The Effect of Informedness and Consensus on Price and Volume Behavior

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ABSTRACT: This paper presents a partially revealing rational expectations model of competitive trading to identify two effects of information releases; an informedness effect and a consensus effect. The informedness effect measures the extent to which agents become more knowledgeable, and the consensus effect measures the extent of agreement among agents at the time of an information release. We demonstrate that informedness and consensus generally occur jointly when information is disseminated, and that unexpected price changes and trading volume are each influenced by both informedness and consensus. Thus, interpretations of unexpected price changes and volume associated with information releases are conceptually similar. Since informedness and consensus each affect both the variance of price changes and volume, our paper provides an economic rationale for examining both price and volume effects at the time of information releases.

EMPirical researchers have traditionally used unexpected price changes to measure the information content of public disclosures of information (e.g., Beaver 1968; Atiase 1985). An announcement is said to contain information if it alters investors' beliefs about the value of an asset (e.g., Beaver 1981, 117). Research in accounting indicates that the variance of unexpected price changes increases at the time of earnings announcements and that unexpected price changes are positively correlated with the unexpected component of earn-

We have benefited from conversations with Doug Diamond, Raffl Indjejikian, Oliver Kim, Rob Vishny, and from comments from Bill Kinney, John Long, several referees (especially Russ Lundholm), and participants at the University of Chicago and Cornell University. Holthausen gratefully acknowledges support from the Center for Research in Security Prices and the Institute of Professional Accounting at the University of Chicago. Verrecchia gratefully acknowledges financial assistance from Arthur Young & Company and the Wharton School of the University of Pennsylvania.

Manuscript received August 1988.
Revision received April 1989.
Accepted August 1989.
nings announcements. Researchers generally agree that unexpected price changes imply that an announcement contains information, that is, that it alters investors' beliefs.

In empirical studies, trading volume has also been used as a measure of information content with a number of researchers explicitly interpreting it as a measure of the extent of agreement or consensus among agents induced by the information.\(^1\) The evidence clearly indicates that trading volume increases at the time of earnings announcements and that trading volume is positively correlated with the absolute value of the unexpected component of earnings announcements. Unlike price changes, researchers have been reluctant to draw strong inferences regarding the interpretation of volume reactions. For example, consider the discussion of Beaver's (1968) paper on observed volume at the time of earnings announcements contained in Watts and Zimmerman (1986, 64):

While Beaver interprets an increase in the volume of trading as evidence of information, there is a problem with this interpretation. Conceptually, information could be conveyed to the market and prices could change by large amounts without a single transaction (trade). . . . On the other hand, there could be substantial trading (e.g., due solely to portfolio rearrangement) without any information release. The problem is the lack of an economic theory of volume. Consequently, as Beaver recognizes, his volume measure of information is \textit{ad hoc}.

In this paper, we identify two effects of information: an informedness effect and a consensus effect. The informedness effect measures the degree to which agents become more knowledgeable, and the consensus effect measures the extent of agreement among agents at the time of an information release. We demonstrate that informedness and consensus effects generally occur jointly when information is disseminated, and that price changes and trading volume are each influenced by both informedness and consensus. Thus, interpretations of price changes and volume associated with information releases are conceptually similar. As such, the paper demonstrates that studies of unexpected price changes or volume are equally relevant means of assessing information "content" assuming that information content is defined as the extent to which an information signal alters investors' beliefs. Moreover, interpretations of volume as solely a measure of consensus appear incomplete since both informedness and consensus influence the behavior of volume and price.

With respect to price changes, we show that an increase in informedness results in an increase in the variance of price changes.\(^2\) It is also the case that an increase in consensus results in an increase in the variance of price changes. Since information typically evidences simultaneous informedness and consensus effects, unexpected price changes are influenced by both phenomena. Consequently, whether the variance of unexpected price changes shifts upward or downward depends upon both the direction of the change in informedness and

\(^1\) Beaver (1968), Morse (1981), Bamber (1987), and Jain (1988) discuss volume as a measure of consensus. Verrecchia (1981) discusses the difficulties associated with interpreting volume reactions to information releases and concludes that no unambiguous inferences can be drawn about consensus when a volume reaction is observed.

\(^2\) In this paper, unexpected change in price is examined, that is, how the realization of price deviates from its expectation. For convenience in exposition, we adopt the convention that the expressions "unexpected price changes" and "price changes" are equivalent.
consensus (which determines whether the effects on the variance of price changes are reinforcing or countervailing), and which effect dominates in a particular informational setting if the effects are countervailing. Trading volume operates in a similar fashion to unexpected price changes. An increase in informedness results in an increase in trading volume, however, an increase in consensus results in a decrease in trading volume. Once again, trading volume is influenced by both informedness and consensus effects. Thus, trading volume may shift upward or downward depending upon whether shifts in informedness and consensus are reinforcing or countervailing, and if countervailing, which effect dominates.

To facilitate discussion, we introduce a partially revealing rational expectations model of competitive trading, in which many agents exchange a single risky asset and a single riskless asset over one period. At the start of the period, agents have homogeneous expectations with respect to the value of the risky asset. Exchange is motivated by agents receiving information about the liquidating value of the risky asset. Investors' interpretation of that information contains a noise term which is common across agents and an agent-specific idiosyncratic noise term. Information is modeled in this way to help assess the extent to which heterogeneous interpretations of a public information release result in price and volume reactions. In this model, agents learn something about other investors' beliefs through the price aggregation process, but they do not learn everything. Hence, investors have heterogeneous expectations after the information is released.

A number of papers have examined the behavior of price changes or volume or both in a rational expectations context (e.g., Pfleiderer 1984; Varian 1985; Grundy and McNichols 1988; Holthausen and Verrecchia 1988; Kim and Verrecchia 1989). One way to characterize broadly these papers is that they develop models that assume a variety of economic environments that lead to changes in price, trading volume, or both. Then, they consider how the various determinants of their models influence price changes and/or volume. The orientation of our work is somewhat different in that it concerns portraying and understanding informedness and consensus as primitive economic notions. How informedness and consensus affect unexpected price changes and volume is also shown and this links our work to what others have done. We are aware of no studies that provide a rigorous characterization of informedness and consensus, and their interrelation in a general economic setting.

3 There is evidence in the accounting literature to suggest that accounting earnings are interpreted heterogeneously. What is relevant here is not that each investor sees the same earnings per share figure, for example, but that investors reach varying conclusions about the revised value of the firm after an earnings announcement. Evidence in Morse et al. (1987) indicates that the variance of analysts' earnings forecasts for the subsequent year increases when the current year's earnings are announced. Moreover, this increased dispersion in analysts' forecasts is larger, the further announced earnings are from the consensus belief. This evidence is consistent with the notion that investors' assessments of the implications of earnings releases for the value of the firm are heterogeneous.

4 These papers are based on the rational expectations paradigm. Other papers which assume different market processes include among others. Kyle (1985), Diamond and Verrecchia (1987), Karpoff (1986), and Admati and Pfleiderer (1988). See Karpoff (1987) for a review of theoretical and empirical work which explores the relation between volume and price changes.

5 Our model only considers how information alters investors' beliefs and, in turn, how price and volume are affected. We abstract from trading which arises because of liquidity considerations, port-
This paper proceeds as follows. Section I describes a market process with many diversely informed agents, a single risky asset, and a single riskless asset over one period of trade. Further, we construct a partially revealing rational expectations equilibrium in this market where information is introduced, and trading arises, during the period. Section II defines measures of informedness and consensus and characterizes the variance of the unexpected change in price and trading volume in terms of informedness and consensus. We discuss how informedness and consensus affect the variance of the unexpected change in price and trading volume in Section III and concluding remarks are contained in Section IV.

I. Partially Revealing Rational Expectations Equilibria

Description of the Market

This section describes a partially revealing, rational expectations model of competitive trading. The model is intended to be as basic and generic as possible, specifically to capture those universal elements of the variety of rational expectations trading models that are variations on this theme.\(^6\)

The market is comprised of a continuum of diversely informed and optimally motivated agents, each indexed by \(a \in [0,1]\). At the beginning of the period, market participants are endowed with risky and riskless assets. The aggregate supply of risky assets is unknown, and uncertainty about the effect of its behavior on prices represents the noise in the economy. Within the period two events occur. First, each agent receives a signal about the liquidating value of the risky asset. Second, agents exchange assets on the basis of their information. This exchange of assets implies an equilibrium price \(p\) for the risky asset. When the period concludes, the risky and riskless assets are liquidated, and agents and traders consume their holdings of each.

The risky and riskless assets pay off in the economy’s single consumption good. The risky asset is represented by the random variable \(\bar{u}\), which has a normal distribution with mean \(m\) and variance \(v\). Realizations of \(\bar{u}\) are represented by \(u\) (i.e., the tilde is used to distinguish a random variable from its realization). The riskless asset is a numeraire commodity, one unit of which returns one unit of the single consumption good when assets are liquidated, independent of when the riskless asset is acquired (i.e., discounting is ignored).

Let the parameters \(D\) and \(B\) represent agent \(a\)'s holdings of the risky and riskless assets, respectively, at the end of the period, where \(D_u\) and \(B_u\) are the agent’s endowments of these assets. Each agent has a utility function for an amount \(w\) of the single consumption good characterized by \(-\exp(-w/r)\). For this (negative) exponential utility function each agent exhibits constant absolute

\(^6\) Models very similar in spirit to the one here include, for example, those by Hellwig (1980), Verrecchia (1982), and Admati (1985).
risk-tolerance of \( r \). The objective of each agent is to maximize his or her expected utility of consumption when the market terminates at the end of the period: that is, \( E_{a}[-\exp(-r^{-1}[D_{a}\bar{u}+B_{a}])] \), where \( E_{a}[\cdot] \) is agent \( a \)'s expectation operator based on his or her information when he or she trades. An agent's ability to trade, however, is constrained by his or her endowment. That is:

\[
D_{a}p + B_{a} = D_{0a}p + B_{0a}.
\]

**Information**

At the beginning of the period, agents believe that \( \bar{u} \) has a normal distribution with mean \( m \) and variance \( \nu \). The realization \( \bar{u} = u \) is the liquidating dividend of the risky asset and, thus, can be thought of as the risky asset's true, economic cash flow. The parameter \( \nu \) is the variance of that cash flow. During the period each agent receives an information signal and interprets what the information signal \( \tilde{y}_{a} \) implies about the liquidating value \( \bar{u} \), such that each investor's interpretation of the signal is given by:

\[
\tilde{y}_{a} = \bar{u} + \bar{\eta} + \tilde{e}_{a},
\]

where \( \bar{\eta} \) is a common noise term that has a normal distribution with mean zero and variance \( n \), and \( \tilde{e}_{a} \) is an idiosyncratic noise term that has a normal distribution with mean zero and variance \( s \). The random variables \( \bar{u}, \bar{\eta} \), and \( \tilde{e}_{a} \) are each independent of one another. Furthermore, the idiosyncratic noise terms \( \tilde{e}_{a} \) are independent across agents: that is, \( E[\tilde{e}_{a} \tilde{e}_{k}] = 0 \) for all \( a \neq k \).

This particular form for the information release is chosen because we focus on the effects of heterogeneous interpretations of a public information release on prices and volume. Evidence suggests that while investors all observe the same reported earnings, their interpretations of those earnings for the value of the firm are not homogeneous. In this environment, each agent observes the same public signal (e.g., an earnings announcement) but each agent's interpretation of what the signal implies about the value of the liquidating dividend varies because of the agent-specific noise term. With respect to an earnings announcement, this is akin to all agents observing the same reported earnings per share figure, and then determining the implication of the reported earnings for the value of the firm. What is relevant is not the earnings number per se, but the implications of the earnings release for the value of the firm. Moreover, the earnings signal per se does not contain a direct statement about the value of the firm which is known by all agents prior to their idiosyncratic assessment. As such, investors do not observe the common and idiosyncratic components of noise separately.

**Definition of an Equilibrium**

We suggest a model of the market process that operates in the following

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\(^7\) Without loss of generality, the results of this analysis are easily extended to the case in which agents have different levels of risk tolerance \( r_{a} \).

\(^8\) Indjejikian (1988), Lundholm (1988), and Pfleiderer (1984), among others, also model information with both a common and idiosyncratic component of noise.

\(^9\) It is also appropriate to interpret the information signal as private information. However, because we are primarily interested in heterogeneous interpretations of a public information signal, the text does not explicitly recognize the private information interpretation.
The Accounting Review, January 1990

stylized fashion. Each period, agents submit demand orders to an auctioneer. The total per capita supply of risky assets is represented by \( z \), where \( z \) is independent of all other random events, and has mean \( z_0 \) and variance \( t \). Therefore, when agents submit their demand orders to the auctioneer, they condition their beliefs on their individual signals, \( \tilde{y}_a \), and price, \( \tilde{p} \). Then, the auctioneer determines the market clearing price \( p \) such that agents’ total per capita demand for the risky asset equals the realization of total per capita supply, \( z \). Before defining an equilibrium, we add one more convention concerning the independent random variables \( \tilde{\varepsilon}_a \). As the idiosyncratic noise terms are aggregated by price across a continuum of agents, they converge almost surely to their common mean of zero, provided their variance is uniformly bounded. The effect of this convention is that when prices aggregate information,

\[
\int_0^1 \tilde{y}_a \, da = \tilde{u} + \tilde{\eta}
\]

almost surely.\(^{10}\) A straightforward interpretation of this result is that as the number of agents becomes large, their idiosyncratic components of noise average to zero through the price aggregation process. This convention permits the following definition of an equilibrium.

**Definition:** An equilibrium with a continuum of agents is a price \( \tilde{p} \) and an allocation function \( D_a(\tilde{y}_a, \tilde{p}), B_a(\tilde{y}_a, \tilde{p}), a \in [0,1] \) such that:

(i) \( \tilde{p} \) is \((\tilde{u} + \tilde{\eta}, \tilde{z}) \) measurable;
(ii) \( (D_a(\tilde{y}_a, \tilde{p}), B_a(\tilde{y}_a, \tilde{p})) \epsilon \arg \max \{D_a p + B_a = D_b p + B_b\}
\]

\[
E[-\exp(-r^{-1}(D_a(\tilde{u} + B_a))| \tilde{y}_a, \tilde{p})]
\]

and

(iii) \( \int_0^1 D_a(\tilde{y}_a, \tilde{p}) \, da = z \) almost surely.

These statements jointly imply that price is a measurable function of the liquidating dividend of the risky asset plus noise and per capita supply, each agent maximizes his or her expected utility subject to his or her budget constraint, and markets clear (i.e., aggregate demand equals aggregate supply).

**Characterization of an Equilibrium**

When prices are of the form \( \tilde{p} = \alpha + \beta(\tilde{u} + \tilde{\eta}) - \gamma \tilde{z} \), the covariance matrix that describes the relation between \( \tilde{u}, \tilde{y}_a, \) and \( \tilde{p} \) is given by:

\[
\begin{bmatrix}
\tilde{u} & v & v & \beta v \\
\tilde{y}_a & v & v + n + s & \beta(v + n) \\
\tilde{p} & \beta v & \beta(v + n) & \beta^2(v + n) + \gamma^2 t
\end{bmatrix}
\]

\(^{10}\) We assume sufficient mathematical regularity to allow for this convention. For a more detailed discussion, see Hellwig (1980) or Admati (1985).
Therefore,

\[ E[\bar{u} | \bar{y}_a = y_a, \bar{p} = p] = m + (g, h)(y_a - m, p - E[\bar{p}])' \]

and,

\[ \Theta^{-1} = \text{Var}[\bar{u} | \bar{y}_a = y_a, \bar{p} = p] = v - (g, h)(v, \beta v)' \]

where the vector \((g, h)\) is as defined by:

\[
(g, h) = (v, \beta v) \begin{bmatrix} v + n + s & \beta(v + n) \\ \beta(v + n) & \beta^2(v + n) + \gamma^2 t \end{bmatrix}^{-1}
\]

This follows from standard formulae for determining a conditional expectation when \((\bar{u}, \bar{y}_a, \bar{p})\) have a tri-variate normal distribution. It is well-known that a feature of the (negative) exponential utility function is that agent \(a\)'s demand is given by:

\[ D_a = r\Theta(E[\bar{u} | \bar{y}_a = y_a, \bar{p} = p] - p). \]

The market-clearing condition equilibrating demand with supply requires:

\[ \int_0^1 D_a da = z \text{ almost surely.} \]

Thus, substituting in the expression for \(D_a\) given in equation (1) into equation (2), and rearranging terms, yields as the price for the risky asset:

\[ p = (r\Theta(1 - h))^{-1}(r\Theta m + \int r\Theta(y_a - m) da - rh\Theta E[\bar{p}] - z). \]

But noting that the idiosyncratic terms \(\epsilon_a\) disappear through the aggregation process (by convention), and the fact that \(E[\bar{p}] = m - (r\Theta)^{-1}z_0\) (which obtains by taking the expectation of both sides of eq. [1]) the expression for price simplifies to:

\[ p = \alpha + \beta(u + \eta) - \gamma z, \]

where \(\alpha\), \(\beta\), and \(\gamma\) are given by \(\alpha = (1-\beta)m + (\gamma - \{r\Theta\}^{-1})z_0\), \(\beta = g(1-h)^{-1}\), and \(\gamma = (r\Theta(1 - h))^{-1}\), respectively. For this representation of prices to be internally consistent (i.e., "rational"), \((\beta / \gamma)\) must equal \(r\Theta g\), which implies that it solves the equation \((\beta / \gamma) s + (s + n)(\beta / \gamma) t = 0\). There exists a unique \((\beta / \gamma)\) which solves this equation, and this solution constitutes the equilibrium. This characterization of an equilibrium is summarized in the following lemma.

**Lemma 1:** When prices are of the form:

\[ \bar{p} = \alpha + \beta(\bar{u} + \bar{t}) - \gamma \bar{z}, \]

then \(\alpha\), \(\beta\), and \(\gamma\) are defined by the relations:

\[
\alpha = (1-\beta)m + (\gamma - \{r\Theta\}^{-1})z_0, \\
\beta = g(1-h)^{-1}, \text{ and} \\
\gamma = (r\Theta(1 - h))^{-1}.
\]
where:
\[
(g, h) = (v, \beta v) \begin{bmatrix} v + n + s & \beta(v + n) \\ \beta(v + n) & \beta^2(v + n) + \gamma^2 t \end{bmatrix}^{-1},
\]
and:
\[
\Theta^{-1} = \text{Var}[\tilde{u} | y_a, p] = v - g v - h \beta v,
\]
and the ratio of \( \beta \) to \( \gamma \) solves the equation:
\[
(\beta / \gamma)^2 s n + (s + n)(\beta / \gamma) t - rt = 0.
\]

Proof: See the discussion above.

The straightforward representation of prices outlined in Lemma 1 is possible because the inherent linearity of the problem is preserved through the assumptions that prices are linear, random outcomes are normally distributed, agents have constant absolute risk tolerance, etc. In effect, price is the (appropriately weighted) sum of the aggregate information revealed during the period, \( \tilde{u} + \tilde{\eta} \) (where the idiosyncratic noise terms \( \tilde{\epsilon} \) average to zero since information is aggregated across agents), and random noise, \( \tilde{z} \).

II. A Representation of Unexpected Price Changes and Volume

Measures of Informedness and Consensus

The ultimate objective in this section is to represent the variance of unexpected price changes and trading volume in terms of concepts of informedness and consensus (as well as other parameters of the model). But an intermediate step is to first introduce and define measures of informedness and consensus, and then (in Lemmas 2 and 5 below) show how they will be used.

To begin, note that agent \( a \)'s expectation of end-of-period economic earnings, \( \tilde{u} = u \), conditional on observing agent-specific information and price is:
\[
E[\tilde{u} | y_a, \bar{p} = \bar{p}] = m + (g, h)(y_a - m, \bar{p} - E[\bar{p}])',
\]
where \( (g, h) \) are defined in Lemma 1. Let the random variable \( \tilde{x}_a \) represent agent \( a \)'s residual error in his or her expectations and define it by:
\[
\tilde{x}_a = \tilde{u} - m - (g, h)(\tilde{y}_a - m, \bar{p} - E[\bar{p}])'.
\]
The variable \( \tilde{x}_a \) can be thought of as an unconditional, or ex ante, measure of the difference between the realized outcome of economic earnings and agent \( a \)'s expectation of that outcome conditional upon agent-specific information and price. For example, \( \tilde{x}_a \) has a normal distribution with mean zero and second moment of:
\[
E[\tilde{x}_a^2] = \text{Var}[\tilde{u} | y_a, p] = v - (g, h)(v, \beta v)'.
\]
The mean zero suggests that agents anticipate no difference between the realization of economic earnings and their expectations of that realization. The second moment around this difference is equal to the variance of economic earnings conditional on observing agent-specific information and price.
We define the *informedness* of each agent, and represent it by $\Theta$, as $\Theta = (E[x_i^2])^{-1}$. This is identical to the definition of $\Theta$ used in Lemma 1. As such, agent $a$'s level of informedness is characterized by the precision (i.e., the inverse of variance) of his or her beliefs conditional on observing the information and price. As $\Theta$ increases (decreases), agent $a$ has more (less) precise information about the asset (holding everything else constant). To avoid limiting cases, we adopt the convention that $\Theta$ is positive and finite.

The consensus effect measures the extent of agreement across agents' beliefs. We define the *consensus* between agents $j$ and $k$, and represent it by $\rho$, as:

$$\rho = \frac{E[x_j x_k]}{(E[x_j^2]E[x_k^2])^{1/2}} = \Theta E[x_j x_k].$$

In effect, the degree of consensus between two agents is governed by the covariance between their residual errors, i.e., $\bar{x}_j$ and $\bar{x}_k$. For example, as agents' information sources become more (less) common, the covariance between $\bar{x}_j$ and $\bar{x}_k$ increases (decreases). By construction, $\rho$ is bounded between 0 and 1. To avoid limiting cases, we assume that $\rho$ is positive and (strictly) less than one.

**Characterization of Unexpected Price Changes in Terms of Informedness and Consensus**

Let the random variable $\tilde{\Delta}$ represent the unexpected change in the price of the risky asset: that is $\tilde{\Delta} = \bar{p} - E[\bar{p}]$. Our concern is with the variance of $\Delta$ conditional on holding noise constant.

**Lemma 2**: The variance of the unexpected change in price, $\tilde{\Delta}$, conditional on the per capita supply of risky assets, is given by:

$$\text{Var}[\tilde{\Delta} | z = z] = v - \Theta^{-1}(2 - \rho) - r^2(1 - \rho^2)t^{-1}.$$

**Proof**: See the Appendix.

The price of the risky asset is an aggregator across agents' demands, beliefs, per capita supply, etc. As such, it also aggregates the exogenous noise in the economy. The rationale for holding constant the noise implicit in the uncertain per capita supply of risky assets, $z = z$, is to isolate that part of the variance of the unexpected change in price that is independent of noise. This allows a cleaner representation of the variance of the unexpected change in price which results from the release of information.

**Characterization of Volume in Terms of Informedness and Consensus**

In addition to prices, researchers commonly examine trading volume to assess the market's behavior. In particular, volume is thought to measure the extent of disagreement among market agents. Let the random variable $\tilde{\varphi}$ represent the trading volume of informed agents. Variable $\tilde{\varphi}$ is equal to one-half the absolute value of each agent's net demand aggregated across all agents. In the notation of our model, trading volume is defined by:

$$\tilde{\varphi} = (1/2) \int_0^1 |D_\omega(\tilde{g}_\omega, \bar{p}) - D_\omega | d\omega,$$
where \( D_a(\bar{y}_a, \bar{p}) - D_{0a} \) is the random variable that characterizes agent \( a \)'s net demand. Agent \( a \)'s net demand, \( D_a(\bar{y}_a, \bar{p}) - D_{0a} \), has a normal distribution with mean \( \mu_a = z_0 - D_{0a} \) and variance \( \sigma^2 \), which we calculate below in Lemma 5.

A volume statistic that parallels our treatment of the variance of the unexpected change in price is expected trading volume, \( E[\bar{\psi}] \). The variance of unexpected price changes at the time of information releases is used to characterize the effects of changes in informedness and consensus on unexpected price changes. Since price effects can be positive or negative, the variance is a more informative characterization for our purposes than, say, the mean. Trading volume results from the absolute value of net demand. Consequently, it is sufficient to understand how expected trading volume behaves when informedness and consensus shift, as opposed to, say, the variance or second moment of trading volume.

Some properties of the expected value of \( \bar{\psi} \) are outlined in the following two lemmas.

**Lemma 3:** Let \( f(x; \mu_a, \sigma^2) \) and \( F(x; \mu_a, \sigma^2) \) represent the density and cumulative density functions, respectively, of a normally distributed random variable with mean \( \mu_a \) and variance \( \sigma^2 \) evaluated at \( x \). Expected trading volume, \( \bar{\psi} \), is given by:

\[
E[\bar{\psi}] = |\{ \sigma^2 f(0; \mu_a, \sigma^2) - \mu_a F(0; \mu_a, \sigma^2) \}| da_a,
\]

where \( \mu_a = z_0 - D_{0a} \) is agent \( a \)'s expected net demand, and \( \sigma^2 \) is the variance of his or her net demand.

**Proof:** It is a simple exercise to show that the expected absolute value of a random variable that has a normal distribution with mean \( \mu \) and variance \( \sigma^2 \) is:

\[
2[\sigma^2 f(0; \mu, \sigma^2) - \mu F(0; \mu, \sigma^2)] + \mu.
\]

This implies, for the case of net demand, that:

\[
E[|D_a(\bar{y}_a, \bar{p}) - D_{0a}|] = 2[\sigma^2 f(0; \mu_a, \sigma^2) - \mu_a F(0; \mu_a, \sigma^2)] + \mu_a.
\]

Recalling that \( \mu_a da_a = 0 \) since per capita supply of the risky asset must equal per capita endowment, aggregating the above over all agents yields the expression for \( E[\bar{\psi}] \) given above. Q.E.D.

Note that expected trading volume is an increasing function of the standard deviation of agents' net demand, \( \sigma \). Consequently, to understand how information affects volume, it is sufficient to examine the behavior of \( \sigma^2 \), since changes in \( \sigma^2 \) imply an identical change in expected trading volume. Thus, the following lemma.

**Lemma 4:** Expected trading volume is an increasing function of the standard deviation (and similarly the variance) of agents' net demand.

**Proof:** From the proof to Lemma 3, the expected absolute value of a random variable that has a normal distribution with mean \( \mu \) and variance \( \sigma^2 \) is:

\[
2[\sigma^2 f(0; \mu, \sigma^2) - \mu F(0; \mu, \sigma^2)] + \mu.
\]
Differentiating this expression with respect to $\sigma$ yields:

$$\left[\sigma + \left(\frac{\mu^2}{\sigma}\right)\right] f(0; \mu, \sigma^2) - \left(\frac{\mu}{\sigma}\right) \int_{-\infty}^{0} \left[\mu^2 - x^2\right] f(x; \mu, \sigma^2) \, dx.$$

Note, however, that the last expression integrates to:

$$-(\mu^2 - x\mu/\sigma) f(x; \mu, \sigma^2) \bigg|_{-\infty}^{0} = -(\mu^2/\sigma) f(0; \mu, \sigma^2).$$

Substituting into the above yields:

$$\left[\sigma + \left(\frac{\mu^2}{\sigma}\right) - \left(\frac{\mu^2}{\sigma}\right)\right] f(0; \mu, \sigma^2) = \sigma f(0; \mu, \sigma^2) > 0.$$

This implies that the expected absolute value of the net demand of each agent is an increasing function of $\sigma$. Thus, expected trading volume (i.e., the expected net demand integrated across all agents) is also an increasing function. Q.E.D.

By virtue of Lemma 4, it is sufficient to examine the behavior of the standard deviation of agents' net demand and, thereby, ignore both prior beliefs and endowments in understanding the effects of informedness and consensus on expected trading volume.\footnote{It should be emphasized that it is sufficient to examine the behavior of $\sigma^2$ provided that agents are comparably informed (i.e., the variance of their idiosyncratic noise terms, $\epsilon$, is identical for all agents). When agents are not comparably informed, it is still the case that each agent's expected net demand is a monotonic function of the standard deviation of his net demand. However, it is not necessarily the case that the standard deviation of all agents' net demands move in the same direction (i.e., for some agents their $\sigma^2$'s might fall, whereas for some other agents their $\sigma^2$'s might increase). Since the $\sigma^2$'s are not moving in parallel, expected trading volume (i.e., the expected net demand integrated across all agents) is not necessarily moving in one direction versus another.}

Therefore, it only remains to represent the variance of agents' net demand in terms of informedness and consensus as in our final lemma.

**Lemma 5:** The variance of agents' net demand is given by:

$$\sigma^2 = \tau^2 \Theta(1-\rho) + \nu.$$

**Proof:** See the Appendix.

In summary, the expression for the variance of the unexpected change in price, given in Lemma 2, and the expression for the variance of agents' net demand, represented in Lemma 5, will form the basis for determining the effects of informedness and consensus on price changes and volume.\footnote{Note that we controlled for noise (by conditioning over it) in computing the variance of the change in price, but found this unnecessary when dealing with the variance of net demand. This difference in approach arises because price aggregates across all features of the economy and, thereby, is influenced significantly by noise. In contrast, an *individual agent*'s net demand, while affected by noise, is not affected as directly. Consequently, the device of demonstrating a monotonic relation between the variance of net demand and expected trading volume (i.e., Lemmas 3 and 4) makes conditioning over noise unnecessary.}
III. The Influence of Informedness and Consensus on Price Changes and Volume

To demonstrate that both informedness and consensus influence the variance of unexpected price changes, consider the effect of each holding the other constant. To elaborate briefly on this issue, imagine that all the exogenous parameters in the model are held fixed except \( n \) and \( s \). Then an increase in informedness, holding consensus constant, is tantamount to \( n \) and \( s \) shifting such that agents’ informedness increases while consensus between agents remains fixed. Similarly, an increase in consensus holding informedness constant is tantamount to \( n \) and \( s \) shifting such that agents’ consensus increases while their informedness stays fixed.\(^\text{13}\)

Using the definitions of informedness and consensus outlined in the previous section, we offer the following results.

**Proposition 1**: As agents’ informedness about the risky asset increases holding the consensus among them constant, the variance of the unexpected change in price increases: that is, as \( \Theta \) increases holding \( \rho \) fixed, \( \text{Var}[\Delta|\bullet] \) increases.

**Proof**: From Lemma 2,

\[
\frac{d \text{Var}[\Delta|\bullet]}{d \Theta} = \Theta^{-2}(2 - \rho) > 0,
\]

since (by convention) \( \Theta > 0 \) and \( \rho < 1 \). Q.E.D.

\(^\text{13}\) In the former case, the way to determine explicitly how \( n \) and \( s \) shift is to propose the following two simultaneous equations,

\[
\frac{d \Theta}{d \Theta} = 1 \quad \text{and} \quad \frac{d \rho}{d \Theta} = 0,
\]

using expressions for \( \Theta \) and \( \rho \) offered in the Appendix, and then solve for:

\[
\frac{dn}{d \Theta} \quad \text{and} \quad \frac{ds}{d \Theta}
\]

as two unknowns. In the latter case,

\[
\frac{dn}{d \rho} \quad \text{and} \quad \frac{ds}{d \rho}
\]

are solved for as two unknowns in the two simultaneous equations:

\[
\frac{d \Theta}{d \rho} = 0 \quad \text{and} \quad \frac{d \rho}{d \rho} = 1,
\]

once again using definitions of \( \Theta \) and \( \rho \). Because the number of unknowns equals the number of (simultaneous) equations, the solutions to:

\[
\frac{dn}{d \Theta}, \quad \frac{ds}{d \Theta}, \quad \frac{dn}{d \rho}, \quad \text{and} \quad \frac{ds}{d \rho},
\]

are unique and well defined. Therefore, with no loss of generality, we assume that \( n \) and \( s \) shift appropriately to ensure increases in informedness holding consensus constant, and vice versa, without explicitly detailing these solutions. Of course, there is no reason to require that the other exogenous parameters, e.g., \( u, r, t \), remain fixed. The only problem here is that there are more unknowns than (simultaneous) equations and, thus, these relations are not unique. Our point is that one need not restrict changes in informedness and consensus to changes in \( n \) and \( s \) alone.
Essentially, Proposition 1 states that as the informedness about the risky asset increases, the realization of price is likely to be farther from its expectation. This is because, given a fixed set of beliefs, agents’ demands increase as they become more knowledgeable. Consequently, this increased demand pushes prices further away from their expected value.

The next result concerns the consensus effect.

**Proposition 2:** As the consensus among agents about the risky asset increases holding their informedness constant, the variance of the unexpected change in price increases: that is, as \( \rho \) increases holding \( \Theta \) fixed, \( \text{Var}[\Delta|\cdot] \) increases.

**Proof:** From Lemma 2,

\[
\frac{d \text{Var}[\Delta|\cdot]}{d \rho} = \Theta^{-1} + 2r^2(1 - \rho) t^{-1} > 0,
\]

since (by convention) \( \Theta > 0 \) and \( \rho < 1 \). Q.E.D.

Essentially, Proposition 2 establishes that as the consensus across agents’ beliefs increases due to a change in the mix of common and idiosyncratic components of information, the more variable is price (and, hence, unexpected price changes). This is because as agents’ residual errors become more correlated, there is less uncertainty resolved through the market aggregation process. Thus, price at the end of the period is more variable because of this decreased resolution of uncertainty.

The results involving volume are equivalent to those involving unexpected price changes, except for the fact that trading volume decreases as consensus increases holding informedness constant. The reason trading volume increases as informedness shifts upward is that, *ceteris paribus*, agents’ demands become more extreme as agents become more knowledgeable. Consequently, trading volume will increase. The reason trading volume decreases as consensus shifts upward is that, *ceteris paribus*, increased consensus implies less diversity of opinion among agents. Consequently, trading volume will decline. This intuition is formalized below.

**Proposition 3:** As agents’ informedness about the risky asset increases holding the consensus among them constant, expected trading volume increases: that is, as \( \Theta \) increases holding \( \rho \) fixed, \( E[\bar{V}] \) increases.

**Proof:** From Lemmas 3 and 4, it is sufficient to examine the behavior of the variance of net demand. But from the definition of the variance,

\[
\frac{d \sigma^2}{d \Theta} = r^2(1 - \rho) > 0,
\]

since by convention \( \rho < 1 \). Q.E.D.

**Proposition 4:** As the consensus among agents about the risky asset increases holding their informedness constant, expected trading volume decreases: that is, as \( \rho \) increases holding \( \Theta \) fixed, \( E[\bar{V}] \) decreases.
Proof: Using the definition of the variance of net demand,

\[ \frac{d\sigma^2}{d\rho} = -r^2 \Theta < 0, \]

since by convention \( \Theta > 0 \). Q.E.D.

The Simultaneous Influence of Information on Informedness and Consensus

It is important to emphasize that when information is disseminated or acquired, the variances of the common and idiosyncratic components of agents' information affect the level of informedness and the degree of consensus simultaneously. For example, consider an increase in the variance of the common component, \( n \). As the variance of \( n \) increases, informedness tends to be reduced because the greater noise in agents' information makes them less knowledgeable. Simultaneously, increased \( n \) is likely to increase the consensus across agents because their signals are more conditionally dependent. Thus, a shift in \( n \) affects both informedness and consensus. Since an increase in \( n \) works to decrease informedness and increase consensus, the effect of increased \( n \) on the variance of unexpected price changes is ambiguous, whereas it is likely to decrease trading volume unambiguously.

Similar effects are observed for a shift in the variance of the idiosyncratic component, \( s \). As the variance of \( s \) increases, informedness is likely to be reduced because of the greater noise in agents' information, and consensus is likely to be decreased because agents' signals are less conditionally dependent. An increase in \( s \) works to decrease informedness and decrease consensus, which is likely to lead to an unambiguous decline in the variance of unexpected price changes, whereas the effect on trading volume is ambiguous. Except for very special cases, simultaneous shifts in the variances of \( n \) and \( s \) result in similar concurrent shifts in informedness and consensus.

Thus, information releases simultaneously affect both price changes and trading volume through informedness and consensus. It would be rare to observe an informedness effect absent a consensus effect, or a consensus effect absent an informedness effect. Since price changes and volume are both affected by simultaneous changes in informedness and consensus at the time of an information release, interpretations of price changes or volume are conceptually similar.

IV. Conclusion

The major results of this paper are as follows. First, we suggest that information manifests both informedness and consensus effects simultaneously. Second, we show that informedness and consensus each affect both the variance of unexpected price changes and trading volume. Hence, interpretations of the variance of price changes and interpretations of volume reactions are conceptually similar. Thus, both price and volume studies are equally relevant means of assessing the information content of an earnings announcement. Third, we indicate exactly how changes in informedness and changes in consensus affect the variance of unexpected price changes and trading volume. More specifically, we show that an increase in informedness results in an increase in the variance of unexpected
price changes and an increase in trading volume. An increase in consensus results in an increase in the variance of price changes and a decrease in trading volume.

Our model helps to interpret existing empirical evidence. Consider, for example, the effect of an earnings announcement on price changes and trading volume. The empirical literature indicates that both the variance of price changes and trading volume increase at the time of an earnings announcement (see, e.g., Beaver 1968). If one defines information content as the ability of an information signal to alter investors' beliefs, evidence on volume reactions is as relevant for assessing information content as evidence on unexpected price changes. Thus, the reluctance of some authors to interpret volume reactions as evidence of information content seems unfounded.

While empiricists have not generally studied the extent to which price changes and volume represent shifts in informedness and consensus, we suggest that by considering price changes and volume in combination, empiricists could learn something about the relative amounts of informedness and consensus. The discussion of informedness and consensus implies that the variance of price changes and trading volume tend to be positively related when the informedness effect dominates the consensus effect, and tend to be negatively related when the consensus effect dominates the informedness effect.

Since informedness and consensus each affect both the variance of price changes and volume, we provide an economic rationale for examining both price and volume effects at the time of information releases. Moreover, viewed within the framework of the model, it is not a surprise that the empirical literature has documented an increase in the variance of price changes and an increase in trading volume at the time of information releases (such as accounting earnings announcements): both of those findings are consistent with accounting earnings containing information.

Appendix

Proof of Lemma 2

The change in price is defined by:

$$\Delta = \bar{p} - E[\bar{p}] = \alpha + \beta(u + \eta) - \gamma \bar{z} - E[\bar{p}] .$$

It is a straightforward exercise to show that the variance of \( \Delta \) conditional on \( \bar{z} = z \) is:

$$\text{Var}[\Delta | \bar{z} = z] = \beta^2 (v + n) .$$

(A1)

The majority of the work involves representing \( \beta \) and \( n \) in terms of informedness and consensus to complete the expression. In the case of \( \beta \), note that by definition \( \Theta = v - \gamma \), and (using the expressions for \( \beta \) and \( \gamma \) in Lemma 1) \( g = (\beta / r \Theta \gamma) \) and \( h = 1 - (r \Theta \gamma)^{-1} \). These expressions imply:

$$\Theta = v - \gamma \Rightarrow \Theta^{-1} = v - \beta v(1 - [r \Theta \gamma]^{-1}) .$$

(A2)
Manipulating equation (A2) yields:

\[ \beta = 1 - (v \Theta)^{-1}. \]  
\[ (A3) \]

Representing \( n \) is considerably more difficult. First, note that by definition,

\[
\rho = \Theta E[\tilde{x}_1 \tilde{x}_4] = \Theta(1, g, h) \begin{bmatrix}
v & -v & -\beta v \\
-v & v + n & \beta(v + n) \\
-\beta v & \beta(v + n) & \beta^2(v + n) + \gamma^2 t
\end{bmatrix} (1, g, h)'
\]
\[ (A4) \]

If, into equation (A4), one substitutes the following expressions for \( \Theta, g, \) and \( h \) (which are implied by the equilibrium outlined in Lemma 1):

\[
\Theta^{-1} = \frac{v(s[n(\beta/\gamma)^2 + t] + nt)}{s([v + n][\beta/\gamma]^2 + t) + (v + n)t},
\]
\[
h = \frac{vs(\beta/\gamma)\gamma^{-1}}{s([v + n][\beta/\gamma]^2 + t) + (v + n)t}, \text{ and}
\]
\[
g = \frac{tv}{s([v + n][\beta/\gamma]^2 + t) + (v + n)t},
\]
then equation (A4) reduces to:

\[
\rho = 1 - \frac{\Theta^{-1} t^2 s}{(s[n(\beta/\gamma)^2 + t] + nt)^2}.
\]
\[ (A5) \]

Let \( k = s[n(\beta/\gamma)^2 + t] + nt \). Note that:

\[
\gamma \Theta h = \frac{s(\beta/\gamma)}{k} = \frac{s\gamma \Theta}{k} = \frac{s}{k^2},
\]

which implies:

\[
\Theta^{-1} \frac{t^2 s}{k^2} = r^{-1} t \gamma h.
\]
\[ (A6) \]

Substituting equation (A6) in equation (A5) yields \( \rho = 1 - r^{-1} t \gamma h \), or:

\[
\gamma h = rt^{-1}(1 - \rho).
\]
\[ (A7) \]

Alternatively, if one substitutes the relation \( \Theta^{-1} = v - gv - \beta hv \) into equation (A4) then it reduces to:

\[
\rho = v^{-1} \Theta^{-1} + n \Theta(1 - v^{-1} \Theta^{-1})^2 + \Theta t(\gamma h)^2, \text{ and}
\]
\[
= v^{-1} \Theta^{-1} + n \Theta(1 - v^{-1} \Theta^{-1})^2 + \Theta r^2 t^{-1}(1 - \rho)^2,
\]
\[ (A8) \]

using the fact that \( \gamma h = rt^{-1}(1 - \rho) \) from equation (A7). Equation (A8), in turn, implies:

\[
n = \Theta^{-1}(1 - v^{-1} \Theta^{-1})^{-2}(\rho - v^{-1} \Theta^{-1} - r^2 t^{-1} \Theta(1 - \rho)^2).
\]
\[ (A9) \]
Finally, substituting the expression for $\beta$ determined in equation (A3) and $\eta$ determined in equation (A9) into equation (A1) yields:

$$\text{Var}[\Delta \beta] = \beta^2 (v + \eta) = v - \Theta^{-1}(2 - \rho) - r^2(1 - \rho)^2 t^{-1}.$$  

We point out that this result and, thus, Propositions 1 and 2 that rely on it, can be extended to the case of signals with idiosyncratic noise terms with different variances: that is, $s_a \neq s_b$ for all agents $a$ and $b$. Q.E.D.

**Proof of Lemma 5**

By definition, net demand is defined by:

$$D_a - D_{oa} = r \Theta (E[u | y, p] - p) - D_{oa},$$

$$= r \Theta (m + (g,h)(y - m, p - E[p])) - p) - D_{oa},$$

$$= r \Theta (g(y - m) - (1 - h)(p - E[p]) + m - E[p]) - D_{oa},$$

$$= r \Theta (g(u + \eta + \epsilon - m) - (1 - h)(u + \eta) - \gamma z - E[p])$$

$$+ m - E[p]) - D_{oa}. \quad (A10)$$

Note, however, that $\beta = g(1 - h)^{-1}$ and $\gamma = (r \Theta [1 - h])^{-1}$, which implies that equation (A10) can be rewritten as:

$$D_a - D_{oa} = r \Theta g \epsilon_a + z - D_{oa}.$$  

Consequently, the variance of $\Delta D_a - D_{oa}$ is:

$$\sigma^2 = E[(r \Theta g \epsilon_a + z)^2],$$

$$= (r \Theta g)^2 s + v.$$  

Note, however, that:

$$(r \Theta g)^2 s = \frac{r^2 t^2 s}{[s \{\beta / \gamma^2 + t\} + nt]^2} = r^2 \Theta (1 - \rho),$$

using the expression developed for $\rho$ in equation (A5). Thus, $\sigma^2$ reduces to:

$$\sigma^2 = r^2 \Theta (1 - \rho) + v.$$  

**References**


