive will select a cutoff point, \( \hat{\theta} \), such that he chooses the risky project if \( r > \hat{\theta} \) and the safe project if \( r < \hat{\theta} \).

Given the contract \((U_0, U_H, U_L)\), the executive's choice of the optimal cutoff point is the solution to the problem

\[
\max_{r} p_0 U_0 + p_H U_H + p_L U_L,
\]

and the first-order condition on \( \hat{\theta} \) is given by

\[
U_0 - [\hat{\theta} U_H + (1 - \hat{\theta}) U_L].
\]

---

\(^7\) If the executive was not required to work in the first-best solution, no welfare loss occurs if his effort is not observable. The principal can achieve the first-best solution by offering the executive a constant wage and asking him to select the first-best action strategy. Therefore, we confine our analysis to the case in which the principal wishes to motivate the executive to work.

\(^8\) The principal can infer which project was selected because the sets of cash flows for the two projects are disjoint. This is not a crucial feature of the model. We can extend the analysis to allow the cash flow \( x_o \) to occur with probability \( q \) when the risky project is selected. We then interpret the information signal \( r \) as representing the probability of \( x_o \)'s occurring, given that \( x_o \) does not occur. In this case if the cash flow is \( x_o \) or \( x_L \), the principal knows that the risky project was selected, but if the cash flow is \( x_o \), he cannot determine which project was selected. All of the results go through under this scenario.
Setting the executive’s first-order condition equal to zero gives the value of \( r \) at which the executive is indifferent between the two projects. Note that the executive’s second-order condition on \( \hat{r} \) is \([-U_H - U_L]\). Under the assumption that \( U_H > U_L \), the first-order condition is therefore necessary and sufficient for the executive’s choice of \( \hat{r} \).

To motivate the executive to expend effort, the contract must satisfy the constraints

\[
\begin{align*}
    p_O U_O + p_H U_H + p_L U_L - V &\geq U_O \quad (6) \\
    p_O U_O + p_H U_H + p_L U_L - V &\geq .5U_H + .5U_L. \quad (7)
\end{align*}
\]

These constraints require the contract to offer the executive at least as much expected utility if he works as he could obtain by not working and either selecting the safe project all of the time or the risky project all of the time. To satisfy these constraints the principal must make the executive’s compensation contingent upon the cash flow that occurs. \( U_H \) must be greater than \( U_O \), or the executive will always select the safe project (and never work), and \( U_L \) must be less than \( U_O \), or the executive will always select the risky project (and never work). In this sense our model is similar to conventional agency models in that risk must be imposed upon the executive to motivate him to work.

The risk imposed upon the executive will also, however, influence his strategy for selecting a project. This complicates matters because his available strategies (do not work and always choose the safe project, do not work and always choose the risky project, and work and choose the risky project if \( r > \hat{r} \)) cannot be ranked according to first-order stochastic dominance. Therefore, increased effort does not move the distribution of the cash flow to the right in the sense of first-order stochastic dominance. For this reason, the role that risk plays in motivating the executive is different in our model from that in the standard agency models.

To see this suppose we start with a riskless contract and simultaneously increase the payment if the high cash flow is observed and decrease the payment if the low cash flow is observed by the same amount. The difference between the executive’s expected utility if he works and his utility if he always selects the safe project (which is his optimal strategy if he does not work, given the contract and the executive’s risk aversion), is \((1 - \hat{r})\{.5(U_H - U_O)\} - V\). The term \((1 - \hat{r})\) is a measure of the \textit{a priori} attractiveness of the risky project to the executive. Specifically, it is the probability that the executive will select the risky project if he works. The term \(.5(U_H - U_O)\) represents the executive’s expected increase in monetary utility relative to the safe project conditional upon \( r \)’s exceeding \( \hat{r} \). We can think of this as the “upside potential” of the risky project. Increasing the amount of risk imposed upon the executive has two effects. First, it makes the risky project less attractive by increasing the value of \( r \) that makes the risky project acceptable. Second, it makes the risky project more attractive for those values of \( r \) for which it is accepted: it increases the “upside potential” of the risky project.

When sufficient risk is imposed, we can show that the probability that the risky project will be acceptable becomes so small that this dominates the fact that the risky project has a big upside potential. As a result, the benefits of working fall below the costs, and the executive’s optimal strategy is not to work and to select the safe project all of the time. Therefore, increasing the risk imposed upon the executive need not make working more attractive to him. For this reason, increasing the risk imposed upon the executive can actually decrease the expected gross cash flow.\(^9\)

\[^9\] Our model therefore avoids problems associated with the first-order condition approach that arise in conventional agency problems. These problems are discussed in Grossman and Hart (1983).

\[^{10}\] In a conventional agency model increasing the risk imposed upon the agent will motivate him to supply more effort. When the agent selects a higher amount of effort, the distribution of the cash flow moves to the right in the sense of first-order stochastic dominance; in particular, the expected gross cash flow increases.
5. Second-best solution

Using a Lagrangian formulation, we can write the principal’s problem as

$$\max_{U_0, U_H, U_L, \hat{r}} p_o(x_o - S(U_0)) + p_H(x_H - S(U_H)) + p_L(x_L - S(U_L))$$

$$\quad + \lambda \{p_o U_o + p_H U_H + p_L U_L - V - \theta\} + \mu_1 \{(p_o - 1) U_o + p_H U_H + p_L U_L - V\}$$

$$\quad + \mu_2 \{p_o U_o + (p_H - .5) U_H + (p_L - .5) U_L - V\} + \mu_3 \{U_o - [\hat{r} U_H + (1 - \hat{r}) U_L]\}.$$  (8)

The first constraint requires the principal to meet the executive’s minimum utility constraint. The second (third) constraint requires the executive’s expected utility weakly to exceed the utility level he could achieve by not working and always selecting the safe (risky) project (see equations (6) and (7)). The last constraint is the executive’s first-order condition on \(\hat{r}\) (see equation (5)). This constraint requires the project selected to be optimal for the executive for each value of the information signal \(r\). The contract must therefore motivate the executive to work hard and to make the desired choice between the two projects.

Differentiating problem (8) with respect to each \(U_j\) and using the fact that \(S'(U_j) = 1/U'(S_j)\), we have the following proposition.

**Proposition 1.** Assuming an interior solution, the optimal contract satisfies

$$\frac{1}{U'(S_o)} = \lambda + \mu_1 \left(\frac{p_o - 1}{p_o}\right) + \mu_2 + \mu_3 \frac{1}{p_o}$$  (9)

$$\frac{1}{U'(S_H)} = \lambda + \mu_1 + \mu_2 \left(\frac{p_H - .5}{p_H}\right) - \mu_3 \frac{\hat{r}}{p_H}$$  (10)

$$\frac{1}{U'(S_L)} = \lambda + \mu_1 + \mu_2 \left(\frac{p_L - .5}{p_L}\right) - \mu_3 \left(1 - \frac{\hat{r}}{p_L}\right).$$  (11)

The following two propositions, which we prove in the Appendix, establish the signs of the Lagrange multipliers.

**Proposition 2.** The solution to problem (8) is such that \(\mu_1 \geq 0\) and \(\mu_2 \geq 0\), with at least one of \(\mu_1\) and \(\mu_2\) strictly greater than zero.

**Proposition 3.** Let \(r\) be the project selection cutoff value in the second-best solution (i.e., the value of \(\hat{r}\) that solves problem (8)). If \(r < .5\), then \(\mu_1 = 0\), \(\mu_2 > 0\), and \(\mu_3 < 0\). If \(r > .5\), then \(\mu_1 > 0\), \(\mu_2 = 0\), and \(\mu_3 > 0\).

These propositions imply that there is a welfare loss relative to the first-best solution. Note that \(\mu_3\), the Lagrange multiplier on the first-order condition on the executive’s choice of a cutoff point \(\hat{r}\), can be negative. In a conventional agency model all of the Lagrange multipliers on the agent’s actions are nonnegative.

Next we examine the executive’s project selection strategy. Specifically, we explore whether the executive always selects the project that is in the principal’s best interests. Differentiating problem (8) with respect to \(\hat{r}\) and using equation (5), we obtain

$$\sum (x_j - s_j) \frac{\partial p_j}{\partial \hat{r}} - \mu_3 (U_H - U_L) = 0.$$  (12)

If \(U_H > U_L\), equation (12) implies that

$$\text{Sign} (\mu_3) = \text{Sign} \left(\sum_j (x_j - s_j) \frac{\partial p_j}{\partial \hat{r}}\right)$$

$$= \text{Sign} \{x_o - s_o - [r_3(x_H - s_H) + (1 - r_3)(x_L - s_L)]\}.$$  (13)
The sign of $\mu_3$ determines the sign of the derivative of the principal's expected utility with respect to the project selection cutoff point $\hat{r}$. In particular, $\mu_3$ determines which project has a higher expected net profitability to the principal at the cutoff point that the executive selects. If $\mu_3$ is negative, the risky project has a higher expected net profitability to the principal at $r = r_s$, so he would prefer the executive to decrease $r_s$ and to select the risky project more frequently. On the other hand, if $\mu_3$ is positive, the safe project is more profitable to the principal at $r = r_s$, and he would prefer the executive to increase $r_s$ and to select the safe project more often.

Since the sign of $\mu_3$ is established in Proposition 3, we therefore have the following proposition.

**Proposition 4.** From the point of view of the principal, the second-best contract motivates the executive to underinvest in the risky project if $r_s < .5$ and to overinvest in the risky project if $r_s > .5$.

In either case the executive does not always choose the project with the highest expected net profitability to the principal. It is important to note that this conflict of interest refers to which project should be selected after the executive observes the value of $r$. The role of the compensation contract, however, is not merely to promote goal congruence with respect to the project selection for each value of $r$; it must also motivate the executive to work to obtain the superior information about the profitability of the risky project. To perform both of these tasks efficiently, the contract trades off the costs of motivating the executive to work with the benefits of having the executive adopt a project selection strategy that the principal desires.

Consider the cost of motivating the executive to work and to select a cutoff point ($r_s$) less than $.5$. In this case if the executive does not work, he will select the risky project all of the time. Therefore, the binding incentive constraint is that the contract must motivate the executive to work versus always choosing the risky project. To accomplish this all of the incentive must be provided from those values of $r$ below $r_s$, because these are the only values of $r$ for which the executive’s payoff differs depending upon whether he works or whether he always selects the risky project. As $r_s$ decreases, the range of $r$ that provides the motivation for the executive to expend effort decreases, and it becomes more expensive to motivate him to work.

To keep the executive’s expected compensation costs low, the principal would therefore like to motivate the executive to pick a cutoff point close to $.5$. On the other hand, the principal also desires to motivate the executive to select the project that is in the principal’s best interest for each value of $r$. To trade off these two factors, the principal allows the executive to select a higher value of $r_s$ than the principal would prefer (considering only the project selection decision) to decrease the cost of motivating the executive to work.

Next, we compare the second-best investment decision with the first-best. Recall that in the first-best solution the project that maximized the expected gross cash flow was chosen (because the executive received a constant wage). The following proposition, which we prove in the Appendix, establishes conditions that lead to an underinvestment (relative to the first-best solution) in the risky project.

**Proposition 5.** If $r_s < .5$ or if $r_f < .5$, then $r_s > r_f$.

Under these conditions there is range of information signals such that $r_f < r < r_s$. For this range of $r$ the risky project would have been chosen in the absence of incentive problems; however, in the second-best solution the executive selects the safe project.

It is also possible for overinvestment in the risky project to occur. To demonstrate this we first present a closed-form solution to the problem of determining the optimal contract

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11 The case in which $r_s$ is greater than $.5$ works similarly.
to motivate the executive to work and to select the risky project only if $r$ exceeds a cutoff point of $\hat{r}$. The solution assumes that all payments to the executive are in the interior of the set of feasible payments. The optimal contract is

$$U_O = \theta + \frac{(2\hat{r} - 1)}{\hat{r}^2} V,$$

$$U_H = \theta + \frac{V}{\hat{r}^2},$$

$$U_L = \theta - \frac{V}{\hat{r}^2} \quad \text{if} \quad \hat{r} < .5,$$

and

$$U_O = \theta, \quad U_H = \theta + \frac{2(1 - \hat{r})}{(1 - \hat{r})^2} V, \quad \text{and} \quad U_L = \theta - \frac{2\hat{r}}{(1 - \hat{r})^2} V \quad \text{if} \quad \hat{r} > .5.$$ 

This characterization of the contract enables us to show the following result, which we prove in the Appendix.

**Proposition 6.** If the executive has a square root utility function and if $r_f > .5$, then $r_s < r_f$.

Under these conditions the second-best solution involves overinvesting in the risky project. Specifically, if $r_s < r < r_f$, the executive selects the risky project in the second-best solution, but he would have selected the safe project in the absence of incentive problems.

Interestingly, whether underinvestment or overinvestment occurs is related to the *a priori* profitability of the risky project. In particular, underinvestment in the risky project occurs in situations in which the risky project is *a priori* more attractive than the safe project. The analysis also suggested that firms that *ex post* frequently invest in risky projects (i.e., firms with $r_s < .5$) also experience underinvestment in the risky project. On the other hand, if the risky project is not viewed to be so attractive as the safe project *a priori*, then an overinvestment in the risky project occurs.\(^{12}\)

### 6. Communication

- One factor contributing to the underinvestment problem may be the inability of the executive to communicate to the principal the information he obtains about the profitability of the risky project.\(^{13}\) The underinvestment problem occurs when the risky contract is excessively risky for the executive, even though the expected profitability of the risky project exceeds the expected profitability of the safe project for the principal. This problem may be reduced by installing a more sophisticated communication system that allows the executive to select from a menu of risky contracts. In particular, if a less risky contract is available for the executive to select for lower values of $r$, he may become more willing to invest in the risky project.

To model this we assume that the principal announces and precommits to a menu of contracts, $(U_O, \{U_H(r), U_L(r)\})$. After the executive observes the signal $r$, he decides which project and which contract to select. It is easy to derive a number of properties of the optimal incentive scheme and the executive’s investment strategy when communication is allowed.

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\(^{12}\) These comparisons assume that it is optimal to motivate the executive to work in both the first-best and the second-best solutions. If the project selection cutoff point when the executive works approaches zero (one), it will become optimal to select the risky (safe) project all of the time, and not to pay the executive to work. Moreover, the expected compensation costs to induce the executive to work will be higher when effort is unobservable for any given project selection cutoff point. This suggests that for ranges of $r_f$ close to zero (one) for which it would be optimal to pay the executive to work in the first-best solution, the second-best solution will be not to pay the executive to work and to select the risky (safe) project all of the time. Therefore, there will be an overinvestment (underinvestment) in the risky project in the second-best solution, relative to the first-best solution, when $r_f$ is close to zero (one).

\(^{13}\) Because the sets of possible cash flows for the two projects are disjoint, one interpretation of the preceding analysis is that a limited form of communication between the principal and the executive is allowed. In particular, the executive chooses between a risky contract $(U_L, U_H)$ and a safe contract $(U_O)$ conditional upon the value of $r$ that he observes.
First, if \((U_H(s), U_L(s))\) and \((U_H(t), U_L(t))\) are the risky contracts selected when \(r = s\) and \(r = t\), respectively, then \(U_H(s) > U_L(t)\) if and only if \(U_L(s) < U_L(t)\). This ensures that no risky contract strictly dominates another one. Similarly, we must have \(U_L(t) \leq U_O \leq U_H(t)\) for each \(t\) to ensure that the safe contract is not strictly dominated by any risky contract. Second, the executive’s expected utility, given \(r\), is a (weakly) increasing and (weakly) convex function of \(r\). This is so since the executive’s expected utility is a linear function of \(r\) for each contract, and he selects the contract to maximize his expected utility conditional upon \(r\).

Third, the executive’s choice of contracts is such that \(U_H(r)\) is a (weakly) increasing function of \(r\) and \(U_L(r)\) is a (weakly) decreasing function of \(r\). The executive’s selection of projects can therefore be characterized as follows. If the executive observes a low value of \(r\), he will select the safe project and the riskless contract. As \(r\) increases, the executive will become willing to accept the risky project and a low risk contract—one with a low value of \(U_H\) and a high value of \(U_L\). As \(r\) further increases, the executive continues to select the risky project and also selects a higher risk contract—one with a higher \(U_H\) and a lower \(U_L\).

We suggested above that this third feature of the optimal contract may induce the executive to be more willing to invest in the risky project. In fact, the above conditions are sufficient to make the underinvestment problem disappear. We prove the following proposition in the Appendix.\(^{14}\)

**Proposition 7.** When communication is allowed, the executive never selects the safe project if the risky project is preferred by the principal.

The principal is able to achieve this result by including a contract that completely shields the executive from risk, regardless of the cash flow that occurs, in the menu of contracts. Let \([0, \hat{r}]\) be the range of \(r\) for which this riskless contract is optimal for the executive. For each value of \(r\) in this range the executive is indifferent between the risky and the safe project, and is therefore willing to adopt whichever project is in the principal’s best interests. For \(r > \hat{r}\) the executive strictly prefers the risky project, and the riskiness of the contract he chooses increases as \(r\) increases.

This result implies that there are strict gains to communication for the case in which underinvestment occurs in the absence of communication. This suggests that the existence of an underinvestment problem may be related to the inability of executives to communicate their information regarding the profitability of their projects. Communication may be infeasible for a number of reasons. First, the solution to the investment problem when communication is allowed critically depends on the principal’s ability to precommit to the announced incentive scheme, and not to renege on the agreement after the executive communicates his information. Second, the cost of communication may be excessive. For example, transmission costs, including regulatory constraints on the type of information that can be conveyed, or the manner in which it is conveyed, may make communication prohibitively costly. Finally, the information may be such that the executive is unable to articulate it, or the principal may be unable to understand it. It is not clear whether a result similar

\(^{14}\) Proposition 7 extends to more general outcome distributions for the risky project (e.g., distributions which result in more than two possible cash flows). The proposition also extends to cases in which more than one risky project is available to the executive. When a riskless contract is included in the menu, the executive never selects the safe project if one of the risky projects has a higher expected profitability. When more than one risky project is available, however, Proposition 7 does not guarantee that the executive always chooses the risky project that is most profitable to the principal. In particular, the executive may select a “safer” risky project than the principal desires. If this is the case, a form of underinvestment in risky projects can result even when communication is allowed.

Proposition 7 critically depends on the assumption that the risky project does not require any more effort to implement than does the safe project.
to Proposition 7 applies to the situation in which there is an overinvestment problem in the absence of communication.

7. Multi-period considerations

To examine further the incentives of managers to invest in new projects, it would be desirable to extend the analysis to a multi-period setting. In the simplest multi-period setting the executive’s effort provides him with information that he uses to select a project in the first period. The project’s return is then realized in the second period. Under this scenario there is no information available in the first period about the executive’s actions, so the executive’s first-period compensation would have to be a flat wage; all of the incentive would have to come from the dependence of the executive’s second-period compensation on the second-period cash flow. This means that it is critical that the executive remain with the firm for both periods. If the executive could leave the firm after the first period and not be held accountable for the second-period cash flow, he would have little incentive to provide any effort in the first period. Instead, he would take his first-period salary and then leave.

To prevent this from occurring, the principal might design a long-term contract for the executive that motivates him to stay with the firm for both periods: the principal could use a “golden handcuffs” approach to motivating the executive. In the first period the executive would be paid a “below market” wage. The second-period contract would be structured so that if the executive works in the first period and remains with the firm, he receives an expected utility in the second period that is above the utility level he could achieve by leaving the firm. In this way the executive would have incentive to work in the first period and remain with the firm for both periods.

A more interesting multi-period setting would involve the executive’s allocating his effort between the type of effort analyzed here and the “productive” effort that is typically analyzed in agency models. Productive effort would have the direct effect of increasing that period’s cash flow. Research effort would provide the executive with information about the profitability of potential projects. In this setting the role of future-period incentives in motivating the executive would become even more critical. For example, consider only the first-period incentives. If the executive’s first-period compensation was increasing in the first-period cash flow, he would allocate all of his effort toward productive uses, since research effort has no effect on the first period’s cash flow. On the other hand, if the executive’s first-period compensation was constant or decreasing, the executive would not supply any productive effort in the first period. Therefore, the only way that the principal can motivate the executive to expend effort both for productive uses and for research is to use the future-period incentives.

Appendix

Proofs of Propositions 2, 3, 5, 6, and 7 follow.

Proof of Proposition 2. Consider the problem of choosing the optimal contract to motivate an arbitrary project selection cutoff point \( \hat{r} \). For any value of \( \hat{r} \) the probabilities in problem (8) are fixed, and problem (8) is a standard nonlinear programming problem of maximizing a concave function subject to a finite number of linear constraints. It follows that for any value of \( \hat{r} \), the Lagrange multipliers on the inequality constraints \( (\lambda, \mu_1, \mu_2) \) are nonnegative. In particular, this is true for the optimal value of \( \hat{r} \).

To prove the second part of the proposition, suppose \( \mu_1 \) and \( \mu_2 \) both equal zero. In this case equations (9)–(11) indicate that the optimal contract is

\[
\frac{1}{U(s_0)} = \lambda + \mu_3 \frac{1}{p_0}, \quad \frac{1}{U(s_H)} = \lambda - \mu_3 \frac{\hat{r}}{p_H}, \quad \text{and} \quad \frac{1}{U(s_L)} = \lambda - \mu_3 (1 - \hat{r}) \frac{1}{p_L}.
\]

Also, note that \( 1/U(s) \) is increasing in \( s \).

If \( \mu_3 < 0 \), then \( s_0 \) is less than \( s_H \) and \( s_L \). This implies that the executive would never select the safe project. But this violates the third constraint of problem (8).
If $\mu_3 = 0$, the optimal contract is a constant; $s_j = s$ for all $x_j$. Again, the executive would prefer to not work, but this violates the second and third constraints of problem (8).

Finally, if $\mu_3 > 0$, then $s_0$ is greater than both $s_H$ and $s_L$. In this case the executive’s optimal strategy is not to work and always to select the safe project, which violates the second constraint of problem (8). Q.E.D.

Proof of Proposition 3. By definition of $r$, the executive prefers the safe (risky) project if $r$ is less than (greater than) $r_f$. If $r < .5$, the executive prefers the risky project if $r = .5$. This implies that if the executive does not work, he strictly prefers the risky project over the safe project. The second constraint to problem (8) is therefore not binding, and its Lagrange multiplier ($\mu_3$) is zero. If $\mu_3 = 0$, however, then Proposition 2 implies that $\mu_2 > 0$.

It remains to establish that $\mu_3 < 0$. Note that for any value of $\hat{r}$, we have $p_H \leq .5$ and $p_L \leq .5$. Suppose that $\mu_3 \geq 0$. The optimal contract given by equations (9)–(11) therefore implies that

\[
\frac{1}{U'(s_0)} = \lambda + \mu_2 + \mu_3 \frac{1}{p_0} \geq \lambda, \\
\frac{1}{U'(s_H)} = \lambda + \mu_3 \left( \frac{p_H - .5}{p_H} \right) - \mu_3 \frac{r}{p_H} \leq \lambda, \quad \text{and} \\
\frac{1}{U'(s_L)} = \lambda + \mu_3 \left( \frac{p_L - .5}{p_L} \right) - \mu_3 \left( 1 - \hat{r} \right) \frac{1}{p_L} \leq \lambda.
\]

But this implies that $s_0$ is at least as great as $s_H$ and $s_L$. The executive will therefore prefer not to work and always to select the safe project. But this violates the second constraint to problem (8).

The case of $r_f > .5$ is similar. Q.E.D.

Proof of Proposition 5. From Proposition 3, if $r < .5$, then $\mu_3 < 0$. We shall therefore show that $\mu_3 < 0$ implies that $r_s > r_f$. From equation (12), we have

\[
\sum_j \frac{\partial p_j}{\partial \hat{r}} x_j \bigg|_{\hat{r} = r_s} = \sum_j \frac{\partial p_j}{\partial \hat{r}} s_j \bigg|_{\hat{r} = r_s} + \mu_3 (U_H - U_L).
\]

By definition of $r_s$ we have $U_0 = r_s U_H + (1 - r_s) U_L$. Since the executive is strictly risk averse, this implies that $s_0 < r_s s_H + (1 - r_s) s_L$. Therefore we have $\sum_j \frac{\partial p_j}{\partial \hat{r}} s_j \bigg|_{\hat{r} = r_s} = s_0 - [r_s s_H + (1 - r_s) s_L] < 0$.

If $\mu_3 < 0$ and $U_H > U_L$, then $\mu_3 (U_H - U_L) < 0$. Therefore, we must have $\sum_j \frac{\partial p_j}{\partial \hat{r}} x_j \bigg|_{\hat{r} = r_s} < 0$. It remains to establish that this implies that $r_s > r_f$.

Lemma A1. Sign \(\left( \sum_j \frac{\partial p_j}{\partial \hat{r}} x_j \bigg|_{\hat{r} = r_f} \right) = \text{Sign} (r_f - r_s).

Proof. By definition, $r_f$ satisfies $\sum_j \frac{\partial p_j}{\partial \hat{r}} x_j \bigg|_{\hat{r} = r_f} = 0$. From Section 3 $p_j x_j$ is a strictly concave function of $\hat{r}$, so this implies that $r_s > r_f$ if and only if the derivative of $\sum p_j x_j$ with respect to $\hat{r}$ is negative at $\hat{r} = r_f$. Q.E.D.

Next we show that $r_f < .5$ implies that $r_s > r_f$. If $r_s > .5$ then $r_f > r_s$. If $r_f < .5$, it was established above that this implies that $r_s > r_f$. Q.E.D.

Proof of Proposition 6. If $r_f < .5$, then $r_s < r_f$, and the result is established. If $r_f > .5$, we use the closed-form solution for the contract to compute the expected payment to the executive necessary to motivate him to select a cutoff point $\hat{r}$. When $U(\hat{r}) = V$, then we have $s_j = \frac{V}{\hat{r}}$, and the expected payment is $\sum_j p_j s_j = \theta^2 + 2\theta V + 2V^2(1 - \hat{r})^2$.

The optimal cutoff value of $\hat{r}$ is therefore the solution to the problem $\max \sum_j p_j x_j - [\theta^2 + 2\theta V + 2V^2(1 - \hat{r})^2]$. The first-order condition on $\hat{r}$ is $\sum_j \frac{\partial p_j}{\partial \hat{r}} x_j = 2V^2(2(1 - \hat{r})^3]$. Setting this equal to zero implies that the second-best cutoff point, $r_s$, is such that

\[
\sum_j \frac{\partial p_j}{\partial \hat{r}} x_j \bigg|_{\hat{r} = r_s} > 0. \tag{A1}
\]

It is also easy to verify that the second-order condition on $\hat{r}$ is negative, so that the first-order condition is necessary and sufficient for an interior optimum.

With Lemma A1 (contained in the proof of Proposition 5), equation (A1) implies that $r_s < r_f$. Q.E.D.
Proof of Proposition 7. Let \( \{U_0, (U_H(r), U_L(r))\} \) be the optimal menu of contracts. Suppose there exists a range of signals, \( T = [l, f] \), such that \( x_0 < r x_H + (1 - r) x_L \) for all \( r \in T \), but the executive selects the safe project upon observing information signals in this range.

Consider a new menu of contracts that includes all of the contracts in the original menu of contracts and that also contains a contract for which the executive's compensation is independent of the cash flow that occurs, i.e., \( U_H = U_L = U_0 \).

Under the new menu of contracts, the executive receives the same expected utility for each signal \( r \). Whenever a risky contract was optimal for the executive under the old menu of contracts, he selects the same contract under the new menu of contracts. When the riskless contract is optimal for the executive, however, he is now indifferent between the two projects. In particular, the executive is now willing to select the risky project for \( r \in T \). The principal's expected utility is the same for all \( r \in T \), but the principal's utility is improved for \( r \in T \). This contradicts the assertion that the original contract was optimal. Q.E.D.

References


